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## SPICE

by
L. W. Nagel and D. O. Pederson

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Simulation Program with Integrated Circuit Emphasis (SPICE)

L. W. Nagel and D. O. Pederson<br>Department of Electrical Engineering and Computer Sciences<br>and the Electronics Research Laboratory University of California, Berkeley, California 94720

## Abstract

A new circuit simulation program, SPICE, is described. The simulation capabilities of nonlinear dc anelysis, small signal analysis, and nonlinear transient analysis are combined in a nodal analysis program to yield a reasonably general purpose electronic circuit simulation program. Particular emphasis is piaced upon the circuit models for the BJT and the FET which are inplemented in SPICE.

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## I. Introduction

The use of a digital computer to simulate the electrical characteristics

- of clectronic circuits has been an important part of circuit design and .evaluation since the advent of integrated circuits. The peculiarities i Of integrated circuit design, such as the need for dc coupling and minimal use of resistances, made hand analysis of even the earliest analog integrated circuits impractical. Increased size and complexity of both analog and digital integrated circuits have made computer simulation an even more important factor in, efficient circuit design. A circuit simulation program has an additional advantage at academic institutions, - In that it provides students with a "dry lab" capability. In essence, each student is supplied with his own "workbeach" where he can design, build, and test circuits in a fraction of the time and expense that a real laboratory would require. This allows for a more relevant and educational set of assigrments than normally would be possible in the time constraints of an academic term.

Because of the several advantages of computer simulation, we have been heavily involved in the development and use of aimulation programs for several years. We found available programs to be either too cumbersome or too inefficient for classroom use, and it became necessary to develop our own set of simulation prograns. The first program developed at our laboratory was BIAS [1]. The need for a transient analysis capability lead to the development of CAVCER [2] and TIME [3]. A new version of TIME, entitled SINC, has beez developed by S. P. Fan at our laboratory. Our program SLIC (Simulator for Inear Integrated Circuits) $[4]$ was developed especially for the sicularion of analog integrated circuits. The latest program developed at our laboratory is SPICE (Simulation Program with Integrated Circiat Emphasis). This program is an improvement of the CANCER program and is used extersively for classroom instruction and graduate research for the large signal simulation of analog and digital integrated circuits.

## II. Description of SPICE

SPICE is a general purpose simulation program for integrated circuits. It contains the three basic analysis capabilities which provide the bulk of information of a circuit's performance: a) nonlinear dc analysis, with the provision for "stepping" an input source to obtain a set of static transfer curves, b) small-signal, sinusoidal steady-state analysis, iacluding a noise analysis [5] to evaluate noise performance, c) nonlinear, time-domain, transient analysis. The circuit size limitarions for SPICE are 400 nodes, and 200 total elements. !. Of which no more than 100 can be semiconductor devices. A user's guide for the SPICE program is included in the Appendix.

Built-in rodels are included for the most common semiconductor devices: diodes, bipolar junction transistors (BJT's), junction fieldeffect transistors (JFET's), and metal-oxide-semiconductor field-effect transistors (MOSFET's). The BJT models arebased on either the EbersYoll [10] or the Gumel-Poon [17] formulations; the models for the FET's are derived from the model of Shichman and Hodges [6].

Because SPICE is used extensively for undergraduate instruction, it was designed to be casy to learn and easy to use. The input language is free format to minimize user errors. inere possible, the program supplies "default" values for circuit parameters that are not specified,
so :hat the brgiuning user need specify only a few model parameters and program control parameters to effect a simulation. Simulation results are available either as tabular listings of the output variables or es line printer plots. The program contains 8000 Fortran IV statements, and requires 40,000 decimal words of core memory to execute on the cac 6400 available at the University of California, Berkeley.

The basic program organization is shown in Figure 1. The ci:c:it is described on a set of punched cards. The program first reads a.:c processes the input deck and checks for input errors. The next step in the sinile:inn is establishing the necessary set of pointers for the sparse matrix rc:iz: ines [7]. These pointers enable the two dimensional $Y$ matrix to be celiapsed into a one dimensional vector containing only the nonzero $Y$-matrix terms. The matrix routines then operate only on the nonzero terms contained in this vector. This saves a substantial emount of core memory and central processor execution time.

The next step in the simulation is the actual analysis. One set of routines is used for the iterative solution of the nodal equations $f c=d c$ analysis or for a given timepoint in the transient analysis [2], and another set of rourines is used for the solution of the complex nodal equations in the small signal analysis.

The final simulation step is the output phase, where the appropriate tabular listings and line printer plots are generated.

As mentioned earlier, SPICE is an improvement of the CANCER program, and many of the algorithms used in SPICE are the same as those used in CANCER and discussed in Ref. [2]. In particular, the sparse matrix routines in both CANCER and SPICE are improvements of those originally developed by Berry [7]. The basic Newton fteracion algorithm of SFICE and CANCER are the same, and both programs use an implicit, trapezoidal integration formula and a fixed, user supplied timestep for transient analysis. The small-signel routines are similar, except that a representation of flicker ( $\frac{1}{f}$ ) noise [8] has been added to the noise analysis of SPICE. The actual Fortran coding has been substantially improved in the transition from CANCER to SPICE.

## III. BJT Models

The most significant development in SPICE is the implementation of adequate device models for the BJT, JFET, and MOSFET. Because the BJT is so important in integrated circuits, the BJT model deserves special attention.

Two BJT models were necessary to accomodate the separate needs for a simple model to be used in classroom use and for a more sophisticated model for graduate research. Both models are represented by the electrical schematic shown in Figure 2. The charge storage elements $Q_{B E}$ and $Q_{B C}$ [9j] represent the stored base charge anc depletion layer charges. The BCasitic elements $r_{c},{ }^{5}{ }^{\prime}, r_{e}$, and $C_{c s}$ are assumed to be constant. The dc characteristics ©f the simpler model are derived from the familiar Ebers-Moll model [10] with an added representation of basewidth modulation [11]. The dc, intrinsic model is defined by the terminal equations:

