CONTROLLING CHAOS WITHOUT FEEDBACK AND CONTROL SIGNALS

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Memorandum No. UCB/ERL M93/14

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CONTROLLING CHAOS WITHOUT FEEDBACK AND CONTROL SIGNALS

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We describe an effective method for controlling chaos by coupling a main chaotic system to a new but simple system with easily changeable parameters. The method is applied to Duffing's oscillator (numerical and analytical study) and to Chua's circuit (experimental study). The effectiveness of our method in controlling noisy systems, as well as an illustrative application in a mechanical and an electrical system, are discussed.

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1. INTRODUCTION

The presence of chaos both in nature and in man-made devices is very common and has been extensively demonstrated in the last decade. Quite frequently chaos is a beneficial feature as in some chemical or heat and mass transport problems [Ottino, 1989]. However, in many other situations chaos is an undesirable phenomenon leading to irregular and possibly catastrophic failures.

The problem of controlling chaos, that is, to convert the chaotic behaviour found in a physical system to a periodic time dependence, or aperiodicity which is predictable, has attracted much recent interest [Ott et al., 1990; Jakson, 1990; Shinbrot et al., 1990; Ditto et al., 1991; Hunt, 1991; Mehta and Henderson, 1991; Tel, 1991; Braiman and Goldhirsch, 1991; Vincent and Yu, 1991; Singer et al., 1991; Shinbrot et al., 1992; Chen and Dong, 1992; Kapitaniak, 1992; Kapitaniak, 1993]. The existing methods used to control chaos can be classified into two main categories: *feedback* and *nonfeedback* methods. Feedback methods [Ott et al., 1990; Jakson, 1990; Shinbrot et al., 1990; Ditto et al., 1991; Hunt, 1991; Mehta and Henderson, 1991; Tel, 1991; Vincent and Yu, 1991; Singer et al., 1991; Shinbrot et al., 1992; Chen and Dong, 1992] make use of the properties of chaotic systems, including their sensitivity to initial conditions, to stabilize orbits already existing in the systems. The initial breakthrough of the feedback technique was made by Ott, Grebogi and Yorke [1990]. They show that permanent chaos can always be suppressed by stabilizing one of many periodic orbits embedded within the chaotic attractor. Non-feedback methods [Hubler, 1989, Braiman and Goldhirsch, 1991; Kapitaniak, 1992; Kapitaniak, 1993], on the other hand, applies a small driving force, or a small modulation to some system parameter. These methods modify the underlying dynamical system such that stable solutions appear.

Chaotic attractors of dynamical systems described by:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \boldsymbol{\mu}) \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$, $n \geq 3$, can be divided into two main classes: (a) hyperbolic attractors and attractors which are similar to hyperbolic ones, like the Lorenz-type attractors, and (b) quasi-attractors

[Shil'nikov, 1993]. The second class of attractors is the most important in practice because of its wide applications. Such attractors are observed in many models; including the logistic map, Henon map, Lorenz system, Duffing's equation, Chua's circuit, etc. The term quasi-attractor denotes the limiting set enclosing the periodic orbits of different topological types, structurally unstable homoclinic Poincare trajectories, which may not be transitive, etc. In systems with quasi-attractors there exists a structurally unstable homoclinic orbit of either system (1), or of a system "close" to it. This implies a sensitive dependence of the attractor structure on any small variation of the parameter μ. We can exploit this property in controlling chaos by changing slightly only one parameter of the system (1), or by coupling a low-dimensional (one- or two-dimensional) linear system to the original system (1).

In this paper we describe a method for controlling chaos in which the control effect is achieved by coupling the chaotic main system to a simpler autonomous system (controller), usually linear, as shown in Figure 1. Our method is developed for chaotic systems in which for some, for example technological, reasons it is difficult if not impossible to change any parameter of the main system. In particular consider the coupling of the chaotic system (1) to another (simpler) asymptotically stable system (controller) described by:

$$\dot{y} = g(y, e) \tag{2}$$

where $x \in \mathbb{R}^m$, e is a vector denoting the controller's parameters, where the value of at least one of the parameters e_i can be easily changed. For practical reasons, the dimension "m" of the controller system (2) should be chosen as low as possible. Since our method is mainly designed for controlling chaos in mechanical systems, in this paper we choose m=2, i.e. one-degree-of-freedom controller (the simplest mechanical system). The equations for the augmented system are given by

$$\dot{x} = f(x, \mu) + \zeta_y \, y \tag{3a}$$

$$\dot{y} = g(y, e) + \zeta_x \, x \tag{3b}$$

where ζ_x and ζ_y are the *coupling* matrices. When $\zeta_x = 0$ and $\zeta_y = 0$ the x and y subsystems in Eq. (3) are uncoupled and for *small* $\|\zeta_x\|$ and $\|\zeta_y\|$ (i.e., all entries are small) only small additional signals are injected into the main system. Since the y-subsystem is asymptotically stable, the role of the controller is to change the behaviour of the system from a chaotic one to some desired periodic, possibly constant, operating regime. If $\|\zeta_x\|$ and $\|\zeta_y\|$ are sufficiently small, $\mathbf{x}(t)$ of the coupled system (3) will evolve in a small neighbourhood of the original attractor of Eq. (1) thereby preserving the qualitative dynamics of the main system. As the evolution of $\mathbf{x}(t)$ given by Eq. (1) takes place on an attractor there exists a vector $\mathbf{M} \in \mathbf{R}^n$ such that $|x_i(t)| \leq M_i$ for all time t. Our control will be effective if $\mathbf{x}(t)$ given by Eqs (3) fulfills the relation $|x_i(t)| \leq M_i + \varepsilon$, where ε is a small parameter. This method is a nonfeedback one. In general it does not stabilize existing unstable orbits, but rather it modifies the dynamical system such that a new stable orbit appears in a neighbourhood of the original attractor.

In most feedback methods the fact that in the chaotic attractor there are many embedded unstable periodic orbits is exploited. In our method we exploit the fact that in a small "parameter" neigbourhood of a quasi-attractor for Eq. (1) there exist many stable periodic orbits. Our goal is to change the dynamics of the system in such a way as to obtain periodic orbits close to the original attractor. Depending on the application, we can stabilize a desired fixed point, or a periodic orbit having a desired period.

The idea of our method is similar to that of the so-called dynamical vibration absorber, long known in linear systems [Shabana, 1991]. A dynamical vibration absorber is a one-degree-of-freedom system, usually a mass m_t attached to a spring (sometimes a viscous absorber is also added), which is connected to the main system as shown in Figure 2. The additional degree of freedom introduces a shifts of the resonance zones, and in some cases can eliminate oscillations of the main mass m. Although such a dynamical absorber can change the overall dynamics substantially, it usually needs only to be physically small in comparison with the main system, and does not require an increase of the excitation force. It can be easily added to an existing system without major changes in design or construction.

The plan of this letter is as follows. Section 2 presents an application of our method for controlling chaos in Duffing's oscillator. First an approximate analytical method which allows us to evaluate appropriate parameters of the controller is presented. Next the effectiveness of our method in the presense of noise is discussed. In Sec. 3 we describe an experimental study of controlling chaos in Chua's circuit by coupling it to a simple linear system. Sume practical suggestions and guidelines on how to couple the controller are given in Sec.4. Finally we summarize our results in Sec.5.

2. CONTROLLING DUFFING'S OSCILLATOR

2.1 ANALITYCAL APPROXIMATION

In this section we consider the problem of controlling the chaotic behaviour of the Duffing oscillator*. To obtain an appropriate control effect Duffing's oscillator is coupled to an additional linear system (controller) in the way shown in Figure 2. The dynamics of the augmented system is described by,

$$\ddot{x} + a\dot{x} + bx + cx^3 + \varepsilon(x-y) = B_0+B_1\cos\Omega t \qquad (4a)$$

$$\ddot{y} + e(y-x) = 0 \qquad (4b)$$

where a,b,c,ε,e,B and Ω are constants. Althought the system (4a) is a nonautonomous one, it can be easily tranformed to an autonomous system. The coefficient ε and e are the characteristic parameters for the controller, and we can take them as the control parameters. The parameters of Eqs (4) are related to those of Figure 2 in the following way: $a=c/m\Omega$, $b=k/m\Omega^2$, $c=k_c/m\Omega^2$, $\varepsilon=k_\varepsilon/m\Omega^2$, $e=k_\varepsilon/m_\varepsilon\Omega^2$, $B_0=F_0/m\Omega$ and $B_1=F_1/m\Omega$. It should be noted here that the parameters ε and e are related to each other by the controller stiffness k_ε. For simplicity in the rest of this letter we assume that ε is constant and consider e as the control parameter, i.e. we assume a constant stiffness k_ε and allow the controller mass m_ε to vary.

