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# Multiple-Output Shared Transistor Logic (MOSTL) Family Synthesized Using Binary Decision Diagram 

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#### Abstract

A new type of logic family, Multiple-Output Shared Transistor Logic (MOSTL) family, is defined and a synthesis method for generating MOSTL is described. The MOSTL implements a logic function not by combining logic gates such as NAND's and OR's, but by combining transistors directly as switches. Since the MOSTL has more freedom in realizing a logic function, it offers a smaller and faster circuit than the standard cell based approach. More concretely speaking, in the MOSTL, transistors are shared among several logic functions and thus the number of MOSFETs are reduced and this in turn may reduce delay time. It is best suited for the Sea-Of-Gates designs and a full manual design where designers are permitted to build a circuit at a transistor level.


A synthesis method presented is based on Binary Decision Diagram (BDD) and usually gives a good solution. The method is demonstrated to generate a sneak-path free circuit and in this sense never fails to produce a solution, which is an important feature when applied to real designs.

A MOSTL together with the synthesis method will provide a systematic way to generate a 'clever' circuit, which could only have been built by the ingenuity of experienced circuit designers otherwise.

## 1. Introduction

Transistor level logic network synthesis has been attracting attentions for a long time[1-5,1416] since the early and important work of Shannon[10,11]. Horn et al. [14, 16] in the middle of 50's proposed a symbolic matrix technique to tackle the problem and it is useful in the analysis of a logic switching network but as for the synthesis it was based on the intuition and gave a limited success.

There are two major advantages in using the transistor-level synthesis. One is the use of 'pass variables' or 'pass transistors', a good example of which is a steering logic family introduced in [17]. The other is a sharing of transistors among different switching paths. The former advantage is pursued by recent researches [5,15] and a big advance has been observed in this area but the latter advantage is not studied well. Wu et al. [3, 4] investigated the sharing problem and a limited success has been reported if the problem is confined to a single-contact, single-output network where one variable can drive only one control gate of a transistor and the number of outputs is one. Even for the general single-output case, the synthesis method is still based on intuition.

In this report, one practically important logic family, namely Multiple-Output Shared Transistor Logic (MOSTL) family, is defined and a systematic way of synthesizing it is described. The MOSTL is a single-stage logic gate and more general than the single-output logic gate. It utilizes both of the pass variables and the transistor sharing and includes usual CMOS complex gates, a steering logic and a barrel shifter.

A synthesis method presented is based on Binary Decision Diagram (BDD)[6,7] and usually gives a good solution. The method is demonstrated to generate a sneak-path free circuit and in this sense never fails to produce a solution, which is an important feature when applied to real designs.

In Section 2, MOSTL is defined and examples are given. A synthesis method based on BDD is described in Section 3, followed by a sneak-path free nature of the the generated circuits is discussed in Section 4. Section 5 and 6 are dedicated for discussions and possible area of future works and conclusions, respectively. In Appendix, a sample program is shown for the BDD-based minimizer.

## 2. Multiple-Output Sharing Transistor Logic (MOSTL)

The schematic diagram of the MOSTL is shown in Fig. 1 and an example is given in Fig.2. In Fig.1, the NMOS and PMOS blocks include a transistor circuit where any number of output terminals are connected to a power line or to pass variable inputs according to the control variables. From the left side of the boxes, control variables are input and from the bottom or the top of the boxes, pass variables are incurred. The NMOS/PMOS block can include non-serial-parallel structure and non-planar structure. Three variations are shown in the figure but several other configurations are also possible.

The salient feature of MOSTL is the exclusion of mixing PMOS's and NMOS's in one circuit block and that the only one power source attached to the NMOS logic part is VSS and the only power line connected to the PMOS logic is VDD. By limiting the structure like this, it is possible to eliminate a multi-stage nature and $\mathrm{V}_{\mathrm{TH}}$ problems from the synthesis. $\mathrm{V}_{\mathrm{TH}}$, threshold voltage of

MOSFET, hinders the output to swing full VDD-VSS range and this degrades circuit margins if it is not treated properly. The multi-stage nature makes the problem intractable. The MOSTL includes most of the practical logic circuits such as CMOS complex gates, a barrel shifter, and a steering logic family.


