Models of Ice Skating for the Development of Robotic Ice Skating Gaits

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Ice skating is a dynamic, creative movement with many possible gaits. There are some, but not many examples of ice skating robots, and even fewer ice skating bipedal robots. For bipedal robotic locomotion, there are standard canonical simple models such as the spring loaded inverted pendulum (SLIP model) and the compass model, which are used to analyze the simplest cases of walking and running. However, for ice skating on a surface, these models are insufficient for generating skating gaits of the kind used by humans. Using a simple controllable non-holonomic model of an ice skate that can slide along a single direction, with friction in the perpendicular direction, we demonstrate how we can use trajectory optimization to generate skating gaits. Additionally we use a hybrid trajectory optimization framework to generate gaits on a simulated bipedal robot.

1 Introduction

Skating is a form of locomotion which takes advantage of a low-friction surface for gliding, while using higher friction contacts to generate forward velocity. It is both a highly efficient mode of travel and a beautiful and graceful sport. Skating gaits are used by human athletes in ice skating, roller skating/blading, and cross-country skiing. Figure skating and roller dancing further exploit these mixed friction characteristics to perform precise and artistic skating motions like spinning and jumping, while hockey players and roller derby skaters rely on agility for competitive play. Interestingly, there is no animal in the wild that ice skates for locomotion, however humans have been ice skating for thousands of years.

The key element for all of these skating settings is ground contact with mixed characteristics: low friction along the longitudinal direction of the foot, and high friction in the lateral direction. In roller skating, this is due to the rotational axis of the wheels; in cross-country skiing, glide wax ensures a low friction contact with the snow while ski edges allow for pushing; and in ice skating a thin blade cuts across the ice. The importance of exploiting the anisotropic friction of the ground contact makes skating an interesting setting for robotic control and dynamic planning.

Ice skating in particular makes for a rich dynamic setting. The friction contact exhibits the highest extremes, with a very low gliding friction but a very high lateral friction due to the cut of the blade. Compared with roller skating, where the slip characteristics of the wheels play a role, and cross country skiing, where skis are several feet long, agility in ice skating relies most directly on exploiting the contact forces. As a result, the range of possible skating motions is quite high, with many possible methods for forward (and backwards) locomotion, as well as a well-defined catalog of skating moves that figure skaters are judged by.

Designing gaits on a surface where slipping is possible presents unique challenges—most bipedal walking gait are designed using a constraint that the foot does not slip once contact is made with the ground. In a trajectory optimization framework for generating walking gaits, this constraint significantly reduces the search space for walking gaits. In skating, by contrast, a slipping motion is desired.

To confront this challenge we use non-holonomic dynamical equations to model the skating contact. Using non-holonomic equations, we do not need to use a constraint to enforce that the skate does not slip perpendicular to the blade—rather this constraint is inherently enforced based on our choice of parameterization of the system. For this work, we target a "slaloming" gait, a one-legged skating gait where the skater keeps one skate on the ice at all times and generates momentum by moving their center of mass and turning side-to-side. Our question is how to build a simple model of ice skating that is physically realistic enough for robotics purposes. We present a simple model of a single ice skate on a surface with infinite friction perpendicular to the blade and zero friction along the direction of the blade. We use a direct collocation framework to generate gaits using this model, and use these trajectories generated by trajectory optimization to derive a simple controller for this ice skating model.

2 Related Work

There is a long history of studying the physics of ice skating. Many works focus on the low friction contact with the ice during gliding, with experimental studies dating back to as early as the 1930s [2]. The physical principles underlying the slipperiness of ice are complex, and require accounting for heat transfer [3] characteristics and fluid dynamics. Surprisingly, there is recent scientific disagreement about the exact mechanisms of this slipperiness, resolved in part by modern experimental studies at the molecular level [4]. However, the macroscopic behavior of ice skates can be described [5] with simpler blade- and temperature-dependent models of sliding friction.

Another body of literature focuses more broadly on the movement of the skater. Biomechanical studies propose dynamic models of speed skating [6], roller skating [7], and jumping [8] in figure skating. These works experimentally evaluate model accuracy towards the goal of understanding human performance and preventing athletic injury. The Chaplygin sleigh is a simplified 2D skater model classically studied [9] in the field of nonholonomic mechanics, which seeks instead a more fundamental understanding of the trajectories of dynamical systems.

There is a body of work on the modelling and design of skating robots, many of which focus on statically-stable robots with more than two legs. Perhaps one of the earliest is the four legged Roller Walker [11] which could either walk or roller skate and was developed in the 90s. Skating gaits for multi-legged robots have been developed as part of an optimization-based design framework [12], leading to an example of a quadruped robot that can locomote across ice using a position controller synthesized through Gauss-Newton like method. Roller and ice skating have also been successfully implemented [13] via motion planning and force control on the quadruped ANYmal robot. There is also a quadruped skating robot [15] with passive wheels using hand-design controllers, trajectory optimization, and reinforcement learning.

