Essays on Expectations-Based Reference-Dependent
Consumption and Portfolio Choice

by

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Abstract

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This dissertation studies the life-cycle consumption and portfolio-choice implications of a new preference specification called “expectations-based reference-dependent preferences” or “news utility.” These preferences formalize the idea that news or changes in expectations about present and future consumption generate instantaneous utility, by which bad news hurts more than good news pleases. The preferences were developed by Koszegi and Rabin (2006, 2007, 2009) to discipline and broadly apply the insights of prospect theory and have since been shown to be consistent with evidence in various behavioral and micro domains. In this dissertation, I show that the preferences not only explain evidence in micro domains but also generate interesting predictions in classic finance and macro models.

In the first part, I incorporate the preferences into a fully dynamic and stochastic life-cycle model to offer a new explanation for three major consumption facts -- excess smoothness and sensitivity in consumption, a hump-shaped consumption profile, and a drop in consumption at retirement. This new explanation relies on believable intuitions that are reminiscent of the micro evidence that the preferences were developed to explain and may provide new foundations for prominent ideas in the macro consumption literature. More precisely, excess smoothness and sensitivity, two prevailing macro consumption puzzles, are explained by loss aversion, a robust experimental risk preference, which has been assumed to explain the most important phenomena in behavioral economics. To explain the other life-cycle phenomena, the preferences combine precautionary savings, which have been studied extensively in the standard consumption literature, and an expectations-based time inconsistency, which is reminiscent of hyperbolic discounting.

In the second part, I explore the quantitative asset-pricing implications of news utility in a canonical Lucas-tree model. I find that the preferences easily succeed in matching historical levels of the equity premium, its volatility, and the degree of predictability in
returns. Moreover, I show that the preferences imply plausible risk attitudes towards small, medium, and large consumption and wealth gambles and thus make another step towards resolving the equity premium puzzle.

In the third part, I extend the news-utility life-cycle model to portfolio choice. Beyond explaining predominant questions in the literature, such as non-participation in the presence of labor income, portfolio shares, and wealth accumulation, I put special emphasis on the question of how often the investor should pay attention to his portfolio. To answer this question, I consider a model in which the investor has access to a brokerage account, which he may or may not look up, and a checking account to finance inattentive consumption. As bad news hurt more than good news pleases, news utility results in a first-order decrease in expected utility and the investor refuses to look up his portfolio and rebalance most of the time. If he looks it up, however, he rebalances extensively. Moreover, the investor is subject to a commitment problem. He would like to precommit to being inattentive even more often because he does not overconsume out of his checking account. Consequently, the investor gains from engaging in mental accounting and would be happy to pay for investment tools that encourage inattention and rebalance actively, such as delegated portfolio management.
Part I

Expectations-Based Reference-Dependent Life-Cycle Consumption

1 Introduction

In the last thirty years, the consumption literature has debated numerous explanations for three major empirical observations about life-cycle consumption: excess smoothness and sensitivity in consumption, a hump-shaped consumption profile, and a drop in consumption at retirement. This paper offers a unified explanation based on expectations-based reference-dependent preferences, which have been developed by Koszegi and Rabin (2006, 2007, 2009) to discipline and broadly apply the insights of prospect theory. The preferences formalize the idea that changes in expectations about consumption generate instantaneous utility and that losses in expectations about consumption hurt more than gains please. While these preferences have been shown to explain evidence in various micro domains, this paper validates the preferences in a classic macro domain. My explanation for the three consumption facts relies on intuitions that are reminiscent of the micro evidence that the preferences were developed to explain and may provide new foundations for prominent ideas in the macro consumption literature. Moreover, I show that the preferences generate new behavior and welfare predictions. These welfare predictions are important, because whether or not consumption, which represents two-thirds of GDP, should be excessively smooth matters for labor market reforms and countercyclical policies.

I first explain the preferences in greater detail. In each period, the agent’s instantaneous utility consists of two components. “Consumption utility” is determined by his level of consumption and corresponds to the standard model of utility. “Gain-loss utility” is determined by his expectations about consumption relative to his reference point and corresponds to a prospect-theory model of utility. The agent’s reference point is determined by his previous beliefs about both his present consumption and his entire stream of future consumption. The agent experiences “contemporaneous” gain-loss utility when he compares his actual present consumption with his probabilistic beliefs about present consumption. In this comparison, he encounters a sensation of gain or loss relative to each consumption outcome that he had previously expected. Additionally, the agent experiences “prospective” gain-loss utility when he compares his updated beliefs about future consumption with his previous beliefs, encountering gain-loss utility over what he has learned about future consumption. Thus, gain-loss utility can be interpreted as utility over good and bad “news” about consumption.

I analyze an agent with such “news-utility” preferences in a life-cycle consumption model. The agent lives for a finite number of periods; at the beginning of each period, he observes the realization of a permanent and a transitory income shock and then decides how much to consume and save. I first assume that the agent’s consumption utility is an exponential CARA function. This assumption produces a closed-form solution, which allows a precise understanding of the

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1Refer to Attanasio and Weber (2010) for a comprehensive survey of the life-cycle consumption literature.

2Prospect theory (Kahneman and Tversky (1979)) states that people care about gains and losses relative to a reference point, where small losses hurt more than small gains give pleasure, i.e., people are loss averse.
intuitions behind the preferences’ implications. I then show that all of the implications hold if I instead assume a power-utility CRRA function.³

As the first key implication, these preferences generate excess smoothness and sensitivity in consumption, which refer to the empirical observations that consumption initially underresponds to income shocks and then adjusts with a delay. This inherently related interpretation of excess smoothness and sensitivity was put forward by Deaton (1986) and Campbell and Deaton (1989), in response to the seminal paper by Flavin (1981).⁴ Such consumption responses are puzzling from the perspective of the standard model, in which consumption fully adjusts immediately, but can be explained by expectations-based loss aversion. A simplified intuition is that, in the event of an adverse income shock, unexpected losses in consumption today are more painful than expected reductions in the future. Accordingly, the agent delays unexpected losses in consumption until his expectations will have adjusted in the future. Losses in present consumption are more painful than losses in future consumption because losses in future consumption depend on future income shocks and are thus still uncertain.

Beyond resolving these puzzles, the preferences are consistent with another stylized fact, namely a hump-shaped life-cycle consumption profile. A hump-shaped profile is characterized by increasing consumption at the beginning but decreasing consumption toward the end of life. Empirical evidence for a hump-shaped consumption profile is provided by Fernandez-Villaverde and Krueger (2007) and Gourinchas and Parker (2002). In my model, this hump results from the interaction of two features of the preferences, a first-order precautionary-savings motive and a beliefs-based time inconsistency. First, the preferences motivate precautionary savings because loss aversion makes fluctuations in consumption painful in expectation. These fluctuations hurt relatively less, however, higher on the concave utility curve, which brings about an additional motive to save. This savings motive depends on concavity and is a first-order consideration, as opposed to the precautionary-savings motive under standard preferences.⁵ Second, the preferences are subject to a beliefs-based time inconsistency. The agent behaves inconsistently because he takes today’s expectations as given when increasing today’s consumption, but takes tomorrow’s expectations into account when increasing tomorrow’s consumption. However, when he will wake up tomorrow, he will take tomorrow’s expectations as given and only consider the pleasure of increasing consumption above expectations rather than increasing consumption and expectations. As a result, the agent overconsumes relative to the optimal pre-committed consumption path that maximizes

³I assume a standard model environment, as proposed by Carroll (2001) and Gourinchas and Parker (2002), but my results are robust to many alternative environmental assumptions such as liquidity constraints, different income profiles, different savings devices, portfolio choice, endogenous labor supply, mortality risk, an endogenous retirement date, different pension designs, and income and expenditure risk after retirement.

⁴I focus on this interpretation because my basic model features only contemporaneous income shocks. A delayed response to last period’s permanent income shock can be interpreted as a response to expectedly high income. The empirical evidence for consumption responses to expected income shocks is surveyed in Jappelli and Pistaferri (2010). In a model that features contemporaneous shocks to future income, excess sensitivity would be explained by news utility, if the contemporaneous shock to future income concerns income in the next period. Other interpretations of excess sensitivity and the model’s explanatory power with regards to them are reviewed in the next section. Recent empirical evidence for excess smoothness using micro data is provided by Attanasio and Pavoni (2011).

⁵In a first-order approximation of savings, the effect of uncertainty depends on the second derivative of the utility function and the precautionary-savings motive does not go to zero when uncertainty becomes small. This result was obtained by Koszegi and Rabin (2009) in a two-period, two-outcome model.
his expected utility. To summarize, the precautionary-savings motive keeps consumption low at the beginning of life. However, the need for precautionary savings decreases when uncertainty resolves over time. At some point, precautionary savings are dominated by the beliefs-based time inconsistency causing overconsumption. Such overconsumption in mid-life will force the agent to choose a declining consumption path by the end of life.

Finally, the preferences predict a drop in consumption at retirement, a highly-debated empirical observation by Battistin, Brugiavini, Rettore, and Weber (2009) and Haider and Stephens (2007) among others. In a standard life-cycle model, income uncertainty is absent during retirement, eliminating both the precautionary-savings motive and the beliefs-based time inconsistency. The inconsistency is eliminated because the agent no longer allocates uncertain labor income but allocates certain income. Certainty implies that time-inconsistent overconsumption today is associated with a certain loss in future consumption. Because the certain loss hurts more than overconsumption gives pleasure, the agent suddenly controls his time-inconsistent desire to overconsume and his consumption drops at retirement. This result is robust to assuming small uncertainty, for instance, inflation or pension risk, or discrete uncertainty, for instance, health shocks.

Beyond these three implications, the preferences generate several new and testable predictions about consumption and savings. For instance, excess smoothness is increasing in the agent’s horizon and prevalent for temporary shocks only if the agent does not face permanent shocks additionally. Moreover, I draw potentially important welfare conclusions by looking at the optimal pre-committed consumption path. Excess smoothness increases welfare and is even more pronounced on the pre-committed path. In contrast, the hump-shaped consumption profile and the drop in consumption at retirement decrease welfare and are absent on the pre-committed path.

Other preferences or model frictions that have been put forward to explain the same phenomena are reviewed in the next section. By way of providing alternative explanations, this paper explores habit-formation, hyperbolic-discounting, temptation-disutility, and standard preferences. However, only news utility provides a unified explanation independent of assumptions other explanations rely on, e.g., a power-utility function, hump-shaped income profiles, liquidity constraints, preference shifters, or non-separabilities of consumption and leisure. Nevertheless, this paper’s main contribution is that the explanation’s intuition connects robust micro evidence on reference-dependent preferences to several compelling concepts in the macro consumption literature. For instance, loss aversion is an experimentally robust risk preference, which explains important behavioral phenomena such as the endowment and disposition effects, as well as macro phenomena such as stock market non-participation and the equity-premium puzzle.

A series of papers, e.g., Banks, Blundell, and Tanner (1998), Bernheim, Skinner, and Weinberg (2001), Battistin, Brugiavini, Rettore, and Weber (2009), Haider and Stephens (2007), Schwerdt (2005) find that consumption drops at retirement taking work-related expenses into account, and my data display such a drop. Moreover, Ameriks, Caplin, and Leahy (2007) and Hurd and Rohwedder (2003) provide evidence that the drop in consumption is anticipated. However, Aguiar and Hurst (2005) find that the drop is absent when properly controlling for health shocks and home production.

The endowment effect refers to the phenomenon that people become less willing to give up an item once they own it. If the item is in people’s possession, foregoing it feels like a loss. The most famous study is Kahneman, Knetsch, and Thaler (1990), in which students are given a mug and then offered the chance to sell it. The authors find that the payment that students’ ask for once they own the mug is twice the payment they are offering to purchase it. More recently, Ericson and Fuster (2011) demonstrate that subjects’ expectations to keep rather than possess an
equity-premium puzzle is nicely related to the explanation of the excess-smoothness puzzle: both rely on loss aversion smoothing consumption relative to movements in asset prices or permanent income. To explain the other life-cycle facts, the preferences intuitively unify precautionary savings, which have been studied extensively in the standard consumption literature, and a beliefs-based time inconsistency, which is reminiscent of hyperbolic discounting.

To quantitatively evaluate the model, I structurally estimate the preference parameters using life-cycle consumption data. I follow the two-stage method-of-simulated-moments approach of Gourinchas and Parker (2002) and use pseudo-panel data from the Consumer Expenditure Survey as provided by the NBER. I can identify all preference parameters because each parameter generates specific variation in consumption growth over the life-cycle. I then compare my estimates to those found in the microeconomic literature by exploiting the fact that all the preference parameters have narrow ranges determined by existing behavioral evidence and common sense. I then show that my estimates are not only in line with the micro literature and generate reasonable attitudes towards small and large wealth bets but also match the empirical evidence for excess smoothness and sensitivity in aggregate data.

The paper is organized as follows. After a literature review, I explain the model environment, preferences, and equilibrium concept in Section 3. Then, I derive the model’s main predictions in closed form under the assumption of exponential utility in Section 4. After briefly outlining the power-utility model, I then calibrate both models to assess whether the quantitative predictions match the empirical evidence and structurally estimate the model’s parameters in Section 5. Section 6 outlines several extensions. Section 7 concludes.

2 Literature Review

The static model of reference-dependent preferences, Koszegi and Rabin (2006, 2007), has been used to explain experimental and other microeconomic evidence in many contexts. In a fully dynamic and stochastic model, I obtain predictions that modify and extend the results item accounts for the gap between the asking and purchase price. The disposition effect (Odean (1998)) is an anomaly related to the tendency of investors to sell winners (stocks that have gone up in value) but keep losers (stocks that have gone down in value) to avoid the realization of losses.

Because consumption is too smooth relative to movements in asset prices, a high equity premium in the canonical asset-pricing economy requires unreasonably high second-order risk aversion.

I use NIPA consumption and income data following Ludvigson and Michaelides (2001).

Heidhues and Koszegi (2008, 2014), Herweg and Mierendorff (2012), and Rosato (2012) explore the implications for consumer pricing, which are tested by Karle, Kirchsteiger, and Peitz (2011), Herweg, Müller, and Weinschenk (2010) do so for principal-agent contracts, and Eisenhuth (2012) does so for mechanism design. An incomplete list of papers providing direct evidence for Koszegi and Rabin (2006, 2007) preferences is Sprenger (2010) on the implications of stochastic reference points, Abeler, Falk, Goette, and Huffman (2012) on labor supply, Gill and Prowse (2012) on real-effort tournaments, Meng (2013) on the disposition effect, and Ericson and Fuster (2011) on the endowment effect (not confirmed by Heffetz and List (2011)). Suggestive evidence is provided by Crawford and Meng (2011) on labor supply, Pope and Schweitzer (2011) on golf players’ performance, and Sydnor (2010) on deductible choice. Moreover, several of the conflicting papers on the endowment effect can be reconciled with the notion of expectations determining the reference point. All of these papers consider the static preferences, but as the dynamic preferences of Koszegi and Rabin (2009) are a straightforward extension, the evidence is valid for the dynamic preferences. Moreover, the notion that agents are loss averse with respect to news about future consumption is indirectly supported by all experiments, which use monetary payoffs because these concern future consumption.
about consumption and savings obtained by Koszegi and Rabin (2009) in a two-period, two-outcome model. In particular, I generalize the implications for precautionary savings and time-inconsistent overconsumption and intuitively combine them to explain the hump-shaped consumption profile and the drop in consumption at retirement. In contrast, the excess-smoothness result is new. It is different from the result by Koszegi and Rabin (2009) that the news-utility agent consumes windfall gains but delays windfall losses, which depend on the windfall gains and losses coming as a surprise, i.e., an initially certain consumption path. It is similarly different from the result by Bowman, Minehart, and Rabin (1999) that the loss averse agent delays losses only to remain at his deterministic reference point. In contrast, the reference point is stochastic in my model because consumption is continuously distributed. The stochastic reference point induces delayed consumption adjustments to both good and bad income shocks because today’s reference point is more sensitive to today’s consumption and savings plan than tomorrow’s reference point, which will adjust to today’s plan.

Excess smoothness in consumption cannot be generated by a time-separable utility function as made clear by Ludvigson and Michaelides (2001) among others. To obtain excess smoothness, a predominant additional assumption is borrowing constraints as analyzed by Deaton (1991) among others. However, this assumption faces two problems. Theoretically, the agent expects these constraints and ensures that they are not binding for most income realizations. Empirically, the implied asymmetry in excess smoothness could not ultimately be confirmed. Borrowing constraints are binding more often in a model that features a time-inconsistency problem, as analyzed by Angeletos, Laibson, Repetto, and Weinberg (2001) and Laibson, Repetto, and Tobacman (2012). In these models, sophisticated hyperbolic-discounting preferences imply that the agent restricts his consumption opportunities with illiquid savings against which he can borrow only up to some constraint. To the extent that his borrowing constraint binds or his liquid asset holdings bunch at

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11 Koszegi and Rabin (2009) develop the dynamic preferences from the static model of Koszegi and Rabin (2006, 2007) by introducing contemporaneous and prospective gain-loss utility in the instantaneous utility function. In so doing, the authors generalize the static “outcome-wise” gain-loss comparison to a “percentile-wise” comparison. I generalize the static comparison slightly differently by assuming that the agent experiences outcome-wise gain-loss utility only over uncertainty that has been realized. Because this comparison preserves an outcome-wise structure and is a linear operator, it is considerably more tractable. Moreover, because the psychological intuition of the separated comparison is also reasonable, I see this modification as a minor contribution to exploring the preferences.

12 This result carries over to environments characterized by discrete income uncertainty. To elaborate on this result and relax the assumption concerning a period’s horizon, which constitutes a calibrational degree of freedom in a model with first-order risk aversion, I outline a model extension that assumes that the agent receives large income shocks every couple periods but merely discrete income shocks in in-between periods. Discrete uncertainty allows the agent to make a credible plan to overconsume less, but implies that he will consume entire small gains and delay entire small losses. There exists evidence that people consume small windfall gains almost entirely (as surveyed in Jappelli and Pistaferri (2010)), which has been related to excess sensitivity in consumption as the permanent income model would predict a marginal propensity to consume out of transitory shocks that is close to zero.

13 These models of loss aversion predict an asymmetric response. The empirical evidence on asymmetric responses to income innovations, as would be predicted by liquidity constraints, is surveyed in Jappelli and Pistaferri (2010) and mixed. For instance, Shea (1995) finds that consumption is more excessively sensitive to expected income declines than increases, which is inconsistent with liquidity constraints or myopia but consistent with loss aversion, but Krueger and Perri (2010) find the opposite result.

14 Demand for commitment is also generated by temptation-disutility preferences, as specified in Gul and Pesendorfer (2004) and analyzed by Bucciol (2012) in a life-cycle context.
zero, his consumption is excessively smooth.\textsuperscript{15}

By explaining excess smoothness and sensitivity with preferences, I resume a literature pioneered by Caballero (1995), who assumes that agents consume near-rationally, and Fuhrer (2000) and Michaelides (2002), who assume internal multiplicative habit formation. The basic concept of news utility appears similar to habit formation. However, the life-cycle implications are very different; most importantly, I confirm the conclusion of Michaelides (2002) that habit formation generates excess smoothness at the cost of unreasonably high wealth accumulation. Furthermore, Flavin and Nakagawa (2008) define a utility function over two consumption goods, one representing non-durable consumption and one representing housing, which is characterized by adjustment costs. As the utility function depends non-separably on the two goods, non-durable consumption is excessively smooth and sensitive. A similar utility function is assumed by Chetty and Szieidl (2010), however, this function is separable in the two goods, which implies that consumption is excessively smooth and sensitive with respect to the durable good only. Moreover, Reis (2006) assumes that agents face costs when processing information and thus optimally decide to update their consumption plans sporadically, Tutoni (2010) assumes that consumers arerationally inattentive as in Sims (2003), and Attanasio and Pavoni (2011) show that excessively smooth consumption results from incomplete consumption insurance due to a moral hazard problem.

Several papers show that the standard and hyperbolic agents’ consumption profiles are hump shaped under the assumption of power utility, sufficient impatience, and a hump-shaped income profile, such as Carroll (1997), Gourinchas and Parker (2002), and Laibson, Repetto, and Tobacman (2012).\textsuperscript{16} Other papers that generate a hump-shaped consumption profile are Caliendo and Huang (2008) with overconfidence, Attanasio (1999) with family size effects, Deaton (1991) with borrowing constraints, Feigenbaum (2008) and Hansen and Imrohoroglu (2008) with mortality risk, Bullard and Feigenbaum (2007) and Heckman (1974) with consumption-leisure choice, and Fernandez-Villaverde and Krueger (2007) with consumer durables. Caliendo and Huang (2007) and Park (2011) show that a hump-shaped profile can be generated by assuming that the agent has a shorter planning horizon, i.e., five to twenty-six years, rather than his true horizon. In partial equilibrium, matching the hump shape is trivial, as preference and environmental parameters are jointly calibrated; thus, Park (2011) shows that short-term planning can generate the hump in a well-calibrated general-equilibrium model.\textsuperscript{17} Last, Caliendo and Huang (2011) show that a hump-shaped profile and an anticipated drop in consumption can be generated by assuming imple-

\textsuperscript{15}Laibson, Repetto, and Tobacman (2012) and Laibson (1997) put forward an interpretation of excess sensitivity that focuses on a high marginal propensity to consume out of transitory income shocks, as the permanent income model would predict this propensity to consume to be close to zero. A high marginal propensity to consume out of transitory income shocks is prevalent in this model if the agent’s time-inconsistent overconsumption dominates precautionary savings or if the model is extended such that the agent has access to an illiquid asset. In such a model, all preference specifications that feature a time-inconsistency problem, i.e., news utility, hyperbolic discounting, or temptation discounting, will predict a high propensity to consume as the agent makes wealth inaccessible to his overconsuming future selves using the illiquid asset.

\textsuperscript{16}The hump-shaped profile constitutes a puzzle as the life-cycle profile of consumption must be monotonic if utility is an additively separable function of consumption, discounting is geometric, and markets are complete. Nevertheless, I argue that the news-utility hump is more robust to alternative assumptions about the discount factor, interest rate, and income profile, and more in line with the empirical hump in consumption.

\textsuperscript{17}A simple real-business-cycle model with expectations-based reference dependence generates realistic moments with the preference parameters that I estimate in this paper as shown by Pagel (2012a).
3 The Life-Cycle Consumption Model

I first define a general life-cycle model environment to formally introduce the preferences and equilibrium concepts.

3.1 The model environment

The agent lives for a total of $T$ discrete periods indexed by $t \in \{1, \ldots, T\}$. At the beginning of each period, a vector $S_t \sim F_S$ realizes that consists of random shocks, which are independent of each other and over time. The realization of $S_t$ is denoted $s_t$. The model’s exogenous state variables are represented by the vector $Z_t$, which evolves according to the following law of motion

$$Z_t = f^Z(Z_{t-1}, S_t).$$  \hspace{1cm} (1)

After observing $s_t$ and $Z_t$, the agent decides how much to consume, $C_t$. The model’s endogenous state variable is cash-on-hand $X_{t+1}$ and is determined by the following budget constraint

$$X_{t+1} = f^X(X_t - C_t, Z_t, S_{t+1}).$$  \hspace{1cm} (2)

All of the model’s variables that are indexed by $t$ realize in period $t$. Because the agent’s preferences are defined over both outcomes and beliefs, I explicitly define his probabilistic “beliefs” about each of the model’s period $t$ variables from the perspective of any prior period as follows.

**Definition 1.** Let $I_t = \{X_t, Z_t, s_t\}$ denote the agent’s information set in some period $t \leq t + \tau$. Then, the agent’s probabilistic beliefs about any model variable $V_{t+\tau}$ conditional on period $t$ information is denoted by $F^{\tau}_{V_{t+\tau}}(v) = \Pr(V_{t+\tau} < v | I_t)$, and $F^{t+\tau}_{V_{t+\tau}}$ is degenerate.

Throughout the paper, I assume rational expectations, i.e., the agent’s beliefs about any of the model’s variables equal the objective probabilities determined by the economic environment.

3.2 Expectations-based reference-dependent preferences

Having outlined the model environment, I now introduce the agent’s preferences. To facilitate the exposition, I first explain the static model of expectations-based reference dependence, as specified in Koszegi and Rabin (2006, 2007), and then introduce the dynamic model of Koszegi and Rabin (2009).

The agent’s utility function consists of two components. First, he experiences consumption utility $u(c)$, which corresponds to the standard model of utility and is solely determined by consumption $c$. Second, he experiences gain-loss utility $\mu(u(c) - u(r))$. The gain-loss utility function $\mu(\cdot)$ corresponds to the prospect-theory model of utility determined by consumption $c$ relative to

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18Throughout most of the paper, I consider a standard life-cycle environment in which the agent’s stochastic labor income is $Y_t = f^F(P_t - 1, S^p_t, S^f_t)$, which depends on his permanent income $P_t - 1$, a permanent shock $S^p_t \sim F^p$, and a transitory shock $S^f_t \sim F^f$. He decides how much to consume $C_t$ and how much to save in a risk-free asset that pays a net return $r$ such that his cash-on-hand $X_{t+1}$ is determined by $X_{t+1} = (X_t - C_t)(1 + r) + Y_{t+1}$. 

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the reference point \( r \). \( \mu(\cdot) \) is piecewise linear with slope \( \eta \) and a coefficient of loss aversion \( \lambda \), i.e., \( \mu(x) = \eta x \) for \( x > 0 \) and \( \mu(x) = \eta \lambda x \) for \( x \leq 0 \). The parameter \( \eta > 0 \) weights the gain-loss utility component relative to the consumption utility component and \( \lambda > 1 \) implies that losses are weighed more heavily than gains, i.e., the agent is loss averse. Koszegi and Rabin (2006, 2007) allow for stochastic consumption, distributed according to \( F_c(c) \), and a stochastic reference point, distributed according to \( F_r(r) \). Then, the agent experiences gain-loss utility by evaluating each possible outcome relative to all other possible outcomes

\[
\int_{-\infty}^{\infty} (\eta \int_{-\infty}^{c} (u(c) - u(r))dF_r(r) + \eta \lambda \int_{c}^{\infty} (u(c) - u(r))dF_r(r))dF_c(c).
\]

(3)

Additionally, the authors make the central assumption that the distribution of the reference point \( F_r \) equals the agent’s fully probabilistic rational beliefs about consumption \( c \).

In the dynamic model of Koszegi and Rabin (2009), the utility function consists of consumption utility, contemporaneous gain-loss utility about current consumption, and prospective gain-loss utility about the entire stream of future consumption. Thus, total instantaneous utility in period \( t \) is given by

\[
U_t = u(C_t) + n(C_t, F_t^{-1}) + \gamma \sum_{\tau=1}^{\infty} \beta^\tau n(F_{t+\tau}^{-1}).
\]

(4)

The first term on the right-hand side of equation (4), \( u(C_t) \), corresponds to consumption utility in period \( t \). Before turning to the subsequent terms in equation (4), which consider consumption and beliefs, I define an “admissible consumption function”. This function allows me to explicitly describe the probabilistic structure of the agent’s beliefs about any of the model’s variables at any future date. Because the agent fully updates his beliefs in each period and because the shocks are independent over time, I consider a stationary function that depends only on this period’s cash-on-hand \( X_t \), the vector of exogenous state variables \( Z_t \), the realization of the vector of shocks \( s_t \), and calendar time \( t \).

Definition 2. The consumption function in any period \( t \) is admissible if it can be written as a function \( C_t = g_t(X_t, Z_t, S_t) \) that is strictly increasing in the realization of each shock \( \frac{\partial g_t(X_t, Z_t, S_t)}{\partial s_t} > 0 \). Repeated substitution of the law of motion, equation (1), and the budget constraint, equation (2), allows me to rewrite \( C_{t+\tau} = g_{t+\tau}(X_{t+\tau}, Z_{t+\tau}, S_{t+\tau}) = h_{t+\tau}(X_t, Z_t, S_t, S_{t+1}, \ldots, S_{t+\tau}) \).

I now return to the two remaining terms on the right-hand side of equation (4). The first term, \( n(C_t, F_t^{-1}) \), corresponds to gain-loss utility over contemporaneous consumption; here, the agent compares his present consumption \( C_t \) with his beliefs \( F_t^{-1} \). According to Definition 1, the agent’s beliefs \( F_t^{-1} \) correspond to the conditional distribution of consumption in period \( t \) given the information available in period \( t - 1 \). The agent experiences gain-loss utility over “news” about contemporaneous consumption as follows
over news, the preferences can be referred to as "news utility". In period $t \in \mathbb{R}$, discounts prospective gain-loss utility relative to contemporaneous gain-loss utility by a factor $F_t$ with $F_t = \mathbb{E}(h_t^{-1}(X_{t-1}, Z_{t-1}, s_{t-1}, s_t) | C_t).$

The third term in equation (4), $\gamma \sum_{t=1}^{\infty} \beta^t n(F_{C_{t+\tau}}^{t+1})$, corresponds to gain-loss utility, experienced in period $t$, over the entire stream of future consumption. Prospective gain-loss utility about period $t + \tau$ consumption depends on $F_{C_{t+\tau}}^{t+1}$, the agent’s beliefs he entered the period with, and on $F_{C_{t+\tau}^{t+1}}$, the agent’s updated beliefs about consumption in period $t + \tau$. Again the probabilistic structure of these beliefs can be explicitly described via the admissible consumption function, i.e., $h_{t+\tau}(X_t, Z_t, s_t, S_{t+1}, ..., S_{t+\tau})$. Importantly, the prior and updated beliefs about $C_{t+\tau}$, $F_{C_{t+\tau}^{t+1}}$ and $F_{C_{t+\tau}}$ are not independent distribution functions because future shocks $S_{t+1}, ..., S_{t+\tau}$ are contained in both. Thus, there exists a joint distribution, which I denote by $F_{C_{t+\tau}^{t+1}} \neq F_{C_{t+\tau}} F_{C_{t+\tau}^{t+1}}$. Because the agent compares his newly formed beliefs with his prior beliefs, he experiences gain-loss utility over "news" about future consumption as follows

$$n(C_t, F_{C_t}^{t-1}) = \eta \int_{-\infty}^{C_t} (u(C_t) - u(c))dF_{C_t}^{t-1}(c) + \eta \lambda \int_{C_t}^{\infty} (u(C_t) - u(c))dF_{C_t}^{t-1}(c)$$  \hspace{1cm} (5)

where $C_t$ and $F_{C_t}^{t-1}(c)$ are explicitly described via the admissible consumption function, i.e., $C_t = g_t(X_t, Z_t, s_t) = h_t^{-1}(X_{t-1}, Z_{t-1}, s_{t-1}, s_t)$ and $F_{C_t}^{t-1}(c) = \mathbb{P}(h_t^{-1}(X_{t-1}, Z_{t-1}, s_{t-1}, s_t) < c)$.

Koszegi and Rabin (2009) generalize the outcome-wise comparison to a separated comparison. Koszegi and Rabin (2009) generalize the outcome-wise comparison to a "percentile-wise" ordered comparison. The separated and ordered comparisons are equivalent for contemporaneous gain-loss utility. However, for prospective gain-loss utility, they are qualitatively similar but quantitatively slightly different. As a linear operator, the separated comparison is more tractable. Moreover, it simplifies the equilibrium-finding process because it preserves the outcome-wise nature of contemporaneous gain-loss utility. 

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19I calculate prospective gain-loss utility $n(F_{C_{t+\tau}}^{t+1})$ by generalizing the "outcome-wise" comparison, specified in Koszegi and Rabin (2006, 2007) and reported in equation (15), to account for the potential dependence of $F_t$ and $F_c$, i.e.,

$$n(F_{C,c}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(u(c) - u(r))dF_{C,c}(c, r) \hspace{1cm} (6)$$

If $F_t$ and $F_c$ are independent, equation (6) reduces to equation (15). However, if $F_t$ and $F_c$ are non-independent, equation (6) and equation (15) yield different values. Suppose that $F_t$ and $F_c$ are perfectly correlated, as though no update in information occurs. Equation (15) would yield a negative value because the agent experiences gain-loss disutility over his previously expected uncertainty, which seems unrealistic. In contrast, equation (6) would yield zero because the agent considers the dependence of prior and updated beliefs, which captures future uncertainty, thereby separating uncertainty that has been realized from uncertainty that has not been realized. Thus, I call this gain-loss formulation the separated comparison. Koszegi and Rabin (2009) generalize the outcome-wise comparison to a "percentile-wise" ordered comparison. The separated and ordered comparisons are equivalent for contemporaneous gain-loss utility. However, for prospective gain-loss utility, they are qualitatively similar but quantitatively slightly different. As a linear operator, the separated comparison is more tractable. Moreover, it simplifies the equilibrium-finding process because it preserves the outcome-wise nature of contemporaneous gain-loss utility.
3.3 The model solution

The news-utility agent’s lifetime utility in each period \( t = \{1, \ldots, T\} \) is

\[
u(C_t) + n(C_t, F_{t-1}^t) + \gamma \sum_{\tau=1}^{T-t} \beta^\tau n(F_{t+\tau}^{t-1}) + E_t[\sum_{\tau=1}^{T-t} \beta^\tau U_{t+\tau}],
\]

(8)

where \( \beta \in [0, 1) \), \( u(\cdot) \) is a HARA utility function, \( \eta \in [0, \infty) \), \( \lambda \in [1, \infty) \), and \( \gamma \in [0, 1] \). I also consider hyperbolic-discounting or \( \beta \delta \) preferences, as developed by Laibson (1997); the \( \beta \delta \) agent’s lifetime utility is given by

\[
u(C^b_t) + bE_t[\sum_{\tau=1}^{T-t} \beta^\tau u(C^b_{t+\tau})]
\]

where \( b \in [0, 1] \) is the hyperbolic-discount factor. Needless to say, standard preferences, as analyzed by Carroll (2001), Gourinchas and Parker (2002), or Deaton (1991), are a special case of the above models for either \( \eta = 0 \) or \( b = 1 \). I now define two equilibrium concepts: the monotone-personal equilibrium and monotone-pre-committed equilibrium.

I define the model’s “monotone-personal” equilibrium in the spirit of the preferred-personal equilibrium solution concept, as defined by Koszegi and Rabin (2009), but within the outlined environment and admissible consumption function as follows.

**Definition 3.** The family of admissible consumption functions \( C_t = g_t(X_t, Z_t, s_t) \) is a monotone-personal equilibrium for the news-utility agent if, in any contingency, \( C_t = g_t(X_t, Z_t, s_t) \) maximizes (8) subject to (2) and (1) under the assumption that all future consumption corresponds to \( C_{t+\tau} = g_{t+\tau}(X_{t+\tau}, Z_{t+\tau}, s_{t+\tau}) \). In each period \( t \), the agent takes his beliefs about consumption \( \{F_{C_{t+\tau}}^{t-1}\}_{\tau=0}^{T-t} \) as given in the maximization problem.

The monotone-personal equilibrium can be obtained by simple backward induction; thus, it is time consistent in the sense that beliefs map into correct behavior and vice versa. In other words, I derive the equilibrium consumption function under the premise that the agent enters period \( t \), takes his beliefs as given, optimizes over consumption, and rationally expects to behave in this manner in the future. If I obtain a consumption function by backward induction that is admissible, then the monotone-personal equilibrium corresponds to the preferred-personal equilibrium as defined by Koszegi and Rabin (2009). For the hyperbolic-discounting agent, the monotone-personal equilibrium corresponds to the solution of Laibson (1997).

The monotone-personal equilibrium maximizes the agent’s utility in each period \( t \) when he takes his beliefs as given. However, if the agent could pre-commit to his consumption in each possible contingency, he would choose a different consumption path. I define this path as the model’s “monotone-pre-committed” equilibrium in the spirit of the choice-acclimating equilibrium concept, as defined by Koszegi and Rabin (2007), but within the outlined environment and admissible consumption function as follows.

20 A utility function \( u(c) \) exhibits hyperbolic absolute risk aversion (HARA) if the risk tolerance \( -\frac{u''(c)}{u'(c)} \) is a linear function of \( c \).
Definition 4. The family of admissible consumption functions, $C_t = g_t(X_t, Z_t, s_t)$ for each period $t$, is a monotone-pre-committed equilibrium for the news-utility agent, if, in any contingency, $C_t = g_t(X_t, Z_t, s_t)$ maximizes (8) subject to (2) and (1) under the assumption that all future consumption corresponds to $C_{t+\tau} = g_{t+\tau}(X_{t+\tau}, Z_{t+\tau}, s_{t+\tau})$. In each period $t$, the agent’s maximization problem determines both his beliefs $\{F^t_{t-1}\}^{T-t}_{\tau=0}$ and consumption $\{C_{t+\tau}\}^{T-t}_{\tau=0}$.

I derive the equilibrium consumption function under the premise that the agent can pre-commit to an optimal, history-dependent consumption path for each possible future contingency and thus jointly optimizes over consumption and beliefs. This equilibrium is not time consistent because the agent would deviate if he were to take his beliefs as given and optimize over consumption alone.

I demonstrate the existence and uniqueness of the monotone-personal and monotone-pre-committed equilibria for special environments, such as exponential utility and permanent and transitory normal shocks, under one parameter condition. The condition is that $F_S$ must be sufficiently dispersed such that the equilibrium consumption functions fall into the class of admissible consumption functions. For the monotone-pre-committed equilibrium, an additional parameter constraint $\eta(\lambda - 1) < 1$ is required to ensure global concavity of the agent’s maximization problem. For other environments, such as power utility and permanent and transitory log-normal shocks, simulations using numerical backward induction suggest that the monotone-personal and monotone-pre-committed equilibria exist and are unique for most reasonable calibrations.

4 Theoretical Predictions about Consumption

In the following, I explain the closed-form solution of the exponential-utility model in detail to illustrate the model’s predictions formally and intuitively. After briefly outlining the model’s monotone-personal equilibrium in Proposition 1, I flesh out the second-to-last period’s decision problem to explain the model’s theoretical predictions. Proposition 2 and Corollary 1 formalize excess smoothness and sensitivity in consumption. Lemma 1 discusses how the precautionary-savings motive competes with the prospective gain-loss discount factor; the net of these forces leads to a hump-shaped consumption profile, which is formalized in Proposition 3. Proposition

21 Moreover, in Section 4.4 and Appendix B.4, I argue that the model’s equilibrium is not affected qualitatively or quantitatively, if this condition does not hold. If the consumption function is decreasing over some range, the agent would simply chose a flat function over this range and the admissible consumption function requirement would be weakly satisfied.

22 Carroll (2011) and Harris and Laibson (2002) demonstrate the existence and uniqueness of equilibria for the standard and sophisticated hyperbolic-discounting agent in similar environments. In these models, the equilibrium consumption functions fall in the class of admissible consumption functions. For the standard agent, the monotone-personal equilibrium corresponds to the pre-committed equilibrium. For the sophisticated hyperbolic-discounting agent, the monotone-personal equilibrium does not correspond to the monotone-pre-committed equilibrium, which instead corresponds to the standard agent’s equilibrium.
4 determines consumption during retirement, and Proposition 5 characterizes when consumption drops at retirement. After these main predictions, I discuss several more subtle consumption implications and new comparative statics. Finally, Proposition 6 characterizes the implications of the monotone-pre-committed equilibrium.

I begin by briefly explaining the model’s environment and stating the equilibrium consumption function to convey a general impression of the model’s solution. The agent’s utility function is exponential \( u(C) = -\frac{1}{\theta} e^{-\theta C} \), where \( \theta \in (0, \infty) \). His additive income process \( Y_t = P_{t-1} + s_{t}^P + s_{t}^T \) is characterized by a permanent \( S_t^P \sim N(\mu_{P_t}, \sigma_{P_t}^2) \) and transitory \( S_t^T \sim N(\mu_{T_t}, \sigma_{T_t}^2) \) normal shocks, and his permanent income is \( P_t = P_{t-1} + s_{t+1}^P \). His end-of-period asset holdings are denoted \( A_t = X_t - C_t \) and his budget constraint is given by

\[
X_{t+1} = (X_t - C_t)(1 + r) + Y_{t+1} \Rightarrow A_{t+1} = A_t R + Y_{t+1} - C_{t+1}.
\] (9)

In Appendix B.2.1, I show that the agent’s optimal consumption function is

\[
C_t = (1 - a(T-t))(1 + r)A_{t-1} + P_{t-1} + s_{t}^P + (1 - a(T-t))s_{t}^T - a(T-t)\Lambda_t.
\] (10)

His consumption depends on his assets, income, horizon, and interest rate; the latter two are captured in the annuitization factor \( a(T-t) = \frac{\sum_{j=0}^{T-t-1}(1+r)^j}{\sum_{j=0}^{T-t}(1+r)^j} \). Moreover,

\[
\Lambda_t = \frac{1}{\theta} \log \left( \frac{1 - a(T-t) \psi_t + \gamma Q_t (\eta F(s_{t}^P + (1 - a(T-t))s_{t}^T) + \eta \lambda (1 - F(s_{t}^P + (1 - a(T-t))s_{t}^T)))}{1 + \eta F(s_{t}^P + (1 - a(T-t))s_{t}^T) + \eta \lambda (1 - F(s_{t}^P + (1 - a(T-t))s_{t}^T))} \right)
\] (11)

where \( F(\cdot) = F_{s_{t}^P + (1 - a(T-t))s_{t}^T}(\cdot) \) and \( \psi_t \) and \( Q_t \) are constant. Thus, \( \Lambda_t \) varies with the shock realizations but is independent of permanent income or assets. The standard and hyperbolic-discounting agents’ monotone-personal equilibria have the same structure except that \( \Lambda_t^s \) and \( \Lambda_t^b \) only vary with the agent’s horizon.

**Proposition 1.** There exists a unique monotone-personal equilibrium in the finite-horizon exponential-utility model if \( \sqrt{\sigma_{P_t}^2 + (1 - a(i))2\sigma_{T_t}^2} \geq \sigma_t^* \) for all \( t \in \{1, \ldots, T\} \).

This proposition’s proof and the proofs of the following propositions can be found in Appendix B.4. All of the following propositions are derived within this model environment and hold in any monotone-personal equilibrium if one exists.

### 4.1 Excess smoothness and sensitivity in consumption

Excess smoothness and sensitivity in consumption are two robust empirical observations, which emerged from tests of the permanent income hypothesis. The permanent income hypothesis postulates that the marginal propensity to consume out of permanent income shocks is one and that future consumption growth is not predictable using past variables. However, numerous studies find that the marginal propensity to consume is less than one because consumption underresponds to
permanent income shocks; thus, consumption is excessively smooth according to Deaton (1986). Moreover, numerous studies find that past changes in income have predictive power for future consumption growth because consumption adjusts with a delay; thus, consumption is excessively sensitive according to Flavin (1985). Campbell and Deaton (1989) explain how these observations are intrinsically related; consumption underresponds to permanent income shocks and thus adjusts with a delay. In this spirit, I define excess smoothness and sensitivity for the exponential-utility model as follows.

**Definition 5.** Consumption is excessively smooth if \( \frac{\partial C_t}{\partial s_t^1} < 1 \) everywhere and excessively sensitive if \( \frac{\partial \Delta C_{t+1}}{\partial s_t^1} > 0 \) everywhere.

This definition has an empirical analogue: an ordinary least squares (OLS) regression of period \( t+1 \) consumption growth on the realization of the permanent shock in periods \( t+1 \) and \( t \); for the two OLS coefficients \( \beta_1 \) and \( \beta_2 \), the above definition implies that consumption is excessively smooth if \( \beta_1 = \frac{\partial C_t}{\partial s_t^1} \bigg|_{s_t^1=\mu_p} < 1 \) and excessively sensitive if \( \beta_2 = \frac{\partial \Delta C_{t+1}}{\partial s_t^1} \bigg|_{s_t^1=\mu_p} > 0 \).

**Proposition 2.** The news-utility agent’s consumption is excessively smooth and sensitive.

I briefly present a simplified intuition for this result to then explain the agent’s first-order condition in greater detail and provide the full intuition.\(^{23}\) The agent’s marginal gain-loss utility today is more sensitive to his savings than his marginal gain-loss utility tomorrow, as his reference point today is invariable while his reference point tomorrow will have adjusted to his savings plan today. As a result, in the event of an adverse shock, the agent prefers to delay the reduction in consumption until his reference point has decreased. Additionally, in the event of a good shock, the agent prefers to delay the increase in consumption until his reference point has increased.

To explain this result in greater detail, I flesh out the agent’s decision-making problem in the second-to-last period assuming that transitory shocks are absent, \( A_{T-2} = P_{T-2} = 0 \), and the permanent income shock is independent and identically distributed (i.i.d.) normal \( s_T^{P} \sim F_P = N(\mu_p, \sigma_p) \). In period \( T-1 \), the agent chooses how much to consume \( C_{T-1} \) and save \( s_T^{P} - C_{T-1} \). His optimal consumption growth is given by

\[
\Delta C_T = s_T^{P} + \frac{1}{\theta} \log((1+r) \frac{\psi_{T-1} + \gamma Q_{T-1} (\eta F_P(s_T^{P}) + \eta \lambda (1-F_P(s_T^{P}))}{1 + \eta F_P(s_T^{P}) + \eta \lambda (1-F_P(s_T^{P}))}).
\]

I explain each component of the fraction in equation (12) in detail. The denominator is marginal consumption and contemporaneous gain-loss utility in period \( T-1 \); the latter consists of two terms. First, the agent compares his actual consumption to all consumption outcomes that would have been less favorable and experiences a gain weighted by \( \eta \), i.e., \( \eta \int_{c_T-1}^{c_T} (u(C_{T-1}) - u(c)) F_{C_T}^{T-1}(c) \). Second, the agent compares his actual consumption to all outcomes that would have been more favorable and experiences a loss weighted by \( \eta \lambda \), i.e., \( \eta \lambda \int_{c_T-1}^{c_T} (u(C_{T-1}) - u(c)) F_{C_T-1}^{T-2}(c) \). Because

\(^{23}\)This result can be generalized to a HARA utility function, arbitrary horizons, and arbitrary income uncertainty.
the agent takes his beliefs as given in the monotone-personal equilibrium, his marginal consumption and marginal contemporaneous gain-loss utility equals

$$u'(C_{T-1}) + u'(C_{T-1}) (\eta F_{C_{T-1}^{T-2}}(C_{T-1}) + \eta \lambda (1 - F_{C_{T-1}^{T-2}}(C_{T-1})))$$.

This expression can be simplified by replacing $F_{C_{T-1}^{T-2}}(C_{T-1})$ with $F_p(s^p_{T-1})$ because any admissible consumption function is increasing in the shock realization.

The second term of the numerator in equation (12) is marginal prospective gain-loss utility over future consumption $C_T = (s^p_{T-1} - C_{T-1})(1 + r) + s^p_{T-1} + S^p_T$. I denote the expected marginal utility of the last period’s income shock $Q_{T-1} = \beta E_{T-1} [u'(S^p_T)]$. As the agent’s admissible consumption is increasing in the shock realization and he takes his beliefs as given, his marginal prospective gain-loss utility corresponds to the same weighted sum of $F_p(s^p_{T-1})$

$$(1 + r)u'((s^p_{T-1} - C_{T-1})(1 + r) + s^p_{T-1}) \gamma Q_{T-1} (\eta F_p(s^p_{T-1}) + \eta \lambda (1 - F_p(s^p_{T-1}))).$$

The first term of the numerator in equation (12) is marginal future consumption and gain-loss utility. I denote the expected marginal consumption and gain-loss utility of the last period’s income shock $\psi_{T-1}$, which equals $Q_{T-1}$ plus $\beta E_{T-1} [\eta (\lambda - 1) \int_{s^p_T}^{s^p_{T-1}} (u'(S^p_T) - u'(s)) dF_p(s)]$. Consequently, marginal expected consumption and gain-loss utility are given by $(1 + r)u'((s^p_{T-1} - C_{T-1})(1 + r) + s^p_{T-1}) \psi_{T-1}$.

The fraction in equation (12) is increasing in $s^p_{T-1}$ for any $\gamma$ iff $\psi_{T-1} > Q_{T-1}$. The difference between $\psi_{T-1}$ and $Q_{T-1}$ corresponds to expected marginal gain-loss utility that is constant because the future reference point adjusts to today’s savings plan. Thus, a positive share of tomorrow’s marginal utility is inelastic to today’s savings, which implies that tomorrow’s marginal utility is less sensitive to changes in savings than today’s marginal utility. Today’s marginal contemporaneous and prospective gain-loss utility is relatively high or low in the event of an adverse or positive shock. In contrast, expected marginal gain-loss utility is constant because tomorrow’s reference point will have adjusted to today’s plan. Thus, the agent will consume relatively more in the event of an adverse shock and relatively less in the event of a positive shock. According to Definition 5, consumption is excessively smooth $\frac{\partial C_{T-1}}{\partial s^p_{T-1}} < 1$ and excessively sensitive $\frac{\partial \Delta C_{T-1}}{\partial s^p_{T-1}} > 0$.

In contrast, the standard agent’s consumption growth is $\Delta C^s_T = s^p_T + \frac{1}{b} \log((1 + r)Q_{T-1})$, and the hyperbolic-discounting agent’s consumption growth is $\Delta C^b_T = s^p_T + b \log((1 + r)b Q_{T-1})$. Thus, the consumption of these agents is neither excessively smooth nor excessively sensitive.

To illustrate the quantitative implications of excess smoothness and sensitivity, I run the linear regression of consumption growth on income

$$\Delta C_{t+1} = \alpha + \beta_1 \Delta Y_{t+1} + \beta_2 \Delta Y_t + \epsilon_{t+1}.$$  

For the news-utility model, I obtain $\beta_1 \approx 0.22$ and $\beta_2 \approx 0.18$ and a marginal propensity to consume out of permanent shocks of approximately 71%. Exponential utility implies that $u'( + \cdot) = u'( + )u'(-)$ and thus works well with additive risk.

24Exponential utility implies that $u'( + \cdot) = u'( + )u'(-)$ and thus works well with additive risk.

25I retain the normal income process outlined in Section 4 assuming that permanent and transitory shocks are i.i.d.,
propensity is one.\textsuperscript{26}

For illustration, Figure 1 displays the news-utility and standard agents’ consumption functions for realizations within two standard deviations of each shock, while the other is held constant. The flatter part of the news-utility consumption function generates excess smoothness and sensitivity.

4.2 The hump-shaped consumption profile

Fernandez-Villaverde and Krueger (2007), among others, show that lifetime consumption profiles are hump-shaped, even when controlling for cohort, family size, number of earners, and time effects.\textsuperscript{27} In the following, I demonstrate that the preferences generate a hump-shaped consumption profile as a result of the net of two competing features – an additional first-order precautionary-savings motive and the agent’s discount factor on prospective gain-loss utility $\gamma$. I choose the environment parameters so as to roughly generate the volatility of the log-normal income process that is typically used in the life-cycle consumption literature. I choose the agent’s horizon $T$, his retirement period $R$, his initial wealth $A_0$ and $P_0$, and the interest rate $r$ in accordance with the life-cycle literature. Additionally, I choose the preference parameters in line with the microeconomic literature and experimental evidence, which is explained in detail in Section 5.3. The parameters are $\mu_P = 0$, $\sigma_P = 5\%$, $\mu_T = 0$, $\sigma_T = 7\%$, $\beta = 0.978$, $r = 2\%$, $\theta = 2$, $\eta = 2$, $\lambda = 2$, $\gamma = 0.75$, $A_0 = 0$, and $P_0 = 0.1$.

\textsuperscript{26}Running the regression

$$\Delta C_{t+1} = \alpha + \beta_1 (s_{t+1} - \mu_P) + \beta_2 \Delta (s_t^P - \mu_P) + \epsilon_{t+1}$$

yields $\beta_1^s \approx 1$ and $\beta_2^s \approx 0$ in the standard model and $\beta_1 \approx 0.71$ and $\beta_2 \approx 0.31$ in the news-utility model. The transitory shock introduces a spurious negative correlation between $\Delta C_{t+1}$ and $\Delta Y_t$ because $\Delta Y_{t+1} = s_{t+1}^P + s_{t+1}^T - s_{t+1}^T$ and $\Delta Y_t = s_t^P + s_t^T - s_{t-1}^T$.

\textsuperscript{27}Moreover, Fernandez-Villaverde and Krueger (2007) find suggestive evidence that non-separability between consumption and leisure, which was promoted by Attanasio (1999) and previous papers, cannot explain more than 20% of the hump in consumption.
Income uncertainty has a first-order effect on savings in the news-utility model. This “first-order precautionary-savings motive” is added to the precautionary savings motive of the standard agent, which is a second-order motive.\(^{28}\) This result is highlighted by Koszegi and Rabin (2009) in a two-period, two-outcome model.

**Definition 6.** There exists a first-order precautionary-savings motive iff \(\frac{\partial (s^p_{T-1} - C_{T-1})}{\partial \sigma_p} |_{\sigma_p=0} > 0\).

However, the agent wishes to increase his consumption and decrease his savings if he discounts prospective gain-loss utility relative to contemporaneous gain-loss utility, i.e., \(\gamma < 1\). This discounting is reminiscent of \(\beta \delta\) – preferences. The following lemma formalizes these two opposing forces.\(^ {29}\)

**Lemma 1.**

1. **Precautionary savings:** News utility introduces a first-order precautionary-savings motive.
2. **Implications for consumption growth:** There exists a \(\bar{\gamma} > 1\), implicitly determined by \(\Delta C_T = \Delta C^p_T\), such that, iff \(\bar{\gamma} < \gamma\), the news-utility agent’s consumption growth in period \(T\) is higher than the standard agent’s consumption growth for any realization of \(s^p_{T-1}\) and \(s^p_T\), and \(\frac{\partial \bar{\gamma}}{\partial \sigma_p} < 0\).

The intuition for the first part of the lemma is as follows. The agent anticipates being exposed to gain-loss fluctuations in period \(T\), which are painful in expectation because losses hurt more than gains give pleasure. Additionally, the painfulness of these fluctuations is proportional to marginal consumption utility, which is lower higher on the utility curve. Thus, the agent has an additional incentive to increase savings. The intuition for the second part of the lemma is straightforward. If \(\gamma < 1\), the agent is more concerned about contemporaneous than prospective gain-loss utility; thus, he wishes to increase his consumption and decrease his savings. Consequently, the presence of news utility might increase or decrease consumption relative to the standard model depending on the net of two parameters \(\sigma_p > 0\) and \(\gamma < 1\).

In the following, I develop a more formal intuition for the standard and additional precautionary-savings motive and demonstrate that the assumption \(\psi_{T-1} > Q_{T-1}\), which I made previously, always holds. As shown above, the marginal value of savings is \((1 + r)u'((s^p_{T-1} - C_{T-1})(1 + r) + s^p_T)\psi_{T-1}\), where \(\psi_{T-1}\) equals the shock’s expected marginal consumption plus expected marginal gain-loss utility

\[
\beta E_{T-1}[u'(S^p_T)] + \beta E_{T-1}[\eta(\lambda - 1)\int_{s^p_T}^{\infty} (u'(S^p_T) - u'(s))dF_P(s)].
\] (15)

The integral in equation (15) reflects the expected marginal utility of all gains and losses, which partly cancel, such that only the overweighted component of the losses remains, i.e., \(\eta(\lambda - 1)(\cdot)\). The key point is that this integral is always positive if \(u'' < 0\) and thus captures the additional precautionary-savings motive, implies that \(\psi_{T-1} > Q_{T-1}\), and is increasing in \(\eta, \lambda\), and

\(^{28}\)Refer to Gollier (2001).