[^0]\[

$$
\begin{aligned}
& I_{C}=I_{S}\left[\exp \left(\frac{q V_{B E}}{k T}\right)-\exp \left(\frac{q V_{B C}}{k T}\right)\right]\left[1-\frac{V_{B C}}{V_{A}}\right]-\frac{I_{S}}{E_{I}}\left[\exp \left(\frac{q V_{B C}}{k T}\right)-1\right] \\
& \vdots \\
& I_{B}=\frac{I_{S}}{B_{I}}\left[\exp \left(\frac{q V_{B E}}{k T}\right)-1\right]+\frac{I_{S}}{B_{I}}\left[\exp \left(-\frac{q V_{B C}}{k T}\right)-1\right]
\end{aligned}
$$
\]

where $q$ is the electronic charge, $k$ is Boltzman's constant, and is the absolute temperature. The remaining variables, $I_{S}, B_{f}, B_{F}$, and $V_{A}$ are user supplied model parameters. The saturation current, $I_{S}$, is the extrapolated intercept current of $108\left(I_{C}\right)$ versus $V_{B E}$ in the forward region and $\log \left(I_{E}\right)$ versus $V_{B C}$ in the reverse region as shown in Figure 3. The parameters $B_{f}$ and $B_{r}$ are the forward and reverse short circuit current gains, respectively, which are assumed not to vary with operationg point. The parameter $V_{A}$, referred to as the "Early voltage", produces a finite value of output conductance, $g_{0}$, due to basewidth modulation. The output conductance is given by the equation:
$g_{0} \equiv \frac{\partial I_{C}}{\partial V_{C E}}=\frac{I_{S}}{V_{A}}\left[\exp \left(\frac{q_{B E}}{k T}\right)-\exp \left(\frac{q V_{B C}}{k T}\right)\right]+\frac{q^{I_{S}}}{k T} \exp \left(\frac{q V_{B C}}{k T}\right)\left[1-\frac{V_{B C}}{V_{A}}\right]=\frac{I_{C}}{V_{A}}$
Hence, output conductance is proportional to $I_{C}$, with the constant of proportionality being $\left(\frac{1}{V}\right)$. A graphical intepretation of $V_{A}$ is shown in Figure 4.

The nonlinear charge storage elements $Q_{B E}$ and $Q_{B C}$ are determined by the equations:

$$
\begin{aligned}
& Q_{B E}=\tau_{F} I_{S}\left[\exp \left(\frac{q V_{B E}}{k T}\right)-1\right]+c_{j e o} \int_{0}^{V_{B E}}\left[1-\frac{V^{\prime}}{\phi_{e}}\right]^{-m} d V \\
& Q_{B C}=\tau_{R} I_{S}\left[\exp \left(\frac{q V_{B C}}{k T}\right)-1\right]+c_{j c o} \int_{0}^{\nabla_{B C}}\left[1-\frac{V}{\phi_{c}}\right]^{-\infty} d V
\end{aligned}
$$

Possibly a more meaningful method of expressing these charge storage element equations is the voltage dependent, small-signal capacitance formulae:

$$
\begin{gathered}
C_{B E}=\frac{\partial Q_{B E}}{\partial V_{B E}}=T_{F}\left(\frac{q I_{S}}{k T}\right) \exp \left(\frac{q V_{B E}}{k T}\right)+C_{j e o}\left[1-\frac{V_{B E}}{\phi_{E}}\right]^{-m^{e}} \\
C_{B C}=\frac{\partial Q_{B C}}{\partial V_{B C}}=T_{R}\left(\frac{q I_{S}}{k T}\right) \exp \left(\frac{q V_{B C}}{k T}\right)+C_{j C O}\left[1-\frac{V_{B C}}{\phi_{C}}\right]^{c}
\end{gathered}
$$

Charge storage is modeled by two base storage terms, which are characterized by the transit times $\tau_{f}$ and $T$, and two depletion terms which are characterized by the parametefs $C_{j e o}{ }^{\circ}{ }^{\circ}$, and $m$ for the einitter depletion region and $C_{j f o} \phi_{c}$, and mf for the colfector depletion region. For the simple Ebers ${ }^{2} 011$ fodel, the parameters $m_{e}$ and $m_{c}$ are fixed at a value of 0.5 . The numerical instability of the depletion layer capacitance formulae is avoided by using a linear approximation for depletion capacitance in forward bias.

The simpler Ebers-Moll transistor model lacks a representation of many: of the important second order effects present in actual devices; the =wo most important neglected effects are high level injection, which causes a drop in $B_{f}$ and $B_{f}$ and an increase in $\tau_{f}$ and $\tau_{r}$ when high level injecticn is reached $\left[12, \mathrm{I}_{3}, 14\right]$, and depletion ${ }^{\text {l }}$ layer fecombination, which causes a drop in $\tau_{f}$ and $\tau_{r}$ at low levels of collector current [15]. These effects are included in the more complicated BJT model that is implemented in SPICE. This model is an adaptation of the model proposed by Gumel an: Poon [16, 17, 18]. The equations for the dc characteristics are:

$$
\begin{aligned}
& I_{B}=\frac{I_{S}}{B_{F M}}\left[\exp \left(\frac{q V_{B E}}{k T}\right)-1\right]+C_{2} I_{S}\left[\exp \left(\frac{q V_{B E}}{n_{e} k T}\right)-1\right] . \\
& +\frac{I_{S}}{B_{R M}}\left[\exp \left(\frac{q_{B C}}{k T}\right)-1\right]+C_{4} I_{S}\left[\exp \left(\frac{q V_{B C}}{n_{C} k T}\right)-1\right]
\end{aligned}
$$

where the normalized base charge, $Q_{B}$, is defined by the equations:

$$
\begin{aligned}
& Q_{1}=1+\frac{\nabla_{B C}}{V_{A}}+\frac{V_{B E}}{\nabla_{B}} \\
& Q_{2}=\frac{I_{S}}{I_{k}}\left[\exp \left(-\frac{q \nabla_{B E}}{k T}\right)-1\right]+\frac{I_{S}}{I_{k I}}\left[\exp \left(\frac{Q V_{B C}}{k T}\right)-1\right] \\
& Q_{B}=\frac{1}{2}\left[Q_{1}+\sqrt{Q_{1}^{2}+4 Q_{2}}\right]
\end{aligned}
$$

The model parameters for the dc portion of the Gummel-Poon model are $I_{S}$, $B_{f m^{\prime}}, B_{r m}, C_{2}, C_{4}, n_{e},{ }^{n_{c}}, I_{k}, I_{k r}, V_{A}$, and $V_{B}$. The charge storage elements $Q_{B E}$ and $Q_{B C}$ are identical to the SPICE Ebers-Moll model.

The second order effect of depletion layer recombination is included in the model with the two nonideal base current components determined by the parameters $C_{2}$ and $n_{e}$ for the emitter depletion region and $C_{4}$ and $n_{c}$ for the collector region. This is shown in the graph of $I_{B}$ against $V_{B E}^{c}$ shown in Figure $/ 5$. Both the effect of basewidth modulation and a simple treatment of high level injection are introduced into the model via the base charge term, $Q_{B}$. This is best understood by considering two limiting cases. First, consider the case when $Q_{2} \approx 0$. Then, in the forward region,

$$
I_{C}=\frac{I_{S}}{1+\frac{V_{B C}}{\nabla_{A}}+\frac{V_{B E}}{V_{B}}} \quad \exp \left(\frac{q \nabla_{B E}}{k T}\right)=I_{S} \exp \left(\frac{q V_{B E}}{k T}\right)\left[1-\frac{V_{B C}}{V_{A}}\right] .
$$

