PRICE-BASED ADAPTIVE SPINNING RESERVE REQUIREMENTS IN POWER SYSTEM SCHEDULING

by

Chung-Li Tseng, Shmuel S. Oren, Alva J. Svoboda,
and Raymond B. Johnson

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College of Engineering
University of California, Berkeley
94720

Price-Based Adaptive Spinning Reserve Requirements in Power System Scheduling

Chung-Li Tseng, Shmuel S. Oren
Department of Industrial Engineering
and Operations Research
University of California at Berkeley
Berkeley, CA 94720 USA

Alva J. Svoboda, Raymond B. Johnson
Pacific Gas & Electric Company
San Francisco, CA 94177
USA

Abstract—**This paper presents a method for adaptively modeling spinning reserve requirements based on prices in power system scheduling. The method is embedded in a Lagrangian relaxation method for solving unit commitment. The spinning reserve requirement responds to a price signal according to a nonincreasing response function at each iteration in the dual optimization. Game theory is used to interpret the proposed algorithms. Numerical test results are also presented.**

Keywords: Power system scheduling, Unit commitment, game theory.

I Introduction

The solution of the power system scheduling problem is important to a power utility like PG&E to determine major unit commitment and transaction decisions. Many optimization methods have been proposed to solve this problem [3]. Among them, Lagrangian relaxation methods are now among the most widely used approaches to solve unit commitment [2,4,7]. At PG&E, the Hydro-Thermal Optimization (HTO) program was developed almost a decade ago based on the Lagrangian relaxation approach [4]. In our recent work, the Lagrangian relaxation-based algorithm has been extended to schedule thermal units under ramp rate constraints [11].

The objective of the unit commitment is to minimize the total generating costs of the power system over the planning horizon subject to system constraints (e.g. load balance constraints and spinning reserve constraints) and unit constraints (e.g. minimum up/down time constraints and ramp constraints.) Most of the unit constraints describe the physical limits of units and should not be violated. These constraints are considered to be 'hard constraints' in this sense. In contrast to the 'hard constraints', the spinning reserve constraints are 'soft'. In order to satisfy a reserve constraint, expensive units must sometimes be turned on. A slight relaxation of these 'soft constraints' might avoid such uneconomic operation without impacting system security too much. In addition to avoiding some uneconomic operations, flexibly setting the reserve requirements can also improve the algorithm's computational speed.

The softness of the reserve constraints has previously been dealt with using fuzzy logic techniques [6]. In [6], Guan et al. proposed an efficient fuzzy optimization-based method to solve the unit commitment problem with soft reserve requirements. They first convert the problem to a 'crisp' one, then take advantage of separability of the problem and solve it by Lagrangian relaxation. They also show the trade-off between cost-minimization and system reserve satisfaction. However, to avoid infeasibility, the aspiration level of the generation cost is obtained by running the crisp problem with the lowest acceptable reserve requirement. This requires multiple runs of the unit commitment algorithm.

The (hourly) spinning reserve requirement (SRR) is usually required to be the greater of a fixed percentage (say 5%) of the total forecast demand and the largest online unit. However under certain conditions, a utility may wish to increase the SRR to say 7% provided this higher level of reliability does not increase costs too much. Our HTO program can be used to handle this situation by first running the program with SRR fixed at 7% of system load. After running the program, if the marginal cost of maintaining that reserve was too high, the operator can run the program again and lower the requirement, presumably to no lower than 5%. This trial and error approach requires multiple runs of the program which is time consuming.

In this paper, we propose a new approach embedded in the Lagrangian relaxation approach. The SRR is defined as adaptive and adjustable between two levels of reserve requirement in all hours. The SRR is adjusted based on the corresponding Lagrange multiplier, which is viewed as price information. Game theory (e.g. [5]) is used to interpret our proposed algorithms. The proposed method in this paper does not require the multiple runs of unit commitment required by other approaches.

We present a formulation of the thermal unit commitment problem, review the Lagrangian relaxation method and describe our adaptive price-based SRR method in Sections II, III and IV respectively. Numerical test results and conclusions are given in Sections V and VI.

II Problem formulation

In this paper the following standard notation will be used. Additional symbols will be introduced when necessary.

i : index for the number of units ($i = 1, \cdots, I$)

t : index for time ($t = 0, \cdots, T$)

u_{it} : zero-one decision variable indicating whether unit i is up or down in time period t

x_{it} : state variable indicating the length of time that unit i has been up or down in time period t

t_i^{on} : the minimum number of periods unit i must remain on after it has been turned on

t_i^{off} : the minimum number of periods unit i must remain off after it has been turned off

p_{it} : state variable indicating the amount of power unit i is generating in time period t

$p_i^{\min}$: minimum rated capacity of unit i

$p_i^{\max}$: maximum rated capacity of unit i

$C_i(p_{it})$: fuel cost for operating unit i at output level p_{it} in time period t

