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A STABILITY CRITERION FOR TUNNEL DIODES

by

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In the design of tunnel diode amplifiers and oscillators a vital and, as yet, unsolved problem is the following: given the complete linear equivalent circuit of the tunnel diode as in Fig. 1 find the necessary and sufficient conditions on the range of values for the normalized series resistance, r, and the normalized series inductance, ℓ, such that the tunnel diode is potentially stable, i. e., all the open circuit natural frequencies can be shifted to the left half of the complex frequency plane, including the imaginary axis, by properly imbedding the tunnel diode in a two terminal passive network. The most general of the previously known stability conditions are summarized in Fig. 2. Since we know that for potential stability we must have $r < 1$, only this range or r is included in the figure. Curves 1 and 2 represent upper bounds on ℓ as function of r for which stabilizing networks can be constructed by known methods.[1, 2] A necessary condition for potential stability is that ℓ be less than the value of the ordinate of curve 3.[3] Thus the entire range of parameters in the hatched area of Fig. 2 represent tunnel diodes for which we do not know whether stabilizing networks exist. A new sufficient condition for potential stability is now derived and is given by the upper bound on ℓ represented by curve 4 in Fig. 2. Thus the area hatched with lines of positive slope gives the range of parameters representing tunnel diodes whose potential stability is newly proved in this note. The area hatched with lines of negative slope represents tunnel diodes whose stability is still ambiguous.

The method of obtaining the new sufficient condition will be to present a new passive imbedding network. This network is to be derived by a uniform distortion technique from an imbedding network used by Smilen[4] for stabilizing tunnel diodes with $r = 0$, as in N_1 in Fig. 3. The squared magnitude of the reflection coefficient $|S_{11}(j\omega)|^2$ in N_1 is given by the Butterworth derived characteristic:

$$|S_{11}(j\omega)|^2 = \frac{1 + (\omega/\omega_c)^{2n}}{K^{2n} + (\omega/\omega_c)^{2n}}, \tag{1}$$

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where K and ω_c are positive real constants such that $K \geq 1$ and n is a positive real integer. The driving point impedance Z_1 can be found from $|S_{11}(j\omega)|^2$ and realized in the first Cauer form as in Fig. 3. The values of C_1 and L_1 are then given in (2) and (3) respectively.

$$C_1 = \frac{2 \sin \pi/2n}{\omega_c R_1 (K^{1/2n} - 1)} \tag{2}$$

$$L_1 = \frac{R_1^2 C_1 (3 - 4\sin^2 \pi/2n)}{1 + 2\omega_c R_1 C_1 \sin \pi/2n + \omega_c^2 R_1^2 C_1^2} . \tag{3}$$

For $n \geq 2$, the stabilizing network in N_1 in Fig. 3 can be constructed for

$$3 \geq L_1/R_1^2 C_1 \tag{4}$$

and any values of C_1 and R_1; where L_1, C_1 and R_1 are interpreted as the parameters of the tunnel diode with $R_s = r = 0$.

Consider next the network N_2 in Fig. 4 which is obtained from N_1 by simply placing in series with each inductor L_i a resistance of value bL_i, and in parallel with each capacitor C_i a conductance of value bC_i, where b is a real positive constant. All poles and zeros in N_2 are then obtained by shifting the poles and zeros in N_1 left by an amount b. Replacing b by d/R_1C_1, we have the network in Fig. 5. The range of validity is therefore given by $0 \leq d < 1$. If $d \geq 1$ we no longer have a negative resistor in the network, and if $d < 0$ we have essentially introduced extraneous negative resistances into the network. Interpreting L_1, $L_1 d/R_1C_1$, C_1 and $-R_1/1-d$ as the tunnel diode parameters and normalizing the capitance and negative resistance to 1 and -1 respectively we have:

$$\ell = \frac{1}{R^2 C} = \frac{L_1}{R_1^2 C_1}(1-d)^2 \tag{5}$$

$$r = \frac{R_s}{R} = \frac{L_1}{R_1^2 C_1} d(1-d) \tag{6}$$

where R, C, L, R_s, ℓ and r are as defined in Fig. 1. A new sufficient condition for potential stability is therefore given by the inequalities:

$$\ell \leq 3(1-d)^2 \qquad (7)$$

$$r \leq 3d(1-d) \qquad (8)$$

$$0 \leq d < 1 \qquad (9)$$

where, from (4), 3 is the maximum value of $L_1/R_1^2C_1$.