\(^{29}\)This result and those following can be generalized to any HARA utility function, arbitrary horizons, and labor income uncertainty.
Because $\frac{\partial (s_{T-1} - C_{T-1})}{\partial \sigma_p} |_{\sigma_p=0} > 0$, this motive is first order, as the news-utility agent is first-order risk averse. In contrast, the standard precautionary-savings motive is captured by $Q_{T-1} = \beta E_{T-1}[u'(S_T')]$, which is larger than $\beta u'(E_{T-1}[S_T])$ if $u''' > 0$, according to Jensen’s inequality. This standard precautionary-savings motive is second order, i.e., $\frac{\partial (s_{T-1} - C_{T-1})}{\partial \sigma_p} |_{\sigma_p=0} = 0$, as the standard agent is second-order risk averse.\(^{30}\)

The two competing news-utility features – the additional precautionary-savings motive and $\gamma < 1$ – make it likely that the life-cycle consumption profile is hump shaped.

**Definition 7.** I say that the agent’s consumption profile is hump shaped if consumption is increasing at the beginning of his life $\Delta C_1 > 0$ and decreasing $\Delta C_T < 0$ at the end of his life.

**Proposition 3.** Suppose $\sigma_{Pt} = \sigma_p$ for all $t$ and $T$ is large; then, there exists a $\sigma_p$ in $[\sigma_p, \sigma_p]$ such that, if $\gamma < 1$, $\log((1+r)\beta) \in [-\Delta, \Delta]$, and $\Delta$ is small, the news-utility agent’s lifetime consumption path is hump shaped.

The basic intuition is illustrated in Lemma 1. The relative strengths of the additional precautionary-savings motive and $\gamma < 1$ determine whether the presence of gain-loss utility increases or decreases the news-utility agent’s consumption relative to the standard model. When the agent’s horizon increases, the precautionary-savings motive accumulates because uncertainty accumulates. Accordingly, at the beginning of life, the presence of gain-loss utility is likely to reduce consumption and increase consumption growth unless $\gamma$ is small. Toward the end of life, however, the additional precautionary-savings motive is relatively small, and $\gamma < 1$ is likely to decrease consumption growth. More formally, the two conditions $\Delta C_{T+1} \leq 0$ reduce to $\Lambda_t \leq 0$ as $T - t$ becomes large or $T - t$ becomes small. The sign of $\Lambda_t$ is determined by the relative values of $\frac{\psi_t}{Q_t > 1}$ and $\gamma < 1$. As $T - t$ increases, $\frac{\psi_t}{Q_t}$ increases such that $\gamma < 1$ loses relative importance and $\Lambda_t$ is more likely to be positive. In contrast, $\frac{\psi_{T-1}}{Q_{T-1}}$ is small such that $\gamma < 1$ is likely to cause $\Lambda_{T-1}$ to be negative.

Figure 2 displays the news-utility and standard agents’ life-cycle consumption profiles.\(^{31}\) The figure displays the average consumption profile of 300 identical agents who encounter different realizations of $s_{Pt}$ and $s_{Tt}$ and the consumption profile if $s_{Pt} = 0$ and $s_{Tt} = 0$ for all $t$. As can be observed from the figure, the news-utility agent’s consumption profile is hump shaped. This hump is very robust to different parameter choices, which I discuss in Section 5.3. In contrast, the standard agent’s profile is V-shaped, which demonstrates that exponential utility and a random-walk income process do not promote the desired hump. Moreover, the figure displays the hump in

\(^{30}\)As shown by Benartzi and Thaler (1995) and Barberis, Huang, and Santos (2001), first-order risk aversion resolves the equity premium puzzle, which highlights that agents must have implausibly high second-order risk aversion to reconcile the historical equity premium because aggregate consumption is smooth compared with asset prices. The excess-smoothness puzzle highlights that aggregate consumption is too smooth compared to labor income, and again, first-order instead of second-order risk aversion is a necessary ingredient for resolving the puzzle.

\(^{31}\)I use the same calibration as in the quantitative exercise in Section 4.1.
the presence of a retirement period, which I explain next.

4.3 News-utility consumption during and at retirement

4.3.1 News-utility consumption during retirement

I now add a retirement period at the end of life. I assume that in periods \( t \in \{T - R, T\}\), the agent earns his permanent income without uncertainty. I first formalize a general prediction of the news-utility agent’s consumption during retirement, in which I generalize a result obtained by Koszegi and Rabin (2009) in a two-period model.\(^\text{32}\)

**Proposition 4.** If uncertainty is absent, both the monotone-personal equilibrium and monotone-pre-committed equilibrium of the news-utility agent correspond to the standard agent’s equilibria iff \( \gamma \geq \frac{1}{\lambda} \). Iff \( \gamma < \frac{1}{\lambda} \) then the monotone-pre-committed equilibrium of the news-utility agent corresponds to the standard agent’s equilibrium and the monotone-personal equilibrium of the news-utility agent corresponds to a \( \beta\delta \)-agent’s monotone-personal equilibrium with the hyperbolic-discount factor given by \( b = \frac{1 + \eta \lambda}{1 + \eta} \).

The news-utility agent is likely to follow the standard agent’s path if uncertainty is absent. The basic intuition is that the agent associates a certain loss in future consumption, which is very painful, with an increase in present consumption. Thus, unless the agent discounts prospective gain-loss utility significantly, he follows the utility-maximizing standard agent’s path. More formally, suppose that the agent allocates his deterministic cash-on-hand between consumption today \( C_{T-1} \) and tomorrow \( C_T \). Under rational expectations, he cannot fool himself; hence, he cannot experience actual gain-loss utility in equilibrium in a deterministic model. Accordingly, his

\(^{32}\)This result can be generalized to a HARA utility function.
expected-utility maximization problem corresponds to the standard agent’s maximization problem, and his monotone-pre-committed equilibrium thus corresponds to the standard agent’s problem determined by \( u'(C_{T-1}) = \beta (1 + r) u'(C_T) \). Suppose that the agent’s beliefs about consumption in both periods correspond to this pre-committed equilibrium path. Taking his beliefs as given, the agent will deviate if the gain from consuming more today exceeds the discounted loss from consuming less tomorrow, i.e.,

\[
u'(C_{T-1})(1 + \eta) > \beta(1 + r) u'(C_T)(1 + \gamma \eta \lambda).
\]

Thus, he follows the standard agent’s path iff \( \gamma \geq \frac{1}{\lambda} \) because the pain of the certain loss in future consumption is greater than the pleasure gained from present consumption. However, if \( \gamma < \frac{1}{\lambda} \), the agent chooses a consumption path that just meets the consistency constraint and behaves as a \( \beta \delta \) or hyperbolic-discounting agent with hyperbolic discount factor \( b = \frac{1 + \gamma \eta \lambda}{1 + \eta} < 1 \).

### 4.3.2 News-utility consumption at retirement

During retirement, the implications of the agent’s prospective gain-loss discount factor \( \gamma \) are simple: it needs to be sufficiently low to overcome the certain loss in future consumption. I now examine the pre-retirement period to derive two additional implications of \( \gamma < 1 \). The first concerns a drop in consumption at retirement, and the second shows how excess sensitivity in consumption arises in the absence of future uncertainty.

The empirical evidence on the prevalence of a drop in consumption at retirement is debated. While a series of papers (see Attanasio and Weber (2010) for a survey) have found that consumption drops at retirement, Aguiar and Hurst (2005) cannot confirm this finding when controlling for the sudden reduction of work-related expenses, the substitution of home production for market-purchased goods and services, and health shocks. In my data, I find such a drop in consumption at retirement even for non-work-related expenditures. Moreover, I consider the evidence provided by Schwerdt (2005) compelling because the author explicitly controls for home production and focuses on German retirees, who receive large state-provided pensions, which require little self-organization, and for whom health is a complement to consumption thanks to proper insurance coverage. Moreover, Ameriks, Caplin, and Leahy (2007) and Hurd and Rohwedder (2003) provide evidence that the drop in consumption is anticipated. I first define a drop in consumption as follows.

**Definition 8.** There occurs a drop in consumption at retirement if consumption growth at retirement \( \Delta C_{T-R} \) is negative and smaller than consumption growth after retirement \( \Delta C_{T-R+1} \).

As an example, if \( \gamma \geq \frac{1}{\lambda} \), the news-utility agent’s post-retirement consumption growth equals that of the standard agent’s, i.e., \( \frac{1}{b} \log(\beta (1 + r)) \approx 0 \), whereas consumption growth at retirement is \( \frac{1}{b} \log(\beta (1 + r)) + \frac{1}{b} g^s \) with \( g^s \in \{ \log(\frac{1 + \gamma \eta \lambda}{1 + \eta \lambda}), \log(\frac{1 + \gamma}{1 + \eta}) \} < 0 \) for the news-utility agent and
remains zero for the standard agent.\footnote{This and the following results can be generalized to a HARA utility function, arbitrary horizons, and arbitrary income uncertainty.}

**Proposition 5.** If $\gamma < 1$, $\log((1 + r)\beta) \in [-\Delta, \Delta]$, and $\Delta$ is small, the news-utility agent’s monotone-personal consumption path is characterized by a drop at retirement.

After the beginning of retirement the agent is less inclined to overconsume than before. The basic intuition for overconsumption in the pre-retirement period is that the agent allocates house money, i.e., labor income that he was not certain that he would receive, and thus wants to consume before his expectations catch up iff $\gamma < 1$. During retirement, the agent associates a certain loss in future consumption with a surprise in present consumption. In contrast, in the pre-retirement period, the agent finds the loss in future consumption merely as painful as a slightly less favorable realization of his labor income, i.e., the agent trades off being somewhere in the gain domain today versus being somewhere in the gain domain tomorrow instead of a sure gain today with a sure loss tomorrow. The agent’s first-order condition in period $T - 1$ absent uncertainty in period $T$ is given by

$$u'(C_{T-1}) = \beta(1 + r)u'(C_T) \frac{1 + \gamma F_p(s^p_{T-1}) + \eta \lambda (1 - F_p(s^p_{T-1}))}{1 + \eta F_p(s^p_{T-1}) + \eta \lambda (1 - F_p(s^p_{T-1}))}. \quad (16)$$

In equation (16), it can immediately be seen that iff $\gamma = 1$, contemporaneous and prospective marginal gain-loss utility cancel. However, iff $\gamma < 1$, the agent reduces the weight on future utility relative to present utility by a factor between $\frac{1 + \gamma \eta \lambda}{1 + \eta \lambda}$ and $\frac{1 + \gamma \eta}{1 + \eta} < 1$. During retirement, the news-utility agent follows the standard agent’s consumption path if $\gamma$ is sufficiently high and a $\beta \delta$–agent’s consumption path with discount factor $b = \frac{1 + \gamma \eta \lambda}{1 + \eta \lambda}$ otherwise. Because $\frac{1 + \gamma \eta}{1 + \eta} < \min\{\frac{1 + \gamma \eta \lambda}{1 + \eta \lambda}, 1\}$ iff $\gamma < 1$, the agent’s factor that reduces the weight on future utility is necessarily lower in the pre-retirement period than after retirement, which implies that consumption drops at retirement.\footnote{What happens if uncertainty in the pre-retirement period becomes small? The drop in consumption depends on the support of uncertainty. First, suppose the agent expects a continuous shock, the variance of which becomes small. So long as a monotone-personal equilibrium exists, there occurs a drop at retirement. However, if the variance of the shock becomes very small, the agent will follow a flat consumption path at some point. Nevertheless, the agent will not be able to follow his deterministic consumption path, but reduces the weight on future marginal value by a factor in the range of $\left\{\frac{1 + \gamma \eta \lambda}{1 + \eta \lambda}, \frac{1 + \gamma \eta}{1 + \eta}\right\}$. Thus, if $\frac{1}{\lambda} \leq \gamma \leq 1$, there occurs a drop for good realizations, and if $\gamma < \frac{1}{\lambda}$, there occurs a drop for all realizations. Second, suppose the agent expects a shock with some probability. If the probability of a shock occurring becomes small, the agent’s consumption in the pre-retirement period approaches his deterministic consumption path; this eliminates the drop because the agent’s first-order condition is no longer subject to a change in the weighting of future versus present marginal value.} The other agents’ consumption paths do not exhibit a drop in consumption at retirement. Quantitatively, Figure 2 displays a substantial drop in consumption at retirement.

The assumption of no uncertainty during retirement is made in all standard life-cycle consumption models, as these abstract from portfolio choice; thus, the drop in consumption at retirement is a necessary artifact of news-utility preferences in the standard environment. However, the drop is robust to three alternative assumptions: small income uncertainty during retirement, due to inflation risk for instance, potentially large discrete consumption uncertainty, due to health shocks for
instance, or mortality risk. Furthermore, if I were to observe a consumption path that is much flatter during retirement than before retirement and interpret this observation from the perspective of the standard model, I may conclude that the agent does not decumulate assets sufficiently rapidly after retirement compared to his pre-retirement asset decumulation. Such a lack of asset decumulation during retirement constitutes another life-cycle consumption puzzle that is observed by Hurd (1989), Disney (1996), and Bucciol (2012) and explained by the model.

In the following, I outline an additional result regarding excess sensitivity in the pre-retirement period. This result is related to Proposition 7 in Koszegi and Rabin (2009), in which the authors find that if \( \frac{1}{2} < \gamma < 1 \), then the news-utility agent might entirely consume small gains but entirely delay small losses when he is surprised by them.

**Corollary 1.** *Iff* \( \gamma < 1 \), *the news-utility agent’s monotone-personal equilibrium consumption is excessively smooth and sensitive in the pre-retirement period.*

The basic intuition is that the agent can effectively reduce his sense of loss by delaying the cut in consumption. *Iff* \( \gamma < 1 \), the agent cares more about contemporaneous than prospective gain-loss utility and thus overconsumes in the presence of uncertainty, as explained above. Moreover, he overconsumes even more when experiencing a relatively bad realization because losses are overweighted. Because the agent overconsumes relatively more in the event of a bad shock and relatively less in the event of a good shock, he delays his adjustment to consumption. Mathematically, the agent behaves like a \( \beta \delta \)-agent, weighting future consumption by a factor of

\[ \frac{1}{\lambda < \gamma < 1} \]

The drop in consumption is due to the fact that the agent overconsumes before retirement but consumes efficiently after retirement. If income uncertainty is very small, the agent is able to credibly plan a flat consumption level independent of the realization of his income shock because the benefits of smoothing consumption perfectly do not warrant the decrease in expected utility from experiencing gain-loss utility. In such a small-uncertainty situation, the agent is able to commit to a flat consumption level that induces less overconsumption after retirement than before retirement such that consumption drops. I formally explain this result about overconsumption in Section 4.4. Moreover, discrete uncertainty after retirement induces less overconsumption than before retirement for the same reason that no uncertainty causes less overconsumption. If uncertainty is discrete, overconsumption is associated with a discrete gain in present consumption and a discrete loss in future consumption. Because the discrete loss hurts more than the discrete gain, the agent may credibly plan a consumption level that induces less overconsumption than the baseline continuous-outcome equilibrium. Finally, mortality risk does not affect the result because the agent would not experience gain-loss utility relative to being dead.

This puzzle can also be explained by bequest motives (Hurd (1989)) and large medical expenditures shocks (Nardi, E. French, and J. Jones (2011)).

This result can be generalized to a HARA utility function, arbitrary horizons, and arbitrary income uncertainty.

In this example, the agent’s consumption is excessively smooth and sensitive for surprise gains. In the same setup, the agent’s consumption would also be excessively smooth and sensitive, according to Definition 5, for gains if they are sufficiently large and thus not entirely consumed or if they are expected. The agent might entirely consume an unexpected gain because it brings about a change in the weighting of future versus present marginal value in the agent’s first-order condition. More formally, absent uncertainty, the agent follows the standard agent’s path, as \( \frac{1}{\lambda < \gamma} \), whereas in the event of a surprise gain, he puts a weight of \( \frac{1+\eta\lambda}{1+\eta} < 1 \) on future consumption. Thus, if the gain is small, the change in the weight the agent places on future marginal consumption utility induces the agent to consume the entire gain.
utility varies with the permanent shock than with the transitory shock because marginal gain-loss utility. In the event of a negative shock, consumption is excessively smooth and sensitive because an increase in the permanent shock increases the fraction determining $\Delta C_T$. Moreover, for any given $\eta$ and $\lambda$, consumption is more excessively smooth and sensitive if $\gamma$ is low.

4.4 New predictions about news-utility consumption

In the following, I highlight several additional news-utility predictions for consumption that are new and testable comparative statics. I first explain the agent’s consumption function, equation (10), in detail to highlight some subtle predictions about how the marginal propensity to consume varies with the realization of the permanent and transitory shocks and the agent’s horizon. To explain the consumption function, I assume that $T - t$ is large such that $a(T - t) \approx \frac{1}{1+\eta}$ Then, in each period $t$, the agent consumes the interest payments of his last period’s asset holdings $rA_{t-1}$, his entire permanent income $P_{t-1} + s^p_t$, and the per-period value of his temporary shock $r A^T_t s^T_t$. $\Lambda_t$ captures the agent’s patience compared to the market, his precautionary savings, and his marginal gain-loss utility. In the event of a negative shock, $\Lambda_t$ is low and the agent consumes more out of his end-of-period asset holdings and thus spreads the consumption adjustment to his entire future. $\Lambda_t$ varies more with the permanent shock than with the transitory shock because marginal gain-loss utility varies with $F_{C_t}^{-1}(C_t)$, which varies little with the transitory shock as the agent only consumes the per-period value of the transitory shock $\frac{r}{1+\eta}s^T_t$ and $\frac{r}{1+\eta}$ is small. This observation constitutes the first novel prediction of the news-utility model: consumption is more excessively sensitive for permanent than for transitory shocks in an environment with permanent shocks. This prediction can be seen in Figure 1. In an environment with transitory shocks alone, however, news-utility consumption is excessively sensitive with respect to transitory shocks.

A second prediction is that the degree of excess smoothness and sensitivity is decreasing in income uncertainty $\sigma_p$. If $\sigma_p$ is small, the agent’s beliefs change more rapidly relative to the change in the realization of the shock; hence, the consumption function is more flat for realizations around $\mu_p$. A third prediction is that any bell-shaped shock distribution induces bounded variation in $\Lambda_t$ and thus the agent’s excess sensitivity. If the agent is affected by a tail realization, the actual value of the low-probability shock matters less because neighboring states have very low probability; thus, the variation in $\Lambda_t$ is bounded. A fourth prediction is that consumption is more excessively smooth when the agent’s horizon increases for two reasons: first the marginal propensity to consume out of permanent shocks declines when the precautionary-savings motive accumulates and second $a(T - t)$ is increasing in $T - t$. However, consumption is relatively less excessively sensitive when the agent’s horizon increases because excess smoothness is proportional to $a(T - t)$ while excess sensitivity is not.

In the following, I explain how the second prediction can be taken to the extreme: the consumption function may be completely flat if $\sigma_p$ is small.\footnote{This prediction about flat consumption is also highlighted by Heidhues and Koszegi (2008).} To formally discuss this result about
flat consumption, I return to the two-period, one-shock model. Suppose that the absolute level of the shock increases; then, holding $C_{T-1}$ constant, the marginal value of savings declines and the agent’s first-order condition implies that consumption should increase. However, $F_P(s^P_{T-1})$ also increases, and marginal gain-loss utility is lower, such that the agent’s optimal consumption should decrease. Suppose that $s^P_{T-1}$ increases marginally but $F_P(s^P_{T-1})$ increases sharply, which could occur if $F_P$ is a very narrow distribution. In this case, the lower marginal gain-loss utility that decreases consumption dominates such that the first-order condition predicts decreasing consumption over some range in the neighborhood of the expected value $\mu_P$ where $F_P$ increases most sharply if $F_P$ is bell shaped. However, a decreasing consumption function cannot be an equilibrium because the agent would unnecessarily experience gain-loss utility over the decreasing part of consumption, which decreases expected utility unnecessarily. In the decreasing-consumption function region, the agent could choose a flat consumption function instead. In such a situation, he does not respond to shocks at all, i.e., his consumption is perfectly excessively smooth and sensitive, which resembles liquidity constraints or adjustment costs to consumption. Moreover, the agent may choose a credible consumption plan with a flat section, which induces less overconsumption than the original plan. Suppose the agent chooses a flat consumption level for realizations of $s^P_{T-1}$ in $\underline{s}$ and $\bar{s}$. Then, $\bar{s}$ is chosen where the original consumption function just stops decreasing, which corresponds to the lowest possible level of the flat section of consumption $\bar{C}_{T-1}$, which I explicitly describe in Appendix 1. Moreover, in Appendix 1, I show that the agent’s consistency constraint for not increasing consumption beyond $\bar{C}_{T-1}$ for any $s^P_{T-1} \in [\underline{s}, \bar{s}]$ always holds. Thus, I can conclude that flat consumption results in less overconsumption than the baseline equilibrium.

4.5 Comparison to the agent’s pre-committed equilibrium and welfare implications

In order to assess the preferences’ welfare implications, I briefly explain the consumption implications of the monotone-pre-committed equilibrium that maximizes expected utility by jointly optimizing over consumption and beliefs. The pre-committed equilibrium is not credible without an appropriate commitment device because the agent overconsumes once he wakes up and takes his beliefs as given. I call this overconsumption phenomenon beliefs-based present bias because the agent prefers to enjoy the pleasant surprise of increasing consumption above expectations today instead of increasing both his consumption and expectations tomorrow. Empirically, there is abundant laboratory and field evidence for time-inconsistent overconsumption, preference reversals, and demand for commitment devices. Theoretically, the hyperbolic-discounting model of Laibson, Repetto, and Tobacman (2012) is very successful in explaining life-cycle consumption. In the next proposition, I formalize how the consumption implications differ in the monotone-pre-committed equilibrium if one exists. Then, I explain beliefs-based present bias in detail and show how it differs from hyperbolic discounting.

**Proposition 6. Comparison to the monotone-pre-committed equilibrium.**

40 Koszegi and Rabin (2009) argue in Proposition 6 that the agent overconsumes relative to the optimal pre-committed path in the presence of uncertainty.

41 See, e.g., DellaVigna (2009), Frederick, Loewenstein, and O’Donoghue (2002), or Angeletos, Laibson, Repetto, and Weinberg (2001) for a survey of the theory and empirical evidence.
1. If $\sigma_{Pt} > 0$ for any $t$, then the monotone-pre-committed consumption path does not correspond to the monotone-personal equilibrium consumption path.

2. The news-utility agent’s monotone-pre-committed consumption is excessively smooth and sensitive.

3. News-utility preferences introduce a first-order precautionary-savings motive in the monotone-pre-committed equilibrium, monotone-pre-committed consumption is lower $C_{T-1}^{c} < C_{T-1}$, and the gap increases in the event of good income realizations $\frac{\partial(1-C_{T-1}-C_{T-1})}{\partial F_{T}} > 0$.

5. The news-utility agent’s monotone-pre-committed consumption path is not necessarily characterized by a hump-shaped consumption profile and consumption does not drop at retirement.

Suppose that the agent can pre-commit to an optimal, history-dependent consumption path for each possible future contingency. Then, the agent’s marginal gain-loss utility is no longer solely composed of the sensation of increasing consumption in one particular contingency; additionally, the agent considers that he will experience fewer sensations of gains and more feelings of loss in all other contingencies. Thus, marginal gain-loss utility has a second component, $-u'(C_{T-1})(\eta(1-F_{C_{t}^{-1}}(C_{t})) + \eta\lambda F_{C_{t}^{-1}}(C_{t}))$, which is negative such that the pre-committed agent consumes less. Moreover, this negative component dominates if the realization is above the median, i.e., $F_{C_{t}^{-1}}(C_{t}) > 0.5$. Thus, in the event of good income realizations, pre-committed marginal gain-loss utility is negative. In contrast, non-pre-committed marginal gain-loss utility is always positive because the agent enjoys the sensation of increasing consumption in any contingency. Therefore, the degree of present bias is reference dependent and less strong in the event of bad income realizations, when increasing consumption is the optimal response even on the pre-committed path. Moreover, this negative component implies additional variation in marginal gain-loss utility such that pre-committed consumption is more excessively smooth and sensitive.

The analysis of the pre-committed equilibrium allows me to draw potentially important welfare conclusions. The result that excess smoothness is an optimal response and even more pronounced on the pre-committed path stands in contrast to the welfare implications of liquidity constraints, the potentially most popular alternative explanation for excess smoothness. In contrast, the result that the life-cycle consumption profile is not necessarily hump shaped and that consumption does not drop at retirement in the pre-committed equilibrium appears to be in line with alternative explanations such as hyperbolic discounting and illiquid savings as proposed by Laibson, Repetto, and Tobacman (2012), inattention as proposed by Reis (2006), incomplete consumption insurance as proposed by Attanasio and Pavoni (2011), an incomplete planning horizon as proposed by Park (2011), or overconfidence as proposed by Caliendo and Huang (2008).

Beyond the observation that the news-utility agent is unable to follow his expected-utility maximizing path, the news-utility implications for welfare and the costs of business cycle fluctuations differ from those of the standard model. In Section 4, I demonstrate that income uncertainty has a first-order effect on savings and thus welfare in the news-utility model; i.e., the news-utility agent dislikes fluctuations in consumption much more than the standard agent. In the spirit of Lucas (1978), I compute the share $\lambda_{W}$ of initial wealth $A_{1}$ that the agent would be willing to give up for a risk-free consumption path. In the power-utility model for the calibration given in Table 1, I obtain a share of approximately 47.83% for the news-utility agent, whereas the standard agent’s share is 8.65%.
Beliefs-based present bias is both conceptually different from hyperbolic-discounting preferences and observationally distinguishable. The four main differences are the following. First, news utility introduces an additional precautionary-savings motive that is absent in the hyperbolic-discounting model. Second, because of this precautionary-savings motive the news-utility agent does not have a universal desire to pre-commit himself to the standard agent’s consumption path, in contrast to hyperbolic-discounting preferences. Third, news utility predicts that the agent’s degree of present bias is reference dependent and lower in bad times; hence, he behaves better in bad times. Fourth, the news-utility agent’s degree of present bias depends on the uncertainty he faces. In the absence of uncertainty, the agent’s present bias is absent so long as $\gamma > \frac{1}{\lambda}$. In the presence of small or discrete uncertainty, the agent’s degree of present bias is less than in the presence of large and dispersed uncertainty, as shown in Section 4.4.

5 Quantitative Predictions about Consumption

In the following, I assess whether the model’s quantitative predictions match the empirical evidence. Because it is commonly argued that exponential utility is unrealistic, I present the numerical implications of a power-utility model, i.e., $u(C) = \frac{C^{1-\theta}}{1-\theta}$, to demonstrate that all of the predictions hold in model environments that are commonly assumed in the life-cycle consumption literature.\footnote{The power-utility model cannot be solved analytically, but it can be solved by numerical backward induction, as shown by Gourinchas and Parker (2002) or Carroll (2001), among others. The numerical solution is illustrated in greater depth in Appendix B.5.4.} In Section 5.1, I first outline the power-utility model. In Section 5.2, I structurally estimate the power-utility model’s parameters. In Section 5.3, I compare my estimates with those in the microeconomic literature and explain each in detail.

5.1 The power-utility model

I follow Carroll (1997) and Gourinchas and Parker (2002), who specify income $Y_t$ to be log-normal and characterized by a deterministic permanent income growth $G_t$, permanent shocks, and transitory shocks, which allow for a low probability of unemployment or illness

$$Y_t = P_t \cdot N_t^T = P_{t-1} G_t P_t N_t^P N_t^T$$

$$N_t^T = \begin{cases} e^{s_t^T} & \text{with probability } 1 - p \text{ and } s_t^T \sim N(\mu_T, \sigma_T^2) \\ 0 & \text{with probability } p \end{cases} N_t^P = e^{s_t^P} s_t^P \sim N(\mu_P, \sigma_P^2).$$

The life-cycle literature suggests fairly tight ranges for the parameters of the log-normal income process, which are approximately $\mu_T = \mu_P = 0$, $\sigma_T = \sigma_P = 0.1$, and $p = 0.01$. $G_t$ typically implies a hump-shaped income profile. Nevertheless, I initially assume that $G_t = 1$ for all $t$ to highlight the model’s predictions in an environment that does not simply generate a hump-shaped consumption profile via a hump-shaped income profile. In addition to the standard and hyperbolic agents, I
display results for internal, multiplicative habit-formation preferences, as assumed in Michaelides (2002), and temptation-disutility preferences, as developed by Gul and Pesendorfer (2004), following the specification of Bucciol (2012). The utility specifications can be found in Appendix B.1. For the habit-formation agent, I roughly follow Michaelides (2002) and choose $h = 0.45$, which matches the excess-smoothness evidence. The tempted agent’s additional preference parameter $\tau = \frac{\lambda_{td}}{1 + \lambda_{td}} = 0.1$ is chosen according to the estimates of Bucciol (2012).43

Figure 3 contrasts the five agents’ consumption paths with the average CEX consumption and income data, which I explain in Section 5.2. The habit-formation agent’s consumption profile is shown only in part because he engages in extremely high wealth accumulation due to his high effective risk aversion, even if I choose a lower value for $h$ than the one that fits the excess-sensitivity evidence.44 Hyperbolic-discounting preferences tilt the consumption profile upward at the beginning and downward at the end of life. Temptation disutility causes severe overconsumption at the beginning of life, which then dies out when alternative consumption opportunities diminish. All of the preference specifications except habit formation generate a hump-shaped consumption profile. The consumption path of all agents is increasing at the beginning of life because power utility renders them unwilling to borrow; however, all agents are sufficiently impatient such that consumption eventually decreases.45 Nevertheless, at first glance, the news-utility agent’s hump looks more similar to the empirical consumption profile with slowly increasing consumption at the beginning of life and decreasing consumption shortly before retirement.

Moreover, Figure 3 shows a substantial drop in consumption at retirement for both the news-utility consumption profile and the CEX consumption data. Thus, I conclude that the news-utility agent’s lifetime consumption profile looks very similar to the average consumption profile from the CEX data, which I explain in greater detail in the next section.

5.2 Structural estimation

I structurally estimate the news-utility parameters using a methods-of-simulated-moments procedure following Gourinchas and Parker (2002), Laibson, Repetto, and Tobacman (2012), and Bucciol (2012). The procedure has two stages. In the first stage, I estimate the structural parameters governing the environment $\Xi = (\mu_P, \sigma_P, \mu_T, \sigma_T, p, G, r, a_0, R, T)$ using standard techniques and

43For the news-utility parameters, I use the same calibration as in the quantitative exercise in Section 4.1.

44This result about wealth accumulation confirms a finding by Michaelides (2002).

45Because power utility eliminates the possibility of negative or zero consumption and because of the small possibility of zero income in all future periods, the agents will never find it optimal to borrow. Moreover, power utility implies prudence such that all agents have a standard precautionary-savings motive. However, this motive is rather weak because the standard agent’s consumption begins to decrease rather early in life. Moreover, in a model with only transitory shocks and no zero-income state, the precautionary savings motive is so weak that the standard agent’s consumption is flat throughout. Attanasio (1999) criticizes this weak motive as lacking realism. In contrast, even if permanent shocks and unemployment are absent, the news-utility model generates a hump-shaped consumption profile.
obtain results perfectly in line with the literature. Given the first-stage estimates $\hat{\Xi}$ and their associated variances $\hat{\Omega}_\Xi$, in the second stage, I estimate the preference parameters $\theta = (\eta, \lambda, \gamma, \beta, \theta)$ by matching the simulated and empirical average life-cycle consumption profiles. The empirical life-cycle consumption profile is the average consumption at each age $a \in [1, T]$ across all household observations $i$. More precisely, it is $\ln \bar{C}_a = \frac{1}{n_a} \sum_{i=1}^{n_a} \ln(\bar{C}_{i,a})$ with $\ln(\bar{C}_{i,a})$ being the household $i$’s log consumption at age $a$ of which $n_a$ are observed. The theoretical population analogue to $\ln \bar{C}_a$ is denoted by $\ln C_a(\theta, \Xi)$ and the simulated approximation is denoted by $\ln \hat{C}_a(\theta, \Xi)$. Moreover, I define $g(\theta, \Xi) = \ln C_a(\theta, \Xi) - \ln \bar{C}_a$ and $\hat{g}(\theta, \Xi) = \ln \hat{C}_a(\theta, \Xi) - \ln \bar{C}_a$.

In turn, if $\theta_0$ and $\Xi_0$ are the true parameter vectors, the procedure’s moment conditions imply that $E[g(\theta_0, \Xi_0)] = E[\ln C_a(\theta, \Xi) - \ln \bar{C}_a] = 0$. In turn, let $W$ denote a positive definite weighting matrix then

$$q(\theta, \Xi) = \hat{g}(\theta, \Xi)W^{-1}\hat{g}(\theta, \Xi)'$$

is the weighted sum of squared deviations of the simulated from their corresponding empirical moments. I assume that $W$ is a robust weighting matrix rather than the optimal weighting matrix to avoid small-sample bias. More precisely, I assume that $W$ corresponds to the inverse of the variance-covariance matrix of each point of $\ln \bar{C}_a$, which I denote by $\Omega_g^{-1}$ and consistently estimate from the sample data. Taking $\hat{\Xi}$ as given, I minimize $q(\theta, \hat{\Xi})$ with respect to $\theta$ to obtain $\hat{\theta}$ the consistent estimator of $\theta$ that is asymptotically normally distributed with standard errors

$$\Omega_\theta = (G'_\theta W G_\theta)^{-1}G'_\theta W[\Omega_g + \Omega_g^s G_\Xi G'_\Xi]WG_\theta (G'_\theta W G_\theta)^{-1}. $$

Here, $G_\theta$ and $G_\Xi$ denote the derivatives of the moment functions $\frac{\partial g(\theta_0, \Xi_0)}{\partial \theta}$ and $\frac{\partial g(\theta_0, \Xi_0)}{\partial \Xi}$, $\Omega_g$ denotes the variance-covariance matrix of the second-stage moments as above that corresponds to $E[g(\theta_0, \Xi_0)g(\theta_0, \Xi_0)']$, and $\Omega_g^s = \frac{n_s}{n_s} \Omega_g$ denotes the sample correction with $n_s$ being the number
of simulated observations at each age \( a \). As \( \Omega_g \), I can estimate \( \Omega_R \) directly and consistently from sample data. For the minimization, I employ a Nelder-Mead algorithm. For the standard errors, I numerically estimate the gradient of the moment function at its optimum. If I omit the first-stage correction and simulation correction the expression becomes \( \Omega_\theta = (G_\theta' \Omega_R^{-1} G_\theta)^{-1} \). Finally, I can test for overidentification by comparing \( \hat{g}(\hat{\theta}, \hat{\Xi}) \) to a chi-squared distribution with \( T - 5 \) degrees of freedom.

I use data from the Consumer Expenditure Survey (CEX) for the years 1980 to 2002 as provided by the NBER.\(^{46} \) The CEX is conducted by the Bureau of Labor Statistics and surveys a large sample of the US population to collect data on consumption expenditures, demographics, income, and assets. As suggested by Harris and Sabelhaus (2001), consumption expenditures consists of food, tobacco, alcohol, amusement, clothing, personal care, housing, house operations such as furniture and housesupplies, personal business, transportation such as autos and gas, recreational activities such as books and recreational sports, and charity expenditures; alternatively, I could consider non-durable consumption only. Income consists of wages, business income, farm income, rents, dividends, interest, pension, social security, supplemental security, unemployment benefits, worker’s compensation, public assistance, foodstamps, and scholarships. The data is deflated to 1984 dollars.

Because the CEX does not survey households consecutively, I generate a pseudo panel that averages each household’s consumption and income at each age. I only consider non-student households that meet the BLS complete income reporter requirement and complete all four quarterly interviews. Furthermore, I only consider households that are older than 25 years; that are retired after age 68, the average retirement age in the US according to the OECD; and that are younger than 78, the average life expectancy in the US according to the UN list.

To control for cohort, family size, and time effects, I employ average cohort techniques (see, e.g., Verbeek (2007), Attanasio (1998), and Deaton (1985)). More precisely, as I lack access to the micro consumption data for each household \( i \) at each age \( a \), I pool all observations and estimate \( \log(C_{i,a}) = \xi_0 + \alpha_a + \gamma_c + f_s + X_{i,a} \beta^a + \epsilon_{i,a} \). Here, \( \xi_0 \) is a constant, \( \alpha_a \) is a full set of age dummies, \( \gamma_c \) is a full set of cohort dummies, and \( f_s \) is a full set of family size and number of earners dummies. Essentially, these sets of dummies allow me to consider the sample means of my repeated cross-section \( C_{c,a} = E[\log(C_{i,a})|c,a] \) and \( X_{c,a} = E[X_{i,a}^*|c,a] \) for each cohort \( c \) at age \( a \). Using the sample means brings about an errors-in-variables problem, which, however, does not appear to make a difference in practice as the sample size of each cohort-age cell is large. Because age \( \alpha_a \) and cohort \( \gamma_c \) effects are not separately identifiable from time effects, I proxy time effects by including the regional unemployment rate as an additional variable in \( X_{i,a}^* \) beyond a dummy for retirement following Gourinchas and Parker (2002). As an alternative to a full set of dummies, I use fifth-order polynomials in order to obtain a smooth consumption profile. After running the pooled regression, I back out the consumption data uncontaminated by cohort, time, family size, and number of earners effects and construct the average empirical life-cycle profile by averaging

\(^{46}\)This data set extraction effort is initiated by John Sabelhaus and continued by Ed Harris, both of the Congressional Budget Office. The data set links the four quarterly interviews for each respondent household and collapses all the spending, income, and wealth categories into a consistent set of categories across all years under consideration.
the data across households at each age. The same exercise is done for income. Figure 3 displays the average empirical income and average empirical consumption profiles.

Theoretically, the functional form of $\Lambda_t$, or the agent's first-order condition in the power-utility model, imply that news utility introduces such specific variation in consumption growth that all preference parameters are identified in the finite-horizon model, i.e., $\eta$, $\lambda$, $\gamma$, $\beta$, and $\theta$, because the Jacobian has full rank. Roughly speaking, the shape of the consumption profile identifies $\beta$ and $\theta$. Because consumption tracks income too closely and peaks too early in the standard model, $\eta > 0$ and $\lambda > 1$ can be identified. Finally, the drop in consumption at retirement identifies $\gamma < 1$. More precisely, I am interested in $\beta$, $\theta$, $\eta$, $\lambda$, $\gamma$. As explained in Appendix B.5.4, the agent’s consumption is determined by the following first-order condition $u'(c_{T-i}) = \Psi_{T-i} + \gamma \Phi_{T-i}(\eta F_{T-i}^{T-1}(c_{T-i}) + \eta \lambda (1 - F_{T-i}^{T-1}(c_{T-i})))$ of which I observe the inverse and log average of all households. $\Phi_{T-i}$ represents future marginal consumption utility, as in the standard model, and is determined by $\beta$ and $\theta$, which can be separately identified in a finite-horizon model. $\Psi_{T-i}$ represents future marginal consumption and news utility and is thus determined by something akin of $\eta(\lambda - 1)$. $\eta F_{T-i}^{T-1}(a_{T-i}) + \eta \lambda (1 - F_{T-i}^{T-1}(a_{T-i}))$ and $\eta F_{T-i}^{T-1}(c_{T-i}) + \eta \lambda (1 - F_{T-i}^{T-1}(c_{T-i}))$ represents the weighted sum of the cumulative distribution function of savings, $a_{T-i}$, and consumption, $c_{T-i}$, of which merely the average determined by $\eta 0.5 (1 + \lambda)$ is observed. Thus, I have two equations in two unknowns and can separately identify $\eta$ and $\lambda$. Finally, $\gamma$ enters the first-order condition distinctly from all other parameters.

I employ a two-stage method-of-simulated-moments procedure. In the first stage, I estimate all of the structural parameters governing the environment $\hat{\mu}_p$, $\hat{\sigma}_p$, $\hat{\mu}_T$, $\hat{\sigma}_T$, $\hat{\rho}$, $\hat{G}$, $\hat{r}$, $\hat{a}_0$, $\hat{R}$, and $\hat{T}$, i.e., $\hat{\mu}_p = -0.002$, $\hat{\sigma}_p = -0.0031$, $\hat{\mu}_T = 0.18$, $\hat{\sigma}_T = 0.16$, and $\hat{\rho} = 0.0031$, which are in accordance with the literature. The mean of Moody’s municipal bond index is $r = 3.1\%$. Moreover, because 25 is chosen as the beginning of life by Gourinchas and Parker (2002), I choose $\hat{R} = 11$ and $\hat{T} = 54$. At age 25, I estimate the mean ratio of liquid wealth to income as 0.0096 under the assumption that $R_0 = 1$.

I estimate the preference parameters $\beta$, $\theta$, $\eta$, $\lambda$, and $\gamma$ and obtain $\hat{\theta} = 0.79$, $\hat{\beta} = 0.97$, $\hat{\eta} = 1.1$, $\hat{\lambda} = 2.4$, and $\hat{\gamma} = 0.53$. I display all first- and second-stage structural parameter estimates as well as their standard errors in Table 1. The second-stage standard errors are adjusted for first-stage uncertainty and the sampling correction; while the former increases the standard errors considerably the latter has very little effect as noted by Laibson, Repetto, and Tobacman (2012). The preference

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47Numerically, I confirm this result in a Monte Carlo simulation and estimation exercise. Moreover, because previous studies cannot separately identify $\eta$ and $\lambda$, I confirm that I obtain similar estimates when I assume $\eta = 1$ and only estimate the other parameters.

48Alternatively, I use a more complex set of moments to estimate the preference parameters, namely the degree of excess smoothness in consumption, the extent of the drop in consumption at retirement, and four other points of the life-cycle consumption profile. The resulting estimates and their standard errors are quantitatively very similar to the original ones.
parameters are estimated very tightly, and I cannot reject the overidentification test, which is a surprisingly positive result given the number of moments $T$ and the number of parameters, which is only five. In contrast, for the standard model, the standard errors are considerably larger and I reject the overidentification test, as do Gourinchas and Parker (2002). Finally, I obtain suggestive evidence for one of the new comparative statics generated by news utility; the excess-smoothness ratio in the CEX data increases from 0.68 at age 25 to 0.82 at the start of retirement.49

### 5.3 Discussion of the estimated preference parameters

I now show that my estimates are perfectly in line with the micro literature, generate reasonable attitudes towards small and large wealth bets, and match the empirical evidence for excess smoothness and sensitivity in aggregate data.

I refer to the literature for the standard preference parameter estimates $\beta \approx 1$ and $\theta \approx 1$ but discuss the news-utility parameter estimates, i.e., $\eta$, $\lambda$, and $\gamma$, in greater detail. In particular, I demonstrate that my estimates are consistent with existing micro evidence on risk and time preferences. In Table 3 in Appendix G, I illustrate the risk preferences over gambles with various stakes of the news-utility, standard, and habit-formation agents. In particular, I calculate the required gain $G$ for a range of losses $L$ to make each agent indifferent between accepting or rejecting a 50-50 win $G$ or lose $L$ gamble at a wealth level of 300,000 in the spirit of Rabin (2001) and Chetty and

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49For comparison, Figure 11 in Appendix G displays the consumption and income data of Gourinchas and Parker (2002) as well as the authors’ fitted consumption profile (i.e. the standard model) and the fitted consumption of the news-utility model using the authors’ baseline estimation results, which are displayed in Table 7 in Appendix G, as well as $\eta = 1$, $\lambda = 2$, and $\gamma = 0.85$ for the news-utility model. As noted by Gourinchas and Parker (2002), the standard agent’s consumption peaks somewhat too early and increases too steeply with income growth. News utility causes consumption to peak later and to increase less steeply at the beginning of life. In the paper Gourinchas and Parker (2002) display the average empirical consumption profile for the average empirical household size profile, whereas I display the profile for a single household, which emphasizes the differences and thus facilitates the comparison.
I now go on to demonstrate that my estimates are not only consistent with those found in the micro literature but generate the degree of excess smoothness found in macro data. I simulate 200 consumption and income data points of 1000 individuals to then aggregate their consumption and income and run the regression

$$\Delta \log(\bar{C}_{t+1}) = \alpha + \beta_1 \Delta \log(\bar{Y}_{t+1}) + \beta_2 \Delta \log(\bar{Y}_t) + \epsilon_{t+1}$$

following Campbell and Deaton (1989).\(^{52}\) The results are displayed in Table 2. In the news-utility model typically range between 0.7 and 0.8 (e.g., Laibson, Repetto, and Tobacman (2012)). Thus, the experimental and field evidence on peoples’ attitudes towards intertemporal consumption trade-offs dictates a choice of $b = \gamma \approx 0.7$ when $\beta \approx 1$, which is roughly in line with my estimate. In Table 3, it can be seen that the news-utility agent’s risk attitudes take reasonable values for small, medium, and large stakes. The habit-formation agent is risk neutral for small and medium stakes and somewhat more risk averse for large stakes than the standard agent, who only exhibits reasonable risk attitudes for very large stakes.

\(^{50}\)In a canonical asset-pricing model, Pagel (2012c) demonstrates that news-utility preferences constitute an additional step towards resolving the equity-premium puzzle, as they match the historical level and the variation of the equity premium while simultaneously implying plausible attitudes towards small and large wealth bets.

\(^{51}\)For illustration, I borrow a concrete example from Kahneman, Knetsch, and Thaler (1990), in which the authors distribute a good (mugs or pens) to half of their subjects and ask those who received the good about their willingness to pay (WTP) if they traded the good. The news-utility agent will only experience prospective gain-loss utility over the gamble’s place. The news-utility agent’s contemporaneous gain-loss utility generates reasonable attitudes towards small and large gambles over immediate consumption. Moreover, $\eta \approx 1$ and $\lambda \approx 2.5$ are consistent with the laboratory evidence on loss aversion over immediate consumption, i.e., the endowment effect literature.\(^{51}\) In contrast, since I assume linear utility over immediate consumption, the standard and habit-formation agents are risk neutral. Second, I elicit the agents’ risk attitudes by assuming that each of them is presented the gamble after all consumption in the current period has taken place. I obtain a similar result for the pen experiment.

\(^{52}\)I simulate data for each individual at normalized wealth level $\frac{\Delta Y_t}{Y_0} = 1$ and date $t = 50$. The regression results are

31
utility model, I obtain a coefficient $\hat{\beta}_2 \approx 0.27$ and the excess smoothness ratio, i.e., $\frac{\sigma(\Delta \log(C_t))}{\sigma(\Delta \log(Y_t))}$ as defined in Deaton (1986), is 0.74, whereas in the standard model, I obtain $\hat{\beta}_2^s \approx 0.01$ and 0.95. Regressing consumption growth on lagged labor income growth in aggregate data, I obtain an OLS estimate for $\hat{\beta}_2$ of approximately 0.23 and an excess-smoothness ratio of approximately 0.68. Unsurprisingly, temptation disutility does not generate excess smoothness and sensitivity, while habit formation does. However, habit formation appears to generate too little excess smoothness and too much excess sensitivity and has unrealistic implications for the life-cycle consumption profile, which I explored in the previous section. I conclude that the estimates obtained from CEX consumption data simultaneously match the degree of excess smoothness and sensitivity found in aggregate data.

6 Extensions

In the following, I briefly outline four extensions of the basic life-cycle model that I have developed separately.

As a first extension, I introduce both illiquid savings and credit-card borrowing to demonstrate that the beliefs-based time inconsistency generates simultaneous demand for illiquid retirement savings and excessive credit-card borrowing. I assume that the agent can borrow against his illiquid savings up to his natural borrowing constraint, which is determined by the discounted value of his accumulated illiquid savings. I again find that only the news-utility model is able to robustly generate the collection of life-cycle consumption facts. My findings differ from those of Laibson, similar for different wealth levels and time horizons.

The aggregate regression results of 1000 individuals with $N = 200$ simulated data points are as follows:

<table>
<thead>
<tr>
<th>Model</th>
<th>news-utility habit standard hyperbolic tempted</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>0.67</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.27</td>
</tr>
<tr>
<td>$\beta_1^h$</td>
<td>0.69</td>
</tr>
<tr>
<td>$\beta_2^h$</td>
<td>0.38</td>
</tr>
<tr>
<td>$\beta_1^s$</td>
<td>0.93</td>
</tr>
<tr>
<td>$\beta_2^s$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\beta_1^{td}$</td>
<td>0.94</td>
</tr>
<tr>
<td>$\beta_2^{td}$</td>
<td>0.01</td>
</tr>
<tr>
<td>$t$-statistic</td>
<td>86.7</td>
</tr>
<tr>
<td>$\text{e-s ratio}$</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Each entry in the table is an average of 1000 simulations with $N = 200$ simulated data points.
Repetto, and Tobacman (2012) because I do not assume the existence of non-natural borrowing constraints. Absent such constraints, only those hyperbolic agents at the margin of zero liquid asset holdings would delay consumption adjustments to shocks or tolerate a drop in consumption at retirement. In a model with illiquid savings, any agent with a time-inconsistency problem, i.e., the news-utility, hyperbolic, or tempted agent, will use illiquid savings to make wealth unavailable in the future and thus reduce future consumption. This unavailability of wealth implies that the future agent will exhibit a high marginal propensity to consume out of permanent as well as transitory income shocks. In contrast, the marginal propensity to consume out of transitory income shocks is close to zero in a model without illiquid savings or absent a time-inconsistency problem. Thus, a high marginal propensity to consume out of transitory income shocks can be interpreted as excess sensitivity in consumption rather than a delayed response to income shocks (Laibson (1997) and Laibson, Repetto, and Tobacman (2012) among others).

As a second extension, I let the agent endogenously determine his work hours in response to fluctuations in wages. In the event of an adverse shock, he can maintain high consumption by working more instead of consuming his savings. Thus, if the agent’s labor supply is relatively elastic, his consumption becomes more excessively smooth and less excessively sensitive.

As a third extension, I allow the agent to invest in a risky asset in addition to his risk-free asset. I obtain four main implications for portfolio choice. First, the agent chooses a low portfolio share or does not participate in the stock market, as he is first-order risk averse even in the presence of labor income.\textsuperscript{56} Second, his optimal portfolio share decreases in the return realization. In the event of a good return realization, the agent chooses a lower portfolio share to realize the good news about future consumption and play safe. Third, the agent exhibits a time-inconsistency for risk. Taking his beliefs as given, the agent is inclined to opt for a higher portfolio share to enjoy the prospect of high future consumption, as he resides on a low-risk path. Fourth, the agent can diversify across time because the expected loss of his investment increases with the square root of his investment horizon whereas his expected return increases linearly. All of these predictions smooth the agent’s risky asset holdings relative to the standard model. Thus, I obtain a novel prediction of stickiness in portfolio choice, which has been observed in household portfolio data by Calvet, Campbell, and Sodini (2009a) or Brunnermeier and Nagel (2008).

As a fourth extension, I assume that the agent receives a large income shocks every couple periods but is subject to merely discrete or small income uncertainty in in-between periods. As I have shown in Section 4.4, in the presence of sufficiently small income uncertainty, the agent will choose a flat consumption level independent of the realization of the income shock. And whenever he is able to discretize his consumption, he overconsumes less than in periods in which he is subject to large income uncertainty that makes flat consumption non-credible. This model extension allows

\textsuperscript{56}The result about first-order risk aversion in the presence of background risk stands in contrast to earlier analyzes, such as Barberis, Huang, and Thaler (2006) and Koszegi and Rabin (2007, 2009). Barberis, Huang, and Thaler (2006) consider utility specifications that exhibit first-order risk aversion at one point. Background risk takes the agent away from this point and he becomes second-order risk averse with respect to additional risk. However, the reference point is stochastic in this paper’s model, so that it exhibits first-order risk aversion over the entire support of background risk. Koszegi and Rabin (2007, 2009) consider situations in which background risk is large and utility potentially linear and find that, in the limit, the agent becomes second-order risk averse. However, labor income risk is not large relative to stock market risk in a life-cycle portfolio framework and the agent’s utility function is unlikely to be linear in a model that is calibrated to realistic labor income and stock-market risk at an annual horizon.
to relax an important calibrational degree of freedom associated with the preferences, that is, the period’s length. Moreover, in a setting with merely discrete uncertainty, I reobtain a result first emphasized by Koszegi and Rabin (2009): the agent may consume entire small windfall gains but delay entire small windfall losses.

7 Conclusion

This paper demonstrates that expectations-based reference-dependent preferences can not only explain micro evidence, such as the endowment effect or cab-driver labor supply, but also offer a unified explanation for major life-cycle consumption facts. Excess smoothness and sensitivity in consumption, two widely analyzed macro consumption puzzles, are explained by loss aversion, a robust risk preference analyzed in experimental research and a popular explanation for the equity premium puzzle. Intuitively, the agent wants to allow his expectations-based reference point to decrease or increase prior to adjusting consumption. Moreover, a hump-shaped consumption profile and a drop in consumption at retirement are explained by the interplay of news-utility risk and time preferences. A hump-shaped consumption profile results from the net of two preference features. The news-utility agent’s consumption path is steeper at the beginning of life because loss aversion generates an additional precautionary-savings motive, which accumulates more rapidly than the standard precautionary-savings motive in the agent’s horizon. However, the news-utility agent’s consumption path declines toward the end of life because the expectations-based reference point introduces a time-inconsistency problem: expected utility is higher in an optimal pre-committed equilibrium in which the agent simultaneously optimizes over consumption and beliefs. The pre-committed equilibrium is non-credible, however, because the agent overconsumes once he wakes up and takes his beliefs as given. Once the agent retires, however, time-inconsistent overconsumption is associated with a certain loss in future consumption. Thus, the agent is suddenly able to behave himself, and his consumption drops at retirement. I explore the intuition for the model’s results in depth by solving an exponential-utility model in closed form. Moreover, assuming power-utility as standard in the literature, I structurally estimate the preference parameters and obtain estimates that are in line with the existing micro evidence and generate the degree of excess smoothness found in aggregate data.

In the future, I wish to further explore expectations-based reference dependence as a potential micro foundation for behavioral biases that have been widely documented. For instance, all of my life-cycle results support the notion that fluctuations in beliefs about consumption are painful. If people have some discretion in choosing how much information to gather, they might choose to “stick their head into the sand” occasionally to avoid fluctuations in beliefs that are painful on average; i.e., people are rationally inattentive. For instance, a long-term investor might choose to not check on his portfolio, particularly when he suspects that it might have decreased in value; this behavior has been termed the Ostrich effect. Similarly, a CEO might choose to not evaluate a project when he suspects that it is performing poorly. An outsider, who acquires all information he does not have a stake in, will perceive the investor’s or CEO’s behavior as overconfident and extrapolative because their expectations are based on an overly favorable and outdated information
set whenever they have received adverse but only incomplete information.
Part II

Expectations-Based Reference-Dependent Preferences and Asset Pricing

8 Introduction

Several leading asset-pricing models assume reference-dependent preferences that evaluate consumption relative to a reference point. Campbell and Cochrane (1999) assume habit-formation preferences, and Benartzi and Thaler (1995), Barberis, Huang, and Santos (2001), and Yogo (2008) use prospect-theory preferences. Each of these models assumes that the reference point is backward-looking and formalizes it in specific ways. Moreover, the prospect-theory models specify utility directly over financial wealth instead of consumption, an assumption known as narrow framing. Because the reference point and its framing can be seen as degrees of freedom associated with prospect theory, a model of reference dependence that is based on consumption and offers an endogenized reference-point specification has been developed by Koszegi and Rabin (2006, 2007, 2009). Here, the reference point corresponds to the agent’s expectations about consumption, with respect to which he is loss averse as in classical prospect theory. Since their introduction, these generally-applicable preferences have shown to explain behavioral and experimental evidence in many domains.