Hence, the parameter has the same interpretation, for small values of $\nabla_{B_{B} C^{\prime}}$ as it does for the simpler SPICE model. A similar argument holds f ot the reverse Early voltage, $V_{B}$, in the reverse region. Now consider the case when $Q_{1} \stackrel{B^{\circ}}{\approx}$. Then, in the forward region, with low level injection,

$$
\frac{4^{I} s}{I_{k}} \exp \left(\frac{\mathrm{~g}_{\mathrm{BE}}}{\mathrm{kI}}\right) \ll 1
$$

the collector current follows the "ideal" law:

$$
I_{C} \simeq I_{S} \exp \left(\frac{q V_{B E}}{k T}\right)
$$

For high levels of injection,

$$
\frac{4^{I_{S}}}{I_{k}} \exp \left(\frac{q V_{B E}}{k T}\right) \gg 1
$$

the collector current becomes "nonideal".

$$
I_{C}=\sqrt{\frac{I_{k}}{I_{S}}} \exp \left(\frac{q V_{B E}}{2 k T}\right)
$$

Hence, the emission coefficient changes from 1 (ideal) to 2 as predicted by first ordcr theory [12]. A similar argument holds for the reverse region, where high level injection occurs when

$$
\frac{4 I_{S}}{I_{k r}} \exp \left(\frac{q V_{B C}}{k I}\right) \gg 1
$$

The parameters $I_{k}$ and $I_{k r}$ are termed the forward knee current and the reverse knee current, ant can be interpreted as the approximate value of collector current (for forward region) and enitter current (for reverse region) where high level injection becomes significant.

Perhaps more insight into the SPICE Gumel-Poon parameters is obtained by inspection of the asymptotic behavior of the short circuit current gain as shown in Figure 6. The current gain is essentially constant at a value of $B_{\text {FM }}$ for collector currents greater than $I$, falls off with a slope of $1-1 / n$ for collector currents 1 itss than $I_{I}$, and fails of with a slope of -1 for collector currents greater than $I_{k} ; I_{L}$, the low current breakpoint, is given by the equation:

$$
I_{L}=I_{S}\left[C_{2} B_{F M}\right]^{\frac{n_{e}}{n^{-1}}}
$$

This, of course, is only an asymptotic relatioc. For cases wheza ion level and high level effects overlap, as is true in most transistors, some trial and error is required to obtain the correct parameters.

Although not obvious from the defining equations, the effective transit times of the device, and hence the $f_{T}$ is also dependent upon collector current. The effective forward transit time is given by:

$$
\tau_{f}\left[e_{f f}\right]=\frac{Q_{B E}}{I_{C}} \approx \tau_{f} Q_{B}
$$

Hence, as high level injection is reached, $\tau_{f}$ is no longer constant but instead is directly proportional to $Q_{B}$ as shown in Figure 7.

The more complex Gumel-Poon model that is implemented in SPICE has been quite adequate for the graduate research in our laboratory [19]. However, we anticipate the necessity of adding current dependent base resistance [12] and base pushout [17] into the model.

Default and typical values for the Ebers Moll and Gumel-Poon models parameters are given in the Appendix.

## IV. Circuit examples

The shunt-series feedback amplifier shown in Figure 8 provides an illustrative example of the difference between the two BJT models in an actual circui: simulation. This circcit was simulated using the .i model parameters shown in Table 1. The graph of the forward current gain against $I_{C}$ is superimposed on the esymptotic curve shown in Figure 6. For the Ebers-Moll model, $B_{F} \because$ as chosen to match the amplifier gain in the Gumrel-Poon simulation. The first stage of the circuit operates at a bias current one decade above the lower knee current, $I_{1}=1 \mu A$ while the second stage is biased at 1 mA . The upper kinee current $I_{k}$ is 3 mA. Since the circuit is current-driven, the effects of a current-dependent beta are more pronounced than with a voltagedriven circuit.

The dc transfer curve of $V_{\text {out }}$ against $I_{\text {in }}$, for the open loop case ( $R_{F 1}$ and $R_{F 2}$ removed), is shownt in Figure $9 n^{\prime}$. The maximum de swing predicted by the two models differs by $48 \%$, while the small signal gain at zero input differs by about 12\%. In the transient analysis, a $0.02 \mu \mathrm{~A}$ (peak-to-peak) sine wave with a frequency of 100 kHz was applied to the input. The Ebers-Moll siculation predicted an output voltage of 2.00 volts (peak-to-peak), while the Gummel-Poon simulation predicted an output voltage of 1.78 volts (peak-to-peak).

When the simulation was repeated for the closed loop case $\overline{\left(R_{F}\right.}$ and $R_{R_{2}}$ ae now included; the loop gain is 2.65) the maximum de swing was of ${ }^{2}$ course the same; the small-signal gain differed by about $6 \%$ for the two models, and the transient output voltage differed by about 7\% for the two models.

The necessity for a more complex BJT nodel is obviously circuit dependent. The above example was purposefully chosen to accentuate the differences between the two models; in meny circuit designs the difference in the predicted waveforms is much less. The central processor time required for these two simulations was 4.6 sec for the Ebers-Moll model and 5.2 sec for the Gumel-Poon model. Of these times, 2.1 sec was required for reading, error checking, matrix setup, and output; the Gumel-Poon model hence requires $25 \%$ additional time on a per Newton iteration basis or $13 \%$ total additional job time.

Additional simulation execution times are shown in table 2 for a SN7400 TTL inverter and in Table 3 for the $\mu \mathrm{A} 741$ operational amplifier. Both of these examples used the Ebers-Moll transistor model. These times were observed on the CDC 6400 computer available at the University of California Computer Center at Berkeley.

## V. FET Models

The increasing use of the MOSFET device in digital integrated circuits, and to a lesser exterit the JFET device in analog integrated circuits, necessitated the inclusion of suitable models for these two devices. The circuit schematic for the JFET model used in SPICE is shown in Figure 10.

The two parasitic ohmic resistances, $r_{d}$ and $r_{s}$, are assumed to be constant. The equation for internal drain current, $I_{D}$, is taken as a simple squarelaw relation with an added parameter to model channel width modulation [6]. The plecewise relations for the various regions of operation of the JFET are:
I. $\quad V_{D S}>0$ (forward region)

$$
I_{D}=\left\{\begin{array}{l}
0 \\
B\left(V_{G S}-\nabla_{T O}\right)^{2}\left(I+\lambda V_{D S}\right) \\
B V_{D S}\left[2\left(V_{G S}-V_{T O}\right)-\nabla_{D S}\right]\left(1+\lambda V_{D S}\right)
\end{array}\right.
$$

$$
\begin{gathered}
\nabla_{G S}-v_{T O}<0 \\
0<V_{G S}-v_{T O}<v_{D S} \\
0<\nabla_{D S}<V_{G S}-v_{T O}
\end{gathered}
$$

II. $\quad V_{D S}<0$ (reverse region)

$$
I_{D}=\left\{\begin{array}{l}
0 \\
-\beta\left(V_{G D}-V_{T O}\right)^{2}\left(1-\lambda \nabla_{D S}\right) \\
B V_{D S}\left[2\left(V_{G D}-V_{T O}\right)+V_{D S}\right]\left(1-\lambda \nabla_{D S}\right)
\end{array}\right.
$$

$$
\begin{gathered}
v_{G D}-v_{T O}<0 \\
0< \\
0< \\
0-v_{G D}-v_{T O}<-v_{D S} \\
\end{gathered}
$$