For bipedal locomotion, some settings do not require a skating gait for propulsion, like bipedal downhill skiing robots (of which there are several examples [17]) or the development of agile roller skating moves downhill using hierarchical MPC [18]. Many works consider the development of skating gaits for small sized robots, including a hand-tuned ice- and roller-skating gait for a DARwIn humanoid robot [22] and ZMP-based motion planning roller skating gait [23]. As far as we could find, there is one human-sized example [24] of bipedal roller skating, where a gait results from COM stabilization using a novel contact wrench cone. Finally, there has been work [25] in Computer Graphics to design dynamically feasible skating trajectories based on videos of human figure skaters, but it is restricted to simulation.

Bipedal locomotion is of course widely studied, but the majority of work [26] focuses on walking and running gaits. Classical approaches lean on simple models for intuition around control strategies and deriving insights about robustness. For walking, this is the linear inverted pendulum [27], while for running, it is the spring-loaded inverted pendulum [28]. Both models can be simply analyzed in two dimensions while still capturing relevant walking phenomena; however, 3D extensions [29] are also studied. The literature on bipedal robot locomotion also studies the problem of slippery surfaces, though usually with the goal of mitigation [30] and robustness rather than exploitation, as we seek for skating.

3 The Skating Model

To understand the dynamics of skating, we first analyzed a simple model that captured the necessary dynamics for realistic skating gaits. The Chaplygin sleigh is a classical model where there is a rigid body attached to a blade in two dimensions. The blade constrains the system to have a velocity in the direction parallel to the blade. The system is non-holonomic because the velocity constraint is not deriveable from a position constraint. The Chaplygin sleigh essentially models in two dimensions a skate with 0 friction along the direction of the blade and infinite friction horizontal to the blade, where the blade is always constrained to have contact with the ice.

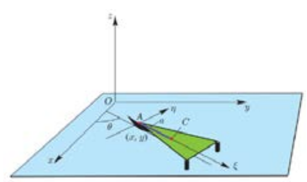

Figure 1: The Chaplygin sleigh model. The knife blade is in contact with the ground and constrains the velocity of the sleigh to be parallel with the knife blade.

We study a modified form of the Chaplygin sleigh system where in addition to the sleigh body, there is another moveable body that is attached to the sleigh which maybe be controlled (see figure 2). The moveable mass represents the skater's body's center of mass which can be moved with respect to the blade. By shifting weight from side-to-side and forward and back, the skater can generate momentum.

Figure 2: The Chaplygin sleigh model with a moveable mass attached. Here the knife is illustrated as a wheel, but the model is the same as in Figure 1, with the addition of a mass separate from the sleigh body mass which can move in relation to the sleigh body.

We use the equations of motion and parameterization from [19]. For the equations of motion below, the notation is the following (see figure 3): (x, y) is the position of the blade, (a, b) is the position of the moving mass relative to the skate, θ is the angle of the skate, p_1 is the angular momentum and p_2 is the projection of the linear momentum along the direction of the blade. See Figure 3.

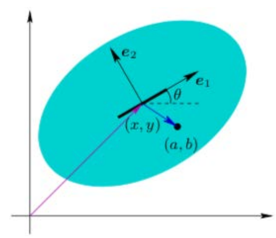

Figure 3: The coordinate system for the Chapyglin sleigh with a moving mass.

The knife blade constraint is

$$-\dot{x}\sin\theta + \dot{y}\sin\theta = 0$$

Defining Ω_1 as the angular velocity of the platform, and Ω_2 the component of the linear velocity along the blade, Ω_3 the velocity of the platform orthogonal to the blade, these satisfy [19]

$$\Omega_1 = \frac{(M+m)p_1 + mbp + 2}{(M+m)(I+ma^2) + Mmb^2}$$
$$\Omega_2 = \frac{mbp_1 + (I + ma^2 + mb^2)p_2}{(M+m)(I+ma^2) + Mmb^2}$$

the equations of motion are

$$\dot{p_1} = -ma\Omega_1\Omega 2 \qquad \dot{p_2} = ma\Omega_1^2 \tag{1}$$
$$\dot{\theta} = \Omega_1 \qquad \dot{x} = \Omega_2\cos\theta \tag{2}$$
$$\dot{y} = \Omega_2\sin\theta \tag{3}$$

4 Gait Generation with Direct Collocation

We would like to use the simple skating model above to model an ice skating gait called one-legged slaloming. In one-legged slaloming, the skater stands on one leg, and generates momentum by alternating turns left and right, shifting the center of mass to turn. Slaloming is a gait that is unlike any walking gait, since the skate stays on the ice and uses the slipping motion to move forward. That the gait is one-legged made it more straightforward to apply the Chaplygin with moving mass model, since that model has only a single skate.