This paper incorporates expectations-based reference-dependent preferences into an otherwise standard Lucas-tree model to analyze their asset-pricing implications. Most importantly, expectations-based loss aversion implies a first-order shift and variation in the consumption-wealth ratio; the latter of which is a distinct prediction in the prospect-theory asset-pricing literature. As a result, the model matches historical levels of the equity premium, the equity premium’s volatility, and the degree of predictability in returns. These implications do not require me to assume a separate process for dividends. Moreover, I show that the preferences imply plausible risk attitudes towards small, medium, and large consumption and wealth gambles and thus make another step toward simultaneously explaining attitudes towards gambles and matching asset-pricing moments. This key contribution to resolving the equity-premium puzzle has been first made by Barberis and Huang (2008, 2009), who propose a preference specification that depends on both consumption and the outcome of a narrowly-framed gamble, for instance, the stock market. I contribute to this literature by assuming a preference specification that has shown to explain microeconomic evidence in domains other than monetary gambles and that is based on consumption, which relaxes the framing assumptions to some extent and implies aversion to both consumption and wealth bets.

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57 Habit formation (Abel (1990)) is a preference theory in which the utility function depends on the change in consumption rather than the level of consumption. Prospect theory (Kahneman and Tversky (1979)) is a behavioral theory aimed at describing risk preferences elicited in experiments. This theory says that people care about gains and losses relative to a reference point, where small losses hurt more than equal-sized gains give pleasure, i.e., people are loss averse.

58 Narrow framing refers to the phenomenon that people appear to evaluate an offered gamble in isolation, rather than mixing it with existing risk and considering its implications for consumption instead of financial wealth.

59 A separate dividend process is typically assumed to reduce the contemporaneous correlation of consumption and returns; this, however, is not necessary in my basic model, which matches the contemporaneous correlation reasonably well.
Expectations-based reference-dependent preferences consist of two components. “Consumption utility” is determined by consumption and corresponds to the standard model of utility. Contemporaneous and prospective “gain-loss utility” is determined by a comparison of current and future consumption with the reference point and corresponds to the prospect-theory model of utility. The latter component incorporates loss aversion; small losses are more painful than equal-sized gains are pleasurable. The reference point is stochastic and corresponds to the agent’s fully probabilistic rational beliefs about current and future consumption formed in the previous period. Then, the agent compares consumption utility for each possible outcome under his updated beliefs with consumption utility for each possible outcome under his prior beliefs, and experiences a corresponding sensation of gain or loss. Accordingly, the agent derives gain-loss utility from unexpected changes in present consumption and revisions in expectations over future consumption; therefore, gain-loss utility can be interpreted as utility over good and bad news.

This paper incorporates such “news-utility” preferences into an otherwise standard consumption-based asset-pricing model and solves for the rational-expectations equilibrium in closed form. The model environment is a simple endowment economy with log-normal consumption growth in the spirit of Lucas (1979). The Mehra and Prescott (1985) model – which shows that constant relative risk aversion preferences are inconsistent with basic financial market moments – is preserved as a special case.

As a stepping stone to describing the model’s asset-pricing implications, I first explain two predictions about the model’s consumption-wealth ratio. First, the consumption-wealth ratio is shifted down relative to the standard model. Because the agent is loss averse, he anticipates uncertain fluctuations in gain-loss utility that are painful on average. But, these fluctuations are less painful on the less steep part of the utility curve, which introduces an additional precautionary-savings motive. Second, the consumption-wealth ratio varies, in contrast to the standard model and despite the i.i.d. environment. Because the agent is loss averse relative to his expectations, he finds unexpected reductions in consumption more painful than expected reductions in consumption; hence, the agent wants to postpone unexpected reductions in consumption until his expectations have adjusted downwards. More precisely, reducing future consumption automatically decreases the future reference point, whereas the present reference point is fixed. Consequently, reductions in future consumption are relatively less painful than reductions in present consumption. Finally, these two effects on the consumption-wealth ratio are first order as they depend on loss aversion.

These findings drive the model’s asset-pricing predictions. First, the shift of the consumption-wealth ratio is reflected in an increased mean equity premium. Because the agent is loss averse, he requires a high compensation for the painful fluctuations in consumption associated with uncertainty. Second, the variation in the consumption-wealth ratio is reflected in variation of expected returns. In bad times, the agent desires to consume more and save less. In general equilibrium, this desire increases the consumption-wealth ratio, decreases the price-consumption ratio, and thus increases expected returns. Intuitively, in bad times, the agent will stick with his low consumption only if expected returns are high and make saving sufficiently valuable. Accordingly, the model generates predictability: In bad times, a high consumption-wealth ratio predicts high future returns. Because high expected returns have a higher variance, which increases the quantity of risk, expected excess returns are higher too. Thus, excess returns are predictable too.

Koszegi and Rabin (2009) anticipate the precautionary-savings result in a two-period, two-outcome problem.
I calibrate the news-utility preference parameters in line with microeconomic evidence and show that this calibration generates realistic attitudes towards small, medium, and large wealth bets. Moreover, this calibration generates a log equity premium of approximately six percent with a standard deviation of nineteen percent and thus matches historical stock market data, even though consumption equals dividends in the basic Lucas-tree model.\(^{61}\) Moreover, I find variation in the consumption-wealth ratio around three percent and \(R^2\)s in the predictability regressions of approximately ten percent. These values match the empirical findings of Lettau and Ludvigson (2001), who document the medium-term predictive power of the consumption-wealth ratio.\(^{62}\) I show that such strong predictive power of the consumption-wealth ratio for the return and excess return on the aggregate consumption claim is not generated by other leading asset-pricing models.

A counterfactual prediction of the model is strong variation in the risk-free rate; this is also commonly predicted by habit-formation models but not borne out in the data. In the event of adverse shock realizations, the agent dislikes immediate reductions in consumption and is unwilling to substitute intertemporally, which increases both the expected risky and risk-free rate of return. Although not reflected in aggregate data, this underlying time-variation in substitution motives may not be implausible in practice. Indeed, because people are sometimes unwilling to substitute intertemporally, they use credit cards and payday loans, thus borrowing at high interest rates. The intent of this paper is not to change the evidence-based utility function; rather, I take the variation in substitution motives seriously and explore three model-environment extensions in which the strong intertemporal substitution effects on the risk-free rate are partly offset by other forces.

First, I assume variation in expected consumption growth, as in Bansal and Yaron (2004). Second, I assume variation in consumption growth volatility, i.e., heteroskedasticity in the consumption process, as in Campbell and Cochrane (1999). Third, I add disaster risk to the consumption process, so that there is a small probability that the agent suffers a large loss in consumption, as in Barro (2006, 2009). I find that news-utility preferences amplify disaster risk, because they feature “left-skewness aversion”, as prospect-theory preferences that assume overweighting of small probabilities. The addition of heteroskedasticity or disaster risk introduces variation in the strength of the precautionary savings motive, which partly offsets the effects of the variation in substitution motives on the risk-free rate, adds variation in the price of risk, and generates long-horizon predictability.

Last, I quickly describe the model’s welfare implications. News utility increases the costs of business cycle fluctuations, in the spirit of Lucas (1978), to realistic levels. Moreover, the first welfare theorem does not hold, because the preferences are subject to a beliefs-based time inconsistency.\(^{63}\)

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\(^{61}\) The model’s predicted equity premium and volatility are increasing in the simulation frequency, which thus constitutes a calibrational degree of freedom. Given the calibration of preference parameters, I choose a one-and-a-half month frequency that matches both the historical equity premium and its volatility. At an annual frequency, which has been argued for by Benartzi and Thaler (1995) and Barberis, Huang, and Santos (2001), the model requires a coefficient of risk aversion around eight to match the historical risk-return tradeoff.

\(^{62}\) Furthermore, Lustig, Nieuwerburgh, and Verdelhan (forthc) and Hirshleifer and Yu (2011) document the volatility of the consumption-wealth ratio and the return on the aggregate consumption claim.

\(^{63}\) Lacking an appropriate commitment device, the agent optimizes in each period, taking his beliefs as given. Thus, he is inclined to positively surprise himself with extra consumption in each period. Consequently, he is forced to choose a sub-optimal consumption path that differs from the expected-utility-maximizing path on which he jointly
After a literature review, I present the preferences, the model environment, and the Markovian rational-expectations equilibrium in Sections 10.1 and 10.2. Then, in Section 10.3, I explain the model’s predictions about the consumption-wealth ratio. In Section 11.1, I discuss the model’s asset-pricing implications and calibrate the model to gauge its quantitative implications in Section 11.2. In Section 12, I extend the model to allow for time-variant expected consumption growth, time-variant volatility, and disaster risk. Section 13 explains the model’s implications for welfare. Finally, Section 14 concludes.

9 Comparison to the Literature

In recent years, loss aversion has become a widely-accepted explanation for the equity premium puzzle. I advance this literature by showing that most of its results carry over to a new preference specification, which has been used in many contexts to explain behavioral and experimental evidence. The preferences aim to resolve several degrees of freedom associated with prospect theory. In particular, they are based on consumption, the reference point is fully endogenous, and tight ranges exist for all preference parameters. However, as noted in the introduction, in dynamic models a different degree of freedom emerges, one that did not receive much attention in static applications, namely the length of each time period. Simulating the model at higher frequencies increases the equity premium because the agent is loss averse or first-order risk averse. First-order risk aversion implies time diversification, i.e., the investment is preferred if his horizon is increased. More specifically, as a first-order effect, the investment’s risk increases with the square root of the investment’s horizon while its return increases linearly, which makes the investment overall more favorable. I calibrate the model in line with microeconomic evidence and then choose a one-and-a-half month frequency that matches both the equity premium and the equity premium’s volatility, i.e., the historical risk-return trade off. Moreover, I show that the preferences are tractable in a multi-period, continuous-outcome framework; this is not readily apparent given their high level of complexity.

While other models are equally able to match asset-pricing moments, the preferences I assume simultaneously explain behavior observed in microeconomic studies. Moreover, I show that the preference parameters induce realistic attitudes towards small, medium, and large wealth bets, which are not well explained by many other preference specifications. Therefore, I take optimizes over consumption and beliefs.

64Heidhues and Koszegi (2008, 2014), Herweg and Mierendorff (2012), and Rosato (2012) explore the implications for consumer pricing, which are tested by Karle, Kirchsteiger, and Peitz (2011), Herweg, Müller, and Weinschenk (2010) do so for principal-agent contracts, and Eisenhuth (2012) does so for mechanism design. An incomplete list of papers providing direct evidence for Koszegi and Rabin (2006, 2007) preferences is Sprenger (2010) on the implications of stochastic reference points, Abeler, Falk, Goette, and Huffman (2012) on labor supply, Gill and Prowse (2012) on real-effort tournaments, Meng (2013) on the disposition effect, and Ericson and Fuster (2011) on the endowment effect. 65Suggestive evidence is provided by Crawford and Meng (2011) on labor supply, Pope and Schweitzer (2011) on golf players’ performance, and Sydnor (2010) on deductible choice. Moreover, the numerous conflicting papers on the endowment effect can be reconciled with the notion of expectations determining the reference point. All of these papers consider the static preferences, but as the dynamic preferences of Koszegi and Rabin (2009) are a straightforward extension, the evidence is equally valid for the dynamic preferences. Moreover, the notion that agents are loss averse with respect to news about future consumption is indirectly supported by all experiments, which use monetary payoffs because these concern future consumption.

65Barberis and Huang (2008, 2009) were the first to show that the asset-pricing theories based on prospect theory
a step forward in developing a framework that can match both macroeconomic and microeconomic behavior. This improved micro foundation has desirable implications, namely variation in the consumption-wealth ratio, expected returns, and predictability, which matches the evidence in Lettau and Ludvigson (2001) better than those of the standard, habit-formation, or long-run risk models. But, the well-known problem that the risk-free rate responds strongly to intertemporal smoothing incentives is not resolved. Time-variant consumption growth, heteroskedasticity, or disaster risk is needed to offset some of the effects on the risk-free rate.

The pioneering prospect-theory asset-pricing papers, Barberis, Huang, and Santos (2001) and Benartzi and Thaler (1995), specify gain-loss utility directly over fluctuations in financial wealth. In so doing, the authors make an assumption about narrow framing. The agent narrowly frames the stock market, because he experiences gain-loss utility directly over financial wealth. In contrast, the news-utility agent experiences gain-loss utility over the implications of his financial wealth for contemporaneous and future consumption. The news-utility model yields a high equity premium without narrow framing, because the agent experiences gain-loss utility over fluctuations in contemporaneous consumption and the entire stream of future consumption, which makes uncertainty sufficiently painful. However, the news-utility model requires a higher frequency or higher coefficient of risk aversion than Barberis, Huang, and Santos (2001) and Benartzi and Thaler (1995).

Yogo (2008) argues also that fluctuations in consumption rather than financial wealth are the relevant measure of risk. The author’s preferences are a mixture of habit formation and prospect theory and yield a high equity premium; variation in the risk-free rate is mitigated via persistence in the habit process. The main difference with respect to Koszegi and Rabin (2009) preferences is that the reference point is backward-looking. In contrast, Andries (2011) incorporates loss aversion into a consumption-based asset pricing model and explains positive skewness premia and a flat security market line. The agent’s value function features a kink at the expected value of consumption, which nicely captures forward-looking reference dependence. However, because the agent exhibits reference-dependent preferences with respect to his future value rather than present consumption, the underlying preference mechanisms and predictions are very different from those I obtain.

Campbell and Cochrane (1999) show that habit formation matches a range of asset-pricing moments. Moreover, one of this paper’s main predictions, the variation in the agent’s willingness to substitute intertemporally, has also been emphasized by Campbell and Cochrane (1999). However, these authors exactly offset the variation in intertemporal substitution motives by a habit process that features variation in the agent’s precautionary-savings motive. Furthermore, because the agent’s habit increases the curvature of the value function, the agent’s effective risk aversion is high and becomes the main variability-driving mechanism. In Barberis, Huang, and Santos (2001), variation in the coefficient of loss aversion introduces predictability, whereas the additively separable gain-loss component over financial wealth yields a constant consumption-wealth ratio and risk-free rate. In the news-utility model, effective risk aversion is constant and equals the coefficient of relative risk aversion. The model retains the power utility property that the curvature of the

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imply plausible attitudes towards small and large wealth bets and thus make an additional step towards resolving the equity premium puzzle. However, standard, habit-formation, or long-run risk preferences do not simultaneously match risk attitudes towards small and large wealth bets, because the agent is second-order risk averse. Similarly, the disappointment-aversion models do not robustly match such risk attitudes, because the agent is not necessarily “at the kink.”
value function is solely determined by the coefficient of relative risk aversion, as gain-loss utility is proportional to consumption utility.

Routledge and Zin (2010) assume generalized disappointment-aversion preferences and show that these are consistent with basic financial market moments. The model has been extended to long-run risk by Bonomo, Garcia, Meddahi, and Tedongap (2010). However, these models rely on high risk aversion in low states of the world when the agent is likely to be disappointed, as habit-formation preferences also do.  

Campanale, Castro, and Clementi (2010) assume disappointment-aversion preferences in a production economy. In this model the excessive volatility of the risk-free rate can be reduced by assuming a high intertemporal elasticity of substitution. However, the variation in returns is acyclical by construction, which rules out predictability.

10 The Model

10.1 Expectations-based reference-dependent preferences

I assume expectations-based reference-dependent preferences, as specified in Koszegi and Rabin (2009). Instantaneous utility in each period is the sum of consumption utility and gain-loss utility. The latter component consists of “contemporaneous” gain-loss utility about current consumption and “prospective” gain-loss utility about the entire stream of future consumption. Thus, total instantaneous utility in period $t$ is given by

$$U_t = u(C_t) + n(C_t, F_{t-1}^C) + \gamma \sum_{\tau=1}^{\infty} \beta^\tau n(F_{t+\tau}^{t-1}).$$  \hspace{1cm} (17)

The first term on the right-hand side of equation (17) corresponds to consumption utility in period $t$, which is a power-utility function $u(c) = \frac{c^{1-\theta}}{1-\theta}$. The remaining terms are defined over both consumption and the agent’s “beliefs” about consumption, which I explicitly define below. Throughout the paper, I assume rational expectations such that the agent’s beliefs about any of the model’s variables equal the objective probabilities determined by the economic environment.

**Definition 9.** Let $I_t$ denote the agent’s information set in some period $t \leq t + \tau$. Then, the agent’s probabilistic beliefs about consumption in period $t + \tau$, conditional on period $t$ information, are denoted by $F_{t, t+\tau}^t(c) = Pr(C_{t+\tau} < c | I_t)$ and $F_{t+\tau}^{t+\tau}$ is degenerate.

To understand the remaining terms in equation (17), first note that the reference point in period $t$ is the fully probabilistic beliefs about consumption in period $t$ and all future periods $t + \tau$, given the information available in period $t - 1$. According to Definition 9, the agent’s beliefs formed in period $t - 1$ about period $t + \tau$ consumption are denoted by $F_{t+\tau}^{t-1}$. Thus, the second term in equation

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66 Strong variation in effective risk aversion has trouble matching the evidence on risk attitudes towards wealth bets and is contradicted by household-level data on portfolio choice (Brunnermeier and Nagel (2008)).

67 Epstein and Zin (1989) preferences are able to rationalize the equity premium with the addition of long-run risk or heterogeneous agents as shown by Bansal and Yaron (2004). Epstein and Zin (1989) preferences feature a constant elasticity of intertemporal substitution that can be chosen as an additional parameter in the model. A stark difference between this approach and my model is that the elasticity of intertemporal substitution is typically chosen to be above one to match financial market moments, whereas the asset-pricing implications of Koszegi and Rabin (2009) preferences and other micro evidence suggest a value below one.
The preferences are referred to as "news utility". Consumption in period $t$ is gain-loss utility, so that the agent puts a weight $\gamma \beta$ on prospective gain-loss utility exponentially by $\beta$, as assumed in Koszegi and Rabin (2006, 2007). Both contemporaneous and prospective gain-loss utility correspond to an outcome-wise comparison of Koszegi and Rabin (2006, 2007) has been generalized to an ordered comparison in Koszegi and Rabin (2009), because the agent would otherwise experience gain-loss disutility over future uncertainty even if no update in information takes place. I circumvent this problem by explicitly noting that prior and new beliefs about consumption are correlated, i.e., I generalize the gain-loss formula of Koszegi and Rabin (2006, 2007) even if no update in information takes place. I circumvent this problem by explicitly noting that prior and new beliefs about period $t$ are correlated distribution functions because future uncertainty is contained in both $F_{C_t}^\tau$ and $F_{C_t+\tau}^\tau$. Because both contemporaneous and prospective gain-loss utility are experienced over news, the ordered comparison yields qualitatively and quantitatively similar results but the model’s solution is not as tractable.

$$n(C_t, F_{C_t}^t) = \int_0^\infty \mu(u(C_t) - u(c))dF_{C_t}^t(c)$$

$$= \eta \int_0^{C_t} (u(C_t) - u(c))dF_{C_t}^t(c) + \eta \lambda \int_{C_t}^\infty (u(C_t) - u(c))dF_{C_t}^t(c). \quad \text{(18)}$$

The third term on the right-hand side of equation (17), $\gamma \sum_{\tau=1}^\infty \beta^\tau n(F_{C_t+\tau}^{t+1-})$, corresponds to prospective gain-loss utility in period $t$ over the entire stream of future consumption. Prospective gain-loss utility about period $t + \tau$ consumption, $n(F_{C_t+\tau}^{t+1-})$, depends on $F_{C_t+\tau}^{t+1-}$, the agent’s beliefs with which he entered the period, and on $F_{C_t+\tau}^\tau$, the agent’s updated beliefs about period $t + \tau$ consumption. $F_{C_t+\tau}^{t+1-}$ and $F_{C_t+\tau}^\tau$ are correlated distribution functions because future uncertainty is contained in both prior and updated beliefs about $C_t+\tau$. Thus, there exists a joint distribution, which I denote by $F_{C_t+\tau}^{t+1-} \neq F_{C_t+\tau}^\tau F_{C_t+\tau}^{t+1-}$. Because the agent compares his new beliefs with his prior beliefs, he experiences gain-loss utility over “news” about future consumption.

$$n(F_{C_t+\tau}^{t+1-}) = \int_0^\infty \int_0^\infty \mu(u(c) - u(r))dF_{C_t+\tau}^{t+1-}(c, r). \quad \text{(19)}$$

Both contemporaneous and prospective gain-loss utility correspond to an outcome-wise comparison, as assumed in Koszegi and Rabin (2006, 2007). Moreover, the agent discounts prospective gain-loss utility exponentially by $\beta$, the standard agent’s consumption utility discount factor; and prospective gain-loss utility is subject to another discount factor $\gamma$ relative to contemporaneous gain-loss utility, so that the agent puts a weight $\gamma \beta^\tau < 1$ on prospective gain-loss utility about consumption in period $t + \tau$.

Because both contemporaneous and prospective gain-loss utility are experienced over news, the preferences are referred to as “news utility”.

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$^{68}$ The outcome-wise comparison of Koszegi and Rabin (2006, 2007) has been generalized to an ordered comparison in Koszegi and Rabin (2009), because the agent would otherwise experience gain-loss disutility over future uncertainty even if no update in information takes place. I circumvent this problem by explicitly noting that prior and new beliefs about consumption are correlated, i.e., I generalize the gain-loss formula of Koszegi and Rabin (2006, 2007).

$$n(F_c, F_r) = \int_0^\infty \int_0^\infty \mu(u(c) - u(r))dF_r(c)dF_r(c) \quad \text{to} \quad n(F_c, r) = \int_0^\infty \int_0^\infty \mu(u(c) - u(r))dF_{c,r}(c, r).$$

The ordered comparison yields qualitatively and quantitatively similar results but the model’s solution is not as tractable.
10.2 The model environment and equilibrium

I consider a Lucas (1979) tree model in which the sole source of consumption is an everlasting tree that produces \( C_t \) units of consumption each period \( t \). I assume that consumption growth is log-normal, following Mehra and Prescott (1985). Thus, the endowment economy’s exogenous consumption process is given by

\[
\log\left(\frac{C_{t+1}}{C_t}\right) = \mu_c + \varepsilon_{t+1} \quad \text{with} \quad \varepsilon_{t+1} \sim N(0, \sigma_c^2).
\]

The price of the Lucas tree in each period \( t \) is \( P_t \). Moreover, there exists a risk-free asset in zero net supply with return \( R_{t+1}^f \). The period \( t+1 \) return of holding the Lucas tree is then \( R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t} \).

Each period \( t \), the agent faces the price of the Lucas tree \( P_t \) and the risk-free return \( R_{t+1}^f \), and, acting as a price taker, optimally decides how much to consume \( C_t^* \) and how much to invest in the risky asset \( \alpha_t^* \).

Because the agent fully updates his beliefs each period and the consumption process is i.i.d., I look for an equilibrium price and risk-free return process that is “Markovian” in the sense that the price-consumption ratio depends on the current shock only.

**Definition 10.** The price process \( \{P_t\}_{t=0}^\infty \) and risk-free return process \( \{R_{t+1}^f\}_{t=0}^\infty \) are Markovian if, in each period \( t \), the price-consumption ratio \( \frac{P_t}{C_t} \) and the risk-free return \( R_{t+1}^f \) depend only on the realization of the shock \( \varepsilon_t \), such that \( \frac{P_t}{C_t} = p(\varepsilon_t) \) and \( R_{t+1}^f = r(\varepsilon_t) \) with the functions \( p(\cdot) \) and \( r(\cdot) \) being independent of calendar time \( t \) and endowment \( C_t \).

Facing Markovian prices and returns, the agent’s maximization problem in period \( t \) is given by

\[
\max_{C_t} \left\{ u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^\infty \beta^\tau n(F_{C_{t+\tau}}^{t+\tau-1}) + E_t[\sum_{\tau=1}^\infty \beta^\tau U_{t+\tau}] \right\}. \tag{21}
\]

The agent’s wealth in the beginning of period \( t \), \( W_t \), is determined by the portfolio return \( R_{t}^p \), which depends on the risky return realization \( R_t \), the risk-free return \( R_t^f \), and the previous period’s optimal portfolio share \( \alpha_{t-1} \). The budget constraint is

\[
W_t = (W_{t-1} - C_{t-1})R_t^p = (W_{t-1} - C_{t-1})(R_t^f + \alpha_{t-1}(R_t - R_t^f)). \tag{22}
\]

In each period \( t \), the agent optimally decides how much to consume \( C_t^* \), how much to invest \( W_t - C_t^* \), and how much to invest in the risky asset \( \alpha_t^* \). In equilibrium, the price of the tree \( P_t = W_t - C_t \) adjusts so that the single agent in the model always chooses to hold the entire tree, i.e., \( \alpha_t^* = 1 \) for

\[A benefit of the Lucas-tree environment is that the correlation structure of consumption, which is left unspecified in equations (18) and (19), is fully determined by the exogenous market-clearing consumption process, i.e., \( F_{C_{t+\tau}}^t(c) = Pr(C_{t+\tau} < c|I_t) \), \( I_t = \{C_t, P_t, \varepsilon_t\} \), and \( \frac{C_{t+\tau}}{C_t} = e^{\mu_c + \varepsilon_{t+\tau}} \sim N(\mu_c, \sigma_c^2) \) for any \( t \in [0, \infty) \) such that \( F_{C_{t+\tau}}^t(c) = log - N(\mu_c + \tau \sigma_c^2) \) for any \( \tau > 0 \).]
all \( t \), and consume the tree’s entire payoff \( C_t^* = C_t \) for all \( t \) as determined by the endowment economy’s exogenous consumption process (20). In the following, I derive the “Markovian rational-expectations equilibrium” recursively; in the Lucas-tree model, it corresponds to the preferred-personal equilibrium, as defined in Koszegi and Rabin (2006).

**Definition 11.** The Markovian rational-expectations equilibrium consists of a Markovian price process \( \{ P_t = C_t \rho(\varepsilon_t) \}_{t=0}^{\infty} \) and a risk-free return process \( \{ R^f_{t+1} = r(\varepsilon_t) \}_{t=0}^{\infty} \) such that the solution \( \{ C_t^*, \alpha_t^* \}_{t=0}^{\infty} \) of the price-taker’s maximization problem (21) subject to the budget constraint (22) satisfies goods market clearing \( \{ C_t^* = C_t \}_{t=0}^{\infty} \) and asset market clearing \( \{ \alpha_t^* = 1 \}_{t=0}^{\infty} \).

**Proposition 7.** A Markovian rational-expectations equilibrium exists.

This and the following propositions’ proofs can be found in Appendices D.1 to D.5.

The equilibrium has a very simple structure and can be derived in closed form. In each period \( t \), optimal consumption \( C_t^* \) is a fraction of current wealth \( W_t \) such that

\[
\rho_t = \frac{C_t^*}{W_t} = \frac{1}{1 + \frac{Q + \Omega + \gamma Q \Omega + \gamma Q (\eta F(\varepsilon_t) + \eta \lambda (1 - F(\varepsilon_t)))}{1 + \eta F(\varepsilon_t) + \eta \lambda (1 - F(\varepsilon_t))}}. 
\]

(23)

Here, \( F(\cdot) \) denotes the cumulative normal distribution function \( N(0, \sigma_c) \) and \( Q \) and \( \Omega \) are determined by exogenous parameters. Thus, \( \rho_t \) varies with the realization of \( \varepsilon_t \), is i.i.d., independent of calendar time \( t \), and the current endowment \( C_t \). The price-consumption ratio is \( \frac{P_t}{C_t} = \frac{1 - \rho_t}{\rho_t} \). The agent’s value function is proportional to the power utility of wealth \( V_t = u(W_t) \Psi_t \). \( \Psi_t \) varies with the realization of \( \varepsilon_t \), is i.i.d., independent of calendar time \( t \), and the current endowment \( C_t \). I now explain the news-utility agent’s first-order condition in detail to build intuition for \( Q \) and \( \Omega \) and to clarify why and how \( \rho_t \) varies with \( \varepsilon_t \).

### 10.3 Predictions about the consumption-wealth ratio

Before turning to the model’s asset pricing implications, I describe the agent’s first-order condition to provide intuition for two predictions about the agent’s consumption-wealth ratio, which
are formalized in Propositions 8 and 9 and illustrated in Figure 4. Although the first-order condition appears complicated, the terms can be easily understood one component at a time. The agent’s consumption-wealth ratio $\rho_t$, equation (23), results from the model’s first-order condition

$$C_t^{-\theta}(1 + \eta F(\varepsilon_t) + \eta \lambda (1 - F(\varepsilon_t)))$$

contemporaneous gain-loss

$$= \left(\frac{\rho_t}{1 - \rho_t}\right)^{-\theta} (W_t - C_t)^{-\theta} (Q + \Omega + \gamma \Omega Q + \gamma Q (\eta F(\varepsilon_t) + \eta \lambda (1 - F(\varepsilon_t))))$$

prospective gain-loss

$$= -\frac{d\beta_t[u(W_{t+1})W'_{t+1}]}{dC_t}$$

First, for $\eta = 0$, the model collapses to the standard consumption-based asset-pricing model with constant relative risk aversion and log-normal consumption growth studied by Mehra and Prescott (1985) among many others. The first-order condition becomes

$$C_t^{-\theta} = \left(\frac{\rho^s}{1 - \rho^s}\right)^{-\theta} (W_t - C_t)^{-\theta} Q$$

and results in a constant consumption-wealth ratio $\rho^s = \frac{1}{1 + \gamma}$. Let me return to news utility and henceforth assume that $\eta > 0$ and $\lambda > 1$. In the following, I describe the news-utility agent’s first-order condition, equation (24), to show that, in contrast to the standard model, the consumption-wealth ratio is shifted down and is not constant.

The left-hand side of the first-order condition, equation (24), is simply determined by marginal consumption and gain-loss utility over contemporaneous consumption. Marginal gain-loss utility is given by the states that would have promised less consumption $F_t^{-1}(C_t)$, weighted by $\eta$, or more consumption $1 - F_t^{-1}(C_t)$, weighted by $\eta \lambda$, i.e.,

$$\frac{\partial n(C_t, F_t^{-1})}{\partial C_t} = u'(C_t)(\eta F_t^{-1}(C_t) + \eta \lambda (1 - F_t^{-1}(C_t))))$$

A key technical insight here allows me to simplify the marginal gain-loss utility term: In the Lucas-tree model, equilibrium consumption is determined by the realization of the shock $\varepsilon_t$, which allows me to simplify $F_t^{-1}(C_t) = F(\varepsilon_t)$.

Let me turn to the right-hand side of equation (24). The first term represents the marginal value of savings $-\frac{d\beta_t[u(W_{t+1})W'_{t+1}]}{dC_t} = u'(W_t - C_t)(Q + \Omega + \gamma \Omega Q)$ with $Q$ and $\Omega$ determined by exogenous parameters. In the standard model, the marginal value of savings is given by $u'(W_t - C_t)Q$. Thus, $Q$ represents the discounted stream of future consumption utility, and $\Omega$ represents expected gain-loss utility; the marginal value of savings is determined by $Q + \Omega + \gamma \Omega Q$, the sum of expected consumption utility, expected contemporaneous gain-loss utility, and expected prospective gain-loss utility discounted by $\gamma$. Accordingly, if expected gain-loss disutility is positive $\Omega > 0$, then the marginal value of saving increases relative to the standard model. The underlying intuition is that the agent anticipates gain-loss disutility that is proportional to marginal consumption utility. Thus, fluctuations are less painful on the less steep part of the utility curve and the agent has an additional incentive to increase savings. Moreover, it can be shown that the additional precautionary-savings motive is first-order, i.e., $\frac{\partial \Omega}{\partial \sigma_c} \mid_{\sigma_c = 0} > 0$, because it depends on the concavity of the utility curve.
rather than on prudence as in the standard model.

However, if the agent discounts news about the future \( \gamma < 1 \) he has an additional reason to consume more today, because positive news about contemporaneous consumption are overweighted. Thus, the additional precautionary-savings motive results in the consumption-wealth ratio being lower than in the standard model if the agent does not discount future news too highly \( \gamma > \bar{\gamma} \). These ideas are formalized in the following proposition.

**Proposition 8.** If \( \theta > 1 \) and \( \gamma > \bar{\gamma} \) with \( \bar{\gamma} = \frac{\eta \lambda - \frac{\Omega}{\Omega + \eta \lambda}}{\Omega + \eta \lambda} < 1 \) then, for all realizations of \( \epsilon_t \), the consumption-wealth ratio in the news-utility model is lower than in the standard model \( \rho_t < \rho_s \). Moreover, \( \bar{\gamma} \) is decreasing in the news-utility parameters \( \frac{\partial \bar{\gamma}}{\partial \lambda}, \frac{\partial \bar{\gamma}}{\partial \eta} \leq 0 \).\(^{71}\)

This result reflects the finding by Koszegi and Rabin (2009) that news utility introduces an additional first-order precautionary savings motive in a two-period two-outcome model. The finding carries over to my setting for \( \theta > 1 \) only because I consider multiplicative instead of additive shocks. Multiplicative shocks imply that savings increase the absolute value of tomorrow’s wealth bet, which the news-utility agent dislikes. For \( \theta < 1 \), this effect dominates the desire for intertemporal smoothing. For log utility \( \theta = 1 \), the two motives exactly offset each other and \( \Omega = 0 \). Thus, if \( \theta = 1 \) and \( \gamma = 1 \), the news-utility model becomes observationally equivalent to the standard model.\(^ {72}\)

Let me move on to the second part on the right-hand side in the first-order condition (24) that represents marginal prospective gain-loss utility. In the absence of expected gain-loss disutility (\( \Omega = 0 \)) and prospective gain-loss discounting (\( \gamma = 1 \)), marginal contemporaneous and prospective gain-loss utilities would cancel out. Then, I would be back in the standard model with a proportional response of consumption to wealth. However, contemporaneous marginal utility is driven above future marginal utility due to the additional marginal value of savings \( \Omega > 0 \) so that \( Q + \Omega + \gamma \Omega = \gamma Q \). Thus, the consumption-wealth ratio \( \rho_t \) varies with the realization of \( \epsilon_t \).

Moreover, the consumption-wealth ratio is decreasing for \( \theta > 1 \). Because unexpected losses are particularly painful, the agent consumes relatively more of his wealth in the event of an adverse shock. I first outline a simplified intuition: If the agent encounters an adverse shock, decreasing consumption below expectations today is more painful than decreasing consumption tomorrow when the reference point will have decreased. If the agent encounters a positive shock, he experiences less painful gain-loss fluctuations today relative to tomorrow when the reference point will have increased. Thus, the agent wants to delay the consumption response to shocks, which makes the consumption-wealth ratio vary. More formally, in the event of an adverse shock, present marginal gain-loss utility is high relative to future marginal gain-loss utility. Today’s reference point is invariant, whereas tomorrow’s reference point will have adjusted to today’s shock.

\(^{71}\)If \( \theta > 0 \) and \( \frac{\eta - \frac{\Omega}{\Omega + \eta \lambda}}{\Omega + \eta \lambda} < \gamma < \bar{\gamma} \) then \( \rho_t \) and \( \rho_s \) cross at \( \epsilon_t = \bar{\epsilon}_t \) and \( \bar{\epsilon}_t \) is decreasing in the news-utility parameters \( \frac{\partial \bar{\epsilon}_t}{\partial \lambda}, \frac{\partial \bar{\epsilon}_t}{\partial \eta} \leq 0 \).

\(^{72}\)This result is analogous to a result for quasi-hyperbolic discounting obtained by Barro (1999).
Thus, future marginal gain-loss utility is constant whereas present marginal gain-loss is high, and the agent wants to consume relatively more today and relatively less tomorrow.\footnote{This prediction about consumption is different from a result in Koszegi and Rabin (2009) predicting that the agent consumes entire small unexpected gains whereas entire small unexpected losses are delayed. The prediction in Koszegi and Rabin (2009) results from the assumption that $\frac{1}{2} < \gamma < 1$ and the small gain and loss coming unexpectedly such that the agent initially planned a certain consumption path absent any gain-loss utility. The prediction would turn into my result, i.e., the agent consumes relatively less in the event of a good shock and relatively more in the event of a bad shock, if the gain or loss in wealth came expectedly.} The following proposition formalizes this idea.

**Proposition 9.** If $\theta \neq 1$, news utility introduces variation in the consumption-wealth ratio $\frac{\partial \rho_t}{\partial \varepsilon_t} \neq 0$. Moreover, for $\theta > 1$, the consumption-wealth ratio is decreasing $\frac{\partial \rho_t}{\partial \varepsilon_t} < 0$.

The model’s implications are illustrated in Figure 4, which displays the consumption-wealth ratio $\rho_t$ as a function of the shock to consumption growth and contrasts it with the standard agent’s ratio for two levels of $\sigma_c$.\footnote{The calibration is displayed in Table 8 and discussed in Section 11.2.2.} News-utility preferences predict a downward shift and variation in the consumption-wealth ratio. The shape is driven by marginal gain-loss utility, which depends on the shock distribution $\eta F(\varepsilon_t) + \eta \lambda (1 - F(\varepsilon_t)) \in [\eta, \eta \lambda]$. As $\varepsilon_t$ is characterized by a bell-shaped distribution, the variation in the consumption-wealth ratio is bounded. The agent experiences gain-loss utility over all other states he might have been in weighted by their probabilities. For extreme realizations of $\varepsilon_t$, the consumption-wealth ratio approaches a limit because the states near these realizations have very low probabilities. $\rho_t$ and $\rho_s$ are displayed for two levels of $\sigma_c$, which illustrates that, for a small increase in $\sigma_c$, the downward shift in $\rho_t$ is larger than the downward shift in $\rho_s$; this is because the additional precautionary savings motive is a first-order effect, i.e., $\frac{\partial \rho_t}{\partial \sigma_c}|_{\sigma_c \rightarrow 0} > 0$, while the standard precautionary-savings motive is second order.
11 Asset Pricing

Now I turn to the model’s asset-pricing implications. First, I derive the expected risky return, the risk-free return, and the equity premium. Then I illustrate the model’s main asset-pricing predictions, namely variation in expected returns, the equity premium, and predictability. I aim to build intuition for these asset-pricing results by connecting them back to my prior theoretical results about the consumption-wealth ratio. Proposition 10 formalizes the main idea. In Section 11.2, I calibrate the model to gauge its quantitative performance and then compare its predictions to those of other models.

11.1 Predictions about expected returns and the equity premium

The return of holding the entire Lucas tree is $R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t}$. I can rewrite the expected risky return in terms of the consumption-wealth ratio $\rho_t$ and consumption growth $\frac{C_{t+1}}{C_t}$ by taking expectations and noting that $P_t = W_t - C_t = C_t \frac{1 - \rho_t}{\rho_t}$, i.e.,

$$E_t[R_{t+1}] = \frac{\rho_t}{1 - \rho_t} E_t \left[ \frac{1}{\rho_{t+1}} \frac{C_{t+1}}{C_t} \right].$$

(26)
$E_t(\frac{1}{\rho_{t+1}} C_{t+1} - C_t)$ is constant because consumption growth $\frac{C_{t+1}}{C_t} = e^{\mu_t + \epsilon_t}$ and next period’s consumption-wealth ratio $\rho_{t+1}$ are i.i.d., as reported in Definition 10 such that $\frac{\rho_{t+1}}{C_t} = p(\epsilon_{t+1}) = \frac{1-\rho_{t+1}}{\rho_{t+1}}$. However, $E_t[R_{t+1}]$ varies with the consumption-wealth ratio $\rho_t$.

I can rewrite the first-order condition as $1 = E_t[M_{t+1}R_{t+1}]$, which gives rise to the agent’s stochastic discount factor $M_{t+1}$ derived in Appendix D.2. The risk-free return is the inverse of the conditional expectation of the stochastic discount factor

$$R^f_{t+1} = \frac{1}{E_t[M_{t+1}]} = \frac{\rho_t}{1-\rho_t} (Q + \Omega + \gamma \Omega Q) E_t[\beta(\frac{C_{t+1}}{C_t})_{\rho_{t+1}}]^{-\theta \Psi_{t+1}}.$$

$E_t[\beta(\frac{C_{t+1}}{C_t})_{\rho_{t+1}}]^{-\theta \Psi_{t+1}}$ is constant because consumption growth $\frac{C_{t+1}}{C_t} = e^{\mu_t + \epsilon_t}$, the next period’s consumption-wealth ratio $\rho_{t+1}$, and the value function’s proportionality factor $\Psi_{t+1}$ are i.i.d. However, $R^f_{t+1}$ varies with the consumption-wealth ratio $\rho_t$. The equity premium

$$E_t[R_{t+1}] - R^f_{t+1} = -\frac{Cov_t(M_{t+1}, R_{t+1})}{E_t[M_{t+1}]} = -\frac{\sigma_t(M_{t+1}) \sigma_t(M_{t+1}) \sigma_t(R_{t+1})}{\sigma_t(M_{t+1}) \sigma_t(R_{t+1})} \sigma_t(R_{t+1})$$

is characterized by a constant price of risk. The price of risk and the conditional Sharpe ratio $S_t = \frac{E_t[R_{t+1} - R^f_{t+1}]}{\sigma_t(R_{t+1})}$ are constant, because the agent holds the entire stock market and thus faces the same risk each period. However, the quantity of risk $\sigma_t(R_{t+1})$ varies with the consumption-wealth ratio $\rho_t$.

I have shown that the expected risky return, the risk-free return, and the equity premium vary with the consumption-wealth ratio $\rho_t$. The news-utility implications about the location and shape of the consumption-wealth ratio $\rho_t$, which are formalized in Propositions 8 and 9, directly carry over to the expected return, the risk-free return, and the equity premium. The variation in the expected risky return is generated by the general-equilibrium nature of the model and driven by variation in the agent’s willingness to substitute intertemporally, as reflected by variation in $\rho_t$.

In bad states of the world, the agent would like to delay adjustments in consumption to let his reference point adjust. To induce the agent to consume his endowment, the price of the Lucas tree must be low and expected returns have to be high. Thus, despite the i.i.d. environment, the expected risky return varies to make the agent willing to hold the entire tree each period. Moreover, the variation in the consumption-wealth ratio generates return predictability. In particular, the realization of $\epsilon_t$ predicts the one-period ahead return $R_{t+1}$. If $\epsilon_t$ is low then $\rho_t$ the consumption-wealth ratio is high and the one-period ahead return is high; hence, the consumption-wealth ratio positively predicts one-period ahead returns. Moreover, this mechanism generates predictability in excess returns through the consumption-wealth ratio. Bad states predict high future returns, and this implies that the standard deviation of returns is also high and the expected equity premium
varies with \( \varepsilon_t \). Using the same argument as above, the realization of \( \varepsilon_t \) then predicts the one-period ahead excess return \( R_{t+1} - R_{t+1}^f \).

The following proposition formalizes the model’s implications for variation and predictability in returns and the equity premium.

**Proposition 10.** If \( \theta > 1 \), the realization of the shock \( \varepsilon_t \) negatively impacts the expected risky return \( \frac{\partial E_t[R_{t+1}]}{\partial \varepsilon_t} < 0 \), risk-free return \( \frac{\partial R_{t+1}^f}{\partial \varepsilon_t} < 0 \), and equity premium \( \frac{\partial (E_t[R_{t+1}] - R_{t+1}^f)}{\partial \varepsilon_t} < 0 \). This implies predictive power of the period \( t \) consumption-wealth ratio \( \rho_t \) for the period \( t+1 \) return \( R_{t+1} \) and excess return \( E_t[R_{t+1}] - R_{t+1}^f \).

For illustration, Figure 12 in Appendix C compares the annualized news-utility return and equity premium with those of the standard model’s ones under the calibration in Table 8 with \( \lambda = 2.75 \). The expected equity premium amounts to approximately ten percent for low values of \( \varepsilon_t \) and three percent for high values of \( \varepsilon_t \). The high equity premium reflects that the news-utility agent perceives uncertain fluctuations in consumption as being much more painful than the standard agent. The equity premium’s variation stems from variation in the quantity of risk as a result of varying intertemporal smoothing incentives. But, the figure also illustrates how the model fails to predict reality: The risk-free return varies considerably, a phenomenon not observed in aggregate data.\(^76\)

### 11.2 Basic model: Calibration and moments

I now calibrate the model to gauge its quantitative performance. Before assessing the model’s ability to match asset-pricing moments, I illustrate the agent’s risk attitudes towards small and large wealth bets. I show that the news-utility model is able to simultaneously match evidence on small-scale and large-scale risk aversion.

#### 11.2.1 Risk attitudes over small and large stakes.

Before moving on to the model’s asset-pricing moments I illustrate which news-utility parameter values, i.e., \( \eta, \lambda, \) and \( \gamma \), are consistent with existing micro-evidence on risk preferences over small and large stakes and time preferences. I first show that the news-utility model does not generate high equity premia by curving the value function to generate high effective risk aversion. On
the contrary, the news-utility model retains a value function with constant curvature because it is proportional to the power utility of wealth, i.e., $V_t = u(W_t)\Psi_t$ such that $RRA_t = -\frac{W_t V_t''}{V_t} = \theta$.\textsuperscript{77}

In Table 3, I illustrate the risk preferences over gambles of various stakes of the standard, news-utility, habit-formation (Campbell and Cochrane (1999)), and long-run risk (Bansal and Yaron (2004)) agents. In particular, I analyze a range of 50-50 win G or lose L gambles at an initial wealth level of $300,000 in the spirit of Rabin (2001), Chetty and Saez (2007), and Barberis and Huang (2008, 2009). I elicit the agents’ risk attitudes by assuming that each of them is presented with the gamble after the shock to period $t$ consumption growth has been realized and all consumption $C_t$ in period $t$ has taken place. Thus, the news-utility agent will experience merely prospective gain-loss utility rather than contemporaneous gain-loss utility over the gamble’s outcome. In Appendix D.6, I show that the news-utility agent is just indifferent to the gamble if

$$
(Q + \Omega + \gamma Q \bar{\Omega})u(\bar{W}_t) = \gamma(0.5\eta(u(\bar{W}_t + G) - u(\bar{W}_t))Q + \eta \lambda 0.5(u(\bar{W}_t - L) - u(\bar{W}_t))Q) + (Q + \Omega + \gamma Q \bar{\Omega})(0.5u(\bar{W}_t + G) + 0.5u(\bar{W}_t - L)). \quad (29)
$$

The first part on the right-hand side of equation (29) represents prospective gain-loss utility, while the second part represents the same value comparison as done by the standard agent, i.e., $u(\bar{W}_t) \leq 0.5u(\bar{W}_t + G) + 0.5u(\bar{W}_t - L)$. Thus, if $\gamma$ were zero, the news-utility agent’s risk attitudes would be exactly the same as those of the standard agent. Moreover, if $L$ and $G$ are small but $G > L$, this second part will certainly be positive as $u(\cdot)$ is almost linear, but the first part will induce prospect-theory risk preferences over future consumption. Although solely $\lambda$ determines the sign of prospective gain-loss utility, there are restrictions on the other parameters, because the positivity of the second part may dominate the negativity of the first part if $\gamma$ is small. Empirical estimates for the quasi-hyperbolic parameter $\beta$ in the $\beta \delta$-model typically range between 0.7 and 0.8 (e.g., Laibson, Repetto, and Tobacman (2012)). Thus, the experimental and field evidence on agent’s attitudes towards intertemporal consumption tradeoffs dictates a choice of $\gamma \approx 0.8$ when $\beta \approx 1$.

Simultaneously, the model should match risk attitudes towards bets about immediate consumption, which are determined solely by $\eta$ and $\lambda$, because it can be reasonably assumed that utility over immediate consumption is linear. Thus, $\eta = 1$ and $\lambda \approx 2.5$ is dictated by the laboratory evidence on loss aversion over immediate consumption, i.e., the endowment effect literature (Kahneman, Knetsch, and Thaler (1990)).\textsuperscript{78}

\textsuperscript{77}The intertemporal elasticity of substitution is disentangled and exhibits variation. The coefficient of relative risk aversion being disentangled from the intertemporal elasticity of substitution is a feature of a broad range of non-time-separable utility functions, such as habit formation.

\textsuperscript{78}Let me take a concrete example from Kahneman, Knetsch, and Thaler (1990) assuming that utility over mugs, pens, and small amounts of money is linear. Kahneman, Knetsch, and Thaler (1990) hand out mugs to half the subjects and ask those who did not receive one about their willingness to pay and those who received one about their willingness to accept when selling the mug. The authors observe that the median willingness to pay for the mug is $2.75 whereas the willingness to accept is $5.25. Accordingly, I can infer $(1 + \eta)u(mug) = (1 + \eta \lambda)2.25$ and $(1 + \eta \lambda)u(mug) = (1 + \eta)5.25$ which implies that $\lambda \approx 3$ when $\eta \approx 1$. For the pen experiment I also obtain $\lambda \approx 3$. Unfortunately, so far I can only jointly identify $\eta$ and $\lambda$. If the news-utility agent exhibits only gain-loss utility I would obtain $\eta \lambda 2.25 \approx 5.25$ and $\eta 2.25 \approx 2.25$, i.e., $\lambda \approx 2.3$ and $\eta \approx 1$ are both identified. Alternatively, if I assume that the market price for mugs (or pens), which is $6 in the experiment (or $3.75), equals $(1 + \eta)u(mug)$ (or $(1 + \eta)u(pen)$), I can estimate $\eta = 0.74$ and $\lambda = 2.03$ for the mug experiment and $\eta = 1.09$ and $\lambda = 2.1$ for the pen experiment. These
Table 3: **Risk attitudes over small and large wealth bets**

<table>
<thead>
<tr>
<th>Loss (L)</th>
<th>standard</th>
<th>news-utility</th>
<th>habit-formation</th>
<th>long-run risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>17</td>
<td>14</td>
<td>10</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>330</td>
<td>270</td>
<td>200</td>
</tr>
<tr>
<td>1000</td>
<td>1007</td>
<td>1650</td>
<td>1359</td>
<td>1362</td>
</tr>
<tr>
<td>5000</td>
<td>5162</td>
<td>8250</td>
<td>7014</td>
<td>19749</td>
</tr>
<tr>
<td>50000</td>
<td>74999</td>
<td>82500</td>
<td>110717</td>
<td>∞</td>
</tr>
<tr>
<td>100000</td>
<td>299524</td>
<td>165000</td>
<td>6200303</td>
<td>∞</td>
</tr>
</tbody>
</table>

For each loss L the table’s entries show the required gain G to make each agent indifferent between accepting and rejecting a 50-50 gamble win G or lose L at wealth level \( W_t = 300,000 \).

In Table 3, I calculate the required G for each value of L to make each agent just indifferent between accepting or rejecting a 50-50 win G or lose L gamble at wealth level \( W_t = 300,000 \). It can be seen that the news-utility agent’s risk attitudes take reasonable values for small, medium, and large stakes.\(^79\) In contrast, the standard and long-run risk agents are risk neutral for small stakes and almost risk neutral for medium stakes. The habit-formation agent is risk neutral for small stakes, reasonably risk averse for medium stakes, but unreasonably risk averse for large stakes. Campbell and Cochrane (1999) also discuss this finding and indicate that the curvature of the habit-formation agent’s value function is approximately 80 at the steady-state surplus-consumption ratio; thus, the habit-formation agent behaves similarly to a standard agent with \( \theta = 80 \). The long-run risk agent behaves similarly to a standard agent with \( \theta = 10 \), the choice of Bansal and Yaron (2004). Moreover, it can be inferred from this discussion that the disappointment-aversion model (Routledge and Zin (2010)) does not robustly match risk attitudes towards small and large wealth bets, because the agent is not necessarily “at the kink”. The asset-pricing theories based on prospect theory (Barberis, Huang, and Santos (2001); Benartzi and Thaler (1995)) imply plausible attitudes towards small and large wealth bets but not consumption bets and are thus inconsistent with the endowment-effect evidence.

### 11.2.2 Calibration and asset-pricing moments

latter assumptions are reasonable given the induced-market experiments of Kahneman, Knetsch, and Thaler (1990). \( \eta = 1 \) and \( \lambda \approx 2.5 \) thus seem to be reasonable choices and have also been typically used in the literature for the static preferences.

\(^79\)While the news-utility agent’s risk preferences over contemporaneous consumption exactly match the findings of Kahneman, Knetsch, and Thaler (1990), the news-utility agent’s required gain for small gambles about future consumption is somewhat lower than the estimates obtained by Tversky and Kahneman (1992), even though the authors consider monetary gambles and thus future consumption. But, the news-utility model predicts that people consume entire small gains when being surprised by risk (Koszegi and Rabin (2009)). Thus, the contemporaneous consumption results might be applicable even for monetary gambles. Moreover, a paper which explicitly considers gambles over future consumption is Andreoni and Sprenger (2012), who find significantly less small-scale risk aversion towards those gambles. In any case, I do not aim to perfectly match experimental evidence here; rather, I want to demonstrate that the model does make a significant step forward in explaining small-stakes risk aversion.

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Table 8 in Appendix C displays the calibration and the resulting moments of the news-utility and standard models, a condensed version of which is Table 4. I begin with the model environment and the well-known preference parameters $\beta$ and $\theta$. I assume a classical Lucas-tree model in which consumption equals dividends, so that the model environment is fully calibrated by $\mu_c$ and $\sigma_c$. I follow Bansal and Yaron (2004) and choose $\mu_c = 1.8\%$ and $\sigma_c = 2.7\%$ in annualized terms. $\beta$ and $\theta$ are then chosen to roughly match the level of the mean risky return, the mean risk-free return, and the risky return volatility as done by Bansal and Yaron (2004). Following Campbell and Cochrane (1999) and Bansal and Yaron (2004), I simulate the model at a higher frequency and then annualize moments. The news-utility equity premium increases in the model’s frequency, as suggested by Benartzi and Thaler (1995). The news-utility agent dislikes fluctuations in beliefs about future consumption. Observing the return realization and readjusting consumption plans at higher frequency, i.e., monthly instead of annually, makes the Lucas tree a less attractive investment opportunity. Therefore, the required compensation for bearing the risk of the Lucas tree increases.

The simulation frequency thus constitutes a calibrational degree of freedom in the news-utility model. At a monthly frequency, $\theta$ has to be close to one to match the historical equity premium. To have a bit more space in picking $\theta$ and be able to aggregate to a quarterly frequency easily, I choose a one-and-a-half month frequency, $\theta = 2$, and $\beta = 0.98$ to match both the historical equity premium and the equity premium’s volatility. Simulating the model at an annual frequency requires a somewhat higher coefficient of risk aversion $\theta$ and comparably high consumption volatility $\sigma_c$ which are, however, not unusual in the literature. For instance, with $\sigma_c = 3.79\%$, as in Barberis, Huang, and Santos (2001), and $\theta = 10$, as in Bansal and Yaron (2004), the annualized news-utility model would roughly match the historical equity premium and its volatility.

The news-utility parameters are calibrated as standard in the prospect-theory literature: $\eta = 1$ and $\lambda \in [2; 2.6]$ to match the large array of experimental evidence on loss aversion and to induce reasonable risk attitudes over small and large stakes, as can be seen in Table 3. These values have also been used in the existing prospect-theory asset-pricing literature; Benartzi and Thaler (1995) assume a coefficient of loss aversion $\lambda = 2.5$ and Barberis, Huang, and Santos (2001) assume a mean coefficient of loss aversion of approximately 2.25. Moreover, to account for the fact that people are present biased, I assume that the agent discounts prospective news utility and set $\gamma = 0.8$. I argue that the existing experimental literature suggests fairly tight ranges for all the news-utility parameters, $\eta$, $\lambda$, and $\gamma$, as well as the standard preference parameters $\theta$ and $\beta$. Thus, news utility does not allow for large parameter ranges that can be used at my discretion. However, the simulation frequency constitutes a more worrisome degree of freedom, because it has been ignored in static applications of Koszegi and Rabin (2006, 2007) preferences.