The three dc parameters which determine the JFET operation are $V_{T O}$, $B$, and $\lambda$. The parameter $V_{T O}$, which is always negative for JFET's, is most commonly referred to as the "pinchoff" voltage.* The parameters $B$ and $V_{T O}$ are probably best understood by examining the graph of transconductance, $\mathrm{gm}_{\mathrm{m}}$, B a function of $\mathrm{V}_{\mathrm{GS}}$ (in the forward, saturated region) as shown in Figure 11. The transconductance is assumed to be a linear function of $V_{G S}$ with a slope of $2 B$. The parameter $\lambda$ is analogous to the parameter' $V_{A}$ for BJT's. The output conductance of the device, in the forward saturated region, is given by the equation:

$$
g_{d} s a t=B \lambda\left(V_{G S}-V_{T O}\right)^{2} \approx \lambda I_{D}
$$

Bence, the output conductance is assumed to vary linearly with drain current, with the constant of proportionality being $\lambda$. The graphic interpretation of the parameter is shown in Figure 12.

The two gate junctions are modeled as ideal diodes with the following defining equations:

$$
\begin{aligned}
& I_{G S}=I_{S}\left[\exp \left(\frac{q V_{G S}}{k T}\right)-1\right] \\
& I_{G D}=I_{S}\left[\exp \left(\frac{q V_{G D}}{k T}\right)-1\right]
\end{aligned}
$$

The charge atorage elements, $Q_{G S}$ and $Q_{G D}$, are modeled as ideal, step junction depletion capacitances. Because the gate functions are normally reverse biased, the diffusion charge storage mechanism is omitted. The charge storage elements are defined by the equations:

[^1]\[

$$
\begin{array}{ll}
Q_{G S}=C_{G S O} & \int_{0}^{\nabla_{G S}} \frac{d V}{\left(1-\frac{\nabla}{\phi_{B}}\right)^{1 / 2}} \\
Q_{G D}=C_{G D O} & \int_{0}^{\nabla_{G D}} \frac{d V}{\left(1-\frac{V}{\phi_{B}}\right)^{1 / 2}}
\end{array}
$$
\]

As for the depletion capacitances in the bipolar models, these charge storage elements can alao be expressed in terms of the voltage dependent, small signal capacitances:

$$
\begin{aligned}
& C_{G S}=C_{G S O}\left[1-\frac{v_{G S}}{\phi_{B}}\right]^{-1 / 2} \\
& C_{G D}=C_{G D O}\left[1-\frac{v_{G D}}{\phi_{B}}\right]^{-1 / 2} .
\end{aligned}
$$

The MOSFET model used in SPICE is very similar to the model proposed by Shichman and Hodges. [6] Although more sophisticated models for the MOSFET device have been proposed, $[23,24]$, we have found this model adequate for our investigations. It includes a representation of the effect of substrate bias on gate threshold voltage and a representation of the effect of channel width modulation. The circuit schematic for the SPICE MOSFET model is shown in Figure 13. The internal drain current generator, $I_{D}$, is given by the following piecewise equations for all regions of operation of the MOSFET device:
I. $\quad \nabla_{D S}>0$ (forward region)

$$
\nabla_{T E}=\nabla_{T O}+\gamma\left[\sqrt{\phi-V_{B S}}-\sqrt{\phi}\right]
$$

$$
I_{D}=\left\{\begin{array}{l}
0 \\
B\left(V_{G S}-V_{T E}\right)^{2}\left(1+\lambda \nabla_{D S}\right) \\
B V_{D S}\left[2\left(V_{G S}-V_{T E}\right)-V_{D S}\right]\left(1+\lambda V_{D S}\right)
\end{array}\right.
$$

$$
\begin{array}{r}
\nabla_{G S}-\nabla_{T E}<0 \\
0<\nabla_{G S}-\nabla_{T E}<\nabla_{D S} \\
0<\nabla_{D S}<\nabla_{G S}-V_{T E}
\end{array}
$$

II. $\quad \nabla_{D S}<0$ (reversed region)

$$
\nabla_{\mathrm{TE}}={v_{\mathrm{TO}}}+\gamma\left[\sqrt{\phi-\nabla_{\mathrm{BD}}}-\sqrt{\phi}\right]
$$

$$
I_{D}=\left\{\begin{array}{lc}
0 & 0<\nabla_{G D}-\nabla_{T E}<0 \\
-B\left(V_{G D}-\nabla_{T E}\right)^{2}\left(1-\lambda \nabla_{T E}\right) & 0<V_{D S} \\
B V_{D S}\left[2\left(V_{G D}-V_{T E}\right)+\nabla_{D S}\right]\left(1-\lambda \nabla_{D S}\right) & 0<-\nabla_{D S}<V_{G D}-V_{T E}
\end{array}\right.
$$

The five dc parameters which determine the operating characteristics of the MOSFET are: $V_{\text {YO }}, \beta, \lambda, \gamma$ and $\phi$. The interpretation of the parameters $V_{T O}, \beta$ and $\lambda$ is similar to the interpretation of these parameters for the JFET. In the case of the MOSFET, VTO is positive for an enhancement mode device (the usual case) and negative for a depletion mode device. The actual threshold voltage of the device depends on the substrate bias, as shown in the graph of $V_{T E}$ against $V_{B S}$ in Figure 14. The parameters $\gamma$ and $\phi$ determine the actual shape of this curve. The parameter $V_{T O}$ is the zero substrate blas value of threshold voltage.

The two substrate (bulk) Junctions are modeled by ideal diodes with the following defining equations:

$$
\begin{aligned}
& I_{B S}=I_{S}\left[\exp \left(\frac{q V_{B S}}{k T}\right)-1\right] \\
& I_{B D}=I_{S}\left[\exp \left(\frac{q V_{B D}}{k T}\right)-1\right]
\end{aligned}
$$

The charge storage effects in the KOSFET device are modeled by five capacitances. The capacitors $C_{G D}, C_{G S}$, and $C_{G B}$ are assumed to be constant. The charge storage elements $Q_{B D}$ and $Q_{B S}$ are treated as ideal, step junction depletion capacitances. Since the substrate jumctions are normally reverse biased, the diffusion charge storage mechanism is omitted. These two charge storage elements are defined by the equations:

$$
\begin{aligned}
& Q_{B D}=C_{B D O} \\
& \int_{0}^{\nabla_{B D}} \frac{d V}{\left[1-\frac{V}{\phi_{B}}\right]^{1 / 2}} \\
& Q_{B S}=C_{B S O} \\
& \int_{0}^{\nabla_{B S}} \frac{d V}{\left[1-\frac{V}{\phi_{B}}\right]^{1 / 2}}
\end{aligned}
$$

These elements can also be expressed in terms of the voltage dependent, small signal capacitances:

$$
\begin{aligned}
& C_{B D}=C_{B D O}\left[1-\frac{\nabla_{B D}}{\phi_{B}}\right]^{-1 / 2} \\
& c_{B S}=C_{B S O}\left[1-\frac{\nabla_{B S}}{\phi_{B}}\right]^{-1 / 2}
\end{aligned}
$$

We have found these FET models quite adequate for both instructional use and research. To attain reasonable convergence in the dc analysis it is necessary to limit the change in $V_{G S}$ and $V_{G D}$ from iteration to iteration, just as it is necessary to lidit junction voltage changes. [2? We have found an acceptable limit to be $0.1+\left|V_{\text {To }}\right|$ (volts). The gate junctions for JFET's and the substrate junctions for MOSFET's are modeled by an equivalent conductance

$$
g_{e q}=\frac{q I_{S}}{k T}
$$

In the reverse blas region (where they normally operate). With these precautions, we have found the convergence propariles of the FET models to be comparable to those of the BJT models.

