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Anode Structures for Cold-Cathode High-Power Magnetrons\*

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Summary-- Rf interaction properties of anode structures suitable for cold-cathode high-power magnetrons have been investigated analytically and experimentally, with special emphasis on increasing the understanding of the interaction and maximizing the area of coherent interaction with the electron beam at a given frequency. The structures analyzed were designed for large mode separation, high interaction impedance, and easy coupling to the output circuit.

## INTRODUCTION

The present contribution describes a project carried out in 1962 as part of an investigation of cold-cathode, high-power, crossed-field devices that has occupied staff members and research assistants at this Laboratory over the past several years, notably under the direction of Prof. D. H. Sloan. Results of the investigation have been variously reported at scientific meetings in the U.S. and Japan and have achieved some circulation through technical reports under U.S. Air Force Contract AF19(628)-324, such as those authored by Hoag, <sup>1</sup> Hartman, <sup>2</sup> Chamran, <sup>3</sup> and (of particular relevance to the present contribution) Ikeda. <sup>4,5</sup>

Numerous interaction structures deemed suitable for the above-mentioned device type have been tested at this Laboratory.

Among the considerations that enter into evaluating their relative merits are ease of fabrication, adequate mode separation, and ease of coupling to the los applicative is to determine he within these often conflicting requirements) the region of cohe and interaction with the beam may be made as large as possible at a given

frequency to maximize output power. At the same time, it is hoped that these investigations may throw new light on the understanding of the nature of crossed-field interaction.

As examples of conflicting requirements, consider that, in a conventional magnetron, the number of resonators is limited by the requirement of reasonable frequency separation of the modes. Cold-cathode interaction structures often have a large number of resonators. As the number of resonators increases, the mode separation of even the "strapped" and "rising-sun" configurations proves to be too low. On the other hand, some circuits (such as a corrugated cylindrical wall with interaction bars and coupling channels<sup>2</sup>) may have as much as 60-percent mode separation--but rather low interaction impedance for the TM<sub>0</sub> mode, since the electric field is minimum at the wall.

The new structures analyzed in this report are free from this difficulty; however, they do suffer from the disadvantage of being susceptible to possible multipactor action along the magnetic field resulting from the rf electric field. This problem probably can be avoided by a careful design of the circuit.

The main assumption made in the present analysis is that the axial length of an individual cavity of the structure is much shorter than a free-space wavelength, so that the electric field inside the cavity is uniform and may be neglected. Moreover, it is assumed that since many bars oriented parallel to the axis are to be used in the structure, the capacity between them and the cylindrical wall may be taken to be uniformly distributed along the cavity circumference. In most cases the capacity between anode and cathode may be safely neglected.

MKS units are used throughout and only the symbols given other than common meaning are explained in the text.

## PROPOSED ANODE STRUCTURES

The types of anode structures to be discussed are illustrated in Figs. 1, 2, and 3. The points at which the plate electrodes and bars are connected have been indicated by dots in the figures. The slowwave structure of Fig. 1 is on the inside and is surrounded by a concentric cold cathode that depends on secondary emission for its electron current. The configuration shown in Fig. 2 is similar to that of Fig. 1, except that it is a conventional magnetron design, with the anode as the outer electrode. The TMo mode that is excited in each cavity makes the bars alternatiely plus and minus and thus provides the circumferential rf fields necessary for magnetron operation. There is a dc magnetic field in the direction of the z axis. The configuration shown in Fig. 3 is suitable for a linear magnetron amplifier. A direct current flowing along the central structure could in principle produce a dc magnetic field, so that electrons would move in the axial direction under the action of both dc electric and magnetic fields. (Even though a large value of current would be required for this purpose, the corresponding dc voltage would be extremely small, so that power dissipation would be quite reasonable.)

The gap between the plates of the anode structures in Fig. 1 to 3 is denoted by  $\delta$ , a parameter chosen so as to minimize the multipactor problem and to optimize the interaction impedance. In general  $\delta$  is small and as a consequence the capacitance for a

unit cavity is large, so that the frequency of the cavity is largely insensitive to the nature of electron-beam interaction or to the proximity of the cathode cylinder.