Fig. 1 Schematic diagram of MOSTL

The example in Fig. 2 is for a parity generator circuit for three input. The functional descriptions are:

$$
\begin{gathered}
f=a b c^{\prime}+a b^{\prime} c+a^{\prime} b c+a^{\prime} b^{\prime} c^{\prime} \\
f^{\prime}=a b c+a b^{\prime} c^{\prime}+a^{\prime} b c^{\prime}+a^{\prime} b^{\prime} c
\end{gathered}
$$

In this expression, prime (') denotes an inverted input. This type of logic function is difficult to minimize by a standard cell approach. Direct implementation of the logic function by a parallel-serial CMOS transistor network needs 48 transistors, while the MOSTL needs 20 or 16 transistors depending on the use of pass variables. If the number of inputs is increased, the advantage becomes more eminent.

a) Load + NMOS

b) CMOS + control variables

c) CMOS + control variables + pass variables

Fig. 2 An example of MOSTL (a parity generator circuit)

## 3. Synthesis of the MOSTL Using Binary Decision Diagram (BDD)

In this section, a synthesis method is described. The synthesis begins by building a BDD. To build the BDD, there are several methods. One method [6] is: first generate logic binary trees for
separate logic functions as shown in Fig. 3 and then merge these trees by merging common subtrees from the bottom. In Fig.3, the left-most $©$ in the left tree means that the function goes to ' 0 ' when $\mathrm{c}=1$ and goes to ' 1 ' when $\mathrm{c}=0$. Consequently, the subgraph which is rooted at the $©$ and the subgraph which is rooted at the second left © in the middle tree is considered to be the same. So the pointer to the second left © can be switched to the left-most © in the left tree. Applying this procedure iteratively, the reduced BDD of the right graph can be obtained. The detailed description of the procedure is found in [6].

The BDD has an important feature that if the input ordering is given, the reduce $\operatorname{BDD}$ is unique so that it can be used as a standard form of logic function. The number of edges included in the BDD depends on the ordering and the optimum ordering is difficult to find without an exhaustive search. However, for less than 5~6 inputs, the exhaustive search is possible and since the MOSTL is a single-stage gate, the number of input is small.

Once the BDD is constructed, it is easy to interpret the graph as a transistor circuit. The edges directed to the terminal 1 basically correspond PMOS block MOSFETs and the edges directed to the terminal 0 correspond to NMOS block MOSFETs. When constructing a PMOS block, $\mathrm{a}=1(0)$ edge should be converted to a PMOSFET whose gate is controlled by a' (a). For a NMOS block, $a=1(0)$ edge should be converted to a NMOSFET whose gate is controlled by a (a'). A literal whose two children are 1 and 0 may be replaced by a pass variable input.

Further reduction in the number of transistors is possible when checks are made for all edges if the edges can omitted or shorted. The example of this further reduction is explained next using a more complicated example.


Fig. 3 Binary Decision Diagram (BDD) for a parity generator

Figure 4 and TABLE I show a Karnaugh map and a truth table of the more complicated example, respectively. There are three output terminals and the functional description is:

$$
\begin{gathered}
f 1=A B^{\prime} C+A^{\prime} D^{\prime}+A^{\prime} B^{\prime} C \\
f 2=A B^{\prime} D^{\prime}+A^{\prime} B \\
f 3=A C+A^{\prime} B C^{\prime}+A B^{\prime} C^{\prime} D^{\prime} .
\end{gathered}
$$



Fig. 4 Karnaugh map of 'relay3' example

TABLE I Truth table of 'relay3' example

| A | B | C | D | $f 1$ | $f 2$ | $f 3$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 | 1 |

After constructing a BDD, the NMOS and PMOS blocks are extracted separately. Then each edge in the graph is tested if it can be omitted or shorted. If it can be omitted or shorted, the transistor can be eliminated. In this example of 'relay3'[1] in Fig.5, five edges are shortable. The shorting process may create a sneak-path (see the next section), so that a careful validity checking of the shorting should be done. One way of doing this is through a simulation, which is adopted in the program listed in Appendix.