---

* It should be noted here that Duffing's oscillator is widely used as a theoretical model for a number of mechanical engineering problems, such as, for example, buckling and stability of elastic beams, shells and plates, rotor dynamics, etc. More details can be found in Moon [1987] and Kapitaniak [1991].

It is well-known that the uncoupled equation (4a) (i.e. without the contoller) exhibits chaotic behaviour for certain parameter region [Ueda, 1979; Kapitaniak, 1991]. In many cases the route to chaos is a sequence of period-doubling bifurcations [Ueda, 1979; Kapitaniak, 1990].

To analyze the system with the controller (ε, $e\neq0$), we first assume that all parameters of equation (1), except the forcing frequency Ω, are constant, and estimate the Ω-domain where chaos exists. The application of the harmonic balance method enables us to determine the stability domain of the associated T-periodic solutions, where $T=2\pi/\Omega$ i.e.,

$$
\begin{aligned}
x &= C_0 + C_1\cos(\Omega t+\psi) \\
y &= D_0 + D_1\cos(\Omega t+\gamma)
\end{aligned}
\tag{5}
$$

and the associated 2T-periodic solutions

$$
\begin{aligned}
x &= A_0 + A_{1/2}\cos[(\Omega/2)t+\rho] + A_1\cos\Omega t \\
y &= E_0 + E_{1/2}\cos[(\Omega/2)t+\beta] + E_1\cos\Omega t
\end{aligned}
\tag{6}
$$

where C_0, C_1, D_0, D_1, A_0, $A_{1/2}$, A_1, E_0, $E_{1/2}$, E_1, ψ, γ, ρ and β are constants which are determined by substituting Eqs (2) or (3) into Eq.(1). The approximate boundaries of stability for these periodic solutions as functions of the forcing frequency Ω for each solution can be estimated by introducing small perturbations dx and dy to x and y, and investigating the associated Hill's equation. The complete procedure is fully described in [Kapitaniak, 1991], so we omit the details here. Knowing the period-doubling bifurcation values Ω_1'and Ω_2' at which we have bifurcation from $T \rightarrow 2T$ periodic solutions, and Ω_1'' and Ω_2'' at which we have bifurcation from $2T \rightarrow 4T$ periodic solutions, we can obtain the approximate values for of the accumulation points Ω_1^∞ and Ω_2^∞ as

$$
\begin{aligned}
\Omega_1^\infty &= \Omega_1' + \Delta\Omega_1/(1-1/\delta) \\
\Omega_2^\infty &= \Omega_2' + \Delta\Omega_2/(1-1/\delta)
\end{aligned}
\tag{7}
$$

where $\Delta\Omega_1=\Omega_1''-\Omega_1'$, $\Delta\Omega_2=\Omega_2'-\Omega_2''$ and $\delta=4.66..$ is a Feigenbaum constant. In [Kapitaniak, 1990, 1991] it has been shown that the interval $[\Omega_1^\infty,\Omega_2^\infty]$ can be considered as an approximation of the Ω frequency domain for which chaos exists.

The above procedure can be easily carried out using any symbolic algebra system (we used Mathematica) and by tracking it for different values of e we are able to obtain a map of the behaviour of Eq.(4) as a function of two parameters: the frequency Ω and the dynamical absorber control parameter e, as shown in Figure 3 (solid lines). The other parameters of Eq.(4) are: a=0.77, b=0, c=1.0, B_0=0.045 and B_1=0.16. This plot is in good agreement with the numerically calculated result shown in Figure 3 (broken lines). The numerical results were obtained using a fourth order Runge-Kutta method with a time step of $\pi/200\Omega$. To determine the chaotic behaviour, the Lyapunov exponents were calculated using the algorithm of Wolf et al. [1985].

From Figure 3 it is clear that, for fixed Ω, we can obtain different types of periodic behaviours by making only small changes in e. As an example, consider a system with Ω=0.98. Changing e from 0.01 to 0.16 it is possible to obtain easily T, 2T, 4T, 8T periodic orbits**. Theoretically, orbits of higher periods are also possible, but their narrow range of existence makes them difficult to find either experimentally or numerically. To justify calling our method a "control" technique, observe that the values of the parameter e $\in$ [0.01, 0.16] can be obtained with a controller mass m_ε approximately 100 times smaller than the main mass for the forcing frequency Ω=0.98 (Figure 2). As it is seen from Figure 3 for the other forcing frequencies in the interval [0.93, 1.05] m_ε can be even smaller.

Similar controlling effects can be obtained by varying the absorber stiffness, i.e., by simultaneous changes of the parameters ε and e [Blazejczyk et al., 1993].

**An easy access to a number of possible periodic orbits can be an advantage in certain mechanical systems, e.g., textile machines, robots, etc. For example, if we allow our system to operate in a chaotic regime during the idling time period, and then switch it to a periodic one when it is ready to perform a useful function, we can significantly reduce the fatique of the materials. Future manufacturing machines could conceivably be designed in this way in order to extend their useful life.

2.2 EFFECT OF NOISE

To show the effectiveness of our method in real experimental conditions we have also considered the effect of quasiperiodic noise, namely

$$h(t) = \sum_{i=1}^{N} \alpha_i \cos(\nu_i t + \eta_i) \tag{8}$$

where $\alpha_i \ll B_{0,1}$ are constant, ν_i and η_i are time independent random variables on the behaviour of Eq. (3). The quasiperiodic noise given by Eq. (8) is an approximation of the realization of a band-limited white noise stochastic process with zero mean and a spectral density:

$$s(\nu) = \begin{cases} s/(\nu_{max}-\nu_{min}) & \nu \in [\nu_{min}, \nu_{max}] \\ 0 & \nu \notin [\nu_{min}, \nu_{max}] \end{cases}$$

where s is the noise intensity and $[\nu_{min}, \nu_{max}]$ is the interval of relevant frequencies and can be easily simulated experimentally [Kapitaniak, 1988].

Considering the perturbed system

$$\ddot{x} + a\dot{x} + bx + cx^3 + \varepsilon(x-y) = B_0 + B_1\cos(\Omega t) + \sum_{i=1}^{N} \alpha_i \cos(\nu_i t + \eta_i) \tag{9a}$$

$$\ddot{y} + e(y-x) = 0 \tag{9b}$$

we have found the interesting property that *the presence of noise actually reduces the range of e necessary to obtain a desired periodic solution*. This property is summarized in Figure 4, where we compared the behaviour of the system (4) for different noise intensity.

3. CONTROLLING CHUA'S CIRCUIT

To verify our method in a real experimental setting we used it to control chaos in one of the simplest chaotic systems - the Chua's circuit [Madan, 1993; Zhong and Ayrom, 1985; Chua et al., 1986]. Chua'a circuit is a remarkably simple and robust electrical circuit made of only four linear elements (one resistor, one inductor, two capacitors) and a nonlinear element. The circuit is shown in Figure 5 while the state equations are given by

$$C_1\frac{dv_{C_1}}{dt}=\frac{1}{R}(v_{C_2}-v_{C_1})-g(v_{C_1})$$

$$C_2\frac{dv_{C_2}}{dt}=\frac{1}{R}(v_{C_1}-v_{C_2})+i_L \tag{10}$$

$$L\frac{di_L}{dt}=-v_{C_2}$$

where v_{C1}, v_{C2} and i_L denote the voltage across the capacitor C_1, the voltage across the capacitor C_2, and the current though the inductor L, respectively, and g($\bullet$) is the voltage versus current characteristic of the nonlinear element shown in Figure 6.

