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# A PHYSICAL POLY-SILICON THIN FILM TRANSISTOR (TFT) MODEL FOR CIRCUIT SIMULATION

by

Chester Li

Memorandum No. UCB/ERL M93/82

22 November 1993

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### **ELECTRONICS RESEARCH LABORATORY**

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### **Abstract**

This report presents a poly-silicon thin film transistors model for circuit simulations. The drain current model includes the effects of hot carrier, drain induced barrier lowering (DIBL), channel length modulation (CLM), and gate induced drain leakage (GIDL). The capacitance model is linked to the drain current and its derivatives. This model has been implemented in SPICE. Simulation and experimental results are compared.

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### **Chapter 1: Introduction**

Poly-silicon Thin Film Transistors (TFTs) are widely used in active matrix liquid crystal displays (LCDs) to drive the pixels and decode the image signals. A common substrate material for TFTs is poly-silicon deposited on a glass substrate (fig 1.1). The typical operating bias in the small size LCD environment is about 12V. The characteristics of the TFTs are severely affected by imperfections at the poly-silicon surface. Moreover, the TFTs operate with a floating substrate. As a result, the characteristics of a TFT cannot be modeled accurately by the common bulk MOSFET model in SPICE.

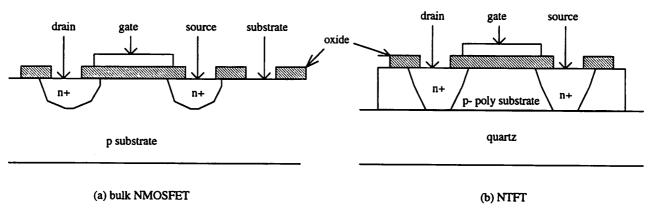


Fig 1.1: Cross sections of a bulk NMOSFET and a NTFT

This report presents a poly-TFT drain current model with an accompanying intrinsic capacitance model. The formulation of the drain current model is similar to BSIM3 [2]. The capacitance model is capacitance based and linked to the drain current. They have been implemented in SPICE. Ring oscillator simulation results will be compared to experimental data. The data used in this study are measured from a LCD wafer with p- substrate and top gate structure. The gate oxide thickness is 76nm. The effective channel length ranges from  $3.3\mu m$  to  $5.3\mu m$ .

Chapter 2 presents the drain current model. Chapter 3 discusses the intrinsic capacitance model. Chapter 4 describes the model implementation in SPICE. Finally, chapter 5 compares the simulation results and experimental data. Equations implemented in SPICE and parameters extraction procedures are summarized in the appendixes.

# **Chapter 2: Drain Current Model**

#### 2.1 Overview

This chapter discusses the TFT drain current model. The model is separated into subthreshold and strong inversion regions. The strong inversion region is further divided into the linear and saturation regions. This physical model describes the hot carrier, drain induced barrier lowering (DIBL), channel length modulation [1], thermal generation, and gate induced rain leakage (GIDL) [4] effects. A parabolic smoothing function [1] is used to ensure continuity of the first order derivative between different regions of operation. Four parameters,  $V_{gtranh}$ ,  $V_{dtranh}$ , and  $V_{dtranh}$  are used to define the transition regions between different bias regions. The transition regions are defined around  $V_{T}$  and  $V_{dsat}$ .  $V_{T}$  is the threshold voltage.  $V_{dsat}$  is the saturation voltage. Figure 2.1a illustrate the boundary values.

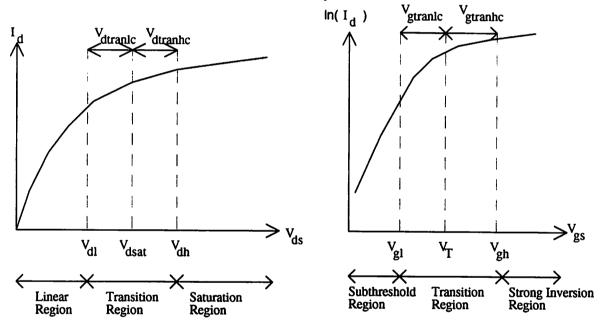


Figure 2.1a: Boundary values for transition region

Section 2.2 and 2.3 discuss the model in the strong inversion and subthreshold region. Section 2.4 describes the transition region between the strong inversion and subthreshold region. Section 2.5 compares the model with the measured data. Appendix A summaries the drain current equations implemented in SPICE. The parameters and their meanings are summarized in chapter 4. The following symbols are defined for model derivation (see figure 2.1a). T is temperature.

$$\begin{aligned} V_{dh} &= V_{dsat} + V_{dtranh} & V_{dl} &= V_{dsat} - V_{dtranl} \\ V_{gh} &= V_T + V_{gtranh} & V_{gl} &= V_T - V_{gtranl} \end{aligned}$$

$$V_{T} = V_{TO} - bT$$

$$V_{dsat} = \left(\frac{1}{V_{gs} - V_{T}} + \frac{1}{E_{sat}L_{eff}}\right)^{-1}$$

#### 2.2 Strong Inversion Region Model

### 2.2.1 Linear Region $(V_{gs} > V_{gh}, 0 < V_{ds} < V_{dl})$

Following Huang's [2] approach, the drain current in the linear region is:

$$I_{d} = \frac{W_{eff}}{L_{eff}} C_{ox} \mu_{eff} \left( V_{gs} - V_{T} - \frac{V_{ds}}{2} \right) \frac{V_{ds}}{1 + \frac{V_{ds}}{E_{sat} L_{eff}}}$$

$$E_{sat} = \frac{2v_{sat}}{\mu_{eff}}$$
  $C_{ox} = \frac{\varepsilon_{ox}}{T_{ox}}$ 

Poly-silicon TFTs have many interface traps at the Si-SiO<sub>2</sub> interface, especially at or near the grain boundaries. As a result, the electrons (or holes in p-channel TFT) have to hop over the barrier formed at the grain boundaries along the channel during electrical conduction (figure 2.2.1a).  $ln(\mu)$ , the logarithm of mobility, is inversely proportional to the barrier height  $(\phi_b)$ . Since  $\phi_b$  is modulated by the gate bias  $(V_{gs})$ , mobility has the form of  $\alpha exp(-\beta/(V_{gs}-V_T))$ .  $\alpha$ ,  $\beta$ , and  $V_T$  are function of temperature.  $\alpha$  and  $\beta$  are  $\mu_0(kT/q)^{-\mu 1}$  and  $-q\mu_2 exp(\mu_3 T)/kTC_{ox}$  [1]. This expression describes the mobility well when  $V_{gs}$  is much bigger than  $V_T$ . An additional  $\mu_4$ , the minimum mobility at low  $V_{gs}$ , is added to the model. Therefore, the effective mobility is modeled as:

$$\mu_{eff} = \mu_o \left(\frac{kT}{q}\right)^{-\mu_1} exp \left(\frac{-q\mu_2 exp(\mu_3 T)}{kTC_{ox}(V_{gs} - V_T)}\right) + \mu_4$$

 $\mu_0$ ,  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$ , and  $\mu_4$  are fitting parameters.  $V_T$  is the threshold voltage, which is approximated as a linear function of temperature [4].

$$V_T = V_{TO} - bT$$

Figure 2.2.1b shows the temperature dependence of  $V_T$ .  $V_T$  decreases when temperature increases, which has the same trend as bulk MOSFET [4]. It indicates that the change in fermi level with temperature is the dominant mechanism. Figure 2.2.1c plots the mobility of a p-channel (PTFT) and a n-channel (NTFT) poly-TFT at  $V_{ds}$ =0.1V at room temperature. When  $V_{gs}$  increases,  $\phi_b$  at the grain boundaries drops. Therefore, mobility increases as  $V_{gs}$  increases. The

mobility of poly-TFTs is lower than that for bulk MOSFET. Therefore the current drive is also smaller. Since the conduction is limited by excitation over the barrier, the mobility rises as temperature increases. Figure 2.2.1d plots the mobility of a NTFT at different temperatures.

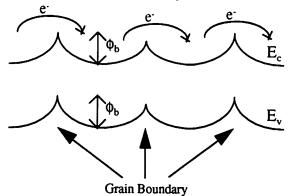


Figure 2.2.1a: Band diagram of a poly-TFT channel and electron conduction of a n-channel device

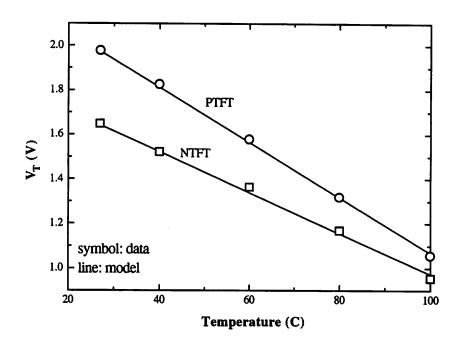


Figure 2.2.1b: Threshold voltage dependence on temperature

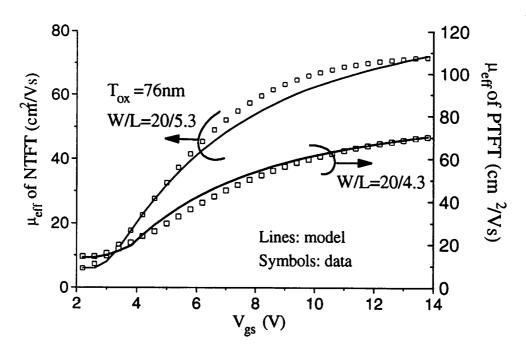


Figure 2.2.1c: Mobility of a PTFT and an NTFT at room temperature

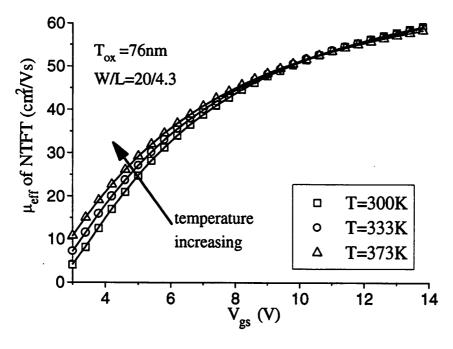


Figure 2.2.1d: Mobility of a NTFT at different temperatures.

Lines are model and symbols are data

### 2.2.2 Saturation Region $(V_{gs} > V_{gh}, V_{ds} > V_{dh})$

Similar to BSIM3 [2] approach, the DIBL, hot carrier, and channel length modulation effect of drain current in the saturation region are modeled. The drain model is expressed as:

$$I_{d} = I_{dsat} \left( 1 + \frac{V_{ds} - V_{dsat}}{V_{A}} \right) f_{hc}$$

$$V_{A} = \left( \frac{1}{V_{ACLM}} + \frac{1}{V_{ADIBL}} \right)^{-1}$$

$$V_{ACLM} = \frac{\left( E_{sat} L_{eff} + V_{gs} - V_{T} \right) (V_{ds} - V_{dsat})}{E_{sat} \ell}$$

$$V_{ADIBL} = \frac{E_{sat} L_{eff} + V_{gs} - V_{T}}{\theta (1 + 2E_{sat} L_{eff} / (V_{gs} - V_{T}))}$$

$$f_{hc} = 1 + s_{1} (V_{ds} - V_{dsat}) exp \left( \frac{-s_{2}}{V_{ds} - V_{dsat}} \right)$$

$$E_{sat} = 2v_{sat} / \mu_{eff}$$

 $I_{dsat}$  is the drain current at  $V_{ds}=V_{dsat}$  using the linear  $I_{d}$  equation.  $V_{ACLM}$  models the channel length modulation effect.  $\ell$  is a fitting parameter.  $V_{ADIBL}$  models the DIBL effect. The hot carrier effect, which is caused by the high electric field at the drain, is modeled by  $f_{hc}$ .  $\theta$  is the DIBL effect coefficient.  $s_1$  and  $s_2$  are fitting parameters for hot carrier effect.