We use an optimization framework to generate an initial swizzling gait for the Chaplygin sleigh. We use direct collocation to optimize for a swizzling trajectory subject to the dynamics. Direct collocation is a trajectory optimization method where the input u is approximated by first-order polynomial splines, and the state is approximated by third-order polynomial splines. The dynamics constraints are enforced at the piecewise spline points. The additional direct collocation constraints are that the derivative of the splines at the collocation points match the dynamics.

We define (x, y) locations for the extreme points on the curve of the desired swizzle trajectory which we will call $(x_0^*, y_0^*), (x_1^*, y_1^*), \ldots, (x_n^*, y_n^*)$ at times $t_0 \ldots t_n$. The cost function for the optimization is then

$$C = R_0 * ||u||_2^2 + \sum_{i=1}^{n} R_1((x(t_i) - x^*(t_i))^2 + (y(t_i) - y^*(t_i))^2)$$

where R_0, R_1 are weights chosen to weigh the input magnitude cost versus the position costs.

The optimization is constrained by the dynamics constraints as in equations 1 - 3. We add an additional constraint to prevent cusps in the trajectory. Cusps are a sudden change in the sign of the linear velocity. Note from equations 2 and 3 that $\dot{x}, \dot{y}$ change signs when Ω_2 changes signs. Therefore we add the constraint on the direct collocation knot points that

$$\Omega_2 \geq c$$

where c is some small constant, in this case $c = .01$. We also have constraints for the initial and final state, on the full state $(p_1, p_2, \theta, x, y, \dot{a}, \dot{b})$. The remaining constraints in the optimization are the direct collocation constraints.

Although the optimization framework above was fairly straightforward, what was not immediately obvious was how to choose the points $(x_0^*, y_0^*), (x_1^*, y_1^*), \ldots, (x_n^*, y_n^*)$, the extreme points of the slalom. However from [19], we can provide high level characteristics of a zero-input trajectory given the initial condition. When the initial conditions are $a = 0$ and $(M + m)p_1 + mbp_2 \neq 0$ and zero input is applied after that, then the trajectory will be circular. Using this, we chose initial conditions such that with zero input the trajectory would be circular. We then use the radius of that circle and the velocities at the extreme points on the turns to choose the target points for the optimization. Using these target points, the trajectory optimization is able to return a slaloming trajectory where the center of mass moves naturally, without moving too far from the skate. Note that although it is physically unrealistic, the center of mass is not constrained to stay close to the gait. However, by putting a cost on the input magnitude, the center of mass stays close to the skate without this constraint.

The resulting gait is illustrated in Figure 4. Although the turns are not perfectly symmetrical, we can see in the right figure that the moveable mass (analogous to an ice skater's body) leans in relation to the blade to facilitate turning. The moveable mass leans on the outer edge of the turn at the beginning of the turn to initiate the turn, and then leans back to the inside of the turn to complete the turn. This model suggests how the center of mass may be controlled to complete ice skating slaloming gaits.

5 Gait Design with Hybrid Trajectory Optimization

In addition to designing a slaloming gait, we also tried to design the common skating gait where one foot is in contact with the ground at the time, while the other is lifted. For this gait, we worked with a simulated bipedal robot Cassie. Cassie is a bipedal walking robot with 20 degrees of freedom, with five actuated motors and two passive joints.

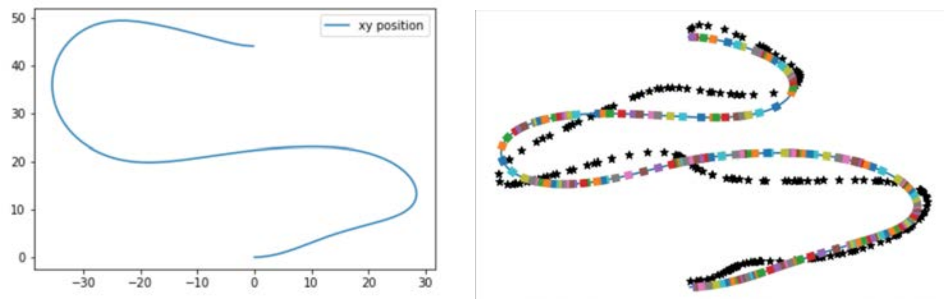

Figure 4: Generated slaloming gaits returned by the direct collocation method. In the figure on the right, the colored rectangles are the sleigh body position and the stars are the positions of the mass that can move freely in relation to the sleigh body.

