PROBABILISTIC FLOWS FOR RELIABILITY EVALUATION OF MULTI-AREA POWER SYSTEM INTERCONNECTIONS

by

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PROBABILISTIC FLOWS FOR RELIABILITY EVALUATION OF
MULTI-AREA POWER SYSTEM INTERCONNECTIONS

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ABSTRACT

The paper reports on the development of an efficient solution method and computer program for the evaluation of multi-area reliability. An interconnected power system is modeled as a capacitated network with probabilistic arc capacities. The proposed solution method consists of an analytic state space decomposition phase and a Monte Carolo simulation phase. An optimization problem is solved to minimize the total computational time for the two phases. The solution of the optimal mix problem determines the termination of the decomposition phase and the size of sample for the Monte Carlo phase. A new reliability index, the indadequate transfer capability, is introduced. This measure indicates the relative effectiveness of either increasing existing capacities or opening new interconnections between two areas. The proposed method has been implemented into a computationally efficient production grade software package, called REMAIN (Reliability Evaluation of Multi-Area INterconnections). The application of REMAIN to a seven area example for planning system enhancement is given. Data on computational times are also presented.

I. INTRODUCTION

This paper reports on the development of an efficient computational tool for the evaluation of multi-area reliability in generation planning studies. An area refers to a utility company or a geographic region within a utility. A single area representation of the power system is commonly used in the reliability evaluation of generation resource plans. In such representations, it is assumed that the transmission system is capable of carrying power flows from generation sources to load points within an area whenever needed. The loss-of-load probability or LOLP method is generally used for single area reliability evaluation [1]. Quite often resource planners are interested in assessing the benefits of interconnection. The power transfer between two areas is limited by the capacities of the tie lines that connect the areas. The objectives of reliability studies of such multi-area power systems are to evaluate the enhancement of reliability due to interconnection and to identify interconnections whose improvement is most effective in increasing the system reliability.

In order to evaluate the reliability enhancement due to interconnection, comparisons are made in terms of the following quantities:

. the probability that the total system demand cannot be met (system loss-of-load probability or system LOLP),

. the expected load demand that the system fails to serve (expected unserved demand or EUD),

. the probability that the area load cannot be met (area LOLP).

The area LOLP index depends on the interconnection policy which determines how the power is routed in case of a loss-of-load. In this paper, the following interconnection policies are considered:

- Load loss sharing (LLS) policy: Whenever loss of load occurs in the system, areas must share the unserved demand to the extent possible.
- No load loss sharing (NLLS) policy: Each area attempts to meet its own demand. If there is excess power, it is supplied to the neighboring areas according to the order specified by a priority list.
- In addition, for purposes of comparison, we consider the case where no power exchange exists between any of the areas of the system and refer to this as the isolation policy.

One important objective of multi-area reliability studies is to identify weak links in the interconnection, i.e., tie lines whose improvement is most effective in increasing the system reliability. Therefore, we introduce a new reliability index, the inadequate transfer capability from area i to area j. It is the probability that a lack of power transfer capability from area i to area j contributes to the loss of load. It can be evaluated for any two areas whether or not they are directly connected. This measure indicates the relative effectiveness of either increasing existing capacities or opening new interconnections between two areas.

In multi-area reliability evaluation a simplified model of the power system is used. Under the assumptions that the bus voltages are constant and the losses in the tie lines are negligible, the real power flows in the multi-area power system can be modeled as flows in a capacitated network with probabilistic arc capacities. Doulliez and Jamoulle in 1972 developed a state space decomposition method for probabilistic flow-network reliability evaluation [2] which was later used for transmission system reliability studies [3]. A decomposition approach was also used for reliability evaluation of composite systems including unconventional energy sources [4]. Pang and Wood [5] developed a computer program for multi-area reliability based on the inclusion and exclusion formula for evaluating the probability of the union of non-disjoint sets. In this paper, we present a composite state space decomposition and Monte Carlo method for multi-area reliability calculations.

Our method provides an estimate of each reliability index. Each estimate is obtained in such a way that its standard deviation is smaller than a specified quantity. This is accomplished by selecting the stopping criterion of the decomposition phase and the number of states in random samples of the Monte Carlo phase using the solution of an optimization problem. The optimization problem is to minimize the total computation time subject to the constraint that the standard deviations of the estimates be smaller than a specified quantity.

We have implemented the combined decomposition-Monte Carlo method into a computationally efficient production grade software package, called REMAIN (Reliability Evaluation of Multi-Area INterconnections). Applications of REMAIN include the study of transmission bottlenecks between regions of a large utility and the investigation of the reliability of power pools. The results of our computational experience with a seven area model of a power pool are presented. The use of the results to plan system enhancements for improving reliability is also described.

The paper discusses in detail the proposed approach and the REMAIN program. The flow-network model of the multi-area power system is discussed in Section II. The next four sections present the composite decomposition-Monte Carlo approach. Section III outlines the proposed scheme. The details of the decomposition phase, Monte Carlo simulation phase, and the selection of the optimal mix of the two phases are given in Sections IV, V, and VI, respectively. Section VII summarizes the implementation of the proposed method in the REMAIN software. Section VIII presents the numerical results.

The notation used in the paper is standard. Vectors are denoted by an underbar, e.g. $\underline{x}$. The ith component of the (row) vector $\underline{x} = (x_1, x_2, \ldots, x_n)$ is x_i. We write $\underline{x} \geq \underline{y}$ if and only if $x_i \geq y_i$ for $i = 1, 2, \ldots, n$. The notation $x \stackrel{\Delta}{=} A$ represents that x is defined by the expression A. The set $\{\underline{x}:A\}$ is the collection of vectors $\underline{x}$ with the property A. Random variables are indicated by underlining with a tilde, e.g., $\utilde{c}$. We write $P\{x:A\}$ for the probability of the set of events x characterized by property A. $P\{A|B\}$ is the conditional probability of event A given that event B has occurred.

II. FLOW-NETWORK MODEL FOR MULTI-AREA RELIABILITY

II.1 Multi-Area Power System

In multi-area reliability evaluation, each area represents a company, a geographic region within a company or jointly owned generation. The total generation capacity in megawatts (MW) within an area is expressed as a discrete random variable to represent the possible forced outages of the generators. The probability distribution of the area generation capacity can be obtained by the convolution formula using forced outage/

partial forced outage rates of individual generators. Between two areas that are directly connected, there is a maximum power in MW that can be transferred due to the limitations on the power carrying capability of the tie lines. Again, because of the possible forced outages of the tie lines, the maximum power transfer capability between two areas can be represented by a discrete random variable. We assume that all these random variables are statistically independent.

For each area, there may be a load demand in MW which is assumed to be a deterministic quantity. When the sum of the area generation and the net power received from all other areas is less than the area load demand, we say the area experiences a loss of load. When there is a loss of load in any area of the system, we say there is a loss of load in the system.

II.2 Probabilistic Flow-Network Model

Power flows from each area's generation to meet its own load and through the tie lines to the other areas. The power flows are limited only by the (random) generation and transfer capacities in the system. We use a flow-network with probabilistic arc capacities to model the multi-area power system. Each area is represented by a node. Two additional nodes are introduced: a generation source node s and a demand sink node t. A directed arc from the source node to each area node is introduced to represent the area generation capacity. A directed arc from each area node to the sink node is introduced to represent the area load demand. The capacity of this arc is the area load demand. Bidirectional arcs between area nodes are used to represent the tie lines between areas. The capacity of each such arc is the random variable of the maximum power transfer capability between the two areas. The power network is assumed to be connected since the subject under study is the multi-area power system interconnections.

Let the number of arcs in the network be n. Let $\underset{\sim}{c}_i$ denote the random variable representing the capacity of arc i. It takes values

$$\underset{\sim}{c}_i = c_{ij} \text{ with probability } p_{ij},\ j = 1,2,\dots,\ell_i$$

where ℓ_i is the number of distinct capacity levels for arc i. For each arc $\alpha(k)$ that joins area node k to the sink node, $\ell_k = 1$ so that $\underset{\sim}{c}_{\alpha(k)}$ becomes a deterministic variable. In this case, $\underset{\sim}{c}_{\alpha(k)}$ equals the area k demand D_k with probability 1. As previously mentioned, we assume that the random variables $\underset{\sim}{c}_i$, $i = 1,2,\dots,n$, are independent.

When each random variable $\underset{\sim}{c}_i$ takes a value, say c_{ix_i}, we have a capacitated flow-network. This corresponds to a system state which we denote by the vector $\underline{x} = (x_1,x_2,\dots,x_n)$. The collection of these sytem states forms the state space X of the multi-area power system model. Based on the definitions and assumptions made, it follows that:

(i) there are $\prod_{i=1}^{n} \ell_i$ states in X;

(ii) there exists a maximum state $\underline{M} = (\ell_1,\dots,\ell_n)$ and a minimum state $\underline{m} = (1,\dots,1)$ in X such that $\underline{M} \geq \underline{x}$ and $\underline{m} \leq \underline{x}$ for all $\underline{x}$ in X;

(iii) the probability associated with each state $\underline{x}$ in X is

$$P\{\underline{x}\} = \prod_{i=1}^{n} p_{ix_i} . \qquad (1)$$

II.3 Maximal Flow and Minimal Cut

For a particular system state $\underline{x}$, a maximal flow from the source to the sink can be found by the Ford-Fulkerson algorithm [6]. Let the resulting flows in the network be denoted by $\underline{f}(\underline{x}) = [f_1(\underline{x}),f_2(\underline{x}), \dots, f_n(\underline{x})]$, where $f_i(\underline{x})$ is the flow through the ith arc. The total amount of flow from the source to the sink is called the value of the maximal flow, $V[\underline{f}(\underline{x})]$. There may be more than one maximal flow through the network; however, all maximal flows have the same value. We use priority lists that order the arcs leaving each node to select a particular maximal flow. When no additional flow can be routed in the network from the source to the sink, there is always a set of arcs whose capacities limit the flow. In other words, associated with each maximal flow $\underline{f}$ there is a minimal cut set of arcs $C[\underline{f}(\underline{x})]$. A minimal cut partitions the nodes in the network into two disjoint sets. We denote these two sets by:

$$N_s\{C[\underline{f}(\underline{x})]\} = \{\text{nodes on the source side of the cut}\} \tag{2}$$

$$N_t\{C[\underline{f}(\underline{x})]\} = \{\text{nodes on the sink side of the cut}\} \tag{3}$$

