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AN EFFICIENT SINGLE-ROW ROUTING ALGORITHM

by

T. T-S. Tarng, M. Marek-Sadowska and E. S. Kuh

Memorandum No. UCB/ERL M83/34

12 April 1983

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ABSTRACT

In this paper we present a heuristic algorithm for single row routing. Our approach is based on the interval graphical representation of the given net list. The objective function for minimization is the street congestion. The problem is known to be intractable in the sense of NPcompleteness, thus, a polynomial-time heuristic algorithm is proposed. It has been implemented and tested with various examples. So far, it has always produced optimal solutions.

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1. INTRODUCTION

The single row routing problem is a very crucial one in the layout design of printed circuit boards. It was first introduced by So in [1] and later generalized by other authors [2-6]. In [4] a novel formulation was proposed, and the conditions of optimum routing were derived. In [7] it was shown that the problem is intractable in the sense of NP-completeness. Here, we propose a heuristic, polynomial routing algorithm based on the approach introduced in [4]. The algorithm has been implemented and tested on various examples. So far, it has always produced the optimal solutions. We have been able to prove that it indeed leads to optimal solutions for some special cases.

2. FORMULATION OF THE PROBLEM

Given a set of n nodes placed on a row and a net list $L = \{N_1, N_2, \dots, N_m\}$ which prescribes the connection pattern of nets to nodes. A net list is to be realized with a set of m nonintersecting nets which consist of only horizontal and vertical paths connecting the nodes according to specification. An example showing a realization of a given net list together with some pertinent terminology is given in Fig. 1. The space above the row R is referred to as the upper street and the space below R is referred to as the lower street. The number of horizontal tracks allowed in the upper street is called the upper street capacity and the number of horizontal tracks required in the realization in the upper street, is called the upper street congestion. We will use the following notation:

 C_{us} - upper street congestion

 C_{1s} - lower street congestion

cui - number of horizontal tracks passing above the ith node in the realization

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cli - number of horizontal tracks passing below the ith node in the realization

From these definitions it follows that:

$$C_{1s} = \max_{i} c_{1i}$$

$$i=1,...n$$

$$C_{us} = \max_{i} c_{ui}$$

$$i=1,2,...n$$

An optimum realization is one which minimizes the street congestion in both streets. Thus we can define Q_0 being the street congestion, i.e., $Q_0 = \max\{C_{us}, C_{ls}\}$ and say that an optimum realization is achieved if Q_0 is minimum.

It has been shown in [4] that to each realization of a given net list L there corresponds a unique interval graphical representation. The interval graphical representation is a set of m horizontal intervals representing the m nets together with an assumed order. Each horizontal line corresponds to the interval specified between the extremal node positions of a given net. Figure 2 shows the interval graphical representation of the single-row realization in Fig. 1.

<u>Definition 1</u>. The <u>cut number</u> c_i <u>at node i</u> is the number of nets cut by the vertical line superimposed on the interval graphical representation at the ith node v_i . From the previous definitions it follows, that

 $c_i = c_{ui} + c_{li}$

<u>Definition 2</u>. A <u>cut number of a net</u> N_j denoted by q_j is the maximum over cut numbers of the nodes which belong to the net N_j .

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For example, in Fig. 2, we have $q_6 = 3$. If in the interval graphical representation the first net from the top has a cut number q then C_{1s} is at least q. If the bottom net has a cut number q, then C_{us} is at least q. Thus, intuitively it is clear that those nets which have the least cut numbers should appear as outer nets in the interval graphical representation corresponding to optimum routing.

Following [4], let us further denote by

$$q_m = \min_j q_j$$

and

In [4] it has been shown that the street congestion Q_0 for the optimum realization satisfies an inequality:

$$Q_0 \ge \max \left\{ q_m, \left\lceil \frac{q_M}{2} \right\rceil \right\},$$

where $\lceil x \rceil$ is the smallest integer not smaller than x. An optimum realization with street congestion $Q_0 = \lceil \frac{q_M}{2} \rceil$ exists if and only if there exists such an ordering that for each v_i with $c_i = \lceil \frac{q_M}{2} \rceil + k$ $(k = 1, 2, \dots, q_M - \lceil \frac{q_M}{2} \rceil)$ the net associated with v_i is covered from above and below by at least k nets. In general there exists an optimum realization with street congestion $Q_0 = \lceil \frac{q_M}{2} \rceil + p$, if and only if p is the least nonnegative integer for which the p-excess property holds. The p-excess property means that there exists an ordering such that for each v_i with $c_i = \lceil \frac{q_M}{2} \rceil + k$ $(k = p+1, \dots, q_M - \lceil \frac{q_M}{2} \rceil)$ the net associated with v_i is covered by at least k-p nets from above and below [4].

