Copyright © 1983, by the author(s).
All rights reserved.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission.

# AN EFFICIENT SINGLE-ROW ROUTING ALGORITHM 

by
T. T-S. Tarng, M. Marek-Sadowska and E. S. Kuh

Memorandum No. UCB/ERL M83/34
12 Apri1 1983

## ELECTRONICS RESEARCH LABORATORY <br> College of Engineering University of California, Berkeley <br> 94720

# AN EFFICIENT SINGLE-ROW ROUTING ALGORITHM 

by
T. T-S. Tarng, M. Marek-Sadowska and E. S. Kuh

Memorandum No. UCB/ERL M83/34
12 April 1983

# an efficient single-row routing algorithm* 

# Tom Tsan-Kuo Tarng, Malgorzata Marek-Sadowska and Ernest S. Kuh <br> Department of Electrical Engineering and Computer Sciences and the Electronics Research Laboratory University of California, Berkeley, California 94720 

## ABSTRACT

In this paper we present a heuristic algorithm for singie row routing. Our approach is based on the interval graphical representation of the given net list. The objective function for minimization is the street congestion. The problem is known to be intractable in the sense of NP.. completeness, thus, a polynomial-time heuristic algorithm is proposed. It has been implemented and tested with various examples. So far, it has always produced optimal solutions.

[^0]
## 1. INTRODUCTION

The single row routing problem is a very crucial one in the layout design of printed circuit boards. It was first introduced by So in [1] and later generalized by other authors [2-6]. In [4] a novel formulation was proposed, and the conditions of optimum routing were derived. In [7] it was shown that the problem is intractable in the sense of NP-completeness. Here, we propose a heuristic, polynomial routing algorithm based on the approach introduced in [4]. The algorithm has been implemented and tested on various examples. So far, it has always produced the optimal solutions. We have been able to prove that it indeed leads to optimal solutions for some special cases.

## 2. FORMULATION OF THE PROBLEM

Given a set of $n$ nodes placed on a row and a net list $L=\left\{N_{1}, N_{2}, \ldots N_{m}\right\}$ which prescribes the connection pattern of nets to nodes. A net list is to be realized with a set of $m$ nonintersecting nets which consist of only horizontal and vertical paths connecting the nodes according to specification. An example showing a realization of a given net list together with some pertinent terminology is given in Fig. 1. The space above the row $R$ is referred to as the upper street and the space below $R$ is referred to as the lower street. The number of horizontal tracks allowed in the upper street is called the upper street capacity and the number of horizontal tracks required in the realization in the upper street, is called the upper street congestion. We will use the following notation:
$C_{\text {us }}$ - upper street congestion
$\mathrm{C}_{1 \mathrm{~s}}$ - lower street congestion
$c_{u i}$ - number of horizontal tracks passing above the ith node in the realization
${ }^{c} 1 i$ - number of horizontal tracks passing below the $i$ th node in the realization

From these definitions it follows that:


An optimum realization is one which mirrimizes the street congestion in both streets. Thus we can define $Q_{0}$ beirg the street congestion, i.e., $Q_{0}=\max \left\{C_{u s}, C_{1 s}\right\}$ and say that an optimum realization is achieved if $Q_{0}$ is minimum.

It has been shown in [4] that to each realization of a given net list $L$ there corresponds a unique interval craphical representation. The interval graphical representation is a set of morizontal intervals representing the $m$ nets together with an assumed order. Each horizontal line corresponds to the interval specified between the extremal node positions of a given net. Figure 2 shows the interval graphical representation of the single-row realization in Fig. 1.

Definition 1. The cut number $c_{i}$ at node $i$ is the number of nets cut by the vertical line superimposed on the interval graphical representation at the $i$ th node $v_{i}$. From the previous definitions it follows, that

$$
c_{i}=c_{u i}+c_{1 i}
$$

Definition 2. A cut number of a net $N_{j}$ denoted by $q_{j}$ is the maximum over cut numbers of the nodes which belong to the net $N_{j}$.