## 2.2.3 Transition Region in Strong Inversion Region ( $V_{gs} > V_{gh}$ , $V_{dl} < V_{ds} < V_{dh}$ )

To improve the convergence property of the model, the first order derivative is made continuous by using a parabolic smoothing function from BSIM3 [2]. The basic concept is illustrated by figure 2.2.3a.  $I_{dl}$  is the current in the linear region at  $V_{dl}$  and the applied  $V_{gs}$ .  $I_{dh}$  is the current in the saturation region at  $V_{dh}$  and the applied  $V_{gs}$ . L1 is the tangent to the linear region at  $V_{ds}=V_{dl}$ . L2 is the tangent to the saturation region at  $V_{ds}=V_{dh}$ . The intersect of L1 and L2 is  $(V_{dp}, I_{dp})$ . Once  $(V_{dh}, I_{dh})$ ,  $(V_{dp}, I_{dp})$ , and  $(V_{dl}, I_{dl})$  are determined, the points between  $V_{dh}$  and  $V_{dl}$  can be computed using the parabolic smoothing function with first order derivative continuity. The expression for  $V_{dp}$ ,  $I_{dp}$ ,  $I_{dp}$ , and the first order derivatives are shown below.

$$\begin{split} V_{ds} &= (1-t)^2 \, V_{dl} + 2 t (1-t) V_{dp} + t^2 V_{dh} \qquad I_d = (1-t)^2 \, I_{dl} + 2 t (1-t) I_{dp} + t^2 I_{dh} \\ V_{dp} &= \frac{I_{dh} - I_{dl} - (g_{dsh} V_{dh} - g_{dsl} V_{dl})}{g_{dsl} - g_{dsh}} \\ I_{dp} &= g_{dsl} (V_{dp} - V_{dl}) + I_{dl} \\ g_{ds} &= \frac{t (I_{dh} - I_{dp}) + (1-t) (I_{dp} - I_{dl})}{t (V_{dh} - V_{dp}) + (1-t) (V_{dp} - V_{dl})} \\ g_{m} &= g_{ml} + \frac{I_{d} - I_{dl}}{I_{dh} - I_{dl}} \frac{V_{dh} - V_{dl}}{V_{ds} - V_{dl}} (g_{mh} - g_{ml}) \\ g_{dsh} &= \frac{\partial I_{d}}{\partial V_{ds}} \bigg|_{V_{dh}, V_{gs}} g_{dsl} &= \frac{\partial I_{d}}{\partial V_{ds}} \bigg|_{V_{dl}, V_{gs}} \\ g_{mh} &= \frac{\partial I_{d}}{\partial V_{gs}} \bigg|_{V_{dh}, V_{gs}} g_{ml} &= \frac{\partial I_{d}}{\partial V_{gs}} \bigg|_{V_{dl}, V_{gs}} \end{split}$$

t can be expressed as a function of  $V_{ds},\,V_{dp},$  and  $V_{dh}$  as:

$$t = \frac{\left(V_{dl} - V_{dp}\right) + \sqrt{\left(V_{dl} - V_{dp}\right)^{2} - \left(V_{dl} - V_{ds}\right)\left(V_{dl} - 2V_{dp} + V_{dh}\right)}}{V_{dl} - 2V_{dp} + V_{dh}}$$

$$I_{dh}$$

$$I_{dp}$$

$$I_{dl}$$

Figure 2.2.3a: Scheme to connect the saturation and linear region

## 2.3 Subthreshold Region Model $(V_{gs} < V_T - V_{gtranl})$

The diffusion current, thermal generation current, and GIDL current are modeled in the subthreshold region. Since the interface traps density is high in TFT, the subthreshold swing (~380mV/decade) of the diffusion current is higher than that in the normal MOSFET (~100mV/decade). The subthreshold drain current is expressed as:

$$\begin{split} I_{d} &= I_{diff} + I_{gidl} + I_{thermal} \\ \\ I_{diff} &= W_{eff} I_{do} \bigg( 1 - exp \bigg( \frac{-V_{ds}}{kT/q} \bigg) \bigg) exp \bigg( \frac{V_{gs} - V_T - V_{off}}{nkT/q} \bigg) \\ \\ I_{gidl} &= W_{eff} A_{gidl} \bigg( V_{dg} - V_i \bigg) exp \bigg( \frac{-B_{gidl}}{V_{dg} - V_i} \bigg) \qquad \qquad V_{dg} = V_{ds} - V_{gs} \\ \\ I_{thermal} &= W_{eff} I_{thermal0} exp \bigg( \frac{-E_a}{kT/q} \bigg) \end{split}$$

 $V_{off}$  is the offset voltage for  $I_{diff}$ . n is the subthreshold slope.  $I_{do}$  is a function of oxide thickness and substrate doping [2]. It is treated as a fitting parameter in this model.  $A_{gidl}$ ,  $B_{gidl}$ , and  $V_i$  are fitting parameters for GIDL current.  $I_{gidl}$  is set to zero when  $(V_{dg}-V_i)$  is less than zero. This equation is same as in [3], except that [3] fixes  $V_i$  to 1.2V, where  $V_i$  here is kept as a parameter because of the abundance of interface traps between the conduction band and valance band.  $E_a$  is the activation energy for the thermal current generation.  $I_{thermal0}$  is a fitting parameter for  $I_{thermal}$ .

# 2.4 Transition Region between Strong Inversion and Subthreshold Region (V $_{gl}\!<\!V_{gs}\!<\!V_{gh}\!)$

To improve the convergence property of the model, the first order derivative is made continuous by using the same scheme as in section 2.2.3. A parabolic smoothing function in the linear-linear scale is used. The basic concept is illustrated by figure 2.4a.

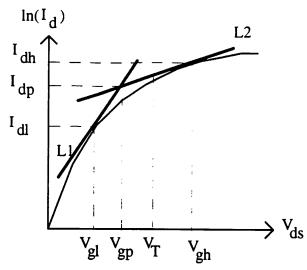


Figure 2.4a: Scheme to connect the subthreshold and strong inversion region

 $I_{dl}$  is the current in the subthreshold region at  $V_{gl}$  and the applied  $V_{ds}$ .  $I_{dh}$  is the current in the strong inversion region at  $V_{gh}$  and the applied  $V_{ds}$ . L1 is the tangent to the subthreshold region at  $V_{gs}=V_{gl}$ . L2 is the tangent to the strong inversion region at  $V_{gs}=V_{gh}$ . The intersect of L1 and L2 is  $(V_{gp}, I_{dp})$ . Once  $(V_{gh}, I_{dh})$ ,  $(V_{gp}, I_{dp})$ , and  $(V_{gl}, I_{dl})$  are determined, the points between  $V_{gh}$  and  $V_{gl}$  can be computed using the parabolic smoothing function with first order derivative continuity. The expression for  $V_{gp}$ ,  $I_{dp}$ ,  $I_{d}$ , and the first order derivatives are shown below.

$$\begin{split} V_{gs} &= (1-t)^2 V_{gl} + 2t(1-t) V_{gp} + t^2 V_{gh} \\ I_d &= (1-t)^2 I_{dl} + 2t(1-t) I_{dp} + t^2 I_{dh} \\ V_{gp} &= \frac{I_{dh} - I_{dl} - \left(g_{mh} V_{gh} - g_{ml} V_{gl}\right)}{g_{ml} - g_{mh}} \\ I_{dp} &= g_{ml} \left(V_{gp} - V_{gl}\right) + I_{dl} \\ g_m &= \frac{t \left(I_{dh} - I_{dp}\right) + (1-t) \left(I_{dp} - I_{dl}\right)}{t \left(V_{gh} - V_{gp}\right) + (1-t) \left(V_{gp} - V_{gl}\right)} \\ g_{ds} &= g_{dsl} + \frac{I_d - I_{dl}}{I_{dh} - I_{dl}} \frac{V_{gh} - V_{gl}}{V_{gs} - V_{gl}} \left(g_{dsh} - g_{dsl}\right) \end{split}$$

$$g_{dsh} = \frac{\partial I_d}{\partial V_{ds}} \bigg|_{V_{ds}, V_{gh}} g_{dsl} = \frac{\partial I_d}{\partial V_{ds}} \bigg|_{V_{ds}, V_{gl}}$$

$$g_{mh} = \frac{\partial I_d}{\partial V_{gs}} \bigg|_{V_{ds}, V_{gh}} g_{ml} = \frac{\partial I_d}{\partial V_{gs}} \bigg|_{V_{ds}, V_{gl}}$$

t can be expressed as a function of  $V_{gs}$ ,  $V_{gp}$ , and  $V_{gh}$  as:

$$t = \frac{\left(V_{gl} - V_{gp}\right) + \sqrt{\left(V_{gl} - V_{gp}\right)^2 - \left(V_{gl} - V_{gs}\right)\left(V_{gl} - 2V_{gp} + V_{gh}\right)}}{V_{gl} - 2V_{gp} + V_{gh}}$$

### 2.5 Verification

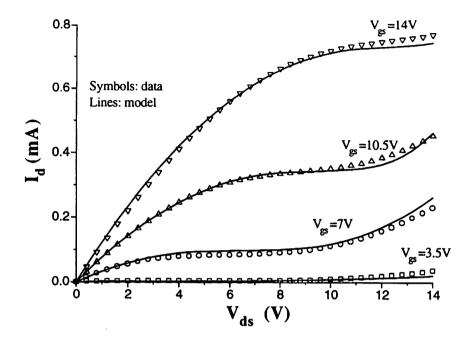


Figure 2.5a:  $I_dV_{ds}$  of a 20/5.3 NTFT with  $T_{ox}$ =76nm

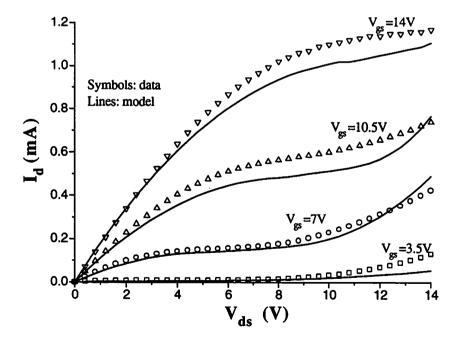


Figure 2.5b:  $I_dV_{ds}$  of a 20/3.3 NTFT with  $T_{ox}$ =76nm. The model curve is generated using the same set of parameters extracted from the 20/5.3 device.  $\Delta L$ =1.7 $\mu$ m (extracted by capacitance method)

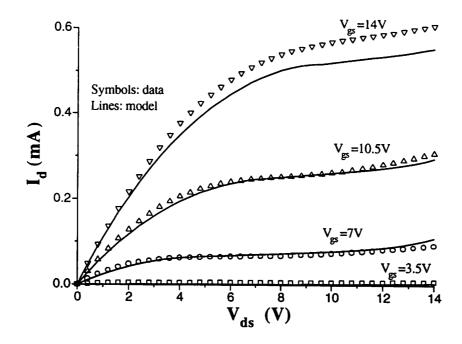


Figure 2.5c:  $I_dV_{ds}$  of a 20/4.42 PTFT with  $T_{ox}$ =76nm

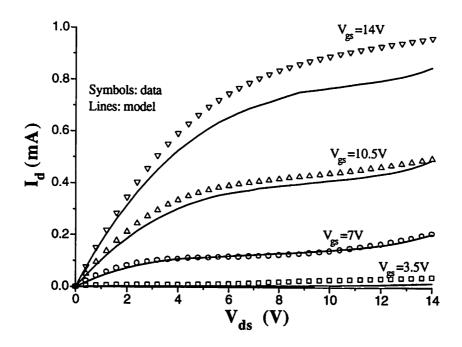


Figure 2.5d:  $I_dV_{ds}$  of a 20/2.42 PTFT with  $T_{OX}$ =76nm. The model curve is generated using the same set of parameters extracted from the 20/4.42 device.  $\Delta L$ =2.58 $\mu$ m (extracted by capacitance method)

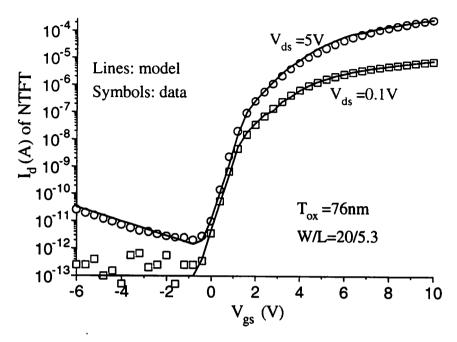


Figure 2.5e:  $I_dV_{gs}$  of a 20/5.3 NTFT with  $T_{ox}$ =76nm

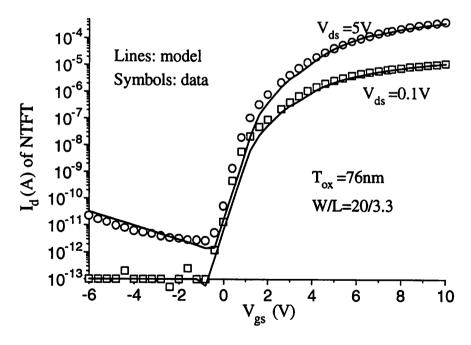


Figure 2.5f:  $I_dV_{gs}$  of a 20/3.3 NTFT with Tox=76nm

# **Chapter 3: Intrinsic Capacitance Model**

#### 3.1 Overview

A capacitance-base model is developed to model  $C_{gs}$  and  $C_{gd}$ . The model is separated into strong inversion and subthreshold regions. The strong inversion region model is developed from charge equations and linked to the drain current model. The subthreshold region model is empirical. The subthreshold region, strong inversion linear region, and strong inversion saturation region are linked by a linear function to model the gradual change in capacitance when the TFT is switched from one bias region to another. Four parameters are used to define the transition region between different bias regions. They are:  $V_{dtranle}$ ,  $V_{dtranle}$ ,  $V_{gtranhe}$ , and  $V_{gtranle}$ . The transition regions are defined around  $V_{T}$  and  $V_{dsat}$ .  $V_{T}$  is the threshold voltage.  $V_{dsat}$  is the saturation voltage, which is  $[1/(V_{gs}-V_{T})+1/(E_{sat}L_{eff})]^{-1}$ . The boundaries are illustrated in figure 3.1a. Two additional parameter,  $A_{cgs}$  and  $A_{cgd}$ , are added to model  $C_{gs}$  and  $C_{gd}$  in the GIDL dominant region.