Default and typical values for JFET and MOSFET model parameters are show in the Appendix.
VI. Conclusions

The implementation of suitable device models for the BJT and FET's, coupled with improved program coding and organization, has provided us with a simulation program which is much more powerful and efficient than its predecessor, CANCER. However, one can never assume that a simulation program is complete, and we forsee many erhancements for SPICE at the time of this writing.

The most desirable addition to the program is a reliable timestep control in the transient analysis. Our program SINC has a timestep control which is based on the iteration count at each timepoint; however, we have found tinis method not totally satisfactory in controling the stability properties of the trapezoidal method. Instead, it is necessary to estimate and control the truncation error at each timepoint to obtain a satisfactory transient solution. Because of the method in which an integration algorithm is implemented in a nodal analysis program [2, 20], the use of variable order integration methods [21, 22] constitute a tremendous increase in the program code. Furthermore, our research has shown that higher order integration algorithms are poorly suited to the highly nonlinear digital logic circuits because of their inherent poor stability properties. The major problem in a timestep control is the estimation of truncation error, which amounts to numerical differentiation. Since higher order formulae require estimation of higher order derivatives, a reliable estifation of truncation error can only become more difficult as order is increased. At this time, it appears that a single order, probably Euler or Trapezoidal, is the most reliable and efficient integration algorithm to use in a nodal analysis program.

## VII. Acknowledgements

We are pleased to acknowledge the coatributions and valuable disCussions of S. Chisholm, R. I. Dowell, S. P. Pan, I. E. Getreau, D. A. Hodges, W. J. McCalla, R. G. Meyer, and R. A. Roarer. We also graceíully acknowledge the many hours of computer time providec us by the Computer Center of the University of California, Berkeley, without which it would have been impossible to develop SPICE. The work on SPICE has been supported by the National Science Foundation, Grant GK-17931.

ADDENDUM

## Flicker Noise Analysis in SPICE

The ability to simulate flicker noise sources in the noise analysis of SPICE (refer to IEEE Journal of Solid State Circuits, August 1971, Pp. 204-215) has been added to SPICE. Flicker noise is included by adding another term to the current generators for the devices:
a. Junction diodes

$$
i_{D}^{2}=2 q I_{D} \Delta f+\frac{K I_{D}^{a}}{f} \Delta f
$$

## b. Bipolar Junction Transistors

$$
i_{B}^{2}=2 q I_{B} \Delta f+\frac{R I_{B}^{a}}{f} \Delta f
$$

c. Field effect transistors (both JFET's and MOSFET's).

$$
i_{D S}^{2}=4 k T\left(\frac{2}{3} g_{m}\right) \Delta f+\frac{R I_{D}^{a}}{f} \Delta f
$$

For bipolar devices, our measurements have shown that $a=1$ for npn devices and $a=1.5$ for pnp devices, and $\mathrm{K}=6.6 \times 10^{-16}$ for $n p n$ devices and $0.3 \times 10^{-13}$ for pnp devices (these measurements are for the devices in a 741 operational amplifier, but should be representative of monolithic devices in general).

Operationally, the flicker noise parameters are defined on the .MODEL card by setting two parameters:

FNK Flicker noise coefficient
Default=0.0
FNA Flicker noise exponent
Default=0.1

These parameter names are the same for all four devices. Hence, a model card for an npn transistor, with flicker noise, might look 1ike:
. $M O D E L$ KI NPN $B F=35 R B=100 \mathrm{RE}=0.1 \mathrm{IS}=1.5 \mathrm{E}-15 \mathrm{FNK}=6.6 \mathrm{E}-16 \mathrm{FNA}=1.0$

## External Models in SPICE

The ability to define an arbitrary device which contains allowable elements has been included in SPICE (versions $1 G$ and later). Hence the gate shown in Figure 1 could be defined to be an element in SPICE just as a bipolar device is a model in SPICE. The restrictions are:
a. External models cannot be nested, that is, an external model cannot contain an external model.
b. An external model cannot have more than 20 external nodes (the gate on the following page has 5 external nodes and three internal nodes).

To define an external model, a group of cards is required. The first card is a .MODEL card which contains the word .MODEL, the name of the model, the letter $X$, and the external nodes of the device. The following cards are the set of element cards which define the model card, and the last card is a .FINIS card, which contains simply the word .FINIS. The following group of cards defines the. TIL gate shown in Figure 1.
. MODEL TILGATE X 12345
Q1 681 Ml
Q2 682 Ml
Q3 683 ML
Q4 467 Mll
Q5 470 ML

R1 54 1K
R2 70 1.5K
R5 58 4K
.MODEL MI NPN BF=80 RB=50 TF=0;1NS TR=10NS CJC=0.3PF CJE=0.5PF .FINIS

Once the model has been defined, it can be referenced just as an internal built-in model is referenced. The "device" name must begin / with the letter $X$. The device is specified by the name, the external nodes, and the model name. For example, the logic circuit shown in Figure 2 could be simulated by the following group of cards:

EXAMPLE LOGIC CIRCUIT
VCC 100 DC 5
VIN1 10 PULSE 05 10NS 10NS 10NS 100NS
VIN2 20 PULSE 05 200NS 10NS 10NS 1OONS
XG1 1113 IO TTLGATE
XG2 112410 TTLGATE
XG5 524510 TTLGATE
.OUT V5 50 PLOT TRAN
.TRAN 5NS 500NS
. MODEL TILGATEX 12345
... 01.6 .8 . $1 . \mathrm{Ml}$
Q2 682 Ml
Q3 683 Ml
Q4 467 Ml
Q5 470 MID
R1 54 1K
R2 70 1.5K
R3 58 4K
.MODEL M1 NPN $\mathrm{BF}=80 \mathrm{RB}=50 \mathrm{TF}=0.1 \mathrm{NS}$ TR=10NS $\mathrm{CJC}=0.3 \mathrm{PF} \mathrm{CJE}=0.5 \mathrm{PF}$ .FINIS
.END

## DC Sensitivity Analysis in SPICE

A de sensitivity analysis capability was also included in SPICE (versions IG and later) but never documented. This option is used to obtain the dc, small-signal sensitivities of a given output variable with respect to every circuit value. The general format for sensitivity analysis is:
.SENS ovar 1 ovar 2. ... ovar 10

Note that from one to ten outputs can be specified. Only one . SENS card should be used in a deck. The syntax for the output variables (ovar 1 ... ovar $n$ ) is identical to the syntax for output variables in the .OUTPUT card.