$S_i(x_{i,t-1}, u_{it}, u_{i,t-1})$: startup cost associated with turning on unit i at the beginning of time period t

D_t : forecast demand requirement in time period t

R_t : spinning capacity requirement in time period t

The unit commitment problem is formulated as the following mixed-integer programming problem: (note that the underlined variables are vectors in this paper, e.g. $\underline{u} = (u_{11}, \cdots, u_{IT})$.)

$$(P) \quad \min_{\underline{u},\underline{x},\underline{p}} \sum_{t=1}^{T} \sum_{i=1}^{I} [C_i(p_{it})u_{it} + S_i(x_{i,t-1}, u_{it}, u_{i,t-1})] \quad (1)$$

subject to the demand constraints,

$$\sum_{i=1}^{I} p_{it}u_{it} = D_t, \quad t = 1, \cdots, T, \quad (2)$$

and the spinning capacity constraints,

$$\sum_{i=1}^{I} p_i^{\max} u_{it} \geq R_t, \quad t = 1, \cdots, T. \quad (3)$$

There are other unit constraints such as unit capacity constraints,

$$p_i^{\min} \leq p_{it} \leq p_i^{\max}, \; i = 1, \cdots, I; \; t = 1, \cdots, T, \quad (4)$$

the state transition equation for $i = 1, \cdots, I$,

$$x_{it} = \begin{cases} \max(x_{i,t-1}, 0) + 1, & \text{if } u_{it} = 1, \\ \min(x_{i,t-1}, 0) - 1, & \text{if } u_{it} = 0, \end{cases} \quad (5)$$

the minimum up/down time constraints for $i = 1, \cdots, I$,

$$u_{it} = \begin{cases} 1, & \text{if } 1 \leq x_{i,t-1} < t_i^{\text{on}}, \\ 0, & \text{if } -1 \geq x_{i,t-1} > -t_i^{\text{off}}, \\ 0 \text{ or } 1, & \text{otherwise}, \end{cases} \quad (6)$$

and the initial conditions on x_{it} at $t = 0$ for $\forall i$.

III The Lagrangian relaxation approach

The Lagrangian relaxation (LR) approach relaxes the demand constraints and the spinning capacity constraints by using Lagrange multipliers. The problem is then decomposed into I subproblems. Let λ_t and μ_t ($t = 1, \cdots, T$) be the corresponding nonnegative Lagrange multipliers to (2) and (3). We have the following dual problem:

$$(D) \quad \max_{\underline{\lambda},\underline{\mu} \geq 0} \theta(\underline{\lambda}, \underline{\mu}; \underline{R}), \quad (7)$$

where

$$\theta(\underline{\lambda}, \underline{\mu}; \underline{R}) = \sum_{i=1}^{I} \theta_i(\underline{\lambda}, \underline{\mu}) + \sum_{t=1}^{T} (\lambda_t D_t + \mu_t R_t), \quad (8)$$

and

$$\theta_i(\underline{\lambda}, \underline{\mu}; \underline{R}) = \min_{\underline{u},\underline{x},\underline{p}} \sum_{t=1}^{T} [C_i(p_{it})u_{it} + S_i(x_{i,t-1}, u_{it}, u_{i,t-1}) - \lambda_t p_{it} u_{it} - \mu_t p_i^{\max} u_{it}]. \quad (9)$$

It is well known that the dual objective function θ is concave and continuous but not necessarily differentiable at all points. A subgradient algorithm is applied to solve the dual problem (D). It can be shown that the vector of the mismatches in the demand constraints and the spinning capacity constraints is a subgradient of the dual objective function θ ([1].)

The subgradient algorithm

Step 0: $k \leftarrow 0$; $\underline{\lambda}^0$ and $\underline{\mu}^0$ are given.

Step 1: Given $\underline{\lambda}^k$ and $\underline{\mu}^k$, solve $\theta(\underline{\lambda}^k, \underline{\mu}^k; \underline{R})$ to obtain $(\underline{u}^k, \underline{p}^k)$.

Step 2: $\lambda_t^{k+1} = \lambda_t^k + s^k(D_t - \sum_{i=1}^{I} p_{it}^k u_{it}^k)$,
$\mu_t^{k+1} = \max(0, \mu_t^k + s^k(R_t - \sum_{i=1}^{I} p_i^{\max} u_{it}^k)), \forall t$.

Step 3: $k \leftarrow k+1$, go to Step 1. ∎

In [8], it is shown that under some conditions on the step size s^k, the subgradient algorithm converges.

III.1 Two-firm model

Each unit subproblem θ_i can be interpreted as a profit maximization problem for unit i [9,10], where unit i is an endogenously priced resource and λ_t and μ_t are the prices paid to the resource.