It can be shown [6, p. 13, Corollary 5.4] that if $N_s^1\{C^1[\underline{f}^1(\underline{x})]\}$ and $N_s^2\{C^2[\underline{f}^2(\underline{x})]\}$ (we will write N_s^1 and N_s^2 for short) are defined by two minimal cuts C^1 and C^2 of the network associated with the system state $\underline{x}$, the arcs joining their intersection $N_s^1 \cap N_s^2$ and its complement $\overline{N_s^1 \cap N_s^2}$ form a minimal cut. Similarly the arcs joining $N_t^1 \cap N_t^2$ and $\overline{N_t^1 \cap N_t^2}$ also form a minimal cut. We now define the sets $N_s^*(\underline{x})$ and $N_t^*(\underline{x})$ as follows:

$$N_s^*(\underline{x}) = \bigcap_{\text{all } C \text{ of } \underline{x}} N_s\{C[\underline{f}(\underline{x})]\}$$

$$N_t^*(\underline{x}) = \bigcap_{\text{all } C \text{ of } \underline{x}} N_t\{C[\underline{f}(\underline{x})]\}$$

Note that $N_s^*(\underline{x})$ itself is one of $N_s\{C[\underline{f}(\underline{x})\}$, therefore $N_s^*(\underline{x})$ is the set having the least number of nodes among all $N_s\{C[\underline{f}(\underline{x})]\}$. Similarly $N_t^*(\underline{x})$ is the set having the least number of nodes among all $N_t\{C[\underline{f}(\underline{x})]\}$. The application of Ford-Fulkerson algorithm starting from node s yields $N_s^*(\underline{x})$, whereas the application of Ford-Fulkerson algorithm backward from node t yields $N_t^*(\underline{x})$.

II.4 Reliability Indices

For D_i being the local demand of area i,

$$D = \sum D_i$$

is the total load demand of the system. A power system is said to be experiencing loss of load whenever there is load that the system fails to supply. The following reliability indices are defined for multi-area systems:

System Loss-of-Load Probability (LOLP). For a particular system state $\underline{x}$, if $V[\underline{f}(\underline{x})]$ is less than the total demand D, there is loss of load in the system. System LOLP is defined to be the probability that there is loss of load in the system.

$$\text{System LOLP} \overset{\Delta}{=} P\{\underline{x} : V[\underline{f}(\underline{x})] < D\} \tag{4}$$

Expected Unserved Demand (EUD). EUD is the expected value of the amount of load demand that the system is unable to meet.

$$\begin{aligned} \text{EUD} &\overset{\Delta}{=} \sum_{\underline{x} \in \{\underline{x}: V[\underline{f}(\underline{x})] < D\}} \{D - V[\underline{f}(\underline{x})]\} P\{\underline{x}\} \\ &= E\{D - V[\underline{f}(\underline{x})] \mid V[\underline{f}(\underline{x})] < D\} P\{\underline{x} : V[\underline{f}(\underline{x})] < D\} \end{aligned} \tag{5}$$

Area LOLP. Area LOLP is the probability that the area fails to meet its load demand. The value of the area LOLP depends on the interconnection policy adopted. Under the NLLS policy, the area LOLP can be calculated as follows:

$$\text{Area i LOLP}\Big|_{\text{NLLS}} \overset{\Delta}{=} P\{\underline{x} : f_{\alpha(i)}(\underline{x}) < D_i\} \tag{6}$$

Here $\alpha(i)$ denotes the arc which connects the area node i to the sink node. Under the LLS policy, the area LOLP can be calculated as follows.

$$\text{Area i LOLP}\Big|_{\text{LLS}} \overset{\Delta}{=} P\{\underline{x} : \text{node } i \in N_t^*(\underline{x})\} \tag{7}$$

<u>Inadequate Transfer Capability (ITC)</u>. For a particular system state $\underline{x}$, suppose there is a loss of load in the system; then an increase in the transfer capability from node i to node j reduces the amount of unserved demand if and only if node i is in $N_s^*(\underline{x})$ and node j is in $N_t^*(\underline{x})$. This is true because of the fact that $N_s^*(\underline{x})$ and $N_t^*(\underline{x})$ are the sets having the least number of nodes among all $N_s\{C[\underline{f}(\underline{x})\}$ and $N_t\{C[\underline{f}(\underline{x})]\}$, respectively. It should be noted that the foregoing statement is not true if the condition is simply that node i is in $N_s\{C[\underline{f}(\underline{x})]\}$ and node j is in $N_t\{C[\underline{f}(\underline{x})]\}$ for some minimal cut C of $\underline{x}$. In other words, when there is a loss of load in the system, node i is in $N_s^*(\underline{x})$ and node j is in $N_t^*(\underline{x})$, the lack of power transfer capability from node i to node j is a contributing factor to the loss of load. If node i is the source node and node j is an area node, the situation described above indicates that the area generation is inadequate. If both node i and node j are area nodes, the situation indicates that the tie line capacity is inadequate. We define the inadequate transfer capability (ITC) for each ordered pair of nodes (node i, node j) as the probability that increasing the capacity from node i to node j reduces the amount of unserved demand. The value of ITC_{ij} can be computed as follows:

$$ITC_{ij} = P\{\underline{x}:V[\underline{f}(\underline{x})] < D, \text{ node } i \in N_s^*(\underline{x}) \text{ and node } j \in N_t^*(\underline{x})\} \qquad (8)$$

The inadequate transfer capability can be interpreted as the probability that a lack of power transfer capability from node i to node j contributes to the loss-of-load. The inadequate transfer capability provides a measure of the inadequacy of area generation or transfer capabilities in various parts of the system. Increases in the capacity of tie lines or area generation with higher ITC values are more effective in improving reliability. The inadequate transfer capability from each area i to the system can be defined as follows:

$$\mathrm{ITC}_{i,\mathrm{system}} = P\{\underline{x} : V[\underline{f}(\underline{x})] < D \text{ and node } i \in N_s^*(\underline{x})\} \quad (9)$$

Attempts have been made in the past [2] to use the probability that the arc from node i to node j is in a minimal cut as a measure of inadequacy of transfer capability. Because of the fact that there may be more than one minimal cut in the network, increasing the transfer capability of an arc in the minimal cut does not necessarily imply that the amount of loss-of-load will be reduced. The new inadequate transfer capability index that we introduced has the desired interpretation. Moreover, since it is defined for any pair of nodes, regardless of whether they are or are not directly connected, the ITC index can be used for comparing the relative effectiveness of increasing existing tie line capacity or building new interconnections.

III. OVERVIEW OF THE DECOMPOSITION-MONTE CARLO APPROACH

The basic idea in the evaluation of each reliability index is to identify the appropriate set of states and to compute its probability. The algorithm we have developed has two phases. The first phase is an analytical state space decomposition and the second phase is a Monte Carlo simulation. The allocation of the time between the two phases is determined by the solution of an optimization problem.

The state space decomposition phase of the algorithm is an extension of the method developed by Doulliez-Jamoulle [2]. It is an iterative process to classify the states in the state space. Initially, all the states in the state space are unclassified. At each iteration, by application of the maximal flow algorithm, the set of unclassified states is decomposed into subsets of states having the same reliability characteristics and subsets of unclassified states. Upper and lower bounds for each reliability index are computed. As the number of iterations increases, the number of states that are classified by each iteration decreases, i.e., the efficiency of the method decreases. Because of this, we switch to a Monte Carlo method in the second phase of our approach. From each subset of unclassified states, a random sample of states is selected. The maximal flow algorithm is applied to each of the sample states. Finally, an estimate of the contribution of the unclassified states to each reliability index is obtained.

The uncertainty in the estimate of each reliability index is measured in terms of its standard deviations. The standard deviation of an estimate depends on the computation time in the two phases of the algorithm. Consider the optimization problem of minimizing the total computation time subject to the condition that the standard deviation does not exceed a specified quantity. Under some reasonable assumptions, we have derived certain relations which must be satisfied by the optimal solution. From these relations, we can determine the stopping criterion for the decomposition phase and the number of states in each random sample for the Monte Carlo phase.

A detailed description of the proposed solution method is given in the next three sections.

IV. STATE-SPACE DECOMPOSITION PHASE

An analytic decomposition schem is used to avoid carrying out a separate maximal flow computation for each of the ($\prod_{i=1}^{n} \ell_i$) states in the state space. The state space is decomposed in such a way that from each maximal flow computation carried out on one state, information concerning the reliability of some other states may be derived.

The iterative decomposition scheme makes use of two basic properties of the model. These are:

(i) Let the maximal flow corresponding to state $\underline{x}$ be $V[\underline{f}(\underline{x})]$. For any state $\underline{y} \geq \underline{x}$, $V[\underline{f}(\underline{y})] \geq V[\underline{f}(\underline{x})]$. In other words the system model is coherent in the sense of reliability theory [9, p. 6]

(ii) Let $\underline{M}$ and $\underline{m}$ be two states in the state space X, $\underline{M} \geq \underline{m}$ For the set of states lying between $\underline{M}$ and $\underline{m}$ is defined as

$$S \triangleq \{\underline{x} : \underline{m} \leq \underline{x} \leq \underline{M}\} \ , \tag{10}$$

we have

$$P\{S\} = \prod_{i=1}^{n} \sum_{m_i \leq x_i \leq M_i} p_{ix_i} \tag{11}$$

We refer to $\underline{M}$ and $\underline{m}$ as the maximum and minimum state of the set S, respectively.

The coherency property (i) enables us to derive from a single maximal flow computation the reliability characteristics of a subset of the states without having to carry out maximal flow calculations for each of these states. Properties (i) and (ii) form the basis in the development of a recursive scheme of decomposing the state space into subsets.

There are two stages in the decomposition phase. The first stage decomposition is for the calculation of system LOLP and the second stage decomposition is for the calculation of the area LOLP, EUD and ITC indices.

IV.1 Recursive Decomposition for System LOLP

The initial set of the decomposition phase is the state space. The state space X has a maximum state $\underline{M} = (M_1, M_2, \ldots, M_n)$ and a minimum state $\underline{m} = (m_1, m_2, \ldots, m_n)$ and is in the form of the set in Eq. (10). We decompose X into subsets of three categories (see Fig. 1).