For example, in Fig. 2, we have $q_{6}=3$. If in the interval graphical representation the first net from the top has a cut number $q$ then $C_{1 s}$ is at least $q$. If the bottom net has a cut number $q$, then $C_{u s}$ is at least q. Thus, intuitively it is clear that those nets which have the least cut numbers should appear as outer nets in the interval graphical representation corresponding to optimum routing.

Following [4], let us further denote by

$$
q_{m}=\min _{j} q_{j}
$$

and

$$
q_{M}=\max _{j} q_{j}
$$

In [4] it has been shown that the street congestion $Q_{0}$ for the optimum realization satisfies an inequality:

$$
Q_{0} \geq \max \left\{a_{m},\left\lceil\frac{a_{M}}{2}\right]\right\}
$$

where $\lceil x\rceil$ is the smallest integer not smaller than $x$. An optimum realization with street congestion $Q_{0}=\left\lceil\frac{q_{M}}{2}\right\rceil$ exists if and only if there exists such an ordering that for each $v_{i}$ with $c_{i}=\left\lceil\frac{{ }^{q} M}{2}\right\rceil+k\left(k=1,2, \ldots q_{M}-\left\lceil\frac{q_{M}}{2}\right\rceil\right)$ the net associated with $v_{i}$ is covered from above and below by at least $k$ nets. In general there exists an optimum realization with street congestion $Q_{0}=\left\lceil\frac{q^{M}}{2}\right\rceil+p$, if and only if $p$ is the least nonnegative integer for which the p-excess property holds. The p-excess property means that there exists an ordering such that for each $v_{i}$ with $c_{i}=\left\lceil\frac{q_{M}}{2}\right\rceil+k$ ( $k=p+1, \ldots q_{M}-\left[\frac{q_{M}}{2}\right\rceil$ ) the net associated with $v_{i}$ is covered by at least $k-p$ nets from above and below [4].

The heuristic algorithm for single-row routing makes use of these necessary and sufficient conditions for optimum realizations.

## 3. FOUNDATIONS OF THE HEURISTIC ALGORITHM

In the previous section we have recalled the necessary and sufficient conditions for optimum single-row routing. These conditions are in such a form that the problem of finding the optimum ordering of intervals in the interval graphical representation remains an open question. However, these conditions help us to understand the problem and to develop an intuitive approach.

First, we will make use of an observation that the nets which have the least cut numbers should appear as outer nets in the interval graphical representation. We classify nets according to their cut numbers and introduce the concept of the zone as follows: We assign nets with maximum cut number $q_{M}$ to the zone $Z_{O}$. Nets with cut number $q_{M}$-i are in zone $Z_{i}$. Thus for the nets in Fig. 2 we have $q_{M}=3$, nets $N_{1}, N_{4}, N_{5}, N_{g}$ and $N_{6}$ are in $Z_{0}, N_{3}, N_{8}$ and $N_{2}$ in $Z_{1}$ and $N_{7}$ in $Z_{2}$. In the optimum realization nets from the zone $Z_{q_{M}}$ should appear as outer nets. Now, we will prove the following:

Lemma 1. Given a net list $L=\left\{N_{1}, N_{2}, \ldots, N_{m}\right\}$ and an ordering $0_{1}$ of the nets resulting in an optimum realization with street congestion $\mathrm{Q}_{0}$. If in $0_{1}$ there is a net $N_{i}$ with cut number $q_{i} \leq Q_{0}$ placed between nets $N_{j}, N_{k}$ with cut numbers larger than $Q_{0}$ then there exists another optimal ordering $\mathrm{O}_{2}$ such that $\mathrm{N}_{\mathrm{i}}$ preceeds both $\mathrm{N}_{\mathrm{j}}, \mathrm{N}_{\mathrm{k}}$ or follows $N_{j}$ and $N_{k}$.