We interpret the dual optimization using the following two-firm model: Firm P, a power utility, facing demand D_t needs to purchase fuel from firm Q for generation. Fuel for different generating units may vary and fuel cost is described by $C_i(\cdot)$. Firm Q also sells power and spinning capacity. It offers firm P the prices λ_t and μ_t for power and spinning capacity at hour t respectively. Firm P's objective is to minimize its total cost given two options: buy fuel and self-generate or directly buy power from firm Q. Firm Q's problem is to adjust the prices of λ_t and μ_t so as to achieve its maximum revenue, considering that customer firm P will minimize its cost.

$$\underbrace{\max_{\underline{\lambda},\underline{\mu}\geq 0} \underbrace{\min_{\underline{u},\underline{x},\underline{p}} \left(\begin{array}{c} \sum_{t=1}^{T}\sum_{i=1}^{I}[C_i(p_{it})u_{it} + S_i(x_{i,t-1}, u_{it}, \\ u_{i,t-1})] + \sum_{t=1}^{T}[\lambda_t(D_t - \sum_{i=1}^{I} p_{it}u_{it}) + \\ \mu_t(R_t - \sum_{i=1}^{I} p_i^{\max} u_{it})] \end{array} \right)}_{\text{firm P's cost minimization problem}}}_{\text{firm Q's revenue maximization problem}}$$

Figure 1: The two-firm model

Remarks:

1. In Figure 1 the dual objective function is divided into two optimization problems of two firms. If firm P decides to purchase fuel, its spending is captured by the two terms in the first bracket in the objective function. If firm P purchases either power or spinning capacity from firm Q, it pays the corresponding amounts in the second bracket. No matter what option firm P chooses, firm Q collects the money.

2. At iteration k, firm Q offers λ_t^k and μ_t^k, and firm P obtains p_{it}^k, u_{it}^k. If the subgradient $(D_t - \sum_{i=1}^{I} p_{it}^k u_{it}^k) > 0$, λ_t^k should be increased at the next iteration. Since from firm Q's perspective if $(D_t - \sum_{i=1}^{I} p_{it}^k u_{it}^k) > 0$ there is an excess demand for power, and firm Q could raise the price.

3. Raising λ_t would stimulate the system to increase total generation at hour t and vice versa. Increasing μ_t would at some point make more units turn on.

This two-firm model can be treated as a 2-player game. Consider a sequential bargaining game of complete and perfect information in which firm Q (player 1) moves first and decides prices λ_t and μ_t $(t = 1, \cdots, T)$; firm P (player 2) observes λ_t and μ_t $(t = 1, \cdots, T)$, and then chooses its optimal generating policy, (u_{it}, x_{it}, p_{it}), $\forall i, t$. We have the following proposition.

Proposition 1 If $(\underline{\lambda}^*, \underline{\mu}^*; \underline{u}^*, \underline{p}^*)$ is the solution to (D), it is a Nash equilibrium of this game. ∎

The solution to (D) is a Nash equilibrium ([5]) because given the other player's strategy, each player has no incentive to deviate. Note that the result of Proposition 1 holds if the game is a simultaneous-move game. However, a sequential game with no need to assume that firm P, a spinning capacity provider, has market power fits reality better than the model of a simultaneous-move game. The subgradient algorithm therefore can be regarded as an interpretation of how each firm adjusts its decision in the face of the other firm's choice. The sequences, $\{(\underline{\lambda}^k, \underline{\mu}^k)\}$ and $\{(\underline{u}^k, \underline{p}^k)\}$, describe a process of adjustment to equilibrium.

III.2 Three-phase algorithm

Despite the efficiency of solving (D) due to its separability, the solution of (D) does not necessarily yield a feasible solution to (P), i.e. satisfying (2) and (3). Lagrangian relaxation algorithms for solving unit commitment have often been implemented with two phases: a dual optimization phase and a feasibility phase. In [13] Zhuang and Galiana first proposed the following overall structure for a unit commitment algorithm.

Phase 1: Dual optimization, i.e., solving (D).

Phase 2: Feasibility phase (continue dual optimization and increase μ_t in hours with insufficient spinning capacity until a reserve feasible solution (satisfying (3)) is found.

Phase 3: Economic dispatch to find a feasible solution (satisfying (2)).

In the Phase 2 proposed in [13], the hour of the most-violated SRR is determined, and the corresponding μ_t is enhanced. A method is proposed to calculate the exact amount of the increase in the value of the corresponding μ_t to make SRR satisfied at the corresponding hour. However, this method seems to take a long time to locate a feasible solution because only one μ_t is updated at a time. In our Phase 2 of the algorithm below, at each iteration the μ_t corresponding to the hours that the SRR is violated are simultaneously updated. This accelerates the speed of the feasibility phase, but could cause overcommitment in the generating units. A unit decommitment method has been developed to resolve the overcommitment problem [12].