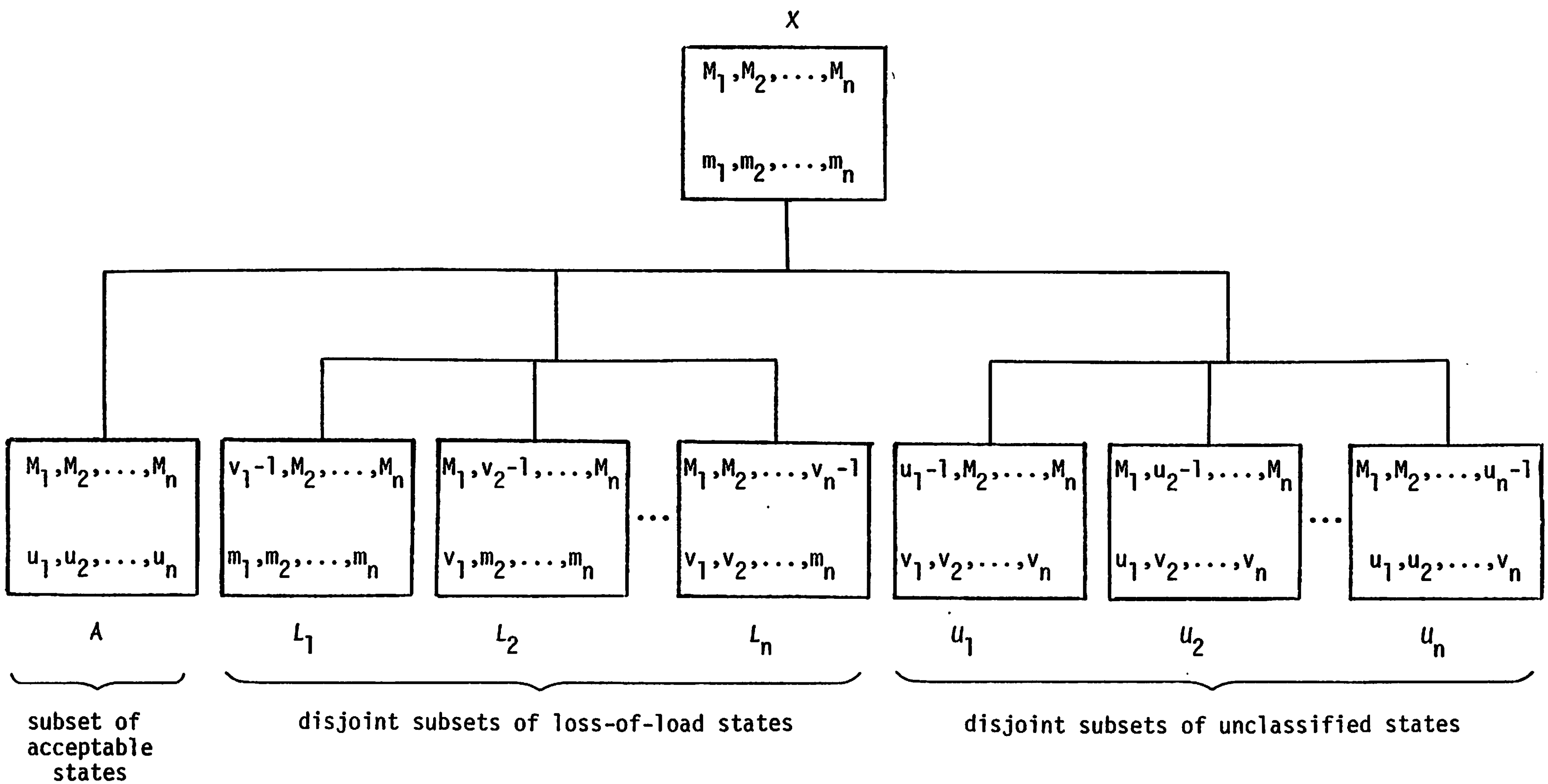

Fig. 1 Recursive decomposition for the evaluation of the system LOLP index.

a) Subset of acceptable states A.

Suppose that the value of the maximal flow for the state $\underline{M}$ is $V[\underline{f}(\underline{M})] = D$, i.e., the state $\underline{M}$ can satisfy the total load demand D of the multi-area system. Based on the flows in the network for the state $\underline{M}$ and the coherency property (i) we are going to determine a collection A of states $\underline{x}$ for which the maximal flow will also be $V[\underline{f}(\underline{x})] = D$. Let f_k denote the flow through arc k. For each arc k, let u_k be the index x_k corresponding to the smallest capacity level c_{kx_k} of the arc which is not less than f_k, i.e.,

$$u_k = \min\{x_k : c_{kx_k} \geq f_k\} \tag{12}$$

By the coherency property of the flow-network model, whenever the capacity $\underset{\sim}{c}_k$ of arc k takes a value between c_{ku_k} and c_{kM_k}, $k = 1,2,\ldots,n$, then a flow D can be sent, i.e., the total load demand D is satisfied. Thus for the set of states

$$A = \{\underline{x} : \underline{u} \leq \underline{x} \leq \underline{M}\} \tag{13}$$

the demand D can be met, i.e., $V[\underline{f}(\underline{x})] = D$. Since the set A is of the form (10), its probability can readily be computed.

b) Subsets of system loss of load states L_k.

We next classify the collection of states $\underline{x}$ for which the total load demand D cannot be satisfied, i.e., $V[\underline{f}(\underline{x})] < D$. For this purpose we first determine for each arc k the minimum capacity level c_{kv_k} such that a flow D can still be sent. The following procedure is used to find the value v_k (see Fig. 2). Let us start with the network corresponding to state $\underline{M}$ with the maximal flow $\underline{f}(\underline{M}) = [f_1(\underline{M}),\ldots,f_n(\underline{M})]$. Suppose that the flow f_k in arc k is from node q to node r. We remove arc k and consider the resulting network having flows $f_\ell(\underline{M})$ in the remaining arcs, $\ell \neq k$. We then apply Ford-Fulkerson algorithm to this network to determine the

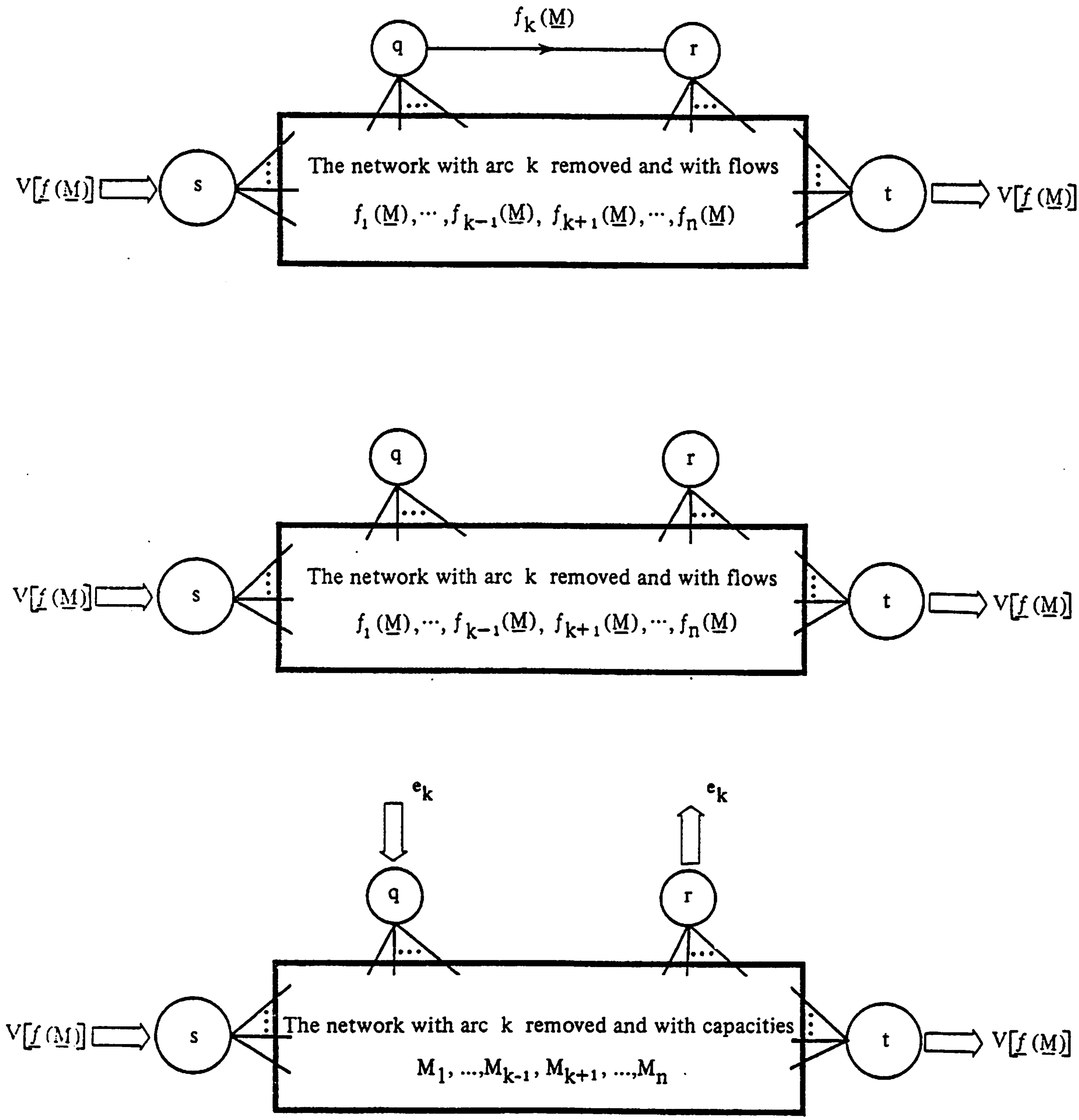

Fig. 2 The procedure for determining the largest decrease in flow e_k such that a maximal flow $V[\underline{f}(\underline{x})] = D$ can still be sent from s to t.

maximal flow from q to r that can be superimposed on the existing flow. The value e_k of this additional flow is the largest decrease in flow for arc k such that the network can still send a maximal flow D from s to t. We define v_k to be

$$v_k = \begin{cases} \min\left\{x_k : c_{kx_k} \geq f_k - e_k\right\} & \text{if } f_k - e_k \geq 0 \\ 1 & \text{if } f_k - e_k < 0 \end{cases} \tag{14}$$

By the coherency property of the flow-network model, system loss of load occurs for any state in the set

$$L = \{\underline{x} : \exists k \ni x_k < v_k\} \tag{15}$$

We may further decompose L into nonintersecting subsets $L_1, L_2, \ldots, L_n$ having the form (10). The states in L are classified into subset L_k if k is the first index of $\underline{x}$ for which $x_k < v_k$, i.e.,

$$L_k = \{\underline{x} : (v_1, \ldots, v_{k-1}, m_k, m_{k+1}, \ldots, m_n) \leq \underline{x} \leq (M_1, \ldots, M_{k-1}, v_k - 1, M_{k+1}, \ldots, M_n)\} \tag{16}$$

Note that the contribution of each subset L_k to the system loss-of-load probability is $P\{L_k\}$, which is readily calculated. Since the subsets $L_1, L_2, \ldots, L_n$ are disjoint, their contributions to system LOLP are additive.

c) Subsets of unclassified states U_k.