Finally using (7) and (8) with equality and eliminating d we have an upper bound on ℓ as a function of r such that if ℓ is less than this bound a stabilizing network can be found. The equation for the bound is

$$\ell = \frac{3 - 2r + \sqrt{(3-4r)3}}{2} \qquad (10)$$

and is plotted as curve 4 in Fig. 2.

The following steps can then be carried out to synthesis a stabilizing imbedding network, for a tunnel diode, given L, R_s, C and -R such that (10) is satisfied.

Step 1: Calculate d from the formula

$$d = \frac{1}{(\ell/r)+1} = \frac{1}{\frac{L}{R R_s C}+1} \qquad (11)$$

which is obtained by dividing (5) by (6) and solving for d.

Step 2: Calculate R_1 from (5) or (6) knowing that $C = C_1$, $L = L_1$.

Step 3: Select an integer n.

Step 4: Using (3) solve for ω_c.

Step 5: Using (2) solve for K.

Step 6: If step 4 results in a negative value of ω_c or step 5 results in a value of $K < 1$ return to step 3 and increase the value of n.

Step 7: Synthesize $|S_{11}(j\omega)|^2$ in (1) in the form shown in Fig. 3.

Step 8: Uniformly distort the network resulting from step 7 by adding a resistance of value bL_i in series with each inductor, L_i; and a conductance of value bC_i in parallel with each capacitance C_i.

It is obvious that the new sufficient condition for potential stability represented by curve 4 is not a necessary condition since the value of the series inductance, L, can always be increased by a finite amount before any open circuit natural frequencies are shifted into the right half plane. This is so because the natural frequencies have been shifted left by amount b to begin with. Further work must therefore be done to find even more general sufficient conditions for potential stability.

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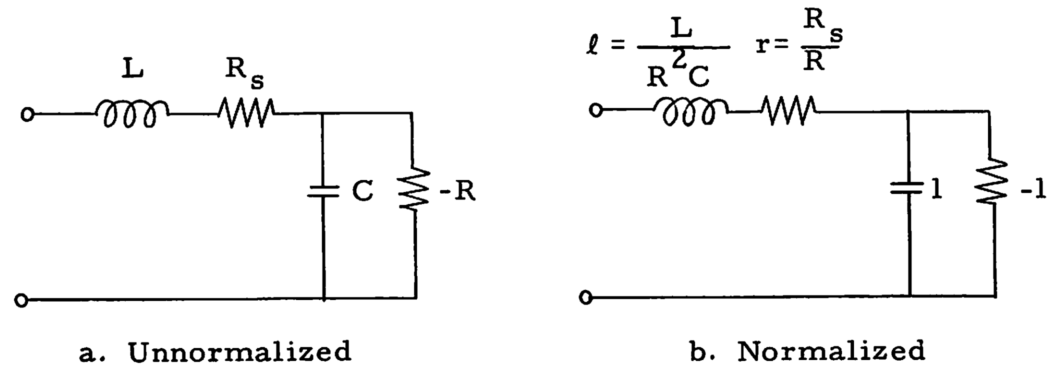

Fig. 1. Tunnel diode equivalent circuit.