This gait, while more similar to typical bipedal walking gaits in some respects, presents challenges to our previous trajectory optimization technique. In the slaloming gait, where the feet are always in contact with the ground, which limits the search space for the trajectory optimization. However, for a more walking-like gait, we must also design when the feet are in and out of contact with the ground. Trajectory optimization through contact is a significantly harder problem due to the increased physical complexity of the many possible interactions between the feet and the ground. A method we use to confront this challenge is hybrid trajectory optimization. A hybrid dynamical system is a system that exhibits both continuous and discrete dynamics. For designing walking gaits, we will have different hybrid "modes" of contact which correspond to the kind of contact at different times in the gait. Prior to the optimization, we specify the order of the modes, that is, the order of the contacts of each foot, and the optimization returns a gait following this specification.

We use the FROST [21] toolkit to perform hybrid trajectory optimization on the bipedal Cassie robot. FROST is an opensource MATLAB toolkit for peforming hybrid trajectory optimization and simulation, particularly for locomotion.

We build off of previous work [20] that has developed walking gaits for Cassie using this framework. For the normal walking gait, we design two modes, a left stance and a right stance. There are guard conditions for left foot impact and right foot impact which trigger switching between the two modes. The optimization problem consists of a cost function

$$L_j = ||u_i(t)||^2$$

where u_i being the control inputs of each joint, and L_j being the integral over time of the cost for stance j. Additional constraints for the optimization are [20]

- Fixed time duraction of 0.4 seconds.
- Swing goot clearance of 15 cm.
- Ground reaction forces respect the friction cone and ZMP condition.
- Zero swing foot horizontal speed at impact.
- The two steps are symmetric on the left and ride side.
- Torso remains upright within degree limit.

To design a bipedal skating gait, we start with the optimization for the walking gait but make some changes. In particular, we change the friction model for the toe so that the toe of foot is frictionless. Although this friction model is not fully realistic for skating, it was a first step in designing a skating-like gait. We change the guard, the condition for switching between the right stance and the left stance, so that upon impact of a foot, the foot will slide forward.

The optimization cost is the same is the same as above, with the stance constraints for the skating-like gait are the following, with distances in meters:

- Average velocity in x-axis of 0.1, average velocity in the y axis of 0.1.
- Average swing toe velocity within [-.02, 0.02].
- Toe-to-toe distance, [-0.4, 0.1].
- Average pitch of 0°, 0°yaw, hip abduction [-70°, 70°], hip rotation [-70°, 70°].
- Pelvis to toe distance constrained within [0.5, 1.0].

The gait which is solution to the optimization is illustrated below in Figure 5. In the gait, the robot moves forward by sliding each foot out and to the front at a time. While this is a first step towards creating a skating gait, the robot isn't using a skating blade to push forward as in human skating. Instead the robot using the tip of the toes, the only part of the foot that has contact with friction (somewhat like a toe pick on figure skating skates), to push forward.

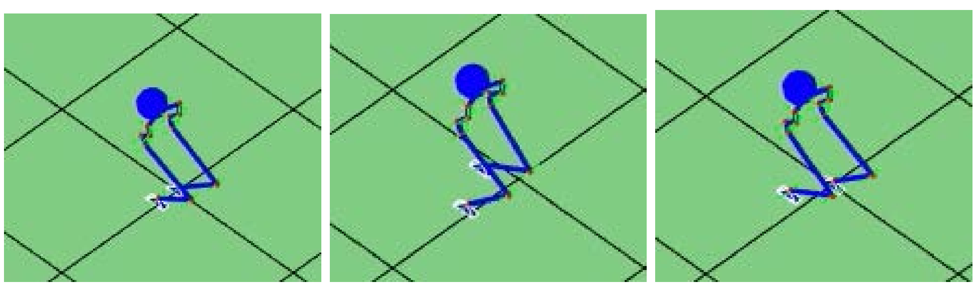

Figure 5: In this gait, the robot starts with even feet. It then slides forward the right foot, then plants the right foot and slides forward the left foot.

6 Conclusion

In this work, we investigated different models of skating particularly for designing bipedal skating gaits. We investigated a simple 2D non-holonomic system in which we can model the swizzling gait. Further work could be done in that direction to design feedback controllers for a swizzling gait. For a concrete robotic system we used hybrid trajectory optimization to design a different skating gait, where the robot slides forward one skate at a time. Additional work could be done to adapt the swizzling gait from before to this robotic system. In both works we investigate setting up optimization problems and modeling friction for the development of skating gaits.

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