The remaining states in X are unclassified. They are characterized by

$$U = \{\underline{x} : x_i \geq v_i \;\; \forall i \;\text{ and }\; \exists j \ni x_j < u_j\} \tag{17}$$

This set generally does not have a maximum state. In order to be able to continue the process of classification recursively, it is necessary to decompose U into subsets of the form in (10). We therefore classify the states in U into the subsets U_k if k is the first index for which $x_k < u_k$, i.e.,

$$\mathcal{U}_k = \{\underline{x}:(v_1,\ldots,v_{k-1},v_k,v_{k+1},\ldots,v_n) \leq \underline{x}$$
$$\leq (M_1,\ldots,M_{k-1},u_k-1,M_{k+1},\ldots,M_n) \quad (18)$$

The subsets $\mathcal{U}_1,\ldots,\mathcal{U}_n$ are disjoint and their union equals $\mathcal{U}$.

The recursive scheme consists of replacing X by $\mathcal{U}_k$ redefining the quantities $\underline{m}$ and $\underline{M}$ and repeating the decomposition into acceptable subset A, loss-of-load subsets $L_1,\ldots,L_n$, and unclassified subsets $\mathcal{U}_1,\ldots,\mathcal{U}_n$. Because all the decomposed subsets are nonintersecting their probabilities can simply be added.

IV.2 Recursive Decomposition for Area LOLP

Each state in the subsets L_ℓ, $\ell = 1,2,\ldots,n$ is a system loss-of-load state. A system loss-of-load state corresponds to the situation of one or more areas experiencing loss of load. While two states may belong to the same subset L_ℓ, their area loss-of-load characteristics may be different. In order to evaluate area LOLP each subset L_ℓ is recursively decomposed into subsets having identical area loss-of-load characteristics (see Fig. 3).

The area LOLP depends on the interconnection policy. Let us consider first the evaluation of the area LOLP under the NLLS policy. Let m* and M* denote the minimum and maximum state, respectively, of L_ℓ, i.e.,

$$L_\ell = \{\underline{x} : \underline{m}^* \leq \underline{x} \leq \underline{M}^*\} \quad (19)$$

The Ford-Fulkerson algorithm is applied to the maximum state $\underline{M}^*$ to determine the set $N_t^*(\underline{M}^*)$ and the corresponding maximal flow. One of the following two cases may occur:

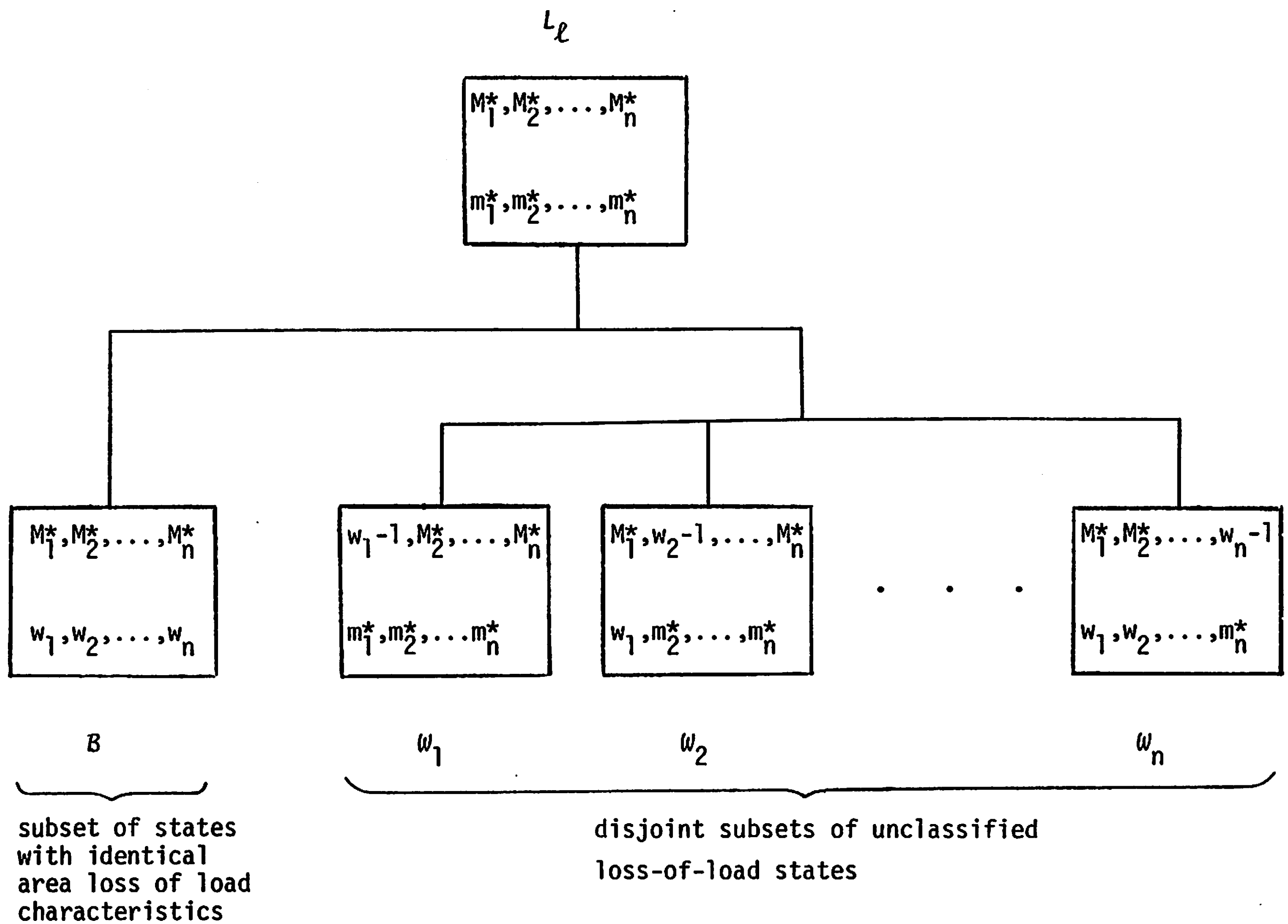

Fig. 3 Recursive decomposition for the evaluation of the area LOLP index.

Case 1. There exists an area node $j \in N_t^*(\underline{M}^*)$ such that the corresponding flow in the arc $\alpha(j)$ from the area node j to the sink is saturated, i.e., $f_{\alpha(j)} = D_j$.

Case 2. For every area node $i \in N_t^*(\underline{M}^*)$, $f_{\alpha(i)} < D_i$, i.e., every area in $N_t^*(\underline{M}^*)$ experiences loss of load.

We consider first Case 1. Under the NLLS policy no loss of load occurs for area j. On the other hand, there exists at least one area node $i \in N_t^*(\underline{M}^*)$ for which $f_{\alpha(i)} < D_i$. Area i thus suffers loss of load. The subset of states of L_ℓ with the same area loss-of-load characteristics, i.e., the same areas satisfy load demand and the same areas suffer loss of load, is

$$B = \{\underline{x} : \underline{w} \leq \underline{x} \leq \underline{M}^*\} \tag{20}$$

where

$$w_k = \min\{x_k : c_{kx_k} \geq f_k(\underline{M}^*)\} \tag{21}$$

Next we consider Case 2. In this case we want to find the set of states $\underline{x}$ for which $N_t^*(\underline{x}) = N_t^*(\underline{M}^*)$, i.e., the states $\underline{x}$ having the same area loss-of-load characteristics. Clearly if a minimal cut $C(\underline{x})$ of $\underline{x}$ is identical to the minimal cut $C(\underline{M}^*)$ at hand, and there is no other minimal cut consisting of arcs in $C(\underline{x})$ and arcs connecting $N_t\{C[\underline{f}(\underline{x})]\}$, then $N_t\{C[\underline{f}(\underline{x})]\} = N_t^*(\underline{x})$ and is equal to $N_t^*(\underline{M}^*)$. Let us define in this case

$$w_k = \begin{cases} \min\{x_k : c_{kx_k} \geq f_k\} & \text{if } k \notin C(\underline{M}^*) \\ m_k^* & \text{if } k \in C(\underline{M}^*) \end{cases} \tag{22}$$

Clearly for each state $\underline{x}$ in the set

$$B = \{\underline{x} : \underline{w} \leq \underline{x} \leq \underline{M}^*\} \tag{23}$$

$N_t^*(\underline{x}) = N_t^*(\underline{M}^*)$ and every area node in $N_t^*(\underline{x})$ experiences loss of load.

Note that in this case, under the NLLS policy, the flow along an arc connecting any two nodes in $N_t^*(\underline{x}) - \{t\}$ is zero. Therefore w_k in Eq. (22) may be alternatively determined by

$$w_k = \begin{cases} \min\{x_k : c_{kx_k} \geq f_k\} & \text{if both nodes connected by arc } k \text{ belong to } N_s - s. \\ m_k^* & \text{otherwise} \end{cases} \qquad (24)$$

The remaining states in L_ℓ can be further decomposed into disjoint subsets $W_1, W_2, \ldots, W_n$ each of which has the form of the set in Eq. (10). A state $\underline{x}$ in $L_\ell - B$ is classified into the subset W_k if k is the first index for which $x_k < w_k$, i.e.,

$$W_k = \{\underline{x} : (w_1, \ldots, w_{k-1}, m_k^*, m_{k+1}^*, \ldots, m_n^*) \leq \underline{x}$$
$$\leq (M_1^*, \ldots, M_{k-1}^*, w_, -1, M_{k+1}^*, \ldots, M_n^*) \qquad (25)$$

The recursive scheme consists of replacing L_ℓ by W_k and repeating the decomposition process.