Proof. Suppose that the ordering $0_{1}$ is such that

$$
N_{j}<N_{i}<N_{k}
$$

and $N_{i}$ cannot be placed just before $N_{j}$ or after $N_{k}$ without affecting optimality. In such a case, there is a $\operatorname{pin} v_{j}$ in $N_{j}$ with upper cut
$Q_{0}$ and there is a pin $v_{k}$ in $N_{k}$ with lower cut number $Q_{0}$ (see Fig. 3). Then $v_{j}$ must be the terminating pin for $N_{j}$, and $v_{k}$ the originating pin for $N_{k}$. If it were not the case, $Q_{0}$ could not be the street congestion or the upper cut in $v_{j}$ would have to be less than $Q_{0}$ or the lower cut in $v_{k}$ would have to be less than $Q_{0}$. Between $v_{j}$ and $v_{k}$ at least one net which covers $v_{j}$ from above have to terminate. It is so because the upper cut number at $v_{k}$ cannot be larger than $Q_{0}$ and $v_{k}$ is already covered by $N_{i}$. For the same reason at least one net covering $v_{k}$ from below has to originate between $v_{i}$ and $v_{k}$. Actually, $N_{j}$ must terminate at $v_{j+7}$, and $N_{k}^{\prime}$ must originate from $v_{k-1}$. If between $N_{j}$ and $N_{j}^{\prime}$ there are $\ell$ nets passing through then the upper cut in $v_{j+1}$ is $Q_{0}-1-\ell$. Suppose that $N_{i}$ is immediately after $N_{j}$ in the ordering $0_{1}$. Let us interchange $N_{j}^{\prime}$ and $N_{i}$ in the $0_{1}$. The upper cut number and lower cut number of $v_{j}$ remain unchanged, and the lower cut number of $v_{j+1}$ is decreased by one and the upper cut number is increased by one but does not exceed $Q_{0}$. Cut numbers of $v_{k-1}$ and $v_{k}$ are not affected by this operation. Following the same reasoning we can show that interchange of $N_{j}$ and $N_{i}$ does not increase the street congestion. If after the interchange we still have a situation that a net with cut number $Q_{0}$ or less is between two nets with cut numbers larger than $Q_{0}$ then the process of interchanges can be repeated.

Using Lemma 1 we can prove the following.
Lemma 2. Given a net list $L=\left\{N_{1}, N_{2}, \ldots N_{m}\right\}$ and an ordering $0_{1}$ of the nets giving an optimum realization with street congestion $Q_{0}$. There exists an ordering $\mathrm{O}_{2}$ such that nets in $\mathrm{O}_{2}$ satisfy the following:
$\mathrm{Z}_{\mathrm{q}_{M}}^{\mathrm{L}}<\mathrm{Z}_{\mathrm{q}_{M-1}}^{\mathrm{L}}<\ldots<\mathrm{Z}_{\mathrm{q}_{M}-\mathrm{Q}_{0}+1}^{\mathrm{L}}<\left(\right.$ Nets with cut numbers $\left.>\mathrm{Q}_{0}\right)<\mathrm{Z}_{\mathrm{q}_{M}-\mathrm{Q}_{0}+1}^{\mathrm{U}}<\ldots<\mathrm{Z}_{\mathrm{q}_{M}}^{\mathrm{U}}$
where $z_{i}^{L} \cup z_{i}^{U}=z_{i}$ and $z_{i}^{L} \cap z_{i}^{U}=\emptyset$, and superscipts denote the lower and upper streets.

Proof. We can transform $0_{1}$ into another ordering $0_{2}$ satisfying Lemma 1 . In $\mathrm{O}_{2}$ we have
$Z^{L}<($ Nets with cut numbers $>Q)<Z^{U}$
where

$$
z^{L}=\bigcup_{i=q_{M}-Q_{0}+1}^{q_{M}} z_{i}^{L} \quad, \quad z^{U}={\underset{i=q_{M}-Q_{0}+1}{q_{M}} z_{i}^{U}}_{U}^{U^{U}}
$$

The ordering within each zone $Z^{L}$ and $Z^{U}$ can be arbitrary, because it does not affect the street congestion. Thus we can make the ordering to be such which satisfies the requirements of the Lemma.