## Examples:

..OSENS..VOUT . 3 . 2
. SENS V1 1 O V2 2 O IX VCC
If a . SENS card is included in the SPICE data deck, the program will compute the dc, small-signal sensitivities (derivatives) at each specified output variable with respect to every circuit value (including the dc parameters for BJT model and diode model). There is no way of selecting specific sensitivities. In the first example, the program will compute and print the derivative of the voltage between node 3 and node 2 with respect to every circuit parameter.


SPICE 10. (1 Mar 74)
The latest version of SPICE, SPICE $1 Q$, is now released. Versions :IM, $1 N, 10$, and $1 P$ were all local versions, so $1 Q$ is the replacement version for SPICE 1L. In contains the following modifications:

1. Overlay organization - The program structure has been modificd slightly to readily accomodate an overlay structure. The prógram requires a 60000 octal region to exccute on the CDC 6400 computer at the University of California, Berkeley.
2. Distortion analysis - A new analysis algorithm for computing the small-signal distortion performance of a linear circuit has been implemented. For further details on the theory of distortion analysis, see Trans. IEEE, Vol. CT-7, November 1973, Pp. 709-717 and PP. 742 - 746.
3. Revised . OUTPUT control card - To accomodate distortion analysis; and to add some flexibility to the $A C$ analysis output, the .OUTPUT format has been modified.
4. Assembly language matrix routines - The matrix decomposition and solution routines have been recoded in COMPASS
. - . assembly_language. .. For .CDC users, .this results in a savings of $10 \%$ - 40\%. For non-CDC users, the FORTRAN code has been included, as comments, in the assembly routines, and can be easily reinstated.
5. Addition of the . RUN card - The . RUN control card has been added to allow the printing of various execution statistics (matrix structure, number of iterations, and timing data for each phase of a simulation).
6. FET convergence - The routines for JFET's and MOSFET's have been modified to improve the convergence of simulations involving JFET's and MOSFET's.
7. Small-signal dc transfer curve errors - An error in the computation of the dc, small-signal input resistance, transfer function, and output resistance has been corrected.
8. Gummel-Poon temperature dependence - An error that caused the saturation currents in the Gumal-Poon transistor model to be computed incorrectly (as a function of temperature) has been corrected.
9. Voltage source polarity error - An error that caused the polarity of voltage sources to sometimes be reversed has been corrected.

## SPICE 10 INPUT FORMAT CHANGES

1. Distortion analysis

SPICE will compute the distortion characteristics of the circuit in a snall-signal mode as a part of the ac small-signal sinusoidal steady-state analysis if requested. The analysis is performed assuming that signals of two frequencies are imposed at the input; let the two frequencies be $f_{1}$ and $f_{2}$.

The program then computes the following distortion measures:
HD2 - The magnitude of the frequency component $2 f_{1}$ assuming that $f_{2}$ is not present.
HD3 - The magnitude of the frequency component $3 f_{1}$ assuming that $f_{2}$ is not present.
$\therefore$ :STM ..- .The.magnitude of the. frequency component $f_{1}+f_{2}$
DIM2 - The magnitude of the frequency component $f_{1}-f_{2}$
DIM3 - The magnitude of the frequency component $2 f_{1}-f_{2}$

All of these distortion measures can be computed at each frequency point (value of $f_{1}$ ) and printed or plotted (either as REAL and IMAG, or as MAG, DB, and PHASE) just as any other output variable. In addition, at specified frequencies, the contribution of every nonlinear device to the total distortion can be printed.

Distortion analysis is specified on the .AC card:
-AC DEC 101 10KHZ DISTO RLOAD INTER REFPWR SKW2 SPW2
any legal freq variation format
optional - defaults supplied if not specified

KHERE:
RLOAD - The name of the output load resistor into which all distortion power products are to be computed (must be specified).

INTER - The interval at which the sumany printout of the contributions of all nonlinear devices to the total distortion is to be printed. Zero implies no printout, 1 implies every point, 2 implies every other point, 3 implies every third point, and so on. Defaults to zero if not specified.

REFPWR - The reference power level used in computing the distorition products. If not specified, a value of 1 mW (that is, $d B m$ ) is used.

SKW2 - The ratio of $f_{2}$ to $f_{1}$. If not specified, a value of 0.9 is used (i.e., $f_{2}=0.9 f_{1}$ ).

SPW2 - The amplitude of $\mathrm{f}_{2}$. A value of 2.0 is used if not specified.

EXARPLE: .AC DEC 10 1.0 100K DISTO RL 2 1.OE-3 0.950 .75
2. Output options (.OUTPUT card)

Some new options have-been added to the .OUTPUT card to accomodate distortion analysis and improve output for AC analysis:
...:QUUTPUT.Vxyxxxx. N1 N2 . PRINT (options) .PLOT a(options) Ixzzuxx Vyyyyy

ONOISE RINOISE HD2 HD3 SIM2 DIM2 SIM 3

The output variable Vxxxoxx is a voltage output (N1 is the positive node, and $N 2$ is the negative node), Ixxxaxx is a current output (Vyyyyy is the voltage source the current is flowing in), ONOISE is the output noise computed in the noise analysis, RINOISE is the reflected input noise computed In the noise analysis, and HD2, HD3, SIM2, DIM2, and DIM3 are the distortion measures mentioned in distortion analysis.

The options available are:
DC de analysis output
TR transient analysis output
MAG. ac analysis output, magnitude
DB ac analysis output, magnitude (in dB)
PHS ac analysis output, phase
RE ac analysis output, real part
IM ac analysis output, imaginary part

Some output examples:
To plot a transient response of node voltage 4
.OUTPUT. V4 40 PLOT TR
To print and plot the de transfer curve for node voltage 17 and the bode plot for node voltage 17
. OUTPUT V17 170 PRINT DC MAG DB PHS PLOT MAG DB PHS DC
To plot the output noise and equivalent input noise in both volts and dB:
.OUTPUT ONOISE PLOT MAG DB .OUTPUT RINOISE PLOT MAG DB

To plot the distortion measures HD2, HD3, DIM2, and DIM3: .OUTPUT HD2 PLOT DB PHS . OUTPUT HD3 PLOT DB PHS .OUTPUT DIM2 PLOT DB PHS .OUTPUT DIM3 PLOT DB PHS
3. . RUN card - If a . RUN card is included in the input deck, -"the program will print matrix statistics, circuit statistics, and timing information. This data is not printed if the . RUN card is absent.