The problem of controlling chaos in Chua's circuit by both *feedback* [Hartley and Mossayebi, 1983; Genesio and Tesi, 1993] and *nonfeedback* [Kapitaniak, 1993] methods has attracted much recent interest. In our method we assume the Chua's circuit is coupled to a 2nd-order linear circuit, as shown in Figure 7 . The state equations of the angmented circuit are given by:

$$C_1\frac{dv_{C_1}}{dt}=\frac{1}{R}(v_{C_2}-v_{C_1})-g(v_{C_1})$$

$$C_2\frac{dv_{C_2}}{dt}=\frac{1}{R}(v_{C_1}-v_{C_2})+i_L+\frac{1}{R_x}(v_C^{(1)}-v_{C_2})$$

$$L\frac{di_L}{dt}=-v_{C_2} \tag{11}$$

$$L^{(1)}\frac{di_L^{(1)}}{dt}=-v_C^{(1)}$$

$$C^{(1)}\frac{dv_C^{(1)}}{dt}=-\frac{1}{R^{(1)}}v_C^{(1)}+i_L^{(1)}+\frac{1}{R_x}(v_{C_2}-v_C^{(1)})$$

where the two angmented state variables $v_C^{(1)}$ and $i_L^{(1)}$ are the voltage across the capacitor $C^{(1)}$ and the current though the inductor $L^{(1)}$, respectively.

To investigate the control of chaos in Chua's circuit, let us fix the parameters of the Chua's circuit so that the system exhibits a chaotic attractor; specifically the so-called double scroll attractor. The following typical values produced the attractor shown in Figure 8: C_1 = 10nF, B_p = 1 V, C_2 = 99.34 nF, G_a = -0.76 mS, G_b = -0.41 mS, L = 18.46 mH, , R = 1.64 kΩ. For the coupled linear system we used off-the shelf components for the inductor and the capacitor: $L^{(1)}$ = 18 mH with tolerance ±10%, $C^{(1)}$ = 100 nF with tolerance ±5%. The value of the resistance $R^{(1)}$ was experimentally chosen to be 7.67 kΩ.

The dimensionless form of Eq.(8) is obtained by rescaling the parameters of the system:

$x = v_{C1}/B_p$ $\quad y = v_{C2}/B_p$ $\quad z = i_L/B_pG$ $\quad x^{(1)} = v^{(1)}/B_p$ $\quad \alpha = C_2/C_1$

$\beta = C_2/LG^2$ $\quad \alpha^{(1)}= C_2/C^{(1)}$ $\quad \beta^{(1)} = C_2/L^{(1)}G^2$ $\quad \gamma^{(1)}=R/R^{(1)}$ $\quad \varepsilon = R/R_x$

which gives the state equations:

$$\dot{x} = \alpha(y - x - g(x))$$

$$\dot{y} = x - y + z + \varepsilon(y^{(1)}-y)$$

$$\dot{z} = -\beta y \qquad (12)$$

$$\dot{y}^{(1)} = \alpha^{(1)}[-\gamma^{(1)}y^{(1)} + z^{(1)} + \varepsilon(y-y^{(1)})]$$

$$\dot{z}^{(1)} = -\beta^{(1)}y^{(1)}$$

where ε denotes the *coupling stiffness*. Note that if ε=0 Eqs (12) described two uncoupled systems. When ε is sufficiently small the dynamics of Eqs (12) is closly related to the dynamics of the original system.

The results of our experimental control to obtain different types of periodic orbits are shown in Fig.9(a-f). Observe that a small change in the coupling stiffness ε allows us to obtain a periodic orbit (ε=0.302) as shown in Figure 9(a), a period-two orbit (ε=0.148) as shown in Figure 9(b), a period-four orbir (ε=0.134) as shown in Figure 9(c), a period-five orbit (ε=0.105) as shown in Figure 9(d) and a period-three orbit (ε=0.097) as shown in Figure 9(e). In Figure 9(f) (ε=0.322) our methods allows us to operate at a fixed point. We can summarize the experimental results as follows:

(1) all of the periodic orbits in Figure 9 are close to the original attractor of Eq. (10). We can compare them with the corresponding periodic orbits obtained from the original Chua's circuit as shown in Figure 10(a-d); note that in the original system the position and the shape of the periodic attractors are similar but have a slightly larger period;

(2) the fixed point shown in Figure 9(f) is one of the unstable fixed points of Eq.(10);

(3) the interval of the system parameter ε when the stable periodic orbits exist in the system (11) is larger than the corresponding interval in the original system (10). As a consequence we cannot experimentally find the period-five orbit in system (10).

We have obtained very similar controlling results when the Chua's circuit is coupled to a simpler one- or zero-dimensional (resistive) controller, thereby demonstrating that the domain of applicability of our method is rather broad. A second-order contoller was chosen in Fig.7 in order to compare the experimental result with the mechanical system example in Fig.2.

4. PRACTICAL CONSIDERATIONS FOR OPTIMAL COUPLING

Since our method is designed mainly for experimental applications, now we shall briefly suggest some guidelines for applyng this method.

(1) The coupled system has to be as simple as possible. In some applications it maybe a one-

dimensional linear system, while in other applications it maybe a two-dimensional linear system. In mechanical systems, it usually consists of a small mass coupled to the main mass by the spring. In electrical systems, it usually consists of a resonant LC circuit.

(2) The coupling stiffness ε should be chosen as small as possible.

(3) If it is possible one could couple the controller in such a way that the location of the fixed points of the original system are not changed. For example, consider the system:

$$
\begin{aligned}
\dot{x}_1 &= f_1(x_1,x_2,x_3) \\
\dot{x}_2 &= f_2(x_1,x_2,x_3) \\
\dot{x}_3 &= f_3(x_1,x_2,x_3)
\end{aligned}
\tag{13}
$$

Suppose that the fixed points of the system (13) are such that the second coordinate is zero: $x_2=0$. Then the following coupled system

$$
\begin{aligned}
\dot{x}_1 &= f_1(x_1,x_2,x_3) \\
\dot{x}_2 &= f_2(x_1,x_2,x_3) + \varepsilon(y_1-x_2) \\
\dot{x}_3 &= f_3(x_1,x_2,x_3) \\
\dot{y}_1 &= a_{11}y_1 + a_{12}y_2 + \varepsilon(x_2-y_1) \\
\dot{y}_2 &= a_{21}y_1 + a_{22}y_2
\end{aligned}
\tag{14}
$$

will have the same fixed points as those of Eq. (13), as can be seen from Eq. (14).

In fact the coupling in the system considered in Sec.3 is chosen in this way. The advantage of such a coupling is obvious: with a relatively small ε we can stabilize the exact fixed point of the original system.

In the mechanical system of Sec. 2 it is difficult to apply this type of coupling but with an additional mass 100 times smaller than the main mass the simple analysis of the augmented system shown in Figure 2 shows that the original fixed points are only slightly perturbed.

5. CONCLUDING REMARKS

In this paper we have shown that chaotic behaviour can be converted into a desired periodic behaviour without feedback by coupling the chaotic system to a simple linear controller. The appropriate control is obtained by changing one of the controller parameters without changing any of the main system parameters. This method offers a way of controlling chaos without the necessity of following a response trajectory and targeting it to some desire domain of the phase space. An additional advantage of our method is that we can stabilize not only periodic orbits, but also fixed points.

The method can be especially useful in mechanical systems, where its simplicity offers important practical advantages in comparison with other controlling methods [Ott et al., 1990; Jakson, 1990; Shinbrot et al., 1990; Ditto et al., 1991; Hunt, 1991; Mehta and Henderson, 1991; Tel, 1991; Braiman and Goldhirsch, 1991; Vincent and Yu, 1991; Singer et al., 1991; Shinbrot et al., 1992; Chen and Dong, 1992; Kapitaniak, 1992; Kapitaniak, 1993, Hartley, 1983; Genesio and Tesi, 1993]. For example, in mechanical systems the feedback controllers are usually very large (sometimes even larger than the controlled system), and have complicated dynamics. In comparison with such controllers the simplicity of our controller (which is realizable by the simplest mechanical system, e.g., mass on a spring with controllable stiffness) offers a straightforward yet effective approach. In practical mechanical applications our method is even simpler than taming chaos with weak periodic or random perturbations [Kapitaniak, 1988; Braiman and Goldhirsch, 1991], as we do not need a constant source of energy. As shown in Sec.3 this method can be applied not only to mechanical systems, but also to such electrical systems as microelectronic and VLSI circuits, where it is difficult, if not impossible to access the internal circuit parameters. It can also be exploited in the design of *foult - tolerant* elecrical systems where a previously built-in linear contoller can be switched on by a remote signal (e.g., from earth to a malfunctioning satellite) to stabilize a system which had become chaotic due to aging, radiation, etc.