Phase 2 Algorithm

Step 0: $k \leftarrow 0$; $\underline{\lambda}^0$ and $\underline{\mu}^0$ are from Phase 1.

Step 1: Given $\underline{\lambda}^k$ and $\underline{\mu}^k$, solve $\theta(\underline{\lambda}^k, \underline{\mu}^k; \underline{R})$ to obtain $(\underline{u}^k, \underline{p}^k)$.

Step 2: If $(\underline{u}^k, \underline{p}^k)$ is reserve feasible (satisfying (3)), stop.

Step 3: $\lambda_t^{k+1} = \lambda_t^k$,
$\mu_t^{k+1} = \mu_t^k + s^k \cdot \min(0, R_t - \sum_{i=1}^I p_i^{\max} u_{it}^k), \forall t$.

Step 4: $k \leftarrow k+1$, go to Step 1. ■

IV Price-based adaptive SRR

As an example, we consider the case where the desired SRR is 7%, i.e., in (3), $R_t = 1.07 D_t$. We then assume that, if the costs are too high, the SRR can be relaxed to 5%. Our problem is how to adaptively define the SRR in a unit commitment problem such that the following system operating phenomena can be avoided:

- Uneconomic solution: some expensive thermal units would not have to be committed if the the SRR at some hours had been slightly relaxed down to 5%; and
- Near infeasibility: the original SRR at some hours are either too restrictive or indeed unachievable. This situation normally retards the computational process.

In the original LR approach, both problems above may be indicated by the value of μ_t. At the hours with either uneconomic solution or near infeasibility explained above, the corresponding μ_t tend to grow to encourage commitment.

IV.1 Two-firm model with adaptive SRR

We now modify the two-firm model defined in Section III.1. Let firm P now also select its SRR. Firm Q, as in the original model offers prices $\underline{\lambda}$ and $\underline{\mu}$. After observing the prices $\underline{\lambda}$ and $\underline{\mu}$, firm P defines its SRR first, then solves the corresponding optimal generating policy, $\{u_{it}, p_{it}\}$ for $\forall i, t$. This scenario defines mappings that take $R_t \in [1.05D_t, 1.07D_t]$ into the solution of firm P's cost-minimization problem, $u_{it}(R_t)$ and $p_{it}(R_t)$. We assume that R_t is selected based on a function of μ_t in the form $r(\mu_t)D_t$, where $r : \Re^+ \rightarrow [1.05, 1.07]$ is a monotone nonincreasing function. For simplicity, we denote it as $R_t(\mu_t)$. $R_t(\cdot)$ is called the *response function* of firm P. This means that when μ_t, the price of spinning capacity offered by Q, is high, firm P not only tends to commit more units (as in the original model) but also wants to lower its SRR, and vice versa. An example of $R_t(\mu_t)$ is depicted in Figure 2, where $R_t(\mu_t) = [1.06 + 0.01\tanh(-\beta(\mu_t - \alpha))]D_t$, $\alpha > 0$, $\beta > 0$.

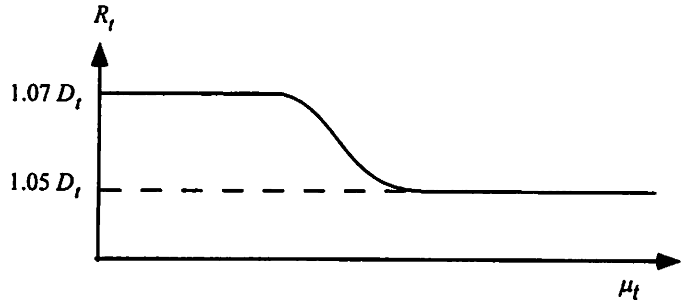

Figure 2: An example of $R_t(\mu_t)$.

IV.2 Sequential bargaining game model

Consider a sequential bargaining game of complete and perfect information in which firm Q moves first and firm P moves second. The timing of the game is as follows: (1) firm Q chooses the prices λ_t and μ_t, $\forall t$; (2) firm P observes λ_t and μ_t, and then chooses its SRR and the corresponding optimal generating policy, u_{it} and p_{it}. Assume the scenario is common knowledge to both firms P and Q, but firm Q does not know the response function of firm P. In the sequel, we use the notation $\underline{R}(\underline{\mu}) = (R_1(\mu_1), \cdots, R_T(\mu_T))$.