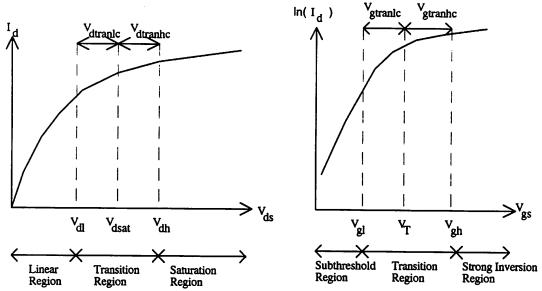


Figure 3.1a: Definition of the boundaries values

Section 3.2 and 3.3 describe the strong inversion and subthreshold model. Section 3.4 verifies the model with measured data. The equations implemented in SPICE is summarized in appendix A. The parameters and their meanings are summarized in chapter 4. The following symbols are defined for the model derivation (see figure 3.1a).

$$\begin{aligned} V_{dl} &= V_{dsat} - V_{dtranlc} & V_{dh} &= V_{dsat} + V_{dtranhc} \\ V_{gl} &= V_T - V_{gtranlc} & V_{gh} &= V_T + V_{gtranhc} \end{aligned}$$

$$V_{gst} = V_{gs} - V_{T} \qquad V_{gstd} = V_{gst} - V_{ds} \qquad V_{gstdsat} = V_{gst} - V_{dsat}$$

$$g_{ds} = \frac{\partial I_{d}}{\partial V_{ds}} \qquad g_{m} = \frac{\partial I_{d}}{\partial V_{gs}}$$

#### 3.2 Strong Inversion Region Model

# 3.2.1 $C_{gd}$ in the Linear Region (0 < $V_{ds}$ < $V_{dl}$ , $V_{gs}$ > $V_{gh}$ )

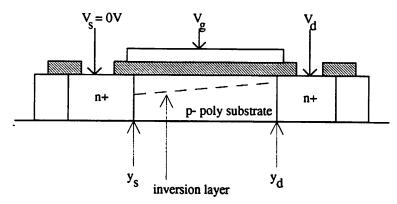


Figure 3.2.1a: Cross section of a NTFT in linear region

Figure 3.2.1a shows the cross section of a NTFT In the linear region ( $V_{ds} < V_{dsat}$  and  $V_{gs} > V_T$ ). The inversion charge density gradually drops from  $C_{ox}(V_{gs} - V_T - V_s)$  at the source  $(y_s)$  to  $C_{ox}(V_{gs} - V_T - V_d)$  at the drain  $(y_d)$ . The charge at the gate is:

$$Q_g = -W_{\text{eff}} \int_{y_s}^{y_d} Q_n(y) dy + Q_{\text{bulk}}$$

 $Q_{bulk}$  is the depletion charge in the substrate.  $Q_n(y)$ , the inversion charge, is:

$$Q_n(y) = C_{ox} \left[ V_{gs} - V_T - V(y) \right]$$

 $Q_g$  can also be re-written as:

$$Q_g = -W_{\text{eff}} \int_{V_s}^{V_d} Q_n(V) \frac{1}{dV/dy} dV + Q_{\text{bulk}}$$

Huang [2] shows that:

$$\frac{dV}{dy} = \frac{I_d}{W_{eff} \mu_{eff} Q_n(V) - I_d / E_{sat}}$$

Therefore, Qg is:

$$Q_g = -W_{eff} \int_{V_s}^{V_d} Q_n(V) \left[ W_{eff} \mu_{eff} Q_n(V) - \frac{I_d}{E_{sat}} \right] / I_d dV + Q_{bulk}$$

 $C_{gd}$  is defined as  $\frac{\partial Q_g}{\partial V_d}$ . Since  $Q_{bulk}$  is not a strong function of  $V_{ds}$ ,  $C_{gd}$  is :

$$\begin{split} C_{gd} &= \frac{g_{ds}}{I_d^2} W_{eff}^2 \mu_{eff} \int\limits_{V_s}^{V_d} Q_n^2(V) dV - \frac{W_{eff}^2 \mu_{eff}}{I_d} Q_n^2(V_d) + \frac{W_{eff}}{E_{sat}} Q_n(V_d) \\ &\int\limits_{V_s}^{V_d} Q_n^2(V) dV = \frac{1}{3} C_{ox}^2 \Big( V_{gst}^3 - V_{gstd}^3 \Big) \\ &E_{sat} = \frac{2 v_{sat}}{\mu_{off}} \end{split}$$

# 3.2.2 $C_{gd}$ in the Saturation Region ( $V_{ds} > V_{dh}, \, V_{gs} > V_{gh}$ )

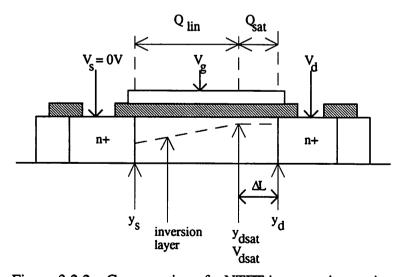


Figure 3.2.2a: Cross section of a NTFT in saturation region

Figure 3.2.2a shows the cross section of a NTFT in the saturation region  $(V_{ds}>V_{dsat})$  and  $V_{gs}>V_T$ . The inversion charge density drops from  $C_{ox}(V_{gs}-V_T-V_s)$  at the source  $(y_s)$  to  $C_{ox}(V_{gs}-V_T-V_{dsat})$  at  $y_{dsat}$ , where  $V(y_{dsat})=V_{dsat}$ . Then the charge density stays constant at  $C_{ox}(V_{gs}-V_T-V_{dsat})$  from  $y_{dsat}$  to the drain  $(y_d)$ , which is known as the velocity saturation region [4]. The gate charge is:

$$Q_g = Q_{lin} + Q_{sat} + Q_{bulk}$$

$$Q_{lin} = -W_{eff} \int_{y_s}^{y_{dsat}} Q_n(y) dy \qquad Q_{sat} = -W_{eff} \int_{y_{dsat}}^{y_d} Q_n(y) dy$$

 $Q_{lin}$  is the charge from  $y_s$  to  $y_{dsat}$ .  $Q_{sat}$  is the charge in the velocity saturation region. Assuming that the gradual channel approximation still holds from  $y_s$  to  $y_{dsat}$ ,  $Q_{lin}$  can be expressed as a function of voltage bias in the same way as section 3.2.1.

$$Q_{lin} = -W_{eff} \int_{V_s}^{V_{dsat}} Q_n(V) \left[ W_{eff} \mu_{eff} Q_n(V) - \frac{I_d}{E_{sat}} \right] / I_d dV$$

$$\frac{\partial Q_{\text{lin}}}{\partial V_{\text{d}}} = \frac{1}{3} W_{\text{eff}}^2 \mu_{\text{eff}} \frac{g_{\text{ds}}}{I_{\text{d}}^2} C_{\text{ox}}^2 \left( V_{\text{gst}}^3 - V_{\text{gstdsat}}^3 \right)$$

Q<sub>sat</sub> is approximated as:

$$Q_{sat} = -W_{eff}C_{ox}(V_{gs} - V_T - V_{dsat})\Delta L$$

$$\frac{\partial Q_{sat}}{\partial V_{d}} = -W_{eff}C_{ox}(V_{gs} - V_{T} - V_{dsat})\frac{\partial \Delta L}{\partial V_{d}}$$

Ko [5] shows that:

$$\Delta L = \ell \ln \left( \frac{\left( V_{d} - V_{dsat} \right) / \ell + E_{m}}{E_{sat}} \right)$$

$$E_{\rm m} = \left[ \left( \frac{V_{\rm d} - V_{\rm dsat}}{\ell} \right)^2 + E_{\rm sat}^2 \right]^{1/2}$$

Therefore,  $\frac{\partial \Delta L}{\partial V_d}$  is:

$$\frac{\partial \Delta L}{\partial V_d} = \frac{1}{\left(V_d - V_{dsat}\right) / \ell + E_m} \left(1 + \frac{V_d - V_{dsat}}{\ell E_m}\right)$$

 $C_{gd}$  is defined as  $\frac{\partial Q_g}{\partial V_d}.$  Since  $Q_{bulk}$  is not a strong function of  $V_d,$   $C_{gd}$  is :

$$C_{gd} = \frac{\partial Q_{lin}}{\partial V_d} + \frac{\partial Q_{sat}}{\partial V_d}$$

# 3.2.3 $C_{gs}$ in the Linear Region (0 < $V_{ds}$ < $V_{dl},\,V_{gs}$ > $V_{gh})$

From charge conservation, one can conclude that  $\frac{\partial Q_g}{\partial V_g} + \frac{\partial Q_g}{\partial V_s} + \frac{\partial Q_g}{\partial V_d} = 0$ . Since  $C_{gd}$  is

 $\frac{\partial Q_g}{\partial V_d}$  and  $C_{gs}$  is  $\frac{\partial Q_g}{\partial V_s},$   $C_{gs}$  can be expressed as :

$$C_{gs} = -\frac{\partial Q_g}{\partial V_g} - C_{gd}$$

Using the same approach as in section 3.2.1 and performing the integration,  $\mathbf{Q}_{\mathbf{g}}$  becomes :

$$Q_{g} = \frac{1}{3} \frac{C_{ox}^{2} W_{eff}^{2} \mu_{eff}}{I_{d}} \left(V_{gstd}^{3} - V_{gst}^{3}\right) - \frac{1}{2} \frac{W_{eff} C_{ox}}{E_{sat}} \left(V_{gstd}^{2} - V_{gst}^{2}\right) + Q_{bulk}$$

Therefore,  $\frac{\partial Q_g}{\partial V_g}$  is:

$$\begin{split} \frac{\partial Q_g}{\partial V_g} &= \frac{1}{3} \frac{C_{ox}^2 W_{eff}^2}{I_d} \Bigg[ \left( \frac{\partial \mu_{eff}}{\partial V_g} - \frac{\mu_{eff}}{I_d} g_m \right) \! \left( V_{gstd}^3 - V_{gst}^3 \right) + 3 \mu_{eff} \left( V_{gstd}^2 - V_{gst}^2 \right) \Bigg] \\ &- \frac{1}{2} W_{eff} C_{ox} \Bigg[ \frac{\partial (1/E_{sat})}{\partial V_g} \left( V_{gstd}^2 - V_{gst}^2 \right) + \frac{2}{E_{sat}} \left( V_{gstd} - V_{gst} \right) \Bigg] \end{split}$$

To compute  $C_{gs}$ , the above equation and the  $C_{gd}$  equation from section 3.2.1 will be substituted

into 
$$C_{gs} = -\frac{\partial Q_g}{\partial V_g} - C_{gd}$$
.