The heuristic algorithm for single-row routing makes use of these necessary and sufficient conditions for optimum realizations.

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3. FOUNDATIONS OF THE HEURISTIC ALGORITHM

In the previous section we have recalled the necessary and sufficient conditions for optimum single-row routing. These conditions are in such a form that the problem of finding the optimum ordering of intervals in the interval graphical representation remains an open question. However, these conditions help us to understand the problem and to develop an intuitive approach.

First, we will make use of an observation that the nets which have the least cut numbers should appear as outer nets in the interval graphical representation. We classify nets according to their cut numbers and introduce the concept of the <u>zone</u> as follows: We assign nets with maximum cut number q_M to the zone Z_0 . Nets with cut number q_M -i are in zone Z_1 . Thus for the nets in Fig. 2 we have $q_M = 3$, nets N_1, N_4, N_5, N_g and N_6 are in Z_0 , N_3, N_8 and N_2 in Z_1 and N_7 in Z_2 . In the optimum realization nets from the zone Z_{q_M} should appear as outer nets. Now, we will prove the following:

<u>Lemma 1</u>. Given a net list $L = \{N_1, N_2, \dots, N_m\}$ and an ordering O_1 of the nets resulting in an optimum realization with street congestion Q_0 . If in O_1 there is a net N_i with cut number $q_i \leq Q_0$ placed between nets N_j, N_k with cut numbers larger than Q_0 then there exists another optimal ordering O_2 such that N_i preceeds both N_j, N_k or follows N_j and N_k .

<u>Proof</u>. Suppose that the ordering O_1 is such that

 $N_j < N_i < N_k$

and N_i cannot be placed just before N_j or after N_k without affecting optimality. In such a case, there is a pin v_j in N_j with upper cut

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 Q_0 and there is a pin v_k in N_k with lower cut number Q_0 (see Fig. 3). Then v_i must be the terminating pin for N_i , and v_k the originating pin for ${\rm N}_{\rm k}.$ If it were not the case, ${\rm Q}_{\rm 0}$ could not be the street congestion or the upper cut in v_{i} would have to be less than Q_0 or the lower cut in v_k would have to be less than Q_0 . Between v_i and v_k at least one net which covers v_i from above have to terminate. It is so because the upper cut number at v_k cannot be larger than Q_0 and v_k is already covered by N_i . For the same reason at least one net covering v_k from below has to originate between v_i and v_k . Actually, N'_i must terminate at v_{j+1} , and N'_k must originate from v_{k-1} . If between N_j and N'_j there are ℓ nets passing through then the upper cut in v_{j+1} is $Q_0 - 1 - \ell$. Suppose that N_i is immediately after N_i in the ordering O_1 . Let us interchange N'_i and N_i in the O_1 . The upper cut number and lower cut number of v_{i} remain unchanged, and the lower cut number of v_{i+1} is decreased by one and the upper cut number is increased by one but does not exceed $\textbf{Q}_{0}.$ Cut numbers of \textbf{v}_{k-1} and \textbf{v}_{k} are not affected by this operation. Following the same reasoning we can show that interchange of N_{j} and N_{i} does not increase the street congestion. If after the interchange we still have a situation that a net with cut number Q_{Ω} or less is between two nets with cut numbers larger than ${\rm Q}_{\rm O}$ then the process of interchanges can be repeated.

Using Lemma 1 we can prove the following.

<u>Lemma 2</u>. Given a net list $L = \{N_1, N_2, \dots, N_m\}$ and an ordering O_1 of the nets giving an optimum realization with street congestion Q_0 . There exists an ordering O_2 such that nets in O_2 satisfy the following:

$$Z_{q_M}^L < Z_{q_{M-1}}^L < \ldots < Z_{q_M}^L - Q_0^{+1} < (Nets with cut numbers > Q_0) < Z_{q_M}^U - Q_0^{+1} < \ldots < Z_{q_M}^U$$

where $Z_i^L \cup Z_i^U = Z_i$ and $Z_i^L \cap Z_i^U = \emptyset$, and superscipts denote the lower and upper streets.