# RESONANT FREQUENCIES FOR INDIVIDUAL CAVITY

The structures proposed are particularly adaptable to multipole configuration up to several tens of poles (or bars). If a large number of bars are installed in a structure of Fig. 1 or 2, their effect on the cavity behavior may be approximated by an equivalent uniform capacitance  $\underline{C}_{\underline{C}}$  per unit length in the circumferential direction. The edge effect of the plates may also be included in this equivalent capacity.

Under the assumptions made in the previous section, Maxwell's equations are solved with the boundary condition at the open edges, where the capacitance  $\underline{C}_{\underline{C}}$  defines the ratio of the radial current to the rf voltage, giving the resonant frequencies with  $\underline{C}_{\underline{C}}$  as parameter.

The frequencies of interest are the lowest possible tone corresponding to  $TM_{001}$  mode and the next one corresponding to  $TM_{011}$  mode; their theoretical values as a function of the ratio  $\xi = R/\mu$  are shown in Figs. 4 and 5 for the two kinds of structures for unit a cell. In these figures the two modes are denoted by  $\underline{n} = 0$  and  $\underline{n} = 1$ ; the ratio is taken as the abscissa. The degree of mode separation is also plotted in Fig. 6, where the parameter  $\underline{C}_{\underline{c}}$  is assumed to be zero. No appreciable change in mode separation is found even if  $\underline{C}_{\underline{c}}$  has an appreciable value (say,  $\underline{C}_{\underline{c}}$   $\xi/\epsilon_0R = 0.2$ , where  $\epsilon_0$  is the permittivity of free space). In Fig. 5 both  $2\pi R/\lambda_0$  and  $2\pi r/\lambda_0$  are plotted for

the conventional magnetron structure because of convenience in taking  $\underline{R}$  or  $\underline{r}$  as a reference length.

If the frequency for the  $\underline{n}=0$  mode is assumed to be fixed, the coordinates of Figs. 4 and 5 directly yield the size of the electrode radius. The smaller the value of  $\xi$ , the larger the size and the worse the mode separation. If both the frequency and the size are fixed, Figs. 4 and 5 show the required value of  $\xi$  and Fig. 6 gives the difference of the mode separation between two types.

As an illustration, for a fixed frequency of 1000 Mc, the ratio R/r and the mode separation are given as a function of the interaction-space radius  $R_0$  in Fig. 7, assuming  $C_c = 0$ . We see that the mode separation is always better for the inverted magnetron structure. Figure 8 gives the ratio of R/r and the mode separation as a function of the geometric mean radius  $R_m = \sqrt{Rr}$ . We see that for a fixed effective size the conventional magnetron has a better mode separation.

The results described in Figs. 4 and 5 approximate the frequencies quite well even on the basis of a rough estimate of C. Many experimental results yield accuracies of the order of a few percent.

# EQUIVALENT-CIRCUIT REPRESENTATION

A. Equivalent Circuits for a Cylindrical Magnetron Structure with Bars

We can synthesize an equivalent circuit for the structure of Fig. 1 or 2 if we assume that neighboring bars constitute a parallel-wire

transmission line periodically loaded by the reactance of the cavities between the parallel plates. This equivalent circuit is shown in Fig. 9. The symbol  $\underline{L}_{\underline{eq}}$  denotes a reactance that changes from inductive to capacitive as the frequency is changed. <sup>5</sup> The symbol may be replaced for convenience by  $\underline{C}_{\underline{eq}}$ , defined by  $\omega^2 \underline{L}_{\underline{eq}} \underline{C}_{\underline{eq}} = 1$ . The symbol  $\underline{C}$  in Fig. 9 denotes the capacitance between bars in a length  $\Delta$  and should also contain the effect of the plate edges. The symbol  $\underline{L}$  represents the inductance of bars per  $\Delta$ , which can be usually estimated.

Figure 9 shows lumped continuous transmission lines made up of elementary four-terminal networks, from which we can estimate the behavior of the circuit. Such an analysis may be taken as an approximation for  $\underline{n} = 0$  mode. However, it is not as good an approximation for higher-order modes because of the variation of field along the circumference of the plates.