For this example, the number of transistors needed is 27 as shown in Fig.5, but a parallelserial implementation of the logic leads to 42 transistors.


Fig. 5 Synthesized MOSTL for 'relay3' example

## 4. Edge-Merging and Sneak-Path

In the synthesis of MOSTL or more general transistor switching network, a sneak-path is a difficult problem. An example of the sneak-path is shown in Fig.6. Suppose two functions $f=\mathrm{a}$ and $g=a+b$ are to be realized. First, the edge controlled by a is connected to $f$ and the edge controlled by b is connected to g realizing that $f=\mathrm{a}$ and $\mathrm{g}=\mathrm{b}$. Then to make g be $\mathrm{a}+\mathrm{b}$, vertices i and j can be connected. Then g becomes correct but $f$ becomes incorrect because there exists a path from $\sqrt{1}$ to $f$ through b . This is a sneak-path. Usually sneak-paths are not obvious and a critical checking should be employed to reveal the sneak-paths.

The essence of the sneak-path is the existence of contradiction on the assignment of logic values on one vertex. In the example, when $\mathrm{a}=0$ and $\mathrm{b}=1, \mathrm{~g}$ expects vertex j to be 1 while $f$ expects vertex j to be 0 , which is a contradiction.


Fig. 6 Sneak-Path

A very powerful transformation in constructing MOSTL is 'edge-merging' as shown in Fig.7. The essence of the edge-merging is to merge two edges with one node common controlled by the same variable into one edge. Other than the BDD based method described above, this edge-merging seems promising. The drawback of the edge-merging, however, is the creation of sneak-path.


Fig. 7 Edge-Merging technique.

It can be demonstrated that the BDD based method generates a network which does not contain sneak-paths. First, separate logic binary trees do not have sneak-paths because only one path is activated at a time which connects an output to the power source. The reduce operation in the BDD reduction scheme does not create any sneak-paths. This latter part is explained in more detail. The reduce operation includes only two kinds of procedures as shown in Fig.8.

One procedure is an elimination procedure and the other is a subgraph sharing procedure. The elimination procedure does not introduce a sneak-path because the only thing this procedure does is to assign one physical vertex instead of two logically shorted vertices. If there exists a sneak-path after the procedure, it must be existed before the procedure.

The subgraph sharing procedure does not introduce any sneak-paths either because whenever vertex $j$ expects $0(1)$ on vertex $n, i$ also expects $0(1)$ on the vertex $n$. So there is no contradiction and thus no sneak-paths.


Fig. 8 Two basic procedures to reduce BDD. The sneak-path free nature is demonstrated by using this figure.

## 5. Discussions and Future Work

The synthesis method presented here based on a BDD usually gives a good-quality solution to a MOSTL generation problem as in the example of relay3 circuit. Sometimes it gives the optimum transistor network as in the example of parity generator circuits. However, the method does not always guarantee the optimality so that sometimes the method generates a bad circuit. In this sense, some procedure is preferable to be taken to improve the generated transistor network. Simulated Diffusion or Simulated Annealing can be a choice.

Other than the synthesis method itself, a research as a VLSI synthesis system is of interest. The total system may look like Fig.9. We can make use of a standard logic minimizer [8,9] and the several outputs of logic functions generated by the logic minimizer which share common inputs are bundled together and input to a MOSTL synthesizer. The transistor sharing and the pass variables are treated properly in the MOSTL synthesizer.

The partitioner in Fig. 9 and MOSTL generator should be working cooperatively or iteratively so as to optimize the area and speed. A research should also to be carried in this area. That is, multi-stage MOSTL optimization is the important next step. As is mentioned in previous section, the inclusion of don't care condition is another area to look into, although simple inclusion is easy.