Proposition 2 If $(\underline{\lambda}^*, \underline{\mu}^*; \underline{u}^*, \underline{p}^*)$ is a solution to the following problem (D^*), $(\underline{\lambda}^*, \underline{\mu}^*; \underline{R}(\underline{\mu}^*), \underline{u}^*, \underline{p}^*)$ is a Nash equilibrium of the sequential bargaining game.
(D^*)

$$\max_{\underline{\lambda},\underline{\mu}\geq 0} \left(\begin{array}{r} \min_{\underline{u},\underline{x},\underline{p}} \sum_{t=1}^T \sum_{i=1}^I [C_i(p_{it})u_{it} + \\ S_i(x_{i,t-1}, u_{it}, u_{i,t-1})] + \\ \sum_{t=1}^T [\lambda_t(D_t - \sum_{i=1}^I p_{it}u_{it}) + \\ \mu_t(R_t^* - \sum_{i=1}^I p_i^{\max} u_{it})] \\ \text{subject to } R_t^* = R_t(\mu_t^*). \end{array} \right) \quad (10)$$

■

To find a Nash equilibrium, we construct an algorithm to simulate the process of adjustment toward equilibrium. At each iteration, as in the subgradient algorithm, firm Q adjusts prices based on the law of supply and firm P minimizes its cost. Also firm P updates its SRR in response to firm Q's price, i.e. $R_t^{k+1} = R_t(\mu_t^k)$. This means that in order to achieve $R_t^* = R_t(\mu_t^*)$ firm P approximates μ_t^* by μ_t^k.

Nash Algorithm

Step 0: $k \leftarrow 0$; $\{\lambda_t^0\}$ and $\{\mu_t^0\}$ are given.

Step 1: Given $\underline{\lambda}^k$, $\underline{\mu}^k$, solve $\theta(\underline{\lambda}^k, \underline{\mu}^k; \underline{R}(\underline{\mu}^k))$ to obtain $(\underline{u}^k, \underline{p}^k)$.

Step 2: $\lambda_t^{k+1} = \lambda_t^k + s^k(D_t - \sum_{i=1}^I p_{it}^k u_{it}^k)$,
$\mu_t^{k+1} = \max(0, \mu_t^k + s^k(R_t - \sum_{i=1}^I p_i^{\max} u_{it}^k)), \forall t$.

Step 3: $k \leftarrow k+1$, go to Step 1. ■

The convergence of Nash Algorithm has not yet been established theoretically, but it converges in all observed cases.

IV.3 Stackelberg game

As in the sequential bargaining game defined in Section IV.2 we now consider the case that firm Q knows the response function of firm P so that firm Q expects firm P's response in its revenue maximization problem. This game is commonly known as a Stackelberg game [5].

Proposition 3 If $(\hat{\underline{\lambda}}, \hat{\underline{\mu}}; \hat{\underline{u}}, \hat{\underline{p}})$ is a solution to the following problem $(\hat{D})$, $(\hat{\underline{\lambda}}, \hat{\underline{\mu}}; \underline{R}(\hat{\underline{u}}), \hat{\underline{u}}, \hat{\underline{p}})$ is a Stackelberg equilibrium of the Stackelberg game.
$(\hat{D})$

$$\max_{\underline{\lambda},\underline{\mu}\geq 0} \min_{\underline{u},\underline{x},\underline{p}} \sum_{t=1}^T \sum_{i=1}^I [C_i(p_{it})u_{it} + S_i(x_{i,t-1}, u_{it}, u_{i,t-1})] +$$

$$\sum_{t=1}^T [\lambda_t(D_t - \sum_{i=1}^I p_{it}u_{it}) + \mu_t(R_t(\mu_t) - \sum_{i=1}^I p_i^{\max} u_{it})]. \quad (11)$$

■

We assume that $R_t(\cdot)$ is continuously differentiable for all t. We can then apply the subgradient algorithm to solve $(\hat{D})$ to obtain a Stackelberg equilibrium.

Stackelberg Algorithm

Step 0: $k \leftarrow 0$; $\underline{\lambda}^0$ and $\underline{\mu}^0$ are given.

Step 1: Given $\underline{\lambda}^k$ and $\underline{\mu}^k$, solve $\theta(\underline{\lambda}^k, \underline{\mu}^k; \underline{R}(\underline{\mu}^k))$ to obtain $(\underline{u}^k, \underline{p}^k)$, for $\forall i, t$.

Step 2: $\lambda_t^{k+1} = \lambda_t^k + s^k(D_t - \sum_{i=1}^I p_{it}^k u_{it}^k)$,
$\mu_t^{k+1} = \max(0, \mu_t^k + s^k(R_t(\mu_t^k) - \sum_{i=1}^I p_i^{\max} u_{it}^k + \mu_t^k R_t'(\mu_t^k))), \forall t$.

Step 3: $k \leftarrow k+1$, go to Step 1. ■

(Note that the objective of $(\hat{D})$ is not necessarily concave because $\sum_{t=1}^T \mu_t R_t'(\mu_t)$ may not be concave.)

IV.4 Phase 2 algorithm

Without much modification, the phase 2 algorithm for the unit commitment with adaptive price-based SRR is as follows.