The same reasoning as in the proofs of Lemmas 1 and 2 can be used to prove:

Lemma 3. Given a net list $L=\left\{N_{1}, N_{2}, \ldots, N_{m}\right\}$ and an ordering $0_{1}$ of the nets which results in an optimum realization with street congestion $Q_{0}$, there exists an ordering $\mathrm{O}_{2}$ such that nets in $\mathrm{O}_{2}$ satisfy the following:

$$
\mathrm{z}_{\mathrm{q}_{M}^{L}}^{\mathrm{L}}<\mathrm{Z}_{\mathrm{q}_{M-1}}^{\mathrm{L}}<\ldots \quad \mathrm{z}_{1}^{\mathrm{L}}<\mathrm{Z}_{0}<\mathrm{z}_{1}^{U}<\mathrm{z}_{2}^{U}<\ldots \quad \mathrm{Z}_{\mathrm{q}_{M}}^{U}
$$

where $z_{i}^{L} \cup z_{i}^{U}=Z_{i}$ and $z_{i}^{L} \cap z_{i}^{U}=\emptyset$.
Lemma 3 is the basis of our heuristic algorithm for single-row routing. The algorithm produces a solution which satisfies conditions of Lemma 3. We apply heuristics to generate $Z_{i}^{L}$ and $Z_{i}^{U}$ for the given $Z_{i}$ and to find ordering within each zone. It can be easily seen that for a net list $L$ where all the nets are 2 pin nets which cover at least
one common point, each zone can have at most 2 nets, and for any division of zone $Z_{i}$ into $Z_{i}^{L}$ and $Z_{i}^{U}$, we always obtain an optimum solution.

## 4. THE BASIC ALGORITHM

The routing algorithm actually produces an ordering of intervals representing nets, because to each net there corresponds a unique realization.

In the following we will make use of the concept of internal and residual cut numbers.

Definition 3. Given a net list $L=\left\{N_{1}, N_{2}, \ldots N_{m}\right\}$ and a division of $L$ into two sublists $L_{1}$ and $L_{2}$ such that $L_{1} \cap L_{2}=\emptyset$ amd $L_{1} U L_{2}=L$. Let $N_{i} \in L_{1}$. The internal cut number. $i q_{i}$ of net $N_{i}$ in $L$ with respect to $L_{1}$ is defined as a cut number of $N_{i}$ in $L_{1}$. Thus, internal cut number has the same meaning as cut number, except that it is calculated as if the net list was composed only of $n \in t s$ which are in $\mathrm{L}_{1}$.

Definition 4. Let $L_{7} \cup L_{2}=L, L_{1} \cap L_{2}=\varnothing$ and $N_{i} \in L_{1}$. The residual cut number $\mathrm{rq}_{\boldsymbol{i}}$ of net $N_{i}$ in $L$ with respect to $L_{1}$ is defined as a cut number of $N_{i}$ in $\left\{L_{2} \cup N_{i}\right\}$.

These definitions can be extended in a natural way to internal and residual cut numbers at a node: $i c_{i}$ and $\mathrm{rc}_{\mathrm{i}}$. Let us consider the net list $L=\left\{N_{1}, N_{2}, N_{3}, \ldots N_{g}\right\}$ shown in Fig. 2. Let $L_{1}=\left\{N_{1}, N_{3}, N_{4}, N_{5}, N_{6}, N_{g}\right\}$, $L_{2}=\left\{N_{2}, N_{7}, N_{8}\right\}$. For nodes of the net $N_{3}$ in the Fig. 4b we have internal cut numbers $(1,1)$, residual cut numbers $[1,0]$. Thus $\mathrm{iq}_{3}=1, \mathrm{ir}_{3}=1$. Now, we are ready to present the basic concept of the algorithm. First, each net is assigned to an appropriate zone. Then division of each zone $Z_{i}$ into upper and lower parts $Z_{i}^{L}$ and $Z_{i}^{U}$ as well as mutual ordering of nets in each of $Z_{i}^{L}$ and $Z_{i}^{U}$ is determined. The algorithm
proceeds until each net is assigned to its proper $Z_{i}^{X}(X:=L, U)$ in a proper place.