Fig. 9 MOSTL generator incorporated in a design system

## 6. Conclusions

A new family of logic circuit is introduced and a synthesis method is presented based on a BDD. Although this method does not guarantee to give the optimum circuit and some extensions are desirable, it usually gives a good solution. The method is demonstrated to generate a sneak-path free circuit and in this sense never fails to produce a solution, which is an important feature when applied to real designs.

A MOSTL together with the synthesis method will provide a systematic way to generate a 'clever' circuit, which could only have been built by the ingenuity of experienced circuit designers otherwise. Sea-Of-Gates and a fully manual design can be benefitted by the proposed method.

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## Appendix A Program Listing

Program source codes are shown in the following pages. The programs are written in QuickBasic Ver.1.0 for Macintosh SE/30. The following is an example of the input to the program.

## Input example of BDD synthesizing program

Two numbers in the first line are number of input and output.
The following lines include function description.
Actually they are the output bit pattern of the function corresponding the input
$0,0,0,0$
$0,0,0,1$
$0,0,1,0$
$0,0,1,1$
-
-
-

4, 3
1,0,0
$0,0,0$
1,0,0
0,0,0
1,1,1
$0,1,1$
1,1,0
1,1,0
0,0,0
$0,1,1$ 1,0,1 $1,1,1$ 0,0,0 $0,0,0$
$0,0,1$ $0,0,1$
option base 0
DEFINT a-w
$E=10000$ : nlaaf $=1:$ ntunc $=1: n \times D=1: n \mathrm{~V}=1$
DIM SHARED bitm(nleaf.ntunc), obilm(nteaf.ntunc)
DIM SHARED XD2YD(nXD). VPat(nV). permV(nV), minPermV(nV)
DIM SHARED XL( $n \times D$ ), XH( $n X D)$, YL( $n \times D)$ ) YH( $n \times D), Y(n \times D)$
DIM SHARED pobitm(nleaf, nuunc). V2XD1(nV), V2XDn(nV). V2YD1(nV).
V2YDn(nV)
DIM SHARED nYFan(nXD), YFan(nXD, INT(nXD / 2)), YFanV(nXD.
INT(nXD / 2)), visihed(nXD)
DIM SHARED queue(500)
DIM SHARED YHbut( $n \times D$ ). YLbuf( $n \times D$ ). Mbuf( $($ KXD)

- V:inpui variable L:low $H$ :high
-... Pat : Patherm
-- X : original data
-- $Y$ : reduced data
Windower:
-.... initialize window 1 ...
WNDOW 2."Graphics Window'. (200,20)-(480, 300). 1
WNDOW 1.'Text Window'. (0.20)-(200, 300), 1
TEXTSIZE 8
TEXTFONT 3
OPEN "scm:" FOR OUTPUT AS \#1
'--- menu ....
MENU 1.0.1."File"
MENU 2,0.1,"Edit"
MENU 3,0,1,"BDD"
MENU 4.0.1,"Params"
MENU 1,1.1,"Load'
MENU 1,2.1."Show Loaded Data"
MENU 1,3,1."Output Select"
MENU 1,4,0."Print"
MENU 1,5,0,.-
MENU 1,6.1. ${ }^{\text {"Quit }}$ ": cmdkey 1,6." $\mathrm{Q}^{\text {" }}$
MENU 2.1,0."Copy":cmdkey 2,1, ${ }^{\circ}{ }^{\circ}$
MENU 3.1.1,"Create \& Reduce"
MENU 3,2,1."Exhaustive Search"
MENU 3.3.1."Minimize"
MENU 3.4.1,"Show BDD"
MENU 4.1.0."Params Set"
ON MENU GOSUB Menucheck: MENU ON
Ide:
GOTO lde
Menucheck:
menumumber $=\operatorname{MENU}(0)$
menuitem $=$ MENU(1)
MENU
ON menunumber GOSUB Filer. ClipBoarder, Bdder, Setter
RETURN
Flar:
ON meruitem GOSUB Loader, Shower, Outer, Quitter, Quiter ,Quiter RETURN