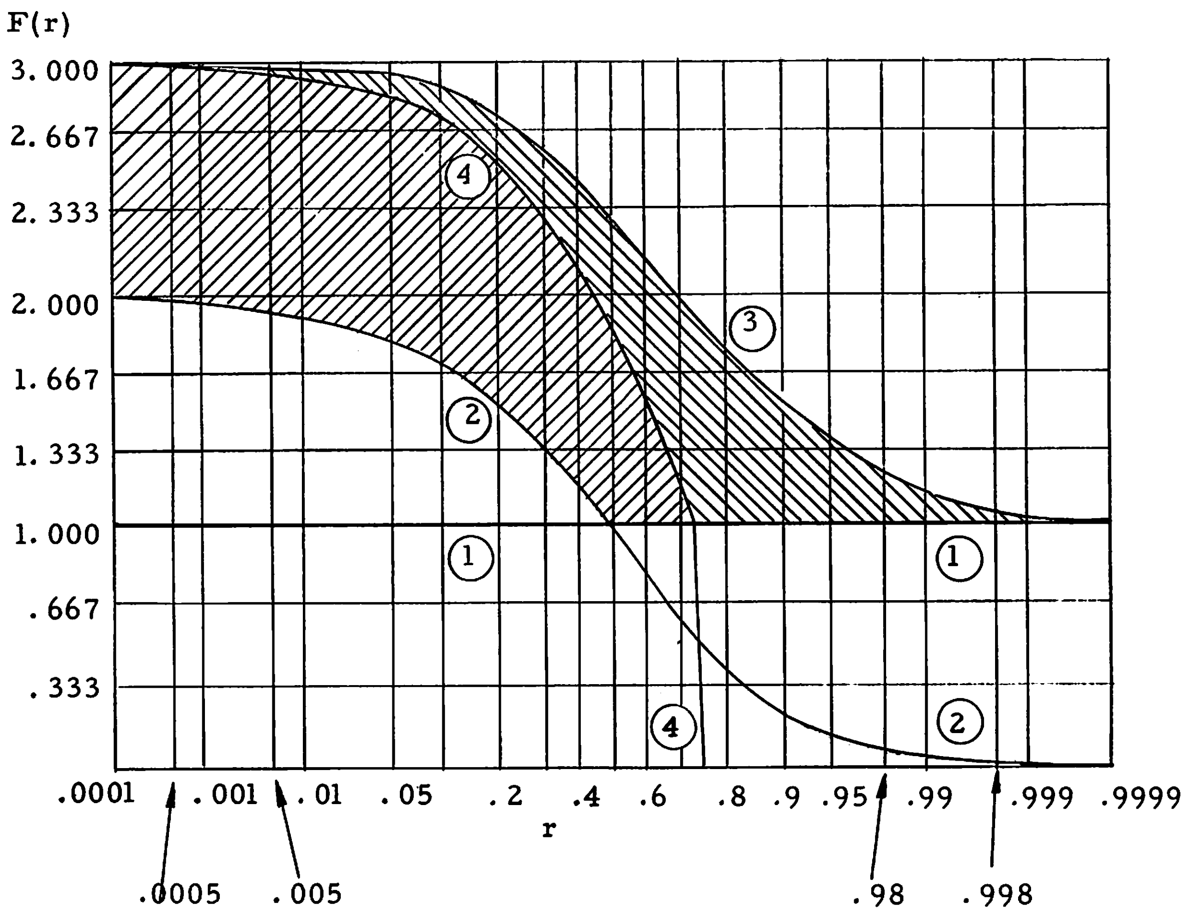

Fig. 2. Conditions for potential stability.

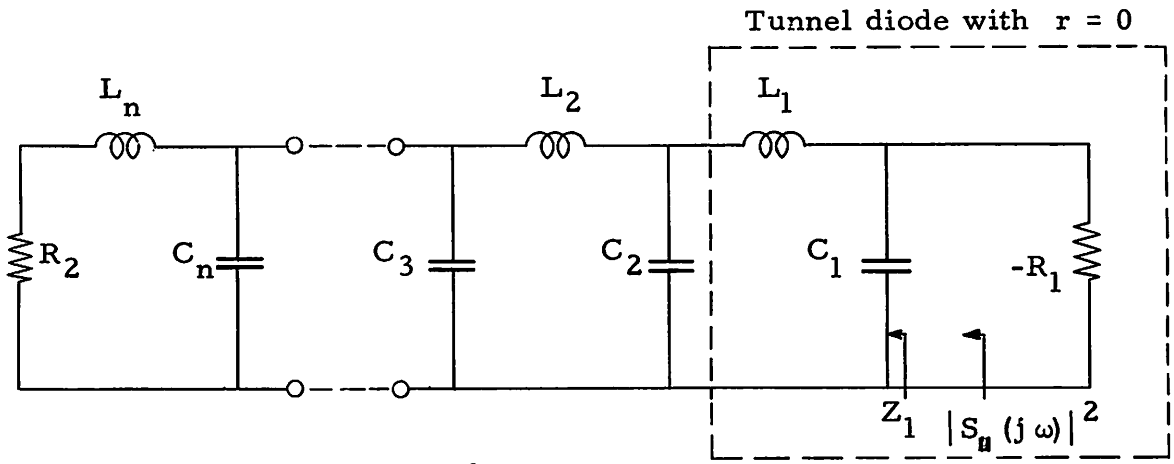

Fig. 3. N_1.