The algorithm starts with nets from $Z_{0}$, i.e., such nets which have the maximum cut number $q_{M} . Z_{0}$ does not have to be divided, thus we have only to determine the net ordering in $Z_{0}$. The ordering of nets in $Z_{0}$ is performed as follows. Nets in this zone are sorted according to descending internal cut numbers with respect to $Z_{0}$. When two overlapping nets have the same internal cut numbers then the one which has larger residual cut number preceeds the one which has smaller residual cut number. If both internal and residual cut numbers are equal then the mutual ordering of these nets can be arbitrary. In the algorithm we use fictitious track $\left\{T_{i}\right\}$ to which nets are being assigned, $i$ is an integer from the interve. $1\left[-Q_{0}, Q_{0}\right]$. The ordering of tracks follows the natural order of the indices. Only nonoverlapping nets can be assigned to the same track. We take the first net from the sequence and place it in $T_{0}$. Then we take corrsecutive nets from the ordered sequence and place them in $T_{0}$ or in some $T_{j}$ so that $|j|$ is minimum. Then nets from consecutive zones are placed in tracks following the same rules as for nets in $Z_{0}$. The division of zones into upper and lower portions is a side effect of minimizing the absolute value of track indices.

Let us consider an example of net list shown in Fig. 2. Figure 4a shows nets from the same net list which belong to $Z_{0}$. In brackets are shown internal cut numbers of nodes for each of nets. For the nets in Fig. $4 a$ we have $N_{4}, N_{1}, N_{6}$ and $N_{9}$ with internal cut number equal 1. They also have the same residual cut numbers, thus their mutual ordering in the ordered sequence is an arbitrary one, let it be: $N_{4}, N_{6}, N_{9}, N_{1}$. After placing them on tracks accordingly to the previously mentioned
rules we have: $N_{1}, N_{5}, N_{9} \rightarrow T_{0}, N_{4} \rightarrow T_{1}, N_{6} \rightarrow T_{-1}$. It gives the following partial ordering in the interval graphical representation: $N_{4},\left(N_{7}, N_{5}, N_{9}\right), N_{6}$. Nets in brackets can have an arbitrary mutual order. In the next step nets from the zone $Z_{1}$ are to be placed in tracks. The nets $N_{2}, N_{8}, N_{3}$ are in $Z_{1}$. Nets $N_{2}$ and $N_{3}$ have the same internal cut numbers equal 2 and residual cut numbers 0 . Net $\mathrm{N}_{8}$ has internal cut number 1 . Thus the ordered sequence for nets in zone $Z_{1}$ can be $N_{2}, N_{3}, N_{8}$. Again, we take consecutive nets from this sequence and place them in ava:lable tracks so that the number of tracks used above and below track $T_{0}$ is as small as possible. We can assign $N_{2} \rightarrow T_{2}, N_{3} \rightarrow T_{-2}, N_{8} \rightarrow T_{2}$. Now, the actual ordering is $\left(N_{2}, N_{8}\right), N_{4},\left(N_{7}, N_{5}, N_{g}\right), N_{6}, N_{3}$. Zone $Z_{2}$ contairs just one net and according to the previous rules it is assigned to $T_{-2}$. The final ordering is $\left(N_{2}, N_{8}\right), N_{4},\left(N_{7}, N_{5}, N_{g}\right), N_{6}, N_{3}$.