## Quitter.

CLOSE
SetCreate "bdd.our, MSWD"
WINDOW CLOSE 1
WNDOW CLOSE 2
END
ClipBoarder:
ON meruitem GOSUB ClipCopier
RETURN
Bdder:
showflag $=111$
ON menuitem GOSUB BDDCreateReducer, BDDExhaustiver, BDDMinimizer, BDDShower RETURN

## Setter:

ON menuitem GOSUB Quitter
RETURN

```
ClipCopier:
    OPEN "CLP:PICTURE FOR OUTPUT AS \#3
    PRINT \#3, imagos
    CLOSE \#3
RETURN
Eraser:
    WNDOW2
    WNDOW1
    images \(={ }^{-}\)
    CLS
RETURN
Loader:
    -.... load data -...
    inflas = FILESS(1,TEXT")
    FF (infles \(=-\) ) THEN RETURN
    OPEN infies FOR INPUT AS \#2
    -... input \# of inpui \& \({ }^{\#}\) of output --
    INPUT \#2, nV, ntunc
    noesf \(=2^{\wedge} n V\)
    ERASE bitm. oblim
    DIM SHARED bitm(nleaf,nV), obitm(nloaf, nfunc)
    --. input output bit pattern ...
    FOR iloaf = 1 TO nloaf
        LINE INPUT *2, inlinos
        FOR iturc = 1 TO nfunc
            obitm(leaf, lifunc) \(=\) VAL(MIDS(inlines. \(2(\) ifunc-1) \(+1,1\) )
        NEXT
    NEXT
    -... initialize bitm --.
    FOR iloat = 1 TO nteof
        remainder \(=\) illeaf - 1
        FOR \(N=n V\) TO 1 STEP - 1
            thisbit \(=\) remainder MOD 2
            bim(lieaf, \(\mathcal{M}\) ) thisbit
            remainder \(=(\) remainder - thisbil) \(/ 2\)
        NEXT
    NEXT
    CLOSE \#2
RETURN
Shower:
    '-.. show loaded data ....
    WNDOW 1
    PRINT \#1, "nVar=";nV, "nuunc=";ntunc
    FOR ilag = 1 TO nlead
        FORN \(=1\) TO nV
            PRINT \#1, bitm(ikaf, M): ":
        NEXT
        PRINT \#1,* -
        FORAunc \(=1\) TO ntunc
            PRINT "1, oblm(liesf, lunc);" ";
        NEXT
        PRINT \#1,
    NEXT
RETURN
Outer:
    -... output device select ....
    WNDOW 1
    PRINT "Ouput to screen(0)"
    INPUT "or new file(1) or append to the file(2)"; outdev
    CLOSE \#1
    ouffies = "bdd.ouf
    SELECT CASE ouldov
    CASE 0
        OPEN "8Cm:" FOR OUTPUT AS \#1
    CASE 1
        'ouffias = FILESS(0)
        IF (outfilos = \({ }^{-}\)) THEN RETURN
```



```
    CASE 2
        'oufiles = FILESS(1,TEXT?
        IF (outfiles \(=-\) ) THEN RETURN
        OPEN outfies FOR APPEND AS \#1
```



```
    CASE ELSE
        OPEN \({ }^{\text {Bcm }}\) :" FOR OUTPUT AS \#1
    END SELECT
RETURN
BDDInitializer:
```

```
    --Intiadize BDD ...
    \(n \times D=\left(2^{\wedge} n V-1\right) \cdot\) nfunc +1
    - define arrarys -..
    ERASE XD2YD. VPat. permV. minPermV. XL XH. YL. YH. Y
    ERASE pobitm. V2XD1, V2XDn, V2YD1, V2YDn
    ERASE nYFan, YFan. YFanV, visited
    ERASE YHbuf, YLbuf. Mout
    DIM SHARED XD2YD(nXD). VPat(nV). permV(nV), minPermV(nV)
```



```
    DM SHARED pobim(ntaaf, nuunc). V2XD1(nV). V2XDn(nV).
V2YD1( \(n \vee+1\) ). V2YD \(n(n \vee+1)\)
    DM SHARED nYFan(nXD). YFan(nXD, INT(nXD / 2)). YFanV(nXD
WT(nXD / 2)). visited( \(n \times D\) )
    DM SHARED YHbuf( \(n \times D\) ). YLbuf( \(n \times D\) ). Wouf( \(n \times D\) )
    \(X L(0)=0: X H(0)=0: X L(1)=1: X H(1)=1\)
    - indititization of V2XO1, V2XDn, I, h -
    \(\mathrm{V} 2 \times \mathrm{D}_{1}(1)=2: \mathrm{V} 2 \times \mathrm{Dn}(1)=2^{\wedge}(\mathrm{nV}-1) *\) ntunc +1
    \(F O R N=2 T O \mathrm{nV}\)
    \(\operatorname{V2XD1}(\mathrm{M})=\operatorname{V2XDn}(\mathrm{N}-1)+1\)
    V2XDn(M) \(=\) V2XD1 \((\mathrm{M})-1+2^{\wedge}(\mathrm{nV}-\mathrm{M})\) * nume
    \(\mathrm{XL}\left(\mathrm{N}_{2} \mathrm{XD} 1(\mathrm{M})\right)=\mathrm{V}_{2 \times D 1}(\mathrm{~N}-1)\)
    \(\mathrm{XH}(\mathrm{V} 2 \mathrm{XD} 1(\mathrm{M})=\mathrm{V} 2 X \mathrm{D} 1(\mathrm{~N}-1)+1\)
    NEXT
RETURN
```