As can be seen in Table 4, the model matches the historical mean equity premium, its volatility, and the mean risk-free rate elicited from CRSP return data. The news-utility model generates the historical equity premium volatility, despite the fact that consumption equals dividends, as in the basic Lucas-tree model. Thus, the model matches the historical risk-return trade-off with a Sharpe ratio of approximately 0.35. Unfortunately, the news-utility model completely mispredicts the

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80At an annual frequency the calibration used here, i.e., $\theta = 2$ and $\lambda = 2.6$, would result in an equity premium around 3%.
Table 4: Moments of the basic model

<table>
<thead>
<tr>
<th>moments</th>
<th>standard and news-utility model</th>
<th>data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E[r_t - r_f^t] )</td>
<td>0.42 3.02 4.31 5.77 6.33</td>
<td>( \lambda = 0 ) ( \lambda = 2 ) ( \lambda = 2.3 ) ( \lambda = 2.6 )</td>
</tr>
<tr>
<td>( \sigma(r_t - r_f^t) )</td>
<td>2.72 14.7 19.2 23.6 19.4</td>
<td></td>
</tr>
<tr>
<td>( E[r_f^t] )</td>
<td>5.48 2.15 0.68 -0.92 0.86</td>
<td></td>
</tr>
<tr>
<td>( \sigma(r_f^t) )</td>
<td>0.00 12.1 16.6 21.0 0.97</td>
<td></td>
</tr>
<tr>
<td>( E[c_t - p_t] )</td>
<td>-5.35 -5.57 -5.61 -5.69 -3.4</td>
<td></td>
</tr>
<tr>
<td>( \sigma(c_t - w_t) )</td>
<td>0.00 0.03 0.04 0.053 0.011</td>
<td></td>
</tr>
<tr>
<td>( AR(c_t - w_t) )</td>
<td>1.00 0.01 0.01 0.01 0.79</td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.00 0.12 0.15 0.17 0.09</td>
<td></td>
</tr>
</tbody>
</table>

Value-weighted CRSP returns are displayed annualized and in percentage terms. The quarterly moments for the consumption-wealth ratio and predictability regression 
\( r_{t+1} = \alpha + \beta (c_t - w_t) + \delta r_f^t \) are taken from Lettau and Ludvigson (2001) Table II and III.

The model’s performance regarding other return moments is mixed, as can be seen in Table 8. The model matches the contemporaneous correlation of consumption growth with returns reasonably well but overpredicts the one-period ahead correlation.\(^{81}\) Predicting too-high correlation between returns and consumption growth is a common failure of leading asset-pricing models, as emphasized by Albuquerque, Eichenbaum, and Rebelo (2012) among others. But, because the variation in the consumption-wealth ratio in the news-utility model is a short-run phenomenon, at longer horizons the correlation between consumption growth and asset returns is very low, thus matching the data. In contrast, the variation in the consumption-wealth ratio in the long-run risk model is a long-run phenomenon and thus implies counterfactually high correlations between consumption growth and asset returns at longer horizons. Moreover, the autocorrelation of returns is negative in the model as opposed to around zero in the data.\(^{82}\) Finally, in Table 8, I display the moments for slightly higher values of \( \theta, \gamma, \) and \( \eta \) to give a quantitative idea of the parameters’ implications.

The model’s simulated consumption-wealth ratio reflects the prior theoretical results. First, the consumption-wealth ratio is lower than in the standard model and exhibits variation. As consumption equals dividends in the classical Lucas-tree model and there is no labor income, the values are difficult to compare with the data. However, the corresponding values in Lettau and Ludvigson

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\(^{81}\) Many asset-pricing models overstate the contemporaneous correlation of consumption and returns, which can be reduced by introducing a separate dividend process. As I roughly match this value I conclude that a separate process for dividends is unnecessary in the basic news-utility model although it will reduce the mispredicted one-period ahead correlation.

\(^{82}\) Although, the aggregation to annualized frequency seems to introduce some spurious correlation as can be seen in the standard model’s moments.
(2001) are displayed as an illustration. Both the standard and news-utility model roughly match the level of the consumption-price ratio, but the standard model mispredicts its variation whereas the news-utility model’s predicted variation is roughly in line with the data.\textsuperscript{83} However, the news-utility consumption-wealth ratio is i.i.d. whereas Lettau and Ludvigson (2001) find relatively high persistence.

With respect to the predictability properties of quarterly returns, the model yields $R^2$ values of approximately 10% to 17%. Lettau and Ludvigson (2001) emphasize the medium-run predictive power of the aggregate consumption-wealth ratio. The authors obtain $R^2$ values for quarterly returns of 9% and of 18% for annual excess returns.\textsuperscript{84} As noted by Lustig, Nieuwerburgh, and Verdelhan (fthc) and Hirshleifer and Yu (2011), traditional leading asset-pricing models have difficulty matching the volatility of the consumption-wealth ratio and the return on the consumption claim, because they rely on a volatile dividend process, and the only variation in the consumption-wealth ratio stems from heteroskedasticity in consumption growth. I can confirm this finding; using the return on the consumption claim, the $R^2$ in the habit-formation model of Campbell and Cochrane (1999) is merely 1.6% and the $R^2$ in the long-run risk model of Bansal and Yaron (2004) is just 2.9%.

Moreover, in Figure 13 in Appendix C, I plot the simulated deviations of the news-utility, standard, habit-formation, and long-run risk consumption-wealth ratio and compare these with the annual $\hat{c}a\hat{y}$ data provided by Lettau and Ludvigson (2005). For the habit-formation and long-run risk model, I use the calibration of Campbell and Cochrane (1999) and Bansal and Yaron (2004) to then aggregate the consumption and wealth time series. Moreover, I feed in the deviations in log consumption growth $\Delta c \sim \mu_c$ of the $\hat{c}a\hat{y}$ data. As can be seen, news utility introduces considerably more rapid variation in the consumption-wealth ratio than the standard model or the model augmented with long-run risk, but much less variation than the habit-formation model. While, the long-run risk consumption-wealth ratio appears to be too smooth and habit-formation consumption-wealth ratio too variable, the variation in the news-utility consumption-wealth ratio matches the $\hat{c}a\hat{y}$ data quite well. Although it is disputable to compare the $\hat{c}a\hat{y}$ data to the simulated data of a Lucas-tree model, I conclude that the rapid variation is supported by the data.\textsuperscript{85}

At first blush, the model’s asset pricing implications appear to be mixed. News utility raises the equity premium and its volatility to historical levels even though I omit a separate dividend process. Moreover, the variation in substitution motives generates strong variation in the consumption-wealth ratio and predictability in returns, matching the data better than leading asset-pricing mod-

\textsuperscript{83}Moreover, the consumption-wealth ratio cannot be used to forecast consumption growth, which is in line with the empirical findings in Lettau and Ludvigson (2001).

\textsuperscript{84}Lettau and Ludvigson (2001) report small-sample statistics which might be biased up. If one argues that the $R^2$ of the model is too high, it could be reduced by decreasing $\theta$, $\lambda$, or increasing $\gamma$.

\textsuperscript{85}Greenwood and Shleifer (2014) compare a variety of survey data on stock market expectations with the predicted expected returns of leading asset-pricing models. The authors show that leading asset-pricing models’ implied expected returns do not correlate highly with the survey evidence on expected returns. In particular, the $\hat{c}a\hat{y}$ model of Lettau and Ludvigson (2001) fits the survey data better than the habit-formation and long-run risk models do. I can confirm this finding using the American Association of Individual Investors Sentiment Survey and also find that the news-utility model is more positively correlated with the survey data than the habit-formation, long-run risk, models or the $\hat{c}a\hat{y}$ data. However, this finding should not be overinterpreted as the annual comparison includes the years 1987 to 2001 only.

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els. However, the model predicts excessive volatility in the risk-free rate, which I address in the following section.

12 Extensions

The news-utility model’s most important shortcoming is the large predicted variation in the risk-free rate. Nevertheless, I want to take the predictions of the evidence-based utility specification seriously and believe that people are very unwilling to substitute consumption intertemporally in some states of the world. The most important evidence is credit-card borrowing or pay-day loans. However, there may be forces at work that offset the effects on the aggregate risk-free rate. What would a consumption process look like which features an almost constant risk-free rate? An adverse shock to contemporaneous consumption growth has to be associated with an adverse prediction about future consumption growth to keep the risk-free rate low. Thus, the model’s risk-free rate process will become more smooth if low values of $\varepsilon_t$ are associated with a decrease in $\mu_c$ or an increase in $\sigma_c$. Variation in the agent’s expected consumption growth $\mu_c$ has been exploited by Bansal and Yaron (2004) and termed long-run risk. Variation in the agent’s expected volatility of consumption growth has been exploited by Campbell and Cochrane (1999) and Bansal and Yaron (2004). 86

In Section 12.1, I reverse-engineer variation in expected consumption growth and its volatility to offset the effect of the variation in the agent’s intertemporal smoothing incentives on the risk-free rate. An adverse shock to consumption growth today is then associated with low consumption growth but high volatility in the future. There exists empirical evidence for countercyclical variation in economic uncertainty, or consumption volatility. 87 The empirical evidence on excess sensitivity suggests that there exists positive autocorrelation in consumption growth. However, it turns out that the variation in the agent’s smoothing incentives require variation in the agent’s expected consumption growth that is too large to be consistent with aggregate consumption data, because the variation in consumption volatility appears to be too weak to significantly affect the strong first-order variation in the agent’s risk-free rate. As another alternative, I extend the model to account for time-varying disaster risk to smooth out the risk-free rate in Section 12.2. Time-varying disaster risk is a very powerful device under news-utility preferences because, as I explain below,

86 Campbell and Cochrane (1999) specify heteroskedasticity in consumption growth to make the risk-free rate exactly constant.

87 Since French, Schwert, and Stambaugh (1987) it is well known that volatility of stock returns fluctuates considerably over time. Moreover, Black (1976) was one of the first to document that stock returns are negatively correlated with future volatility, an empirical observation which has been referred to as the leverage or volatility-feedback effect. More recently, Lettau and Ludvigson (2004) document that the countercyclical and highly volatile Sharpe ratio is not replicated by leading consumption-based asset pricing models. The Sharpe ratio becomes both more countercyclical and volatile if low returns imply high expected returns and low volatility, as I assume in the extended model. The authors find that the consumption-wealth ratio predicts stock market volatility and provide evidence for variation in aggregate consumption volatility. Furthermore, Tauchen (2011) connects the negative correlation in stock returns and volatility back to the consumption process underlying a standard Lucas-tree model. Finally, robust evidence for heteroskedasticity is provided by Bansal, Khatchatrian, and Yaron (2002).
they feature left-skewness aversion: The news-utility agent hates the left tail and thus disaster risk. It turns out that time-varying disaster risk is powerful enough to successfully offset the variation in the risk-free rate. Moreover, Barro (2006) provides compelling evidence for the existence of a small probability of economic disaster.

It is important to note that introducing another source of variation does not eliminate the variation in substitution motives; it merely offsets its effects on the risk-free rate. Moreover, the extended models feature two sources of variation: The news-utility variation in substitution motives and heteroskedasticity in consumption growth or time-varying disaster risk. While the first source of variation concerns intertemporal substitution, the latter works via variation in the price of risk.

12.1 Time-varying consumption growth and volatility

A decrease in expected consumption growth $\mu_t$ or an increase in expected volatility $\sigma_t$ make the agent consume less and save more. Thus, if an adverse shock is associated with a decrease in expected consumption growth or an increase in expected volatility, the agent’s intertemporal substitution effects on the risk-free rate will be partially offset. Let consumption growth be given by

$$\log\left(\frac{C_{t+1}}{C_t}\right) = \mu_t + \sigma_t \epsilon_{t+1}$$

with $\mu_{t+1} = \mu_c + \nu_\mu (\mu_t - \mu_c) + \tilde{\mu} (\epsilon_{t+1}) + u_{t+1}$, $u_{t+1} \sim (0, \sigma_u^2)$, and $\tilde{\mu} (\epsilon_{t+1}) = \bar{\mu} (\log(\frac{1 - \rho}{1 - \rho_{t+1}}) - E[\log(\frac{1 - \rho}{1 - \rho_t})])$. Moreover, $\sigma_{t+1}^2 = \sigma_c^2 + \tilde{\sigma} (\epsilon_{t+1}) + \nu_\sigma (\sigma_t^2 - \sigma_c^2) + w_{t+1}$, $w_t \sim (0, \sigma_w^2)$, and $\tilde{\sigma} (\epsilon_t) = \bar{\sigma} (0.5 - F(\epsilon_t))$. The variation in $\tilde{\sigma} (\epsilon_{t+1})$ aims to reflect the variation in the homoskedastic consumption-wealth ratio, because heteroskedasticity is intended to offset the general-equilibrium impact on the risk-free rate. Note that $\sigma_t$ is a Markovian process, increases in the event of an adverse shock and is characterized by a shape similar to the consumption-wealth ratio determined by the variation in intertemporal substitution motives. Moreover, the conditional expectation of economic volatility is characterized by an AR(1) process with persistence $\nu_\sigma$. $\mu_t$ is chosen to fine-tune the remaining variation in the risk-free rate. The functional form of $\tilde{\mu} (\epsilon_t)$ is reverse-engineered such that if $\bar{\mu} = 1$ and $\nu_\mu = 0$ the variation in the risk-free rate brought about by the variation in the price-consumption ratio will be exactly offset, as can be seen in equation (27). If $\nu_\mu > 0$ the conditional expectation of consumption growth is characterized by an AR(1) process with persistence $\nu_\mu$. The model’s simple structure is unaffected by variation in expected consumption growth and derived in Appendix E.

I slightly modify the calibration presented in Table 8 to roughly match the basic asset-pricing moments in the modified model which are displayed in Table 9 in Appendix C. In particular, I simulate the model at a monthly frequency and decrease $\beta$ and $\theta$ slightly. For illustration, I chose the simplest possible process for expected consumption growth with $\nu_\sigma = \nu_\mu = 0$, $\sigma_w = \sigma_u = 0$ and $\bar{\sigma} = 2$. Moreover, I chose $\bar{\mu} = 0.8$ to smooth out 80% of the variability in the risk-free rate. As can be seen in Table 9, the basic asset-pricing moments are matched well in the model with variation in expected consumption growth. Importantly, countercyclical variation in consumption growth does not reduce the variation in the consumption-wealth ratio, such that the model continues to fit the
cây data provided by Lettau and Ludvigson (2005) and the model’s predictability properties are still present. Thus, the variation in expected consumption growth does not eliminate the variation in intertemporal substitution motives but rather introduces a second channel that offsets the impact on the risk-free rate. If $\nu_\sigma > 0$, a positive shock to economic volatility today implies high volatility in the future because the heteroskedasticity process is autocorrelated. Then, the size of the excess returns will be autocorrelated and the model is able to generate autocorrelation in the returns and long-horizon predictability.  

However, $\mu_c$, $\sigma_c$, and $\mu(\cdot)$ jointly determine the moments of the annualized consumption growth process, which I also display in Table 9 following Bansal and Yaron (2004). Unfortunately, the required variation in $\mu(\cdot)$ significantly changes the moments of the annualized consumption growth process which then fails to match the data even if lower levels for both $\mu_c$ and $\sigma_c$ are chosen. The annualized standard deviation of the simulated consumption process should be at most 3.5%. This value is far exceeded because of the extent of variation in expected consumption growth required to smooth out 80% of the variability of the risk-free rate.

12.2 Time-varying disaster risk

An increase in the probability of disaster makes the agent value a unit of safe consumption more highly. Thus, if adverse shocks are associated with disaster risk, the risk-free rate smooths out. Thus, I introduce a small time-varying probability of disaster according to Barro (2006, 2009). In each period $t$, there is a probability $p_t$ that a disaster occurs in period $t+1$ in which case consumption drops by $d$ percent. Thus, consumption growth is given by $log(C_{t+1}/C_t) = \mu_v + \nu_{t+1} + \sigma_{t+1}$ with $\nu_{t+1} \sim N(0, \sigma_c^2)$ and $\nu_{t+1} = log(1 - d)$ with probability $p_t$ and zero otherwise. I assume that $\nu_{t+1}$ and $\sigma_{t+1}$ are independent. The simple process governing the variability in disaster risk is $p_{t+1} = p + \nu_t(p - p) + u_{t+1} + \tilde{g}(\epsilon_t)$ with $u_{t+1} \sim N(0, \sigma_d^2)$ and $\tilde{g}(\epsilon_t) = p\tilde{p}(0.5 - F(\epsilon_t))$. Note that $p_t$ is a Markovian process, increases in the event of an adverse shock, and is characterized by a similar shape as the consumption-wealth ratio determined by the variation in intertemporal substitution motives. Moreover, the conditional expectation of disaster risk is characterized by an AR(1) process with persistence $\nu$. The model’s simple structure is unaffected by the addition of disaster risk and derived in Appendix F.

The news-utility agent is more affected by the probability of disaster than the standard agent, because the news-utility agent dislikes disaster risk more. The utility function’s gain-loss component over news is inspired by prospect theory. Classical prospect theory assumes a value function

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88 Koszegi and Rabin (2007) find that news utility causes variation in risk attitudes. In particular, the authors state that the agent becomes less risk averse when moving from a fixed to a stochastic reference point. With a stochastic reference point, a gamble does not appear as daunting, because some potential losses were previously expected. Thus, the equity premium in period $t$ depends negatively on $\sigma_{-1}$, because it is determined by the price of risk, i.e., $\frac{Cov(M_{t+1} + R_{t+1})}{E_t[M_{t+1} + R_{t+1}]}$, which varies with $\sigma$ and $\sigma_{-1}$. If high volatility is expected, $p_t$ is less steep and thus less responsive to a shock to consumption growth, which tends to reduce the required equity premium. Hence, news-utility preferences introduce two sources of variation in the price of risk and thus the required equity premium: The price of risk varies with economic volatility $\sigma$ as in the standard model. Furthermore, for any given $\sigma$, the price of risk varies inversely with the variability of beliefs determined by $\sigma_{-1}$.
of the form $v(c - r)$, defined over the actual consumption level $c$ relative to the reference point $r$. Typically, the value function features a kink at the reference point $r$, concavity over gains $c > r$, convexity over losses $c < r$, and probability weighting. In contrast, Koszegi and Rabin (2009) specify gain-loss utility as the linear difference in utility values $\mu(u(c) - u(r))$ with $\mu(\cdot)$ being some type of prospect-theory value function. The authors note that diminishing sensitivity or probability weighting may be introduced via $\mu(\cdot)$. However, thus far I have followed the literature and will retain news-utility preferences in their most basic form with $\mu(\cdot)$ being piecewise linear. Interestingly though, using a piecewise linear $\mu(\cdot)$ function results in left-skewness aversion: The news-utility agent hates the left tail. Because the agent assesses gain-loss utility as the linear difference in utility values $u(c) - u(r)$, the left tail, where $u(\cdot)$ becomes steep, is relatively overweighted. In classical prospect theory, left-skewness aversion can only be caused by low-probability over-weighting. Thus, the basic form of Koszegi and Rabin (2009) preferences is likely to yield very interesting dynamics with respect to a small disaster probability.

The extended model yields a realistic set of moments as shown in Table 9 in Appendix C. The parameters of disaster risk are calibrated according to Barro (2009) with $p = 1.7\%$ and $d = 0.29$. Most importantly, the additional variation does not eliminate the variation in intertemporal substitution motives or the variation in the consumption-wealth ratio. Rather it introduces another channel that offsets the impact of varying intertemporal smoothing incentives on the risk-free rate. A positive autocorrelation in the probability of disaster $\nu > 0$ will generate long-horizon predictability in returns and excess returns. However, for simplicity, I again omit persistence and additional noise in the disaster risk process $\nu = 0$ and $\sigma_u = 0$. The model generates a high equity premium that exhibits considerable variation, although, the model now requires the addition of a separate dividend process to match historical levels of the equity premium’s variability. The reason is the low $\theta$ I have to choose to not increase the equity premium over and beyond historical levels. But, the variation in the risk-free rate is successfully reduced to approximately 3\%, which is reasonable for international data.

13 Welfare and Beliefs-based Present Bias

Last, I illustrate the model’s welfare implications. In the spirit of Lucas (1978) and Reis (2009) I show that the news-utility agent would be willing to give up a fraction $\lambda_W$ of consumption in exchange for a risk-free consumption path, i.e., $E_t[\sum_{\tau=1}^{\infty} \beta^\tau u(C_{t+\tau}(1 + \lambda_W))] = \sum_{\tau=1}^{\infty} \beta^\tau u(C_{t+\tau})$ with $C_{t+\tau} = E_t[C_{t+\tau}] = C_t e^{\tau \mu_u + \tau^2 \sigma_u^2}$ for all $\tau$. This fraction determines the costs of business cycle fluctuations and is much higher for the news-utility agent than the standard agent. For the calibration in Table 8 with $\lambda = 2$, the news-utility agent would be willing to give up 17.02\% of his consumption in exchange for a stable consumption path whereas the standard agent would give up merely 3.37\%.

The preferences give rise to a time-inconsistent desire for immediate consumption, which I call beliefs-based present bias. A simplified intuition is that the agent prefers to raise consum-
tion above expectations today instead of increasing consumption and expectations tomorrow. The preferred-personal solution concept requires the agent to choose an equilibrium path that is credible in the sense that beliefs map into the correct behavior and vice versa. However, the agent likes to surprise himself with some extra consumption, taking his beliefs as given in each period. In contrast, on the optimal pre-committed path that maximizes expected utility the agent jointly chooses optimal consumption and beliefs. Hence, the time-consistent equilibrium path does not correspond to the expected-utility maximizing one and the first welfare theorem does not hold. In Appendix 6, I elaborate on the properties of the optimal pre-committed path and how beliefs-based present bias differs from hyperbolic discounting.

14 Conclusion

This paper incorporates expectations-based reference-dependent preferences into the canonical Lucas-tree model. In so doing, I contribute to the prospect-theory asset-pricing literature, pioneered by Benartzi and Thaler (1995), Barberis, Huang, and Santos (2001), and Yogo (2008), by assuming a generally-applicable utility function that is based on consumption, does not require a narrow-framing assumption, has a fully developed reference point, and has been shown to be consistent with behavior in a various micro domains. News utility generates both desirable and undesirable implications. Most importantly, the preferences shift and introduce strong variation in the consumption-wealth ratio, which is reflected in an increase and variation in the equity premium matching historical levels despite the fact that consumption equals dividends. Intuitively, in bad states of the world, reducing consumption below expectations is particularly painful and the agent becomes unwilling to substitute present for future consumption - as is likely to be true for people engaging in too much credit-card borrowing. However, in a general-equilibrium setup, this translates into large variability in the risk-free rate, a phenomenon not observed in aggregate data. Moreover, I contribute to the asset-pricing literature by making an additional step towards resolving the equity premium puzzle following Barberis and Huang (2008, 2009). In particular, I show that the agent exhibits plausible risk attitudes towards small, medium, and large wealth bets simultaneously.
Part III

A News-Utility Theory for Inattention and Delegation in Portfolio Choice

15 Introduction

Standard finance theory says that investors should rebalance their portfolios to realign their stock shares with their target shares frequently because stock prices display large fluctuations. However, Bonaparte and Cooper (2009), Calvet, Campbell, and Sodini (2009a), Karlsson, Loewenstein, and Seppi (2009), Alvarez, Guiso, and Lippi (2012), and Brunnermeier and Nagel (2008) find that investors are either inattentive or undertake rebalancing efforts that do not seem to aim for well-defined target shares. Additionally, such inattention has been shown to matter in the aggregate by Gabaix and Laibson (2001), Reis (2006), and Duffie and Sun (1990). Moreover, French (2008) and Hackethal, Haliassos, and Jappelli (2012) show that investors overpay for delegated portfolio management, Calvet, Campbell, and Sodini (2009b) and Meng (2013) document that investors have a tendency to sell winning stocks (the disposition effect), and Choi, Laibson, and Madrian (2009) and Thaler (1980) argue that investors mentally separate their accounts.

This paper offers an explanation for inattention and demand for delegated portfolio management that simultaneously speaks to the disposition effect and mental accounting by assuming that the investor experiences utility over news or changes in expectations about consumption. Such news-utility preferences were developed by Koszegi and Rabin (2006, 2007, 2009) to discipline the insights of prospect theory and have since been shown to explain a broad range of micro evidence. The preferences’ central idea is that bad news hurts more than good news pleases, which makes fluctuations in news utility painful in expectation and provides a micro foundation for inattention. This micro foundation has many behavior and welfare implications. If the investor has access to both a brokerage and a checking account, he chooses to ignore his portfolio in the brokerage account and, instead, fund his consumption out of the checking account most of the time. Moreover, the investor has a first-order willingness to pay for a portfolio manager who rebalances his portfolio actively on his behalf, as rebalancing reduces the portfolio’s risk. Occasionally, however, the investor has to rebalance his portfolio himself to smooth his consumption plans; in this case, he engages in behavior reminiscent of realization utility and the disposition effect. Moreover, the investor’s desire to separate accounts, his consumption, and his self-control problems are reminiscent of mental accounting. Additionally, I present the preferences’ implications for non-participation and stock shares in the presence of stochastic labor income to show that news utility also addresses pertinent questions in life-cycle portfolio theory.

I first explain the preferences in detail. The investor’s instantaneous utility in each period consists of the following components. First, “consumption utility” is determined by the investor’s consumption as in the standard model. Second, “news utility” is determined by comparing the investor’s updated expectations about consumption and his previous expectations about consumption. More specifically, the investor experiences “contemporaneous news utility” by comparing his present consumption with his expectations about consumption. In this comparison, he experiences a sensation of good or bad news over each previously expected consumption outcome, whereby
bad news hurts more than good news pleases. Moreover, the investor experiences “prospective news utility” by comparing the updated expectations about future consumption with his previous expectations. In so doing, he experiences news utility over what he has learned about future consumption.

To build intuition for my results in the dynamic model, I first highlight two fundamental implications of news utility for portfolio choice in a static framework. First, as bad news hurts more than good news pleases, even small risks, resulting in good and bad news, cause a first-order decrease in expected utility. Consequently, the investor is first-order risk averse. Thus, he does not necessarily participate in the stock market and, when he does, he chooses a lower portfolio share of stocks than under standard preferences. Moreover, first-order risk aversion and non-participation are prevalent even in the presence of stochastic labor income. Second, first-order risk aversion implies that the investor can diversify over time, i.e., his portfolio share is increasing in his investment horizon. He considers the accumulated stock-market outcome of a longer investment horizon as less risky relative to its return because the expected first-order news disutility increases merely with the square root of his horizon whereas the return increases linearly in his horizon.

These two implications extend to a fully dynamic life-cycle model, in which the investor chooses how much to consume and how much to invest in a risk-free asset and a risky asset. To allow for inattention, I modify the standard life-cycle model by assuming that the investor adjusts his portfolio via a brokerage account that he can choose to ignore. If the investor plans to ignore his brokerage account, he uses a separate checking account to finance consumption in those inattentive periods.

The news-utility investor prefers to be inattentive for some periods if a period’s length becomes sufficiently short. Looking up the portfolio implies fluctuations in good and bad news, causing a first-order decrease in expected utility as explained above. Not looking up the portfolio implies that the investor cannot smooth consumption perfectly. However, imperfect consumption smoothing has only a second-order effect on expected utility because the investor deviates from an initially optimal path. Moreover, in inattentive periods, the investor cannot rebalance his portfolio. This increases the portfolio’s risk and thus generates a first-order willingness to pay a portfolio manager to rebalance the portfolio according to standard finance theory. With plausible parameter values, these two effects have quantitative implications that corroborate the empirical evidence. In particular, the model matches the findings of Alvarez, Guiso, and Lippi (2012) and Bonaparte and Cooper (2009) that the typical investor rebalances his portfolio approximately once a year and of French

89 The result about non-participation in the presence of background risk stands in contrast to earlier analysis, such as Barberis, Huang, and Thaler (2006) and Koszegi and Rabin (2007, 2009). Nevertheless, labor income makes stock-market risk more bearable as news utility is proportional to consumption utility, such that fluctuations in good and bad news hurt less on the flatter part of the concave utility curve. Jointly, these implications generate increasing participation and portfolio shares in the beginning of life, consistent with empirical evidence.

90 In the dynamic model, time diversification implies that the investor chooses a lower portfolio share toward the end of life. Empirically, participation and portfolio shares are hump-shaped over the life cycle as shown in Section 20 and in Ameriks and Zeldes (2004). A lower portfolio share later in life is advised by financial planners; as a rule of thumb, it should equal 100 minus the investor’s age. This advice is explained in the standard model via variation in the wealth-income ratio, variation in risk aversion due to changes in wealth, or mean reversion in stock prices.

91 Beyond allowing for inattention, this model relaxes the degree of freedom associated with calibrating a period’s length; this would otherwise crucially affect the results due to the possibility for time diversification.
(2008), who finds that the typical investor forgoes approximately 67 basis points of the market's annual return for portfolio management.

Occasionally, the news-utility investor has to reconsider his consumption plans and rebalance his portfolio. I show that his optimal portfolio share decreases in the return realization, whereas the standard investor’s portfolio share remains constant; therefore, the news-utility investor rebalances extensively. Intuitively, in the event of a good return realization, the investor wants to realize the good news about future consumption and sells the risky asset. In the event of a bad return realization, however, the investor has to come to terms with bad news about future consumption. This bad news can be kept uncertain by increasing the portfolio share, which allows the investor to delay the realization of bad news until the next period by which point his expectations have adjusted. Such behavior is reminiscent of the empirical evidence on the disposition and break-even effects, but originates from news utility about future consumption rather than risk-lovingness in the loss domain.\(^{92}\) However, extensive rebalancing implies that the investor holds on to rather than buys the risky asset after the market goes down. The reason is that the investor’s end-of-period asset holdings may increase in the return realization due to a decrease in the consumption-wealth ratio. Intuitively, if an adverse return realizes, the investor consumes relatively more from his wealth to delay the decrease in consumption until his expectations have decreased.

Beyond portfolio choice, this model makes predictions about consumption, which are reminiscent of the concept of mental accounting.\(^{93}\) In my model, the investor’s two accounts finance different types of consumption. The brokerage account finances future consumption whereas the checking account finances current consumption. Moreover, the accounts exhibit different marginal propensities to consume. An unexpected windfall gain in the brokerage account is integrated with existing stock-market risk and consumed partially while a windfall gain in the checking account is consumed entirely immediately.\(^{94}\) However, with respect to expected gains, the investor has a higher marginal propensity to consume out of the brokerage account than from the checking account because he overconsumes time inconsistently out of the brokerage account, but consumes efficiently out of the checking account, which I explain next.\(^{95}\)

The investor may behave time inconsistently because he takes his expectations as given when he thinks about increasing his consumption today. Yesterday, however, he took into account how such increases would have increased his expectations too. Thus, today, he is inclined to increase his consumption above expectations compared to the optimal precommitted plan that maximizes expected utility. This time inconsistency depends on news utility and, therefore, uncertainty. Be-

\(^{92}\)The disposition effect (Odean (1998)) is an anomaly related to the tendency of investors to sell winners (stocks that have gone up in value) but keep losers (stocks that have gone down in value) to avoid the realization of losses. Odean (1998) argues in favor of the purchase price as a reference point, but Meng (2013) shows that expectations as a reference point seem to explain the data even better. The break-even effect refers to the observation that people become less risk averse in the loss domain. This effect is documented by Lee (2004) for professional poker players and Post, van den Assem, Baltussen, and Thaler (2008) for Deal-or-No-Deal game show participants.

\(^{93}\)Mental accounting (Thaler (1980)) describes the process whereby people mentally categorize financial assets as belonging to different accounts and treat these accounts as non-fungible.

\(^{94}\)This windfall result was first obtained by Koszegi and Rabin (2009). Moreover, this result relates to narrow framing, the phenomenon that people evaluate an offered gamble in isolation rather than mixing it with existing risk.

\(^{95}\)This result may speak to the puzzle that people simultaneously borrow on their credit cards and hold liquid assets. The investor would happily pay additional interest to separate his wealth in order to not reconsider his consumption plans too frequently and overconsume less.

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cause inattentive consumption is deterministic and restricted by the amount of funds in the checking account, the investor consumes efficiently when he is inattentive. Intuitively, time-inconsistent overconsumption would result in good news about today’s consumption but bad news about tomorrow’s consumption. As the investor dislikes bad news more than he likes good news, he does not overconsume. As a result, the investor would like to precommit to looking up his portfolio less frequently, lowering his attentive consumption and portfolio share, but rebalancing even more extensively. Exploring the investor’s commitment problem for both consumption and portfolio choice allows me to conclude that inattention and separate accounts result in a first-order increase in welfare and less overconsumption, which are important results for normative household finance.

Even if people are deliberately inattentive, it seems unrealistic to assume that they do not receive any signals about what is happening in their brokerage account. Therefore, I extend the model such that the investor receives a signal about the value of his brokerage account and then decides whether he will remain inattentive. I first argue that the model’s implications are completely unaffected by signals with low information content. In the presence of such signals, the investor chooses not to look up his portfolio regardless of the realization of the signal. In this case, he does not increase his current consumption, which is restricted by his funds in the checking account, because he would have to consume less in the next period. In turn, I outline the implications of signals with information content that is high enough to affect the investor’s attentiveness and, thus, consumption behavior using simulations. I find that, adverse signals induce the investor to ignore his portfolio; thus, he behaves according to the “Ostrich effect” (Karlsson, Loewenstein, and Seppi (2009)).

To evaluate the model quantitatively, I structurally estimate the preference parameters by matching the average empirical life-cycle profile of participation and portfolio shares using household portfolio data from the Survey of Consumer Finances (SCF) from 1992 to 2007. I show that the estimated parameters are consistent with existing microeconomic estimates, generate reasonable attitudes towards small and large wealth bets, and generate reasonable intervals of inattention. In the data, I control for time and cohort effects using a technique that solves the identification problem associated with the joint presence of age, time, and cohort effects with minimal assumptions (Schulhofer-Wohl (2013)).

Moreover, the model’s quantitative predictions about consumption and wealth accumulation are consistent with the empirical profiles inferred from the Consumer Expenditure Survey (CEX).

I first review the literature in Section 16. In Section 17, I then explore news utility in a static portfolio setting to illustrate several fundamental results. I proceed to dynamic portfolio theory in Section 18. I first outline the model environment, preferences, and solution in Sections 18.1 to 18.3. In Section 19, I extend my previous results about inattention and time diversification to the dynamic setting and outline several more subtle comparative statics about inattention (Section 19.1), explain the investor’s motives for rebalancing (Section 19.2), illustrate the model’s welfare implications (Section 19.3), and explore a model extension in which the investor receives signals about his portfolio (Section 19.4). In Section 20, I empirically assess the quantitative performance of the model. Finally, I conclude the paper and discuss future research in Section 21.
Comparison to the Literature


Information aversion has recently been explored by Andries and Haddad (2013), Artstein-Avidan and Dillenberger (2011), and Dillenberger (2010) under the assumption of disappointment-aversion preferences, as formalized by Gul (1991). Several related papers study information and trading frictions in continuous-time portfolio-selection models.\(^97\) Abel, Eberly, and Panageas (2013) assume that the investor faces information and transaction costs when transferring assets between accounts. Huang and Liu (2007) and Moscarini (2004) explore costly information acquisition, Ang, Papanikolaou, and Westerfield (forth.) assume illiquidity. Rational inattention also refers to a famous concept in macroeconomics introduced by Sims (2003), which postulates that people acquire and process information subject to a finite channel capacity. Luo (2010) explores how inattention affects consumption and investment in an infinite-horizon portfolio-choice model. A somewhat similar idea is pursued by Nieuwerburgh and Veldkamp (2013), who explore how precisely an investor wants to observe a signal about each asset’s payoffs when he is constrained in the total amount of signal precision he can observe. Moreover, Mondria (2010) and Peng and Xiong (2006) study attention allocation of individual investors among multiple assets. Among many others, Mankiw and Reis (2002), Gabaix and Laibson (2001), and Chien, Cole, and Lustig (2011) explore the aggregate implications of assuming that a fraction of investors are inattentive.

I combine these two literatures by exploring the implications of inattention for portfolio choice in a standard discrete-time framework, which simultaneously explains life-cycle consumption and wealth accumulation, participation, and shares.\(^98\) Inattention and sporadic rebalancing have been documented empirically by Alvarez, Guiso, and Lippi (2012), Bonaparte and Cooper (2009), Calvet, Campbell, and Sodini (2009a,b), Karlsson, Loewenstein, and Seppi (2009), Brunnermeier and Nagel (2008), Bilias, Georgarakos, and Haliassos (2010), Agnew, Balduzzi, and Sunden (2003), Dahlquist and Martinez (2012), and Mitchell, Mottola, Utkus, and Yamaguchi (2006). Moreover,

\(^97\) An argument against information constraints is that there exist quite simple portfolio strategies that are consistent with the standard model, i.e., people could invest their entire wealth in an index fund.

\(^98\) Hereby, I confirm the analysis in Pagel (2012b), which shows how news utility generates a realistic hump-shaped life-cycle consumption profile in addition to other life-cycle consumption phenomena. Moreover, Pagel (2012c) shows that news utility makes an additional step towards solving the equity premium puzzle by explaining asset prices and simultaneously generating reasonable attitudes over small and large wealth bets; an important point in favor of the prospect-theory asset-pricing literature that has been first emphasized by Barberis and Huang (2008, 2009).
Engelberg and Parsons (2013) show that after stock market declines hospital admissions for mental conditions in California increase. Lab and field evidence for myopic loss aversion is provided by Gneezy and Potters (1997), Gneezy, Kapteyn, and Potters (2003), Haigh and List (2005), and Fellner and Sutter (2009). Relatedly, Anagol and Gamble (2011), Bellemare, Krause, Kroeger, and Zhang (2005), and Zimmermann (2013) tests if subjects prefer information clumped or piecewise. The prediction of extensive rebalancing once the investor looks up his portfolio contradicts the intuitive idea that people sell stocks when the market is going down. The latter behavior would imply an increasing portfolio share as predicted by herding behavior or habit formation. Habit formation is tested in Brunnermeier and Nagel (2008), who, however, find evidence in favor of a constant or slightly decreasing portfolio share.99 I briefly extend their analysis to provide suggestive evidence for a decreasing portfolio share, as predicted by news utility, using Panel Study of Income Dynamics (PSID) household portfolio data.

This paper analyzes a generally-applicable preference specification that has been used in various contexts to explain experimental and other microeconomic evidence.100 The preferences’ explanatory power in these other contexts is important because I put emphasis on the potentially normative question of how often people should look up and rebalance their portfolios. While most of the existing applications and evidence consider the static model of Koszegi and Rabin (2006, 2007), I incorporate the preferences into a fully dynamic and stochastic model. The inattention result has been anticipated by Koszegi and Rabin (2009) in a two-outcome model featuring consumption in the last period and signals in all prior periods. Several of my additional results extend and modify Koszegi and Rabin (2007), who analyze the preferences’ implications for attitudes towards risk in a static setting.

17 News Utility in Static Portfolio Theory

I start with static portfolio theory to introduce the preferences and quickly illustrate several important predictions in a simple framework. I first outline the news-utility implications for par-
Lemma 2. The news-utility agent’s optimal portfolio share can be approximated by

\[ \alpha^* = \frac{\mu - r^f + \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \hat{s})dF_s(\hat{s})]}{\sigma^2}. \]

Proof. I can rewrite the maximization problem as

\[ r^f + \alpha(\mu - \frac{\sigma^2}{2} - r^f) + \alpha(1 - \alpha)\frac{\sigma^2}{2} + \alpha \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \hat{s})dF_s(\hat{s})] \]
which results in the optimal portfolio share stated in equation (31) if $0 \leq \alpha^* \leq 1$ and $\alpha^* = 0$ if the expression (31) is negative or $\alpha^* = 1$ if the expression (31) is larger than one.

17.1.1 Samuelson’s colleague, time diversification, and inattention

In order to illustrate the model’s implications for time diversification, I define $\mu \triangleq h \mu_0$, $\sigma \triangleq \sqrt{h} \sigma_0$, and $r_f \triangleq h r_{f0}$. The parameter $h$ can be interpreted as the period’s length, i.e., if $\mu_0$ and $\sigma_0$ were originally calibrated to a monthly frequency, $h = 3$ would imply a quarterly frequency. In the following proposition, I formalize that the news-utility investor prefers to not invest into the risky asset if a period’s length is too short, can diversify over time, and gains from being inattentive.

Proposition 11. (Horizon effects on portfolio choice)

1. (Samuelson’s colleague and time diversification) There exists some $h$ such that the news-utility agent’s portfolio share is zero for $h < h$, whereas the standard agent’s portfolio share is always positive. Moreover, the news-utility agent’s portfolio share is increasing in $h$, whereas the standard agent’s portfolio share is constant in $h$.

2. (Inattention) The news-utility agent’s normalized expected utility, i.e., $\frac{EU}{h}$ with $W = 1$, is increasing in $h$, whereas the standard agent’s normalized expected utility is constant in $h$.

This and the following propositions’ proofs can be found in Appendix J.

To illustrate this proposition more formally, I plug the redefined terms into equation (31). As can be easily seen, the news-utility agent’s portfolio share is positive if and only if $h$ is high enough, i.e.,

$$\frac{\mu_0 - r_f}{\sigma_0} > -\frac{\sqrt{h}}{h} E[\eta(\lambda - 1) \int_{\tilde{s}}^{\infty} (s - \tilde{s})dF_s(\tilde{s})] > 0.$$ 

As the integral is always negative the news-utility agent requires a higher excess return to invest in the stock market if $\lambda > 1$ and $\eta > 0$, i.e., he is first-order risk averse. Thus, he refuses to invest in the stock market if $h$ is low, as $\frac{\sqrt{h}}{h}$ is high and increases the expected bad news of the investment relative to its expected return. Moreover, $\alpha^*$ is decreasing in $h$ because the expected mean and

101 Alternatively, $h$ could be interpreted as the number of i.i.d. draws of an independent gamble of which the agent observes the overall outcome. Or $h$ can be interpreted as a restriction on consumption smoothing.

102 The standard agent’s portfolio share is positive whenever $\mu > r_f$ because he is approximately risk neutral for small risks, i.e., for small risks his concave utility function becomes approximately linear. The investment’s risk becomes small if $\alpha$ is positive but small. A zero portfolio share for $\mu > r_f$ requires a kink in the utility function, which is introduced by news utility if $\eta > 0$ and $\lambda > 1$. An increase in $\sigma$ is then associated with a first-order decrease in expected utility and the portfolio share. More formally, the terms of the agent’s first-order condition that depend on $\sigma$, i.e., $-\alpha \sigma^2 + \sigma E[\eta(\lambda - 1) \int_{\tilde{s}}^{\infty} (s - \tilde{s})dF_s(\tilde{s})]$, can be approximated by a second-order Taylor expansion around $\sigma = 0$, i.e., $(E[\eta(\lambda - 1) \int_{\tilde{s}}^{\infty} (s - \tilde{s})dF_s(\tilde{s})])\sigma + (-2\alpha)\sigma^2$. In this second-order approximation, the news-utility term is proportional to $\sigma$ while the standard agent’s term is proportional to $\sigma^2$; thus, the former is a first-order and the latter is a second-order effect of uncertainty on the portfolio share and expected utility.
variance of the investment increase by $h$ while its expected sensation of bad news increases merely by $\sqrt{h}$. Thus, if $h$ increases the investment’s expected sensation of bad news increases by less than its expected return, which makes the investment more attractive. Thus, the news-utility agent can diversify over time. In contrast, the standard agent will invest some fraction of his wealth whenever $\mu > r_f$ and his portfolio share is independent of $h$ as the expected mean of the investment increases in $h$ proportional to its variance.

Figure 5: News-utility and standard agents’ portfolio share as a function of the investment horizon $h$ and the coefficient of loss aversion $\lambda$.

Figure 5 displays the static portfolio share of the news-utility and standard agents as a function of $h$ and the coefficient of loss aversion $\lambda$ for $h = 10$.\textsuperscript{103}

I now give some background on these two results starting with the Samuelson’s colleague story. In Samuelson’s seminal paper, his colleague refuses to accept a 50/50 bet lose $100 win $200 but would accept 100 of such bets. In turn, Samuelson (1963) showed that, if one bet was rejected at all wealth levels, any number of such bets should be rejected under standard expected-utility theory. The reason is that the mean and variance of the sum of 100 bets increase by 100. The same logic applies to the investors who believe in diversification over time, i.e., who believe that stock-market risk is decreasing in the investment’s horizon. Because the accumulated stock-market return over a given horizon is the sum of the individual outcomes in each time period, time diversification does not exist under standard assumptions, and investors’ portfolio shares should be independent of their investment horizon.

To draw the connection to the welfare benefits of inattention, I consider the implications of $h$ for expected utility per unit of time or investment, i.e., normalized by $h$, and $W = 1$. The news-utility agent’s normalized expected utility, i.e.,

$$
\frac{EU}{h} = r_f + \alpha(\mu - r_f) - \frac{\alpha^2 \sigma^2}{2} + \frac{\sqrt{h} \alpha \sigma E[\eta(\lambda - 1) \int_s^{\infty} (s - \tilde{s})dF_s(\tilde{s})]}{h}
$$

\textsuperscript{103}The parameter values for the annual horizon are $\mu_0 = 8\%$, $r_f = 2\%$, and $\sigma_0 = 20\%$ and the preference parameters are $\eta = 1$ and $\lambda = 2$. These are standard parameters in the prospect-theory literature as I argue in Section 20.1.2 and generate realistic portfolio shares as can be seen in Figure 5.
is increasing in \( h \). Again, the investment’s expected return increases at a higher rate than the expected sensation of bad news. In contrast, the standard agent’s normalized expected utility is constant in \( h \). I now give some background on this result. Benartzi and Thaler (1995) explain the intuition of Samuelson’s colleague by formally showing that people find individual gambles inherently less attractive than the accumulated outcome of several of them, if they are loss averse and myopically evaluate the outcome of each gamble. Myopically loss averse investors thus gain from evaluating their portfolio at long rather than short horizons and can diversify over time. Hereby, myopic loss aversion assumes that the accumulated outcome of the gamble is evaluated rather than each individual gamble to explain the behavior of Samuelson’s colleague. In other words, Samuelson’s colleague has to be inattentive to each individual gamble. Here, I show that the intuition generalizes to a setting in which the reference point is endogenous and stochastic. An increase in \( h \) implies that the agent integrates all risk, i.e., he ignores and experience news utility over each realization of the independent gambles \( h \) represents; thus, he gains from being inattentive.

17.1.2 Labor income and wealth accumulation

Before moving on to dynamic portfolio theory, I make a short digression to illustrate the implications of labor income and wealth accumulation in the static framework to briefly show that my theoretical results do not depend on the absence of labor income. I simply assume that the agent will receive riskless labor income \( \bar{Y} \geq 0 \) and risky labor income \( Y = e^y \sim log - N(\mu_y, \sigma_y) \). The risky labor income may be correlated with the risky return with covariance \( \text{Cov}(r, y) = \sigma_{ry} \). Lowercase letters denote logs.

**Lemma 3.** In the presence of risky labor income, the news-utility agent’s optimal portfolio share can be approximated by

\[
\alpha^* = \frac{1}{\rho} \frac{\mu - r_f + \sigma E[\eta(\lambda - 1) \int_{s}^{\infty} (s - \tilde{s}) dF_s(\tilde{s})]}{\sigma^2} - \frac{1 - \rho}{\rho} \frac{\sigma_{ry}}{\sigma^2}. \tag{32}
\]

The derivation of \( \alpha^* \) and the consumption-function’s log-linearization parameter \( \rho \) is delegated to Appendix H.\textsuperscript{104} The portfolio share is potentially zero, decreasing in \( \sigma \), \( \eta \), and \( \lambda \), increasing in riskless labor income, and decreasing in wealth. Riskless labor income simply transforms the portfolio share by \( 1 + \frac{\bar{Y}}{R/W} \). This transformation does not affect participation because the news-utility agent refuses to invest in the stock market, if the expected excess return is not high enough, at any wealth level. Because the inverse of the consumption-function’s log-linearization parameter \( \frac{1}{\rho} \) is decreasing in age but \( \frac{1 - \rho}{\rho} \) decreases faster, labor income becomes relatively less important for older agents’ portfolio shares. Nevertheless, in higher order approximations, additional wealth that enters in an additive manner buffers stock-market risk. Risk generates fluctuations in news utility, which are proportional to consumption utility. Accordingly, these fluctuations hurt less on a high part of the concave utility curve. Because labor income is increasing in the beginning of life, this consideration implies that both shares and participation are increasing in the beginning of life.

\textsuperscript{104}See, e.g., Campbell and Viceira (2002).
consistent with empirical evidence.\textsuperscript{105}

In this approximation, the presence of stochastic labor income does not affect the agent’s participation constraint if $\sigma_{ry} \geq 0$, i.e., the agent’s first-order risk aversion is preserved. Moreover, this result about first-order risk aversion in the presence of background risk does not depend on the approximation and can be illustrated via the agent’s risk premium when stock market risk goes to zero, which I do in Appendix H. This result stands in contrast to earlier analyzes, such as Barberis, Huang, and Thaler (2006) and Koszegi and Rabin (2007, 2009). Barberis, Huang, and Thaler (2006) consider utility specifications that exhibit first-order risk aversion at one point. Background risk takes the agent away from this point and he becomes second-order risk averse with respect to additional risk. However, the reference point is stochastic in this paper’s model, so that it exhibits first-order risk aversion over the entire support of background risk. Koszegi and Rabin (2007, 2009) consider situations in which background risk is large and utility potentially linear and find that, in the limit, the agent becomes second-order risk averse. However, labor income risk is not large relative to stock market risk in a life-cycle portfolio framework and the agent’s utility function is unlikely to be linear in a model that is calibrated to realistic labor income and stock-market risk at an annual horizon.

18 News Utility in Dynamic Portfolio Theory

The previous results about time diversification illustrate that the model’s implications are highly dependent on the length of one time period, which remains a worrisome degree of freedom in the application of news utility. In order to relax this degree of freedom and further elaborate on the interesting implications of the model’s period length, I now introduce a life-cycle portfolio-choice model that deviates from other dynamic models in that it allows the agent to look up or refuse to look up his portfolio. I start with the model environment to then explain the dynamic preferences of Koszegi and Rabin (2009). Then, I extend my previous results to the dynamic setting, further describe the agent’s preference for inattention, explain the agent’s motives for rebalancing, and illustrate the agent’s time inconsistency and its implications for inattention and rebalancing. Finally, I consider an extension to signals about the market to illustrate more refined results about information acquisition.

18.1 The life-cycle model environment

The agent lives for $T$ periods indexed by $t \in \{1,\ldots,T\}$. In each period $t$, the agent consumes $C_t$ and may invest a share $\alpha_t$ of his wealth $W_t$ in a risk-free investment with deterministic return $\log(R_f) = r^f$ or a risky investment with stochastic return $\log(R_t) = r_t \sim N(\mu - \frac{\sigma^2}{2}, \sigma^2)$. I assume that short sale and borrowing are prohibited. In each period $t$, the agent may be inattentive and choose to not observe the realization of the risky asset $r_t$. If the agent invests a positive amount of wealth in the risky asset in period $t-i$, but is inattentive in periods $t-i+1,\ldots,t$, he cannot

\textsuperscript{105}Refer to Fernandez-Villaverde and Krueger (2007) for an examination of hump-shaped income and consumption profiles and Ameriks and Zeldes (2004) for an analysis of portfolio share and participation profiles. Moreover, this paper’s analysis of SCF data in Section 20 confirms these results.
observe the realization of his wealth $W_t$; therefore, his wealth for consumption in inattentive periods
$C_{t-i+1}, \ldots, C_t$ has to be stored in a risk-free checking account that pays interest $R^d = e^{rt}$. Thus, his budget constraint in any period $t$, when having observed the realization of his risky return in period $t - i$, is given by
\[ W_t = (W_{t-i} - C_{t-i} - \sum_{k=1}^{i-1} \frac{1}{R^d}^k C_{t-i+k})((R^d)^i + \alpha t - \alpha (R^d)^i)). \tag{33} \]

All the model’s variables that are indexed by period $t$ realize in period $t$. As preferences are defined over outcomes as well as beliefs, I explicitly define the agent’s probabilistic “beliefs” about each of the model’s period $t$ variables from the perspective of any prior period. Throughout the paper, I assume rational expectations such that the agent’s beliefs about any of the model’s variables equal the objective probabilities determined by the economic environment.

**Definition 12.** Let $I_t$ denote the agent’s information set in some period $t \leq t + \tau$. Then, the agent’s probabilistic beliefs about any model variable, call it $X_{t+\tau}$, conditional on period $t$ information is denoted by $F_{X_{t+\tau}}(x) = Pr(X_{t+\tau} < x | I_t)$.

### 18.2 The dynamic preferences

In the dynamic model of Koszegi and Rabin (2009), the utility function consists of consumption utility, “contemporaneous” news utility about current consumption, and “prospective” news utility about the entire stream of future consumption. Thus, total instantaneous utility in period $t$ is given by
\[ U_t = u(C_t) + n(C_t, F_t^{t-1}) + \sum_{\tau=1}^{T-t} \beta^\tau n(F_{X_{t+\tau}}^{t-1}). \tag{34} \]

The first term on the left-hand side of equation (34), $u(C_t)$, corresponds to consumption utility in period $t$. The first of the two remaining terms on the left-hand side of equation (34), $n(C_t, F_t^{t-1})$, corresponds to news utility over contemporaneous consumption; here, the agent compares his present consumption $C_t$ with his beliefs $F_{X_{t+\tau}}^{t-1}$ about present consumption. According to Definition 12, the agent’s beliefs $F_{X_{t+\tau}}^{t-1}$ correspond to the conditional distribution of consumption in period $t$ given the information available in period $t - 1$. Contemporaneous news utility is given by
\[ n(C_t, F_t^{t-1}) = \eta \int_{C_t} (u(C_t) - u(c))dF_{C_t}^{t-1}(c) + \eta \lambda \int_{C_t} (u(c) - u(C_t))dF_{C_t}^{t-1}(c). \tag{35} \]

---

106 The separate checking account allows the agent to be inattentive and consume without risking zero or negative wealth, which is associated with infinitely negative utility and would thus prohibit inattentive behavior. Without the separate checking account, the agent would not consume more than $(1 - \alpha_{t-1})W_{t-1}(R^d)^i$ in any inattentive period $t$, which might impose a binding restriction on his consumption maximization problem. To avoid such binding restrictions, the inattention model of Reis (2006) assumes exponential instead of power utility, which allows consumption to take negative values.
The third term on the left-hand side of equation (34), \( \gamma \sum_{\tau=1}^{\infty} \beta^\tau n(F_{C_t+\tau}^{t-1}) \), corresponds to prospective news utility, experienced in period \( t \), over the entire stream of future consumption. Prospective news utility about period \( t + \tau \) consumption depends on \( F_{C_t+\tau}^{t-1} \), the agent’s beliefs he entered the period with, and on \( F_{C_t+\tau}^t \), the agent’s updated beliefs about consumption in period \( t + \tau \). The prior and updated beliefs about \( C_{t+\tau}, F_{C_t+\tau}^{t-1} \) and \( F_{C_t+\tau}^t \), are not independent distribution functions because future uncertainty \( R_{t+1}, \ldots, R_{t+\tau} \) is contained in both. Thus, there exists a joint distribution, which I denote by \( F_{C_t+\tau}^{t-1} \neq F_{C_t+\tau}^t \). Because the agent compares his newly formed beliefs with his prior beliefs, he experiences news utility about future consumption as follows

\[
n(F_{C_t+\tau}^{t-1}) = \int_{-\infty}^{\infty} (\eta \int_{-\infty}^{c} (u(c) - u(r)) + \eta \lambda \int_{-\infty}^{\infty} (u(c) - u(r))) dF_{C_t+\tau}^{t-1}(c, r). \tag{38}
\]

The agent exponentially discounts prospective news utility by \( \beta \in [0, 1] \). Moreover, he discounts prospective news utility relative to contemporaneous news utility by a factor \( \gamma \in [0, 1] \). Thus, he puts the weight \( \gamma \beta^\tau < 1 \) on prospective news utility about \( t + \tau \) consumption.