The simplicity of controlling chaos by this and similar methods and the possibility of easy access to different periodic orbits can foster wider applications of chaotic systems in practice.

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CAPTIONS

Figure 1. Chaotic system coupled to an appropriately design controller without any external inputs.

Figure 2. The dynamical absorber.

Figure 3. Behaviour of the equations (1) for different values of e and Ω: a=0.77, b=0, c=1.0, d=0.002, B_0=0.045 and B_1=0.16; analytical approximation (solid line), numerical simulation (broken line).

Figure 4. Effect of noise on the behaviour of Eqs (4).

Figure 5. The Chua's circuit.

Figure 6. Nonlinear resistor characteristic.

Figure 7. The Chua's circuit coupled to a two-dimensional linear system.

Figure 8. The double scroll attractor; vertical axis: v_{c2} 200mv/div, horizontal axis v_{c1} 1v/div.

Figure 9. Effects of experimental controlling procedure; vertical axis: v_{c2} 200mv/div, horizontal axis v_{c1} 1v/div: (a) periodic orbit (ε=0.322), (b) period-two orbit (ε=0.302), (c) period-four orbit (ε=0.148), (d) period-five orbit (ε=0.105), (e) period-three orbit (ε=0.097), (f) fixed point (ε=0.322).

Figure 10.Experimental periodic orbits in the Chua's circuit, (a) periodic orbit (R=1.811kΩ), vertical axis: v_{c2} 200mv/div, horizontal axis v_{c1} 2v/div (b) period-two orbit (R=1.777kΩ), (c) period-four orbit (R=1.769kΩ), (d) period-three orbit (R=1.749kΩ); (b-d) vertical axis: v_{c2} 200mv/div, horizontal axis v_{c1} 1v/div