### 3.2.4 $C_{gs}$ in the Saturation Region $(V_{ds} > V_{dh}, V_{gs} > V_{gh})$

Following the same approach in section 3.2.3 and 3.2.2 (figure 3.2.2a), Qg is expressed as:

$$Q_g = Q_{bulk} + Q_{lin} + Q_{sat}$$

$$\begin{aligned} Q_{lin} &= \frac{1}{3} \frac{\mu_{eff} W_{eff}^2 C_{ox}^2}{I_d} \left( V_{gstdsat}^3 - V_{gst}^3 \right) - \frac{1}{2} \frac{W_{eff} C_{ox}}{E_{sat}} \left( V_{gstdsat}^2 - V_{gst}^2 \right) \\ Q_{sat} &= -W_{eff} C_{ox} V_{gstdsat} \Delta L \end{aligned}$$

 $Q_{sat}$  is the charge in the velocity saturation region.  $Q_{lin}$  is the charge from  $y_s$  to  $y_{dsat}$  (figure 3.2).  $Q_{bulk}$  is the depletion charge. Since  $Q_{bulk}$  is a weak function of  $V_g$ ,  $\partial Q_g/\partial V_g$  is :

$$\frac{\partial Q_g}{\partial V_g} = \frac{\partial Q_{lin}}{\partial V_g} + \frac{\partial Q_{sat}}{\partial V_g}$$

$$\begin{split} \frac{\partial Q_{lin}}{\partial V_g} &= \frac{1}{3} \frac{C_{ox}^2 W_{eff}^2}{I_d} \Bigg[ \Bigg( \frac{\partial \mu_{eff}}{\partial V_g} - \frac{\mu_{eff}}{I_d} g_m \Bigg) \Big( V_{gstdsat}^3 - V_{gst}^3 \Big) + 3 \mu_{eff} \Bigg( V_{vgstdsat}^2 \Bigg( 1 - \frac{\partial V_{dsat}}{\partial V_g} \Bigg) - V_{gst}^2 \Bigg) \Bigg] \\ &- \frac{1}{2} W_{eff} C_{ox} \Bigg[ \frac{\partial (1/E_{sat})}{\partial V_g} \Big( V_{gstdsat}^2 - V_{gst}^2 \Big) + \frac{2}{E_{sat}} \Bigg( V_{gstdsat} \Bigg( 1 - \frac{\partial V_{dsat}}{\partial V_g} \Bigg) - V_{gst} \Bigg) \Bigg] \\ &\frac{\partial Q_{sat}}{\partial V_g} = - W_{eff} C_{ox} \Bigg( 1 - \frac{\partial V_{dsat}}{\partial V_g} \Bigg) \Delta L - W_{eff} C_{ox} V_{gstdsat} \frac{\partial \Delta L}{\partial V_g} \\ &\frac{\partial V_{dsat}}{\partial V_g} = V_{dsat}^2 \Bigg( \frac{1}{V_{gst}^2} - \frac{1}{L} \frac{\partial (1/E_{sat})}{\partial V_g} \Bigg) \end{split}$$

$$\frac{\partial \Delta L}{\partial V_{g}} = \frac{\ell E_{sat}}{(V_{d} - V_{dsat}) / \ell + E_{m}} \left[ \frac{-\frac{1}{\ell} \frac{\partial V_{dsat}}{\partial V_{g}} + \frac{\partial E_{m}}{\partial V_{g}}}{E_{sat}} + \left( \frac{V_{d} - V_{dsat}}{\ell} + E_{m} \right) \frac{\partial (1 / E_{sat})}{\partial V_{g}} \right]$$

$$\frac{\partial E_{m}}{\partial V_{g}} = \frac{1}{E_{m}} \left[ -\frac{\partial V_{dsat}}{\partial V_{g}} \frac{V_{d} - V_{dsat}}{\ell^{2}} + E_{sat} \frac{\partial E_{sat}}{\partial V_{g}} \right]$$

To compute  $C_{gs}$ , the above equations and the  $C_{gd}$  equations from section 3.2.2 will be substituted

into 
$$C_{gs} = -\frac{\partial Q_g}{\partial V_g} - C_{gd}$$
.

# 3.2.5 Connection Scheme between Linear and Saturation Region ( $V_{gs} > V_{gh}, V_{dl} < V_{ds} < V_{dh}$ )

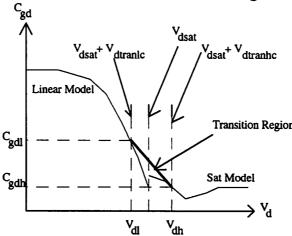


Figure 3.2.5a: Strong inversion transition region model scheme

To reduce the discontinuity between the linear and saturation region, a linear function is used to connect the two regions. A straight line is drawn between  $C_{gd}(C_{gs})$  at  $V_{dh}$  to  $C_{gd}(C_{gs})$  at  $V_{dl}$ . Figure 3.2.5a below illustrates this idea. The linear function used is

$$C_{gd} = a_{cgd}V_{ds} + b_{cgd} \qquad a_{cgd} = \frac{C_{gdh} - C_{gdl}}{V_{dh} - V_{dl}} \qquad b_{cgd} = C_{gdh} - a_{cgd}V_{dh}$$

$$C_{gs} = a_{cgs}V_{ds} + b_{cgs} \qquad a_{cgs} = \frac{C_{gsh} - C_{gsl}}{V_{dh} - V_{dl}} \qquad b_{cgs} = C_{gsh} - a_{cgs}V_{dh}$$

 $C_{gsh}$  and  $C_{gdh}$  are computed using the equations in the saturation region at the applied  $V_{gs}$  and  $V_{dh}$ .  $C_{gsl}$  and  $C_{gdl}$  are computed using the equations in the linear region at the applied  $V_{gs}$  and  $V_{dl}$ .

#### 3.3 Subthreshold Region Model

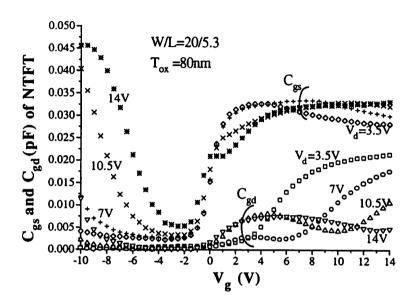


Figure 3.3.1a :  $C_{gs}$  and  $C_{gd}$  vs  $V_{g}$ 

### $3.3.1 C_{gd} Model (V_{gs} < V_{gh})$

Figure 3.3.1a shows the  $C_{gd}$  and  $C_{gs}$  data in a  $V_g$  sweep. Following the behavior of a bulk MOSFET,  $C_{gd}$  gradually decreases to 0 when the TFT is switched from the strong inversion to the weak inversion region. Figure 3.3.1b shows the scheme of the empirical model from the

strong inversion to weak inversion region. A linear function,  $C_{gd} = \frac{C_{gdh}}{V_{gtran}} (V_{gs} - V_{gl})$ , is used to modeled this region. The boundaries of the region are  $V_{gh}$  and  $V_{gl}$ .  $V_{gtran}$  is  $V_{gtranhc} + V_{gtranlc}$  (figure 3.3.1b).  $C_{gdh}$  is computed using the equations in the strong inversion region at the applied  $V_{ds}$  and  $V_{gh}$ .

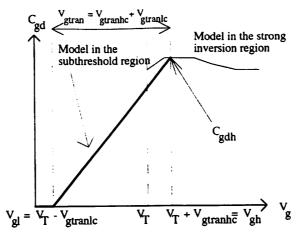


Figure 3.3.1b: Modeling scheme for C<sub>gd</sub> weak inversion region

However, the RC coupling between the drain and substrate makes  $C_{gd}$  increases in the GIDL [3] dominant region (figure 3.3.1a). This does not happen in bulk MOSFET because of the presense of the bulk contact. The coupling efficiency depends of the junction leakage.  $A_{cgd}$  describes this coupling efficiency. Therefore,  $C_{gd}$  at  $V_{gs} < V_{gl}$  is modeled as:

$$C_{gd} = \left(\frac{1}{W_{eff} L_{eff} C_{ox}} + \frac{1}{A_{cgd} I_{d}}\right)^{-1}$$

The maximum  $C_{gd}$  possible is  $W_{eff}L_{eff}C_{ox}$ . The above formulation will limit  $C_{gd}$  to  $W_{eff}L_{eff}C_{ox}$ .

### $3.3.2 C_{gs} Model (V_{gs} < V_{gh})$

 $C_{gs}$ , similar to  $C_{gd}$  (figure 3.3.1a), also gradually drops to 0 from strong inversion to weak inversion region. The same scheme from section 3.3.1 is applied to model the gradual change in  $C_{gs}$  when  $V_{gs}$  is between  $V_{gl}$  and  $V_{gh}$ , i.e.  $V_{gl} < V_{gs} < V_{gh}$ .  $V_{gtran}$  is  $V_{gtranhc} + V_{gtranlc}$  (figure 3.3.2a).  $C_{gsh}$  is computed using the equations in the strong inversion region at the applied  $V_{ds}$  and  $V_{gh}$ .

$$C_{gs} = \frac{C_{gsh}}{V_{gtran}} (V_{gs} - V_{gl})$$

The RC coupling effect is higher in  $C_{gs}$  (figure 3.3.1a).  $A_{cgs}$  describes this coupling efficiency.  $C_{gs}$  at  $V_{gs} < V_{gl}$  is modeled as :

$$C_{gs} = \left(\frac{1}{W_{eff}L_{eff}C_{ox}} + \frac{1}{A_{cgs}I_{d}}\right)^{-1}$$

The maximum  $C_{gs}$  possible is  $W_{eff}L_{eff}C_{ox}$ . The above formulation will limit  $C_{gs}$  to  $W_{eff}L_{eff}C_{ox}$ .

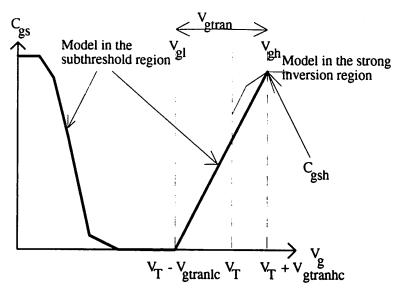


Figure 3.3.2a: Modeling scheme for subthreshold  $C_{\rm gs}$ 

#### 3.4 Verification

The parasitic capacitance from the measuring equipment and the drain-source overlapped capacitance are subtracted from the data in the following graphs. Figure 3.4a and b plot the  $C_{gs}$  and  $C_{gd}$  vs  $V_{ds}$  of a n-channel TFT. Figure 3.4c and d plot the  $C_{gs}$  and  $C_{gd}$  vs  $V_{ds}$  of a p-channel TFT. Figure 3.4e and f plot the  $C_{gs}$  and  $C_{gd}$  vs  $V_{gs}$  of a n-channel TFT.

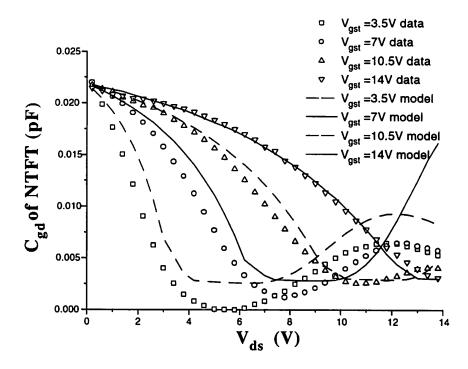


Figure 3.4a :  $C_{gd}$  vs  $V_{ds}$  for NTFT with  $W_{eff}/L_{eff}$ =20/5.38 and  $T_{ox}$ =76nm

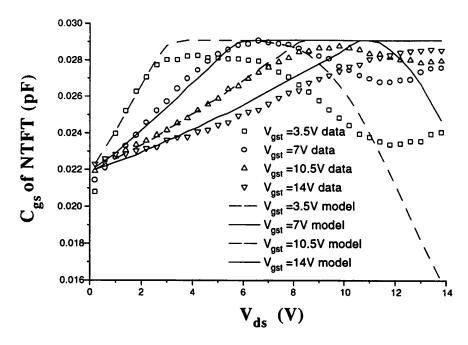


Figure 3.4b :  $C_{gs}$  vs  $V_{ds}$  for NTFT with  $W_{eff}/L_{eff}$ =20/5.38 and  $T_{ox}$ =76nm

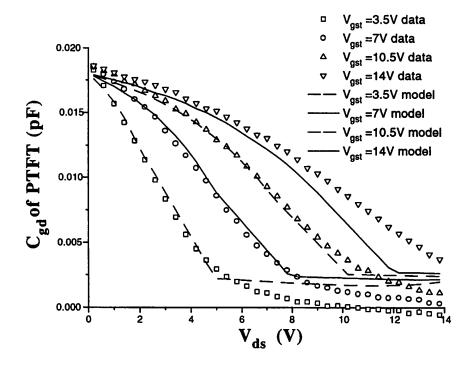


Figure 3.4c :  $C_{gd}$  vs  $V_{ds}$  for PTFT with  $W_{eff}/L_{eff}$ =20/4.42 and  $T_{ox}$ =76nm

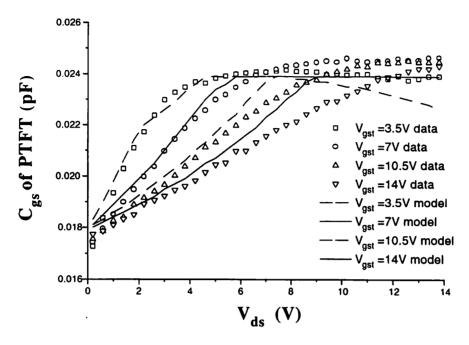


Figure 3.4d :  $C_{gs}$  vs  $V_{ds}$  for PTFT with  $W_{eff}/L_{eff}$ =20/4.42 and  $T_{ox}$ =76nm

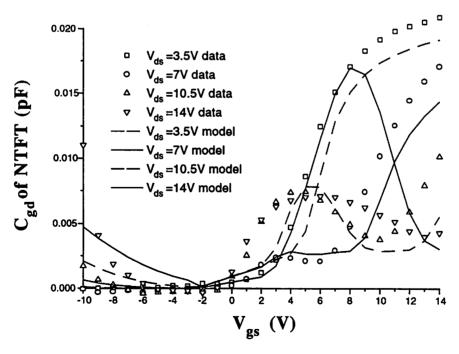


Figure 3.4e :  $C_{gd}$  vs  $V_{gs}$  for NTFT with  $W_{eff}/L_{eff}$ =20/5.38 and  $T_{ox}$ =76nm

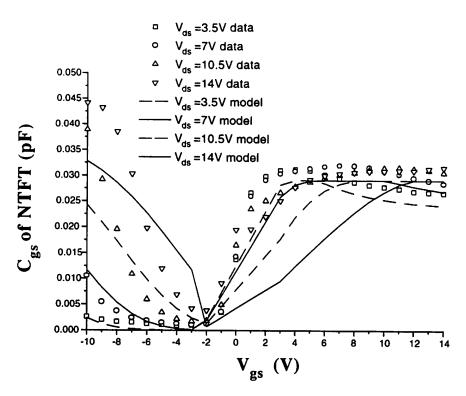


Figure 3.4f :  $C_{gs}$  vs  $V_{gs}$  for NTFT with  $W_{eff}/L_{eff}$ =20/5.38 and  $T_{ox}$ =76nm

# **Chapter 4: Spice Implementation**

#### 4.1 Overview

The special features of SPICE implementation of the model and the model parameters names are discussed in this chapter. Section 4.2 lists the model parameters, their meanings and their default values. Section 4.3 describes special features in the implementation of the model.