18.3 The life-cycle model’s solution

In order to obtain analytical results, I assume log utility \( u(c) = log(c) \) and approximate the log portfolio return \( log(R_t^f + \alpha(R_t - R_{t-1}^f)) \) by \( r^F + \alpha(r_t - r_{t-1}^f) + \alpha(1 - \alpha) \frac{\sigma^2}{2} \). The agent’s life-time utility in each period \( t \) is

\[
u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^{T-t} \beta^\tau n(F_{C_{t+\tau}}^{t-1}) + E_t[T \sum_{\tau=1}^{T-t} \beta^\tau U_{t+\tau}] \tag{39},
\]

\(^{107}\)Koszegi and Rabin (2006, 2007) allow for stochastic consumption, represented by the distribution function \( F_c \), and a stochastic reference point, represented by the distribution function \( F_r \). Then, the agent experiences news utility by evaluating each possible outcome relative to all other possible outcomes

\[
n(c, F_r) = \int_{-\infty}^{\infty} (\eta \int_{-\infty}^{c} (u(c) - u(r)) dF_r(r) + \eta \lambda \int_{-\infty}^{\infty} (u(c) - u(r)) dF_r(r) dF_c(c). \tag{36}
\]

I calculate prospective news utility \( n(F_{C_{t+\tau}}^{t-1}) \) by generalizing this “outcome-wise” comparison, equation (36), to account for the potential dependence of \( F_r \) and \( F_c \), i.e.,

\[
n(F_{C_{t+\tau}}, c) = \int_{c=-\infty}^{c=\infty} (\eta \int_{t=\tau}^{\tau} (u(c) - u(r)) dF_{C_{t+\tau}}(r) + \eta \lambda \int_{t=\tau}^{\tau} (u(c) - u(r)) dF_{C_{t+\tau}}(r) dF_c(c). \tag{37}
\]

If \( F_c \) and \( F_r \) are independent, equation (37) reduces to equation (36). However, if \( F_c \) and \( F_r \) are non-independent, equation (37) and equation (36) yield different values. Suppose that \( F_c \) and \( F_r \) are perfectly correlated, as though no update in information occurs. Equation (36) would yield a negative value because the agent experiences news disutility over his previously expected uncertainty, which is unrealistic. In contrast, equation (37) would yield zero because the agent considers the dependence of prior and updated beliefs, which captures future uncertainty, thereby separating uncertainty that has been realized from uncertainty that has not been realized. Thus, I call this comparison the separated comparison. Koszegi and Rabin (2009) generalize the outcome-wise comparison slightly differently to a “percentile-wise” ordered comparison. The separated and ordered comparisons are equivalent for contemporaneous news utility. However, for prospective news utility, they are qualitatively similar but quantitatively slightly different. As a linear operator, the separated comparison is more tractable. Moreover, it simplifies the equilibrium-finding process because it preserves the outcome-wise nature of contemporaneous news utility.

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with $\beta \in [0,1]$, $\eta \in (0,\infty)$, $\lambda \in (1,\infty)$, and $\gamma \in [0,1]$. I define the model’s “monotone-personal” equilibrium solution concept in the spirit of the preferred-personal equilibrium solution concept as defined by Koszegi and Rabin (2009). In period $t$, the agent has looked up his portfolio in periods $t - i$ and $t - i - j_0$.

**Definition 13.** The attentive consumption function in any period $t$ is admissible if it can be written as $C_t = g(W_t, T - t, r_t, \ldots, r_{t-1})$ and satisfies $\frac{\partial \log(C_t)}{\partial (\sum_{j=1}^{T} r_{t-i+j})} > 0$. The inattentive consumption function in any period $t$ is admissible if it can be written as $C_t^{in} = g^{in}(W_{t-i} - C_{t-i}, T - t)$ and satisfies $\frac{\partial \log(C_t^{in})}{\partial (\sum_{j=1}^{T} r_{t-i+j})} > 0$. The portfolio function in any period $t$ is admissible if it can be written as $\alpha_t = g_{\alpha}(T - t, j_1, r_t, \ldots, r_{t-i-1})$. $\{C_t^{in}, C_t, \alpha_t\}_{t \in \{1,\ldots,T\}}$ is a monotone-personal equilibrium if, in all periods $t$, the admissible consumption and portfolio functions $C_t^{in}, C_t$, and $\alpha_t$ maximize (39) subject to (33) under the assumption that all future consumption and portfolio functions correspond to $C_t^{in}, C_t^{out}, C_{t+1}, \text{and } \alpha_{t+1}$. In each period $t$, the agent takes his prior beliefs about consumption $\{F_{C_{t+1}}\}_{t=0}^{T-t}$ as given in the maximization problem.

The monotone-personal equilibrium solution can be obtained by simple backward induction. The first-order condition is derived under the premise that the agent enters period $t$, takes his beliefs as given, optimizes over consumption, and expects to behave like this in the future. Thus, the equilibrium is time consistent in the sense that beliefs map into correct behavior and vice versa.\(^{108}\)

I now briefly state the equilibrium consumption and portfolio functions to convey a general idea of the model’s solution. The derivation is explained in detail in Appendix I.

**Proposition 12.** There exists a unique monotone-personal equilibrium if $\sigma \geq \sigma_i^*$ for all $t \in \{1,\ldots,T\}$. The agent’s optimal level of attentive consumption is $C_t = W_t \rho_t$ with

$$\rho_t = \frac{1}{1 + \sum_{\tau=t}^{T-t} \beta^\tau \frac{1 + \gamma F_t(r_t)\ldots F_t(r_{t-1}) + \eta \lambda (1 - F_t(r_t)\ldots F_t(r_{t-1}))}{1 + \gamma F_t(r_t)\ldots F_t(r_{t-1}) + \eta \lambda (1 - F_t(r_t)\ldots F_t(r_{t-1}))}}.$$  

(40)

If the agent is attentive, his optimal portfolio share is

$$\alpha_t = \frac{\mu - \rho_t}{\sigma^2} + \frac{\sum_{\tau=t}^{T-t-1} \beta^\tau \sqrt{\frac{1 + \gamma F_t(r_t)\ldots F_t(r_{t-1}) + \eta \lambda (1 - F_t(r_t)\ldots F_t(r_{t-1}))}{1 + \gamma F_t(r_t)\ldots F_t(r_{t-1}) + \eta \lambda (1 - F_t(r_t)\ldots F_t(r_{t-1}))}}}{\gamma F_t(r_t)\ldots F_t(r_{t-1}) + \eta \lambda (1 - F_t(r_t)\ldots F_t(r_{t-1}))}}.$$  

(41)

if he plans to look up his portfolio next time in periods $t + j_1$ and has looked up his portfolio in periods $t - i$ and $t - i - j_0$. Moreover, the agent’s optimal level of inattentive consumption is $C_t^{in} = (W_{t-i} - C_{t-i})(R^d)^i \rho_t^{in}$ with $\rho_t^{in}$ for $k = 1,\ldots,j_1 - 1$ determined by the following recursion

$$\rho_t^{in} = \frac{1}{1 + \beta^j \sum_{\tau=0}^{T-t-j_1} \beta^\tau (1 - \sum_{k=1}^{i-1} \rho_{t-i+k}^{in} - \sum_{k=1}^{j_1-1} \rho_{t-i+k}^{in})}.$$  

(42)

\(^{108}\)If the consumption function obtained by backward induction falls into the class of admissible consumption functions, then the monotone-personal equilibrium corresponds to the preferred-personal equilibrium as defined by Koszegi and Rabin (2009).
Here, \(0 \leq \alpha_t \leq 1\) and \(\alpha_t = 0\) if the expression (41) is negative or \(\alpha_t = 1\) if the expression (41) is larger than one, as I assume that short sale and borrowing are prohibited. Moreover, note that \(j_1\), i.e., the number of periods the agent is inattentive after he looked up his brokerage account in period \(t\), depends on \(T - t\) and \(\alpha_t\) and is not determined in closed form. All of the following propositions are derived within this model environment under the first-order approximation for the portfolio return and hold in any monotone-personal equilibrium if one exists.\(^{109}\)

18.4 Comparison of the news-utility and standard policy functions

I now highlight three observations about the news-utility agent’s policy functions in comparison to the standard agent’s ones. The standard agent is attentive in every period \(t\), his optimal consumption function is \(C_t^s = W_t^s \frac{1}{1 + \sum_{i=1}^T \beta_i}\), and his portfolio share is \(\alpha^s = \frac{\mu - r_t^f}{\sigma^2}\). First, the news-utility agent overconsumes relative to the standard agent in attentive periods, as the standard agent’s consumption-wealth ratio is lower than the news-utility agent’s consumption-wealth ratio \(\rho_t\), i.e., equation (40). This overconsumption results from news utility as determined by the history of returns \(F_r(r_t) \cdots F_r(r_{t-\tau_1+1})\) and \(\gamma < 1\), i.e., the agent cares more about contemporaneous than prospective news, in combination with a time inconsistency that I explain in Section 19.3. Second, the news-utility agent does not overconsume in inattentive periods. In inattentive periods, the agent does not experience news utility over inattentive consumption and \(\gamma\) is irrelevant, as can be seen in equation (42). The agent does not overconsume in inattentive periods because the good news in present consumption would be outweighed by the bad news in future consumption.\(^{110}\) Absent overconsumption, I can assume that the agent’s optimal level of inattentive consumption has been stored in the checking account previously. Third, the news-utility agent’s portfolio share, equation (41), is reminiscent of the static model’s portfolio share. However, in the dynamic model it depends on the agent’s horizon \(T - t\), the amount of periods the agent will remain inattentive \(j_1\), and the history of returns \(F_r(r_t) \cdots F_r(r_{t-\tau_1+1})\). I now explain all these implications in greater detail. Hereby, I assume that \(\gamma < 1\), however, all the results that depend on \(\gamma < 1\) hold for \(\gamma = 1\) too, if, instead of log utility, I assume power utility with the relative coefficient of risk aversion larger than one.

19 Theoretical Predictions

I first extend the static model’s implications to the dynamic environment. Then, I explain the model’s predictions about inattention and rebalancing as well as the agent’s time inconsistency in more detail to finally consider signals about the market.

\(^{109}\)Most of my results can be easily extended to a model with a CRRA utility function and without the approximation for the portfolio return. Moreover, I can confirm my results by solving the more complicated model numerically. Finally, my main proposition is derived for \(h\) small and the portfolio approximation becomes accurate for \(h \to 0\).

\(^{110}\)In inattentive periods, the agent does not experience news utility in equilibrium because no uncertainty resolves and he cannot fool himself. Therefore, he is not going to deviate from his inattentive consumption path in periods \(t - i + 1\) to \(t - 1\) so long as \(u'(C_{t-i+1})^\gamma (1 + \eta) < (\beta R^s)^{i-1} u'(C_{t-i})^\gamma (1 + \gamma \sigma).\) As \(u'(C_{t-i+1})^\gamma \approx (\beta R^s)^{i-1} u'(C_{t-i})\), this condition boils down to \(\gamma \lambda > 1\). In the derivation, I assume that this condition holds, such that, in inattentive periods, the agent does not deviate from his consumption path and overconsumes time inconsistently.
19.1 Predictions about inattention

19.1.1 Samuelson’s colleague, time diversification, and inattention

To extend the model’s predictions for the Samuelson’s colleague story, time diversification, and inattention to the dynamic environment, I define $\mu \triangleq h\mu_0$, $\sigma \triangleq \sqrt{h}\sigma_0$, $r^f \triangleq hr^f_0$, and $\beta \triangleq \beta_0^h$.

**Corollary 2. (Horizon effects on portfolio choice)** For $\beta \in (1-\Delta, 1)$ with $\Delta$ small and assuming that the agent has to look up his portfolio every period.

1. (Samuelson’s colleague and time diversification) There exists some $h$ such that the news-utility agent’s portfolio share is zero for $h < h_1$, whereas the standard agent’s portfolio share is always positive. The news-utility agent’s portfolio share is increasing in $h$, whereas the standard agent’s portfolio share is constant in $h$. Moreover, if $\gamma < 1$ the news-utility agent’s portfolio share is increasing in the agent’s horizon $T-t$, whereas the standard agent’s portfolio share is constant in $T-t$.

2. (Inattention) The news-utility agent’s normalized expected utility, i.e., $\frac{E_t-1}{h}[\log(W_t\rho_t)] = 0$, is increasing in $h$, whereas the standard agent’s normalized expected utility is constant in $h$.

The intuitions presented in Section 17.1 carry over to the dynamic model directly. I now proceed to the main proposition in the dynamic model.

**Proposition 13.** For $T$ large, there exist some $\bar{h}$ and some $h$ with $\bar{h} > h$ such that if $h > \bar{h}$ the news-utility agent will be attentive in all periods and if $h < h$ the news-utility agent will be inattentive in at least one period. The standard agent will be attentive in all periods independent of $h$.

The basic intuition for this proposition is that the agent will look up his portfolio in every period, if a period’s length is very long, say ten years. However, if a period’s length is very short, say one day, the agent will find it optimal to be inattentive for a positive number of periods. The agent trades off the benefits from consumption smoothing and the costs from experiencing news utility. The benefits from consumption smoothing are proportional to the length of a period $h$ and second-order because the agent deviates from his optimal consumption path. The costs from experiencing news utility are proportional to $\sqrt{h}$ and first-order. Thus, as $h$ becomes small the benefits from consumption smoothing decrease relative to the costs of news utility. Moreover, inattention has the additional benefit that the agent overconsumes less.\footnote{Thus, avoiding overconsumption and news utility are two forces that drive inattention. Which of these forces dominates cannot be inferred from the proof of Proposition 13. However, for the parameter ranges I consider in the quantitative section, the avoidance of news utility is the more important force.}

To facilitate understanding of the model’s implications when the agent can choose to look up his portfolio, I will explain the agent’s decision-making problem in four periods in his end of life, i.e., periods $T$, $T-1$, $T-2$, and $T-3$. Let me start with the agent’s portfolio share in period $T-1$ assuming that he has looked up his portfolio in period $T-2$, i.e.,

$$\alpha_{T-1} = \frac{\mu_0 - r^f_0 + \sigma_0 \frac{E_t[\eta(\lambda-1) \int_{s-t}^\infty (s-\tilde{s})dF(\tilde{s})]}{1+\gamma(\eta(1-\tilde{F}(T-1)) + \eta\lambda(1-\tilde{F}(T-1)))}}{\sigma_0^2}. $$


For now, I ignore the term containing $F_t(r_{T-1})$, which I explain in Section 19.2, and focus on the terms that are known from the static model. As can be seen, there exists a lower bound for $h$ such that the agent’s portfolio share in period $T-1$ is zero for any realization of $r_{T-1}$; i.e., he behaves consistent with Samuelson’s colleague. I now assume that $h = h$ and ask if the agent would look up his portfolio in period $T-1$ or be inattentive. Abstracting from the difference in consumption utility terms, he will look up his portfolio iff

$$\alpha_{T-2}(1 + \gamma \beta)\sigma E[\eta(\lambda - 1) \int_{s}^{\infty} (s - \tilde{s})dF(\tilde{s})] > \alpha_{T-2}\sqrt{2} \sigma E[\eta(\lambda - 1) \int_{s}^{\infty} (s - \tilde{s})dF(\tilde{s})]$$.

Contemporaneous and prospective news utility today is proportional to $1 + \gamma \beta$ while news utility tomorrow over the realization of returns today and tomorrow is proportional to $\beta \sqrt{2}$. Because $1 + \gamma \beta > \beta \sqrt{2}$ is a reasonable parameter combination and the agent consumes more if he looks up his portfolio $E_{T-2}[log(C_{T-1})] > log(C_{T-1}^m)$, as can be seen in equations (40) and (42), I conclude that he is not unlikely to look up his portfolio. Let me simply suppose he does so and move on to the optimal portfolio share in period $T-2$, which differs from the one in period $T-1$ in that the expected sensation of bad news, i.e., $\sigma E[\eta(\lambda - 1) \int_{s}^{\infty} (s - \tilde{s})dF(\tilde{s})]$, is multiplied by $\frac{1 + \gamma \beta}{1 + \beta} < 1$. As I picked $h = h$, the portfolio share in period $T-2$ has to be positive for some realizations of $r_{T-2}$; thus, the agent chooses a higher portfolio share early in life because he can diversify over time.

Now, again omitting differences in consumption utilities, the agent will find it optimal to look up his portfolio in period $T-2$ iff

$$(\alpha_{T-3}(1 + \gamma (\beta + \beta^2)) + \beta \alpha_{T-2}(1 + \gamma \beta))\sigma E[\eta(\lambda - 1) \int_{s}^{\infty} (s - \tilde{s})dF(\tilde{s})] > \beta \alpha_{T-3}\sqrt{2}(1 + \gamma \beta)\sigma E[\eta(\lambda - 1) \int_{s}^{\infty} (s - \tilde{s})dF(\tilde{s})].$$

This consistency constraint is much less likely to hold than the previous one because $\alpha_{T-2}$ is positive whereas $\alpha_{T-1}$ is zero. Thus, the agent experiences news utility, which is painful in expectation, in period $T-2$ and $T-1$ as opposed to period $T-1$ only. Thus, being inattentive in period $T-2$ causes a first-order increase in expected utility that is proportional to $\sigma = \sqrt{h} \sigma_0$. To not smooth consumption perfectly in period $T-2$, however, causes a decrease in expected utility that is proportional to his consumption level and thus proportional to the portfolio return, which is proportional to $\mu = h \mu_0$. Finally, let me suppose that the condition does not hold, i.e., the agent remains inattentive in period $T-2$, and move on to his portfolio share in period $T-3$, if he has looked up his portfolio in period $T-4$. The mere difference of his portfolio share in period $T-3$ relative to the one in period $T-1$ is that the expected sensation of bad news is multiplied by $\frac{1 + \gamma \beta}{1 + \beta} < 1$. Thus, the mere difference to his portfolio share in period $T-2$ is that the expected sensation of bad news is multiplied by $\frac{\sqrt{2}}{2} < 1$. Accordingly, the agent chooses a higher portfolio share if he will be inattentive in period $T-2$ because the expected return of two periods is $2\mu$ while the expected sensation of bad news increases by $\sqrt{2}$. 

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19.1.2 Comparative statics about inattention

The agent trades off the benefits from smoothing consumption perfectly with the costs of experiencing more news utility. Moreover, his portfolio share is higher if \( j_1 \) is high, i.e., if he plans to be inattentive for a while. \( j_1 \), in turn, is determined by the agent’s decision of whether or not to look up his portfolio. But what affects the agent’s consideration whether or not he should look up his portfolio? I now illustrate several comparative statics about the cost and benefit from inattention. Suppose the agent plans to look up his portfolio in period \( t + j_1 \) and has looked up his portfolio in period \( t - i \). When the agent decides whether or not to look up his portfolio in period \( t \), he considers that he would experience news utility if he looks up his portfolio but also that he would consume more. Thus, he compares the “benefit of delaying news utility” in period \( t \) given by

\[
-\sqrt{\alpha_{t-i}}(1 + \gamma \sum_{\tau=1}^{T-t} \beta^\tau) \sigma E[\eta(\lambda - 1) \int_s^{\infty} (s - \tilde{s})dF(\tilde{s})],
\]

with the “cost of less consumption utility” in period \( t \), i.e., \( \log(\rho^{\text{in}}) - E[\log(\rho_t)] \). However, he also considers that he will experience more news utility in period \( t + j_1 \). The “cost of delayed news utility” in period \( t + j_1 \) is given by

\[
-(\sqrt{j_1}E[\alpha_t] - \alpha_{t-i}\sqrt{i + j_1})(1 + \gamma \sum_{\tau=1}^{T-t-j_1} \beta^\tau) \sigma E[\eta(\lambda - 1) \int_s^{\infty} (s - \tilde{s})dF(\tilde{s})].
\]

Thus, in terms of news utility, the “benefit of inattention” is the “benefit of delaying news utility” plus the “cost of delayed news utility”. I first formalize the implications in the following proposition to then explain each in detail.

**Corollary 3. In the beginning of life, if \( t \) is small, the expected benefit of inattention is positive. Toward the end of life, if \( t \) is large, the expected benefit of inattention is decreasing. Moreover, the benefit of inattention is decreasing in \( r_{t-i} + \ldots + r_{t-i-j_0} \).**

In the beginning of life the optimal portfolio share has converged such that \( E[\alpha_{t-i}] = E[\alpha_t] \) and the expected benefit of inattention is necessarily positive. As \( \sqrt{i + j_1} > \sqrt{i + j_1} \), the agent prefers to ignore his portfolio in order to reduce his overall expected disutility from fluctuations in news. Toward the end of life the expected benefit of inattention is decreasing, as \( E[\alpha_{t-i}] > E[\alpha_t] \). The optimal portfolio share converges if the agent’s horizon becomes large and thus decreases rapidly toward the end of life. Therefore, \( i \) and \( j_1 \) are small toward the end of life, which itself results in more frequent readjustments as the benefit of inattention, which is proportional to \( \sqrt{i + j_1} \), is decreasing. Thus, looking up the portfolio more often itself results in a reduction of the portfolio share as the agent cannot benefit from inattention as much. Last, if the agent experienced an adverse return realization in period \( t - i \) such that \( \alpha_{t-i} > E[\alpha_t] \), which I will explain in the next section, the benefit of inattention is reduced and the agent looks up his portfolio again earlier. The optimal level of \( j_1 \) is increasing in the agent’s horizon and the history of returns \( r_{t-i} + \ldots + r_{t-i-j_0} \). Note that, the agent is attentive every period if his portfolio share is zero in the end of life as \( R^f > R^d \). However, the portfolio share is not necessarily zero in the last ten years of life, which depends on the realization of returns. In the beginning of life, the portfolio share is
typically nonzero and the average interval of inattention will be eight to fourteen months depending on the parametrization. Figure 6 displays the news-utility and standard agents portfolio shares at different points in their life cycle and indicates how long the news-utility agent is planning to be inattentive.\footnote{The parameter values are $12\mu = 7\%$, $12r_f = 2\%$, and $\sqrt{12}\sigma_r = 20\%$ and the preference parameters are $\beta = 0.995 \approx 0.98\pi$, $\eta = 1$, $\lambda = 2$ or $\lambda = 2.5$, and $\gamma = 0.8$.}

![Figure 6: News-utility and standard agents’ portfolio shares at different points over the life cycle with $\lambda = 2$ and $\lambda = 2.5$.](image)

It can be seen that the news-utility agent’s portfolio share is not only increasing in his horizon but also decreasing in the return realization, which I explain next.

19.2 Predictions about rebalancing

If the agent stays inattentive, he does not rebalance his portfolio as the return on his wealth is given by $(\left(\frac{R_f}{i}\right) + \alpha_t - i(\prod_{k=1}^{i}R_{t-i+k} - \left(\frac{R_f}{i}\right)))$. However, if he looks up his portfolio, he has a motive for extensive rebalancing or buy-low-sell-high investing because his portfolio share is decreasing in the return realization. However, the agent does not necessarily buy the risky asset in bad times, rather, his end-of-period asset holdings may increase in the market return by more than implied by a constant portfolio share.

19.2.1 Extensive rebalancing in life-cycle portfolio theory

I refer to extensive rebalancing as a buy-low-sell-high investment strategy, i.e., the portfolio share is decreasing rather than constant in the return realization.

**Definition 14.** The agent rebalances extensively if $\frac{\partial\alpha_t}{\partial (r_{t-\lambda} + \ldots + r_{t-1})} < 0$.

**Proposition 14.** If he looks up his portfolio, the news-utility agent rebalances extensively. Moreover, iff $\gamma < 1$, the degree of extensive rebalancing is decreasing in the agent’s horizon.
The basic intuition for extensive rebalancing is that, upon a favorable return realization, the agent wants to realize the good news about consumption and liquidates his risky asset holdings. In contrast, upon an adverse return realization, the agent prefers to not realize all of the bad news associated with future consumption. Rather, he wants to keep the bad news in future consumption more uncertain and thus increases his portfolio share. This allows him to effectively delay the realization of bad news until the next period by which point his expectations will have decreased.

To illustrate this result, I now explain the derivation of the optimal portfolio share in period $T - 1$ assuming that the agent has looked up his portfolio in period $T - 2$. The first derivative of the agent’s continuation value is explained in the static portfolio choice application (Section 17.1) and as above given by $\beta(\mu - r^f - \alpha_{T-1} \sigma^2 + E[\eta(\lambda - 1) \int_{r_1}^{r_T} (r_T - \tilde{r}) dF(r(\tilde{r}))])$. Moreover, the portfolio share affects prospective news utility, as $\log(C_T) = \log(W_{T-1} - C_{T-1}) + r^f + \alpha_{T-1}(r_T - r^f) + \alpha_{T-1}(1 - \alpha_{T-1}) \frac{\sigma^2}{2}$. Note that, $W_{T-1} - C_{T-1}$ and $\alpha_{T-1}$ are stochastic from the perspective of period $T - 2$ but deterministic from the perspective of period $T - 1$, whereas the realization of $r_T$ is stochastic from the perspective of period $T - 2$ and $T - 1$. Because the agent takes his prior beliefs $F^T_{r_{T-1}} = W_{T-1} - C_{T-1}$ and $F^T_{r_{T-1}}$ as given and $\log(C_T)$ is increasing in $r_{T-1}$, the first derivative of prospective news utility with respect to the portfolio share is

$$\frac{\partial \gamma \beta n(F^T_{r_{T-1}}, T - 2)}{\partial \alpha_{T-1}} = \gamma \beta (\mu - r^f + \alpha_{T-1} \sigma^2)(\eta F(r_{T-1}) + \eta \lambda (1 - F(r_{T-1}))).$$

This term implies that $\alpha_{T-1}$ is decreasing in the realization of $r_{T-1}$. If $F(r_{T-1})$ is high, then the agent experiences relatively good news about future consumption and $\alpha_{T-1}$ is relatively low. Or, if $F(r_{T-1})$ is low then the agent experiences relatively bad news about future consumption and $\alpha_{T-1}$ is relatively high.\(^{113}\)

19.2.2 End-of-period asset holdings in life-cycle portfolio theory

Extensive rebalancing, however, does not necessarily imply that the agent buys the risky asset in the event of a bad return realization. Rather, he might leave his risky wealth untouched or decrease his risky asset holdings. In fact, his end-of-period risky asset holdings, i.e., $W_t(1 - \rho_t) \alpha_t$, may increase more with the market return than a constant portfolio and consumption share, as displayed by the standard agent, would imply.

\(^{113}\)Additionally, a decreasing portfolio share implies that the news-utility agent portfolio holdings are predictable by past shocks. The standard agent’s risky asset holdings in the end of period $t$ are given by $(W_t - C_t) \alpha_t = ((W_{t-1} - C_{t-1}) R_t - C_t) \alpha_t$ and thus linearly increasing in the standard agent’s portfolio return in period $t$. In contrast, the news-utility agent’s risky asset holdings in the end of period $t$, assuming he has looked up his portfolio in period $t - 1$, are given by $(W_t - C_t) \alpha_t = ((W_{t-1} - C_{t-1}) R_t - C_t) \alpha_t$ and are thus increasing in the portfolio return. However, they are not increasing linearly but less than linearly in the neighborhood of the return’s mean, as $\frac{\partial \alpha}{\partial r} < 0$. Moreover, the period $t + 1$ change in risky asset holdings is given by $(R_t + \alpha_t (R_{t+1} - R_t) (1 - \rho_{t+1}) \alpha_{t+1} - \alpha_t$ and thus predictable by the period’s $t$ return realization, whereas it only depends on the period $t + 1$ return realization in the standard model. Therefore, news-utility risky asset holdings are more smooth than the standard agent’s risky asset holdings.
Corollary 4. If \( \gamma < \bar{\gamma} \), the news-utility agent’s end-of-period asset holdings increase with the market return by more than a constant portfolio share would imply

\[
\frac{\partial}{\partial (R_t \ldots R_{t-1})} \left( (R^f_t + \alpha_t - (R^f_{t-1})) (1 - \rho_t) \alpha_t \right) > 0.
\]

In any period \( t \), the agent’s end-of-period risky asset holdings are given by \( W_t (1 - \rho_t) \alpha_t \) whereas the beginning-of-period asset holdings are given by \( W_{t-1} (1 - \rho_{t-1}) \alpha_{t-1} R_t \). I have shown that \( \alpha_t \) is decreasing in the return realization. But, \( 1 - \rho_t \), i.e., one minus the agent’s consumption-wealth ratio, is increasing in the return realization as can be easily seen in equation (40). If \( 1 - \rho_t \) is increasing in the return realization then optimal end-of-period asset holdings in the event of a favorable shock are relatively high. Intuitively, the consumption-wealth ratio \( \rho_t \) is increasing in the return realization because, in the event of an adverse shock, the agent wishes to delay the reduction in consumption until his expectations have decreased.\(^{114}\) This increase may dominate the desire for extensive rebalancing, if \( \gamma \) is small. For illustration, suppose \( \gamma = 0 \), in which case \( \alpha_t \) is constant but \( 1 - \rho_t \) is increasing in the return realization; in this case, the agent engages in insufficient rebalancing.\(^{115}\)

19.2.3 The net effect on the agent’s asset holdings

Now, I want to assess the net effect of the two motives, i.e., does the news-utility agent move money in or out of his risky account in the event of favorable or adverse shocks? I start with the standard agent, whose change in risky asset holdings is proportional to \( (R^f_t + \alpha^s(R_t - R^f))(1 - \rho^s) \alpha^s - \alpha^s R_t \). Both terms are negative whenever the portfolio share is not larger than one, i.e., \( \alpha^s < 1 \), and the return realization is not too low, such that the standard agent will typically move money out of the risky account and the amount of the money transfer is monotonically increasing in the return realization.

The news-utility agent’s change in risky asset holdings is proportional to \( (R^f_t + \alpha_{t-1}(R_t - R^f))(1 - \rho_t) \alpha_t - \alpha_t R_t \). Again, both terms are likely to be negative as \( \alpha_{t-1} < 1 \), however, as both \( \alpha_t \) and \( \rho_t \) are decreasing in the realization of \( R_t \) the overall response becomes ambiguous instead of uniformly decreasing as the case for the standard agent. Figure 7 illustrates that the news-utility variation in \( \alpha_t \) and \( \rho_t \) may simultaneously lead to the change in his risky asset holdings being biased towards zero for each \( t \), as in the first scenario, or induce the news-utility agent to sell stocks when the market is going down because \( \gamma \) is low, as in the second scenario.\(^{116}\)

\(^{114}\)This result about delaying consumption adjustments is analyzed in Pagel (2012b), as it brings about excess smoothness in consumption, and Pagel (2012c), as it brings about predictability in stock returns.

\(^{115}\)In a general-equilibrium asset-pricing model, in which consumption is exogenous and returns are endogenous, the latter motive drives strongly countercyclical expected returns as shown by Pagel (2012b).

\(^{116}\)The parameter values for the annual horizon are \( \mu = 8\% \), \( r^f = 2\% \), and \( \sigma_r = 20\% \) and the preference parameters are \( \eta = 1 \), \( \lambda = 2 \), and \( \gamma = 0.8 \) versus \( \gamma = 0.2 \).
Commitment and welfare implications

The monotone-personal equilibrium is different from the one the agent would like to precommit to, thus he is subject to a commitment or time inconsistency problem. The agent behaves time inconsistently because he enjoys the pleasant surprise of increasing his consumption or portfolio share above expectations today, whereas yesterday he also considered that such an increase in his consumption and portfolio share would have increased his expectations. Thus, today’s self thinks inherently differently about today’s consumption and portfolio share than yesterday’s self. And moreover, today’s self wants to consume and enjoy the good news of potentially higher future consumption before his expectations catch up. In the next proposition, I formalize that the agent would like to precommit to consume less, invest less, look up his portfolio less often, but if he does, rebalance more extensively.

Proposition 15. (Comparison to the monotone-precommitted equilibrium)

1. The monotone-precommitted consumption share does not correspond to the monotone-personal consumption share, if the agent is attentive and $\gamma < 1$. In the monotone-precommitted equilibrium, the agent chooses a lower consumption share and the gap increases in good states, i.e., $\rho_t > \rho_c^t$ and $\frac{\partial(\rho_t - \rho_c^t)}{\partial(r_t + \ldots + r_{t-1} + i)} > 0$. If the agent is inattentive, he chooses the same consumption share, i.e., $\rho_t^{in} > \rho_c^{in}$.

2. The monotone-precommitted portfolio share does not correspond to the monotone-personal portfolio share. In the monotone-precommitted equilibrium, the agent chooses a lower portfolio share and the gap increases in good states, i.e., $\alpha_t > \alpha_c^t$ and $\frac{\partial(\alpha_t - \alpha_c^t)}{\partial(r_t + \ldots + r_{t-1} + i)} > 0$.

3. The cost of less consumption utility from not looking up the portfolio are lower on the precommitted path.

4. Extensive rebalancing is more pronounced on the precommitted path.
I first explain the precommitted equilibrium in greater detail. The monotone-precommitted equilibrium maximizes expected utility and is derived under the premise that the agent can precommit to an optimal history-dependent consumption path for each possible future contingency and thus jointly optimizes over consumption and beliefs. In contrast, the monotone-personal equilibrium is derived under the premise that the agent takes his beliefs as given, which is why he would deviate from the optimal precommitted path. I define the model’s “monotone-precommitted” equilibrium in the spirit of the choice-acclimating equilibrium concept in Koszegi and Rabin (2007) as follows.

**Definition 15.** \( \{ C^i_t, C_t, \alpha_t \}_t \in \{1, \ldots, T \} \) is a monotone-precommitted equilibrium if, in all periods \( t \), the admissible consumption and portfolio functions \( C^i_t \), \( C_t \), and \( \alpha_t \) maximize (39) subject to (33) under the assumption that all future consumption and portfolio functions correspond to \( C^i_{t+\tau}, C_{t+\tau} \), and \( \alpha_{t+\tau} \). In each period \( t \), the agent’s maximization problem determines both the agent’s fully probabilistic rational beliefs \( \{ F^t_{C_t+\tau} \}_{\tau=0}^{T-t} \) as well as consumption \( \{ C^i_{t+\tau}, C_{t+\tau} \}_{\tau=0}^{T-t} \).

The monotone-precommitted equilibrium is derived in Appendix I.2. Suppose that the agent can precommit to an optimal consumption path for each possible future contingency. In his optimization problem, the agent’s marginal news utility is no longer solely composed of the sensation of increasing consumption in that contingency; additionally, the agent considers that he will experience fewer sensations of good news and more bad news in all other contingencies. Thus, marginal news utility has a second component, \( -u'(C_{T-1}) (\eta(1-F^t_{C_t}^{-1}(C_t)) + \eta \lambda F^t_{C_t}^{-1}(C_t)) \), which is negative such that the precommitted agent consumes and invests less than the non-precommitted agent, as can be easily seen in equations (40) and (41). The additional negative component dominates if the consumption realization is above the median, i.e., \( F^t_{C_t}^{-1}(C_t) > 0.5 \). Thus, in the event of good return realizations, precommitted marginal news utility is negative. In contrast, non-precommitted marginal news utility is always positive because the agent enjoys the sensation of increasing consumption in any contingency. Thus, the additional negative component in marginal news utility implies that the precommitted agent does not overconsume even if \( \gamma < 1 \). Moreover, the difference between the precommitted and non-precommitted consumption paths is less large in the event of adverse return realizations because increasing risky asset holdings is the optimal response even on the precommitted path. Thus, the degree of the agent’s time inconsistency is reference dependent, which also implies that the motive for extensive rebalancing is more pronounced on the precommitted path. However, in inattentive periods, the agent does not overconsume so long as \( \gamma > \frac{1}{\lambda} \), as explained in Section 18.3. Therefore, the difference between inattentive and attentive consumption is less large for the precommitted agent, which decreases the benefits from looking up his portfolio.

A simple calculation reveals the quantitative magnitude of the welfare implications of inattention. Suppose a period’s length is one month. If the news-utility agent has to look up his portfolio every period, his portfolio share would be zero. If he can be inattentive, however, his wealth would achieve a return of around four percent per year which accumulates over time. Thus, he would be willing to give up a considerable share of his initial wealth or the return to his wealth to separate his accounts. Moreover, he has a first-order willingness to pay for a portfolio manager who rebalances actively. The simple reason is that his log portfolio return is given by \( \log((R^f)^i + \alpha_{t-i}((\prod_{k=1}^i R_{t-i+k} - (R^f)^i))) \) while his return under active rebalancing would be given by \( \log(\prod_{k=1}^i (R^f + \alpha_{t-i}(R_{t-i+k} - R^f))) \). The variance of the former is strictly higher than the vari-
ance of the latter and the news-utility agent cares about risk to a first-order extent. This effect matches the empirical evidence provided by French (2008), who finds that the typical investor forgoes about 67 basis points of the market’s annual return for active investing. Furthermore, the news-utility agent would pay a portfolio manager who commits him to be inattentive more often, as it prevents overconsumption. More generally, as the separate accounts help the agent to exercise self control, my welfare results relate to the idea of mental accounting (Thaler (1980)).

19.4 Extension to signals about the market

Even if people are deliberately inattentive, it seems unrealistic to assume that they do not receive any news about what is happening in their brokerage account. Therefore, I extend the model such that, in each period, the agent receives a signal about the value of his asset holdings in the brokerage account and then decides if he stays inattentive or not. In Section 19.4.1, I first argue that the equilibrium under consideration will be completely unaffected by the signal if its information content is low. In this case, the signals do not affect the agent’s attentiveness and inattentive consumption at all but affect his attentive consumption and rebalancing. His attentive consumption and rebalancing behavior will depend on how his portfolio compares to the signals he received. In Section 19.4.2, I then outline the implications of signals that have large information content such that they would affect the agent’s attentiveness and thus consumption behavior. Nevertheless, I argue that the signal’s effect on consumption and attentiveness is modest and confirm my conjectures using simulations.

19.4.1 Signals with low information content

In the following, I consider signals about the market that have low information content and argue that the agent’s attentiveness and thus consumption behavior are completely unaffected. The basic idea is that the potential good news from looking up the portfolio even if the signal happens to be particularly favorable are outweighed by the expected disutility from looking up the portfolio. If the presence of the signal does not affect the agent’s plans to look up his portfolio, they do not affect his consumption out of the checking account either. The agent does not want to consume more in the event of a favorable signal because such an increase in current consumption would imply a decrease in future consumption, if the agent does not plan to change the date of when to look up his brokerage account. Since the bad news about the decrease in future consumption outweigh the good news of overconsumption, the agent sticks to his original consumption plan independent of the signal. Thus, the agent experiences news utility merely over his consumption in the future after he has looked up his brokerage account.

19.4.2 Signals with high information content and the Ostrich effect

In the following, I will outline what happens if the signal’s information content is so large that it does affect the agent’s plans to look up his portfolio. I assume that the agent has looked up

\[\text{As a back-of-the-envelope calculation, suppose the agent’s portfolio share is 0.4. Monthly as opposed to yearly rebalancing will result in a reduction of risk given by 0.3% and an increase in the expected return of around 0.04%.

Thus, the agent would be willing to give up } \left(-0.3E[\eta(\lambda-1)\int_{-\bar{s}}^{\bar{s}}(s-\bar{s})dF(\bar{s})]\right)-0.04 \right) \text{ of the annual stock-market return, which matches the empirical evidence for } \eta = 1 \text{ and } \lambda \approx 2.6.\]
his portfolio in period \( t - i \), plans to look up his portfolio in period \( t + j_1 \), and receives a signal \( \hat{r}_t = r_t + \epsilon_t \) with \( \epsilon_t \sim N(0, \sigma^2_\epsilon) \) about \( r_t \) in period \( t \). In this section, I will show that the agent’s willingness to look up his portfolio increases in the realization of the signal. This allows me to conjecture how the equilibrium looks like given my previous findings.

In period \( t \), the agent will look up his portfolio after receiving a particularly favorable signal, will not react to signals in some middle range, and will want to refuse to look up his portfolio for particularly bad signals. If he ignores his portfolio, he does not consume time-inconsistently because he does not experience news utility over inattentive consumption but merely over future consumption after having looked up the portfolio. Absent time-inconsistent overconsumption, his previous selves have no reason to restrict the funds in the checking account. However, his previous selves may want to affect his decision whether or not to look up his portfolio in period \( t \). But, his previous selves cannot affect his period \( t \) decision to look up his portfolio. Although his previous selves can force his period \( t \) self to look up his portfolio, namely, when his period \( t \) self runs out of funds in the checking account. However, his previous selves will never want to make him look up his portfolio earlier than his period \( t \) self wants as the precommitted path features looking up the portfolio fewer times than the non-precommitted path. Because there is no time inconsistency associated with inattentive consumption, I can assume that the previous selves stored sufficient funds in the checking account to allow the investor to remain inattentive longer, in the event of adverse signals, until he would look up his portfolio on the precommitted path.

Now, I show that the agent is more likely to look up his portfolio after a favorable realization of the signal; a behavior that has been termed the Ostrich effect. If the agent looks up his portfolio, he will experience news utility over the signal. In particular, the agent expects contemporaneous and prospective news utility in period \( t \) assessed conditional on the signal. In particular, the agent expects contemporaneous and prospective news utility from looking up his return as follows

\[
\alpha_{t-i}(1 + \gamma \sum_{\tau=1}^{T-t} \beta^\tau) \int_{-\infty}^{\infty} \eta(\lambda - 1) \int_{-\infty}^{\infty} (r - \tilde{r}) dF_{r_{t+1} + \ldots + r_{t-j_1+1} | \hat{r}_{t-1} \ldots \hat{r}_{t-j_1+1}}(\tilde{r}) dF_{r_{t+1} + \ldots + r_{t-j_1+1} | \hat{r}_{t-1} \ldots \hat{r}_{t-j_1+1}}(r).
\]

If \( \hat{r}_t \) is high then \( r_t \) is more likely to be high and the agent is likely to experience positive news utility. If the agent decides to ignore his portfolio, he will experience prospective news utility over the signal. In that case, prospective news utility in period \( t \) is

\[
\sum_{j=1}^{j_1} \beta_j \cdot p_{t+j} \cdot \alpha_{t-i}(1 + \gamma \sum_{\tau=1}^{T-t-j} \beta^\tau) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \eta(\lambda - 1) \mu(r - \tilde{r}) dF_{r_{t+1} + \ldots + r_{t-j_1+1}}(r, \tilde{r}).
\]

Here, \( p_{t+j} \) denotes the probability of looking up the portfolio in period \( t + j \) conditional on period \( t \) information, such that \( \sum_{j=1}^{j_1} p_j = 1 \), i.e., the investor knows he cannot delay to look up his portfolio beyond period \( t + j_1 \). For a simplified comparison, suppose the agent will look up his portfolio in period \( t + 1 \), i.e., \( j_1 = 1 \) and \( p_{j_1} = 1 \). In expectation, it is more painful to look up the true return than merely experiencing prospective news utility simply because more uncertainty will be resolved. But, the difference between the two is smaller when the agent received a more favorable signal. The reason is that expected marginal news utility from resolution of \( \epsilon_t \) is less if \( \hat{r}_t \) is high because the
agent considers news fluctuations high up on the utility curve. Accordingly, after having received a favorable signal, the agent is more likely to look up his portfolio. This behavior is commonly known as the “Ostrich effect” and supported by empirical evidence (Karlsson, Loewenstein, and Seppi (2009)).

Nevertheless, this example also suggests that it is not unlikely that the agent will choose to ignore the signal and not adjust his attentiveness and thus consumption. After all, in the event of a favorable signal, he experiences news utility over the signal or news utility over the actual realization. Thus, the sole reason that he is more likely to look up the realization is that the expected costs of receiving more information are lower conditional on a favorable signal. A simple simulation confirms this conclusion quantitatively. Figure 8 compares the news utility experienced over the signal to the expected news utility conditional on the signal. It can be seen that the quantitative difference is decreasing in the realization of the signal, but is very small in comparison to the overall variation in news utility. More details can be found in Appendix 19.4.

![Figure 8: Comparison of experienced news utility over the signal and expected news utility conditional on the signal.](image)

However, this comparison does not take into account that the agent cares more about contemporaneous than prospective news, i.e., the expected news utility is weighted by $1 + \gamma \sum_{\tau=1}^{T-1} \beta^\tau$ while prospective news utility is weighted by $\sum_{j=1}^{T-1} \beta^j p_{r+j}(1 + \gamma \sum_{\tau=1}^{T-t+j} \beta^\tau)$. But, if the agent’s horizon is long and a period is short the difference between the two are small. Moreover, if a period is short the contemporaneous realization of the return has very small quantitative implications for immediate consumption. Additionally, if a period’s length is small the agent compares experiencing news utility over a one-period realization of the signal to experiencing news utility over the multi-period uncertainty left.

I choose a quantitative example that should produce a larger difference in experienced and expected news utility than the model under consideration. I choose an annual horizon as a period’s length such that the signal has large information content and choose a variance of the signal that is equally variable as the stock-market variance because more noisy signals increase this difference in news utility. The parameter values are $\mu = 8\%$, $r_f = 2\%$, and $\sigma_r = \sigma_e = 20\%$ and the preference parameters are $\eta = 1$ and $\lambda = 2.5$. 

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Now, I ask how the agent’s behavior is perceived by an outsider. The outsider acquires all available information because he does not experience any news utility over the agent’s portfolio or consumption. Will the outsider perceive the agent’s behavior as overconfident or extrapolative? After all, whenever the agent decides to ignore his portfolio, his behavior, as reflected in his portfolio share, is based on a different information set than the outsider’s one. The agent’s information set in each period \( t \) is denoted by \( I_t \) and might contain today’s return \( r_t \) and all past returns or only the returns past \( r_{t-i} \). Even though the agent receives the signals \( \hat{r}_{t-i+1}, \ldots, \hat{r}_t \), his behavior, as reflected in his portfolio share will be based on the returns past \( r_{t-i} \) only. This portfolio share is denoted by \( f_\alpha(I_t) \). In contrast, the outsider’s information is denoted by \( I_o^t \) and contains \( r_t \) and all past \( r_{t-i} \).

I say that if \( f_\alpha(I_t) > f_\alpha(I_o^t) \) the agent is perceived to be overconfident by an outsider. Whenever the agent ignores his portfolio because the return realization is likely to be bad, he will have chosen a higher portfolio share as if he expects high returns. Thus, an outsider would perceive his behavior as overconfident.

I say that if \( f_\alpha(I_t) = \rho f_\alpha(I_o^t) + (1 - \rho) f_\alpha(I_o^{t-i}) \) the agent is perceived to have extrapolative expectations by an outsider. Whenever the agent decides to ignore his portfolio, but the outsider acquires all information, then the agent’s behavior, as reflected in his portfolio share, is based on an outdated information set; thus, he looks extrapolative. Overconfidence and extrapolative expectations are two descriptive theories for beliefs that have been assumed in a variety of behavioral-finance papers to explain stock prices, e.g., Scheinkman and Xiong (2003), Malmendier and Tate (2005, 2008), Choi (2006), Hirshleifer and Yu (2011), and Barberis, Greenwood, Jin, and Shleifer (2013).

**20 Quantitative Implications for Life-Cycle Consumption and Portfolio Choice and Empirical Evidence**

In Section 20.1, I assess the quantitative performance of the model with a structural estimation exercise using household portfolio data on participation and shares. I first choose an empirically plausible parametrization of the environmental parameters and the period’s length to then estimate the preference parameters. For the parametrization, I explore how often the investor chooses to look up his portfolio given a plausible calibration of the environmental and preference parameters. I then use the implied average length of inattention as the period’s length in a standard life-cycle model. This standard model assumes power utility \( u(c) = \frac{c^{1-\theta}}{1-\theta} \) with a coefficient of risk aversion \( \theta \) instead of relying on log utility and is outlined in Appendix I.4. Section 20.1.1 quickly describes the household portfolio data, Section 20.1.2 presents results for different calibrations of the period’s length, Section 20.1.3 provides details on the identification, and Section 20.1.4 describes the estimation procedure and compares the estimates with the existing literature. Finally, in Section 20.2, I provide some suggestive empirical evidence for extensive rebalancing in portfolio choice using PSID household portfolio data.
20.1 Structural estimation

To validate the model quantitatively, I structurally estimate the preference parameters by matching the average empirical life-cycle profile of participation and portfolio shares using household portfolio data of the Survey of Consumer Finances (SCF) from 1992 to 2007.

20.1.1 Data

The SCF data is a statistical survey of the balance sheet, pension, income and other demographic characteristics of families in the United States sponsored by the Federal Reserve Board in cooperation with the Treasury Department. The SCF is conducted of six survey waves from 1992 to 2007 but does not survey households consecutively. I construct a pseudo-panel by averaging participation and shares of all households at each age following the risk-free and risky-asset definitions of Flavin and Nakagawa (2008).

In addition to the age effects of interest, the data is contaminated by potential time and cohort effects, which constitutes an identification problem as time minus age equals cohort. In the portfolio-choice literature, it is standard to solve the identification problem by acknowledging age and time effects, as tradable and nontradable wealth varies with age and contemporaneous stock-market happenings are likely to affect participation and shares, but omitting cohort effects (Campbell and Viceira (2002)). In contrast, it is standard to omit time effects but acknowledge cohort effects in the consumption literature (Gourinchas and Parker (2002)). I employ a new method, recently invented by Schulhofer-Wohl (2013), that solves the age-time-and-cohort identification problem with minimal assumptions. In particular, the method merely assumes that age, time, and cohort effects are linearly related. I first estimate a pooled OLS model, whereby I jointly control for age, time, and cohort effects and identify the model with a random assumption about its trend. Then, I estimate this arbitrary trend together with the structural parameters, which results in consistent estimates using data that is uncontaminated by time and cohort effects. This application of Schulhofer-Wohl (2013) to household portfolio data is an important contribution, as portfolio profiles are highly dependent on which assumptions the identification is based on, as made clear by Ameriks and Zeldes (2004).

Figure 9 displays the uncontaminated empirical profiles for participation and portfolio shares as well as labor income. Both participation and portfolio shares are hump shaped over the life cycle. The predicted income profile is lower than the profile containing the disturbances because the SCF oversamples rich households but provides weights to adjust the regressions.

Moreover, the model’s quantitative predictions about consumption and wealth accumulation are compared to the empirical profiles as inferred from the Consumer Expenditure Survey (CEX). The CEX is a survey of the consumption expenditures, income, balance sheet, and other demographic characteristics of families in the United States sponsored by the Bureau of Labor Statistics.

20.1.2 Calibrating the period's length

I estimate a standard life-cycle model in which the investor looks up his portfolio each period. As can be inferred from the theoretical analysis, an important calibrational degree of freedom constitutes the model’s period length, which I determine first. As a first step, I will calibrate the risky
and risk-free return moments, i.e., $h\mu_0$, $\sqrt{h}\sigma_0$, and $hr_0^f$, to a monthly investment horizon if $h = 1$. Then, $h = 12$ would recalibrate the model to an annual horizon. The literature suggests fairly tight ranges for the parameters of the log-normal return, i.e., $12\mu_0 = 8\%$, $\sqrt{12}\sigma_0 = 20\%$, and the log risk-free rate, i.e., $12r_0^f = 2\%$. Additionally, I choose the agent’s horizon $T = 60$ years (720 months) and his retirement period $R = 10$ years (120 months), in accordance with the life-cycle literature. Moreover, I set $\eta = 1$, $\lambda = 2.5$, and $\gamma = 0.8$, which are reasonable preference parameter choices given the literature on prospect theory and reference dependence, as I will discuss in Section 20.1.4.

Under this calibration, I find that the agent looks up his portfolio approximately once a year early in life and chooses a zero portfolio share after the start of retirement. Figure 6 in Section 19.1.2 displays the news-utility and standard agents’ optimal portfolio shares. As can be seen, the news-utility agent’s share is increasing in the agent’s horizon and decreasing in the return realization. In contrast, the standard agent’s share is constant in the horizon and return realizations. Not surprisingly, the standard agent accumulates wealth more rapidly, as his portfolio share is one. Beyond these implications for portfolio choice, the news-utility agent’s consumption profile is hump shaped whereas the standard agent’s consumption profile is increasing throughout. Figure 10 displays the theoretical and empirical consumption profiles estimated from CEX data.\footnote{Note that this empirical profile implicitly assumes that households do not retire, which is why consumption is not decreasing too much toward the end of life. I consider this comparison to be more adequate, as the model I use in this section abstracts from labor income.}
I conclude that a yearly investment horizon seems a reasonable calibrational choice that has also been assumed in similar contexts (Benartzi and Thaler (1995) and Barberis, Huang, and Santos (2001)).

20.1.3 Identification

Are the empirical life-cycle participation and portfolio shares profiles able to identify the preference parameters? I am interested in five preference parameters, namely $\beta$, $\theta$, $\eta$, $\lambda$, $\gamma$. As shown in Appendix I.4, both participation and the portfolio share are determined by the following first-order condition

$$
\gamma \frac{\partial \Phi}{\partial \alpha_t} (\eta F_{A_t}^{-1}(A_t) + \eta \lambda (1 - F_{A_t}^{-1}(A_t))) + \frac{\partial \psi}{\partial \alpha_t} = 0
$$

of which I observe the average of all households. $\frac{\partial \Phi}{\partial \alpha_t}$ represents future marginal consumption utility, as in the standard model, and is determined by $\beta$ and $\theta$, which can be separately identified in a finite-horizon model. $\frac{\partial \psi}{\partial \alpha_t}$ represents future marginal consumption and news utility and is thus determined by something akin of $\eta(\lambda - 1)$. $\eta F_{A_t}^{-1}(A_t) + \eta \lambda (1 - F_{A_t}^{-1}(A_t))$ represents the weighted sum of the cumulative distribution function of savings, $A_t$, of which merely the average determined by $\eta 0.5(1 + \lambda)$ is observed. Thus, I have two equations in two unknowns and can separately identify $\eta$ and $\lambda$. Furthermore, participation is determined by the value of $F_{A_t}(A_t)$ at which the average portfolio share becomes zero, which provides additional variation identifying $\eta$ and $\lambda$ separately. Finally, $\gamma$ enters the first-order condition distinctly from all other parameters, and I conclude that the model is identified which can also be verified by deriving the Jacobian that has full rank.