BDDCreateReducer:
-- BDD create and reduce according to the input perm --.
GOSUB BDDtritiaizer
WNDOW1
PRINT "Enter permutation pattern."
PRINT There should be":nV:" numbers separated by blank."
UNE ENPUT intines
FOR $\mathrm{N}=1 \mathrm{TO} \mathrm{nV}$
permV(iV) $=$ VALMIDS(inineS. $2(\mathbb{N}-1)+1,1)$ )

NEXT
GOSUB BDDTreaCreator
GOSUB BDDReducer
GOSUB BDDChecker
GOSUB BDDMinimizer
GOSUB BDDCoster
RETURN
BDDExhasativer:
GOSUB BDDinitiaizer
- permunation matrix initialize -
FOR $N=1$ TO nV
permV(M) $=\mathrm{N}$
NEXT
endPemflag $=0$
startPemflag $=1$
-- permutaion loop -..
minBDDtotalCost $=E$
WHILE (endPermflag $=0$ )
GOSUB GeneratePerm
GOSUB BDDTreaCrastor
GOSUB BDDReducer
'GOSUB BDDChecker
GOSUB BDDCoster
IF (todaCost < minBDDRotalCost) THEN
minBDDictalCost = totaiCost
FORN $=1$ TO nV
minPermV(M) $=$ permV(V)
NEXT
ENDF
WEND
-... re-generate minimum BDD ...
FORN $=1$ TO nV
permV $(\mathbb{M})=$ minPermV(M)
NEXT
GOSUB BDDTreeCremtor
GOSUB BDDReducer
GOSUB BDDChecker
GOSUB BDDCoster
RETURN
BDDTreeCreator.
- - . scramble function data according to permutation -
FOR ileaf = 1 TO ntead
FOAN $=1$ TO nV
VPat(M) $=\operatorname{bil} m$ (ileat. permV(V)
NEXT
target $=1$

```
    FORN = 1 TO nV
        target = target + VPat(M)* 2^(nV -M)
    NEXT
    FOR turnc = 1 TO rtunc
        pobitm(target, lunc) = obim(deaf, itunc)
    NEXT
    NEXT
    -..- create BDD leaf -.
    FOR leaf = 1 TO nlesf STEP 2
        FOR lunc = 1 TO nfunc
        XD = (func-1)* (nloat/2) + (llegf-1)/2 +2
        XL(XD) = pobim(fisaf, lunc)
        XH(iXD) = pobitm(lleaf+1, lunc)
    NEXT
    NEXT
    -mintiatize XL & XH --
    FOR N=2TO nV
        FOR XXD = V2XD1(M)+1 TO V 2XDn(M)
        XL(XD) = XL(DD-1 + +2
        XH(XD) = XL(XDD) +1
    NEXT
NEXT
FORIYD = OTO NXD
    XDEYD(YD) = MD
    YL(YD) = E:YH(MD) = E
    NEXT
    YL(0) = 0:YH(0) =0:YL(1) = 1:YH(1) =1
RETURN
BDDReducer:
    \because-- raduce BDD --
    V2YD1(1) = 2: V2YDn(1) = 2: speciaVertex =0
    FORN=1TO nV
        FOR DOD = V2XD1(M) TO V2XDn(N)
            -_.- high child = low child, eliminate the vertex ...
            IF (XD2YD(XL(IXD)) = XD2YD(XH(IXD))) THEN
                XD2YD(XD) = XD2YD(XL(XD))
                OF (N = nV THEN
                    YL(VZYDn(N)) = XD2YD(XL(XD))
                    YH(V2YDNX(N)) = XD2YD(XH(XD))