The final track assignment is shown in Fig. 5. All arderings which follow from it correspond to the optimum realization in Fig. 1. More precisely, the ordering algorithm is as follows:

Step 0: $\mathrm{i}:=0, \mathrm{j}:=0$, assign each net to an appropriate zone.
Step 1: Calculate internal and residual cut numbers for nets in $Z_{i}$ with respect to $L_{1}=\bigcup_{j=0} Z_{j}$.
Step 2: Sort the nets from $Z_{i}$ into a sequence $0_{i}$ such that for $N_{a}, N_{b} \in Z_{i}$, $N_{a}<N_{b}$ if $i q_{a}>i q_{b}$ or $i q_{a}=i q_{b}$ and $i r_{a}>i r_{b}$. If $N_{a}$ and $N_{b}$ have equal both internal and residual cut numbers then their mutual ordering in $0_{i}$ is an arbitrary one.
Step 3: $\mathrm{j}=0$ ? yes $\rightarrow$ follow 4, no $\rightarrow$ go to Step 5 .
Step 4: Assign the first net from $0_{i}$ to track $T_{0}$. Are there any nets in $0_{j}$ which are not assigned yet? Yes $\rightarrow$ follow $5, n o \rightarrow$ go to 8 .

Step 5: Take the consecutive net from $0_{i}$ which does not have the track assignment. Can this net be assigned to $T_{x}$, such that $|x| \leq j$ ? Yes $\rightarrow$ follow 6, no $\rightarrow$ go to 7 .

Step 6: Assign this net to such track that $|x|=$ min. Are there any nets in $0_{i}$ which do not have track assignment? Yes $\rightarrow$ go to 5 , no $\rightarrow$ go to 8.

Step 7: $\mathrm{j}:=\mathrm{j}+1$, go to Step 6.
Step 8: Is there any net in the net list $L$ which is unassigned to any track yet? Yes $\rightarrow$ follow 9, no $\rightarrow$ go to 10 .

Stẹ 9: $\mathbf{i}:=\mathfrak{i}+1$, go to Step 1.
Step 10: Exit.

## 5. MODIFICATIONS OF THE BASIC ALGORITHM

Up to now we have assumed that in the realization of a single row the nets are unidirectional, i.e., there does not exist a net which has two horizontal segments crossing a vertical line. It turns out, however, that when this requirement is relaxed, better results are possible in some cases. If we obtain a realization where street congestion equals to $\left[\frac{q^{M}}{2}\right\rceil$ then no further improvement is possible. However, when the obtained ordering yields a realization with street congestion larger than $\left[\frac{{ }^{9}}{2}\right\rceil$ then allowing some nets to have more than one segment in the horizontal graphical representation can sometimes improve the result. We will explain this mechanism using the following examples. Let us consider a situation shown in the Fig. 6a. The node $A$ is covered from below by $q_{M^{-1}}$ nets, node $B$ is covered from above by $q_{M}$ nets and from below by no more than $\mathrm{q}_{\mathrm{M}}-2$ nets and there exists a net $N_{D}$ which covers $A$ from below and $B$ from above such that cut number $C_{D 1} \leq\left[\frac{q_{M}}{2}\right\rceil$. In such a case
we can replace one horizontal interval representing net $N_{D}$ with a different pattern as shown in Fig. 6b. Net $N_{D}$ covers $A$ from below and does not cover node B from above but rather from below. Thus the node B has upper cut number decreased by one and lower cut number increased by one. If $B$ were the only node where the street congestion occurred and the replacement does not introduce such new nodes then it reduces street congestion. The type of net representation as shown in the Fig. 6 b is called an external detour. For some net lists two or more external detours are possible. Figure 7 illustrates an example, where external detours have been applied to two overlapping nets. In this example $C_{E 1} \gg C_{E 2}, C_{D 1} \ll C_{D 2}$. Sometimes application of external detour to one net can result in decreasing of, say, upper cut number at more than one critical node, i.e., in a node where upper cut number equals to street congestion (see Fig. 8). However, in general, it is quite difficult to apply external detours. The reason for this is due to the nature of external detour: if we want to use it we have first to.find some ordering and then check whether street congestion can be reduced by external detour. It results in general in an exhaustive search. For a given net list we may have a case that for two different ordering resulting in the same street congestion, in one case external detours may decrease the congestion while for the other may not.