20.1.4 Estimation

At an annual horizon, the literature suggests fairly tight ranges for the parameters of the log-normal return that I match by estimating $\hat{\mu} - \hat{r}^f = 6.33\%$, $\hat{\sigma} = 19.4\%$, and the log risk-free rate, i.e., $\hat{r}^f = 0.86\%$, using value-weighted CRSP return data. Moreover, the life-cycle consumption literature suggests fairly tight ranges for the parameters determining stochastic labor income, i.e.,
labor income is log-normal, characterized by shocks with variance $\sigma_Y$ and a trend $G$ that I roughly match by estimating $\hat{\delta}_Y = 0.12$ and $\hat{G}$ from the SCF data. Moreover, because 25 is chosen as the beginning of life by Gourinchas and Parker (2002), I choose $\hat{R} = 11$ and $\hat{T} = 54$ in accordance with the average retirement age in the US according to the OECD and the average life expectancy in the US according to the UN list. After having calibrated the structural parameters governing the environment $\tilde{E} = (\mu, \sigma, \sigma_Y, G, r^f, R, T)$, I now estimate the preference parameters $\theta = (\eta, \lambda, \gamma, \beta, \theta)$ by matching the simulated and empirical life-cycle profiles for participation and shares. The empirical profiles are the average participation and shares at each age $a \in [1, T]$ across all household observations $i$. More precisely, it is $\alpha_a = \frac{1}{n_a} \sum_{i=1}^{n_a} \alpha_{i,a}$ with $\alpha_{i,a}$ being the household $i$’s portfolio share at age $a$ of which $n_a$ are observed. The theoretical population analogue to $\tilde{\alpha}_a$ is denoted by $\alpha_a(\theta, \Xi)$ and the simulated approximation is denoted by $\hat{\alpha}_a(\theta, \Xi)$. Similarly, I define the empirical percentage of participating households at each age as $\tilde{p}_a$ and its theoretical population analogue is denoted by $p_a(\theta, \Xi)$ and the simulated approximation is denoted by $\hat{p}_a(\theta, \Xi)$. Moreover, I define

$$g(\theta, \Xi) = \left( \begin{array}{c} \tilde{\alpha}_a(\theta, \Xi) - \tilde{\alpha}_a \\ \tilde{p}_a(\theta, \Xi) - \tilde{p}_a \end{array} \right).$$

In turn, if $\theta_0$ and $\Xi_0$ are the true parameter vectors, the procedure’s moment conditions imply that $E[g(\theta_0, \Xi_0)] = 0$. In turn, let $W$ denote a positive definite weighting matrix then

$$q(\theta, \Xi) = g(\theta, \Xi)W^{-1}g(\theta, \Xi)'$$

is the weighted sum of squared deviations of the simulated from their corresponding empirical moments. I assume that $W$ is a robust weighting matrix rather than the optimal weighting matrix to avoid small-sample bias. More precisely, I assume that $W$ corresponds to the inverse of the variance-covariance matrix of each point of $\tilde{\alpha}_a$ and $\tilde{p}_a$, which I denote by $\Omega_{\tilde{\alpha}}^{-1}$ and consistently estimate from the sample data. Taking $\tilde{\Xi}$ as given, I minimize $q(\theta, \tilde{\Xi})$ with respect to $\theta$ to obtain $\hat{\theta}$ the consistent estimator of $\theta$ that is asymptotically normally distributed with standard errors

$$\Omega_{\theta} = (G_{\theta}^tW G_{\theta})^{-1}G_{\theta}^tW[\Omega_{\tilde{\alpha}} + \Omega_{\tilde{p}} + G_{\tilde{\alpha}} \Omega_{\tilde{\alpha}} G_{\tilde{\alpha}}^t] W G_{\theta}(G_{\theta}^tW G_{\theta})^{-1}.$$ 

Here, $G_{\theta}$ and $G_{\tilde{\alpha}}$ denote the derivatives of the moment functions $\frac{\partial g(\theta, \Xi_0)}{\partial \theta}$ and $\frac{\partial g(\theta, \Xi_0)}{\partial \tilde{\Xi}}$, $\Omega_{\tilde{\alpha}}$ denotes the variance-covariance matrix of the second-stage moments as above that corresponds to $E[g(\theta_0, \Xi_0)g(\theta_0, \Xi_0)'$, and $\Omega_{\tilde{p}}^p = \frac{n_a}{n_t} \Omega_{\tilde{p}}$ denotes the sample correction with $n_a$ being the number of simulated observations at each age $a$. As $\Omega_{\tilde{\alpha}}$, I can estimate $\Omega_{\tilde{\alpha}}$ directly and consistently from sample data. For the minimization, I employ a Nelder-Mead algorithm. For the standard errors, I numerically estimate the gradient of the moment function at its optimum. If I omit the first-stage correction and simulation correction the expression becomes $\Omega_{\theta} = (G_{\theta}^t \Omega_{\tilde{\alpha}}^{-1} G_{\theta})^{-1}$. Finally, I can test for overidentification by comparing $\hat{g}(\hat{\theta}, \hat{\Xi})$ to a chi-squared distribution with $T - 5$ degrees of freedom. The calibration and estimated parameters can be found in Table 5.

I refer to the literature regarding the standard estimates for $\beta$ and $\theta$, but discuss the plausibility.
of the news-utility parameter values, i.e., $\eta$, $\lambda$, and $\gamma$, in more detail. In the following, I demonstrate that my estimates are consistent with existing micro evidence on risk and time preferences. In Table 3 in Appendix G, I illustrate the risk preferences over gambles with various stakes of the news-utility and standard agents. In particular, I calculate the required gain $G$ for a range of losses $L$ to make each agent indifferent between accepting or rejecting a 50-50 win $G$ or lose $L$ gamble at a wealth level of 300,000 in the spirit of Rabin (2001) and Chetty and Szeidl (2007).\footnote{In a canonical asset-pricing model, Pagel (2012c) demonstrates that news-utility preferences constitute an additional step towards resolving the equity-premium puzzle, as they match the historical level and the variation of the equity premium while simultaneously implying plausible attitudes towards small and large wealth bets.}

First, I want to match risk attitudes towards bets regarding immediate consumption, which are determined solely by $\eta$ and $\lambda$ because it can be reasonably assumed that utility over immediate consumption is linear. Thus, $\eta \approx 1$ and $\lambda \approx 2.5$ are suggested by the laboratory evidence on loss aversion over immediate consumption, i.e., the endowment effect literature.\footnote{For illustration, I borrow a concrete example from Kahneman, Knetsch, and Thaler (1990), in which the authors distribute a good (mugs or pens) to half of their subjects and ask those who received the good about their willingness to accept (WTA) and those who did not receive it about their willingness to pay (WTP) if they traded the good. The median WTA is $5.25$, whereas the median WTP is $2.75$. Accordingly, I infer $(1 + \eta)(mug) = (1 + \eta \lambda)2.25$ and $(1 + \eta \lambda)(mug) = (1 + \eta)5.25$, which implies that $\lambda \approx 3$ when $\eta \approx 1$. I obtain a similar result for the pen experiment. Unfortunately, thus far, I can only jointly identify $\eta$ and $\lambda$. If the news-utility agent were only to exhibit news utility, I would obtain $\lambda 2.25 \approx 5.25$ and $\eta 2.25 \approx 2.25$, i.e., $\lambda \approx 2.3$ and $\eta \approx 1$ both identified. Alternatively, if I assume that the market price for mugs (or pens), which is $6$ in the experiment (or $3.75$), equals $(1 + \eta)(mug)$ (or $(1 + \eta)(pen)$), then I can estimate $\eta = 0.74$ and $\lambda = 2.03$ for the mug experiment and $\eta = 1.09$ and $\lambda = 2.1$ for the pen experiment. These latter assumptions are reasonable given the induced-market experiments of Kahneman, Knetsch, and Thaler (1990). $\eta \approx 1$ and $\lambda \approx 2.5$ thus appear to be reasonable estimates that are typically used in the literature concerning the static preferences.}

Table 5: This table displays the calibrated and estimated parameters as well as the standard errors of the estimated parameters (in parentheses).

<table>
<thead>
<tr>
<th>$\hat{\mu} - \hat{\mu}'$</th>
<th>$\hat{\sigma}$</th>
<th>$\hat{\sigma}_y$</th>
<th>$\hat{\theta}_t$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\gamma}'$</th>
<th>$\hat{\theta}$</th>
<th>$\hat{\eta}$</th>
<th>$\hat{\lambda}$</th>
<th>$\hat{\gamma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.33%</td>
<td>19.4%</td>
<td>0.12</td>
<td>$e^{Y_t+1-Y_t}$</td>
<td>0.98</td>
<td>0.86%</td>
<td>2.01</td>
<td>1.11</td>
<td>2.53</td>
<td>0.78</td>
</tr>
<tr>
<td>(0.008)</td>
<td>(0.054)</td>
<td>(0.052)</td>
<td>(0.083)</td>
<td>(0.103)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Beyond matching micro estimates, the implied life-cycle profiles of shares and participation fit the hump-shaped empirical profiles. The profiles are decreasing in the end of life because risk is not as well diversified across time and expected labor income is decumulating. The profiles are increasing in the beginning of life because high expected labor income makes labor-income and stock-market risk more bearable as news utility is proportional to consumption utility, such that fluctuations in good and bad news hurt less on the flatter part of the concave utility curve. In the beginning of life, however, the implied profiles tend to be slightly too high and not as steeply increasing relative to what I find in the SCF data. The implied life-cycle consumption profiles display a hump similar to what I find in the CEX data.

20.2 Testing the model’s implications using household portfolio data

In the following, I provide some suggestive empirical evidence for extensive rebalancing in portfolio choice. I use the same data set as Brunnermeier and Nagel (2008) from the PSID, which contains household characteristics, wealth, income, stock market holdings, business equity, and home equity. Brunnermeier and Nagel (2008) aim to test if portfolio shares are increasing in wealth as would be predicted by a habit-formation model and other asset-pricing models that rely on increasing risk aversion to explain countercyclical equity premia. The authors find that, if anything, the risky asset share is slightly decreasing in wealth. In the following, I show that the risky asset share seems to be decreasing in the innovation to wealth for the group of households that adjust their risky asset holdings, as predicted by the news-utility model. Nevertheless, in an asset-pricing model, news utility predicts countercyclical equity premia as shown in Pagel (2012c).

I follow the methodology of Brunnermeier and Nagel (2008). However, instead of the risky asset share’s response to changes in wealth, I am interested in its response to innovations or unexpected changes in wealth. The unexpected change in wealth, \( \tilde{\Delta}w_t \), corresponds to the residual of a predictive regression of the change in wealth on all other variables used by Brunnermeier and Nagel (2008) including last period’s change and level in wealth, i.e.,

\[
\Delta w_t = \beta q_{t-2} + \gamma \Delta h_t + \tilde{\Delta}w_t.
\]

\( q_{t-2} \) consists of a vector of ones and constant or lagged variables that are known at date \( t \), such as age, education, gender, marital status, employment, inheritances, etc.\(^{122}\) \( \Delta h_t \) is a vector of variables that captures major changes in family composition or asset ownership, such as changes

---

\(^{122}\) Following Brunnermeier and Nagel (2008), I include age and age\(^2\); indicators for completed high school and college education, respectively, and their interaction with age and age\(^2\); dummy variables for gender and their interaction with age and age\(^2\); marital status, health status; the number of children in the household, the number of people in the household; dummy variables for any unemployment in the two years leading up to and including year \( t-2 \), and for coverage of the household heads job by a union contract. In addition, I include the log of the equity in vehicles owned by the household, log family income at \( t-2 \) and \( t-4 \), and a variable for inheritances received in the two years leading up to and including year \( t-2 \). Moreover, I include time fixed effects to control for aggregate wealth changes, region fixed effects to control for local wealth changes, and an interaction of each of them.
in family size, house ownership, etc.\textsuperscript{123} Moreover, I restrict the regression to those households who did change their risky asset holdings, as the model predicts that some people are inattentive. Furthermore, Brunnermeier and Nagel (2008) analyze the change in the risky asset share, whereas I use the level of the risky asset share as the dependent variable and the lagged risky asset share as an additional independent variable. I restrict myself to the second subsample that contains the PSID waves of 1999, 2001, and 2003, as the first subsample has a five year difference, which is likely to be too long for analyzing an expectations-based reference point. I run a pooled regression of the form

$$\alpha_t = b\alpha_{t-2} + \beta q_{t-2} + \gamma \Delta h_t + \rho \Delta w_t + \varepsilon_t$$

with $\alpha_{t-2}$ denoting the lagged risky asset share.\textsuperscript{124} Consistent with the results of Brunnermeier and Nagel (2008), the coefficient on the unexpected change in wealth for those households who did change their risky asset holdings in that period is negative but relatively small. I obtain a coefficient of approximately -.13 with a t-statistic of 3.34 which implies that an unexpected decrease in wealth of 20% leads to an increase in the risky asset share by 2.6%. The coefficient is more negative and significant if I restrict the sample to households that changed their risky asset holdings, consistent with the model. However, the model predicts that an innovation in wealth by $\sigma_r$, i.e., $e^{\mu_r + \sigma_r} - e^{\mu_r} \approx 20\%$, yields to a reduction in the risky asset share by roundabout 10% when I omit the variation in $1 - \rho_t$. Alternatively, I run a 2SLS regression in which I instrument the unexpected change in wealth by the unexpected change in labor income using the same methodology as Brunnermeier and Nagel (2008) obtaining similar results.

\textbf{21 Discussion and Conclusion}

This paper provides a comprehensive analysis of the portfolio implications of news utility, a recent theoretical advance in behavioral economics, which successfully explains micro evidence in a broad range of domains. The preferences' robust explanatory power in other domains is of special importance because I put emphasis on a potentially normative issue. In particular, I ask how often people should look up and rebalance their portfolios. Several of my results are applicable to financial advice. First, the preferences predict that people prefer to stay inattentive and can diversify over time. Therefore, people should look up their portfolios only once in a while and choose lower portfolio shares toward the end of life. Moreover, people should delegate the management of their portfolios to an agent who encourages inattention and rebalances actively. Furthermore, the preferences make specific predictions about how investors should rebalance their portfolios if they look it up. Most importantly, people should choose a lower portfolio share after

\textsuperscript{123}Following Brunnermeier and Nagel (2008), I include changes in some household characteristics between $t - 2$ and $t$: changes in family size, changes in the number of children, and a sets of dummies for house ownership, business ownership, and non-zero labor income at $t$ and $t - 2$.

\textsuperscript{124}Following Brunnermeier and Nagel (2008), I define liquid assets as the sum of holdings of stocks and mutual funds plus riskless assets, where riskless assets are defined as the sum of cash-like assets and holdings of bonds. Subtracting other debts, which comprises non-mortgage debt such as credit card debt and consumer loans, from liquid assets yields liquid wealth. I denote the sum of liquid wealth, equity in a private business, and home equity as financial wealth. I then calculate the risky asset share as the sum of stocks and mutual funds, home equity, and equity in a private business, divided by financial wealth. Alternatively, I could exclude home equity and equity in a private business.
the market goes up and thus follow a buy-low-sell-high investment strategy.

The intuitions behind my results are immediately appealing. If the investor cares more about bad news than good news, then fluctuations in expectations about future consumption hurt on average. Thus, the investor prefers to be inattentive most of the time and does not rebalance his portfolio. Once in a while, however, the investor has to look up his portfolio and then rebalances extensively. After the market goes down, the investor finds it optimal to increase his portfolio share temporarily to not realize all the bad news associated with future consumption. Hereby, the investor effectively delays the realization of bad news until the next period by which point his expectations will have decreased. On the other hand, after the market goes up, the investor finds it optimal to play safe and wants to realize the good news by liquidating his asset holdings. The investor may ignore his portfolio because he has access to two different accounts, a brokerage account, through which he can invest a share of his wealth into the stock market, and a checking account, which finances inattentive consumption. These two accounts relate to the notion of mental accounting because the accounts finance different types of consumption, they feature different marginal propensities to consume, they allow less overconsumption or to exercise self control, and the investor treats windfall gains in each differently.

Theoretically, I obtain analytical results to explain these phenomena under the assumption of log utility and the Campbell and Viceira (2002) approximation of log portfolio returns. Quantitatively, I structurally estimate the preference parameters and show that the model’s predictions match the empirical life-cycle evidence on participation, portfolio shares, consumption, and wealth accumulation and provide some suggestive evidence for extensive rebalancing. In the future, I would like to further explore cross-sectional asset pricing, as the theory predicts that more newsy investments should carry higher risk premia.
References


Part IV

Expectations-Based Reference-Dependent Life-Cycle Consumption

A  More Figures and Tables

Table 6:

<table>
<thead>
<tr>
<th>Loss (L)</th>
<th>standard contemp.</th>
<th>news-utility prospective</th>
<th>habit-formation</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>15</td>
<td>22</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>300</td>
<td>435</td>
</tr>
<tr>
<td>1000</td>
<td>1000</td>
<td>1500</td>
<td>2166</td>
</tr>
<tr>
<td>5000</td>
<td>5000</td>
<td>7500</td>
<td>10719</td>
</tr>
<tr>
<td>50000</td>
<td>50291</td>
<td>75000</td>
<td>105487</td>
</tr>
<tr>
<td>100000</td>
<td>100406</td>
<td>150000</td>
<td>2066770</td>
</tr>
</tbody>
</table>

For each loss L, the table’s entries show the required gain G to make each agent indifferent between accepting and rejecting a 50-50 gamble win G or lose L at a wealth level of 300,000 and a permanent income of 100,000 (power-utility model).

Table 7:

<table>
<thead>
<tr>
<th>( \mu_n )</th>
<th>( \sigma_n )</th>
<th>( \mu_u )</th>
<th>( \sigma_u )</th>
<th>( p )</th>
<th>( r )</th>
<th>( \beta )</th>
<th>( \theta )</th>
<th>( \gamma_0 )</th>
<th>( \gamma_1 )</th>
<th>( P_0 )</th>
<th>( A_0 )</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ( \sqrt{0.044} )</td>
<td>0 ( \sqrt{0.0212} )</td>
<td>0.00302</td>
<td>0.0344</td>
<td>0.9598</td>
<td>0.514</td>
<td>0.0701</td>
<td>0.071</td>
<td>1</td>
<td>0.3</td>
<td>40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 11: Consumption and income profiles and the fitted model’s consumption from Gourinchas and Parker (2002).

The news-utility consumption follows the same specification except for the choice of news-utility parameters $\eta = 1$, $\lambda = 2$, and $\gamma = 0.85$.

B Derivations and Proofs

B.1 Summary of utility functions under consideration

I briefly summarize the lifetime utility of all preference specifications that I consider. I define the “news-utility” agent’s lifetime utility in each period $t = \{0, \ldots, T\}$ as

$$u(C_t) + n(C_t, F_{t-1}) + \gamma \sum_{\tau=1}^{T-t} \beta^\tau n(F_{C_{t+\tau}}, t) + E_t \left[ \sum_{\tau=1}^{T-t} \beta^\tau U_{t+\tau} \right]$$

with $\beta \in [0, 1]$, $u(\cdot)$ a HARA utility function, $\eta \in (0, \infty)$, $\lambda \in (1, \infty)$, and $\gamma \in [0, 1]$. Additionally, I first consider standard preferences as analyzed by Carroll (2001), Gourinchas and Parker (2002), and Deaton (1991), among many others. The “standard” agent’s lifetime utility is given by

$$u(C_t^s) + E_t \left[ \sum_{\tau=1}^{T-t} \beta^\tau u(C_{t+\tau}^s) \right].$$

Second, I consider internal, multiplicative habit-formation preferences as assumed in Michaelides (2002). The “habit-forming” agent’s lifetime utility is given by

$$u(C_t^h) - hu(C_{t-1}^h) + E_t \left[ \sum_{\tau=1}^{T-t} \beta^\tau u(C_{t+\tau}^h) - hu(C_{t+\tau}^h - 1)) \right]$$

with $h \in [0, 1]$.

A utility function $u(c)$ is said to exhibit hyperbolic absolute risk aversion (HARA) if the level of risk tolerance, $-\frac{u''(c)}{u'(c)}$, is a linear function of $c$. 

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A solution for period utility model can be solved through backward induction. In the following, I outline the model’s B.2.1 The finite-horizon model B.2

with \( \tilde{C}_t \) following the specification of Bucciol (2012). The “tempted” agent’s lifetime utility is given by

\[
u(C_t^b) + bE_t[\sum_{\tau=1}^{T-t} \beta^\tau u(C_{t+\tau}^b)]
\]

with \( b \in [0, 1] \) corresponding to the \( \beta \delta - \)agent’s \( \beta \).

Fourth, I consider temptation-disutility preferences as developed by Gul and Pesendorfer (2004) following the specification of Bucciol (2012). The “tempted” agent’s lifetime utility is given by

\[
u(C_{t}^{\text{ed}}) - \lambda^{td}(u(C_{t}^{\text{ed}}) - u(C_{t}^{\text{ed}})) + E_t[\sum_{\tau=1}^{T-t} \beta^\tau (u(C_{t+\tau}^{\text{ed}}) - \lambda^{td}(u(\tilde{C}_{t+\tau}^{\text{ed}}) - u(C_{t+\tau}^{\text{ed}})))]
\]

with \( \tilde{C}_t^{\text{ed}} \) being the most tempting alternative consumption level and \( \lambda^{td} \in [0, \infty) \).

B.2 Derivation of the exponential-utility model

B.2.1 The finite-horizon model

A simple derivation of the second-to-last period can be found in the text. The exponential-utility model can be solved through backward induction. In the following, I outline the model’s solution for period \( T-i \) in which the agent chooses how much to consume \( C_{T-i} \) and how much to invest in the risk-free asset \( A_{T-i} \). I guess and verify the model’s consumption function

\[
C_{T-i} = \left(1 + \frac{r}{f(i)}\right)^i (1 + r)A_{T-i-1} + P_{T-i-1} + s_{T-i}^P + (1 - \frac{f(i-1)}{f(i)})s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i}
\]

with

\[
\Lambda_{T-i} = \frac{1}{\theta} \log\left(\frac{(1 + r)^i}{f(i)} \frac{\psi T_{T-i} + \gamma Q_{T-i} \xi F(s_{T-i}^P + (1 + r)^i f(i)) s_{T-i}^T}{1 + \eta F(s_{T-i}^P + f(i)(1 + f(i)) s_{T-i}^T)} + \eta \lambda (1 - F(s_{T-i}^P + (1 + r)^i f(i)) s_{T-i}^T)\right)
\]

and \( f(i) = \sum_{j=0}^i (1+r)^j = (1+r)^i \frac{1-r^{i+1}}{1-r} \) (in the text \( a(i) = \frac{f(i-1)}{f(i)} \)). Then, the budget constraint

\[
A_{T-i} = (1+r)A_{T-i-1} + P_{T-i-1} - C_{T-i}
\]

determines end-of-period asset holdings

\[
A_{T-i} = \frac{f(i-1)}{f(i)} (1 + r)A_{T-i-1} + \frac{f(i-1)}{f(i)} s_{T-i}^P + \frac{f(i-1)}{f(i)} \Lambda_{T-i}.
\]

\( \Lambda_{T-i} \) is a function independent of \( A_{T-i-1} \) and \( P_{T-i-1} \) but dependent on \( s_{T-i}^P \) and \( s_{T-i}^T \). In the last period the agent consumes everything such that \( \Lambda_T = 0 \). As a first step to verify the solution guess, I sum up the expectation of the discounted consumption function utilities from period \( T-i \) to \( T \)

\[
\beta E_{T-i-1} \left[ \sum_{\tau=0}^i \beta^\tau u(C_{T-i+\tau}) \right] = u(P_{T-i-1} + \frac{(1+r)^i}{f(i)} (1 + r)A_{T-i-1}) Q_{T-i-1}
\]

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\[
Q_{T-i-1} = \beta E_{T-i-1} \left[ \exp \left\{ - \theta (s_{T-i}^p + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i} ) \right\} + \exp \left\{ - \theta (s_{T-i}^p + (1 + \frac{r}{f(i)}) s_{T-i}^T ) \right\} \right] 
\]

with \( Q_{T-i-1} \) given by

\[
Q_{T-i-1} = \beta E_{T-i-1} \left[ \exp \left\{ - \theta (s_{T-i}^p + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i} ) \right\} + \exp \left\{ - \theta (s_{T-i}^p + (1 + \frac{r}{f(i)}) s_{T-i}^T ) \right\} \right] 
\]

\( Q_{T-i-1} \) is a constant if \( \Lambda_{T-i} \) depends only on \( s_{T-i}^p \) and \( s_{T-i}^T \). To derive the above sum, I simply plug in the asset-holding function into each future consumption function. For instance, \( C_{T-i+1} \) is given by

\[
C_{T-i+1} = \frac{(1 + r)^i}{f(i)} (1 + r) A_{T-i} + P_{T-i} + s_{T-i+1}^p + (1 - \frac{f(i-2)}{f(i-1)}) s_{T-i+1}^T - \frac{f(i-2)}{f(i-1)} \Lambda_{T-i+1} 
\]

The consumption function and its sum allows me to write down the agent’s continuation utility in period \( T - i - 1 \) as follows

\[
u(P_{T-i-1} + \frac{(1 + r)^i}{f(i)} (1 + r) A_{T-i-1}) \psi_{T-i-1} = - \frac{1}{\theta} \exp \left\{ - \theta (P_{T-i-1} + \frac{(1 + r)^i}{f(i)} (1 + r) A_{T-i-1}) \right\} \psi_{T-i-1} 
\]

with \( \psi_{T-i-1} \) given by

\[
\psi_{T-i-1} = \beta E_{T-i-1} \left[ \exp \left\{ - \theta (s_{T-i}^p + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i} ) \right\} + \exp \left\{ - \theta (s_{T-i}^p + (1 + \frac{r}{f(i)}) s_{T-i}^T ) \right\} \right] 
\]

and \( \omega(x) \) for any random variable \( X \sim F_X \), where the realization is denoted by \( x \), is

\[
\omega(x) = \eta \int_{-\infty}^{x} (x - y) dF_X(y) + \eta \lambda \int_{x}^{\infty} (x - y) dF_X(y). 
\]

The above expression for \( \psi_{T-i-1} \) can be easily inferred from the agent’s utility function. The first component in \( \psi_{T-i-1} \) corresponds to the expectation of consumption utility in period \( T - i \), the second to contemporaneous gain-loss in period \( T - i \), the third to prospective gain-loss in period \( T - i \) that depends on the sum of future consumption utilities \( Q_{T-i} \), and the last to the agent’s continuation value. Moreover, for any random variable \( Y \sim F_Y = F_X \) note that

\[
\int_{-\infty}^{\infty} \omega(g(x)) dF_X(x) = \int_{-\infty}^{0} \{ \eta \int_{-\infty}^{x} (g(x) - g(y)) dF_Y(y) + \eta \lambda \int_{x}^{\infty} (g(x) - g(y)) dF_Y(y) \} dF_X(x) > 0 
\]

in order to ensure that \( g'(<0) < 0 \) if \( g'(>0) > 0 \)
\[
\int_{-\infty}^{\infty} \eta \int_{-\infty}^{x} (g(x) - g(y)) dF_Y(y) + \eta \int_{x}^{\infty} (g(x) - g(y)) dF_Y(y) + \eta(\lambda - 1) \int_{x}^{\infty} (g(x) - g(y)) dF_Y(y) \] dF_X(x) > 0
\]
\[
\int_{-\infty}^{\infty} \eta(\lambda - 1) \int_{x}^{\infty} (g(x) - g(y)) dF_Y(y) \] dF_X(x) > 0
\]

if \( \lambda > 1, \eta > 0, \) and \( g'(\cdot) < 0. \) The above consideration implies that \( \psi_{T-i} > Q_{T-i} \) necessarily if \( \theta > 0 \) such that \( u(\cdot) \) is concave. Now, I turn to the agent’s maximization problem in period \( T - i, \) which is given by

\[
u(C_{T-i}) + n(C_{T-i}, F_{C_{T-i}}^{T-i-1}) + \gamma \sum_{\tau=1}^{i} \beta^{\tau} n(F_{C_{T-i+i}}^{T-i-1}) + u(P_{T-i} + \frac{(1+r)^{i-1}}{f(i-1)} A_{T-i}) \psi_{T-i}.
\]

I want to find the agent’s first-order condition. I begin by explaining the first derivative of contemporaneous gain-loss utility \( n(C_{T-i}, F_{C_{T-i}}^{T-i-1}). \) The agent takes his beliefs about period \( T - i \) consumption \( F_{C_{T-i}}^{T-i-1} \) as given such that

\[
\frac{\partial n(C_{T-i}, F_{C_{T-i}}^{T-i-1})}{\partial C_{T-i}} = \frac{\partial (\eta \int_{-\infty}^{C_{T-i}} (u(C_{T-i}) - u(c)) dF_{C_{T-i}}^{T-i-1}) + \eta \lambda \int_{-\infty}^{C_{T-i}} (u(C_{T-i}) - u(c)) dF_{C_{T-i}}^{T-i-1}(c)}{\partial C_{T-i}}
\]

\[
= u'(C_{T-i})(\eta F_{C_{T-i}}^{T-i-1}(C_{T-i}) + \eta \lambda (1 - F_{C_{T-i}}^{T-i-1}(C_{T-i}))) = u'(C_{T-i})(\eta F(s_{T-i}^{p} + (1+r)^i s_{T-i}^{T}) + \eta \lambda (1 - F(s_{T-i}^{p} + (1+r)^i s_{T-i}^{T})
\]

the last step results from the guessed consumption function and the assumption that admissible consumption functions are increasing in both shocks. Here, I abuse notation somewhat by writing \( F(\cdot) = F_{s_{T-i}^{p} + (1+r)^i s_{T-i}^{T}}(\cdot). \) The first derivative of the agent’s prospective gain-loss utility

\[
\sum_{\tau=1}^{i} \beta^{\tau} n(F_{C_{T-i+i}}^{T-i-1})
\]

over the entire stream of future consumption utilities \( u(P_{T-i} + \frac{(1+r)^i}{f(i-1)} A_{T-i}) Q_{T-i} \) can be inferred in a similar manner. Recall that \( Q_{T-i} \) is a constant under the guessed consumption function; thus, the agent only experiences gain-loss utility over the realized uncertainty in period \( T - i, \) i.e.,

\[
\frac{\partial \sum_{\tau=1}^{\infty} \beta^{\tau} n(F_{C_{T-i+i}}^{T-i-1})}{\partial A_{T-i}} = \sum_{\tau=1}^{\infty} \beta^{\tau} \int_{-\infty}^{\infty} \mu(t - u(r)) dF_{C_{T-i+i}}^{T-i-1}(t, r)
\]

\[
= \frac{\partial}{\partial A_{T-i}} \int_{-\infty}^{\infty} \mu(P_{T-i} + \frac{(1+r)^i}{f(i-1)} A_{T-i}) Q_{T-i} + u(x) Q_{T-i} dF_{P_{T-i} + \frac{(1+r)^i}{f(i-1)} A_{T-i}}^{T-i-1}(x)
\]

\[
= \frac{(1+r)^i}{f(i-1)} \exp\{\theta(P_{T-i} + \frac{(1+r)^i}{f(i-1)} A_{T-i}) Q_{T-i}(\eta F(s_{T-i}^{p} + (1+r)^i s_{T-i}^{T}) + \eta \lambda (1 - F(s_{T-i}^{p} + (1+r)^i s_{T-i}^{T}))
\]

and again, \( F(s_{T-i}^{p} + (1+r)^i s_{T-i}^{T}) \) results from the solution guess for \( A_{T-i} \) times \( \frac{(1+r)^i}{f(i-1)} \) and the fact that future consumption is increasing in both shocks. The derivative of the agent’s continuation
utility with respect to \( A_{T-i} \) is simply given by

\[
\frac{(1+r)^i}{f(i-1)} \exp\{-\theta \frac{(1+r)^i}{f(i-1)} A_{T-i}\} \psi_{T-i}.
\]

In turn, in any period \( T - i \) the news-utility agent’s first-order condition (normalized by \( P_{T-i} \)) is given by

\[
\exp\{-\theta ((1+r)A_{T-i-1} + s_{T-i}^P - A_{T-i})\}(1 + \eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T) + \eta \lambda(1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T)))
\]

\begin{equation}
= \frac{(1+r)^i}{f(i-1)} \exp\{-\theta \frac{(1+r)^i}{f(i-1)} A_{T-i}\}(\psi_{T-i} + \gamma Q_{T-i}(\eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T) + \eta \lambda(1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T))).
\end{equation}

The first-order condition can be rewritten to obtain the optimal consumption and end-of-period asset holdings functions and the function \( A_{T-i} \)

\[
A_{T-i} = \frac{1}{\theta} \log\left(\frac{(1+r)^i}{f(i-1)} \psi_{T-i} + \gamma Q_{T-i}(\eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T) + \eta \lambda(1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T))\right)
\]

and the guessed consumption function can be verified.

**B.2.2 The infinite-horizon model**

Suppose \( \sigma_{P_t} = \sigma_p \) and \( \sigma_{T_t} = \sigma_T \) for all \( t \) and \( T, i \rightarrow \infty \). I use a simple guess and verify procedure to find the infinite-horizon recursive equilibrium; alternatively, the solution can be obtained by simple backward induction taking \( T \) and \( i \) to infinity. The infinite-horizon model consumption and asset-holding functions are given by

\[
C_t = Y_t + rA_{t-1} - \frac{1}{1 + r} s_t^T - A_t = P_{t-1} + s_t^P + rA_{t-1} + \frac{r}{1 + r} s_t^T - A_t \quad \text{and} \quad A_t = A_{t-1} + \frac{1}{1 + r} s_t^T + A_r.
\]

The first-order condition normalized by \( P_t \) is given by

\[
\exp\{-\theta (1+r)A_{t-1} - \theta s_t^P + \theta A_r\}(1 + \eta F(s_t^P + \frac{r}{1 + r} s_t^T) + \eta \lambda(1 - F(s_t^P + \frac{r}{1 + r} s_t^T)) \}
\]

\[
= r \exp\{-\theta r A_t\}(\psi + \gamma Q(\eta F(s_t^P + \frac{r}{1 + r} s_t^T) + \eta \lambda(1 - F(s_t^P + \frac{r}{1 + r} s_t^T))).
\]

Solving for optimal end-of-period asset holdings yields

\[
A_t = A_{t-1} + \frac{1}{1 + r} s_t^T + \left(\frac{1}{\theta (1 + r)} \log(r \eta F(s_t^P + \frac{r}{1 + r} s_t^T) + \eta \lambda(1 - F(s_t^P + \frac{r}{1 + r} s_t^T)))\right).
\]
Consumption is then determined by the budget constraint

\[ C_t = Y_t + rA_t_{-1} - \frac{1}{1+r}s_t^T - \Lambda_t = P_{t-1} + s_t^P + rA_t_{-1} + \frac{r}{1+r}s_t^T - \Lambda_t. \]

\( Q \) and \( \psi \) are constant in an i.i.d. environment and given by

\[ Q = \frac{\beta E_t[\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r}s_{t+1}^T - \Lambda_{t+1})\}]}{1 - \beta E_t[\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r}s_{t+1}^T + r\Lambda_{t+1})\}]} \]

\[ \psi = \frac{\beta E_t[\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r}s_{t+1}^T - \Lambda_{t+1})\} + \omega(\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r}s_{t+1}^T - \Lambda_{t+1})\}) + \gamma Q \omega(\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r}s_{t+1}^T + r\Lambda_{t+1})\})]}{1 - \beta E_t[\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r}s_{t+1}^T + r\Lambda_{t+1})\}]} \]

**B.2.3 The optimal pre-committed equilibrium**

Suppose the agent has the ability to pick an optimal history-dependent consumption path for each possible future contingency in period zero when he does not experience any gain-loss utility. Thus, in period zero the agent chooses optimal consumption in period \( t \) in each possible contingency jointly with his beliefs, which of course coincide with the agent’s optimal state-contingent plan. For instance, consider the joint optimization over consumption and beliefs for \( C(Y^*) \) when income \( Y^* \) has been realized

\[
\frac{\partial}{\partial C(Y^*)} \left\{ \int \int \mu(u(C(Y))) - u(C(Y'))dF_Y(Y')dF_Y(Y) \right\}
\]

\[
= \frac{\partial}{\partial C(Y^*)} \left\{ \eta \int_{-\infty}^{Y} \{u(C(Y)) - u(C(Y'))\}dF_Y(Y') + \eta \lambda \int_{Y}^{\infty} \{u(C(Y)) - u(C(Y'))\}dF_Y(Y') \right\}dF_Y(Y)
\]

\[
= u'(C(Y^*)) \eta F_Y(Y^*) + \eta \lambda (1 - F_Y(Y^*)) - u'(C(Y^*)) \eta (1 - F_Y(Y^*)) + \eta \lambda F_Y(Y^*)
\]

\[
= u'(C(Y^*)) \eta (\lambda - 1)(1 - 2F_Y(Y^*)) \text{ with } \eta (\lambda - 1)(1 - 2F_Y(Y^*)) > 0 \text{ for } F_Y(Y^*) < 0.5.
\]

Consider the difference to the term in the initial first-order condition \( u'(C_t)(\eta F_{C_t}^{-1}(C_t) + \eta \lambda (1 - F_{C_t}^{-1}(C_t))) \): when choosing the pre-committed plan the additional utility of increasing consumption a little bit is no longer only made up of the additional step in the probability distribution; instead the two additional negative terms consider that in all other states of the world the agent experiences less gain feelings and more loss feelings because of increasing consumption in that contingency. The equation says that the marginal utility of state \( Y^* \) will be increased by news utility if the realization is below the median. For realizations above the median the marginal utility will be decreased and the agent will consume relatively less.

Unfortunately there is a problem arising in the pre-commitment optimization problem that has been absent in the non-pre-committed one: When beliefs are taken as given the agent optimizes over two concave functions consumption utility and the first part of gain-loss utility, accordingly the first-order condition pins down a maximum. In contrast, when the agent chooses his beliefs simultaneously to his consumption additionally the second convex part of gain-loss utility is optimized over. The additional part determining marginal utility \( -u'(C_t)(\eta (1 - F_{C_t}^{-1}(C_t))) + \)
\( \eta \lambda F_{C_t}^{-1}(C_t) \) is largest for particular good income realizations, since increasing consumption in these states implies additional loss feelings in almost all other states of the world. It can be easily shown that the sufficient condition of the optimization problem holds if the parameters satisfy following simple condition: \( \eta (\lambda - 1) \left( 2F_{C_t}^{-1}(C_t) - 1 \right) < 1 \). Accordingly, for \( \eta (\lambda - 1) < 1 \), which is true for a range of commonly used parameter combinations, the first-order condition pins down the optimum.

From the above consideration it can be easily inferred that the optimal pre-committed consumption function in the exponential-utility model is thus given by

\[
\Lambda_{T-i}^\sigma = \frac{1}{\theta} \log \left( \frac{(1+r)^i}{f(i-1)} \psi_{T-i} + \gamma Q_{T-i}^\sigma \eta (\lambda - 1) (1 - 2F(s_{T-i}^{p} + \frac{(1+r)^i}{f(i)} s_{T-i}^T)) \right)
\]

with

\[
Q_{T-i}^\sigma = E_{T-i-1} \left[ \beta \exp \left\{ -\theta (s_{T-i}^{p} + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i}^\sigma) \right\} + \beta \exp \left\{ -\theta (s_{T-i}^{p} + \frac{(1+r)^i}{f(i)} s_{T-i}^T + (1+r)^i \Lambda_{T-i}^\sigma) \right\} \right]
\]

and

\[
\psi_{T-i}^\sigma = \beta E_{T-i-1} \left[ \exp \left\{ -\theta (s_{T-i}^{p} + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i}^\sigma) \right\} + \omega \exp \left\{ -\theta (s_{T-i}^{p} + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i}^\sigma) \right\} \psi_{T-i} \right] + \gamma Q_{T-i}^\sigma \omega \left( \exp \left\{ -\theta (s_{T-i}^{p} + \frac{(1+r)^i}{f(i)} s_{T-i}^T + (1+r)^i \Lambda_{T-i}^\sigma) \right\} \right) + \frac{1}{\theta} \exp \left\{ -\theta (s_{T-i}^{p} + \frac{(1+r)^i}{f(i)} s_{T-i}^T + (1+r)^i \Lambda_{T-i}^\sigma) \right\} \psi_{T-i}.
\]

**B.3 The other agent’s exponential-utility consumption functions**

By the same arguments as for the derivation of the news-utility model, the “standard” agent’s consumption function in period \( T - i \) is

\[
A_{T-i}^\sigma = \frac{f(i-1)}{f(i)} (1+r) A_{T-i-1}^\sigma + \frac{f(i-1)}{f(i)} s_{T-i}^T + \frac{f(i-1)}{f(i)} \Lambda_{T-i}^\sigma
\]

\[
C_{T-i}^\sigma = \frac{(1+r)^i}{f(i)} (1+r) A_{T-i-1}^\sigma + P_{T-i-1} + s_{T-i}^p + \frac{f(i-1)}{f(i)} s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i}^\sigma
\]

\[
\Lambda_{T-i}^\sigma = \frac{1}{\theta} \log \left( \frac{(1+r)^i}{f(i-1)} Q_{T-i}^\sigma \right)
\]

\[
Q_{T-i}^\sigma = E_{T-i-1} \left[ \exp \left\{ -\theta (s_{T-i}^{p} + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i}^\sigma) \right\} + \beta \exp \left\{ -\theta (s_{T-i}^{p} + \frac{(1+r)^i}{f(i)} s_{T-i}^T + (1+r)^i \Lambda_{T-i}^\sigma) \right\} \right]
\]

In the infinite-horizon equilibrium \( Q^\sigma = \psi^\sigma \) in an i.i.d. environment with \( \sigma_{Pr} = \sigma_P \) and \( \sigma_{Tr} = \sigma_T \) for all \( t \)

\[
A_t^\sigma = A_{t-1}^\sigma + \frac{1}{1+r} s_t^T + \frac{1}{\theta (1+r)} log(r^\psi)
\]

\[
= \Lambda^\sigma
\]

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\[
\psi^s = Q^s = \frac{\beta E_t[\exp\{-\theta (s^p_{t+1} + \frac{r}{1+r} s^T_{t+1} - \Lambda^s)\}]}{1 - \beta E_t[\exp\{-\theta (s^p_{t+1} + \frac{r}{1+r} s^T_{t+1} + r \Lambda^s)\}]}
\]

\[
C_i^s = Y_i + r A^s_{i-1} - \frac{1}{1+r} s^T_i - \Lambda^s = P_i + s^p_i + r A^s_{i-1} + \frac{r}{1+r} s^T_i - \Lambda^s.
\]

The “tempted” agent’s maximization problem is given by

\[
\max_{C^d_t} \{u(C^d_t) - \lambda^{td}(u(\tilde{C}^d_{t+i}) - u(C^d_t)) + E_t[\sum_{\tau=1}^{T_t} \beta^\tau (u(C^d_{t+i}) - \lambda^{td}(u(\tilde{C}^d_{t+i}) - u(C^d_{t+i})))]\}
\]

with \(C_{t+i}^d\) being the most tempting alternative. In period \(T\) as the agent cannot die in debt the most tempting alternative is \(C_T^d = X_T^d\) but the agent will consume \(X_T\) anyway thus temptation disutility is zero and \(Q_{T-1}^d = Q^s_{T-1}\). In period \(T-1\) the agent’s consumption is then given by

\[
A^d_{T-1} = \frac{1+r}{2+r} A^d_{T-2} + \frac{1}{2+r} s^T_{T-1} + \frac{1}{2+r} \lambda^{td}_{T-1}
\]

\[
C^d_{T-1} = \frac{1+r}{2+r} (1+r) A^d_{T-2} + P_{T-2} + s^p_{T-1} + \frac{1+r}{2+r} s^T_{T-1} - \frac{1}{2+r} \Lambda^{td}_{T-1}
\]

with \(\Lambda^{td}_{T-1} = \frac{1}{\theta} \log((1+r) \frac{1}{1+\lambda^{td}} Q^d_{T-1})\) and \(Q^d_{T-1} = \beta E_{T-1}[\exp\{-\theta (s^p_i + s^T_i)\}].\)

What’s the agent’s most tempting alternative in period \(T-1\)? The value of cash-on-hand is \(X_{T-1}^d\) but the most tempting alternative is \(\tilde{C}_T^d \rightarrow \infty\) as consumption could be negative in the last period \(C_T \rightarrow -\infty\), which would yield \(\lim_{C_T^d \rightarrow -\infty} u(C_T^d) = \lim_{C_T^d \rightarrow -\infty} -\frac{1}{\theta} e^{-\theta C_T^d} \rightarrow -\infty\). Accordingly, \(Q^d_{T-2} \rightarrow 0\) and \(Q^d_{T-2} = \beta E_{T-2}[\exp\{-\theta (s^p_{T-1} + \frac{1+r}{2+r} s^T_{T-1} - \frac{1}{2+r} \Lambda^{td}_{T-1})\}] - \lambda^{td}(\exp\{-\theta (s^p_{T-1} + \frac{1+r}{2+r} s^T_{T-1} - \frac{1}{2+r} \Lambda^{td}_{T-1})\})\)

\[
= \exp\{\theta \frac{1+r}{2+r} (1+r) A^d_{T-2} - \lambda^{td}_{T-2}\} + \exp\{-\theta (s^p_{T-1} + \frac{1+r}{2+r} s^T_{T-1} + \frac{1+r}{2+r} \Lambda^{td}_{T-1})\} Q^d_{T-1}
\]

\[
Q^d_{T-2} = E_{T-2}[\beta \exp\{-\theta (s^p_{T-1} + \frac{1+r}{2+r} s^T_{T-1} - \frac{1}{2+r} \Lambda^{td}_{T-1})\}(1-\lambda^{td}) + \beta \exp\{-\theta (s^p_{T-1} + \frac{1+r}{2+r} s^T_{T-1} + \frac{1+r}{2+r} \Lambda^{td}_{T-1})\} Q^d_{T-1}]
\]

And in period \(T-i\)

\[
A^d_{T-i} = \frac{f(i-1)}{f(i)} (1+r) A^d_{T-i-1} + \frac{f(i-1)}{f(i)} s^T_{T-i} + \frac{f(i-1)}{f(i)} \Lambda^{td}_{T-i}
\]

\[
C^d_{T-i} = \frac{(1+r)^i}{f(i)} (1+r) A^d_{T-i-1} + P_{T-i-1} + s^p_{T-i-1} + (1 - \frac{f(i-1)}{f(i)}) s^T_{T-i} - \frac{f(i-1)}{f(i)} \Lambda^{td}_{T-i}
\]

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\[ \mathcal{N}_{T-i}^d = \frac{1}{\theta} \log \left( \frac{(1+r)^i}{f(i-1)} \frac{1}{1+\lambda^d} Q_{T-i}^d \right) \]

\[ Q_{T-i-1}^d = \beta E_{T-i-1} \exp \{-\theta(s_{T-i}^p + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T - \frac{f(i-1)}{f(i)} \mathcal{N}_{T-i}^d)\} \]

And for \( T \to \infty \)

\[ A_i^d = A_{i-1}^d + \frac{1}{1+r} n_i + \frac{1}{\theta (1+r)} \log (\frac{1}{1+\lambda^d} Q_i^d) = A_i^d \]

\[ C_i^d = Y_i + r A_i^d - \frac{1}{1+r} m_i - \Lambda_i^d = P_{i-1} + s_i^p + r A_i^d + \frac{r}{1+r} s_i^T - \Lambda_i^d \]

\[ Q_i^d = \frac{\beta E_i [\exp \{-\theta(s_{i+1}^p + \frac{r}{1+r} s_{i+1}^T - \Lambda_i^d)\} (1 - \lambda^d)]}{1 - \beta E_i [\exp \{-\theta(s_{i+1}^p + \frac{r}{1+r} s_{i+1}^T + r \Lambda_i^d)\}]} . \]

The “hyperbolic-discounting” agent’s consumption in period \( T-i \) is

\[ A_{T-i}^b = \frac{f(i-1)}{f(i)} (1+r) A_{T-i-1}^b + \frac{f(i-1)}{f(i)} s_{T-i}^T + \frac{f(i-1)}{f(i)} \Lambda_{T-i}^b \]

\[ C_{T-i}^b = \frac{(1+r)^i}{f(i)} (1+r) A_{T-i-1}^b + P_{T-i-1} + s_{T-i}^p + \frac{f(i-1)}{f(i)} s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i}^b \]

\[ \Lambda_{T-i}^b = \frac{1}{\theta} \log \left( \frac{(1+r)^i}{f(i-1)} b Q_{T-i}^b \right) \]

\[ Q_{T-i-1}^b = \beta E_{T-i-1} \exp \{-\theta(s_{T-i}^p + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i}^b)\} + \beta \exp \{-\theta(s_{T-i}^p + \frac{r}{1+r} s_{T-i}^T + \Lambda_{T-i}^b)\} \]

and for \( T \to \infty \)

\[ A_i^b = A_{i-1}^b + \frac{1}{1+r} s_i^T + \frac{1}{\theta (1+r)} \log (rb Q_i^b) = A_i^b \]

\[ C_i^b = Y_i + r A_i^b - \frac{1}{1+r} s_i^T - \Lambda^b = P_{i-1} + s_i^p + r A_i^b + \frac{r}{1+r} s_i^T - \Lambda^b \]

\[ Q_i^b = \frac{\beta E_i [\exp \{-\theta(s_{i+1}^p + \frac{r}{1+r} s_{i+1}^T - \Lambda^b)\}]}{1 - \beta E_i [\exp \{-\theta(s_{i+1}^p + \frac{r}{1+r} s_{i+1}^T + r \Lambda^b)\}]} . \]

**B.4 Proofs of Section 4:**

**B.4.1 Proof of Proposition 1**

If the consumption function derived in Section B.2.1 belongs to the class of admissible consumption functions then the equilibrium exists and is unique as the equilibrium solution is obtained by maximizing the agent’s objective function, which is globally concave, and there is a finite period that uniquely determines the equilibrium. Please refer to Section B.2.1 for the derivation of
the consumption function. \( \sigma_i^* \) is implicitly defined by the two admissible consumption function restrictions \( \frac{\partial C_{T-i}}{\partial T_{T-i}} > 0 \) and \( \frac{\partial C_{T-i}}{\partial s_{T-i}} > 0 \) as

\[
C_{T-i} = \frac{(1 + r)^i}{f(i)} (1 + r) A_{T-i-1} + p_{T-i-1} + s_{T-i}^P + (1 - a(i)) s_{T-i}^T - a(i) \Lambda_{T-i}
\]

the restrictions are equivalent to \( \frac{\partial a(i) \Lambda_{T-i}}{\partial s_{T-i}} < 1 \) and \( \frac{\partial a(i) \Lambda_{T-i}}{\partial s_{T-i}} < 1 - a(i) \) as \( \frac{\partial \Lambda_{T-i}}{\partial s_{T-i}} > 0 \) (since \( \psi_{T-i} > \gamma Q_{T-i} \) for any concave utility function which I have shown in Section B.2)). Recall that

\[
a(i) = 1 - \frac{(1 + r)^i}{f(i)} = \frac{f(i-1)}{f(i)}.
\]

Thus, \( \sigma_i^* \) is implicitly defined by the two restrictions

\[
\frac{\partial a(i) \Lambda_{T-i}}{\partial s_{T-i}^P} = \frac{a(i) \theta (1 - a(i))}{\psi_{T-i} + \gamma Q_{T-i}} \left( 1 + \eta F(s_{T-i}^P + \frac{(1 + r)^i}{f(i)} s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + \frac{(1 + r)^i}{f(i)} s_{T-i}^T)) \right) < 1
\]

and

\[
\frac{\partial a(i) \Lambda_{T-i}}{\partial s_{T-i}^T} = \frac{a(i) \theta (1 - a(i))}{\psi_{T-i} + \gamma Q_{T-i}} \left( 1 + \eta F(s_{T-i}^P + \frac{(1 + r)^i}{f(i)} s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + \frac{(1 + r)^i}{f(i)} s_{T-i}^T)) \right) < 1 - a(i).
\]

Here, the normal pdf of any random variable \( X \) is denoted by \( f_X \). Increasing \( \sigma_{P_i} \) and \( \sigma_{T_i} \) unambiguously decreases \( f_{s_{T-i}^P + \frac{(1 + r)^i}{f(i)} s_{T-i}^T} \) and thereby \( \frac{\partial a(i) \Lambda_{T-i}}{\partial s_{T-i}^P} \) and \( \frac{\partial a(i) \Lambda_{T-i}}{\partial s_{T-i}^T} \). Thus, there exists a condition \( \sigma_{P_i}^2 + \left( \frac{(1 + r)^i}{f(i)} \right)^2 \sigma_{T_i}^2 \geq \sigma_i^* \) for all \( i \) which ensures that an admissible consumption function exists that uniquely determines the equilibrium (given the admissible consumption functions in each future period until the final period) because the optimization problem is globally concave.

If uncertainty is small then the consumption function may be decreasing over some range, i.e., \( \frac{\partial C_{T-i}}{\partial s_{T-i}^P} < 0 \) or \( \frac{\partial C_{T-i}}{\partial s_{T-i}^T} < 0 \). I now show that the agent would pick a consumption function that is instead of decreasing flat and thus weakly increasing in the shock realizations, i.e., \( \frac{\partial C_{T-i}}{\partial s_{T-i}^P} \geq 0 \) or \( \frac{\partial C_{T-i}}{\partial s_{T-i}^T} \geq 0 \). To discuss this result in a simple framework, I return to the two-period, one-shock model. Suppose that the absolute level of the shock increases; then, holding \( C_{T-1} \) constant, the marginal value of savings declines and the agent’s first-order condition implies that consumption should increase. However, \( F_p(s_{P_{T-1}}^P) \) also increases, and marginal gain-loss utility is lower, such that the agent’s optimal consumption should decrease. Suppose that \( s_{T-1}^P \) increases sharply, which could occur if \( F_p \) is a very narrow distribution. In this case, the lower marginal gain-loss utility that decreases consumption dominates such that the first-order condition predicts decreasing consumption over some range in the neighborhood of the expected
value $\mu_P$ where $F_P$ increases most sharply if $F_P$ is bell shaped. However, a decreasing consumption function cannot be an equilibrium because the agent would necessarily experience gain-loss utility over the decreasing part of consumption, which decreases expected utility unnecessarily. In the decreasing-consumption-function region, the agent could choose a flat consumption function instead. In the following I show that the agent may choose a credible consumption plan with a flat section. Suppose the agent chooses a flat consumption level for realizations of $s_{T-1}^P$ in $\bar{s}$ and $\bar{s}$. Then, $\bar{s}$ is chosen where the original consumption function just stops decreasing, which corresponds to the lowest possible level of the flat section of consumption $C_{T-1}$. In that is then determined by

$$u'(C_{T-1}) = (1 + r)u'((\bar{s} - C_{T-1})(1 + r) + \bar{s}) \frac{\Psi_{T-1} + \gamma Q_{T-1}(\eta F_P(\bar{s}) + \eta \lambda(1 - F_P(\bar{s})))}{1 + \eta F_P(\bar{s}) + \eta \lambda(1 - F_P(\bar{s}))}$$

in which case $\bar{s}$ is determined by

$$u'(C_{T-1}) = (1 + r)u'((\bar{s} - C_{T-1})(1 + r) + \bar{s}) \frac{\Psi_{T-1} + \gamma Q_{T-1}(\eta F_P(s_{T-1}^P) + \eta \lambda(1 - F_P(s_{T-1}^P)))}{1 + \eta F_P(\bar{s}) + \eta \lambda(1 - F_P(\bar{s}))}$$

The agent’s consistency constraint for not increasing consumption beyond $C_{T-1}$ for any $s_{T-1}^P \in [\bar{s}, \bar{s}]$ is given by

$$u'(C_{T-1}) < (1 + r)u'((s_{T-1}^P - C_{T-1})(1 + r) + s_{T-1}^P) \frac{\Psi_{T-1} + \gamma Q_{T-1}(\eta F_P(s_{T-1}^P) + \eta \lambda(1 - F_P(s_{T-1}^P)))}{1 + \eta F_P(s_{T-1}^P) + \eta \lambda(1 - F_P(\bar{s}))}$$

and always holds as can be easily inferred. This result can be easily generalized to any horizon, thus, if the consumption function is decreasing over some range, the agent can credibly replace the decreasing part with a flat section as described above.