We next consider the evaluation of area LOLP under the LLS policy. The following Fact shows that the set of states in B defined by either Eqs. (20-21) or Eqs. (22-23) has the same area loss-of-load characteristics under the LLS policy.

Fact. Each state $\underline{x}$ in the set B, whether constructed as in Case 1 or as in Case 2, has the property that all the area nodes in $N_t^*(\underline{x}) = N_t^*(\underline{M}^*)$ suffer loss of load under the LLS policy.

Proof. For Case 2, the same as under the NLLS policy, each area node in $N_t(\underline{x})$ suffers loss of load. For Case 1, let us suppose that area node $j \in N_t^*(\underline{x})$ and $f_{\alpha(j)} = D_j$. We are going to show that the flow in the network can be rerouted so that area j will also experience loss of load. Indeed, we claim that some of the flow in $f_{\alpha(j)}$ can be rerouted from node j to the sink node t through other nodes in $N_t^*(\underline{x})$. In other words we claim that there is a flow augmenting path [6, p. 12] with respect to the current flow from node j to node t through other nodes in $N_t^*(\underline{x})$. If the claim is not true, then there must be a minimal cut consisting of arcs connecting nodes in $N_t^*(\underline{x}) = N_t^*(\underline{M}^*)$, which contradicts the definition of $N_t^*(\underline{M}^*)$. ¤

IV.3 Computatation of EUD and ITC

The expected unserved demand (EUD), which measures the shortfall of supply for the demand D, is defined by

$$\begin{aligned} \text{EUD} &= E\{D-V[\underline{f}(\underline{x})] \mid V[\underline{f}(\underline{x})] < D\}\; P\{\underline{x}:V[\underline{f}(\underline{x})] < D\} \\ &= D\cdot P\{\underline{x}:V[\underline{f}(\underline{x})] < D\} - E\{V[\underline{f}(\underline{x})] \mid V[\underline{f}(\underline{x})] < D\}\; P\{\underline{x}:V[\underline{f}(\underline{x})] < D\} \end{aligned} \tag{26}$$

Since each $\mathcal{B}$ obtained from the decomposition process is a subset of the set $\{\underline{x}:V[\underline{f}(\underline{x}) < D\}$, we can write

$$\text{EUD} = D \cdot \sum_{\mathcal{B}} P\{\mathcal{B}\} - \sum_{\mathcal{B}} E\{V[\underline{f}(\underline{x})] \mid \underline{x}\in\mathcal{B}\}\cdot P\{\mathcal{B}\}, \tag{27}$$

where the summation is over all the subsets $\mathcal{B}$ obtained in the decomposition process. Since the set $\mathcal{B}$ has the form of $\mathcal{S}$ in Eq. (10) the first term in Eq. (27) can be readily computed,

$$P\{\mathcal{B}\} = \prod_{i=1}^{n} \sum_{x_i=w_i}^{M_i^*} p_{ix_i}$$

Consider the term $E\{V[\underline{f}(\underline{x})] \mid \underline{x}\in\mathcal{B}\}$. For each state $\underline{x}\in\mathcal{B}$ constructed as in Case 1 the value of the maximal flow $V[\underline{f}(\underline{x})]$ is constant. Let $V[\underline{f}(\underline{x})] = Q$, so that

$$E\{V[\underline{f}(\underline{x})] \mid \underline{x}\in\mathcal{B}\} = Q \tag{28}$$

On the other hand, in Case 2, for each state $\underline{x}$ in $\mathcal{B}$, the minimal cut $C[\underline{f}(\underline{x})]$ has the same set of arcs. Hence

$$
\begin{aligned}
E\{V[\underline{f}(\underline{x})] \mid \underline{x} \in B\} &= E\{\sum_{i \in C} \underline{c}_i(\underline{x}) \mid \underline{w} \le \underline{x} \le \underline{M}^*\} \\
&= \sum_{i \in C} E\{\underline{c}_i(\underline{x}) \mid \underline{w} \le \underline{x} \le \underline{M}^*\} \\
&= \sum_{i \in C} \frac{\sum_{x_i = w_i}^{M_i} p_{ix_i}\, c_{ix_i}}{\sum_{x_i = w_i}^{M_i^*} p_{ix_i}} \qquad (29)
\end{aligned}
$$

The inadequate transfer capability between two nodes i and j is defined by

$$
\begin{aligned}
ITC_{ij} &= P\{\underline{x} : V[\underline{f}(\underline{x})] < D, \text{ node } i \in N_s^*(\underline{x}) \text{ and node } j \in N_t^*(\underline{x})\} \\
&= \sum_{B} P\{\text{node } i \in N_s^*(\underline{x}) \text{ and node } j \in N_t^*(\underline{x}) \mid \underline{x} \in B\}\, P\{B\} \qquad (30)
\end{aligned}
$$

Note that P{node $i \in N_s^*(\underline{x})$ and node $j \in N_t^*(\underline{x}) \mid \underline{x} \in B$} is either 0 or 1. The above expression can then be readily computed.

IV.4 Bounds on Reliability Indices

Each time a subset L_k is identified, its contribution to the system LOLP index is accumulated. Similarly each time a subset B is identified, its contribution to the area LOLP, EUD and ITC indices are accumulated. When the decomposition is completed, i.e., there is no set with unclassified states, we obtain the true values of these reliability indices. However if the decomposition is terminated earlier so that there are non-empty subsets U_k and W_k, the cumulative value of each reliability index is clearly a lower bound for that index. On the other hand if we add the probabilities of the remaining unclassified subsets U_k to the system LOLP we obtain an upper bound of the system LOLP. Similarly if we add the probabilities of the remaining unclassified subsets U_k and W_k to each of the other reliability indices we obtain an upper bound of that index. Thus at each stage of the recursive process a lower bound and an upper bound for each reliability index are available.

V. MONTE-CARLO PHASE

The number of states in each subset that is classified in the process of the state space decomposition decreases rapidly as the number of iterations increases. When the efficiency of the decomposition phase decreases below the level specified by the solution of the optimal mix problem (Section VI), we switch to Monte Carlo simulation. The details of the Monte Carlo phase of the proposed approach are described below.

Let r denote any reliability index defined in Sec. II.4 other than the EUD. At the termination of the decomposition phase, we have a lower bound r^m and an upper bound r^M for the true value r^* of the index r. The problem is to estimate the contribution to r^* of the remaining unclassified subsets $\mathcal{U}_k$ and $\mathcal{W}_k$ so as to estimate the true value of r. The Monte-Carlo phase of the algorithm consists of picking a random sample of N states $\{\underline{x}^1, \underline{x}^2, \ldots, \underline{x}^N\}$ from the unclassified subsets $\mathcal{U}_k$ and $\mathcal{W}_k$ and estimating the contribution to r^*.

Let us define

$$p = \frac{r^* - r^m}{r^M - r^m} \tag{31}$$

to be the fraction of states in the unclassified subsets that contribute to the index r. Then

$$r^* = r^m + p(r^M - r^m) \tag{32}$$

We may think of p as the probability that each state in the unclassified subsets makes a contribution to the index r. Thus we can define an indicator random variable $\underset{\sim}{I}$ for each state $\underline{x}$ in the unclassified subsets as to its contribution to r,

$$\underset{\sim}{I}(x) = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases} \tag{33}$$

Clearly $E\{\underset{\sim}{I}\} = p$ and $\mathrm{var}\{\underset{\sim}{I}\} = p(1-p)$.

Since the true value of p is not known a priori, the problem is to determine an estimate of p. We use the following unbiased estimator of p,

$$\underset{\sim}{\hat{p}} = \frac{1}{N} \sum_{i=1}^{N} \underset{\sim}{I}(\underline{x}^i) \tag{34}$$

The variance of this estimator is [7, pp. 228]

$$\mathrm{var}\{\underset{\sim}{\hat{p}}\} = \frac{p(1-p)}{N} \tag{35}$$

which has an upper bound of $(\frac{1}{4N})$. Based on the estimator $\underset{\sim}{\hat{p}}$ in Eq. (34) we obtain

$$\underset{\sim}{\hat{r}} = r^m + \underset{\sim}{\hat{p}}(r^M - r^m) \tag{36}$$

as an estimator of r. The variance of this estimator is

$$\mathrm{var}\{\underset{\sim}{\hat{r}}\} = (r^M - r^m)^2 \, \frac{p(1-p)}{N} \tag{37}$$

with an upper bound of

$$\mathrm{var}\{\underset{\sim}{\hat{r}}\} \leq \frac{(r^M - r^m)^2}{4N} \tag{38}$$

To estimate the value of the EUD index, let us define the conditional expectation Q of the value of the maximal flow given that the states belong to the family $\mathcal{R}$ of the remaining unclassified subsets $\mathcal{U}_k$ and $\mathcal{W}_k$.

$$Q = \frac{\sum_{\underline{x} \in \mathcal{R}} V(\underline{x}) P(\underline{x})}{P\{\underline{x} : \underline{x} \in \mathcal{R}\}} \tag{39}$$

We use the following estimator of Q from the sample.

$$\hat{Q} = \frac{1}{N} \sum_{i=1}^{N} V[\underline{f}(\underline{x}^i)] \tag{40}$$

Let EUD^m be the value of the EUD index at the termination of the decomposition phase. We use the following estimator for EUD,

$$\widehat{EUD} = EUD^m + (D-\hat{Q})\ P\{\underline{x}:\underline{x}\in R\} \tag{41}$$

VI. THE OPTIMAL MIX

There are two outstanding issues that must be dealt with in the composite decomposition-Monte Carlo method:

(i) When to terminate the decomposition phase

and

(ii) What size should the sample be in the Monte Carlo phase.

The resolution of these questions can be expressed in terms of the follow-int two parameters:

α = the threshold probability of an unclassified subset for decomposition, i.e., no further decomposition will be carried out for any unclassified set S (a U_k or W_k) whenever $P\{S\} < \alpha$

N = the number of states in the random sample of the Monte Carlo phase.

The selection of the parameters α and N is based on the solution of a simple optimization problem which is described next.