#### **4.2 Model Parameters**

Table 4.2 lists all the parameters used in this model. Symbol is the symbol used in the equations summarized in appendix A. Spice Name is the spice model parameter name.

Table 4.2: Table of SPICE parameters

Symbol   Spice Name   Meaning   Unit   Defaul
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$μ_1$ $u1$ mobility parameter for temperature dependence0.134 $μ_2$ $u2$ mobility parameter in exponential term $pFV^2/cm^2$ 1750 $μ_3$ $u3$ mobility parameter for temperature dependence $T^{-1}$ 0.0030 $μ_4$ $u4$ mobility parameter for low gate bias fitting $cm^2/Vs$ 2 $v_{sat}$ $v_{max}$ maximum drift velocity $m/s$ $10^5$ $ℓ$ $12$ channel length modulation parameter $m$ $10^{-10}$ $θ$ phitaDIBL effect parameter $V$ $0.05$ $s_1$ $s_1$ hot carrier pre-exponential parameter $V^{-1}$ $1.2$ $s_2$ $s_2$ hot carrier exponential parameter $V$ $30$ $n$ subslopediffusion current subthreshold slope $6.5$ $V_{off}$ voffoffset voltage of diffusion current $V$ $0$ $I_{do}$ idodiffusion current pre-exponential factor per unit width $A/m$ $0.0006$ $A_{eidl}$ gidlaGIDL effect pre-exponential parameter per unit width $A/V/m$ $0.0018$ $B_{eidl}$ gidlbGIDL effect offset voltage $V$ $1.12$ $V_i$ gidlvGIDL effect offset voltage $V$ $1.12$ $I_{thermal0}$ thermalithermal current pre-exponential parameter $I$ unit width $A/m$ $62.5n$ $E_a$ eaactivation energy for $I_{thermal}$ eV $0.5$ $I_{thermal}$ ldlateral diffusion for channel length $m$ $0$ $I_{therm$
$μ_2$ $u2$ mobility parameter in exponential term $pFV^2/cm^2$ $1750$ $μ_3$ $u3$ mobility parameter for temperature dependence $T^{-1}$ $0.0030$ $μ_4$ $u4$ mobility parameter for low gate bias fitting $cm^2/Vs$ $2$ $v_{sat}$ $v_{max}$ maximum drift velocity $m/s$ $10^5$ $\ell$ $12$ channel length modulation parameter $m$ $10^{-10}$ $\theta$ phitaDIBL effect parameter $V$ $0.05$ $s_1$ $s_1$ hot carrier pre-exponential parameter $V^{-1}$ $1.2$ $s_2$ $s_2$ hot carrier exponential parameter $V$ $30$ $n$ subslopediffusion current subthreshold slope $6.5$ $V_{off}$ voffoffset voltage of diffusion current $V$ $0$ $I_{do}$ idodiffusion current pre-exponential factor per unit width $A/m$ $0.0006$ $A_{pidl}$ gidlaGIDL effect pre-exponential parameter per unit width $A/V/m$ $0.0018$ $B_{pidl}$ gidlbGIDL effect exponential parameter $V$ $90$ $V_i$ gidlvGIDL effect offset voltage $V$ $1.12$ $I_{thermal0}$ thermalthermal current pre-exponential parameter $/$ unit width $A/m$ $62.5n$ $E_a$ eaactivation energy for $I_{thermal}$ eV $0.5$ $I_{diff}$ $I_{di$
$μ_3$ u3 mobility parameter for temperature dependence $μ_4$ u4 mobility parameter for low gate bias fitting $μ_4$ u4 mobility parameter for low gate bias fitting $μ_5$ $μ_4$ $μ_4$ mobility parameter for low gate bias fitting $μ_5$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
n subslope diffusion current subthreshold slope $V_{off}$ voff offset voltage of diffusion current $V_{off}$ on $V_{off}$ diffusion current pre-exponential factor per unit width $V_{off}$ on $V_{off}$ gidla GIDL effect pre-exponential parameter per unit width $V_{off}$ on $V_{off}$ gidlb GIDL effect exponential parameter $V_{off}$ of $V_{off}$ gidly GIDL effect offset voltage $V_{off}$ 1.12 $V_{off}$ thermal thermal current pre-exponential parameter $V_{off}$ unit width $V_{off}$ decrease a activation energy for $V_{off}$ lid lateral diffusion for channel length $V_{off}$ lid lateral diffusion for channel width $V_{off}$ lid lateral diffusion for channe
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$L_{diff}$ ldlateral diffusion for channel lengthm0 $W_{diff}$ lwlateral diffusion for channel widthm0 $T_{ox}$ toxgate oxide thicknessm85cmodcmodcapacitance model selection flag1
$L_{diff}$ ldlateral diffusion for channel lengthm0 $W_{diff}$ lwlateral diffusion for channel widthm0 $T_{ox}$ toxgate oxide thicknessm85cmodcmodcapacitance model selection flag1
$W_{diff}$ lwlateral diffusion for channel widthm0 $T_{ox}$ toxgate oxide thicknessm85cmodcmodcapacitance model selection flag1
Toxtoxgate oxide thicknessm85cmodcmodcapacitance model selection flag1
cmod         cmod         capacitance model selection flag         1
A good gubbhashald Carried 1-11
A <sub>CSS</sub> acgs subthreshold Cgs modeling parameter F/A 10-8
A <sub>ced</sub> acgd subthreshold Cgd modeling parameter F/A 10-8
V <sub>gtranl</sub> vgtranl transition parameter for I <sub>d</sub> in V <sub>gs</sub> domain V 1.5
V <sub>gtranh</sub> vgtranh transition parameter for I <sub>d</sub> in V <sub>gs</sub> domain V 0.5
V <sub>dtranh</sub> vdtranh transition parameter for I <sub>d</sub> in V <sub>ds</sub> domain V 0.1
V <sub>dtranl</sub> vdtranl transition parameter for I <sub>d</sub> in V <sub>ds</sub> domain V 0.1
V <sub>gtranhc</sub> vgtranhc transition parameter for cap model in the V <sub>gs</sub> domain V 1
V <sub>gtranlc</sub> vgtranlc transition parameter for cap model in the V <sub>gs</sub> domain V 1.5
V <sub>dtranhc</sub> vdtranhc transition parameter for cap model in the V <sub>ds</sub> domain V 0.5
V <sub>dtranlc</sub> vdtranlc transition parameter for cap model in the V <sub>ds</sub> domain V 0.5

#### Section 4.3: Implementation of Model

### 4.3.1: Cgd and Cgs Model Implementation

 $C_{gd}$  and  $C_{gs}$  are proportional to  $1/I_d^2$  in the strong inversion region. When  $V_{ds}$  is small,  $I_d$  is very small. This may cause  $C_{gd}$  to become unreasonably big and inaccurate. Therefore  $C_{gd}$  and  $C_{gd}$  are set to  $0.5C_{ox}$  when  $V_{ds} < 0.1V$  to prevent this situation.

Two capacitance models are implemented. They are (1) the one described in chapter 3, and (2) a simplified version of chapter 3 for speed consideration. The flag cmod is used to specified which model to use. When cmod is 1, (1) will be used. When cmod is 2, (2) will be used. (1) and (2) are identical in the subthreshold region and strong inversion linear region. The difference is in the saturation region.  $C_{gd}$  and  $C_{gs}$  of (2) in the saturation region are set to 0 and  $2/3C_{ox}L_{eff}W_{eff}$  (see fig 4.3.1a).

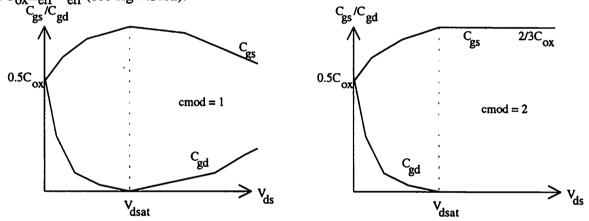


Figure 4.3.1.a: Difference between the two capacitance model (cmod=1 and 2)

## **Chapter 5: Simulation Results**

#### 5.1 Overview

This chapter compares the results of a 33-stage ring oscillator simulation with measured data. The drawn size of all p-channel TFT is  $16\mu m/7\mu m$ . The drawn size of all n-channel TFT is  $9\mu m/7\mu m$ . The oxide thickness is 76nm. Section 5.2 compares and discusses the simulation results and measured data. The data will be presented in both tables and graphs.

#### 5.2 Simulation Results and Measured Data

Table 5.2a tabulates the simulation results and measured data. Figure 5.2a and 5.2b plots the results in the table. A positive error means the simulation overestimate the data. A negative error means the simulation underestimate the data.

Table 5.2a: Simulations Results and Measured data

Vcc	Freq from	Freq from	% Error	Power from	Power from	% Error
(V)	Simulation	Measured data		Simulation	Measured	
	(MHz)	(MHz)		(mW)	Data (mW)	
6	0.62	0.277	124	0.075	0.033	127
7	0.83	0.504	65	0.145	0.084	73
8	1.13	0.788	43	0.255	0.171	49
9	1.48	1.11	33	0.42	0.318	32
10	1.8	1.52	18	0.65	0.548	19
11	2.15	1.92	12	0.935	0.854	9.5
12	2.36	2.42	-2	1.38	1.31	5
13	2.78	2.99	-7	1.82	1.92	-5
14	3.2	3.66	-12	2.45	2.82	-13
15	3.5	4.33	-19	3.15	4.02	-21

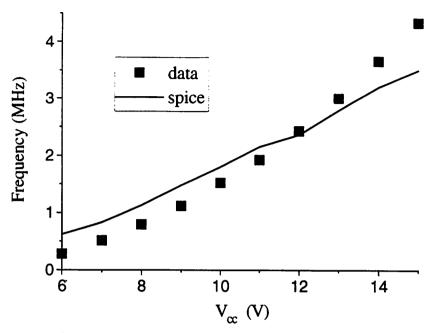


Figure 5.2a: Compare simulated and measured frequency of a 33-stage ring oscillator.

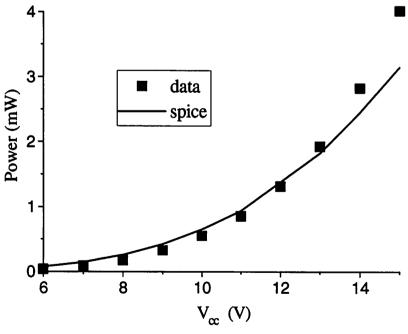


Figure 5.2b: Compare simulated and measured power of a 33-stage ring oscillator.

The simulation results in the higher  $V_{cc}$  region is more accurate then those in the lower  $V_{cc}$  region. In the lower  $V_{cc}$  region, the simulations overestimate the frequency and power, because the capacitance model underestimates the capacitance data by a large margin. Therefore the percentage error is big. In the higher  $V_{cc}$  region, the simulations underestimate the frequency and power, because the drain current model underestimates the drain current data by a small margin. Therefore the percentage error is small.

#### Appendix A: Equations Implemented in SPICE

This appendix summaries all equations implemented in spice3e1. Section 1 lists the drain current model equations. Section 2 lists the capacitance model equations.  $g_m$  is  $\partial I_d / \partial V_{gs}$ .  $g_{ds}$  is  $\partial I_d / \partial V_{ds}$ .