### B.4.2 Proof of Proposition 2

Please refer to the derivation of the exponential-utility model Section B.2 for a detailed derivation of $\Lambda_{T-1}$. According to Definition 5 consumption is excessively smooth if $\frac{\partial C_{T-i}}{\partial s_{T-i}^P} < 1$ and excessively sensitive if $\frac{\partial \Delta C_{T-i}}{\partial s_{T-i}^P} > 0$. Consumption growth is

$$\Delta C_{T-i} = s_{T-i}^P + (1 - a(i))s_{T-1}^P - a(i)\Lambda_{T-1} + \Lambda_{T-1}$$

so that $\frac{\partial C_{T-i}}{\partial s_{T-i}^P} < 1$ if $\frac{\partial \Delta C_{T-i}}{\partial s_{T-i}^P} > 0$ and $\frac{\partial \Delta C_{T-i}}{\partial s_{T-i}^P} > 0$ if $\frac{\partial \Delta C_{T-i}}{\partial s_{T-i}^P} > 0$. Since $\psi_{T-i} > \gamma Q_{T-i}$ (for any concave utility function which I have shown in Section B.2) it can be easily seen that $\frac{\partial \Delta C_{T-i}}{\partial s_{T-i}^P} > 0$, i.e.

$$\frac{\partial \Lambda_{T-1}}{\partial s_{T-i}^P} = \frac{1}{\theta(\frac{1 - a(i)}{a(i)}) \psi_{T-i} + \gamma Q_{T-i}(\eta F(s_{T-i}^P + \frac{(1 + r)y}{f(i)}s_{T-i}^P)) + \eta \lambda(1 - F(s_{T-i}^P + \frac{(1 + r)y}{f(i)}s_{T-i}^P))) > 0.$$
The same holds true for the infinite-horizon model

$$\Delta C_t = s^P_t + \frac{r}{1+r} s^T_t - \Lambda_t + (1+r) \Lambda_{t-1}$$

as $\Lambda_t$ is increasing in the permanent shock

$$\frac{\partial \Lambda_t}{\partial s^P_t} = \frac{1}{\theta (1+r) r} \frac{(\psi - \gamma Q) \eta f s^P_t + \frac{r^2}{1+r} s^T_t (s^P_t + \frac{r}{1+r} s^T_t) (\lambda - 1)}{\lambda + \eta F(s^P_t + \frac{r}{1+r} s^T_t) + \eta (1-F(s^P_t + \frac{r}{1+r} s^T_t))} > 0.$$ 

Accordingly, $\frac{\partial \Lambda_t}{\partial s^P_t} > 0$ as $\psi > \gamma Q$. Thus, if $s^P_t \uparrow$ then $\Lambda_t \uparrow$ and the shock induced change in consumption is less than one and the period $t$ shock induced change in one-period ahead consumption $\Delta C_{t+1}$ is larger than zero.

The difference to the standard, tempt, and quasi-hyperbolic discounting agents is that $\frac{\partial \Lambda_t}{\partial s^P_t} = 0$ for all $t$ such that consumption is neither excessively sensitive nor excessively smooth.

**B.4.3 Proof of Lemma 8**

I start with the first part of the lemma, the precautionary-savings motive. In the second-to-last period of the simple model outlined in the text, the first-order condition is given by

$$u'(C_{T-1}) + u'(C_{T-1}) (\eta F_{p}(s^P_{T-1}) + \eta \lambda (1-F_{p}(s^P_{T-1})))$$

$$= (1+r)u'((s^P_{T-1} - C_{T-1})(1+r) + s^P_{T-1}) \gamma \beta E_{T-1}[u'(S^p_{T})](\eta F_{p}(s^P_{T-1}) + \eta \lambda (1-F_{p}(s^P_{T-1})))$$

$$+ (1+r)u'((s^P_{T-1} - C_{T-1})(1+r) + s^P_{T-1}) \beta E_{T-1}[u'(S^p_{T}) + \eta (\lambda - 1) \int_{S^p_{T}}^{\infty} (u'(S^p_{T}) - u'(y)) dF_{p}(y)].$$

From Section B.2 I know that $\psi_{T-1} > Q_{T-1}$ because for any two random variables $X \sim F_X$ and $Y \sim F_Y$ with $F_X = F_Y$ in equilibrium

$$\int_{-\infty}^{\infty} \{ \eta \int_{-\infty}^{\infty} (g(x) - g(y)) dF_{Y}(y) + \eta \lambda \int_{x}^{\infty} (g(x) - g(y)) dF_{Y}(y) \} dF_{X}(x) > 0$$

if $\lambda > 1$, $\eta > 0$, and $g'(\cdot) < 0$. Thus, $\beta E_{T-1}[\eta (\lambda - 1) \int_{S^p_{T}}^{\infty} (u'(S^p_{T}) - u'(y)) dF_{p}(y)] > 0$ if $u''(\cdot) > 0$ the agent is risk averse or $u(\cdot)$ is concave. Moreover, it can be easily seen that $\frac{\partial \beta E_{T-1}[\eta (\lambda - 1) \int_{S^p_{T}}^{\infty} (u'(S^p_{T}) - u'(y)) dF_{p}(y)]}{\partial \eta} > 0$ and $\frac{\partial \beta E_{T-1}[\eta (\lambda - 1) \int_{S^p_{T}}^{\infty} (u'(S^p_{T}) - u'(y)) dF_{p}(y)]}{\partial \lambda} > 0$. Then, for any value of savings $A_{T-1} = s^P_{T-1} - C_{T-1}$ the right hand side of the first-order condition is increased by the presence of expected gain-loss disutility if $\sigma > 0$ whereas if $\sigma = 0$ then $\psi_{T-1} = Q_{T-1}$. The increase of the agent’s marginal value of savings by the presence of expected gain-loss disutility depends on $\sigma > 0$, 

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but does not go to zero as $\sigma_p \to 0$ so that the additional precautionary savings motive is first-order $\frac{\partial(x_T - C_{T-1})}{\partial \sigma_p} \big|_{\sigma_p=0} > 0$ as can be easily shown for any normally distributed random variable $X \sim F_X = \text{N}(\mu, \sigma^2)$.

$$E_T - 1 \left[ \eta(\lambda - 1) \int_X (u'(x) - u'(y))dF_X(y) \right]$$

$$= e^{-\theta \mu} \int_{-\infty}^{z}(e^{-\theta \sigma z} - e^{-\theta \sigma z})dF_{01}(\varepsilon) + \eta \lambda \int_{z}^{\infty}(e^{-\theta \sigma z} - e^{-\theta \sigma z})dF_{01}(\varepsilon) dF_{01}(z)$$

$$= e^{-\theta \mu} \eta(\lambda - 1) \int_{-\infty}^{\infty} \{(1 - F_{01}(z))e^{-\theta \sigma z} - e^{\frac{1}{2}\theta^2 \sigma^2}F_{01}(\theta \sigma - z)\} dF_{01}(z)$$

$$\frac{\partial(\cdot)}{\partial \sigma} \big|_{\sigma=0} = e^{-\theta \mu} \eta(\lambda - 1) \int_{-\infty}^{\infty} \{-z(1 - F_{01}(z))e^{-\theta \sigma z} - \theta e^{\frac{1}{2}\theta^2 \sigma^2}F_{01}(\theta \sigma - z) + \theta e^{\frac{1}{2}\theta^2 \sigma^2}F_{01}(\theta \sigma - z)\} dF_{01}(z)$$

$$= e^{-\theta \mu} \theta \eta(\lambda - 1) \int_{-\infty}^{\infty} \{-z + zF_{01}(z) + F_{01}(-z)\} dF_{01}(z) \approx e^{-\theta \mu} \theta \eta(\lambda - 1)0.7832 > 0.$$  

Thus, news-utility introduces a first-order precautionary-savings motive.

In the second part of the lemma the implications for consumption can be immediately seen by comparing the agents’ first-order conditions. The standard agent’s first-order condition in period $T - 1$ is given by

$$u'(C_{T-1}) = Ru'((s_{T-1}^P - C_{T-1})R + s_{T-1}^p)Q_{T-1}.$$  

The difference to the news-utility model can be seen easily: First, $\frac{\psi_{r-1}}{Q_{r-1}} > 1$ implies that

$$\frac{\psi_{r-1}}{Q_{r-1}} + \gamma(\eta F_p(s_{T-1}^P) + \eta \lambda (1 - F_p(s_{T-1}^P)))$$

$$\frac{1 + \eta F_p(s_{T-1}^P) + \eta \lambda (1 - F_p(s_{T-1}^P))}{1 + \eta F_p(s_{T-1}^P) + \eta \lambda (1 - F_p(s_{T-1}^P))} > 1$$

for $\gamma$ high enough such that the news-utility agent consumes less than the standard agent if he does not discount prospective gain-loss utility very highly. Moreover, as $\frac{\psi_{r-1}}{Q_{r-1}}$ is increasing in $\sigma_p$ the threshold value for $\gamma$, i.e., $\gamma$, in each comparison is decreasing in $\sigma_p$.

B.4.4  Proof of Proposition 3

The agent optimally chooses consumption and asset holdings in periods $T - i = 1, ..., T$ for any horizon $T$. I defined a hump-shaped consumption profile as characterized by increasing consumption and asset holdings in the beginning of life $C_1 < C_2$ and decreasing consumption in the end of life $C_T < C_{T-1}$ (note that, I derive the thresholds $\sigma_p$ and $\overline{\sigma_p}$ for $s_t^P = 0$ and $s_t^T = 0$ in all periods, since $\lambda T_{T-1}$ is skewed this is not exactly the average consumption path but the difference is minor). The first characteristic requires $C_1 < C_2$ which implies that

$$\frac{(1 + r)^{T-1}}{f(T-1)}(1 + r)A_0 + P_0 - \frac{f(T - 2)}{f(T - 1)}A_1 < \frac{(1 + r)^{T-2}}{f(T - 2)}(1 + r)A_1 + P_0 - \frac{f(T - 3)}{f(T - 2)}A_2.$$
so that \( \Lambda_1 > \frac{f(T-3)}{f(T-2)} \Lambda_2 \) and since \( \frac{f(T-3)}{f(T-2)} < 1 \) this holds always if \( \Lambda_1 > 0 \) as \( T \) becomes large since in the limit \( \Lambda_1 = \Lambda_2 \). Recall that if \( \lambda > 1 \) and \( \eta > 0 \) then \( \psi_{T-i} > Q_{T-i}, \psi_{T+i} > \psi_{T+i+1} \) and \( Q_{T-i} > Q_{T-i+1} \) and \( \psi_{T-i} - Q_{T-i} > \psi_{T+i+1} - Q_{T+i+1} \) for all \( i \) and \( \psi_{T-i} \) approaches it’s limit \( \frac{\psi}{Q_{T-i}} \) as \( i \) and \( T \) become large. \( \sigma_{r} \) is then implicitly defined by the requirement \( \Lambda_1 > 0 \) which is equivalent to

\[
\frac{(1+r)^{T-1}}{f(T-2)} \psi + \gamma Q_{1} \frac{1}{2} (1+\lambda) \frac{1}{1+\eta \frac{1}{2}(1+\lambda)} = \frac{r}{1- \left( \frac{1+r}{1+r} \right)^{T-1}} \psi + \gamma Q_{1} \frac{1}{2} (1+\lambda) \frac{1}{1+\eta \frac{1}{2}(1+\lambda)} > 1.
\]

Accordingly, if \( \frac{\psi}{Q_{1}} \) (which is determined by expected marginal gain-loss utility) is large enough relative to \( \gamma \) the agent chooses an increasing consumption path. For \( T \to \infty \) the condition boils down to

\[
r \frac{\psi + \gamma Q_{1} \frac{1}{2} (1+\lambda)}{1+\eta \frac{1}{2}(1+\lambda)} > 1 \Rightarrow r (\psi + \gamma Q_{1} \frac{1}{2} (1+\lambda)) > 1 + \eta \frac{1}{2}(1+\lambda)
\]

for which a sufficient condition is \( \gamma Q > \frac{1}{r} \).

The second characteristic requires \( C_T < C_{T-1} \) which implies that

\[
(1+r)A_{T-1} < \frac{1+r}{2+r} (1+r)A_{T-2} - \frac{1}{2+r} \Lambda_{T-1}
\]

and is equivalent to \( \Lambda_{T-1} < 0 \). Thus, \( \sigma_{r} \) is implicitly defined by \( \Lambda_{T-1} < 0 \)

\[
\frac{1}{\theta} \log ((1+r) \frac{\psi_{T-1} + \gamma Q_{T-1} \frac{1}{2} (1+\lambda)}{1+\eta \frac{1}{2}(1+\lambda)}) < 0 \Rightarrow (1+r) \frac{\psi_{T-1} + \gamma Q_{T-1} \frac{1}{2} (1+\lambda)}{1+\eta \frac{1}{2}(1+\lambda)} < 1.
\]

Note that, because \( \beta (1+r) \approx 1 \) the standard agent will choose an almost flat consumption path such that \( (1+r)Q_{T-1} \approx 1 \). Thus, the news-utility agent chooses a mean falling consumption path in the end of life as long as \( \frac{\psi_{T-1}}{Q_{T-1}} \) is not too large or \( \gamma \) is not too close to one.

**B.4.5 Proof of Proposition 4**

In the deterministic setting, \( s_{t}^{r} = s_{t}^{T} = 0 \) for all \( t \) such that the news-utility agent will not experience actual news utility in a subgame-perfect equilibrium because he cannot fool himself and thus \( \psi_{t} = Q_{t} \) for all \( t \). Thus, the expected-utility maximizing path corresponds to the standard agent’s one which is determined in any period \( T - i \) by the following first-order condition

\[
\exp \left\{-\theta (1+r)A_{T-i-1} + \theta A_{T-i} \right\} = \frac{(1+r)^{i}}{f(i-1)} \exp \left\{-\theta \frac{(1+r)^{i}}{f(i-1)} A_{T-i} \right\} Q_{T-i}^{r}. \]

If the agent believes he follows the above path then the consistency constraint (increasing consumption is not preferred) has to hold

\[
\exp \left\{-\theta (1+r)A_{T-i-1} + \theta A_{T-i} \right\} (1+\eta) < \frac{(1+r)^{i}}{f(i-1)} \exp \left\{-\theta \frac{(1+r)^{i}}{f(i-1)} A_{T-i} \right\} Q_{T-i}^{r} \eta \lambda.
\]

Thus, if \( \eta < \gamma \eta \lambda \Rightarrow \gamma > \frac{1}{\lambda} \) the agent follows the expected-utility maximizing path. Whereas
for $\gamma \leq \frac{1}{\lambda}$ news-utility consumption is characterized by equality of the consistency constraint, because the agent will choose the lowest consumption level that just satisfies it. Then, the first-order condition becomes equivalent to a $\beta \delta$-agent’s first-order condition with $b = \frac{1 + \gamma \eta \lambda}{1 + \eta} < 1$.

In the infinite-horizon model, a simple perturbation argument gives the following consistency constraint

$$\exp(-\theta(1+r)A_{t-1} + \theta A_t)(1 + \eta) < \exp(-\theta r A_t)Q(1 + \gamma \eta \lambda),$$

because $\psi = Q$. However, if $\gamma > \frac{1}{\lambda}$ the news-utility agent finds it optimal to follow the expected-utility-maximizing standard agent’s path

$$\exp(-\theta(1+r)A_{t-1} + \theta A_t) = \exp(-\theta r A_t)Q^s \Rightarrow A_t = A_{t-1} + \Lambda^s \Rightarrow C_t = rA_{t-1} + Y_t - \Lambda^s$$

$$\Lambda^s = \frac{1}{\theta(1+r)}\log(rQ^s) \text{ with } Q^s = \frac{\beta \exp(-\theta(-\Lambda^s))}{1 - \beta \exp(-\theta r \Lambda^s)}.$$ 

Whereas for $\gamma \leq \frac{1}{\lambda}$ news-utility consumption will choose the lowest consumption level that just satisfies his consistency constraint. Then, the first-order condition becomes equivalent to a $\beta \delta$-agent’s first-order condition with $b = \frac{1 + \gamma \eta \lambda}{1 + \eta} < 1$.

**B.4.6 Proof of Proposition 5**

I say that the news-utility agent’s consumption path features a drop in consumption at retirement, if the change in consumption at retirement is negative and smaller than it is after the start of retirement, i.e., $\Delta C_{T-R}$ is negative and smaller than $\Delta C_{T-R+1}$. In general, after the start of retirement the news-utility agent’s consumption growth follows the standard model or the hyperbolic-discounting model. Thus, the news-utility agent’s implied hyperbolic-discount factor after retirement is $b^R \in \{\frac{1 + \eta \lambda}{1 + \eta}, 1\}$, which is larger than the news-utility agent’s implied hyperbolic-discount factor at retirement. In the at-retirement period the weight on future marginal value versus current marginal consumption is between $\{\frac{1 + \eta \lambda}{1 + \eta}, \frac{1 + \eta \lambda}{1 + \eta}\}$ and since $\frac{1 + \eta \lambda}{1 + \eta} < \frac{1 + \gamma \eta \lambda}{1 + \eta} \leq 1$ the hyperbolic-discount factor implied by at-retirement consumption growth is necessarily lower than the hyperbolic-discount factor implied by post-retirement consumption growth. Thus, consumption growth at retirement will necessarily be less than consumption growth after retirement. Moreover, if consumption growth after retirement is approximately zero, because $\log((1 + r)\beta) \in [-\Delta, \Delta]$ with $\Delta$ small then consumption growth at retirement will be negative.

Let me formalize the agent’s consumption growth at and after retirement. After retirement the news-utility agent’s consumption growth is $\Delta C_{T-R+1} = C_{T-R+1} - C_{T-R} = -a(R-1)\Lambda_{T-R+1} + \Lambda_{T-R}$ and will correspond to a hyperbolic-discounting agent’s consumption with $b \in \{\frac{1 + \eta \gamma \lambda}{1 + \eta}, 1\}$ such that the agent’s continuation utilities in period $T - R$ and $T - R + 1$ (which determine $\Lambda_{T-R}$ and $\Lambda_{T-R+1}$) correspond to

$$Q_{T-R} = \psi_{T-R} = Q^b_{T-R} \text{ and } Q_{T-R+1} = \psi_{T-R+1} = Q^b_{T-R+1}$$

such that

$$\Lambda_{T-R} = \frac{1}{\theta} \log\left(\frac{(1+r)^R}{f(R-1)} b^{Q^b_{T-R}}\right) \text{ and } \Lambda_{T-R+1} = \frac{1}{\theta} \log\left(\frac{(1+r)^{R-1}}{f(R-2)} b^{Q^b_{T-R+1}}\right)$$
thus if \( \log((1+r)\beta) \in [-\Delta, \Delta] \) with \( \Delta \) small then consumption growth after retirement will be approximately zero (if \( b = 1 \) and the news-utility agent follows the standard agent’s path) or negative if \( b < 1 \) (if the news-utility agent follows a hyperbolic-discounting path but \( \log((1+r)\beta) \approx 0 \)). Consumption growth at retirement is

\[ \Delta C_{T-R} = C_{T-R} - C_{T-R-1} = -a(R)\Lambda_{T-R} + \Lambda_{T-R-1}. \]

\( \Lambda_{T-R} \) will correspond to a hyperbolic-discounting agent’s value with \( b \in \{ \frac{1+\eta\gamma}{1+\eta}, 1 \} \) as above. But, \( \Lambda_{T-R-1} \) will correspond to a hyperbolic-discounting agent’s value with \( b \in \{ \frac{1+\eta\gamma}{1+\eta}, \frac{1+\eta\gamma}{1+\eta} \} \) and it can be easily seen that \( \frac{1+\eta\gamma}{1+\eta} < \frac{1+\eta\gamma}{1+\eta} < \frac{1+\eta\gamma}{1+\eta} < 1 \). Thus, from above

\[ \Lambda_{T-R} = \frac{1}{\theta} \log \left( \frac{(1+r)^R}{f(R-1)} \right) b Q_{T-R}^b \]

and if the news-utility agent would continue this hyperbolic path implied by the past retirement \( b \in \{ \frac{1+\eta\gamma}{1+\eta}, 1 \} \) then

\[ \Lambda_{T-R-1}^b = \frac{1}{\theta} \log \left( \frac{(1+r)^{R+1}}{f(R)} \right) b Q_{T-R-1}^b \]

whereas in fact his \( \Lambda_{T-R-1} \) is given by

\[ \Lambda_{T-R-1} = \frac{1}{\theta} \log \left( \frac{(1+r)^{R+1}}{f(R)} \right) \psi_{T-R-1} + \gamma Q_{T-R-1} \left( \eta F(s_{T-R-1}^p + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T) + \eta \lambda (1 - F(s_{T-R-1}^p + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T)) \right) \]

with \( \psi_{T-R-1} = Q_{T-R-1} = Q_{T-R-1}^b \) because there is no uncertainty from period \( T-R+1 \). As can be easily seen iff \( \gamma < 1 \) then \( \Lambda_{T-R-1} < \Lambda_{T-R-1}^b \) because

\[ Q_{T-R-1}^b + \gamma Q_{T-R-1} \left( \eta F(s_{T-R-1}^p + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T) + \eta \lambda (1 - F(s_{T-R-1}^p + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T)) \right) < Q_{T-R-1} \]

for instance, if \( F(\cdot) = 0.5 \) then \( \frac{1+\gamma}{1+\eta} \frac{1+\lambda}{1+\eta} < 1 \) iff \( \gamma < 1 \). Thus, news-utility consumption growth is smaller at retirement than after retirement. Moreover, it is negative because it is either approximately zero after retirement (if \( b^R = 1 \)) or negative after retirement (if \( b^R = \frac{1+\eta\gamma}{1+\eta} < 1 \)).

**B.4.7 Proof of Corollary 1**

After retirement the news-utility agent’s consumption from period \( T-R \) on will correspond to a hyperbolic-discounting agent’s consumption with \( b \in \{ \frac{1+\eta\gamma}{1+\eta}, 1 \} \) such that the agent’s continuation utilities correspond to

\[ Q_{T-R-1} = \psi_{T-R-1} = Q_{T-R-1}^b \]
thus $\Lambda_{T-R-1}$ is given by

$$\Lambda_{T-R-1} = \frac{1}{\theta} \log \left( \frac{(1+r)^{R+1} \psi_{T-R-1} + \gamma Q_{T-R-1} (\eta F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T) + \eta \lambda (1 - F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T))}{1 + \eta F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T) + \eta \lambda (1 - F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T))} \right)$$

as can be easily seen iff $\gamma < 1$ then $\frac{\partial \Lambda_{T-R-1}}{\partial s_{T-R-1}} > 0$ and consumption is excessively smooth and sensitive in the pre-retirement period.

**B.4.8 Proofs of the new predictions about consumption (Section 4.4)**

The new predictions can be easily inferred from the above considerations.

1. Consumption is more excessively sensitive for permanent than for transitory shocks in an environment with permanent shocks. In an environment with transitory shocks only, however, news-utility consumption is excessively sensitive with respect to transitory shocks. $\Lambda_t$ varies more with the permanent shock than with the transitory shock, because the agent is consuming only the per-period value $\frac{1}{1+r}s_t^T$ of the period $t$ transitory shock, such that $F_{C_t}^{-1}(C_t)$ varies little with $s_t^T$. However, in the absence of permanent shocks $\Lambda_t$ would vary with $F_{s_{T-i}^P}^{-1}(s_{T-i}^T)$ which fully determines $F_{C_t}^{-1}(C_t)$ even though consumption itself will increase only by the per-period value $\frac{1}{1+r}s_t^T$ of the transitory shock. Thus, consumption is excessively sensitive for transitory shocks when permanent shocks are absent. With permanent shocks, however, consumption is excessively sensitive for transitory shocks only when the horizon is very short or the permanent shock has very little variance such that the transitory shock actually moves $F_{s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T} (s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T)$ despite the fact that it is discounted by $\frac{(1+r)^i}{f(i)}$.

2. The degree of excess smoothness and sensitivity is decreasing in the amount of economic uncertainty $\sigma_P$. If $\sigma_P$ is small, the agent’s beliefs change more quickly relative to the change in the realization of the shock; hence, the consumption function is more flat for realizations around $\mu_P$. The consumption function $C_{T-i}$ is less increasing in the realizations of the shocks $s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T$ if $\frac{\partial \Lambda_{T-i}}{\partial s_{T-i}}$ is relatively high. As can be seen easily, $\frac{\partial \Lambda_{T-i}}{\partial s_{T-i}}$ is increasing in $f_{s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T} (s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T)$ which is high if $f_{s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T}$ is very high at $s_{T-i}^P = \mu_P$ which happens if $f_{s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T}$ is a very narrow distribution, i.e., $\sigma_P$ is small.

3. Any bell-shaped shock distribution induces the variation in $\Lambda_{T-i}$ and thereby the amount of excess sensitivity to be bounded. If the agent is hit by an extreme shock, the actual value of the low probability realization matters less because neighboring states have very low probability. The expression $\eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T))$ is bounded if the two shocks’ distributions are bell shaped. Thus, the variation in $\Lambda_{T-i}$ is bounded.

4. Consumption is more excessively sensitive and excessively smooth when the agent’s horizon
increases, because the marginal propensity to consume out of permanent shocks declines when the additional precautionary-savings motive accumulates. $\psi_{T-i}$ is increasing in $i$ and approaches a constant $\frac{\psi}{Q_{T-i}}$ when $T \to \infty$ and $i \to \infty$. Then, the variation in $\Lambda_{T-i}$ is increasing in $i$. And since consumption growth $\Delta C_{T-i}$ is determined by $-a(i)\Lambda_{T-i} + \Lambda_{T-i-1}$ on average the larger variation in $\Lambda_{T-i}$ translates into a higher coefficient in the OLS regression. This can be seen by looking at $\frac{\partial \Lambda_{T-i}}{\partial s_{T-i}}$, i.e.,

$$\frac{\partial a(i)\Lambda_{T-i}}{\partial s_{T-i}} = \frac{a(i)}{\theta(1-a(i))} \psi_{T-i} + Q_{T-i} \gamma(\eta F(s_{T-i}^i + (1+r)^i(s_{T-i} + s_{T-i-1}^j)(\lambda-1))}{1 + \eta F(s_{T-i}^i + (1+r)^i(s_{T-i} + s_{T-i-1}^j)(\lambda-1))} > 0.$$  

As $a(i) = \frac{f(i-1)}{f(i)}$ is increasing in $i$ because $f(i) = \sum_{j=0}^{i} (1+r)^j$ and thus $\frac{(a(i))^2}{1-a(i)}$ is increasing in $i$ and approaching a constant and $\frac{\psi_{T-i}}{Q_{T-i}}$ is increasing in $i$ and approaching a constant it can be easily seen that $\frac{\partial a(i)\Lambda_{T-i}}{\partial s_{T-i}}$ is increasing in $i$ which means that consumption becomes more excessively smooth as the agent’s horizon increases. Moreover, as $a(i) = \frac{\psi_{T-i}}{Q_{T-i}}$ is increasing in $i$ too $\frac{\partial \Lambda_{T-i}}{\partial s_{T-i}}$ is increasing in $i$ which means that consumption becomes more excessively sensitive as the agent’s horizon increases.

B.4.9 Proof of Proposition 6

In the following, I assume that the following parameter condition (which ensures that the agent’s maximization problem is globally concave) holds $\eta(\lambda - 1) < 1$. All of the following proofs are direct applications of the prior proofs for the monotone-personal equilibrium just using $\Lambda_{T-i}^c$ instead of $\Lambda_{T-i}$. Thus, I make the exposition somewhat shorter.

1. The personal and pre-committed consumption functions are different in each period as can be seen in Section B.2. But, if there’s no uncertainty and $\gamma > \frac{1}{2}$ then the personal and pre-committed consumption functions both correspond to the standard agent’s consumption function as shown in the proof of Proposition 4.

2. Please refer to the derivation of the exponential-utility pre-committed model in Section B.2 for a detailed derivation of $\Lambda_{T-i}^c$. According to Definition 5 consumption is excessively smooth if $\frac{\partial C_{T-i}^c}{\partial s_{T-i}^{c+1}} < 1$ and excessively sensitive if $\frac{\partial \Delta C_{T-i}^c}{\partial s_{T-i}^{c+1}} > 0$. Consumption growth is

$$\Delta C_{T-i}^c = s_{T-i}^c + (1 - a(i))s_{T-i}^c - a(i)\Lambda_{T-i}^c + \Lambda_{T-i-1}^c$$

so that $\frac{\partial C_{T-i}^c}{\partial s_{T-i}^{c+1}} < 1$ iff $\frac{\partial \Lambda_{T-i}^c}{\partial s_{T-i}^{c+1}} > 0$ and $\frac{\partial \Delta C_{T-i}^c}{\partial s_{T-i}^{c+1}} > 0$ iff $\frac{\partial \Lambda_{T-i}^c}{\partial s_{T-i}^{c+1}} > 0$. Since $\psi_{T-i}^c > \gamma Q_{T-i}^c$ it can
be easily seen that $\frac{\partial \Lambda_{T-i}}{\partial s_{T-i}} > 0$, i.e.

$$
\frac{\partial \Lambda_{T-i}^c}{\partial s_{T-i}} = \frac{1}{\theta(1-a(i))} \psi_{T-i} + \gamma Q_{T-i} \eta (\lambda-1) (1-2F(s_{T-i}^p + (1+r)^i s_{T-i}^p)) > 0.
$$

Thus, optimal pre-committed consumption is excessively smooth and sensitive.

3. The first-order condition of the second-to-last period in the exemplified model of the text is

$$
u'(C_{T-i}^c) = (1+r)u'((s_{T-i}^p - C_{T-i}^c)(1+r) + s_{T-i}^p) \frac{\psi_{T-i} + \gamma Q_{T-i} \eta (\lambda-1) (1-2F(s_{T-i}^p))}{1 + \eta (\lambda-1) (1-2F(s_{T-i}^p))}.
$$

By the exact same argument as above $\psi_{T-i} > Q_{T-i}$ such that news utility introduces a first-order precautionary-savings motive in the pre-committed equilibrium. Compare the above first-order condition with the one for personal-monotone consumption $C_{T-i}$, i.e.,

$$
u'(C_{T-i}) = (1+r)u'((s_{T-i}^p - C_{T-i})(1+r) + s_{T-i}^p) \frac{\psi_{T-i} + \gamma Q_{T-i} \eta F_p(s_{T-i}^p) + \eta \lambda (1-F_p(s_{T-i}^p))}{1 + \eta F_p(s_{T-i}^p) + \eta \lambda (1-F_p(s_{T-i}^p))}.
$$

Because $\eta F_p(s_{T-i}^p) + \eta \lambda (1-F_p(s_{T-i}^p)) > \eta (\lambda-1) (1-2F(s_{T-i}^p))$ for all $s_{T-i}^p$ and $\psi_{T-i} > \gamma Q_{T-i}$ monotone personal consumption is higher $C_{T-i} > C_{T-i}^c$ than pre-committed consumption. Moreover, the difference $\eta F_p(s_{T-i}^p) + \eta \lambda (1-F_p(s_{T-i}^p)) - \eta (\lambda-1) (1-2F(s_{T-i}^p)) = \eta (1-F_p(s_{T-i}^p)) + \eta \lambda F_p(s_{T-i}^p)$ is increasing in $s_{T-i}^p$ such that the difference in consumption $C_{T-i} - C_{T-i}^c$ is increasing in $s_{T-i}^p$.

4. Consider the pre-committed first-order conditions before and after retirement. After retirement the news-utility agent’s consumption from period $T-R$ on will correspond to the standard agent’s one, i.e.,

$$Q_{T-R} = \psi_{T-R} = Q_{T-R}^c
$$

thus $\Lambda_{T-R-1}$ is given by

$$\Lambda_{T-R-1} = \frac{1}{\theta} \log \left( \frac{(1+r)^{R+1}}{f(R)} \psi_{T-R-1} + \gamma Q_{T-R-1} \eta (\lambda-1) (1-2F(s_{T-R-1}^p + (1+r)^{R+1} s_{T-R-1}^p)) \right).
$$

it can be easily seen that

$$\frac{\partial \Lambda_{T-R-1}}{\partial \gamma} < 0 \text{ only if } F(s_{T-R-1}^p + (1+r)^{R+1} s_{T-R-1}^p) < 0.5.
$$

Thus, $\gamma < 1$ does not necessarily increase or decrease $\Lambda_{T-R-1}$ and thereby consumption growth at retirement is not necessarily negative and smaller than consumption growth after
retirement. There is no systematic underweighting of marginal utility before or after retirement and there does not occur a drop in consumption at retirement for \( \gamma < 1 \). The same argument that \( \gamma < 1 \) does not necessarily lead to a reduction in consumption growth even in the end of life when \( \psi_{T-i} \) and \( Q_{T-i} \) are small implies that the pre-committed consumption path is not necessarily hump shaped.

### B.5 Derivation of the power-utility model

In the following, I outline the numerical derivation of the model with a power-utility specification \( u(C_t) = \frac{C_t^{1-\theta}}{1-\theta} \). I start with the standard model to then explain the news-utility model in detail.

#### B.5.1 The standard model

The standard agent’s maximization problem in any period \( T-i \) is

\[
\max \{ u(C_{T-i}) + \sum_{\tau=1}^{i} \beta^{\tau} E_{T-i}[u(C_{T-i+\tau})] \}
\]

subject to \( X_t = (X_{t-1} - C_{t-1})R + Y_t \) and \( Y_t = P_{t-1} e^\tau e^{i\tau} = P_i e^\tau \).

The maximization problem can be normalized by \( P^{1-\theta}_{T-i} \) and then becomes in normalized terms \((X_t = P_i x_t \) for instance)

\[
\max \{ u(e^{iT}) + \sum_{\tau=1}^{i} \beta^{\tau} E_{T-i}\prod_{j=1}^{\tau} (G_{T-i+j} e^{j\tau})^{1-\theta} u(e^{iT}) \}
\]

subject to \( x_t = (x_{t-1} - c_{t-1}) \frac{R}{G_i e^{T\tau}} + y_t \) and \( y_t = e^{i\tau} \).

The model can be solved by numerical backward induction as done by Gourinchas and Parker (2002) and Carroll (2001). The standard agent’s first-order condition is

\[
u'(c_{T-i}) = \Phi'_{T-i} = \beta R E_{T-i}\left[ \frac{\partial c_{T-i+\tau}}{\partial X_{T-i+1}} (G_{T-i+1} e^{\tau})^{-\theta} u'(c_{T-i+\tau}) + (1 - \frac{\partial c_{T-i+1}}{\partial X_{T-i+1}}) (G_{T-i+1} e^{\tau})^{-\theta} \Phi'_{T-i-1} \right]
\]

with his continuation value

\[
\Phi'_{T-i-1} = \beta R E_{T-i-1}\left[ \frac{\partial c_{T-i}}{\partial X_{T-i}} (G_{T-i} e^{\tau})^{-\theta} u'(c_{T-i}) + (1 - \frac{\partial c_{T-i-1}}{\partial X_{T-i-1}}) (G_{T-i} e^{\tau})^{-\theta} \Phi'_{T-i} \right]
\]

where it can be easily noted that

\[
P_{T-i} \Phi'_{T-i} = E_{T-i}\left[ \frac{\partial X_{T-i+1}}{\partial A_{T-i}} \frac{\partial \sum_{\tau=1}^{i} \beta^\tau u(C_{T-i+1})}{\partial X_{T-i+1}} \right] = E_{T-i}\left[ \frac{\partial X_{T-i+1}}{\partial A_{T-i}} \left( \frac{\partial \beta u(C_{T-i+1})}{\partial X_{T-i+1}} + \frac{\partial \sum_{\tau=1}^{i} \beta^\tau u(C_{T-i+1+\tau})}{\partial X_{T-i+1}} \right) \right]
\]

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\[
E_T^{-i} \left[ \frac{\partial X_{T-i+1}}{\partial A_{T-i}} \frac{\partial u(C_{T-i+1})}{\partial A_{T-i}} + \frac{\partial X_{T-i+2}}{\partial A_{T-i}} \frac{\partial u(C_{T-i+1})}{\partial A_{T-i}} + \frac{\partial X_{T-i+1}}{\partial X_{T-i+1}} \frac{\partial u(C_{T-i+1})}{\partial X_{T-i+1}} \right]
\]

\[
= E_T^{-i} \left[ \frac{\partial X_{T-i+1}}{\partial A_{T-i}} \frac{\partial u(C_{T-i+1})}{\partial X_{T-i+1}} + \frac{\partial X_{T-i+2}}{\partial X_{T-i+1}} \frac{\partial u(C_{T-i+1})}{\partial X_{T-i+2}} + \frac{\partial X_{T-i+1}}{\partial X_{T-i+1}} \frac{\partial u(C_{T-i+1})}{\partial X_{T-i+2}} \right]
\]

\[
= \beta RE_T^{-i} \left[ \frac{\partial u(C_{T-i+1})}{\partial X_{T-i+1}} + \frac{\partial X_{T-i+2}}{\partial X_{T-i+1}} \frac{\partial u(C_{T-i+1})}{\partial X_{T-i+2}} + \frac{\partial X_{T-i+1}}{\partial X_{T-i+1}} \frac{\partial u(C_{T-i+1})}{\partial X_{T-i+2}} \right]
\]

\[
= \beta RE_T^{-i} \left[ \frac{\partial u(C_{T-i+1})}{\partial X_{T-i+1}} + \frac{\partial X_{T-i+1}}{\partial X_{T-i+1}} \left( E_T^{-i} \frac{\partial X_{T-i+2}}{\partial A_{T-i+1}} \frac{\partial u(C_{T-i+1})}{\partial X_{T-i+2}} \right) \right]
\]

\[
= \beta RE_T^{-i} \left[ \frac{\partial u(C_{T-i+1})}{\partial X_{T-i+1}} + \frac{\partial X_{T-i+1} - C_{T-i+1}}{\partial X_{T-i+1}} \frac{\partial u(C_{T-i+1})}{\partial X_{T-i+1}} \right] = \beta RE_T^{-i} \left[ \frac{\partial u(C_{T-i+1})}{\partial X_{T-i+1}} + (1 - \frac{\partial C_{T-i+1}}{\partial X_{T-i+1}}) \frac{\partial u(C_{T-i+1})}{\partial X_{T-i+1}} \right].
\]

\( \Phi'_{T-i} \) is a function of savings \( a_{T-i} \) thus I can solve for each optimal consumption level \( c_{T-i} \) on a grid of savings \( a_{T-i} \) as \( c_{T-i} = (\Phi'_{T-i} - a_{T-i}) \), \( \Phi'_{T-i} = (\Phi'_{T-i} - a_{T-i}) \). To then find each optimal level of consumption for each value of the normalized cash-on-hand grid \( x_{T-i} \) by interpolation. This endogenous-grid method has been developed by Carroll (2001). Alternatively, I could use the Euler equation instead of the agent’s continuation value but this solution illustrates the upcoming solution of the news-utility model of which it is a simple case.

**B.5.2 The monotone-personal and pre-committed equilibrium in the second-to-last period**

Before starting with the fully-fledged problem, I outline the second-to-last period for the case of power utility. In the second-to-last period the agent allocates his cash-on-hand \( X_{T-1} \) between contemporaneous consumption \( C_{T-1} \) and future consumption \( C_T \), knowing that in the last period he will consume whatever he saved in addition to last period’s income shock \( C_T = X_T = (X_{T-1} - C_{T-1})R + Y_T \). According to the monotone-personal equilibrium solution concept, in period \( T-1 \) the agent takes the beliefs about contemporaneous and future consumption he entered the period with \( \{F^{T-2}_{C_{T-1}}, F^{T-2}_{C_T}\} \) as given and maximizes

\[
u(C_{T-1}) + n(C_{T-1}, F^{T-2}_{C_{T-1}}) + \gamma \beta n(F^{T-1,T-2}_{C_T}) + \beta E_{T-1} [u(C_T) + n(C_T, F^{T-1}_{C_{T-1}})]
\]

which can be rewritten as

\[
u(C_{T-1}) + \eta \int_{-\infty}^{C_{T-1}} (u(C_{T-1}) - u(c)) F^{T-2}_{C_{T-1}}(c) + \eta \lambda \int_{C_{T-1}}^{\infty} (u(C_{T-1}) - u(c)) F^{T-2}_{C_{T-1}}(c)
\]

\[+ \gamma \beta \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (u(c) - u(r)) F^{T-1,T-2}_{C_T}(c, r) + \beta E_{T-1} [u(C_T) + \eta (\lambda - 1) \int_{C_T}^{\infty} (u(C_T) - u(c)) F^{T-1}_{C_T}(c)]]
\]

To gain intuition for the model’s predictions, I explain the derivation of the first-order condition

\[
u'(C_{T-1})(1 + \eta F^{T-2}_{C_{T-1}}(C_{T-1}) + \eta (1 - F^{T-2}_{C_{T-1}}(C_{T-1}))) = \gamma \beta E_{T-1} [u'(C_T)](\eta F^{T-2}_{A_{T-1}}(A_{T-1}) + \eta \lambda (1 - F^{T-2}_{A_{T-1}}(A_{T-1})))
\]

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\[ + \beta R E_{T-1} [u'(C_T) + \eta (\lambda - 1) \int_{C_T}^{\infty} (u'(C_T) - u'(c)) F_{C_T}^{T-1}(c)]. \]

The first two terms in the first-order condition represent marginal consumption utility and gain-loss utility over contemporaneous consumption in period \( T - 1 \). As the agent takes his beliefs \( \{F_{C_T}^{T-2}, F_{C_T}^{T-2}\} \) as given in the optimization, I apply Leibniz’s rule for differentiation under the integral sign. This results in marginal gain-loss utility being the sum of states that would have promised less consumption \( F_{C_{T-1}}^{T-2}(C_{T-1}) \), weighted by \( \eta \), or more consumption \( 1 - F_{C_{T-1}}^{T-2}(C_{T-1}) \), weighted by \( \eta \lambda \).

\[
\frac{\partial n(C_{T-1}, F_{C_{T-1}}^{T-2})}{\partial C_{T-1}} = u'(C_{T-1})(\eta F_{C_{T-1}}^{T-2}(C_{T-1}) + \eta \lambda (1 - F_{C_{T-1}}^{T-2}(C_{T-1}))).
\]

Note that, if contemporaneous consumption is increasing in the realization of cash-on-hand then I can simplify \( F_{C_{T-1}}^{T-2}(C_{T-1}) = F_{X_{T-1}}^{T-2}(X_{T-1}) \). Returning to the maximization problem the third term represents prospective gain-loss utility over future consumption \( C_T \) experienced in \( T - 1 \). As before, marginal gain-loss utility is given by the weighted sum of states \( u'(C_T)(\eta F_{A_{T-1}}^{T-2}(A_{T-1}) + \eta \lambda (1 - F_{A_{T-1}}^{T-2}(A_{T-1}))) \). Note that \( F_{C_{T-1}}^{T-2}(c) \) is defined as the probability \( Pr(C_T < c | I_{T-2}) \). Applying a logic similar to the law of iterated expectation

\[
Pr(C_T < c | I_{T-2}) = Pr(A_{T-1}R + Y_T < c | I_{T-2}) = Pr(A_{T-1} < \frac{c - Y_T}{R} | I_{T-2})
\]

thus if savings are increasing in the realization of cash-on-hand then I can simplify \( F_{A_{T-1}}^{T-2}(A_{T-1}) = F_{X_{T-1}}^{T-2}(X_{T-1}) \).

The last term in the maximization problem represents consumption and gain-loss utility over future consumption \( C_T \) in the last period \( T \), i.e., the first derivative of the agent’s continuation value with respect to consumption or the marginal value of savings. Expected marginal gain-loss utility \( \eta (\lambda - 1) \int_{C_T}^{\infty} (u'(C_T) - u'(c)) F_{C_T}^{T-1}(c) \) is positive for any concave utility function such that

\[
\Psi'_{T-1} = \beta R E_{T-1} [u'(C_T) + \eta (\lambda - 1) \int_{C_T}^{\infty} (u'(C_T) - u'(c)) F_{C_T}^{T-1}(c)] > \beta R E_{T-1} [u'(C_T)] = \Phi'_{T-1}.
\]

As expected marginal gain-loss disutility is positive, increasing in \( \sigma_Y \), absent if \( \sigma_Y = 0 \), and increases the marginal value of savings, I say that news-utility introduces an “additional precautionary-savings motive”. The first-order condition can now be rewritten as

\[
u'(C_T) = \frac{\Psi'_{T-1} + \gamma \Phi'_{T-1}(\eta F_{X_{T-1}}^{T-2}(X_{T-1}) + \eta \lambda (1 - F_{X_{T-1}}^{T-2}(X_{T-1})))}{1 + \eta F_{X_{T-1}}^{T-2}(X_{T-1}) + \eta \lambda (1 - F_{X_{T-1}}^{T-2}(X_{T-1}))}.
\]

Beyond the additional precautionary-savings motive \( \Psi'_{T-1} > \Phi'_{T-1} \) implies that an increase in
$F_{X_{T-1}}^{T-2}(X_{T-1})$ decreases

$$\frac{\psi_{T-1} + \gamma(\eta F_{X_{T-1}}^{T-2}(X_{T-1}) + \eta \lambda (1 - F_{X_{T-1}}^{T-2}(X_{T-1})))}{1 + \eta F_{X_{T-1}}^{T-2}(X_{T-1}) + \eta \lambda (1 - F_{X_{T-1}}^{T-2}(X_{T-1}))},$$

i.e., the terms in the first-order condition vary with the income realization $X_{T-1}$ so that consumption is excessively smooth and sensitive.

**B.5.3 The monotone-pre-committed equilibrium in the second-to-last-period**

The first-order condition for pre-committed consumption in period $T - 1$ is

$$u'(C_{T-1}) = \frac{\psi_{T-1} + \gamma \Phi_{T-1} \eta (\lambda - 1)(1 - 2F_{X_{T-1}}^{T-2}(X_{T-1}))}{1 + \eta (\lambda - 1)(1 - 2F_{X_{T-1}}^{T-2}(X_{T-1}))}$$

by the same arguments as in the exponential-utility model derivation of the pre-committed equilibrium.

**B.5.4 The monotone-personal equilibrium path in all prior periods**

The news-utility agent’s maximization problem in any period $T - i$ is given by

$$u(C_{T-i}) + n(C_{T-i}, F_{C_{T-i}}^{T-i-1}) + \gamma \sum_{\tau=1}^{i} \beta^{\tau} n(F_{C_{T-i+\tau}}^{T-i-1,F_{T-i-1}}) + \sum_{\tau=1}^{i} \beta^{\tau} \tau E_{T-i}[U(C_{T-i+\tau})]$$

again, the maximization problem can be normalized by $P_{T_i}^{\theta}$ as all terms are proportional to consumption utility $u(\cdot)$. In normalized terms, the news-utility agent’s first-order condition in any period $T - i$ is given by

$$u'(c_{T-i}) = \frac{\psi_{T-i} + \gamma \Phi_{T-i} (\eta F_{C_{T-i}}^{T-i-1}(c_{T-i}) + \eta \lambda (1 - F_{C_{T-i}}^{T-i-1}(c_{T-i})))}{1 + \eta F_{a_{T-i}}^{T-i-1}(a_{T-i}) + \eta \lambda (1 - F_{a_{T-i}}^{T-i-1}(a_{T-i}))}$$

I solve for each optimal value of $c_{T-i}$ for a grid of savings $a_{T-i}$, as $\psi_{T-i}$ and $\Phi_{T-i}$ are functions of $a_{T-i}$ until I find a fixed point of $c_{T-i}^{*}, a_{T-i}, F_{a_{T-i}}^{T-i-1}(a_{T-i}),$ and $F_{a_{T-i}}^{T-i-1}(c_{T-i})$. The latter two can be inferred from the observation that each $c_{T-i} + a_{T-i} = x_{T-i}$ has a certain probability given the value of savings $a_{T-i-1}$ I am currently iterating on. However, this probability varies with the realization of permanent income $e_{T-i}^{p}$ thus I cannot fully normalize the problem but have to find the right consumption grid for each value of $e_{T-i}^{p}$ rather than just one. The first-order condition can be slightly modified as follows

$$u'(e_{T-i}^{p}, c_{T-i}) = \frac{e_{T-i}^{p}\psi_{T-i} + \gamma e_{T-i}^{p}\Phi_{T-i} (\eta F_{C_{T-i}}^{T-i-1}(c_{T-i}) + \eta \lambda (1 - F_{C_{T-i}}^{T-i-1}(c_{T-i})))}{1 + \eta F_{a_{T-i}}^{T-i-1}(a_{T-i}) + \eta \lambda (1 - F_{a_{T-i}}^{T-i-1}(a_{T-i}))}$$

to find each corresponding grid value. Note that, the resulting two-dimensional grid for $c_{T-i}$ will be the normalized grid for each realization of $s_{T-i}^{p}$ and $s_{T-i}^{p}$, because I multiply both sides of the
first-order conditions with $e^{\psi_{T-i}}$. Thus, the agent’s consumption utility continuation value is

$$\Phi'_{T-i-1} = \beta RE_{T-i-1}(\frac{\partial c_{T-i}}{\partial x_{T-i}}(G_{T-i}e^{\psi_{T-i}}) - \theta u'(c_{T-i}) + (1 - \frac{\partial c_{T-i}}{\partial x_{T-i}})(G_{T-i}e^{\psi_{T-i}}) - \theta \Phi'_{T-i-1}).$$

The agent’s news-utility continuation value is given by

$$p_{T-i-1}^\gamma \psi'_{T-i-1} = \beta RE_{T-i-1}(\frac{dc_{T-i}}{dx_{T-i}}u'(c_{T-i}) + \eta(\lambda - 1) \int_{c_{T-i}}^{x_{T-i}} (\frac{dc_{T-i}}{dx_{T-i}}u'(c_{T-i}) - x)dF_{T-i-1}^{\gamma}(x) + (1 - \frac{dc_{T-i}}{dx_{T-i}}p_{T-i-1}^\gamma \psi'_{T-i-1})] \psi'_{T-i-1}. \quad (here, \int_{c_{T-i}}^{x_{T-i}} \text{ means the integral over the loss domain})$$

(here, $\int_{c_{T-i}}^{x_{T-i}}$ means the integral over the loss domain) or in normalized terms

$$\Psi'_{T-i-1} = \beta RE_{T-i-1}(\frac{dc_{T-i}}{dx_{T-i}}u'(c_{T-i})(G_{T-i}e^{\psi_{T-i}}) - \theta$$

$$+ \eta(\lambda - 1) \int_{c_{T-i}}^{x_{T-i}} (\frac{dc_{T-i}}{dx_{T-i}}u'(c_{T-i})(G_{T-i}e^{\psi_{T-i}}) - \theta - x)dF_{T-i-1}^{\gamma}(x) + (1 - \frac{dc_{T-i}}{dx_{T-i}}p_{T-i-1}^\gamma \psi'_{T-i-1})] \psi'_{T-i-1}.$$

B.5.5 Risk attitudes over small and large stakes

First, I suppose the agent is offered a gamble about immediate consumption in period $t$ after that period’s uncertainty has resolved and that period’s original consumption has taken place. I assume that utility over immediate consumption is linear. Then, the agent is indifferent between accepting or rejecting a 50-50 win $G$ or lose $L$ gamble if $0.5G - 0.5L + 0.5\eta G - 0.5\eta L = 0$. Second, I suppose the agent is offered a monetary gamble or wealth bet that concerns future consumption. Suppose $T \to \infty$. I assume that his initial wealth level is $A_t = 100,000$ and $P_t = 300,000$. Let $f^\psi(A)$ and $f^\Phi(A)$ be the agent’s continuation value as a function of the agent’s savings $A_t$. Then, the agent is indifferent between accepting or rejecting a 50-50 win $G$ or lose $L$ gamble if $0.5\eta(f^\Phi(A_t + G) - f^\Phi(A_t)) + 0.5\eta \lambda(f^\Phi(A_t - L) - f^\Phi(A_t)) + 0.5f^\psi(A_t + G) + 0.5f^\psi(A_t - L) = f^\psi(A_t - L)$.

B.6 Habit formation

Consider an agent with internal, multiplicative habit formation preferences $u(c_t, H_t) = \frac{(c_t/ H_t)^{1-\theta}}{1-\theta}$ with $H_t = H_{t-1} + \vartheta(C_{t-1} - H_{t-1})$ and $\vartheta \in [0, 1]$ (Michaelides (2002)). Assume $\vartheta = 1$ such that
$H_t = C_{t-1}$. For illustration, in the second-to-last period his maximization problem is

\[ u(C_{T-1}, H_{T-1}) + \beta E_{T-1}[u(R(X_{T-1} - C_{T-1}) + Y_T, H_T)] = \frac{(C_{T-1})^{1-\theta}}{1-\theta} + \beta E_{T-1}[\frac{1}{1-\theta}(R(X_{T-1} - C_{T-1}) + Y_T)^{1-\theta}] \]

which can be normalized by $P_T^{(1-\theta)(1-h)}$ (then $C_T = P_Tc_T$ for instance) and the maximization problem becomes

\[ P_T^{(1-\theta)(1-h)}(\frac{C_{T-1}}{h_T^{1-\theta}})^{1-\theta} + \beta P_T^{(1-\theta)(1-h)}E_{T-1}[\frac{1}{1-\theta}(G_T e_T^p)^{(1-\theta)(1-h)}(\frac{x_{T-1} - c_{T-1}}{h_T} + \frac{R}{h_T} + \frac{c_T}{h_T})]^{1-\theta}] \]

which results in the following first-order condition

\[ c_{T-1}^{1-\theta} = h_T^{1-\theta} + \beta E_{T-1}[(G_T e_T^p)^{-\theta(1-h)}(\frac{C_T}{h_T})^{-\theta}(R + \frac{c_T}{h_T})] = \Phi_{T-1}' \]

with $\Phi_{T-1}'$ being a function of savings $x_{T-1} - c_{T-1}$ and habit $h_T$. The first-order condition can be solved very robustly by iterating on a grid of savings $a_{T-1}$ assuming $c_{T-1}' = (\Phi_{T-1}')^{-\frac{1}{h}}$ and $h_T = c_{T-1}' \frac{1}{G_T e_T^p}$ until a fixed point of consumption and habit has been found. The normalized habit-forming agent’s first-order condition in any period $T - i$ is given by

\[ c_{T-i}^{1-\theta} = h_T^{1-\theta} + \beta E_{T-i}[(G_{T-i+1} e_{T-i+1}^p)^{-\theta(1-h)}(\frac{C_{T-i+1}}{h_T})^{-\theta}(R \frac{dc_{T-i+1}}{dx_{T-i+1}} + \frac{c_{T-i+1}}{h_T})]^{1-\theta} \]

\[ + (1 - \frac{dc_{T-i+1}}{dx_{T-i+1}})(G_{T-i+1} e_{T-i+1}^p)^{-\theta(1-h)}] \Phi_{T-1}' \]

\[ = \Phi_{T-i}' \]

where $\Phi_{T-i}'$ is a function of savings $x_{T-i} - c_{T-i}$ and habit $h_{T-i}$. The first-order condition can be solved very robustly by iterating on a grid of savings $a_{T-i}$ assuming $c_{T-i}' = (\Phi_{T-i}')^{-\frac{1}{h}}$ and $h_{T-i} = c_{T-i}' \frac{1}{G_T e_T^p}$ until a fixed point of consumption and habit has been found.
Part V

Expectations-Based Reference-Dependent Preferences and Asset Pricing

C More Figures and Tables

Figure 12: Annualized expected risky $E_t[R_{t+1}]$ and risk-free returns $R_{t+1}^f$ in the news-utility and standard models and annualized equity premium $E_t[R_{t+1}] - R_{t+1}^f$ in the news-utility and standard models.
Table 8: **Calibration and moments of the basic model**

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<th>Calibration</th>
<th>( \mu_c )</th>
<th>( \sigma_c )</th>
<th>( \beta )</th>
<th>( \theta )</th>
<th>( \eta )</th>
<th>( \lambda )</th>
<th>( \gamma )</th>
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<td>.98</td>
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<td>0.51</td>
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<td>0.004</td>
<td>0.005</td>
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<tr>
<td>( R^2 )</td>
<td>0.00</td>
<td>0.12</td>
<td>0.15</td>
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Return and consumption moments are inferred from value-weighted CRSP return data and BEA data on-real per-capita consumption of nondurables and services for the period 1929-1998 (as in Bansal and Yaron (2004)). All return moments are annualized and in percentage terms. The parameters \( \mu_c, \sigma_c, \) and \( \beta \) are annualized. The quarterly moments for the consumption-wealth ratio, the consumption-price ratio, and predictability regression are taken from Lettau and Ludvigson (2001) Table II. The \( R^2 \) corresponds to a forecasting regression of quarterly stock returns on the quarterly consumption-wealth ratio \( r_{t+1} = \alpha + \beta (c_t - w_t) + \delta r_t' \) (Table III in Lettau and Ludvigson (2001)).
Figure 13: Simulated consumption-wealth ratio and comparison to the \( \hat{cay} \) data as provided by Lettau and Ludvigson (2005).