<u>Proof</u>. We can transform 0_1 into another ordering 0_2 satisfying Lemma 1. In 0_2 we have

 Z^{L} < (Nets with cut numbers > Q) < Z^{U}

where

$$z^{L} = \bigcup_{i=q_{M}-Q_{0}+1}^{q_{M}} z^{L}_{i}$$
, $z^{U} = \bigcup_{i=q_{M}-Q_{0}+1}^{q_{M}} z^{U}_{i}$

The ordering within each zone Z^L and Z^U can be arbitrary, because it does not affect the street congestion. Thus we can make the ordering to be such which satisfies the requirements of the Lemma.

The same reasoning as in the proofs of Lemmas 1 and 2 can be used to prove:

<u>Lemma 3</u>. Given a net list $L = \{N_1, N_2, ..., N_m\}$ and an ordering O_1 of the nets which results in an optimum realization with street congestion Q_0 , there exists an ordering O_2 such that nets in O_2 satisfy the following:

$$Z_{q_{M}}^{L} < Z_{q_{M-1}}^{L} < \dots \quad Z_{1}^{L} < Z_{0} < Z_{1}^{U} < Z_{2}^{U} < \dots \quad Z_{q_{M}}^{U}$$

where $Z_i^L \cup Z_i^U = Z_i$ and $Z_i^L \cap Z_i^U = \emptyset$.

Lemma 3 is the basis of our heuristic algorithm for single-row routing. The algorithm produces a solution which satisfies conditions of Lemma 3. We apply heuristics to generate Z_i^L and Z_i^U for the given Z_i and to find ordering within each zone. It can be easily seen that for a net list L where all the nets are 2 pin nets which cover at least

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one common point, each zone can have at most 2 nets, and for any division of zone Z_i into Z_i^L and Z_i^U , we always obtain an optimum solution.

4. THE BASIC ALGORITHM

The routing algorithm actually produces an ordering of intervals representing nets, because to each net there corresponds a unique realization.

In the following we will make use of the concept of internal and residual cut numbers.

<u>Definition 3</u>. Given a net list $L = \{N_1, N_2, \dots, N_m\}$ and a division of L into two sublists L_1 and L_2 such that $L_1 \cap L_2 = \emptyset$ and $L_1 \cup L_2 = L$. Let $N_i \in L_1$. The <u>internal cut number</u> iq_i of net N_i in L with respect to L_1 is defined as a cut number of N_i in L_1 . Thus, internal cut number has the same meaning as cut number, except that it is calculated as if the net list was composed only of nets which are in L_1 .

<u>Definition 4</u>. Let $L_1 \cup L_2 = L$, $L_1 \cap L_2 = \emptyset$ and $N_i \in L_1$. The <u>residual cut</u> <u>number</u> rq_i of net N_i in L with respect to L_1 is defined as a cut number of N_i in $\{L_2 \cup N_i\}$.

These definitions can be extended in a natural way to internal and residual cut numbers at a node: ic_i and rc_i . Let us consider the net list $L = \{N_1, N_2, N_3, \dots, N_g\}$ shown in Fig. 2. Let $L_1 = \{N_1, N_3, N_4, N_5, N_6, N_g\}$, $L_2 = \{N_2, N_7, N_8\}$. For nodes of the net N_3 in the Fig. 4b we have internal cut numbers (1,1), residual cut numbers [1,0]. Thus $iq_3 = 1$, $ir_3 = 1$.

Now, we are ready to present the basic concept of the algorithm. First, each net is assigned to an appropriate zone. Then division of each zone Z_i into upper and lower parts Z_i^L and Z_i^U as well as mutual ordering of nets in each of Z_i^L and Z_i^U is determined. The algorithm

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proceeds until each net is assigned to its proper Z_i^X (X = L,U) in a proper place.