#### **Section 1: Drain Current Equations**

First of all, we need to define the boundaries of different regions (Figure 2.1a) and some common symbols.

$$\begin{split} V_T &= V_{To} - bT & V_{gst} &= V_{gs} - V_T & C_{ox} &= \epsilon_{ox} / T_{ox} \\ V_{dsat} &= \left(\frac{1}{V_{gst}} + \frac{1}{E_{sat}L_{eff}}\right)^{-1} & V_{dss} &= V_{ds} - V_{dsat} & E_{sat} &= \frac{2v_{sat}}{\mu_{eff}} \\ L_{eff} &= L - 2L_{diff} & W_{eff} &= W - 2W_{diff} \\ & \mu_{eff} &= \mu_o \left(\frac{kT}{q}\right)^{-\mu_1} \exp\left(\frac{-q\mu_2 \exp(\mu_3 T)}{kTC_{ox}V_{gst}}\right) + \mu_4 \\ V_{dl} &= V_{dsat} - V_{dtranl} & V_{dh} &= V_{dsat} + V_{dtranh} \\ V_{gl} &= V_T - V_{gtranh} & V_{gh} &= V_T + V_{gtranh} \end{split}$$

#### 1.1 Strong Inversion Linear Region: $V_{gs}$ > $V_{gh}$ and 0< $V_{ds}$ < $V_{dl}$

$$I_{d} = \frac{W_{eff}}{L_{eff}} C_{ox} \mu_{eff} \left( V_{gst} - \frac{V_{ds}}{2} \right) \frac{V_{ds}}{1 + \frac{V_{ds}}{E_{sat} L_{eff}}}$$

$$g_{ds} = \frac{W_{eff}}{L_{eff}} \frac{\mu_{eff} C_{ox}}{1 + \frac{V_{ds}}{E_{sat} L_{eff}}} \left[ \left( V_{gst} - V_{ds} \right) - \frac{\left( V_{gst} - V_{ds} / 2 \right) V_{ds}}{E_{sat} L_{eff} \left( 1 + \frac{V_{ds}}{E_{sat} L_{eff}} \right)} \right]$$

$$g_{m} = \frac{W_{eff}}{L_{eff}}C_{ox}\frac{1}{1 + \frac{V_{ds}}{E_{sat}L_{eff}}} \left[ \left(V_{gst} - V_{ds}/2\right)V_{ds}\frac{\partial \mu_{eff}}{\partial V_{gs}} + \mu_{eff}V_{ds} - \frac{\mu_{eff}V_{ds}^{2}\left(V_{gst} - V_{ds}/2\right)}{L_{eff} + \frac{V_{ds}}{E_{sat}}}\frac{\partial (1/E_{sat})}{\partial V_{gs}} \right]$$

$$\frac{\partial (1/E_{sat})}{\partial V_{gs}} = \frac{1}{2v_{sat}} \frac{\partial \mu_{eff}}{\partial V_{gs}}$$

$$\frac{\partial \mu_{\text{eff}}}{\partial V_{gs}} = \frac{(\mu_{\text{eff}} - \mu_4) q \mu_2 \exp(\mu_3 T)}{kTC_{ox}V_{gst}^2}$$

# 1.2 Strong Inversion Saturation Region: $V_{gs}$ > $V_{gh}$ and $V_{ds}$ > $V_{dh}$

$$I_{d} = I_{dsat} \left( 1 + \frac{V_{dss}}{V_{A}} \right) f_{kink}$$

$$V_A = \left(\frac{1}{V_{ACHM}} + \frac{1}{V_{ADIBL}}\right)^{-1}$$

$$V_{ACHM} = \frac{\left(E_{sat}L_{eff} + V_{gst}\right)V_{dss}}{E_{sat}\ell}$$

$$V_{ACHM} = \frac{\left(E_{sat}L_{eff} + V_{gst}\right)V_{dss}}{E_{sat}\ell}$$

$$V_{ADIBL} = \frac{E_{sat}L_{eff} + V_{gst}}{\theta\left(1 + \frac{2E_{sat}L_{eff}}{V_{gst}}\right)}$$

$$f_{kink} = 1 + s_1 V_{dss} \exp\left(\frac{-s_2}{V_{dss}}\right)$$

$$I_{dsat} = \frac{W_{eff}}{L_{eff}} C_{ox} \mu_{eff} \left( V_{gst} - \frac{V_{dsat}}{2} \right) \frac{V_{dsat}}{1 + \frac{V_{dsat}}{E_{sat} L_{eff}}}$$

$$g_{ds} = I_{dsat} s_1 \left(1 + \frac{V_{dss}}{V_A}\right) \left(1 + \frac{s_2}{V_{dss}}\right) exp\left(\frac{-s_2}{V_{dss}}\right) + I_{dsat} \left(\frac{1}{V_A} - \frac{V_{dss}}{V_A^2} \frac{\partial V_A}{\partial V_{ds}}\right) f_{kink}$$

$$\frac{\partial V_A}{\partial V_{ds}} = \frac{V_A^2 E_{sat} \ell}{\left(E_{sat} L_{eff} + V_{gst}\right) V_{dss}^2}$$

$$g_{m} = \frac{\partial I_{dsat}}{\partial V_{gs}} \left( 1 + \frac{V_{dss}}{V_{A}} \right) f_{kink} - I_{dsat} \left( \frac{\partial V_{dsat}}{\partial V_{gs}} \frac{1}{V_{A}} + \frac{V_{dss}}{V_{A}^{2}} \frac{\partial V_{A}}{\partial V_{gs}} \right) f_{kink} + I_{dsat} \left( 1 + \frac{V_{dss}}{V_{A}} \right) \frac{\partial f_{kink}}{\partial V_{gs}}$$

$$\frac{\partial V_{dsat}}{\partial V_{gs}} = V_{dsat}^{2} \left( \frac{1}{V_{gst}^{2}} - \frac{1}{2v_{sat}L_{eff}} \frac{\partial \mu_{eff}}{\partial V_{gs}} \right)$$

$$\frac{\partial \mu_{eff}}{\partial V_{gs}} = \frac{(\mu_{eff} - \mu_4)q\mu_2 \exp(\mu_3 T)}{kTC_{ox}V_{gst}^2} \qquad \qquad \frac{\partial (1/E_{sat})}{\partial V_{gs}} = \frac{1}{2v_{sat}} \frac{\partial \mu_{eff}}{\partial V_{gs}}$$

$$\frac{\partial f_{kink}}{\partial V_{gs}} = -\left(1 + \frac{s_2}{V_{dss}}\right) s_1 \exp\left(\frac{s_2}{V_{dss}}\right) \frac{\partial V_{dsat}}{\partial V_{gs}}$$

$$\frac{\partial I_{dsat}}{\partial V_{gs}} = \frac{W_{eff}}{L_{eff}} C_{ox} \frac{1}{1 + \frac{V_{dsat}}{E_{sat} L_{eff}}} \left[ \left( V_{gst} - V_{dsat} / 2 \right) V_{dsat} \frac{\partial \mu_{eff}}{\partial V_{gs}} + \mu_{eff} V_{dsat} - \frac{\mu_{eff} V_{dsat}^2 \left( V_{gst} - V_{dsat} / 2 \right)}{L_{eff} + \frac{V_{dsat}}{E_{sat}}} \frac{\partial (1 / E_{sat})}{\partial V_{gs}} \right]$$

$$\frac{\partial V_{A}}{\partial V_{gs}} = -V_{A}^{2} \left( \frac{\partial (1/V_{ACHM})}{\partial V_{gs}} + \frac{\partial (1/V_{ADIBL})}{\partial V_{gs}} \right)$$

$$\frac{\partial (1/V_{ACHM})}{\partial V_{gs}} = \frac{E_{sat}\ell}{\left(E_{sat}L_{eff} + V_{gst}\right)V_{dss}} \left(\frac{1}{V_{dss}} \frac{\partial V_{dsat}}{\partial V_{gs}} - \frac{1 + L_{eff}}{E_{sat}} \frac{\partial E_{sat}}{\partial V_{gs}}\right)$$

$$\frac{\partial E_{sat}}{\partial V_{gs}} = \frac{-2v_{sat}}{\mu_{eff}^2} \frac{\partial \mu_{eff}}{\partial V_{gs}}$$

$$\frac{\partial (1/V_{ADIBL})}{\partial V_{gs}} = \frac{2\theta \left(\frac{L_{eff}}{V_{gst}} \frac{\partial E_{sat}}{\partial V_{gs}} - \frac{E_{sat}L_{eff}}{V_{gst}^2}\right)}{E_{sat}L_{eff} + V_{gst}} - \frac{\theta \left(1 + \frac{2E_{sat}L_{eff}}{V_{gst}}\right) \left(L_{eff} \frac{\partial E_{sat}}{\partial V_{gs}} + 1\right)}{\left(E_{sat}L_{eff} + V_{gst}\right)^2}$$

# 1.3 Subthreshold Region: $V_{gs} < V_{gl}$

$$\begin{split} E_{sat} &= \frac{2 v_{sat}}{\mu_{eff}} \qquad \qquad \mu_{eff} = \mu_o \bigg(\frac{kT}{q}\bigg)^{-\mu_1} \qquad V_{dsat} = \bigg(\frac{1}{V_{gtranh}} + \frac{1}{E_{sat}L_{eff}}\bigg) \\ & I_d = I_{diff} + I_{thermal} + I_{gidl} \\ & I_{diff} = W_{eff}I_{do} \bigg(1 - exp\bigg(\frac{-V_{ds}}{kT/q}\bigg)\bigg) exp\bigg(\frac{V_{gs} - V_T - V_{off}}{(kT/q)n}\bigg) \\ & I_{gidl} = W_{eff}A_{gidl} \bigg(V_{dg} - V_i\bigg) exp\bigg(\frac{-B_{gidl}}{V_{dg} - V_i}\bigg) \\ & I_{thermal} = W_{eff}I_{thermal0} exp\bigg(\frac{E_a}{kT/q}\bigg) \end{split}$$

If  $(V_{dg}-V_i) > 0$ , then

$$g_{ds} = \frac{W_{eff} I_{do}}{kT/q} exp \left(\frac{-V_{ds}}{kT/q}\right) exp \left(\frac{q \left(V_{gs} - V_T - V_{off}\right)}{kTn}\right) + A_{gidl} \left(1 + \frac{B_{gidl}}{V_{dg} - V_i}\right) exp \left(\frac{-B_{gidl}}{V_{dg} - V_i}\right)$$

$$g_{m} = \frac{W_{eff} I_{do}}{(kT/q)n} \left(1 - exp\left(\frac{-V_{ds}}{kT/q}\right)\right) exp\left(\frac{V_{gs} - V_{T} - V_{off}}{(kT/q)n}\right) - A_{gidl} \left(1 + \frac{B_{gidl}}{V_{dg} - V_{i}}\right) exp\left(\frac{-B_{gidl}}{V_{dg} - V_{i}}\right)$$

else

$$g_{ds} = \frac{W_{eff} I_{do}}{kT/q} exp \left( \frac{-V_{ds}}{kT/q} \right) exp \left( \frac{q \left( V_{gs} - V_T - V_{off} \right)}{kTn} \right)$$

$$g_{m} = \frac{W_{eff} I_{do}}{(kT/q)n} \left(1 - exp\left(\frac{-V_{ds}}{kT/q}\right)\right) exp\left(\frac{V_{gs} - V_{T} - V_{off}}{(kT/q)n}\right)$$

# 1.4 Transition between Linear and Saturation Region: $V_{gs} > V_{gh}$ and $V_{dl} < V_{ds} < V_{dh}$

$$V_{ds} = V_{dl}(1-t)^2 + 2V_{dp}t(1-t) + V_{dh}t^2$$
$$I_d = I_{dl}(1-t)^2 + 2I_{dp}t(1-t) + I_{dh}t^2$$

$$\begin{split} I_{dl} &= I_{d}\big|_{V_{gs},V_{dl}} & I_{dh} = I_{d}\big|_{V_{gs},V_{dh}} \\ g_{dsl} &= g_{ds}\big|_{V_{gs},V_{dl}} & g_{dsh} = g_{ds}\big|_{V_{gs},V_{dh}} \\ g_{ml} &= g_{m}\big|_{V_{gs},V_{dl}} & g_{mh} = g_{m}\big|_{V_{gs},V_{dh}} \\ t &= \frac{\left(V_{dl} - V_{dp}\right) + \sqrt{\left(V_{dp} - V_{dl}\right)^{2} - \left(V_{dl} - 2V_{dp} + V_{dh}\right)\left(V_{dl} - V_{ds}\right)}}{V_{dh} - 2V_{dp} + V_{dl}} \\ V_{dp} &= \frac{I_{dh} - I_{dl} - \left(g_{dsh}V_{dh} - g_{dsl}V_{dl}\right)}{-g_{dsh} + g_{dsl}} \\ I_{dp} &= g_{dsl}\left(V_{dp} - V_{dl}\right) + I_{dl} \\ g_{ds} &= \frac{t\left(I_{dh} - I_{dp}\right) + \left(1 - t\right)\left(I_{dp} - I_{dl}\right)}{t\left(V_{dh} - V_{dp}\right) + \left(1 - t\right)\left(V_{dp} - V_{dl}\right)} \\ g_{m} &= g_{ml} + \frac{I_{d} - I_{dl}}{I_{dh} - I_{dl}} \frac{V_{dh} - V_{dl}}{V_{ds} - V_{dl}} (g_{mh} - g_{ml}) \end{split}$$