Phase 2 Algorithm

Step 0: $k \leftarrow 0$; $\underline{\lambda}^0$ and $\underline{\mu}^0$ are from Phase 1.

Step 1: Given $\underline{\lambda}^k$ and $\underline{\mu}^k$, solve $\theta(\underline{\lambda}^k, \underline{\mu}^k; \underline{R}(\underline{\mu}^k))$ to obtain $(\underline{u}^k, \underline{p}^k)$.

Step 2: If $(\underline{u}^k, \underline{p}^k)$ is reserve feasible, stop.

Step 3: $\lambda_t^{k+1} = \lambda_t^k$,
$\mu_t^{k+1} = \mu_t^k + s^k \min(0, R_t(\mu_t^k) - \sum_{i=1}^I p_i^{\max} u_{it}^k), \forall t$.

Step 4: $k \leftarrow k+1$, go to Step 1. ∎

It can be shown that if there exists an $\epsilon > 0$ and the step size $s^k > \epsilon$ for $\forall k$, the Phase 2 algorithm terminates within a finite number of steps. Also this phase 2 algorithm is of the sequential bargaining game model. The Stackelberg-type phase 2 algorithm does not necessarily work because the extra term in the subgradient $\mu_t R_t'(\mu_t) < 0$ may outweigh the mismatch of the corresponding SRR, $R_t(\mu_t) - \sum_{i=1}^I p_i^{\max} u_{it}$, and may either cause the value of μ_t to be unchanged in the following iteration or slow down the process.

The three-phase algorithm can be applied to solve the unit commitment with price-based adaptive SRR. Nash or Stackelberg Algorithms can be applied to solve the phase 1 dual optimization. We shall compare the their numerical results in a later section.

IV.5 Discussion

Define $L(\underline{x}, \underline{u}, \underline{p}; \underline{\lambda}, \underline{\mu}, \underline{R}) \equiv$

$$\sum_{t=1}^{T}\sum_{i=1}^{I}[C_i(p_{it})u_{it} + S_i(x_{i,t-1}, u_{it}, u_{i,t-1})] +$$

$$\sum_{t=1}^{T}[\lambda_t(D_t - \sum_{i=1}^{I} p_{it}u_{it}) + \mu_t(R_t - \sum_{i=1}^{I} p_i^{\max} u_{it})]. \quad (12)$$

L is the Lagrangian of the unit commitment problem (P). Suppose $1.05 D_t = R_t^{\min} \le R_t(\mu_t) \le R_t^{\max} = 1.07 D_t$ for all $\mu_t \ge 0$, $\forall t$, and $(\underline{\lambda}^*, \underline{\mu}^*)$ and $(\hat{\underline{\lambda}}, \hat{\underline{\mu}})$ solve (D^*) and $(\hat{D})$ respectively. We have

$$\max_{\underline{\lambda},\underline{\mu}\ge 0} \min_{\underline{x},\underline{u},\underline{p}} L(\underline{x}, \underline{u}, \underline{p}; \underline{\lambda}, \underline{\mu}, \underline{R}^{\min}) \quad (13)$$

$$\le (D^*) \max_{\underline{\lambda},\underline{\mu}\ge 0} \min_{\underline{x},\underline{u},\underline{p}} L(\underline{x}, \underline{u}, \underline{p}; \underline{\lambda}, \underline{\mu}, \underline{R}(\underline{\mu}^*)) \quad (14)$$

$$\le (\hat{D}) \max_{\underline{\lambda},\underline{\mu}\ge 0} \min_{\underline{x},\underline{u},\underline{p}} L(\underline{x}, \underline{u}, \underline{p}; \underline{\lambda}, \underline{\mu}, \underline{R}(\underline{\mu})) \quad (15)$$

$$\le \max_{\underline{\lambda},\underline{\mu}\ge 0} \min_{\underline{x},\underline{u},\underline{p}} L(\underline{x}, \underline{u}, \underline{p}; \underline{\lambda}, \underline{\mu}, \underline{R}(\hat{\underline{\mu}})) \quad (16)$$

$$\le \max_{\underline{\lambda},\underline{\mu}\ge 0} \min_{\underline{x},\underline{u},\underline{p}} L(\underline{x}, \underline{u}, \underline{p}; \underline{\lambda}, \underline{\mu}, \underline{R}^{\max}). \quad (17)$$

To see the inequality between (14) and (15), note that the optimal solution $(\underline{\lambda}^*, \underline{\mu}^*)$ of (D^*) is feasible to $(\hat{D})$. This shows that the revenue of the leader (firm Q) in the Stackelberg game exceeds that in the sequential bargaining game defined in Section IV.2. This is because in the Stackelberg game firm Q has more information about firm P than in the Sequential bargaining game.