                    WNZYDN(N) =N
                XD2YD(ODD = YD 
                V2YDn(M) = V2YDn(N) + 1
                speciaiVertex = speciaNertex +1
            END [F
        ELSE
            MD = V2YD1(M)
            BDDReduceLoop:
                '.-. check for the same subtree which already exists --
                IF (XD2YD(XL(XXD)) = YL(MD)) AND (XD2YD(XH(XXD)) = YH(MOD))
THEN
                XD2YD(DD) = MDD
                GOTO BreakBDDReduceLoop
            ESE
                F (YD }>=\mp@subsup{V}{2YON(N)) THEN}{
                    YLN2YDn(M) = XD2YD(XL(XD))
                    YH(N2YDn(N)) = XD2YD(XH(XD))
                    W(V2YDr(N)}=
                    XDZYD(DD) = YD 
                    V2YDn(V) = V2YDr(M) + 1
                    GOTO BreakBDDPeduceLoop
                END IF
            ENDF
            MD=MD +1
        GOTO BDDReduceLoop
        BreakBDDReduceLoop:
        ENDIF
    NEXT
        '.-.- set frrst and last IYD for the next level - .-
        V2YD1(N+1) = V2YDn(M)
        V2YDn(N+1) = V2YDn(iV)
    NEXT
    nYD = V2YDn(nV)-1
RETURN
BDDCoster:
    .-- cost calculation and output --
    zeroedge =0: oneedge =0
    zeroedge = 0: oneed
            (F YL(YD) =0 OR YH(YD) =0 THEN zeroedge = zeroedge +1
            IF YL(YD) = 1OR YH(YD) = 1 THEN oneedge = oneedge + }
    IF
    NEXT
    ncost = 2*(nYD - 1-speciaNertex) - onoedge
```



```
        Y(MD) \(=1 \mathrm{YD}\)
        GOSUB ShortopenOK
            -- il not openable. resume the edge --
            FI (retSOOK \(=0\) ) THEN YL(YO) \(=0 \mathrm{Ol} \mathrm{Y}\)
        END IF
        '-- for high edge -
        \(\mathrm{old}=\mathrm{YH}(\mathrm{IYD})\)
        .... skip terminal open edge -
        IF (odoY <> termOpen) THEN
            \(\mathrm{YH}(\mathrm{YD})=\mathrm{YD}\)
            GOSUB ShortopenOK
            '...- il nol openable. resume the edge ....
            IF \((\) retSOOK \(=0)\) THEN YH \((Y D)=0 l d Y\)
        ENDIF
    NEXT
    -.-. store separate BDD ..-
    FOR IYD \(=0\) TO \(\cap Y D\)
        Y01H(oneFlag, iYD) \(=\mathrm{YH}(\mathrm{YOD})\)
        Y01L(oneFlag. MD ) \(=\) YL(iYD)
        Y01V(oneFlag. IYD\()=\mathrm{W}(\mathrm{iYD})\)
        '- restore YH. YL, W -
```



```
    NEXT
NEXT
RETURN
MakeFanMatrix
    -... YFan example ....
        \(v\) (stored in nYFan)