Let T_d and T_m denote the computational time spent in the decomposition phase and Monte Carlo phase, respectively, of the combined method. Let

$$L = r^M - r^m \tag{42}$$

denote the uncertainty interval of the reliability index r upon the termination of the decomposition phase. Intuitively, it is reasonable to expect that the smaller the threshold probability α is, the shorter the uncertainty interval L is and the longer the computation time T_d is. Moreover, the computation time T_m should be proportional to the number of states N in the random sample. Based on the experimental results such as those shown in Fig. 4 we make the following assumptions

$$\text{(i)} \quad L = a\alpha^b \tag{43}$$

$$\text{(ii)} \quad T_d = c\alpha^d \tag{44}$$

$$\text{(iii)} \quad T_m = hN \tag{45}$$

As indicated in Fig. 4, we assume that the parameters b and d are independent of the system demand D, whereas a and c are functions of D. While b and d are expected to be system dependent, our experience indicates that the variation between systems is rather slight.

The variance $\operatorname{var}\{\hat{\underset{\sim}{r}}\}$ of the estimator $\hat{\underset{\sim}{r}}$ is a measure of the uncertainty associated with the estimated value $\hat{r}$. Let σ^2 be the maximum tolerable uncertainty of the estimate, i.e., we want to impose the constraint

$$\operatorname{var}\{\hat{\underset{\sim}{r}}\} \leq \sigma^2 \tag{46}$$

The objective for the selection of the parameters α and N is to minimize the total computation time $(T_d + T_m)$. We may therefore formulate this as the constrained optimization problem:

$$\min_{\alpha,N} (T_d + T_m)$$

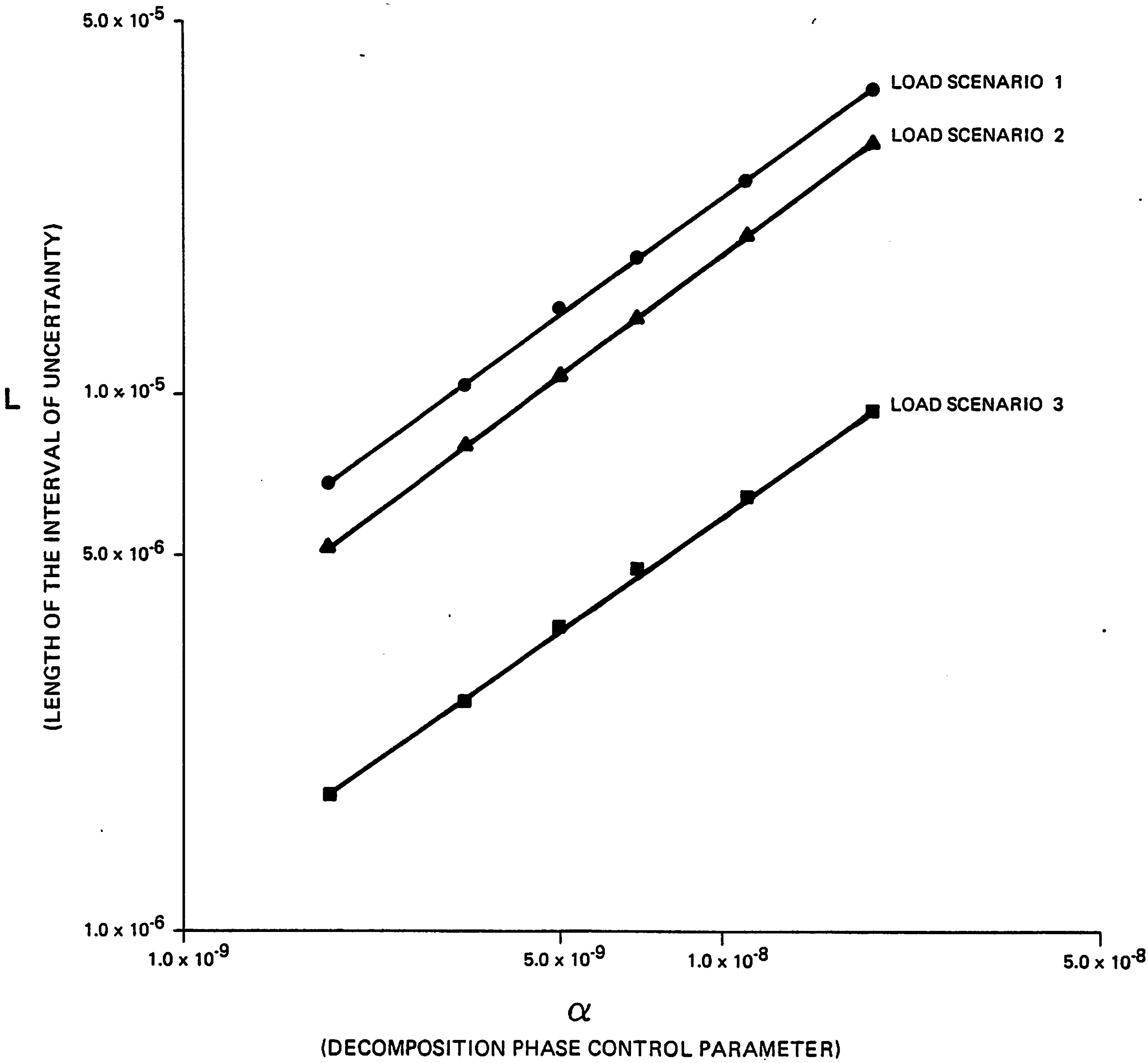

Fig. 4(a) Uncertainty after the decomposition phase as a function of the control parameter α.

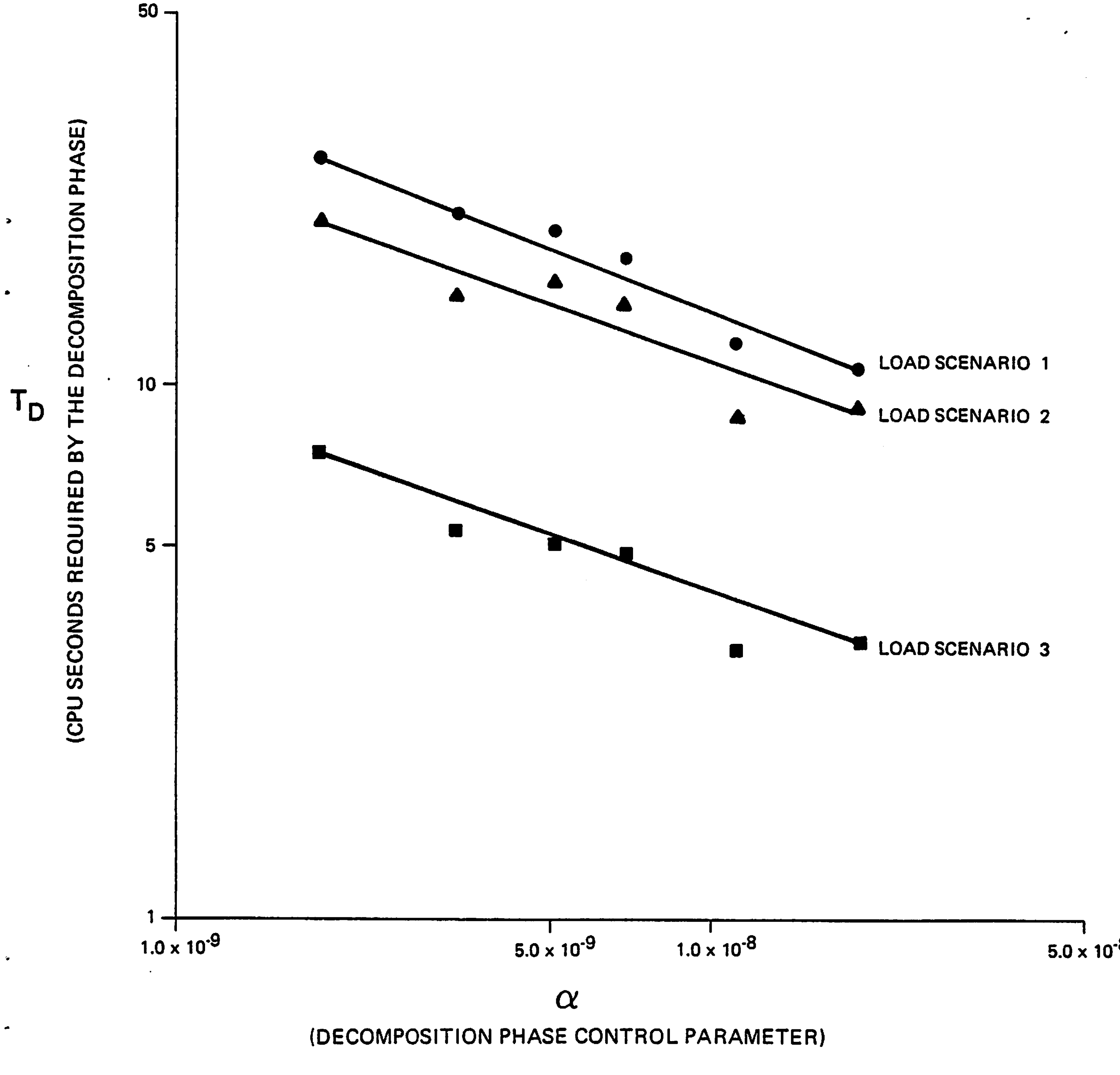

Fig. 4(b) Computation (CPU) time of the decomposition phase as a function of the control parameter α.