Under these considerations the phase diagram and the characteristic impedance as a function of frequency have been calculated for an inverted magnetron structure whose dimensions are illustrated in Fig. 10, where the results of this calculation are shown both for the n=0 (TM $_{001}$  and TM $_{002}$ ) and the  $\underline{n}=1$  (TM $_{011}$ ) modes. The frequency  $\underline{f}$  is one coordinate and the phase parameter per unit section  $\beta\Delta$  is the other. Also plotted along the same axis as  $\beta\Delta$  is the characteristic impedance  $\underline{Z}_{\underline{c}}$  divided by the bar number  $\underline{N}$  (here, 60).

Points  $\underline{A}_0$  and  $\underline{A}_1$  denote the conditions for no energy transmission along the bars (parallel resonance:  $\underline{v}_{\underline{p}} = \infty$ ,  $\underline{v}_{\underline{g}} = 0$ ); and  $\underline{C}$  and

equivalent circuit that is a good approximation to the strucutre is shown in Fig. 1lb. The quantity  $\omega_{\text{C}}$  is the lowest resonant frequency of the TE coaxial line mode. The effective capacitance  $C_1$  is determined from the maximum value of electric energy per unit volume stored in the space between the anode and the cathode for the  $\underline{n}=1$  mode. The circuit of Fig. 1lb has its cutoff at  $\omega_{\underline{C}}$ . The effective loading inductance  $\underline{L}_{eql}$  can also be determined from energy considerations.

(3) Numerical Calculation and Experiment. The phase characteristics and the characteristic admittances for the  $\underline{n}=0$  and  $\underline{n}=1$  modes for a model representing the structure of Fig. 3 are shown in Fig. 12. Experiments show that all the resonances of this structure can be predicted from the phase characteristic diagram with an error smaller than a few percent. 5

### ADDITIONAL CONSIDERATIONS

# A. Cavity Losses and Q

Distribution of loss along the radial plates of an individual cavity and the approximate Q of the structure are important, since the gap length of a cavity might have to be less than a few millimeters for a reasonable frequency range owing to multipactor considerations; concentration of loss at a particular location must be avoided for high-power applications.

The loss-density distributions on the plate surfaces have been calculated<sup>5</sup> for certain models from the electromagnetic field in the cavity (Fig. 13). The loss density for an inverted magnetron

which could lead to difficulties in cooling a high-power magnetron structure. By way of comparison, the losses for conventional magnetron structures are relatively spread out over the plates; however, one can visualize circumstances under which heat concentrated in the central structure might be more easily dealt with than distributed heat.

Values of  $\underline{Q}$  of the order of 1000 may be expected for a gaplength of 1.5 mm and a radial size of several centimeters, and are roughly proportional to the gap length and inversely proportional to the root of the radial size for a given  $\underline{R}/\underline{r}$  ratio. The total circuit impedance as seen from the open end of the cavity is of the order of a few thousand ohms for the same structures and is approximately proportional to the square of the gap length and inversely proportional to the 3/2 power of the radial size.

## B. Improvement of Mode Separation

Mode separation can be improved by changing the gap length as a function or radius, as shown in Fig. 14; the frequency dependence changes from mode to mode and is different from the uniform-gap case.

The exact field solution for this case is rather involved because of the additional boundary conditions at  $\underline{r}_1$ , where the gap length changes. With the assumption that current and potential difference are continuous at  $\underline{r}_1$ , calculations show that for  $\eta$  > 1 (Fig. 14) the mode separation may be strongly increased at the expense of frequency. Conversely, a structure with  $\eta$  < 1 may yield a higher frequency for a given size.

#### C. Other Considerations

An important design consideration is the prevention of multipactor action. Since the rf voltage between the two radial plates constituting an individual cavity changes along the radial path from the maximum at the open rim to zero at the closed rim, one procedure might be to choose the value of  $\delta/\lambda_0$  so large that the maximum amplitude of the rf voltage may be smaller than the multipactor voltage for an appropriate high-order mode. Another (and more effective) procedure is to choose  $\delta/\lambda_0$  small enough to allow the multipactor relation only under the condition of trivial rf voltage, for which no current amplification occurs.