Both Algorithm 1 and 2 can be applied to obtain a *price-based adaptive* SRR. For example, if we solve $(\hat{D})$ and obtain $(\hat{\underline{\lambda}}, \hat{\underline{\mu}})$, we can use $\underline{R}(\hat{\underline{\mu}})$ as SRR to define a unit commitment $(\bar{P})$ as below.

$$(\bar{P}) \min_{\underline{u},\underline{x},\underline{p}} \sum_{t=1}^{T}\sum_{i=1}^{I}[C_i(p_{it})u_{it} + S_i(x_{i,t-1}, u_{it}, u_{i,t-1})]$$

$$\text{s.t. } \sum_{i=1}^I p_{it}u_{it} = D_t;\ \sum_{i=1}^I p_i^{\max} u_{it} \ge R_t(\hat{\mu}_t), \forall t.$$

The corresponding dual problem $(\bar{D})$ of $(\bar{P})$ is as follows.

$$(\bar{D}) \max_{\underline{\lambda},\underline{\mu}\ge 0} \min_{\underline{x},\underline{u},\underline{p}} L(\underline{x}, \underline{u}, \underline{p}; \underline{\lambda}, \underline{\mu}, \underline{R}(\hat{\underline{\mu}})) \quad (18)$$

Without really solving $(\bar{D})$, we approximate the duality gap between $(\bar{P})$ and $(\bar{D})$ by the 'duality' gap between $(\bar{P})$ and $(\hat{D})$. Note that this approximation is more restrictive (with a larger value) than the value of the actual duality gap. In our experiments, the 'duality' gap between $(\hat{D})$ and $(\bar{P})$ is achieved within 1.5% for 10-unit-168-hour cases, and within 0.3% for 30-unit-168-hour cases. This means the actual duality gaps between $(\bar{P})$ and $(\bar{D})$ for both cases are actually smaller.

V Numerical results

The algorithms are implemented in FORTRAN on an HP 700 workstation. A 30-unit thermal model problem over a one week planning horizon is tested. The total system capacity is 15,515 MW which is much higher than the peak load. We apply the three-phase algorithm to solve the unit commitment problem. The original fixed SRR method is compared with the adaptive SRR method. We also have tested both Nash and Stackelberg Algorithms in the Phase 1 in the adaptive SRR case. With

$R_t(\mu_t) = 1.06 + 0.01 \tanh(-4(\mu_t - 0.5))$ for $\forall t$, the algorithm performances are summarized in Table 1.

Both adaptive SRR algorithms have lower total generating cost due to the relaxation of SRR, and achieve better solutions in terms of the duality gap. Also the Stackelberg model outperforms the sequential bargaining model (Nash Algorithm) in terms of cost saving because of having more information as explained in Section IV.

The system spinning capacity profiles obtained by fixed SRR and adaptive SRR (Stackelberg model) methods are depicted in Figure 3, in which only one day is shown. The corresponding data can be found in Tables 2 and 3. In Figure 3 it can be seen at hour 12, the SRR has been relaxed down to 5% of the load and yield an actual reserve of 5.8% of the load. Slight relaxation of the SRR at hour 12 affects the commitment in the following hours (due to minimum up/down time constraints). In Figure 3 it can be seen that during the period between hours 15 and 19, a commitment with adequate but more economic spinning reserve is achieved with adaptive SRR.

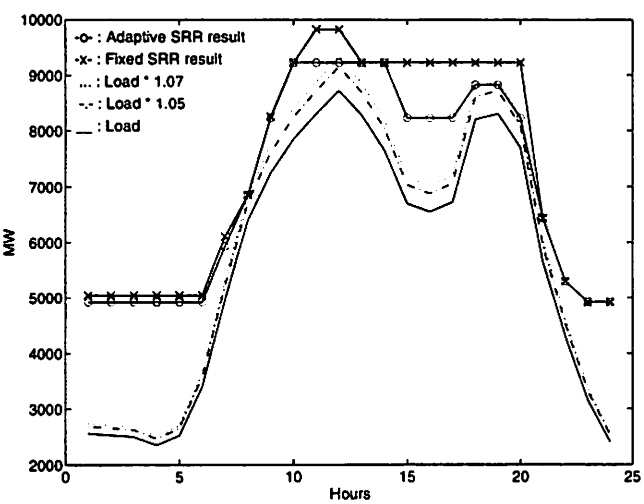

Figure 3: System spinning capacities and SRR.

Table 1: Comparison of methods

methods	total cost ($)	CPU time (sec.) Ph. 1	Ph. 2	Ph. 3	Duality Gap (%)
Fixed SRR*	9333747	11.53	10.1	0.13	0.70
Adp. SRR1†	9296353	10.17	7.64	0.29	0.40
Adp. SRR2‡	9291727	11.56	7.22	0.12	0.35

*Fixed SRR: 7%×Load.

†Adp. SRR 1: Nash Algorithm in Phase 1.