MUTUAL INDUCTORS

SPICE $1 Q$ also contains provision for specifying a coupling between inductors in the circuit. A mutual inductance is specified by the following card:

Uxxxxxx Lyyyyyy Lzzzzzz, value
The name must begin with aU. Lyyyyyy and Lzzzzzz are the two coupled inductors, and 'value' is the value of mutual inductance between the inductors. The coefficient of coupling, $k$, is defined by:

$$
k=\frac{M}{L_{1} L_{2}}
$$

where $H$ is the mutual inductance, and $L_{1}$ and $L_{2}$ are the values of inductance for the two coupled inductors. This coeff.i, client of coupline.must.always he less than unity. in absolute value. A negative value of $M$ inverts the direction in which current flows. The following data deck defines an ac analysis of the simple transformer circult shown below.
test of mutual inductance 1101 AC 1 RIH1O2K 11101014 $L 2203 \mathrm{UH}$ U12 Li 12 0.99 UH RLOAD 20 1K -AC DEC 1010100 kHz .OUTPUT VI 10 PLUT hAG PHS .OUTPUT VI 20 PLOT MAG PHS -END


## TYPICAL BJT AND FET PARAMETERS

For bipolar transistors, forward and inverse current gains, output conductance or Early voltage, and emitter saturation current are measured at a typical active region operating point. Collector series resistance Is measured at a typical saturated region operating point. Base series resistance is calculated from mask dimensions and base sheet resistance. The three depletion layer capacitances are calculated at zero bias from mask dimensions and process specifications. Forward and inverse transit times are calculated from these capacitances (under bias) and measured values of forward and inverse current gain-bandwidth ( $f_{T}$ ), taken at a typical operating point. These eleven paramenters are adequate to characterize integrated circuit bipolar transistors in almost all analog and digital applications. Representative values for a small IC transistor are given in Table I.

Once these parameters are known for one transistor made by a give: process, parameters for devices of differing mask geometry may be determined without further measurements. Current gains, Early voltage, and transit times are to first order independent of mask geometry for npn IC.transistors. Emitter saturation current, junction capacitances, and series resistances are simple functions of mask dimensions and process specifications. The most troublesome parameters in this scaling process are the inverse current gain and inverse transit time; due to the trend toward non-saturating and Schottky clamped circuits, the significance of these parameters is decreasing. In such circuits, inverse parameters have virtually no influence on circuit performance.

This approach to modeling bipolar IC components usually will produce circuit simulation results accurate to within .05 V in DC levels and to within $10 \%$ in time and frequency domain characteristics. Substantially more effort is needed to reduce errors by a factor of two.

A similar approach works well for MOS transistor model parameter determination. A sample device made by the chosen process is evaluated. Threshold voltage, gain factor (k or Beta), body effect coefficient, and output conductance or £arly voltage are measured. The five interelectrode capacitances are calculated from mask dimensions and process specifications. Source-body and drain-body capacitances are those of normal pn junctions. Gate area and oxide capacitance determine the total capacitance from gate to source, body, and drain. This capacitance may be divided into two or three parts. For static MOS circuits, - dividing this capacitance into equal, constant gate-source and gatedrain components is simple and adequate. For dynamic circuits (particularly the ratioless type), gate capacitance should be divided into three voltage-dependent parts. Representative parameters for a small pchannel silicon-gate MOS transistor are shown in Table II.

Parameters for other MOS transistors, differing in mask geometry from the sample device, are obtained by scaling. Gain factor and capacitances are simple functions of geometry; the other parameters are independent of geometry in first-order approximation.
FORWARD BETA ..... 100INVERSE BETAEMITTER SATURATION CURPENTEARLY VOLTAGEbase series resistaice$2 \times 10^{-16} \mathrm{~A}$50 V
COLLECTOR SERIES RESISTANCE$50 \Omega$
FORWARD TRANSIT TINE
INVERSE TRANSIT TIME$50 \Omega$EMITTER JURCTION CAPACITANCECOLLECTOR JUNCTION CAPACITANCE0.3 nS
10 ns0.5 pF
SUBSTRATE JUNCTION CAPACITANCE0.5 pF
1.0 pF

TABLE I

| CHANAEL WIDTH TO LENGIH PATIO |  | 1.0 |
| :---: | :---: | :---: |
| THRESHOLD VOLTAGE |  | 2 V 2 |
| GAIN FACTOR |  | $2 \mu A / V_{1 / 2}^{2}$ |
| BODY EFFECT |  | $0.75 \mathrm{~V}^{1 / 2}$ |
| EARLY VOLTAGE | $\cdots$ | 50 V |
| SOURCE-BODY CAPACITANCE |  | 0.05 pF |
| DRAIN-BODY CAPACITANCE |  | 0.05 pF |
| GATE-SOURCE CAPACITANCE |  | 0.01 pF |
| GATE-DRAIN CAPACITANCE |  | 0.01 pF |
| GATE-BODY CAPACITANCE |  | - 0 |

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## TABLE 1

BJT MODEL PARAMETERS FOR SHUNT-SERIES FEEDBACK AMPLIFIER


## TABLE 2

## EXECUTION TIMES FOR SN7400 TTL INVERTER (27 NODES, 8 BJT'S, 101 TIMEPOINTS)

READIN
SETUP
DC ANALYSIS
(ITERATIONS)
TRANSIENT
(ITERATIONS)
OUTPUT

TOTAL

TIME PER NEWTON ITERATION:
TIME PER NEWTON INTERATION PER BJT:
9.20 sec
0.42 sec
0.08 sec
0.52 sec
(22)
7.26 sec (340)
0.92 sec
21.4 msec
2.7 msec

## TABLE 3

EXECUTION TIMES FOR 741 OP AMP
(49 NODES, 22 BJTS, 101 TIMEPOINTS)

| READIN | 0.8 sec |
| :--- | ---: |
| SETUP | 0.5 sec |
| DC ANALYSIS |  |
| $\quad$(ITERATIONS) | 0.9 sec |
| (12) |  |
| TRANSIENT <br> $\quad$ ITERATIONS) | 13.6 sec <br> OUTPUT |
|  | 0.9 sec |
| TOTAL | 16.7 sec |

TIME PER NEWTON ITERATION:
67 msec
tIME PER NEWTON INTERATION PER BJT:


Figure 1


Figure 2


Pigure 3


Figure 4


Pigure 5

$I_{k}=3 \mathrm{~mA}$
$C_{2}=100$
$\beta_{F M}=100$

$$
\because
$$



Figure 7


Pigure 8


Pigure 9


Pigure 10


Pigure 11



Pigure 13


$$
\begin{aligned}
& V_{T 0}=1 \text { volt } \\
& \gamma=1 \frac{1}{\sqrt{\text { vet }}} \\
& \phi=0.7 \text { volt }
\end{aligned}
$$

$\square$

APPENDIX
PROGRAN Linitarions



> UNIVERSITY OF CALIFORNIA COLLEGE OF ENGINERING OEPARTMENT OF ELECRICAL ENGINEERING AND COMPUTER SCIENCES

## L M MAEE 0 O PEDERSOM

USERS GUIDE FOR SPICE 1
PAES 2

 the freguency range and the notse analysis options are spectified on the
-actontrol card page 15).