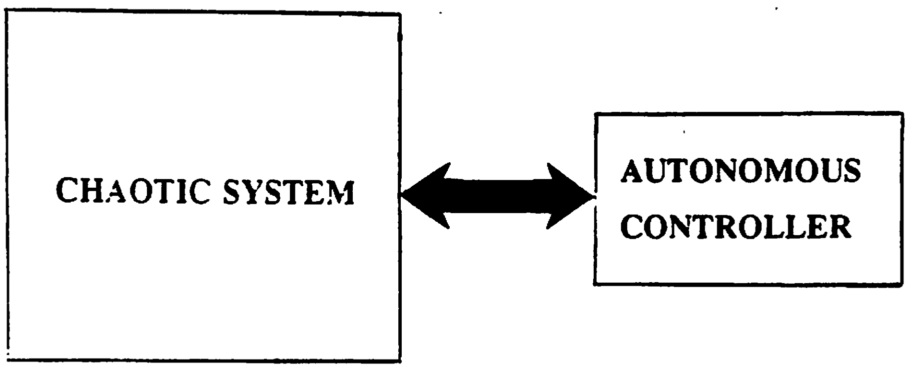


Figure 1

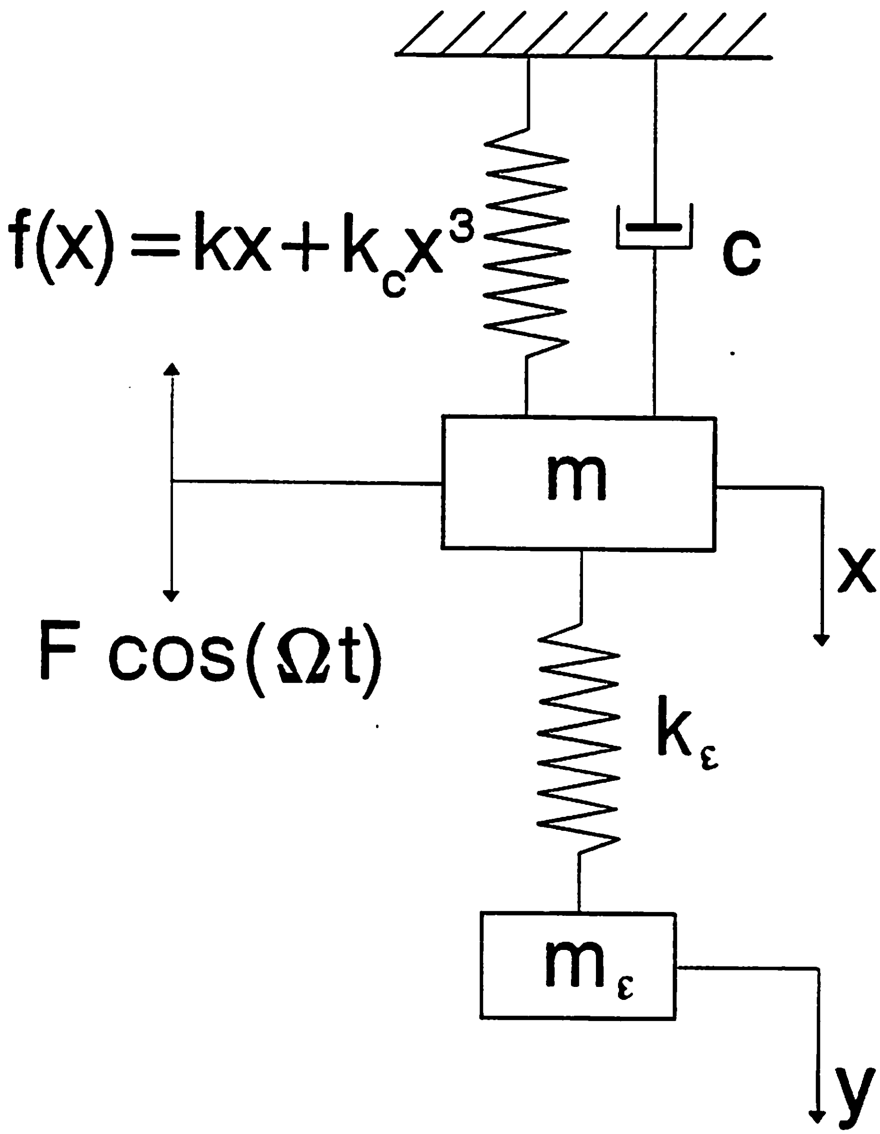


Figure 2

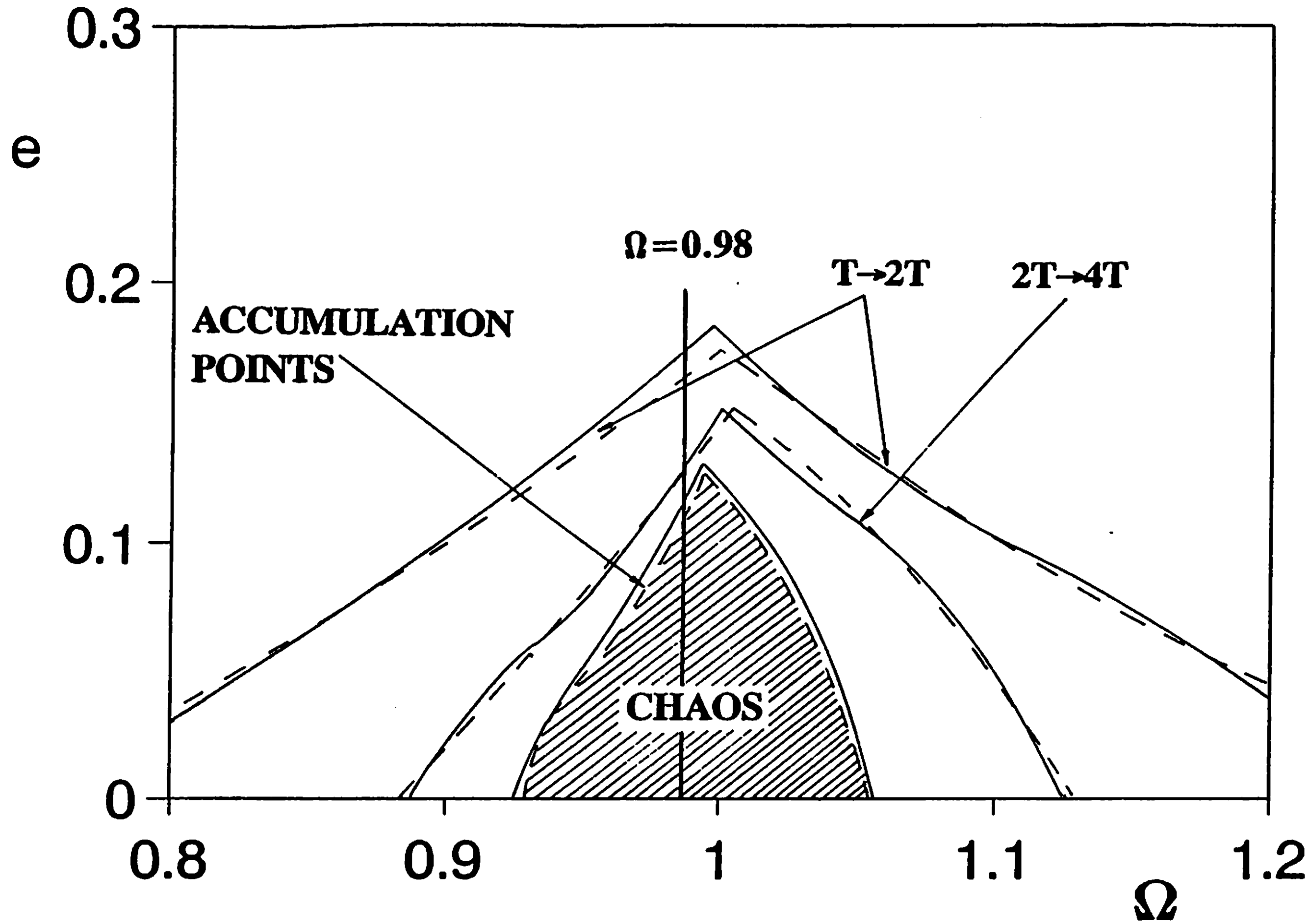

e
0.3
0.2
0.1
0
0.8
0.9
1
1.1
1.2
Ω
Ω=0.98
T→2T
2T→4T
ACCUMULATION
POINTS
CHAOS

Figure 3

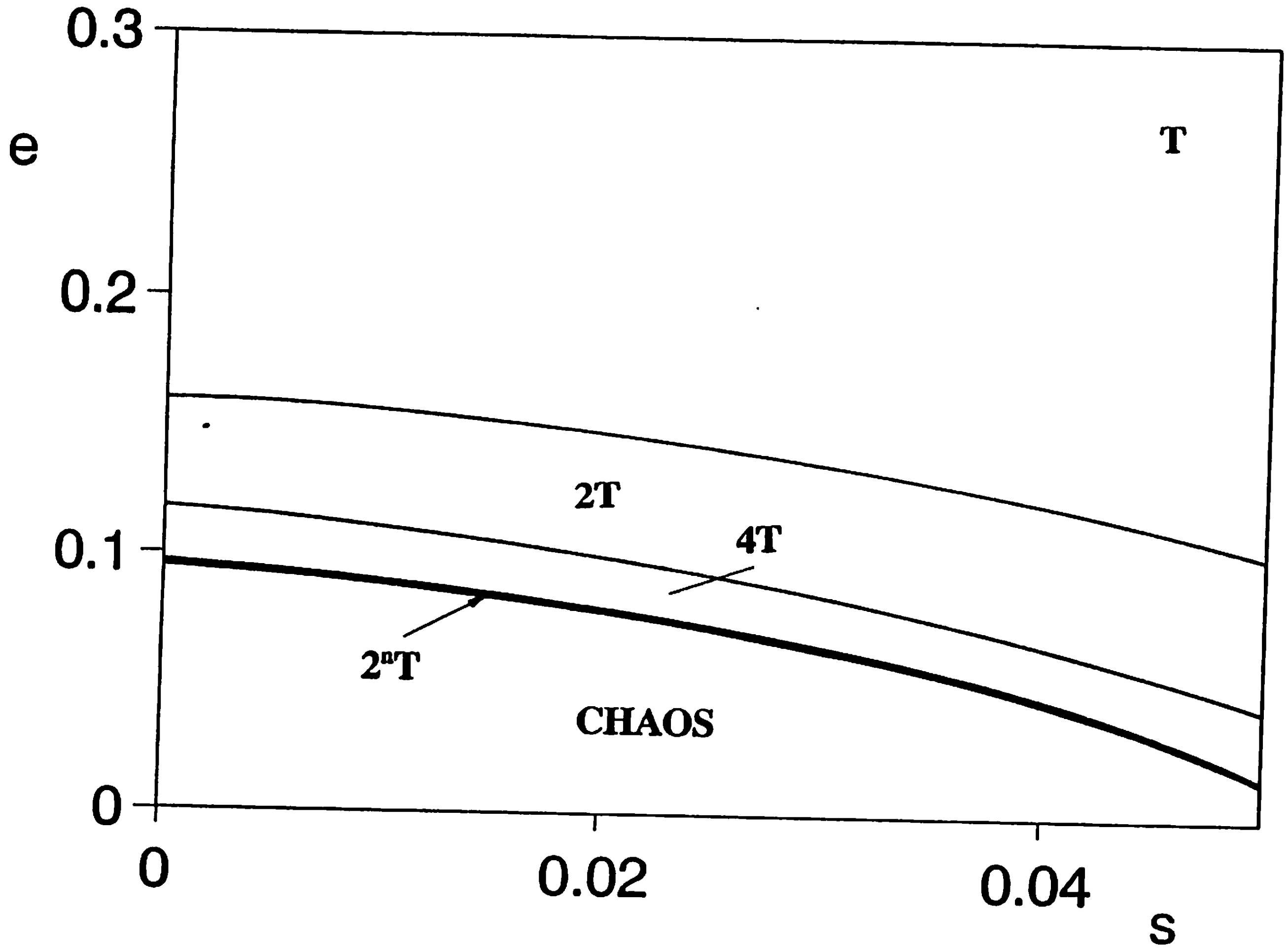
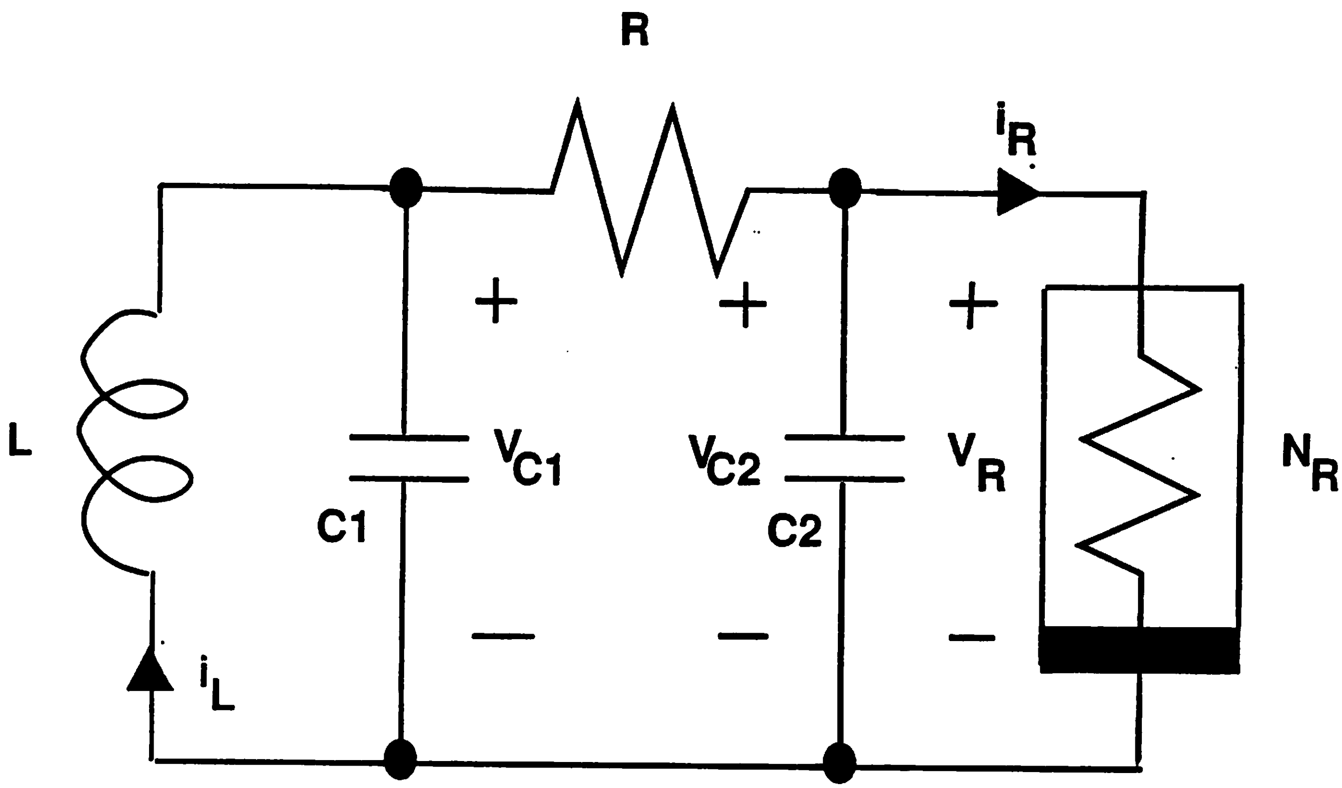