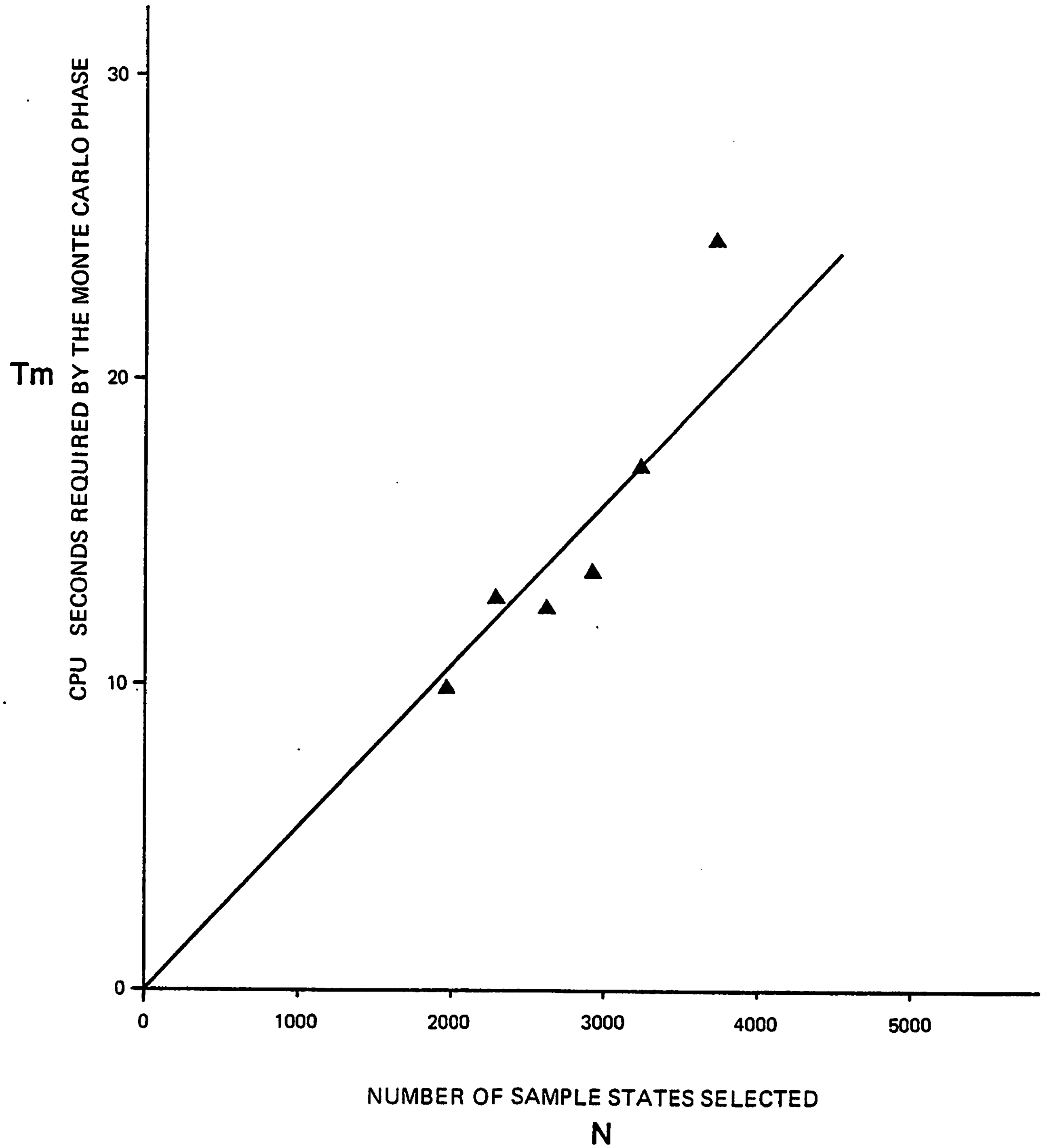

Fig. 4(c) Computation (CPU) time of the Monte Carlo phase as a function of the number of samples selected.

subject to

$$\frac{L^2}{4N} = \sigma^2$$

where L, T_d, T_m are related to α, N through Eqs. (43)-(45). The optimal solution of this problem is

$$\alpha = \left(- \frac{a^2 h\, b}{2\sigma^2 cd} \right)^{1/(d-2b)} \tag{47}$$

$$N = \frac{a^2}{4\sigma^2} \left(- \frac{a^2 h\, b}{2\sigma^2 cd} \right)^{2b/(d-2b)} \tag{48}$$

In the actual implementation of the method, whenever we find an unclassified set S such that $P\{S\} < \alpha$ we immediately proceed to the Monte Carlo phase rather than wait until the entire collection of unclassified subsets is obtained. Therefore, we need to know how many states γ should be selected from the unclassified subset S. We let the number of states selected from S be proportional to the probability of the subset S i.e.,

$$\gamma = \frac{N}{L} P\{S\} \tag{49}$$

The optimal value of γ can be obtained by substituting Eqs. (43), (47) and (48) into Eq. (49):

$$\gamma = P\{S\} \quad \frac{a}{4\sigma^2} \cdot \left(- \frac{a^2 h\, b}{2\sigma^2 cd} \right)^{b/(d-2b)} \tag{50}$$

VII. IMPLEMENTATION

We have implemented the decomposition-Monte Carlo approach into a production grade program REMAIN (Reliability Evaluation of Multi-Area INterconnection The software package includes an effective implementation of the Ford-Fulkerson maximal flow algorithm [6]. REMAIN is capable of handling systems without any restrictions on the network topology. The number of areas and the number of states in each area generation and each tie line capacity distribution are only limited by the burdens they impose on the computational effort. The results for the seven area system presented in the next section give a notation of typical computing times with this program.

REMAIN has the capability of studying multi-area reliability over extended periods. The study period is subdivided into a number of subperiods defined by the events determined by a discrete event simulator. The events simulated include new units coming on-line, old units retiring, the beginning and end of maintenance periods, and changes in the generation and intertie capacity random variables' distributions caused by seasonal factors or by policy decisions. In the subperiods defined by two successive events, the probability distributions of the generation capacity and tie line capacity random variables are fixed. These subperiods are further subdivided into shorter time units to account for changing load patterns in the areas and load diversity. For each of these time units, each area has a deterministic (fixed) load. The solution scheme evaluates the reliability indices for this set of fixed loads. The yearly values of the reliability indices are computed by weighted averaging the values of these indices for each time unit with the weights being the ratio of the duration of the time unit to the total time duration.

A flowchart of REMAIN is presented in Fig. 5. We set the initial values of $\alpha = \gamma = 10^{-5}$, $b = 0.4$ and $d = -0.6$. Our experience shows that only for a small number of peak load demand time units, α and γ need to be recalculated. In actual implementation a depth-first search scheme [8] is used so that the decomposition phase and Monte Carlo phase are completed along each branch of the search tree. For simplicity we assume that for each state the minimal cut is unique so that $N_s^*(\underline{x}) = N_s\{C[\underline{f}(\underline{x})]\}$, $N_t^*(\underline{x}) = N_t\{C[\underline{f}(\underline{x})]\}$, and the Ford-Fulkerson algorithm needs to be run only once.

VIII. APPLICATION EXAMPLE

An important application of REMAIN is the study of existing power pools, in particular for the planning of tie line capacity. To illustrate the application of REMAIN to the study of interconnection enhancement and the performance of REMAIN, the reliability of a seven area power system is evaluated as a function of intertie capacity.

The system configuration is found in Fig. 6. This system models an interconnection of four utilities which are represented by areas A, B, C, and D. The remaining three areas E, F, and G have no native load demand. Each of these areas represents generation resources that are jointly owned by two of the utilities. Table 1 gives the generation levels and associated probabilities for each of the seven areas. The interconnection data giving the tie line capacities and associated probabilities are presented in Table 2. In this example, for the sake of simplicity, all unidirectional ties are represented by deterministic variables and all bidirectional ties are represented by random variables. In Fig. 6 the numbers on the directed branches leaving an area give the

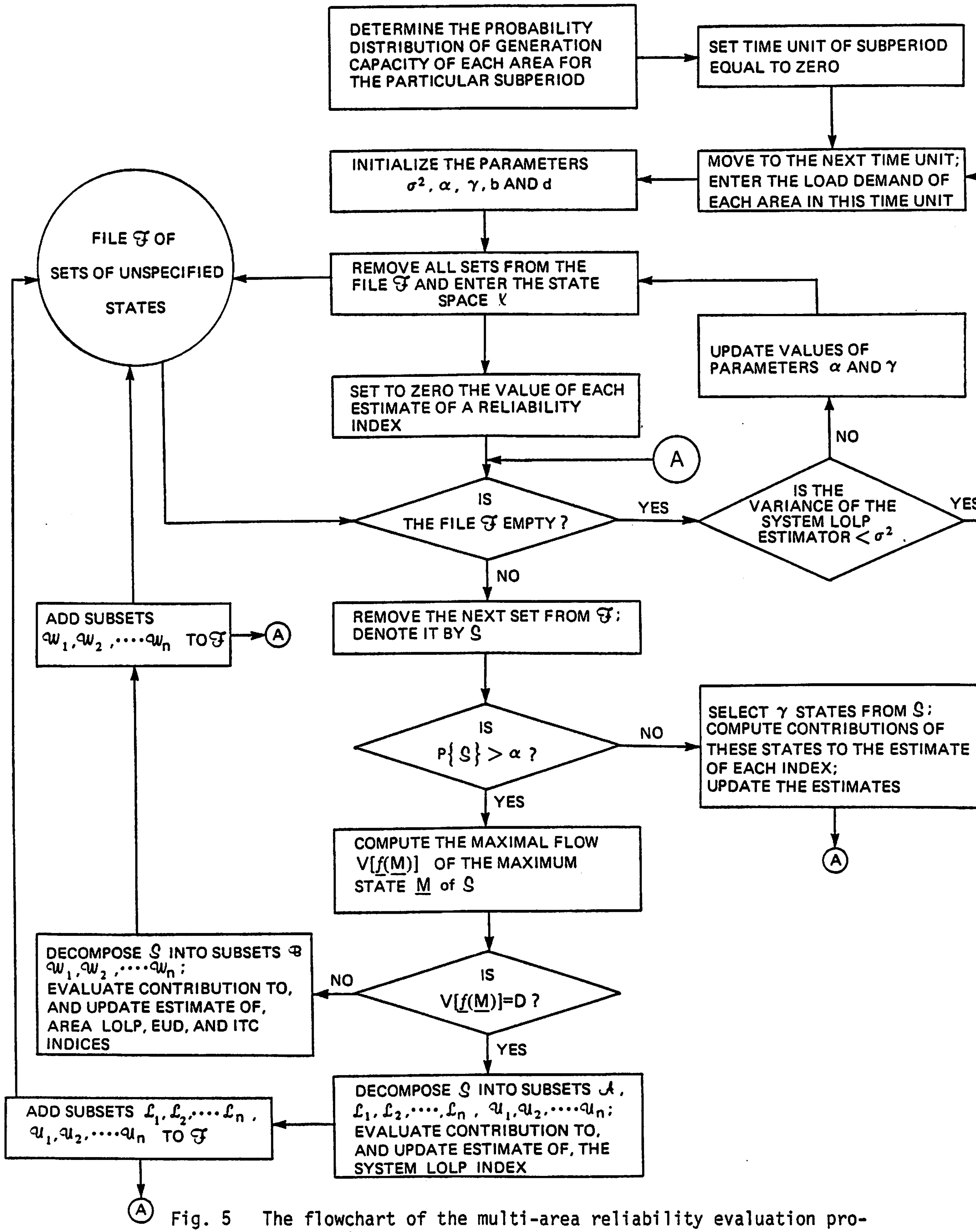

Fig. 5 The flowchart of the multi-area reliability evaluation program REMAIN.