The multipactor phenomenon might well limit the design of the structures seriously, particularly at the higher frequencies and/or higher voltages. Structures such as that of Fig. 14 could be expected to be easily effective in that regard. The gap length corresponding to the closed-edge side of a cavity may be selected as large as necessary, and that corresponding to the open-edge side may be chosen as small as necessary, or vice versa.

A slight strucutral modification may have additional advantages. Suppose that every bar is curved slightly in the direction of electron-beam rotation (Fig. 15). The component of the rf electric field parallel to the dc magnetic field might then be expected to provide a confining effect on the electron bunches, thus improving the bunching action.

Finally, in any complete cold-cathode-magnetron design, an effective way must be developed of damping out the extraneous modes

that may be expected to occur in the interaction space because of the cold-cathode configuration.

#### CONCLUSIONS

Resonant frequencies and mode separation have been determined for an inverted magnetron structure and for a conventional one. These structures were chosen because of the desirability of habing appropriate circuits for high-power magnetrons from the viewpoint of having the largest possible interaction volume. The proposed structures can be larger than conventional structures such as the "strapped" or the "rising sun" for a fixed frequency (Table I). The structures proposed are easier to fabricate than conventional circuits. Moreover, they may be of interest in connection with a traveling wave amplifier, in which the wave travels along the axis of symmetry.

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TABLE I

Type of Anode Structure		Wavelength (cm)	Mode Separation %	Anode-Surface Radius at Interaction Space (cm)	Anode-Surface Radius at Other Side (cm)	Remarks
HP10V (double ring strap)		10.7	5	1.5	2. 86	From
BM50 (single ring strap)		3.2	27. 5	0.128	0.58	Collins 6
New anode structure:	Inverted .	10.7	5	4.1	2.41	$\frac{\underline{C}_{c}}{=} \frac{\delta}{0} \frac{\underline{R}}{2}$ $= 0.2$
		3.2	27.5	0.66	0.194	
		3.2	5	1. 14	0.67	
	cs Conventional type	10.7	5	3. 4	5.61	$\frac{C_{\underline{c}}\delta/o^{\underline{r}}}{=0.2}$
		3.2	27.5	0.35	1.26	
		3.2	5	1. 1	1. 82	

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(FIGURE CAPTIONS--Ikeda and Susskind)

Fig. 1--Inverted magnetron structure.

Fig. 2 -- Conventional structure for sylindrical magnetron.

Fig. 3 -- Structure for linear magnetron.

Fig. 4--Resonant frequencies for inverted magnetron structure.

Values of  $\underline{C}_c \delta / \epsilon_0 \underline{R}$  denoted by x for 0, · for 0.1, o for 0.2.

Fig. 5--Resonant frequencies for conventional magnetron structure.

Values of  $\underline{\underline{C}}_{\underline{c}} \delta/\epsilon_{0}\underline{\underline{r}}$  denoted by x for 0, o for 0.2;  $\underline{\underline{C}}_{\underline{c}} \delta/\epsilon_{0}\underline{\underline{R}} = 0.2$  denoted by  $\nabla$ .

Fig. 6--Degree of mode separation.

Fig. 7--Mode separation (solid line) and R/r ratio (dashed line) at 1 Gc as a function of  $R_0$ .

Fig. 8--Mode separation (solid line) and R/r ratio (dashed line) at 1 Gc as a function of mean radius  $(Rr)^{1/2}$ .

Fig. 9--Equivalent-circuit representation for various resonant conditions: (a) parallel resonance, (b) series resonance, (c) combined resonance.

Fig. 10--Phase characteristics and characteristic impedance for structure of Fig. 1.

Fig. 11--Equivalent circuits: (a) for  $\underline{n}=0$  mode, (b) for  $\underline{n}=1$  mode. Fig. 12--Phase characteristics and characteristic impedance for structure of Fig. 3.  $\underline{R/r}=4$ ,  $\underline{r}=1.5$  cm,  $\underline{a}=1.2$ ,  $\delta=0.1$  cm,  $\Delta=0.6$  cm.

Fig. 13--Loss-density distribution over plate surfaces: (a) for inverted structures. (b) for conventional structures. (p is the radial coordinate.)

Fig. 14--Structure used for mode-separation improvement.

Fig. 15--Developed view of modified structure.