Figure 4



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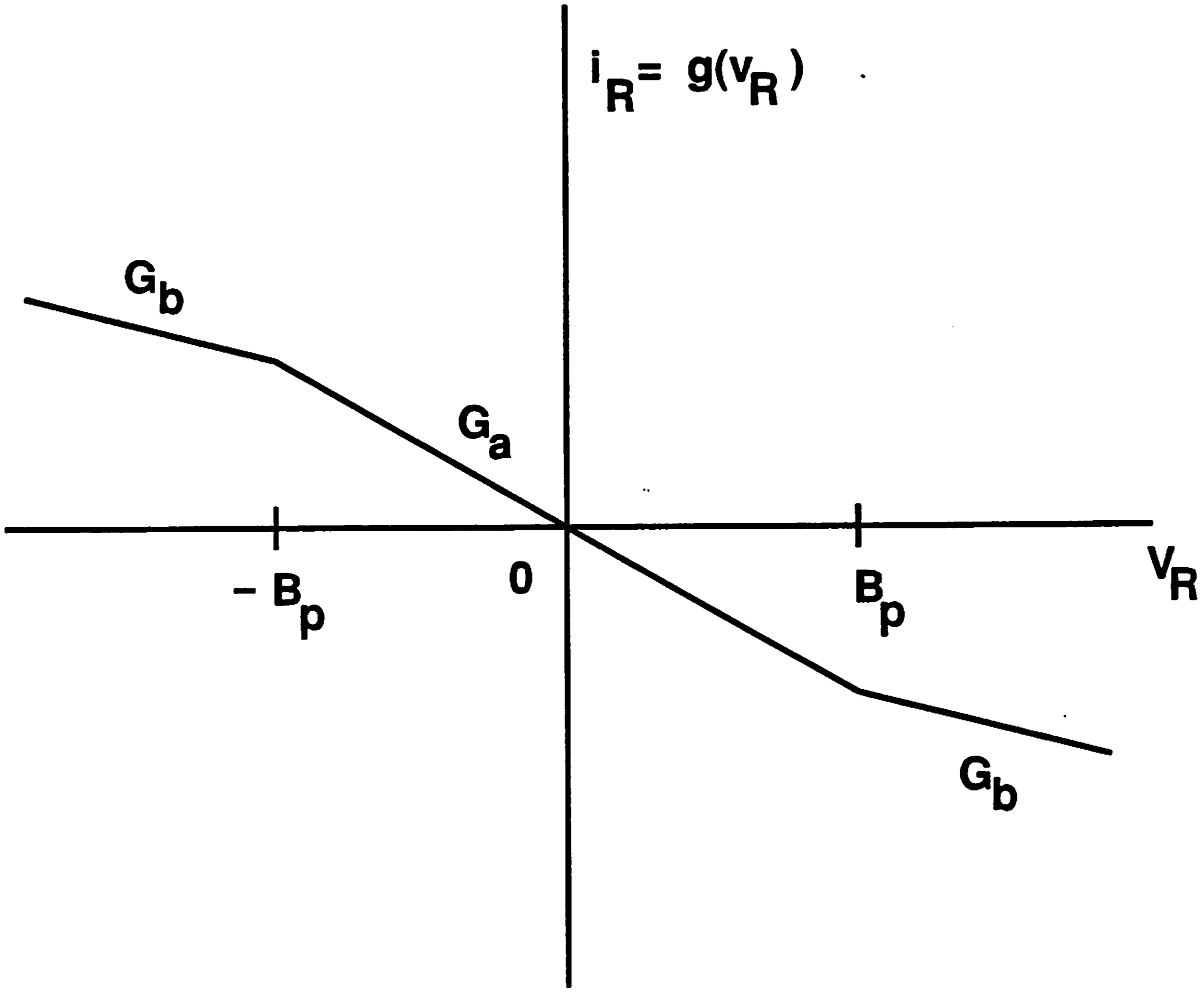

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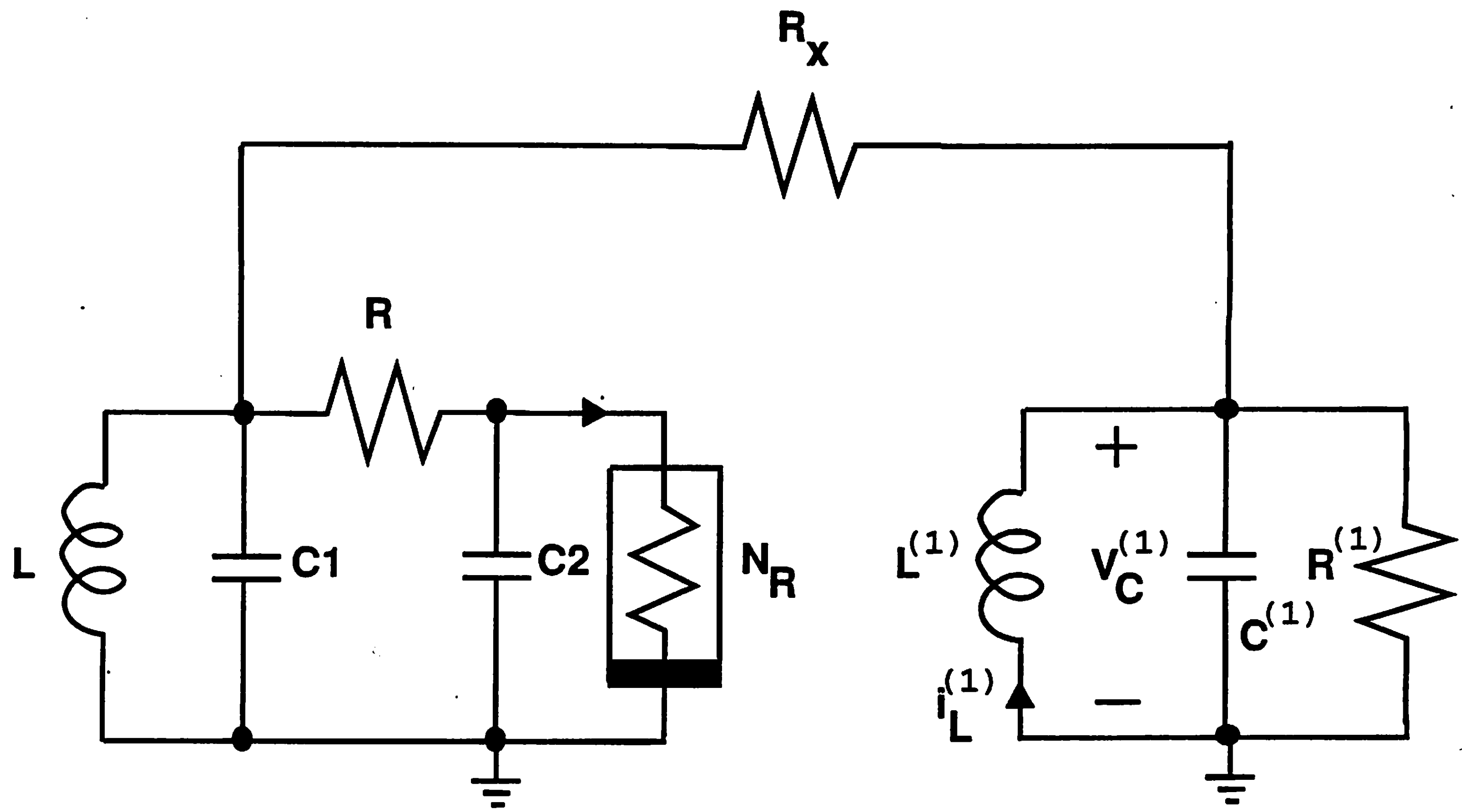