‡Adp. SRR 2: Stackelberg Algorithm in Phase 1.

Table 2: μ_t of selected hours (fixed SRR)

hour	2	4	6	8	10	12
μ_t	0	0	0	0.35	0.595	1.724
SRR(%)	7.0	7.0	7.0	7.0	7.0	7.0
Load(MW)	2530	2350	3400	6400	7850	8720
SSC(MW)†	5045	5045	5045	6855	9225	9825
reserve(%)‡	99.4	114.6	48.4	7.1	17.5	12.6
hour	14	16	18	20	22	24
μ_t	0.437	0	0.892	0.523	0	0
SRR(%)	7.0	7.0	7.0	7.0	7.0	7.0
Load(MW)	7650	6550	8200	7700	4300	2400
SSC(MW)	9225	9225	9225	9225	5295	4920
reserve(%)	20.5	40.8	12.5	19.8	23.1	105

†SSC: System Spinning Capacity

‡reserve(%)=100×(SSC - Load)/Load

Table 3: μ_t of selected hours (Adaptive SRR)

hour	2	4	6	8	10	12
μ_t	0	0	0	0	0.469	1.281
SRR(%)	7.0	7.0	7.0	7.0	6.1	5.0
Load(MW)	2530	2350	3400	6400	7850	8720
SSC(MW)	4920	4920	4920	6855	9225	9225
reserve(%)	94.4	109.3	44.7	7.1	17.5	5.8
hour	14	16	18	20	22	24
μ_t	0.317	0	0.788	0.375	0	0
SRR(%)	6.5	7.0	5.3	6.4	7.0	7.0
Load(MW)	7650	6550	8200	7700	4300	2400
SSC(MW)	9225	8225	8825	8225	5295	4920
reserve(%)	20.5	25.5	7.6	6.8	23.1	105

VI Conclusion

In this paper we have presented a price-based adaptive spinning capacity requirements for power system scheduling. A game theory interpretation is given to the Lagrangian relaxation method for solving unit commitment. We show that this method uses the Lagrange multipliers as price information. The numerical test results show that our method can yield both cost savings and improved algorithm performance.

VII Acknowledgement

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Reference

[1] M. S. Bazaraa and C. M. Shetty, *Nonlinear Programming - Theory and Algorithms*, John Wiley & Sons

Inc., New York, 1979.

[2] D. P. Bertsekas, G. S. Lauer, N. R. Sandell, Jr., and T. A. Posbergh, "Optimal short-term scheduling of large-scale power systems," *IEEE Tran. Automatic Control*, vol. AC-28, no. 1, pp. 1-11, 1983.

[3] A. Cohen and V. Sherkat, "Optimization-based methods for operations scheduling," *Proceedings of IEEE*, vol. 75, no. 12, pp. 1574-1591, 1987.

[4] L. A. F. M. Ferreira, T. Andersson, C. F. Imparato, T. E. Miller, C. K. Pang, A. Svoboda, and A. F. Vojdani, "Short-term resource scheduling in multi-area hydrothermal power systems," *Electrical Power & Energy Systems*, vol. 11, no. 3, 1989.

[5] R. Gibbons, *Game theory for applied economists*, Princeton University Press, Princeton, NJ, 1992.

[6] X. Guan, P. B. Luh, and B. Prasannan, "Power system scheduling with fuzzy reserve requirements", presented in 1995 IEEE/PES summer meeting, Portland, OR, July, 1995.

[7] X. Guan, P. B. Luh, and H. Yan, "An optimization-based method for unit commitment," *Electrical Power & Energy Systems*, vol. 14, no. 1, pp. 9-17, February 1992.

[8] B. T. Polyak, "Minimization of unsmooth functionals," *USSR Comput. Math. Math. Phys.*, vol. 9, pp 14-29, 1969.

[9] A. J. Svoboda, "Simulation of dispatchable demand-side management in electric power system operations planning", Ph.D. Thesis, University of California at Berkeley, Nov. 1992.

[10] A. J. Svoboda and S. S. Oren, "Integrating price-based resources in short-term scheduling of electric power systems", *IEEE Trans. Energy Conversion*, vol. 9, no. 4, pp. 760-9, Dec. 1994.

[11] A. J. Svoboda, C. L. Tseng, C. A. Li, and R. B. Johnson, 'Short term resource scheduling with ramp constraints', presented at the 1995 IEEE/PES Winter meeting.

[12] C. L. Tseng, S. S. Oren, A. J. Svoboda, and R. B. Johnson, 'A unit decommitment method in power system scheduling', Report UCB/ERL/IGCT M96/31, UC Berkeley, May 1996.

[13] F. Zhuang and F. D. Galiana, "Towards a more rigorous and practical unit commitment by Lagrangian relaxation," *IEEE Trans Power Systems*, vol. PWRS-3, no. 2, pp. 763-770, 1988.