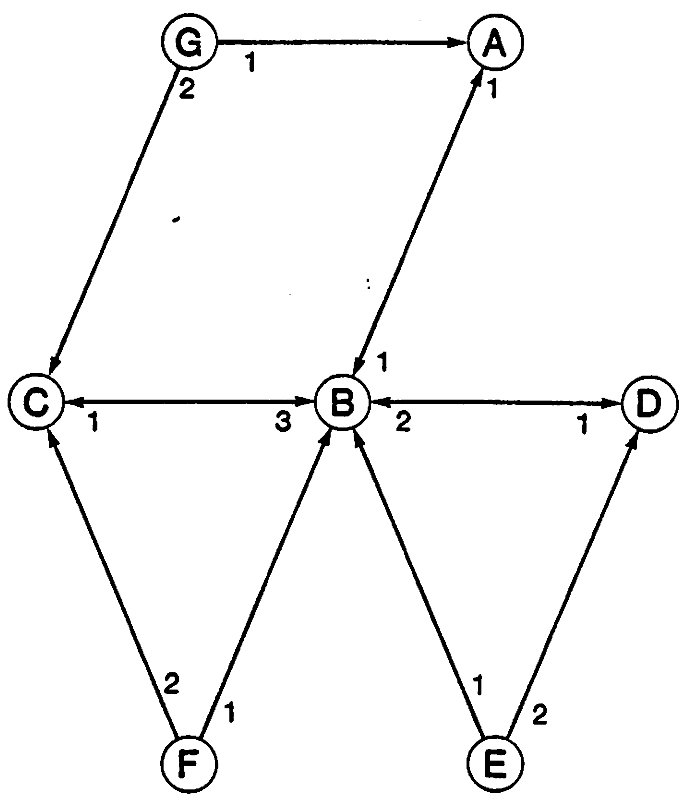

Fig. 6 Seven area power system configuration for the application example.

priority associated with assiting the neighboring areas to which the area is directly connected. The ties leaving each area are numbered in decreasing priority order; when an area has excess capacity, the area connected by the tie with the number 1 receives assistance first. The assistance priority lists are needed for evaluating the area LOLP values under the NLLS policy.

Table 3 gives the area loads for the eleven hours corresponding to the eleven largest monthly peaks of the total pool-wide load for the planning year considered in this example. Consider the hour in which the annual peak system load occurs. The generation data in Table 1 are for the period to which this hour belongs. The reliability indices for this hour are evaluated and presented in Tables 4 and 5. The dramatic improvement in reliability brought about by interconnecting the four utilities is seen by comparing the reliability indices for the isolation policy with those of the interconnected systems under the LLS and NLLS policies.

It is clear from the data in Table 4 that most of the unreliability of the interconnected system is due to the high LOLP of area D (two orders of magnitude more severe than that of any other area). The values of the ITC index indicate the high probability that the two tie lines into area D restrict power flow even though excess power is available from the other areas. We conclude that any attempt to improve the reliability of the pool must concentrate on relief for area D. Either additional native generation capacity must be added, or the tie line capacity into area D must be expanded by increasing existing ties or establishing new ties. We consider the expansion of the tie from area B to area D from 200 MW to 600 MW with the outage probability remaining unchanged. All other system parameters remain unchanged. This results in a considerable improvement of the reliability indices. For example, the first line in Table 6 gives the area LOLP values under the NLLS policy and the system LOLP and EUD values with the increased tie capacity. For comparison purposes, the last line in Table 6 gives these indices when the bidirectional ties between areas have unlimited and completely reliable capacity. All other system parameters remain unchanged. This case yields the maximum benefit that the system can derive from interconnection. Further improvements in system reliability can only come from increases in the generation capacity of the system.

The CPU time required on an IBM 3033 system for evaluating the reliability indices for the annual peak hour is given in Table 7. Less than 15% of this time was spent on determining maximum flows. For purposes of comparison, the times for exhaustive enumeration and pure decomposition are also presented. The computational times and accuracies of the other ten hours studied are given in Table 3.

In our extensive testing of REMAIN, we observed that the contributions of relatively few unreliable hours overshadow those of the remaining, generally reliable hours. This is borne out by the data presented in Table 3. Note that the system LOLP of hour 10 is five orders of magnitude smaller than that in the annual peak hour. Data for hour 12 are not presented since the system LOLP is effectively 0.

IX. CONCLUSION

We have presented a computationally efficient scheme for evaluating the reliability of multi-area power system interconnections. We have demonstrated the use of a newly defined reliability index in planning enhancements of the reliability of interconnections. Numerical results illustrating the performance of this new analysis on a system of practical interest were presented.

Acknowledgement

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TABLE 1

Probability densitites of area generation random variables

AREA A	
Generation capacity level in MW	Probability
21,000	0.100000
20,500	0.020000
20,000	0.300000
19,500	0.200000
19,000	0.140000
18,500	0.040000
18,000	0.010000
17,500	0.006000
17,000	0.003500
16,500	0.000450
16,000	0.000045
13,500	0.000005

AREA C	
Generation capacity level in mW	Probability
6,500	0.080000
6,200	0.200000
5,900	0.300000
5,600	0.230000
5,300	0.150000
5.000	0.025000
4.700	0.012500
4,400	0.002200
4,100	0.000250
3,900	0.000045
3,500	0.000005

AREA E	
Generation capacity level in MW	Probability
2,200	0.722500
2,000	0.170000
1,800	0.010000
1,100	0.085000
900	0.010000
0	0.002500

AREA B	
Generation capacity level in MW	Probability
15,500	0.080000
15,000	0.150000
14,500	0.270000
14,000	0.250000
13,500	0.160000
13,000	0.050000
12,500	0.035000
12,000	0.004500
11,500	0.000450
11,000	0.000045
10,000	0.000005

AREA D	
Generation capacity level in MW	Probability
3,300	0.280000
3,100	0.430000
2,900	0.250000
2,700	0.025000
2,500	0.012500
2,300	0.002200
2,100	0.000250
1,900	0.000045
1,700	0.000005

AREA F	
Generation capacity level in MW	Probability
2,500	0.600000
2,000	0.200000
1,500	0.150000
1,000	0.049500
0	0.000500

AREA G	
Generation capacity level in MW	Probability
3,900	1.000000

TABLE 2

Intertie Capacities

<table>
<tr><td colspan="6">Probability densities of bidirectional intertie capacity random variables</td></tr>
<tr><td colspan="2">Between Areas A and B</td><td colspan="2">Between Areas B and C</td><td colspan="2">Between Areas B and D</td></tr>
<tr><td>Intertie Capacity in MW</td><td>Probability</td><td>Intertie Capacity in MW</td><td>Probability</td><td>Intertie Capacity in MW</td><td>Probability</td></tr>
<tr><td>2,000</td><td>0.9801</td><td>1,000</td><td>0.9999</td><td>200</td><td>0.9999</td></tr>
<tr><td>1,000</td><td>0.0198</td><td>0</td><td>0.0001</td><td>0</td><td>0.001</td></tr>
<tr><td>0</td><td>0.0001</td><td></td><td></td><td></td><td></td></tr>
</table>

Unidirectional intertie capacities in MW					
From Area E to B	From Area E to D	From Area F to B	From Area F to C	From Area G to A	From Area G to C
1,750	450	1,500	900	2,500	1,400

TABLE 3

Area loads, system LOLP and computation times for 11 hours of the planning period

HOUR	Loads in MW for Area				System LOLP Evaluation		CPU time in seconds for evaluation of all indices
	A	B	C	D	System LOLP (10^{-6})	Standard Deviation (10^{-6})	
1	19,550	15,000	6,650	2,850	347	0.180	30.41
2	19,450	14,700	6,600	2,800	343	0.160	23.64
3	18,400	14,400	6,400	2,700	61	0.090	8.04
4	17,500	14,200	6,060	2,600	56	0.065	3.90
5	16,550	13,500	5,850	2,500	6	0.037	0.97
6	16,450	13,450	5,500	2,400	6	0.033	0.71
7	15,800	13,200	5,300	2,300	0.12	0.020	0.21
8	15,700	12,400	5,100	2,200	0.13	0.016	0.13
9	15,600	12,000	5,000	2,100	0.01	0.010	0.07
10	15,400	11,500	4,900	2,000	0.01	0.008	0.05
11	14,900	11,000	4,850	1.800	0.00	0.006	0.02

TABLE 4

Reliability indices for the annual peak system load hour

Inter-connection Policy	System		AREA LOLP			
	LOLP	EUD in MW	AREA A	AREA B	AREA C	AREA D
Isolation	1.000000	1846.991	0.400000	0.770000	1.000000	0.040000
LLS	0.000347	14.087	0.000009	0.000003	0.000003	0.000338
NLLS	0.000347	14.087	0.000008	0.000001	0.000001	0.000337

TABLE 5

The ITC index for the annual peak system load hour

From Area \ To Area	A	B	C	D	System
A	0.000000	0.000000	0.000001	0.000337	0.000337
B	0.000006	0.000000	0.000000	0.000337	0.000343
C	0.000006	0.000000	0.000000	0.000337	0.000343
D	0.000008	0.000003	0.000002	0.000000	0.000009
E	0.000007	0.000001	0.000001	0.000299	0.000307
F	0.000006	0.000000	0.000000	0.000337	0.000343
G	0.000006	0.000001	0.000001	0.000337	0.000344

TABLE 6

The impact of improved tie capacities on reliability

CASE	System		Area LOLP Under the NLLS Policy			
	LOLP	EUD (MW)	AREA A	AREA B	AREA C	AREA D
Increased tie capacity	0.000015	0.406	0.000007	0.000001	0.000001	0.000006
Unlimited tie capacity	0.000003	0.001	0.000002	0.000001	0.000000	0.000000

TABLE 7

Computational effort required for the annual peak system load hour

Approach	Number of Maximal Flows	CPU Time (sec)	Standard Deviation
Enumeration	4,704,480	4,704*	(Exact)
Decomposition	22,700	82	0.0000002
Proposed Method	17,480	30	0.0000002