## 1.5 Transition between Strong Inversion and Subthreshold Region: $V_{gl} < V_{gs} < V_{gh}$

$$\begin{split} V_{gs} &= V_{gl}(1-t)^2 + 2V_{gp}t(1-t) + V_{gh}t^2 \\ &\ln(I_d) = \ln(I_{dl})(1-t)^2 + 2\ln(I_{dp})t(1-t) + \ln(I_{dh})t^2 \\ I_{dl} &= I_d\big|_{V_{gl},V_{ds}} & I_{dh} = I_d\big|_{V_{gh},V_{ds}} \\ g_{dsl} &= g_{ds}\big|_{V_{gl},V_{ds}} & g_{dsh} = g_{ds}\big|_{V_{gh},V_{ds}} \\ g_{ml} &= g_{m}\big|_{V_{gl},V_{ds}} & g_{mh} = g_{m}\big|_{V_{gh},V_{ds}} \\ t &= \frac{\left(V_{gl} - V_{gp}\right) + \sqrt{\left(V_{gp} - V_{gl}\right)^2 - \left(V_{gl} - 2V_{gp} + V_{gh}\right)\left(V_{gl} - V_{gs}\right)}}{V_{gh} - 2V_{gp} + V_{gl}} \\ V_{gp} &= \frac{I_{dh} - I_{dl} - \left(g_{mh}V_{dh} - g_{ml}V_{dl}\right)}{-g_{mh} + g_{ml}} \end{split}$$

$$\begin{split} I_{dp} &= g_{ml} \Big( V_{gp} - V_{gl} \Big) + I_{dl} \\ g_{m} &= \frac{t \Big( I_{dh} - I_{dp} \Big) + (1 - t) \Big( I_{dp} - I_{dl} \Big)}{t \Big( V_{gh} - V_{gp} \Big) + (1 - t) \Big( V_{gp} - V_{gl} \Big)} \\ g_{ds} &= g_{dsl} + \frac{I_{d} - I_{dl}}{I_{dh} - I_{dl}} \frac{V_{gh} - V_{gl}}{V_{gst} - V_{gl}} \Big( g_{dsh} - g_{dsl} \Big) \end{split}$$

#### **Section 2: Capacitance Model Equations**

First of all, we need to define the boundaries of different regions (Figure 3.1a) and some common symbols.

$$\begin{aligned} V_T &= V_{To} - bT & V_{dsat} = \left(\frac{1}{V_{gst}} + \frac{1}{E_{sat}L_{eff}}\right)^{-1} & C_{ox} = \epsilon_{ox} / T_{ox} \\ V_{gst} &= V_{gs} - V_T & V_{gstd} = V_{gst} - V_{ds} & V_{gstdat} = V_{gst} - V_{dsat} \\ V_{dl} &= V_{dsat} - V_{dtranlc} & V_{dh} = V_{dsat} + V_{dtranhc} \\ V_{gl} &= V_T - V_{gtranlc} & V_{gh} = V_T + V_{gtranhc} \end{aligned}$$

#### 2.1 $C_{gd}$ in Strong Inversion Linear Region: $V_{gs} > V_{gh}$ and $0 < V_{ds} < V_{dl}$

$$C_{gd} = \frac{g_{ds}}{I_d^2} W_{eff}^2 \mu_{eff} \int_{V_s}^{V_d} Q_n^2(V) dV - \frac{W_{eff}^2 \mu_{eff}}{I_d} Q_n^2(V_{ds}) + \frac{W_{eff}}{E_{sat}} Q_n(V_{ds})$$

$$\int_{V_s}^{V_d} Q_n^2(V) dV = \frac{1}{3} C_{ox} \left( V_{gst}^3 - V_{gstd}^3 \right)$$

# 2.2 $C_{gd}$ in the Strong Inversion Saturation Region: $V_{gs}>V_{gh}$ and $V_{ds}>V_{dh}$ If cmod=1, then

$$C_{gd} = \frac{\partial Q_{lin}}{\partial V_d} + \frac{\partial Q_{sat}}{\partial V_d}$$

$$\frac{\partial Q_{lin}}{\partial V_d} = \frac{1}{3} W_{eff}^2 C_{ox}^2 \mu_{eff} \frac{g_{ds}}{I_d^2} \left( V_{gst}^3 - V_{gstdsat}^3 \right)$$

$$\frac{\partial Q_{sat}}{\partial V_d} = -W_{eff} C_{ox} V_{gstdsat} \frac{\partial \Delta L}{\partial V_d}$$

$$\frac{\partial \Delta L}{\partial V_d} = \frac{1}{(V_{ds} - V_{dsat}) / \ell + E_m} \left( 1 + \frac{V_{ds} - V_{dsat}}{\ell E_m} \right)$$

$$E_{\rm m} = \left[ \left( \frac{V_{\rm ds} - V_{\rm dsat}}{\ell} \right)^2 + E_{\rm sat}^2 \right]^{1/2}$$

else if cmod=2, then

$$C_{gd} = 0$$

### 2.3 $C_{gd}$ in the Subthreshold Region: $V_{gs} < V_{gh}$

If  $V_{gs}>V_{gl}$ , then

$$C_{gd} = \frac{C_{gdh}}{V_{gtran}} (V_{gs} - V_{gl})$$

$$V_{gtran} = V_{gtranlc} + V_{gtranhc}$$

else,

$$C_{gd} = \left(\frac{1}{W_{eff} L_{eff} C_{ox}} + \frac{1}{A_{cgd} I_d}\right)^{-1}$$

 $C_{gdh}$  is the  $C_{gd}$  computed at  $V_{gs}=V_{gh}$  and the applied  $V_{ds}$ . If  $V_{ds}>V_{dsat}$ , then the equations in section 2.2 are used. Otherwise, the equations in section 2.1 are applied.

# 2.4 $C_{gd}$ in the Strong Inversion Transition Region: $V_{gs}\!\!>\!\!V_{gh}$ and $V_{dl}\!\!<\!\!V_{ds}\!\!<\!\!V_{dh}$

$$C_{gd} = a_{cgd}V_{ds} + b_{cgd}$$

$$a_{cgd} = \frac{C_{gdh} - C_{gdl}}{V_{dh} - V_{dl}}$$

$$b_{cgd} = C_{gdh} - a_{cgd}V_{dh}$$

 $C_{gdl}$  is the  $C_{gd}$  computed at  $V_{ds}=V_{dl}$  and the applied  $V_{gs}$  using equations in section 2.1.  $C_{gdh}$  is the  $C_{gd}$  computed at  $V_{ds}=V_{dh}$  and the applied  $V_{gs}$  using equations in section 2.2.

## 2.5 $C_{gs}$ in the Strong Inversion Linear Region: $V_{gs}\!\!>\!\!V_{ghcgs}$ and $0\!\!<\!\!V_{ds}\!\!<\!\!V_{dl}$

$$\begin{split} C_{gs} &= -\frac{\partial Q_g}{\partial V_g} - C_{gd} \\ \frac{\partial Q_g}{\partial V_g} &= \frac{1}{3} \frac{C_{ox}^2 W_{eff}^2}{I_d} \left[ \left( \frac{\partial \mu_{eff}}{\partial V_{gs}} - \frac{\mu_{eff}}{I_d} g_m \right) \left( V_{gstd}^3 - V_{gst}^3 \right) + 3 \mu_{eff} \left( V_{gstd}^2 - V_{gst}^2 \right) \right] \\ &- \frac{1}{2} W_{eff} C_{ox} \left[ \frac{\partial (1/E_{sat})}{\partial V_{gs}} \left( V_{gstd}^2 - V_{gst}^2 \right) + \frac{2}{E_{sat}} \left( V_{gstd} - V_{gst} \right) \right] \end{split}$$

 $C_{gd}$  is the  $C_{gd}$  computed at the applied  $V_{gs}$  and  $V_{ds}$ .

# 2.6 $C_{gs}$ in the Strong Inversion Saturation Region: $V_{gs} > V_{gh}$ and $V_{ds} > V_{dh}$ If cmod=1, then

$$\begin{split} C_{gs} &= -\frac{\partial Q_g}{\partial V_g} - C_{gd} & \frac{\partial Q_g}{\partial V_g} = \frac{\partial Q_{lin}}{\partial V_g} + \frac{\partial Q_{sat}}{\partial V_g} \\ & \frac{\partial Q_{lin}}{\partial V_g} = \frac{1}{3} \frac{C_{ox}^2 W_{eff}^2}{I_d} \Bigg[ \bigg( \frac{\partial \mu_{eff}}{\partial V_{gs}} - \frac{\mu_{eff}}{I_d} g_m \bigg) \bigg( V_{gstdsat}^3 - V_{gst}^3 \bigg) + 3 \mu_{eff} \bigg( V_{gstdsat}^2 \bigg( 1 - \frac{\partial V_{dsat}}{\partial V_{gs}} \bigg) - V_{gst}^2 \bigg) \Bigg] \\ & - \frac{1}{2} W_{eff} C_{ox} \Bigg[ \frac{\partial (1/E_{sat})}{\partial V_{gs}} \bigg( V_{gstdsat}^2 - V_{gst}^2 \bigg) + \frac{2}{E_{sat}} \bigg( V_{gstdsat} \bigg( 1 - \frac{\partial V_{dsat}}{\partial V_{gs}} \bigg) - V_{gst} \bigg) \Bigg] \\ & \cdot \frac{\partial Q_{sat}}{\partial V_{gs}} = -W_{eff} C_{ox} \bigg( 1 - \frac{\partial V_{dsat}}{\partial V_{gs}} \bigg) \Delta L - W_{eff} C_{ox} V_{gstdsat} \frac{\partial \Delta L}{\partial V_{gs}} \\ & \Delta L = \ell \ln \bigg( \frac{(V_{ds} - V_{dsat})/\ell + E_m}{E_{sat}} \bigg) \end{split}$$

$$\frac{\partial \Delta L}{\partial V_{gs}} = \frac{\ell E_{m}}{(V_{ds} - V_{dsat}) / \ell + E_{m}} \left[ \frac{-\frac{1}{\ell} \frac{\partial V_{dsat}}{\partial V_{gs}} + \frac{\partial E_{m}}{\partial V_{gs}}}{E_{sat}} \right] + \left( \frac{V_{ds} - V_{dsat}}{\ell} + E_{m} \right) \frac{\partial (1 / E_{sat})}{\partial V_{gs}}$$

$$\frac{\partial E_{m}}{\partial V_{gs}} = \frac{1}{E_{m}} \left[ -\frac{\partial V_{dsat}}{\partial V_{gs}} \frac{V_{ds} - V_{dsat}}{\ell^{2}} + E_{sat} \frac{\partial E_{sat}}{\partial V_{gs}} \right]$$

else if cmod=2, then

$$C_{gs} = \frac{2}{3}C_{ox}L_{eff}W_{eff}$$

## 2.7 $C_{gs}$ in the Subthreshold Region: $V_{gs}\!\!<\!\!V_{gh}$

If  $V_{gs}>V_{gl}$ , then

$$C_{gs} = \frac{C_{gsh}}{V_{gtran}} (V_{gs} - V_{gl})$$

$$V_{gtran} = V_{gtranhc} + V_{gtranlc}$$

else,

$$C_{gs} = \left(\frac{1}{W_{eff} L_{eff} C_{ox}} + \frac{1}{A_{cgs} I_{d}}\right)^{-1}$$

 $C_{gsh}$  is the  $C_{gs}$  computed at  $V_{gs}=V_{gh}$  and the applied  $V_{ds}$ . If  $V_{ds}>V_{dsat}$ , then the equations in section 2.6 are used. Otherwise, the equations in section 2.5 are applied.

## 2.8 $C_{gs}$ in the Strong Inversion Transition Region: $V_{gs} > V_{gh}$ and $V_{dl} < V_{ds} < V_{dh}$

$$C_{gs} = a_{cgs}V_{ds} + b_{cgs}$$

$$a_{cgs} = \frac{C_{gsh} - C_{gsl}}{V_{dh} - V_{dl}}$$

$$b_{cgs} = C_{gsh} - a_{cgs}V_{dh}$$

 $C_{gsl}$  is the  $C_{gs}$  computed at  $V_{ds}$ = $V_{dl}$  and the applied  $V_{gs}$  using equations in section 2.5.  $C_{gsh}$  is the  $C_{gs}$  computed at  $V_{ds}$ = $V_{dh}$  and the applied  $V_{gs}$  using equations in section 2.6.