    - 0 (YD): 3 (" of fans) 4153 (fans) -.
    1-- 1 (YD): 2 (" of fans) \(4 \quad 7 \quad\) (fans) -
    - 2 (YD): 1 (" of fans) 6 (fans) --
    -... YFanV example ...
    -
    .... The YFanVs equals to 0 is shorted path ...
    '-. The \(\mathrm{YFanVs}<0\) is low child path --.
    -.... initialize nYFan ...
    showFlag = 100: GOSUB BDDShower
    FOR MVD \(=0\) TO nYD
        \(\mathrm{nYFan}(\mathrm{YD})=0\)
    NEXT
    FOR IYD \(=0\) TO nYD
        FOR iHL = 0 TO 1
            '--. for high and low edge ...-
            IF \((H L=0)\) THEN \(\mathcal{Y D}=Y L(Y D) E L S E ~ Y D=Y H(Y D)\)
            -- skip open path --
            IF (YV \(<>\) iYD) AND (YD \(<>\) termOpen) THEN
                -.. add low or high child to YFan --
                    PRINT "1. "nYFan. IYD. JYD. iHL chido:nYFan(IYD);iYD;jYD;iHL
            \(n \mathrm{YFan}(\mathrm{YD})=\mathrm{nYFan}(\mathrm{YD})+1\)
            \(\mathrm{YFan}(\mathrm{YD} . \operatorname{nYFan}(\mathrm{YD}))=j Y 0\)
            (F \((\mathrm{iHL}=1)\) THEN YFanV(YD, nYFan(iYD \())=W(i Y D)\) ELSE
YFanV(iYD. \(\operatorname{nYFan(1YD))=-Y(iYD)}\)
            -- consider short path --
            EF OYD < short) THEN
            PRINT "1, "nYFan, MD. MYD, iHL, non-8";nYFan(IYD);YD;jYD;iHL
            \(n \mathrm{VFan}(\mathrm{YD})=n \mathrm{VFan}(\mathrm{YD})+1\)
            YFan(YD. nYFan(jYD)) = iYD
            IF (iHL = 1) THEN YFanV(YD, nYFan(YD) \()=W(\) IYD \()\) ELSE
YFanV(JYD. nYFan(jYD)) \(=-\mathbf{W}(\mathbf{i Y D})\)
            ESE
            MD = MD - short
            PAINT \#1, "nYFan. IYD. YYD. iHL, short";nYFan(iYD):YD;jYD;iHL
            \(n Y F a n(Y D D)=n Y F a n(j Y D)+1\)
            \(\mathrm{YFan}(Y \mathrm{~V}, \mathrm{n} \mathrm{VFan}(\mathrm{YD}) \mathrm{I})=\mathrm{YD}\)
            YFanV(Y)D, nYFan(jYD)) \(=0\)
            ENDIF
        END \({ }^{2}\)
    NEXT
    NEXT
    GOSUB FanShower:
RETURN
```

```
FanShower:
```

FanShower:
-.-show fanout matrix --
-.-show fanout matrix --
FORIYD =0TO NYD
FORIYD =0TO NYD
PRINT \#1. 'YFan (YFanV): ":
PRINT \#1. 'YFan (YFanV): ":
FOR jYD = 1 TO nYFan(YOD)
FOR jYD = 1 TO nYFan(YOD)
PRINT *1. USING "\#\# ("*) ";YFan(IYD, MD):YFanV(MD_YD);
PRINT *1. USING "\#\# ("*) ";YFan(IYD, MD):YFanV(MD_YD);
NEXT
NEXT
PRINT %1.-
PRINT %1.-
NEXT
NEXT
RETURN

```
RETURN
```