## tramsient amalrsis

 VAITIALES CONOITIONS ARE AUTOMATICALLY DETERMINED OY A OC ANALYSIS. ALL SOURCES YHICH ARE NOT TIME OEPENDENT (FOR EXAMPLE, POMER SUPPLIES) ARE SET TS THEIR
VALUE. FOR LARGE SIGMAL SINUSOIOAL SIMULATIONS, A FOURIER ANALYSIS OF THE
 ake specified on the itran control card lpage 161.
c 324d

CONVER̈TENCE




PEMPERATURE APPEARS EXPLIEITLY IN THE EXPONENTIAL TERMS DF THE BIT AND
OIODE MODEL EOUATIONS. IM ADOITIONP SATURATION CURRENTS HAVE AUILT-IN
TENPERATURE DEPENDENCE. THE TEMPERATURE DEPENDENCE OF THE SATURATION CURRENT
IN THE OJT MODEES IS DETERMINED EY:

 TOL CARO IPAEE 151
analvsis at olfcerent temperatures
 CERCENT OR SO MICROVOLTS, WHICHEVER IS LARGER. ALTHDUGH TME PARTICULAR
 OUT THE LAST MODE VOLTAGES ANO TERNINATE THE JOO. THE MODE VOLTACES THAT
PRINTED ARE MOT MECESSARILY CORRECT OR EVEN CLOSE TO THE CORRECT SOLUTION.

FAILURE TO CONVERGE IN THE OC ANALYSIS IS USUALLY OUE TO AN ERROR IN REEEMERATIVE SWITCNING CIRCUITS OR CIRCUITS WITH POSITIVE FEEDBACK PROBAALY MILL COM CONVERGE IM THE DC INE STEP WHICH IS TCO LARGE. SPICE PAESENTLY DOES NOT MANE AN AUYOMATIC TIME STEP CONTROL. AND SIGNIFICCANT ERROR ANDIDR NONCONVERG


[^2]
## CIACUIT DESCRIPTION

WOOES MEED NOT OE MUMBERED SEDUENTIALLY. THE CIRCUIT CATOF CO CURRENT SOURC
$\begin{aligned} & \text { WODES NEEO NOT GE NOIOR INDUCTORS AND CANNOT CCNTAIN A CUTSET OFTCHE } \\ & \text { VOLTAGE SOURCES ANDI } \\ & \text { AMDOR CAPACITORS. EACH NODE IM THE CIRCUIT. INCLUDING THE DATUY NODE, MUST }\end{aligned}$
GNAVE AT LEAST TMO CONNECTIONS.

## PACE

# GM - value e EXP (-J * 6.20310 * freo - TOI 

the ozlay is igmoned in the de and transient amalyses.






C


$$
\begin{aligned}
& \text { OUTPUT INPEDANCE. IF THE TRANSFER FUNCTINN VALUE IS NOT DESIREO, ONT } \\
& \text { OUTPUT AND INPUT SPECIFICATIONS. IF THE OC OPERATING PONT IS MOT OESIRED, } \\
& \text { OMIT THE LETTERS OP. HOWEVER. A OC OPERATING POINT WILL ALWAYS BE COMPUTED }
\end{aligned}
$$

$$
\begin{aligned}
& \text { OMIT THE LETTERS OP. HOGENER. ALYC OPERATING TRANSIENT AMALVSIS. } \\
& \text { PRIOR TO AN AC SMALL SIGAL ANALYSIS OR A }
\end{aligned}
$$

FOR TMANSFER CUAVE, ELMAME IS THE NAME OF THE VARIAELE SOURCE, VSTART IS
THE STARTIMG SOURCE VALUE, VSTOP IS THE FIMAL SOURCE VALUE AND VIMCR IS THE
 PARAMETERS.


## 

GENERAL FORM

## Examples

OE.
VIING FR
OUENCY POINTS TO CE COMPUTED CANNOT EXCEED IOL.
FOR MOISE AMALYSIS, OUTPUT IS THE MAME OF A VOLTAGE OUTPUT VARIABLE. THIS
INPUT IS
THE NAME OF AN IMDEPENDENT VOLTAGE OR CURRENT SOURCE. THE TOTAL OUTPUT NOISE IS OIVIDED OY THE TRANSFER FUNCTION SOUTPUT/INPUTI TO O日TAIN THE EOUIVALENT INPUT NNISE LEVEL. NUMS IS THE SU EACH ELEMENT ARE PRINTED OUT. IF MUNS IS OWITTEO OR SET TO ZERO, NO SUMMARY PRINROUT MIL PAINTOUT, MUHS SHOULD BE AS LARGE AS POSSIBLE. IF THE NOISE ANALYSIS IS NOT

## -cooe .tpaam caro


IMTEAVAL IZEAO. SSTAATI. TME CIRCUIT IS ANLTLEE ITO TSTOP). THE CIRCUIT IS AMALVIED AMD OUTPUTS ARE STORED. THE NUMER OF TINEPOINTS IN THE INTE INEREVAL ITSTART. TSTOPI CANNOT EXCEED 101.
COEPONENTS ARE DETERHINED. FOR MAXIMUM ACCURACY, THE NUMBER OF PERIODS IN THE
$\begin{aligned} & \text { ONE POSSHIIE. IF IHE FOURIER } \\ & \text { TME FOURIER SPECPICATIONS. }\end{aligned}$


 THE TIME NTERVACH SUBIMTERVAL THE INTERNAL TINE STEPS AND SUBINTERVAL ENDPOINTS ARE SPE
anal SIS OPTIONS:

[^3]$$
\text { .tran lws loons } 0 \text { O.1NS lows } 0.5 \mathrm{SNS} \text { loows }
$$

OI is THE FIRST INTERHAL TIMESTEP AMD EEL IS TME ENDPOINT OE THE FIRST SUEINO SUSINTERVAL. ANO SO ON. IN THIS EXAMPLE, THE PROGGAM MILL USE AN TME STEP
 OF O.SNS FOR THE INTERVAL TMUEER OF TIMEPOINTS TO BE COMPUTEO CANNOT EXCEED 1001.

[^4]example this example, the program will use an internal time step of ooivi over the


[^5]


[^0]:    *An overlaid version can be executed in approximately 25,000 decimal words.

[^1]:    ${ }^{*} \mathbf{V}_{\text {TO }}$ is assumed to be negative for all depletion mode devices iritespective of channel polarity.

[^2]:    
     REPRESENT THE SAME SCALE FACTOR. NOTE THAT.
    IKHZ, AHD IK ALL REPRESENT THE SAME NUMBER.

[^3]:    gemeral form .than tstep tstop tstart ol ei 02 ez ... DS es foun output fate

[^4]:    .tRan ius loous oo.ius hoous

[^5]:    the program stores and outputs every tenth timepoint.
    the program stores and outpurs every tenh thator