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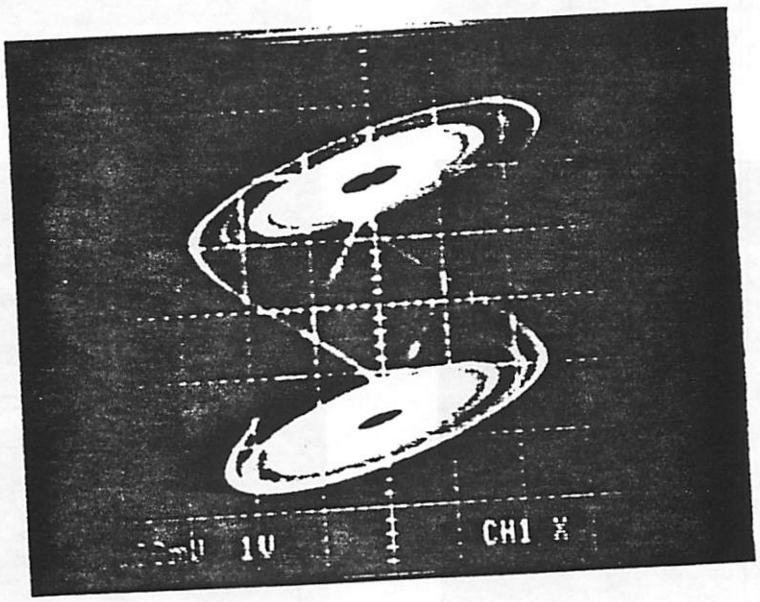
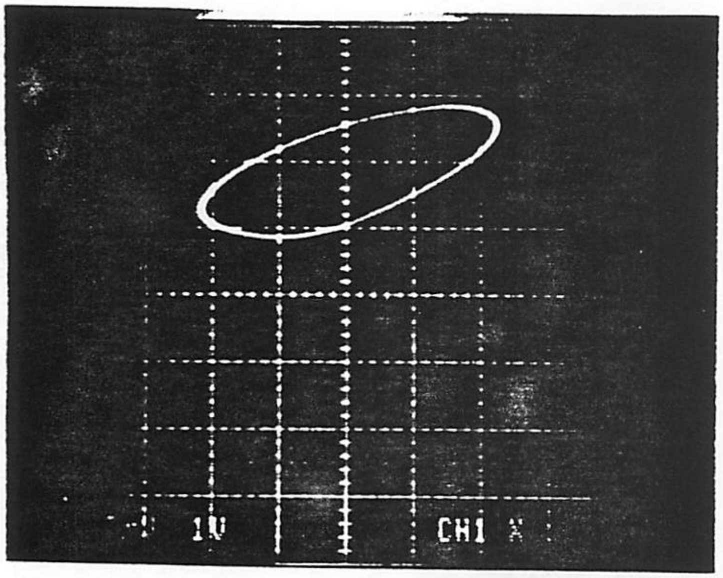
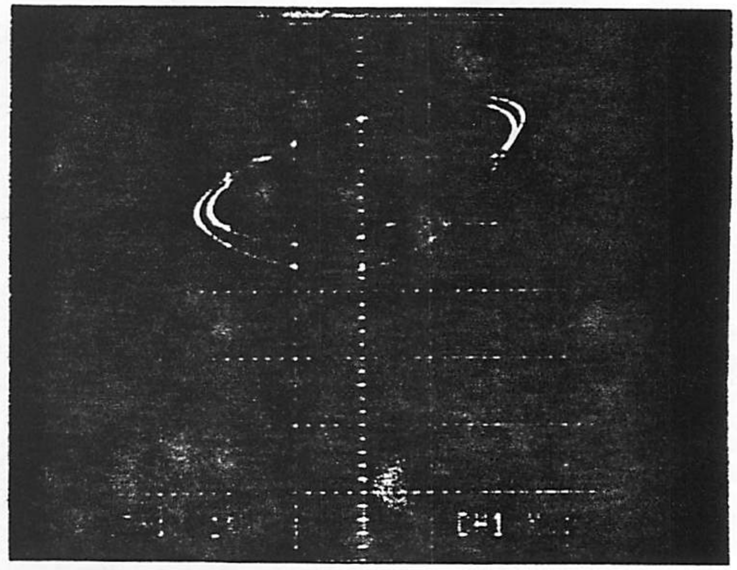


FIG 8



(a)



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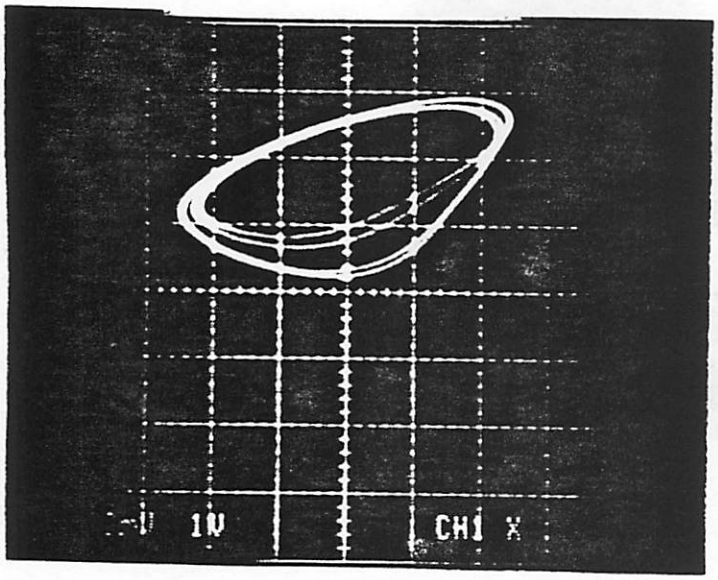
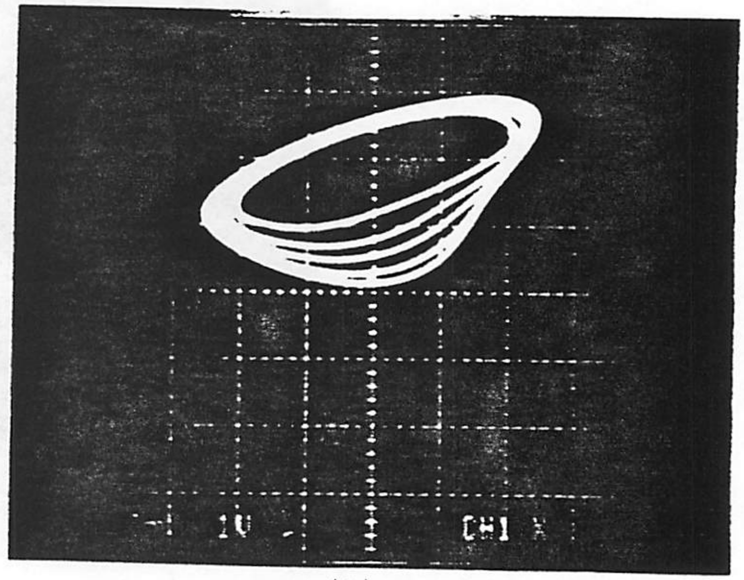


(c)