#### **Appendix B: Parameter Extraction**

This section discusses the parameter extraction procedures used in this project. A spreadsheet program, EXCEL 4.0, is used to visually fit the model with the measured data for both the drain current and capacitance model parameters locally. Temperatures are in unit of Kelvin. Section 1 discusses the drain current parameters extraction. Section 2 discusses the capacitance parameter extraction.

#### **Section 1: Drain Current Model Extraction**

To extract the drain current parameters with temperature dependence, the following measurements are needed at different temperatures (e.g. 300K, 325K, 350K, and 375K) are needed. If temperature dependence is ignored, only one set of data is necessary.

- 1)  $C_gV_{gs}$  data with both drain and source grounded (e.g.  $V_{gs}$  = -3V to 12V)
- 2)  $I_dV_{ds}$  data with several  $V_{gs}$  bias bigger than  $V_T$  (e.g.  $V_{ds} = 0V$  to 12V and  $V_{gs} = 3V$ , 6V, 9V, and 12V)
- 3)  $I_dV_{gs}$  data with different  $V_{ds}$  bias (e.g.  $V_{gs} = -5V$  to 12V and  $V_{ds} = 0.1V$  and 5V) The gate oxide thickness and process lateral diffusion length must be extracted first. Section 1.1 to 1.5 describes the extraction of the parameters in different regions. Section 1.6 discusses the order of extraction.

#### 1.1: Threshold Voltage (V<sub>TO</sub> and b)

We use the equation  $Q_n = C_{ox} (V_{gs} - V_T)$  to define  $V_T$ . First of all,  $C_g - V_{gs}$  data are measured with both the drain and source grounded. Then the parasitic capacitance is subtracted

from  $C_g$ .  $Q_n$  is computed by integrating the  $C_g$ - $V_{gs}$  curve using the relation  $Q_n(V_{gs}) = \int\limits_{-\infty}^{V_{gs}} C_g dV$ . A straight line will be fitted to  $Q_n$  and the x-intercept is  $V_T$  (figure b1).

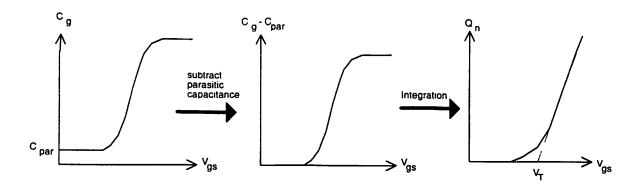


Figure b1: V<sub>T</sub> extraction procedure

If the temperature dependence is ignored,  $V_{TO}$  is  $V_{T}$  and b is 0. To extract the temperature dependence, b,  $V_{T}$ 's at several temperature are measured. Then a linear fit is used to determine  $V_{TO}$  and b, where  $V_{T} = V_{TO} - bT$ .

#### 1.2: Mobility $(\mu_0, \mu_1, \mu_2, \mu_3, \text{ and } \mu_4)$

At a particular temperature,  $\mu_{eff}$  is modeled with an expression in the form of  $A \exp(-B/(V_{gs}-V_T)) + \mu_4$ , where A and B are function of temperature, oxide thickness, and mobility parameters. An estimate of A, B, and  $\mu_4$  are obtained by fitting the  $\mu_{eff}$  at low drain bias (e.g. 0.1V) using the  $I_dV_{gs}$  data. When A increases and B decreases,  $\mu_{eff}$  increases.  $\mu_4$  determines  $\mu_{eff}$  at  $V_{gs}$  close to  $V_T$ .  $\mu_{eff}$  increases when  $\mu_4$  increases. Since the objective is to fit the drain current in the strong inversion linear region, the estimated A and B are further optimized by fitting the  $I_dV_{ds}$  data in the strong inversion linear region (figure b2) with  $V_{gs} > V_T$ . Using the IV curves at different temperatures, different sets of A and B are found. Then we can compute  $\mu_0$ ,  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$ , and  $\mu_4$ .

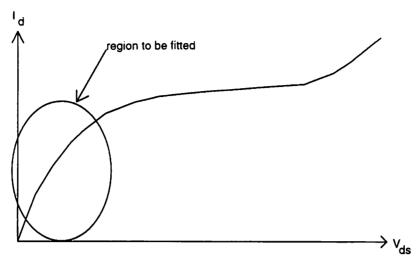


Figure b2: strong inversion linear region

#### 1.3: Saturation Velocity (v<sub>sat</sub>)

 $v_{sat}$  affects the magnitude of the drain current when  $V_{ds}$  is near  $V_{dsat}$  and the location of  $V_{dsat}$ . When  $v_{sat}$  increases,  $V_{dsat}$  and  $I_{d}$  increase.  $v_{sat}$  is extracted by visually fitting the drain current near  $V_{dsat}$  and the location of  $V_{dsat}$  (figure b3).

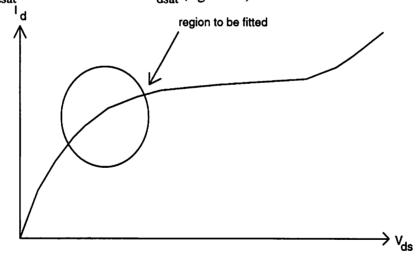


Figure b3: extracting v<sub>sat</sub>

### 1.4: DIBL, Channel Length Modulation, and Hot Carrier Effects ( $\ell$ , $\theta$ , $s_1$ , and $s_2$ )

 $\ell$  and  $\theta$  affects  $V_A$ . When  $\ell$  and  $\theta$  increase,  $V_A$  drops and  $I_d$  increases.  $s_1$  and  $s_2$  affect the hot carrier tail at high  $V_{ds}$ . When  $s_1$  increases and  $s_2$  decreases, the hot carrier effect will be more pronounced (figure b4).  $s_1$  is set to 1.2 for NTFT and 2.2 for PTFT in this study. Only  $s_2$  is varied to fit the data. However, the user can vary both  $s_1$  and  $s_2$  as they see fit.

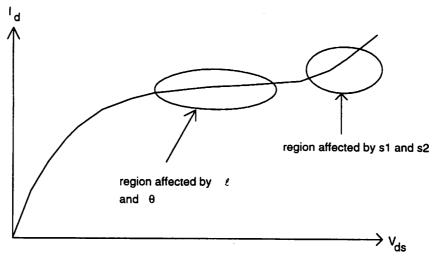


Figure b4: effect of  $s_1$ ,  $s_2$ ,  $\ell$ , and  $\theta$ 

## 1.5: Subthreshold Region (n, $V_{off}$ , $I_{do}$ , $A_{gidl}$ , $B_{gidl}$ , $V_{i}$ , $E_{a}$ , and $I_{thermal0}$ )

 $I_{do}$ , n, and  $V_{off}$  affect the region where the diffusion current dominates. When n and  $I_{do}$  increase and  $V_{off}$  decreases, the diffusion current increases. Huang [1] shows that  $I_{do}$  is a function of substrate doping.  $I_{do}$  is treated as a model parameter in this model.  $A_{gidl}$ ,  $B_{gidl}$ , and  $V_i$  affects the region where the GIDL effect dominates ( $V_{dg}$  is big). When  $A_{gidl}$  increases,  $B_{gidl}$  decreases, and  $V_i$  decreases, the GIDL current increases.  $I_{thermal0}$  and  $E_a$  set the minimum leakage current. The thermal generation current is not a function of bias. When  $E_a$  decreases and  $I_{thermal0}$  increases, the thermal generation current increases. To extract  $I_{thermal0}$  and  $E_a$ ,  $I_{thermal}$  at different temperature are extracted first. Then  $I_{thermal0}$  and  $E_a$  are extracted from the  $I_{thermal}$  found. If temperature dependence are not extracted, then  $I_{thermal0}$  is  $I_{thermal}$  and  $E_a$  is 0. Figure b5 illustrates the regions that the above parameters affect.

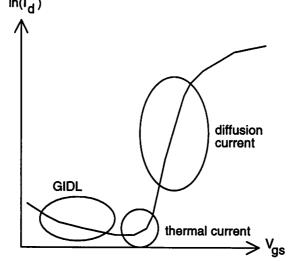


Figure b5: different regions of the subthreshold region

#### 1.6: Order of Extraction

- 1) Extract threshold voltage parameter (V<sub>TO</sub> and b).
- 2) Extract mobility parameters in the linear region of  $I_dV_d$  ( $\mu_0$ ,  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$ , and  $\mu_4$ ).
- 3) Adjust  $v_{sat}$  to fit the  $V_{dsat}$  location and  $I_{d}$  near  $V_{dsat}$ .
- 4) Adjust  $\ell$ ,  $\theta$ ,  $s_1$ , and  $s_2$  to fit the saturation region of the  $I_dV_d$  curves.
- 5) Extract n and Voff for diffusion current.
- 6) Extract Agidl, Bgidl, Vi, and Ithermal together for GIDL and thermal generation current.
- 7) Repeat the extraction of  $I_{thermal}$  for different temperature to extract  $I_{thermal0}$  and  $E_a$ .
- 8) Adjust  $V_{gtranh}$ ,  $V_{gtranh}$ ,  $V_{dtranh}$ , and  $V_{dtranh}$  to improve continuity.

#### **Section 2: Capacitance Model Extraction**

For  $C_{gs}$  and  $C_{gd}$ , only  $A_{cgs}$ ,  $A_{cgd}$ , and the transition region parameters need to be extracted. The following measurements are needed.

- 1)  $C_{gs}$  and  $C_{gd}$  in a  $V_{ds}$  sweep with different  $V_{gs}$  bias (e.g.  $V_{ds} = 0V$  to 12V and  $V_{gs} = 3V$ , 6V, 9V, and 12V)
- 2)  $C_{gs}$  and  $C_{gd}$  in a  $V_{gs}$  sweep with different  $V_{ds}$  bias (e.g.  $V_{gs} = -5V$  to 12V and  $V_{ds} = 3V$ , 6V, 9V, and 12V)

The gate oxide thickness and process lateral diffusion length must be known. The parasitic capacitance and the overlap capacitance should be subtracted from the data. Drain current model parameters can also be slightly altered to fit the capcaitance data more accurately.

#### $2.1 A_{cgs}$ and $A_{cgd}$

 $A_{cgs}$  and  $A_{cgd}$  affects  $C_{gs}$  and  $C_{gd}$  in the GIDL dominant region ( $V_{dg}>0$ , e.g. high  $V_{ds}$  in accumulation region). When  $A_{cgs}$  and  $A_{cgd}$  increase,  $C_{gs}$  and  $C_{gd}$  increase.  $A_{cgs}$  and  $A_{cgd}$  are extracted by fitting the model with the data in the GIDL. The GIDL dominant region in 3.3.1a of chapter 3 is from between -10V and -2V. The figure is re-drawn below.

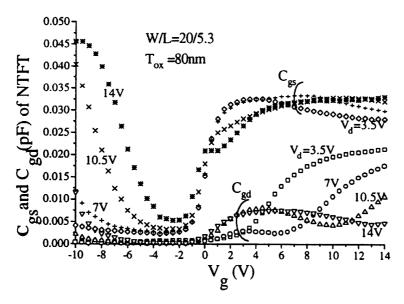


Figure 3.3.1a :  $C_{gs}$  and  $C_{gd}$  vs  $V_{g}$ 

# 2.2 Transition Region Parameters ( $V_{dtranlc}$ , $V_{dtranhc}$ , $V_{gtranhc}$ , $V_{gtranhc}$ )

To extract those transition region parameters, we can just examine the data and get a reasonable estimate.  $V_{dtranlc}$  and  $V_{dtranhc}$  are the transition width from saturation to linear region on the  $V_{ds}$  domain in  $C_{gs}$ - $V_{ds}$  and  $C_{gd}$ - $V_{ds}$  data. A default value of 0.5V for both  $V_{dtranlc}$  and  $V_{dtranhc}$  works well for the data used in this project.

When the gate bias decreases below  $V_T$ ,  $C_{gs}$  and  $C_{gd}$  gradually drop to 0.  $V_{gtranlc}$  is the voltage below  $V_T$  that  $C_{gs}$  and  $C_{gd}$  drop to 0. The  $C_{gd}$  and  $C_{gs}$  model assume a linear drop from  $\left(V_T + V_{gtranhc}\right)$  to 0 at  $\left(V_T - V_{gtranlc}\right)$ . Therefore,  $V_{gtranhc}$  is the voltage above  $V_T$  that the model begin the linear drop (see figure 3.3.2a of chapter 3).

## Reference

- [1] J. Levinson, et al, Journal of Appl. Phys, Feb 1982, p. 1193
- [2] J. H. Huang, et al, IEDM Technical Digest, 1992, p. 569
- [3] T. Y. Chan, et al, IEDM Technical Digest, 1987, p.718
- [4] S. M. Sze, Physics of Semiconductor Devices, 2nd Edition
- [5] P. K. Ko, et al, VLSI Electronics: Microstructure Science, Vol. 18, Chapter 1

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