The algorithm starts with nets from Z_{Ω} , i.e., such nets which have the maximum cut number $\boldsymbol{q}_{M}.$ \boldsymbol{Z}_{0} does not have to be divided, thus we have only to determine the net ordering in Z_{Ω} . The ordering of nets in $\rm Z_{0}$ is performed as follows. Nets in this zone are sorted according to descending internal cut numbers with respect to Z_0 . When two overlapping nets have the same internal cut numbers then the one which has larger residual cut number preceeds the one which has smaller residual cut number. If both internal and residual cut numbers are equal then the mutual ordering of these nets can be arbitrary. In the algorithm we use fictitious track $\{T_i\}$ to which nets are being assigned, i is an integer from the interval $[-Q_{\Omega},Q_{\Omega}]$. The ordering of tracks follows the natural order of the indices. Only nonoverlapping nets can be assigned to the same track. We take the first net from the sequence and place it in T_0 . Then we take consecutive nets from the ordered sequence and place them in T_0 or in some T_{j} so that |j| is minimum. Then nets from consecutive zones are placed in tracks following the same rules as for nets in Z_0 . The division of zones into upper and lower portions is a side effect of minimizing the absolute value of track indices.

Let us consider an example of net list shown in Fig. 2. Figure 4a shows nets from the same net list which belong to Z_0 . In brackets are shown internal cut numbers of nodes for each of nets. For the nets in Fig. 4a we have N_4 , N_1 , N_6 and N_9 with internal cut number equal 1. They also have the same residual cut numbers, thus their mutual ordering in the ordered sequence is an arbitrary one, let it be: N_4 , N_6 , N_9 , N_1 . After placing them on tracks accordingly to the previously mentioned

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rules we have: N_1 , N_5 , $N_9 + T_0$, $N_4 + T_1$, $N_6 + T_{-1}$. It gives the following partial ordering in the interval graphical representation: N_4 , (N_1, N_5, N_9) , N_6 . Nets in brackets can have an arbitrary mutual order. In the next step nets from the zone Z_1 are to be placed in tracks. The nets N_2 , N_8 , N_3 are in Z_1 . Nets N_2 and N_3 have the same internal cut numbers equal 2 and residual cut numbers 0. Net N_8 has internal cut number 1. Thus the ordered sequence for nets in zone Z_1 can be N_2 , N_3 , N_8 . Again, we take consecutive nets from this sequence and place them in available tracks so that the number of tracks used above and below track T_0 is as small as possible. We can assign $N_2 + T_2$, $N_3 + T_{-2}$, $N_8 + T_2$. Now, the actual ordering is (N_2, N_8) , N_4 , (N_1, N_5, N_g) , N_6 , N_3 . Zone Z_2 contains just one net and according to the previous rules it is assigned to T_{-2} . The final ordering is (N_2, N_8) , N_4 , (N_1, N_5, N_9) , N_6 , N_3 .

The final track assignment is shown in Fig. 5. All orderings which follow from it correspond to the optimum realization in Fig. 1. More precisely, the ordering algorithm is as follows:

Step 0: i:=0, j:=0, assign each net to an appropriate zone.

- Step 1: Calculate internal and residual cut numbers for nets in Z_i with i respect to $L_1 = \bigcup_{i=0}^{I} Z_j.$
- Step 2: Sort the nets from Z_i into a sequence O_i such that for $N_a, N_b \in Z_i$, $N_a < N_b$ if $iq_a > iq_b$ or $iq_a = iq_b$ and $ir_a > ir_b$. If N_a and N_b have equal both internal and residual cut numbers then their mutual ordering in O_i is an arbitrary one.

Step 3: j = 0? yes \rightarrow follow 4, no \rightarrow go to Step 5.

Step 4: Assign the first net from 0_i to track T_0 . Are there any nets in 0_i which are not assigned yet? Yes \rightarrow follow 5, no \rightarrow go to 8.

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- Step 5: Take the consecutive net from 0_i which does not have the track assignment. Can this net be assigned to T_x , such that $|x| \le j$? Yes \ne follow 6, no \rightarrow go to 7.
- Step 6: Assign this net to such track that $|x| = \min$. Are there any nets in O_i which do not have track assignment? Yes \neq go to 5, no \neq go to 8.

Step 7: j := j+1, go to Step 6.

Step 8: Is there any net in the net list L which is unassigned to any track yet? Yes \rightarrow follow 9, no \rightarrow go to 10.

Step 9: i := i+1, go to Step 1.

Step 10: Exit.