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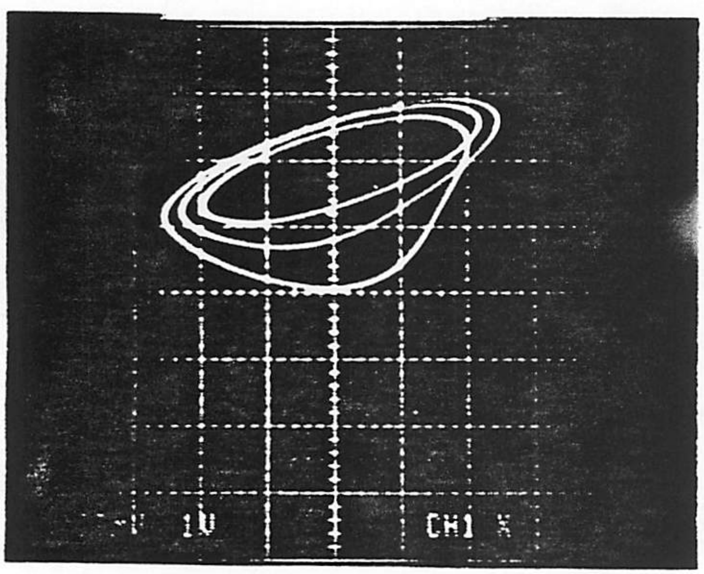

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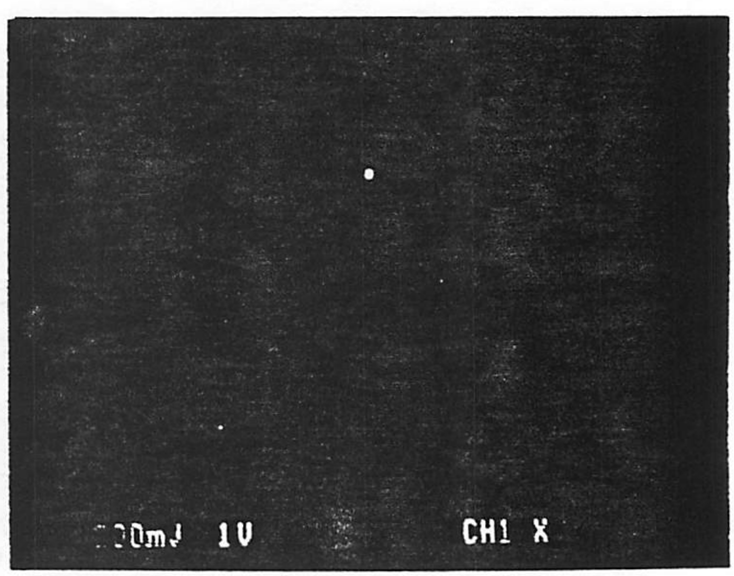
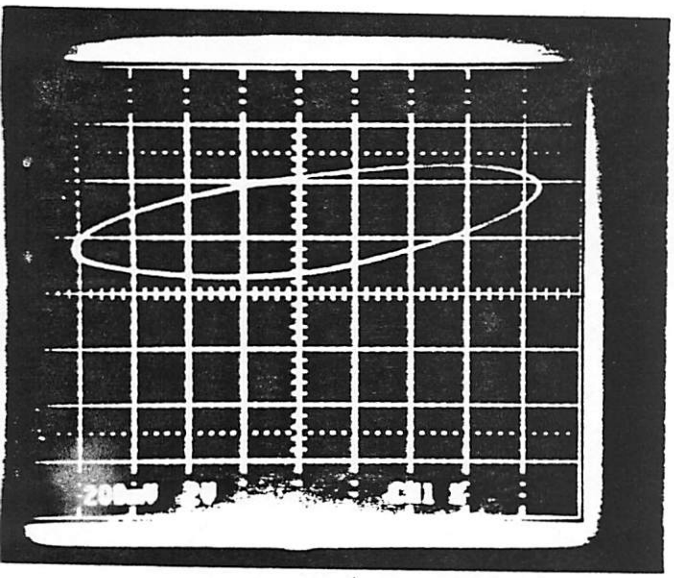
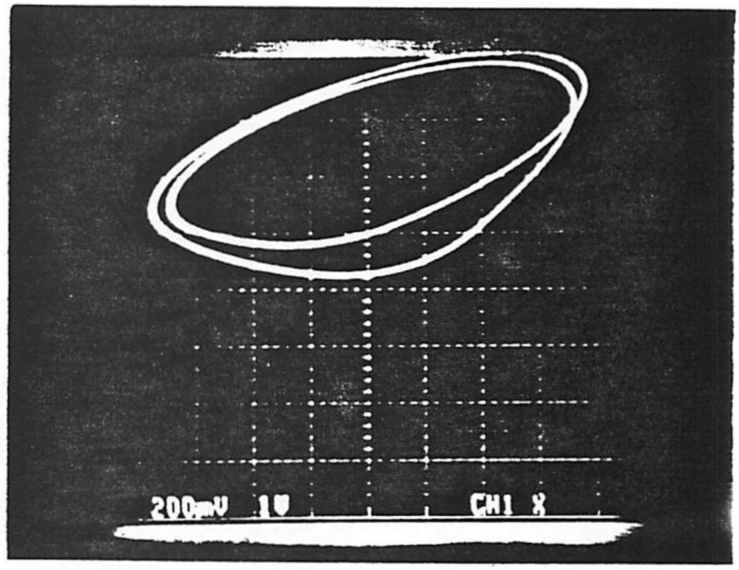


(f)

FIG. 9



(a)



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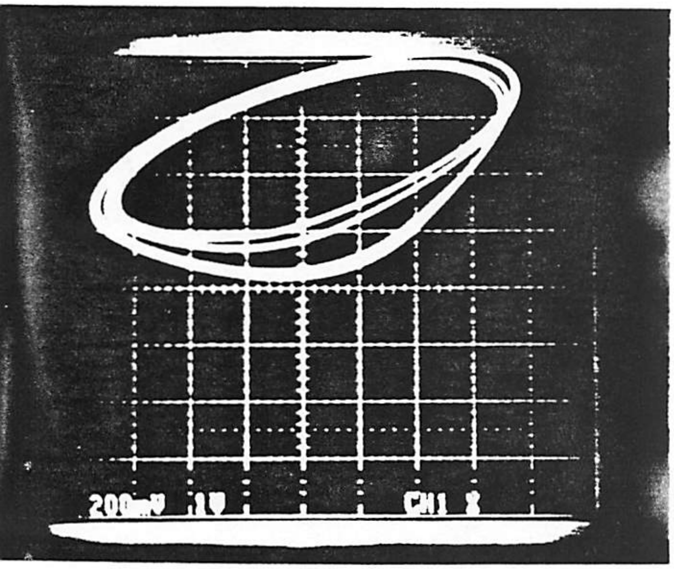

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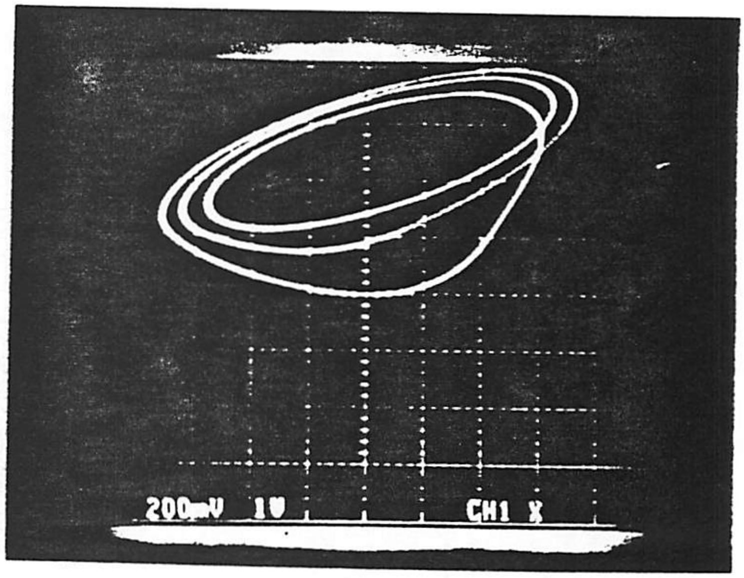

(d)

Fig. 10