At the present moment we do not have a good heuristic implementing external detours. Here, we just want to point out that interval graphical representation may sometimes lead to single row realizations with larger street congestion than a more sophisticated representation, like using the external detour (which is a special case of general Steiner tree).

## 6. EXPERIMENTAL RESULTS

The computer program implementing the proposed approach has been written and tested on number of examples. It allows both multipin and 2-pin nets. Although the algorithm is heuristic, it always managed to find optimum solutions for all examples it was tested with. Figure 9 shows examples from the computer program. The example (a) is taken from [4] where it is shown that the optimum street congestion is 5 here. In the example (b) the $q_{t}=11$ and since $\left\lceil\frac{11}{2}\right\rceil=6$ thus the solution presented is also optimal.

## REFERENCES

[1] H.C. So, "Some theoretical results on the routing of multilayer printed-wiring boards," Proc. 1974 IEEE Int. Symposium on Circuits and Systems, pp. 296-303.
[2] B.S. Ting, E.S. Kuh, and I. Shirakawa, "The multilayer routing problem: algorithms and necessary and sufficient conditions for the single-row, single layer case," IEEE Trans. Circuits and Systems, Vol. CAS-23, pp. 768-778, Dec. 1976.
[3] B.S. Ting and E.S. Kuh, "An approach to the routing of multilayer printed circuit boards," Proc. 1978 IEEE. Int. Symp. on Circuits and Systems.
[4] E.S. Kuh, T. Kashiwabana, and T. Fujisawa, "On optimum single-row routing," IEEE Trans. Circuits and Systems, Vol. CAS-26, pp. 361368, June 1979.
[5] S. Tsukijama, E.S. Kuh, and I. Shirakawa, "An algorithm for singlerow routing with prescribed street congestion," IEEE Trans. Circuits and Systems, Vol. CAS-27, pp. 765-772, Sept. 1980.
[6] M.T. Doreau and L.C. Abel, "A topologically based non-minimum distance routing algorithm," Proc. 15th Design Automation Conf., pp. 92-99, 1978.
[7] S. Tsukiyama and E.S. Kuh, "Double row planar routing and permutation layout," Networks, Vol. 12, pp. 286-316, 1982.
[8] R. Raghavan and S. Sahni, "Single-row routing," Tech. Rep. 80-20, Computer Sci. Dep., University of Minnesota, 1980.

## FIGURE CAPTIONS

Fig. 1. An example of a single row realization and basic terminology.

Fig. 2. The interval graphical representation for the net list realization from the Fig. 1. Numbers in brackets show cut numbers for nodes which belong to the given net.

Fig. 3. Illustration for Lemma 1.

Fig. 4. (a) Subset of nets from Fig. 2 with internal cut numbers.
(b) Internal and residual cut numbers for the net $N_{3}$ with respect to the shown subset of the net list from Fig. 2 .

Fig. 5. Track assignment for the example of Fig. 1.

Fig. 6. Concept of external detour.

Fig. 7. External detour for two overlapping nets.

Fig. 8. External detour resulting in decrease of critical upper cut number at 3 nodes.

Fig. 9. Examples calculated by the computer program.

Fig. 1

Fig. 2

(a)

(b)

Fig. 3

(a)

(b)

Fig. 4


Fig. 5

(a)

(b)

Fig. 6

(a)

Fig. 7
(b)


(a)

Fig. 9



```
    CUTNUMIER CMaX = 11
    UIPER STEEET CONGESTION QU : 6 LOWER STEEET CONGESTION OL : G
    (b)
```

Fig. 9


[^0]:    *Research sponsored by the National Science Foundation Grant ECS-8201580 and the State of California Microelectronics Innovation and Computer Research Opportunities Program (G.E., Intel, TRW and Xerox).