5. MODIFICATIONS OF THE BASIC ALGORITHM

Up to now we have assumed that in the realization of a single row the nets are unidirectional, i.e., there does not exist a net which has two horizontal segments crossing a vertical line. It turns out, however, that when this requirement is relaxed, better results are possible in some cases. If we obtain a realization where street congestion equals to $\lceil \frac{q_M}{2} \rceil$ then no further improvement is possible. However, when the obtained ordering yields a realization with street congestion larger than $\lceil \frac{q_M}{2} \rceil$ then allowing some nets to have more than one segment in the horizontal graphical representation can sometimes improve the result. We will explain this mechanism using the following examples. Let us consider a situation shown in the Fig. 6a. The node A is covered from below by q_M -1 nets, node B is covered from above by q_M nets and from below by no more than q_M -2 nets and there exists a net N_D which covers A from below and B from above such that cut number $C_{D1} \leq \lceil \frac{q_M}{2} \rceil$. In such a case

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we can replace one horizontal interval representing net N_{D} with a different pattern as shown in Fig. 6b. Net N_{D} covers A from below and does not cover node B from above but rather from below. Thus the node B has upper cut number decreased by one and lower cut number increased by one. If B were the only node where the street congestion occurred and the replacement does not introduce such new nodes then it reduces street congestion. The type of net representation as shown in the Fig. 6b is called an external detour. For some net lists two or more external detours are possible. Figure 7 illustrates an example, where external detours have been applied to two overlapping nets. In this example $C_{E1} >> C_{E2}$, $C_{D1} << C_{D2}$. Sometimes application of external detour to one net can result in decreasing of, say, upper cut number at more than one critical node, i.e., in a node where upper cut number equals to street congestion (see Fig. 8). However, in general, it is quite difficult to apply external detours. The reason for this is due to the nature of external detour: if we want to use it we have first to find some ordering and then check whether street congestion can be reduced by external detour. It results in general in an exhaustive search. For a given net list we may have a case that for two different ordering resulting in the same street congestion, in one case external detours may decrease the congestion while for the other may not.

At the present moment we do not have a good heuristic implementing external detours. Here, we just want to point out that interval graphical representation may sometimes lead to single row realizations with larger street congestion than a more sophisticated representation, like using the external detour (which is a special case of general Steiner tree).

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6. EXPERIMENTAL RESULTS

The computer program implementing the proposed approach has been written and tested on number of examples. It allows both multipin and 2-pin nets. Although the algorithm is heuristic, it always managed to find optimum solutions for all examples it was tested with. Figure 9 shows examples from the computer program. The example (a) is taken from [4] where it is shown that the optimum street congestion is 5 here. In the example (b) the $q_t = 11$ and since $\lceil \frac{11}{2} \rceil = 6$ thus the solution presented is also optimal.

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FIGURE CAPTIONS

- Fig. 1. An example of a single row realization and basic terminology.
- Fig. 2. The interval graphical representation for the net list realization from the Fig. 1. Numbers in brackets show cut numbers for nodes which belong to the given net.
- Fig. 3. Illustration for Lemma 1.
- Fig. 4. (a) Subset of nets from Fig. 2 with internal cut numbers.
 - (b) Internal and residual cut numbers for the net N_3 with respect to the shown subset of the net list from Fig. 2.
- Fig. 5. Track assignment for the example of Fig. 1.
- Fig. 6. Concept of external detour.
- Fig. 7. External detour for two overlapping nets.
- Fig. 8. External detour resulting in decrease of critical upper cut number at 3 nodes.
- Fig. 9. Examples calculated by the computer program.



Fig. I



Fig. 2



Fig. 3









(b)





Fig.5





(a)











(q)

(D)

Fig. 8



1 2 3 4 5 6 1 4 7 8 2 9 5 10 11 12 11 8 12 13 10 14 15 3 6 14 13 15 9 7

CUTNUMBER : Cmax = 7

UPPER STREET CONGESTION OU + 4

LOWER STREET CONGESTION OL : 5

(a)

Fig. 9



 $\begin{smallmatrix} 13 & 16 & 17 & 14 & 15 & 12 & 13 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 6 & 4 & 7 & B & 7 & B & 3 & 9 & 10 & 11 & 10 & 11 & 2 & 9 & 1 & 12 & 13 & 14 & 1 \\ 7 & 15 & 12 & 16 & 14 & 15 \end{smallmatrix}$

CUTNUMBER Cmax = 11

ŧ.,

UPPER STREET CONGESTION QU : 6 LOWER STREET CONGESTION QL : 6

(b)

Fig. 9