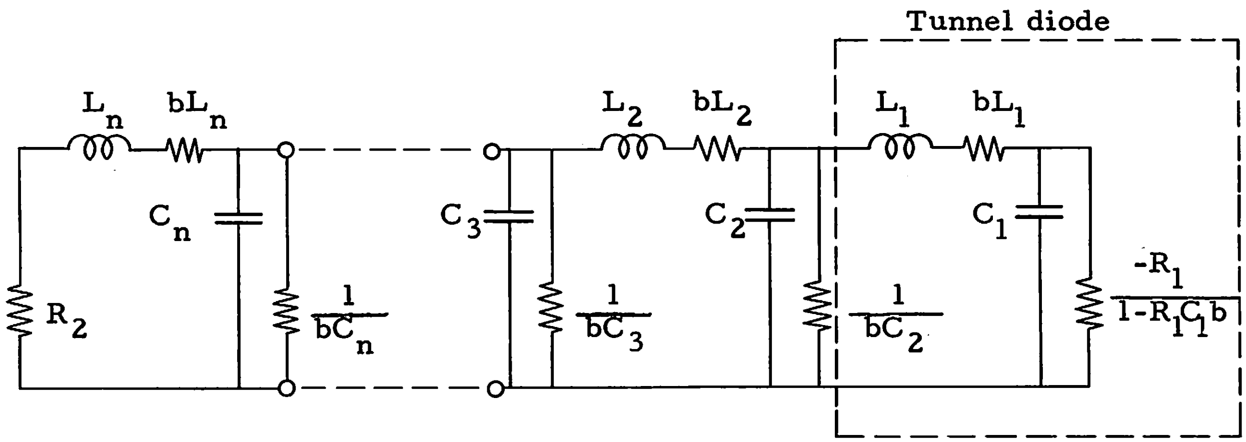

Fig. 4. N_2.

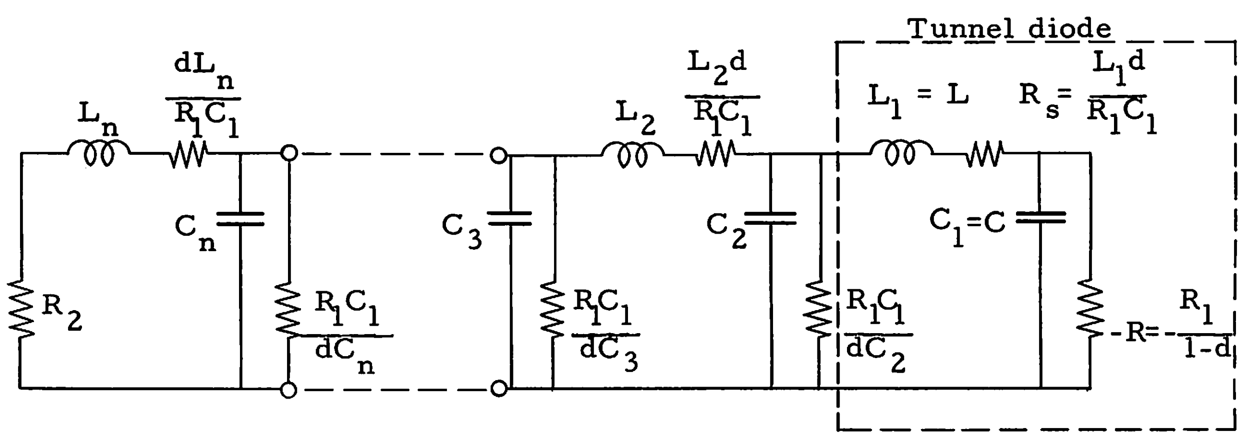

Fig. 5. N_3.