### D Derivation and proofs

#### D.1 Proof of Proposition 7

In the following, I quickly guess and verify the model’s equilibrium. In Section D.2, I derive the model’s equilibrium in greater detail and more comprehensively. The exogenous consumption process is \( \frac{C_{t+1}}{C_t} = e^{\mu_c + \epsilon_{t+1}} \) and, in equilibrium, the agent beliefs about consumption are fully determined by it, i.e., \( F_{C_{t+\tau}}^{\tau} = \log - N(\log(C_t) + \tau \mu_c, \tau^2 \sigma^2_c) \). First, I define the following two constants determined by the exogenous parameters only

\[
Q = E_t \left[ \sum_{\tau=1}^\infty \beta^\tau \left( \frac{C_{t+\tau}}{C_t} \right)^{1-\theta} \right] = E_t \left[ \sum_{\tau=1}^\infty \beta^\tau (e^{\tau \mu_c + \sum_{j=1}^\tau \epsilon_{t+j}})^{1-\theta} \right] = \frac{\beta e^{\mu_c (1-\theta)} + \frac{1}{2} (1-\theta)^2 \sigma^2_c}{1 - \beta e^{\mu_c (1-\theta)} + \frac{1}{2} (1-\theta)^2 \sigma^2_c}
\]

and

\[
\psi = \beta e^{\mu_c (1-\theta)} E_t \left[ (e^{\epsilon_{t+1}})^{1-\theta} + (1 + \gamma Q)(\eta \int_{-\infty}^{\epsilon_{t+1}} ((e^{e^{e_{t+1}}})^{1-\theta} - (e^e)^{1-\theta}) dF(e) + \eta \lambda \int_{\epsilon_{t+1}}^\infty ((e^{e_{t+1}})^{1-\theta} - (e^e)^{1-\theta}) dF(e)] + (e^{e_{t+1}})^{1-\theta} \psi \right].
\]
Table 9: Calibration and moments of the extended models

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<th>2.7%</th>
<th>.95</th>
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<td>( \sigma_c )</td>
<td>( \beta )</td>
<td>( \theta )</td>
<td>( \eta )</td>
<td>( \lambda )</td>
<td>( \gamma )</td>
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<td>( \bar{\mu} )</td>
<td>( \bar{\sigma} )</td>
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calibration disaster risk

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<th>2</th>
<th>0.8</th>
<th>0</th>
<th>2</th>
</tr>
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<tbody>
<tr>
<td>( \mu_c )</td>
<td>( \sigma_c )</td>
<td>( \beta )</td>
<td>( \theta )</td>
<td>( \eta )</td>
<td>( \lambda )</td>
<td>( \gamma )</td>
<td>( \nu )</td>
<td>( \bar{\mu} )</td>
<td>( \bar{\sigma} )</td>
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<tr>
<td>5.04</td>
<td>6.17</td>
<td>6.33</td>
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<td></td>
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</tbody>
</table>

\( E[r_t - r_{f_t}^t] \)
\( \sigma(r_t - r_{f_t}^t) \)
\( E[r_{f_t}^t] \)
\( \sigma(r_{f_t}^t) \)

<table>
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<td>( corr(\Delta c_t, r_t) )</td>
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<tr>
<td>( corr(\Delta c_t, r_{f_t+1}) )</td>
<td>0.60</td>
<td>0.06</td>
<td>0.09</td>
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<tr>
<td>( AR(r_t) )</td>
<td>0.18</td>
<td>-0.44</td>
<td>0.011</td>
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</table>

\( E[c_t - p_t] \)
\( \sigma(c_t - w_t) \)
\( AR(c_t - w_t) \)
\( R^2 \)

| 2.25       | 1.86 | 1.89 |     |     |   |   |     |   |   |
|            | 13.5 | 2.24 | 2.93 |     |     |   |   |     |   |

Return and consumption moments are inferred from value-weighted CRSP return data and BEA data on-real per-capita consumption of nondurables and services for the period 1929-1998 (as in Bansal and Yaron (2004)). All return moments are annualized and in percentage terms. The parameters \( \mu_c \), \( \sigma_c \), and \( \beta \) are annualized. The quarterly moments for the consumption-wealth ratio, the consumption-price ratio, and predictability regression are taken from Lettau and Ludvigson (2001) Table II. The \( R^2 \) corresponds to a forecasting regression of quarterly stock returns on the quarterly consumption-wealth ratio \( r_{t+1} = \alpha + \beta (c_t - w_t) + \delta r_{f_t}^t \) (Table III in Lettau and Ludvigson (2001)).
The agent’s maximization problem is
\[
\max_{C_t} \{ u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^{\infty} \beta^\tau n(F_{C_{t+\tau}}^{t-1}) + E_t[\sum_{\tau=1}^{\infty} \beta^\tau U_{t+\tau}] \}. 
\]

Now, it can be easily noted that \( E_t[\sum_{\tau=1}^{\infty} \beta^\tau U_{t+\tau}] = u(C_t) \) and \( \gamma \sum_{\tau=1}^{\infty} \beta^\tau n(F_{C_{t+\tau}}^{t-1}) = \gamma(\eta \int_{-\infty}^{C_t}(u(C_t)Q - u(c)Q)dF_{C_t}^{t-1}(c)) \) in equilibrium.

The agent is a price-taker. In the beginning of each period, the agent observes the realization of his wealth \( W_t \) and decides how much to consume \( C_t \) and how much to invest into the Lucas tree \( P_t = C_t - W_t \). I guess the model’s solution as \( C_t = W_t \rho_t \) with \( \rho_t \) being i.i.d., independent of calender time \( t \), or wealth \( W_t \). Thus, next period’s consumption is given by \( C_{t+1} = (W_t - C_t)\rho_{t+1} \) with \( R_{t+1} = \frac{\rho_{t+1} + C_{t+1}}{\rho_t} = \frac{\rho_t C_{t+1} + 1}{\rho_t} \) so that \( C_{t+1} = (W_t - C_t)\frac{\rho_t C_{t+1}}{\rho_t} \). From this consideration it can be easily seen that the agent’s future value \( u(C_t)Q \) and \( u(C_t)Q \) can be rewritten as \( u(W_t - C_t)(\frac{\rho_t}{1 - \rho_t})^{1-\theta} \) and \( u(W_t - C_t)(\frac{\rho_t}{1 - \rho_t})^{1-\theta} Q \) whereby \( \frac{\rho_t}{1 - \rho_t} \) stems from the return and is thus taken as exogenous by the agent. In turn, the maximization problem can be rewritten as

\[
\max_{C_t} \{ u(C_t) + \eta \int_{-\infty}^{C_t} (u(C_t) - u(c))dF_{C_t}^{t-1}(c) + \eta \lambda \int_{C_t}^{\infty} (u(C_t) - u(c))dF_{C_t}^{t-1}(c) \\
+ \gamma \mathcal{Q}(\eta \int_{-\infty}^{C_t} (u(W_t - C_t)(\frac{\rho_t}{1 - \rho_t})^{1-\theta} - u(c))dF_{C_t}^{t-1}(c) \\
+ \eta \lambda \int_{C_t}^{\infty} (u(W_t - C_t)(\frac{\rho_t}{1 - \rho_t})^{1-\theta} - u(c))dF_{C_t}^{t-1}(c) + u(W_t - C_t)(\frac{\rho_t}{1 - \rho_t})^{1-\theta} \}
\]

which yields the following first-order condition
\[

C_t^{-\theta} (1 + \eta F(\epsilon_t) + \eta \lambda (1 - F(\epsilon_t))) = (W_t - C_t)^{-\theta} (\frac{\rho_t}{1 - \rho_t})^{1-\theta} (\gamma \mathcal{Q}(\eta F(\epsilon_t) + \eta \lambda (1 - F(\epsilon_t))) + \psi)
\]

as the agent takes his prior beliefs about consumption \( F_{C_t}^{t-1} \) as given in the optimization and since \( F_{C_t}^{t-1}(C_t) = F(\epsilon_t) \) with \( F \sim N(0, \sigma_e^2) \), because \( C_t = C_{t-1}e^{\mu_t + \epsilon_t} \). Rewriting the first-order condition allows me to verify the solution guess
\[
C_t = \frac{\rho_t}{1 - \rho_t} = \frac{1}{1 + \frac{\psi + \gamma \mathcal{Q}(\eta F(\epsilon_t) + \eta \lambda (1 - F(\epsilon_t)))}{\eta F(\epsilon_t) + \eta \lambda (1 - F(\epsilon_t))}}.
\]

### D.2 Detailed derivation of the model’s equilibrium

In the following I derive the model’s equilibrium in greater detail. The agent optimally chooses his consumption \( C_t \) to maximize his life-time utility
\[
\max_{C_t} \{ u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^{\infty} \beta^\tau n(F_{C_{t+\tau}}^{t-1}) + E_t[\sum_{\tau=1}^{\infty} \beta^\tau U_{t+\tau}] \}. 
\]

(43)
The agent’s wealth in the beginning of the period $W_t$ is determined by the portfolio return $R_t^p = R_t^f + \alpha_{t-1}(R_t - R_t^f)$, which depends on the risky return realization $R_t$, the risk-free return $R_t^f$, and last period’s optimal portfolio share $\alpha_{t-1}$. I impose the equilibrium condition $\alpha_t = 1$ for all $t$ to simplify the maximization problem. Now the agent’s problem can be thought of as an infinite-horizon cake-eating problem with a single risky savings device. Thus, the budget constraint is

$$W_t = (W_{t-1} - C_{t-1})R_t$$

which results in the following first-order condition

$$u'(C_t)(1 + \eta F_{C_t}^{t-1}(C_t) + \eta \lambda (1 - F_{C_t}^{t-1}(C_t))) = u'(W_t - C_t)Q_t^0 + u'(W_t - C_t)\psi_t^0$$

(45)

I explain each term in the first-order condition, equation (45), subsequently. The left hand side in equation 45 represents the agent’s marginal utility due to consumption utility and gain-loss utility over contemporaneous consumption. Because the agent takes the reference point as given in the optimization and assuming optimal consumption is monotonically increasing in the return realization only the probability masses of states ahead and beneath remain to be considered. As an illustration, consider the following optimization

$$\frac{d}{dC_t}(\eta \int_{-\infty}^{C_t} (u(C_t) - u(c)))dF_{C_t}^{t-1}(c) + \eta \lambda \int_{C_t}^{\infty} (u(C_t) - u(c)))dF_{C_t}^{t-1}(c))$$

$$= \eta \int_{-\infty}^{C_t} u'(C_t)dF_{C_t}^{t-1}(c) + \eta \lambda \int_{C_t}^{\infty} u'(C_t)dF_{C_t}^{t-1}(c) = u'(C_t)(\eta F_{C_t}^{t-1}(C_t) + \eta \lambda (1 - F_{C_t}^{t-1}(C_t)))$$

$$= u'(C_t)(\eta F_{R_t}^{t-1}(R_t) + \eta \lambda (1 - F_{R_t}^{t-1}(R_t)) = u'(C_t)(\eta F(\varepsilon_t) + \eta \lambda (1 - F(\varepsilon_t)))$$

if $C_t$ is monotonically increasing in the realization of $R_t$ then $F_{R_t}^{t-1}(R_t) = F_{C_t}^{t-1}(C_t)$. In a preferred personal equilibrium the agent would know ex ante if the first-order condition induces him to “jump” realizations of $R_t$, and expectations over optimal consumption would adjust accordingly such that in equilibrium $F_{C_t}^{t-1}(C_t) = F_{R_t}^{t-1}(R_t)$ for each corresponding realization of $C_t$ and $R_t$. Moreover, in general equilibrium the agent’s beliefs have to match the model environment and hence $F_{R_t}^{t-1}(R_t) = F_{C_t}^{t-1}(C_t) = F(\varepsilon_t)$ for each corresponding realization of $C_t$, $R_t$, and $\varepsilon_t$ such that both $C_t$ and $R_t$ are necessarily increasing in $\varepsilon_t$.

To explain the right hand side in equation 45 I guess and verify the equilibrium’s structure. In each period $t$, the agent will consume a fraction $\rho_t$ of his wealth $W_t$, i.e., $C_t = \rho_t W_t$. In the first-order condition, equation 45, the first term on the right hand side represents prospective gain-loss utility over the entire stream of future consumption. Note that, each future optimal consumption as a fraction of wealth can be iterated back to the current savings decision

$$C_{t+\tau} = (W_t - C_t)R_{t+\tau}\rho_{t+\tau} \prod_{j=1}^{\tau-1} R_{t+j}(1 - \rho_{t+j})$$

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Then, taking the reference point as given and assuming that optimal savings are monotonically increasing in the return realization results in

\[-(W_t - C_t)^{-\theta} Q_t^0 = \frac{\partial}{\partial C_t} \sum_{t=1}^{\infty} \beta^t \mathbb{E}_t \left[ \prod_{j=1}^{\infty} R_{t+j}^{1-\theta} \right] \int_{-\infty}^{\infty} \mu(u(c) - u(r)) dF_{C_t}^{t+1}(c, r)\]

\[
= -\sum_{t=1}^{\infty} \beta^t (W_t - C_t)^{-\theta} E_t \left[ \prod_{j=1}^{\infty} R_{t+j}^{1-\theta} \right] \int_{-\infty}^{\infty} \mu(\eta F_t^{t+1}(r) + \eta \lambda (1 - F_t^{t+1}(r)))
\]

\[-(W_t - C_t)^{-\theta} Q_t^0 = -(W_t - C_t)^{-\theta} E_t \left[ \prod_{j=1}^{\infty} R_{t+j}^{1-\theta} \right] \int_{-\infty}^{\infty} \mu(\eta F_t^{t+1}(r) + \eta \lambda (1 - F_t^{t+1}(r))).\]

Moreover,

\[R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t} = \frac{\rho_t}{1 - \rho_t} \frac{C_{t+1}}{C_t} \frac{1}{\rho_{t+1}}\]

such that

\[R_{t+1}^{1-\theta} \rho_{t+1}^{1-\theta} = \left( \frac{C_{t+\tau}}{C_{t+\tau-1}} \frac{\rho_{t+\tau-1}}{1 - \rho_{t+\tau-1}} \right)^{1-\theta} \text{ and } R_{t+j}^{1-\theta} (1 - \rho_{t+j})^{1-\theta} = \left( \frac{C_{t+j}}{C_{t+j-1}} \frac{\rho_{t+j-1}}{1 - \rho_{t+j-1}} \frac{1 - \rho_{t+j}}{\rho_{t+j}} \right)^{1-\theta}.\]

Recall that, the model’s exogenous consumption process implies \(\frac{C_{t+\tau}}{C_t} = e^{\psi \mu + \sum_{j=1}^{\tau} \epsilon_{t+j}}.\) Because in a rational-expectations equilibrium, the agent’s expectational terms have to match the model’s specification \(\frac{\partial}{\partial C_t} \sum_{t=1}^{\infty} \beta^t \mathbb{E}_t \left[ \prod_{j=1}^{\infty} F_{C_t}^{t+1}(c, r) \int_{-\infty}^{\infty} \mu(u(c) - u(r)) dF_{C_t}^{t+1}(c, r)\right]\) can be rewritten as \(-(W_t - C_t)^{-\theta} Q_t^0\)

\[-(W_t - C_t)^{-\theta} Q_t^0 = -(W_t - C_t)^{-\theta} \left( \frac{\rho_t}{1 - \rho_t} \right)^{1-\theta} \gamma \frac{\beta e^{\psi (1-\theta) + 1/2 (1-\theta)^2 \sigma^2}}{1 - \beta e^{\psi (1-\theta) + 1/2 (1-\theta)^2 \sigma^2}} (\eta F(\epsilon_t) + \eta \lambda (1 - F(\epsilon_t)))\]

\[-(W_t - C_t)^{-\theta} \gamma Q(\eta F(\epsilon_t) + \eta \lambda (1 - F(\epsilon_t))).\]

Returning to equation 45, the second term on the right hand side \(-(W_t - C_t)^{-\theta} \psi_t^0\) refers to next period’s marginal value, which turns out to be linear in the marginal utility of wealth. As above, iterating back next period’s marginal utility, i.e., \(\frac{\partial}{\partial C_t} \left( W_t - C_t \right)^{-\theta} R_{t+1}^{1-\theta} \rho_{t+1}^{1-\theta}\) and similarly
for future consumption, for instance \( \frac{\partial u(C_{t+2})}{\partial C_t} = (W_t - C_t)^{-\theta} R_{t+1}^{1-\theta} (1 - \rho_{t+1}^{1-\theta}) R_{t+2}^{1-\theta} \), yields

\[
(W_t - C_t)^{-\theta} \beta E_t[R_{t+1}^{1-\theta} \Psi_{t+1}] = (W_t - C_t)^{-\theta} \beta E_t[R_{t+1}^{1-\theta} \rho_{t+1}^{1-\theta} + \eta \int_{-\infty}^{R_{t+1}^{1-\theta}} (R_{t+1}^{1-\theta} \rho_{t+1}^{1-\theta} - (R\rho)^{1-\theta})dF_{R\rho}(R\rho) + \\
+ \eta \lambda \int_{R_{t+1}^{1-\theta}}^{\infty} (R_{t+1}^{1-\theta} \rho_{t+1}^{1-\theta} - (R\rho)^{1-\theta})dF_{R\rho}(R\rho) + \\
+ \gamma(\frac{\rho_{t+1}}{1 - \rho_{t+1}})^{1-\theta} Q(\eta \int_{-\infty}^{R_{t+1}^{1-\theta}} (1 - \rho_{t+1})^{1-\theta} - (R(1 - \rho))^{1-\theta})dF_{R(1-\rho)}(R(1 - \rho)) + \\
+ \eta \lambda \int_{R_{t+1}^{1-\theta}}^{\infty} (1 - \rho_{t+1})^{1-\theta} - (R(1 - \rho))^{1-\theta})dF_{R(1-\rho)}(R(1 - \rho)) + \beta R_{t+1}^{1-\theta} (1 - \rho_{t+1}^{1-\theta}) E_{t+1}[R_{t+2}^{1-\theta} \Psi_{t+2}]].
\]

Now, let \( \psi = \beta E_t[(C_{t+1}^{1-\theta} - C_t^{1-\theta})^{1-\theta} \Psi_{t+1}] = \beta E_t[(C_{t+2}^{1-\theta} - C_{t+1}^{1-\theta})^{1-\theta} \Psi_{t+2}] \) which is constant for any period \( t \) because \( C_{t+1}^{1-\theta}, \rho_{t+1}, \) and \( \Psi_{t+1} \) are all solely determined by the realization of \( \varepsilon_{t+1} \) and exogenous parameters. Then, the last term in the equation of \( (W_t - C_t)^{-\theta} \beta E_t[R_{t+1}^{1-\theta} \Psi_{t+1}] \) is

\[
\beta R_{t+1}^{1-\theta} (1 - \rho_{t+1}^{1-\theta}) E_{t+1}[R_{t+2}^{1-\theta} \Psi_{t+2}] = R_{t+1}^{1-\theta} (1 - \rho_{t+1}^{1-\theta}) (\frac{\rho_{t+1}}{1 - \rho_{t+1}}) \Psi_{t+1}^{1-\theta} = R_{t+1}^{1-\theta} \rho_{t+1}^{1-\theta} \Psi_{t+1}.
\]

And, moreover

\[
\beta E_t[R_{t+1}^{1-\theta} \Psi_{t+1}] = \beta E_t[(C_{t+1}^{1-\theta} - C_t^{1-\theta})^{1-\theta} \Psi_{t+1}] = (\frac{\rho_t}{1 - \rho_t})^{1-\theta} \Psi
\]

such that it follows for the first-order condition, equation 45, that \( \psi^0 = (\frac{\rho_t}{1 - \rho_t})^{1-\theta} \Psi. \)

Plugging in \( R_{t+1} = \frac{C_{t+1}}{C_t} \rho_{t+1} \rho_{t+1} \) in the equation for \( E_t[R_{t+1}^{1-\theta} \Psi_{t+1}] \) and recalling that \( \frac{C_{t+1}}{C_t} = e^{\mu + \varepsilon_{t+1}} \) or alternatively simply dividing next period’s \( C_{t+1} \) terms by \( C_t \) allows to express \( \Psi \) in much simpler terms

\[
(\frac{\rho_t}{1 - \rho_t})^{1-\theta} \Psi = (\frac{\rho_t}{1 - \rho_t})^{1-\theta} \beta e^{\mu(1-\theta)} E_t[(e^{\varepsilon_{t+1}})^{1-\theta} + (1 + \gamma Q)(\eta \int_{-\infty}^{\varepsilon_{t+1}} ((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon_{t+1}})^{1-\theta})dF(e) + \\
+ \eta \lambda \int_{\varepsilon_{t+1}}^{\infty} ((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon_{t+1}})^{1-\theta})dF(e) + (e^{\varepsilon_{t+1}})^{1-\theta} \Psi)]
\]

accordingly \( \Psi = Q + (1 + \gamma Q)\Omega = Q + \Omega + \gamma \Omega Q \) with \( \Omega \) given by

\[
\Omega = \frac{\beta e^{\mu(1-\theta)} E_t[(\eta \int_{-\infty}^{\varepsilon_{t+1}} ((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon_{t+1}})^{1-\theta})dF(e) + \eta \lambda \int_{\varepsilon_{t+1}}^{\infty} ((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon_{t+1}})^{1-\theta})dF(e)]}}{1 - \beta e^{\mu(1-\theta) + \frac{1}{2}(1-\theta)^2\sigma_c^2}}
\]

\[
\Omega = \frac{\beta e^{\mu(1-\theta)} \omega(\sigma_c)}{1 - \beta e^{\mu(1-\theta) + \frac{1}{2}(1-\theta)^2\sigma_c^2}} \text{ with}
\]
And the general equilibrium consumption-wealth ratio is then given by

\[
\omega(\sigma) = \int_{-\infty}^{\infty} (e^z)^{-\theta} \left( (e^z)^{-1} - (e^z)^{-\theta} \right) dF(z) + \eta \lambda \int_{-\infty}^{\infty} \left( (e^z)^{-1} - (e^z)^{-\theta} \right) dF(z)dz, \quad z, e \sim N(0, \sigma^2)
\]

\[
= \int_{-\infty}^{\infty} \{ \eta F(z) e^{(1-\theta)z} - \eta e^{z(1-\theta)^2/\sigma^2} (1 - F(1 - \theta) \sigma^2 - z)) \}
\]

\[
+ \eta \lambda (1 - F(z)) e^{(1-\theta)z} - \eta \lambda e^{z(1-\theta)^2/\sigma^2} F(1 - \theta) \sigma^2 - z)) dF(z)
\]

In turn, the first-order condition can be rewritten as

\[
u'(C_t) (1 + \eta F(\varepsilon_t) + \eta \lambda (1 - F(\varepsilon_t))) = u'(W_t - C_t) \left( \frac{\rho_t}{1 - \rho_t} \right)^{-\theta} (\gamma Q(\eta F(\varepsilon_t) + \eta \lambda (1 - F(\varepsilon_t)))) + \psi.
\]

And the general equilibrium consumption-wealth ratio is then given by

\[
\frac{C_t}{W_t} = \rho_t = \frac{1}{1 + \frac{Q_0 + \gamma Q + \gamma Q \varepsilon_t}{1 + \eta F(\varepsilon_t) + \eta \lambda (1 - F(\varepsilon_t))}}.
\]

Now, the solution guess \(C_t = \rho_t W_t\) and \(W_t - C_t = (1 - \rho_t)W_t\) can be verified. The agent’s value function is given by \(V_t(W_t) = u(W_t) \Psi_t\). Obviously, \(C_t, W_t - C_t,\) and \(R_t\) are all increasing in the realization of \(\varepsilon_t\). Finally, note that solving the model using backward induction and taking it to its limit yields this exact same solution.

The stochastic discount factor can be inferred from the first-order condition

\[
1 = E_t[M_{t+1}R_{t+1}] = E_t[\frac{\beta u'(W_{t+1})\Psi_{t+1}}{u'(C_t) (1 + \eta F(R_t) + \eta \lambda (1 - F(R_t)))} - E_t[u'(W_t - C_t)\Omega_t] \]
\]

\[
\Rightarrow M_{t+1} = (1 + (\eta F(\varepsilon_t) + \eta \lambda (1 - F(\varepsilon_t)))) \left( 1 - \frac{\rho_t}{1 - \rho_t} \right) \left( \frac{C_{t+1}}{C_t} \right) \left( \frac{1}{\rho_{t+1}} \right) - \beta \left( \frac{C_{t+1}}{C_t} \right) \left( \frac{1}{\rho_{t+1}} \right) \Psi_{t+1}
\]

\[
\beta \left( \frac{C_{t+1}}{C_t} \right) \left( \frac{1}{\rho_{t+1}} \right) \Psi_{t+1} = \beta e^{\mu(1-\theta)} \{ (e^{\bar{\varepsilon}_{t+1}})^{1-\theta} + (1 + \gamma Q)(\eta \int_{-\infty}^{\bar{\varepsilon}_{t+1}} ((e^{\bar{\varepsilon}_{t+1}})^{1-\theta} - (e^z)^{1-\theta}) dF(z) + \]

\[
+ \eta \lambda \int_{\varepsilon_{t+1}}^{\infty} ((e^{\bar{\varepsilon}_{t+1}})^{1-\theta} - (e^z)^{1-\theta}) dF(z) + (e^{\bar{\varepsilon}_{t+1}})^{1-\theta} \psi
\]

If \(\eta(\lambda - 1) > 1\), the stochastic discount factor in the news utility model has a somewhat irritating feature: The existence of gain-loss utility generates negative values of the stochastic discount factor in particularly good states of the world. For any parameter choice, increasing the realization of \(\varepsilon_{t+1}\) will result in negative values of \(M_{t+1}\) at some point. The agent dislikes it if a return pays out in particularly good states of the world because he will experience adverse news-utility in all other states. Therefore, ex-ante, the agent would prefer to burn consumption in those particularly pleasurable states. Although a negative stochastic discount factor implies arbitrage opportunities, non-satiated agents would not choose to buy consumption in these states at negative prices because they would experience adverse news utility in all other states. Therefore, the equilibrium is still
valid. Moreover, the negativity of the stochastic discount factor in these states is unlikely to matter for the model’s implications because, for reasonable parameter combinations, negativity only occurs in the range of four to five standard deviations from the mean. This positive probability of negative state prices is not new to the literature, Chapman (1998) elaborates on the possibility arising in habit-formation endowment economies and Dybvig and Ingersoll (1989) show how it arises in the CAPM.

Note that, for \( \eta = 0 \) the model reduces to non-news or plain power utility in which the consumption-wealth ratio \( \rho^s \) is constant:

\[
\left( \frac{1 - \rho^s}{\rho^s} \right)^{1-\theta} \psi = \beta E_t[R_{t+1}^{1-\theta} \Psi_{t+1}] \Rightarrow \psi = \beta e^{\mu_t(1-\theta)} E_t[(e^{\epsilon_t+1})^{1-\theta} + (e^{\epsilon_t+1})^{1-\theta} \Psi_t] \Rightarrow \psi = Q
\]

\[
1 = E_t[M_{t+1}R_{t+1}] = E_t\left[\frac{\beta u'(W_{t+1})((\rho^s)^{1-\theta} + (1 - \rho^s)(\rho^s)^{1-\theta} \psi)}{\beta u'(C_t)}\right] R_{t+1}
\]

\[
M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\theta} (\rho^s)^{1-\theta} (1 + \psi) = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\theta}
\]

### D.3 Proof of Proposition 8

The marginal value of savings is given by \(-\frac{d\beta E_t[u'(W_t)Q^t_{t+1}]}{dC_t} = u'(W_t - C_t)(Q + \Omega + \gamma \Omega Q)\) whereas in the standard model \( \eta = 0 \Rightarrow \Omega = 0 \) and the marginal value of savings is given by \( u'(W_t - C_t)Q \). If \( \eta > 0, \lambda > 1 \) and \( \theta > 1 \) then \( \Omega > 0 \) such that \( Q + \Omega + \gamma \Omega Q > Q \) because:

\[
\Omega = \frac{\beta e^{\mu_t(1-\theta)}}{1 - \beta e^{\mu_t(1-\theta) + \frac{1}{2}(1-\theta)^2 \sigma_c^2}} \omega(\sigma_c) > 0 \text{ for } \theta > 1
\]

since \( \omega(\sigma) = \int_{-\infty}^{\infty} (\int_{-\infty}^{z} ((e^x)^{1-\theta} - (e^x)^{1-\theta}) \sigma x dF(x) + \eta \lambda \int_{z}^{\infty} ((e^x)^{1-\theta} - (e^x)^{1-\theta}) dF(x) dF(z) \]

Therefore, news-utility introduces an additional precautionary savings motive. Moreover, the consumption-wealth ratio is given by

\[
\rho_t = \frac{1}{1 + \frac{Q + \Omega + \gamma \Omega Q + \gamma \Omega (\eta F(\epsilon_t) + \eta \lambda (1 - F(\epsilon_t)))}{1 + \eta F(\epsilon_t) + \eta \lambda (1 - F(\epsilon_t))}} \text{ whereas in the standard model } \rho = \frac{1}{1 + Q} \cdot
\]

Thus, the consumption-wealth ratio is unambiguously lower than in the standard model for \( \gamma = 1 \) because \( \frac{Q + \Omega + \gamma \Omega Q + \gamma \Omega (\eta F(\epsilon_t) + \eta \lambda (1 - F(\epsilon_t)))}{1 + \eta F(\epsilon_t) + \eta \lambda (1 - F(\epsilon_t))} > 1 \). For \( \gamma < 1 \), the consumption-wealth ratio is lower if \( \gamma > \breve{\gamma} \) with

\[
\frac{Q + \Omega + \gamma \Omega Q + \gamma \Omega (\eta F(\epsilon_t) + \eta \lambda (1 - F(\epsilon_t)))}{1 + \eta F(\epsilon_t) + \eta \lambda (1 - F(\epsilon_t))} = Q \Rightarrow \breve{\gamma} = \frac{\eta \lambda - \frac{\Omega}{\rho}}{\Omega + \eta \lambda}.
\]

As can be easily seen, \( \breve{\gamma} < 1 \). I chose \( F(\epsilon_t) = 1 \) to obtain \( \breve{\gamma} \) because \( F(\epsilon_t) = 1 \) maximizes \( \rho \) if
\( \theta > 1 \). Moreover, as can be easily seen \( \frac{\partial \Omega}{\partial \eta}, \frac{\partial \Omega}{\partial \lambda} > 0 \) if \( \theta > 1 \). Then

\[
\frac{\partial \gamma}{\partial \eta} = \frac{\partial \eta \lambda - \frac{\partial \gamma}{\partial \Omega}(\Omega + \eta \lambda) - (\frac{\partial \gamma}{\partial \eta} + \lambda)(\eta \lambda - \frac{\partial \gamma}{\partial \lambda})}{(\Omega + \eta \lambda)^2} \leq 0 \text{ if } \Omega \leq \frac{\partial \Omega}{\partial \eta} \eta,
\]

\[
\frac{\partial \gamma}{\partial \lambda} = \frac{\partial \eta \lambda - \frac{\partial \gamma}{\partial \Omega}(\Omega + \eta \lambda) - (\frac{\partial \gamma}{\partial \eta} + \eta)(\eta \lambda - \frac{\partial \gamma}{\partial \lambda})}{(\Omega + \eta \lambda)^2} < 0 \text{ if } \Omega < \frac{\partial \Omega}{\partial \lambda} \lambda.
\]

Additionally, by looking at \( \Omega \) it is clear that \( \frac{\partial \Omega}{\partial \eta} = \frac{\Omega}{\eta} \) and \( \frac{\partial \Omega}{\partial \lambda} \lambda > \Omega \) if \( \theta > 1 \) so that the two conditions always hold.

If \( \theta > 0 \) then \( \Omega > 0 \) and \( \frac{\partial \Omega}{\partial \eta} > 0 \) and \( \frac{\partial \Omega}{\partial \lambda} > 0 \) such that \( \frac{\partial \rho_t}{\partial \eta} < 0 \) and \( \frac{\partial \rho_t}{\partial \lambda} < 0 \) for any \( \varepsilon_t \). As \( \frac{\partial \rho_t}{\partial \varepsilon_t} < 0 \) and if \( \frac{\rho - \frac{\partial \rho_t}{\partial \varepsilon_t}}{\Omega + \eta \lambda} < \gamma \), such that \( \rho^s \) and \( \rho_t \) cross at some point \( \varepsilon_t = \tilde{\varepsilon}_t \) determined by \( \rho_t = \rho^s \), then it can be easily inferred that \( \tilde{\varepsilon}_t \) is decreasing in the news-utility parameters \( \frac{\partial \tilde{\varepsilon}_t}{\partial \lambda}, \frac{\partial \tilde{\varepsilon}_t}{\partial \eta} \leq 0 \).

**D.4 Proof of Proposition 9**

The slope of the consumption-wealth ratio is given by

\[
\frac{\partial \rho_t}{\partial \varepsilon_t} = -\rho_t^2 \left( \frac{Q + \Omega + \gamma \Omega Q - \gamma Q}{(1 + \eta F(\varepsilon_t) + \eta \lambda (1 - F(\varepsilon_t)))^2} \right).
\]

Accordingly, \( \frac{\partial \rho_t}{\partial \varepsilon_t} \neq 0 \) iff \( \lambda > 1 \) and \( Q + \Omega + \gamma \Omega Q \neq \gamma Q \), additionally, \( \frac{\partial \rho_t}{\partial \varepsilon_t} < 0 \) iff \( \lambda > 1 \) and \( Q + \Omega + \gamma \Omega Q > \gamma Q \) which is necessarily true for \( \theta > 1 \) or for \( \theta < 1 \) if \( \gamma < \tilde{\gamma} \) with \( \tilde{\gamma} = \frac{Q + \Omega}{Q(1 - \Omega)} \).

Furthermore, if \( \theta = 1 \) and \( \gamma = 1 \) then \( \frac{\partial \rho_t}{\partial \varepsilon_t} = 0 \). If \( \theta < 1 \) and \( \gamma > \tilde{\gamma} \) then \( \frac{\partial \rho_t}{\partial \varepsilon_t} > 0 \).

**D.5 Proof of Proposition 10**

The news-utility equity premium

\[
E_t[R_{t+1}] - R_{t+1}^f = \left( \frac{\rho_t}{1 - \rho_t} E_t \left[ \sum_{t=1}^{C_t+1} \frac{1}{(C_t+1) \rho_{t+1}} \right] \right) - \psi E_t[\beta(C_{t+1} - C_t) \rho_{t+1}] - \theta \psi E_t[1 - \theta \psi C_t],
\]

Clearly, \( E_t[R_{t+1}], R^f_{t+1}, \) and \( E_t[R_{t+1}] - R^f_{t+1} \) vary with \( \frac{\rho_t}{1 - \rho_t} \) whereas the other terms are constant in an i.i.d. world. As \( \frac{\partial \rho_t}{\partial \varepsilon_t} < 0 \) for \( \eta > 0 \), \( \lambda > 1 \), and \( \theta > 1 \) so are \( \frac{\partial E_t[R_{t+1}]}{\partial \varepsilon_t} < 0 \), \( \frac{\partial R_{t+1}}{\partial \varepsilon_t} < 0 \), and \( \frac{\partial E_t[R_{t+1}] - R_{t+1}^f}{\partial \varepsilon_t} < 0 \).

**D.6 Risk attitudes towards wealth bets**

Recall that \( \beta E_t[V_{t+1}(W_{t+1})] = E_t \left[ \sum_{t=1}^{C_t} \beta^t U_{t+1} \right] = u(C_t) \psi \) and \( \gamma \sum_{t=1}^{C_t} \beta^t n(F_{t+1}^{C_t-C_{t-1}}) = \gamma(\eta \int_{-\infty}^{C_t} u(C_t)Q - u(c)Q)\tau F_{C_t}^{C_t-1}(c) + \eta \lambda \int_{C_t}^{\infty} (u(C_t)Q - u(c)Q)\tau F_{C_t}^{C_t-1}(c)) \) such that the news-utility agent will accept.
the gamble iff

$$\gamma(0.5\eta(u((W_t + G)\rho_t)Q - u((W_t)\rho_t)Q) + \eta\lambda 0.5(u((W_t - L)\rho_t)Q - u((W_t)\rho_t)Q) + 0.5u((W_t + G)\rho_t)\psi + 0.5u((W_t - L)\rho_t)\psi > u(W_t)\rho_t)\psi$$

$$\Rightarrow \gamma(0.5\eta(u(W_t + G) - u(W_t)) + \eta\lambda 0.5(u(W_t - L) - u(W_t)))\frac{Q}{Q + \Omega + \gamma\Omega} > 0.5u(W_t + G) + 0.5u(W_t - L) > u(W_t).$$

whereas the standard agent will accept the gamble iff

$$0.5u((W_t + G)\rho^s)Q + 0.5u((W_t - L)\rho^s)Q > u(W_t)\rho^s)Q \Rightarrow 0.5u(W_t + G) + 0.5u(W_t - L) > u(W_t).$$

### E Variation in consumption growth

The model’s simple structure is unaffected, $$C_t = \rho_t W_t$$ and $$V_t(W_t) = u(W_t)\Psi_t$$, with $$\rho_t$$ given by

$$\rho_t = \frac{1}{1 + \psi_t + \gamma Q_t(\eta F_{t-1}(\epsilon_t) + \eta \lambda (1 - F_{t-1}(\epsilon_t)))}.$$

Fluctuations in beliefs about economic volatility make the exogenous parameters $$\psi_t = f^\psi(\mu_t, \sigma_t)$$ and $$Q_t = f^Q(\mu_t, \sigma_t)$$ variant, and the calculation of $$\psi_t$$ and $$Q_t$$ thus becomes somewhat more complicated, by the same argument as above

$$Q_t = E_t[\beta(e^{\mu_t + \sigma_t\epsilon_{t+1}})^{1-\theta}] + E_t[\beta(e^{\mu_t + \sigma_t\epsilon_{t+1}})^{1-\theta}Q_{t+1}] = e^{(1-\theta)\mu_t}E_t[\beta(e^{\sigma_t\epsilon_{t+1}})^{1-\theta}] + E_t[\beta(e^{\sigma_t\epsilon_{t+1}})^{1-\theta}Q_{t+1}].$$

And $$\psi_t$$ is

$$\psi_t = \beta E_t[((e^{\mu_t + \sigma_t\epsilon_{t+1}})^{1-\theta} + \eta \lambda - 1) \int_{-\infty}^{\epsilon_{t+1}} ((e^{\mu_t + \sigma_t\epsilon_{t+1}})^{1-\theta} - (e^{\mu_t + \sigma_t x})^{1-\theta})dF(x) +$$

$$\eta \lambda \int_{-\infty}^{\epsilon_{t+1}} ((e^{\mu_t + \sigma_t\epsilon_{t+1}})^{1-\theta} Q_{t+1} - (e^{\mu_t + \sigma_t x})^{1-\theta} f^Q(\mu_t, x, \sigma_t(x))dF(x) + \beta (e^{\mu_t + \sigma_t\epsilon_{t+1}})^{1-\theta} \psi_{t+1}]].$$

Note that in the standard model $$\rho_t = \frac{1}{1 + \psi_t}$$ with

$$\psi_t = f^\psi(\mu_t, \sigma_t) = \beta E_t[((e^{\mu_t + \sigma_t\epsilon_{t+1}})^{1-\theta}] + \beta E_t[((e^{\mu_t + \sigma_t\epsilon_{t+1}})^{1-\theta} \psi_{t+1}]].$$

Unfortunately, the heteroskedasticity model can no longer be solved analytically. But, thanks to the geometric-sum nature of $$Q_t$$ and $$\psi_t$$ they can be computed numerically using a simple interpolation procedure that iterates until convergence. The numerical solution procedure appears to be very robust and pricing errors in $$1 = E_t[M_{t+1}R_{t+1}]$$ are very small.

### F Disaster risk

The model’s simple structure is unaffected by disaster risk in the consumption process, $$C_t =$$
\( \rho_t W_t \) and \( V_t(W_t) = u(W_t)\Psi_t \), but \( \rho_t \) now depends on the probability of disaster \( p_t \) and if disaster happened \( v_t \) and is given by

\[
\rho_t = \frac{1}{1 + \frac{\psi_t + \gamma Q_t(\eta F_{t-1}(\varepsilon_t, v_t) + \eta \lambda (1 - F_{t-1}(\varepsilon_t, v_t)))}{1 + \eta F_{t-1}(\varepsilon_t, v_t) + \eta \lambda (1 - F_{t-1}(\varepsilon_t, v_t))}}.
\]

Note that \( F_{t-1}(\varepsilon_t, 0) = p_{t-1} F(\varepsilon_t - \log(1 - d)) + (1 - p_{t-1}) F(\varepsilon_t) \) if a disaster does not occur with probability \( 1 - p_{t-1} \) and \( F_{t-1}(\varepsilon_t, \log(1 - d)) = p_{t-1} F(\varepsilon_t) + (1 - p_{t-1}) F(\varepsilon_t + \log(1 - d)) \) if a disaster occurs with probability \( p_{t-1} \).

If disaster risk is invariant, \( Q \) and \( \psi \) are constant, from the same arguments as above

\[
Q = E_t \left[ \sum_{\tau=1}^{\infty} \beta^\tau (e^{\tau \mu_c + \sum_{j=1}^{\tau} e_{t+j} + \sum_{j=1}^{\tau} v_{t+j})^{1-\theta}} \right]
\]

with \( E_t[e^{(1-\theta)\sum_{j=1}^{\tau} v_{t+j}}] = E_t[e^{((1-\theta)v_{t+1}}] = (1 - p + p(1 - d)^{1-\theta})^\tau \)

such that \( Q = \frac{\beta e^{\mu_c(1-\theta) + \frac{1}{2}(1-\theta)^2 \sigma_c^2}}{1 - \beta e^{\mu_c(1-\theta) + \frac{1}{2}(1-\theta)^2 \sigma_c^2}} \)

\[
\psi = Q + \Omega + \gamma Q \Omega \text{ and } \Omega = \frac{\beta e^{\mu_c(1-\theta)((1 - p) \omega(p) + p \omega_p(p))}}{1 - \beta e^{\mu_c(1-\theta) + \frac{1}{2}(1-\theta)^2 \sigma_c^2}} \text{ with }
\]

\[
\omega(p) = \int_{-\infty}^{\infty} \eta(\lambda - 1) \int_{z} (1 - p)((e^z)^{1-\theta} - (e^z)^{1-\theta}) + p((e^z)^{1-\theta} - (e^z(1 - d))^{1-\theta}) dF(z)
\]

\[
\omega_p(p) = \int_{-\infty}^{\infty} + \eta(\lambda - 1) \int_{e^{(1-d)}} (1 - p)((e^z(1 - d))^{1-\theta} - (e^z)^{1-\theta}) + p((e^z(1 - d))^{1-\theta} - (e^z(1 - d))^{1-\theta}) dF(z)
\]

For time-variation in disaster risk, \( v_{t+1} \sim (p_t, d) \), \( Q_t = f^{Q_t}(p_t) \) and \( \psi_t = f^{\psi_t}(p_t) \) become variant with \( p_t \)
\[ Q_t = \beta E_t[(e^{\mu_t + \sigma_t \xi_{t+1} + \nu_{t+1}})^{1-\theta}] + \beta E_t[(e^{\mu_t + \sigma_t \xi_{t+1} + \nu_{t+1}})^{1-\theta} Q_{t+1}] = E_t[\sum_{\tau=1}^\infty \beta^\tau (e^{\mu_t + \sum_{j=1}^\tau \xi_{t+j} + \sum_{j=1}^\tau \nu_{t+j}})^{1-\theta}]
\]
\[ = \beta (1 - p_t + p_t (1 - d)^{1-\theta})(E_t[(e^{\mu_t + \sigma_t \xi_{t+1}})^{1-\theta}] + E_t[(e^{\mu_t + \sigma_t \xi_{t+1}})^{1-\theta} Q_{t+1}])
\]
\[ \psi_t = \beta e^{(1-\theta)\mu_t} E_t[(1 - p_t)(e^{\sigma_t \xi_{t+1}})^{1-\theta} + p_t(e^{\sigma_t \xi_{t+1}})^{1-\theta} Q_{t+1}]
\]
\[ + (1 - p_t) \eta (\lambda - 1) \int_{\xi_{t+1}}^\infty ((1 - p_t)((e^{\sigma_t \xi_{t+1}})^{1-\theta} - (e^{\sigma_t x})^{1-\theta} + p_t((e^{\sigma_t \xi_{t+1}})^{1-\theta} - (e^{\sigma_t x} (1-d)^{1-\theta})) dF(x)
\]
\[ + p_t \eta (\lambda - 1) \int_{\log(e^{\xi_{t+1}(1-d)})}^\infty (1 - p_t)((e^{\sigma_t \xi_{t+1}}(1-d))^{1-\theta} - (e^{\sigma_t x})^{1-\theta} + p_t((e^{\sigma_t \xi_{t+1}}(1-d))^{1-\theta} - (e^{\sigma_t x} (1-d)^{1-\theta})) dF(x)
\]
\[ + \gamma(1 - p_t) \eta (\lambda - 1) \int_{\xi_{t+1}}^\infty (1 - p_t)((e^{\sigma_t \xi_{t+1}})^{1-\theta} Q_{t+1} - (e^{\sigma_t x})^{1-\theta} f^Q(\bar{\theta}(x, p_t))) + p_t((e^{\sigma_t \xi_{t+1}})^{1-\theta} Q_{t+1}
\]
\[ - (e^{\sigma_t x} (1-d)^{1-\theta} f^Q(\bar{\theta}(x, p_t))) dF(x) + \gamma p_t \eta (\lambda - 1) \int_{e^{\sigma_t \xi_{t+1}(1-d)}}^\infty ((1 - p_t)((e^{\sigma_t \xi_{t+1}}(1-d))^{1-\theta} Q_{t+1} - (e^{\sigma_t x} (1-d)^{1-\theta}) f^Q(\bar{\theta}(x, p_t))) dF(x)
\]
\[ + (1 - p_t + p_t (1-d)^{1-\theta}) e^{1-\theta} \sigma_t \xi_{t+1} \psi_{t+1}]
\]
Part VI

A News-Utility Theory for Inattention and Delegation in Portfolio Choice

G  Attitudes over Wealth Bets

<table>
<thead>
<tr>
<th>Loss (L)</th>
<th>standard</th>
<th>news-util</th>
<th>contemp.</th>
<th>prospective</th>
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<td>317490</td>
<td>255050</td>
<td></td>
</tr>
</tbody>
</table>

For each loss L, the table’s entries show the required gain G to make each agent indifferent between accepting and rejecting a 50-50 gamble win G or lose L at a wealth level of 300,000.

H  Derivations of the portfolio share approximation in the presence of labor income

I first derive the news-utility agent’s optimal portfolio share in greater detail. To obtain a closed-form portfolio solution, I assume log utility $u(c) = \log(c)$. The news-utility agent wants to maximize

$$E[\log(W) + (\log(R^f + \alpha(R - R^f)) + \eta(\lambda - 1)\int_R^\infty (\log(R^f + \alpha(R - R^f)) - \log(R^f + \alpha(\tilde{R} - R^f)) dF_R(\tilde{R}))]

\Rightarrow E[(r^f + \alpha(r - r^f) + \alpha(1 - \alpha)\frac{\sigma^2}{2}) + \eta(\lambda - 1)\int_r^\infty (r^f + \alpha(r - r^f) + \alpha(1 - \alpha)\frac{\sigma^2}{2} - (r^f + \alpha(\tilde{r} - r^f) + \alpha(1 - \alpha)\frac{\sigma^2}{2}))dF_r,]

E[(r^f + \alpha(r - r^f) + \alpha(1 - \alpha)\frac{\sigma^2}{2}) + \alpha\eta(\lambda - 1)\int_r^\infty (r - \tilde{r})dF_r(\tilde{r}))]

(r^f + \alpha(\mu - \frac{\sigma^2}{2} - r^f) + \alpha(1 - \alpha)\frac{\sigma^2}{2}) + \alpha\mu[\eta(\lambda - 1)\int_r^\infty (r - \tilde{r})dF_r(\tilde{r})] = 0

\mu - r^f - \alpha\sigma^2 + E[\eta(\lambda - 1)\int_r^\infty (r - \tilde{r})dF_r(\tilde{r})] = 0
\[ \alpha^* = \frac{\mu - r_f + E[\eta(\lambda - 1) \int_{\tilde{r}}^{\infty} (r - \tilde{r})dF_r(\tilde{r})]}{\sigma^2}. \]

I now consider the case in which labor income is riskless, i.e., \( Y > 0 \) and \( F_Y \) degenerate. In this case, I can simply transform the problem to the one above by noting that \( Y \) dollars of labor income is equivalent to \( \frac{1}{r_f}Y \) dollars invested in the risk-free asset. Thus, the agent wants to invest the fraction \( \alpha^* \) out of his transformed wealth \( W + \frac{1}{r_f}Y \) in the risky asset; accordingly, his actual share invested into the risky asset is

\[ \alpha^*_{y > 0} = \frac{\alpha^*(W + \frac{1}{r_f}Y)}{W} = \alpha^*(1 + \frac{Y}{r_fW}). \]

Now, I move on to stochastic labor income. Stochastic income makes the model considerably more complicated, as its solution requires numerical techniques. In order to provide analytical insights into the model’s mechanisms, I will continue to follow Campbell and Viceira (2002) and employ an approximation strategy for the log portfolio return, the consumption function, and the agent’s first-order condition. Logs are denoted by lower case letters. More specifically, the log portfolio return \( r^p \) is approximated by \( r^f + \alpha(r - r^f) + \alpha(1 - \alpha)\frac{\sigma^2}{2} \) and the log of the consumption function \( C = g^c(r, y) = W(e^{r^f} + \alpha(e^r - e^{r^f})) + e^y \) can be approximated by

\[ \log(C) - \log(Y) = c - y = \log(e^{r^p+w-y} + 1) \Rightarrow c \approx k + \rho(w + r^p) + (1 - \rho)y \]

with \( \rho \) being the log-linearization parameter, i.e., \( \rho = \frac{e^{r^p+w-y}}{1+e^{r^p+w-y}} \). The agent’s first-order condition is given by

\[ E[e^{-c}(e^r - e^{r^f})] + \eta(\lambda - 1) \int_y^{\infty} \int_r^{\infty} (e^{-c}(e^r - e^{r^f}) - e^{-\log(e^{\tilde{r},\tilde{y}})}(e^\tilde{r} - e^{\tilde{r}^f}))dF_r(\tilde{r})dF_y(\tilde{y}) = 0. \]

The first part of this equation can be approximated by

\[ E[e^{-c}(e^r - e^{r^f})] = 0 \Rightarrow E[e^{-(\rho \alpha r + (1 - \rho)y)r}] - E[e^{-\rho \alpha(1 - \rho)y + r^f}] = 0 \]

\[ \Rightarrow E[e^{(1 - \rho \alpha)(1 - \rho)y}] - E[e^{-\rho \alpha(1 - \rho)y + r^f}] = 0 \]

\[ e^{(1 - \rho \alpha)(\mu - \frac{1}{2}\sigma^2) - (1 - \rho)\mu_y + 0.5(1 - \rho)\alpha^2 \sigma^2 + 0.5(1 - \rho)^2 \sigma_y^2 - (1 - \rho)\alpha(1 - \rho)\sigma^2} \]

\[ -e^{-\rho \alpha(\mu - \frac{1}{2}\sigma^2) - (1 - \rho)\mu_y + 0.5(1 - \rho)\alpha^2 \sigma^2 + 0.5(1 - \rho)^2 \sigma_y^2 + \rho \alpha(1 - \rho)\sigma^2 + r^f} = 0 \]

\[ e^{\mu - \rho \alpha \sigma^2 - (1 - \rho)\sigma_y - r^f} = 1 \Rightarrow \mu - r^f - \rho \alpha \sigma^2 - (1 - \rho)\sigma_y = 0 \]

therefore, the news-utility first-order condition can be approximated by

\[ e^{\mu - \rho \alpha \sigma^2 - (1 - \rho)\sigma_y - r^f} - 1 + e^{\rho \alpha(\mu - \frac{1}{2}\sigma^2) + (1 - \rho)\mu_y - 0.5(1 - \rho)\alpha^2 \sigma^2 + 0.5(1 - \rho)^2 \sigma_y^2 - \rho \alpha(1 - \rho)\sigma^2 - r^f} \]

\[ \eta(\lambda - 1)E[\int_r^{\infty} \int_y^{\infty} (e^{-\rho \alpha(1 - \rho)y}(e^r - 1) - e^{-\rho \alpha(1 - \rho)y}(e^r - 1))dF_r(\tilde{r})dF_y(\tilde{y})] = 0. \]
which can be rewritten as

\[
e^{\mu - \rho \alpha \sigma^2 - (1 - \rho) \sigma_y - r_f} - 1 + e^{-0.5\rho^2 \sigma^2 - 0.5(1 - \rho)^2 \sigma^2_{\gamma} - \rho \alpha (1 - \rho) \sigma^2_{\gamma} - r_f}
\]

\[
\eta(\lambda - 1) E \left[ \int_{s}^{\infty} \int_{\tilde{s}}^{\infty} \left( e^{\rho \alpha \sigma s - (1 - \rho) \sigma y} (e^{\sigma s} - 1) - e^{\rho \alpha \sigma \tilde{s} - (1 - \rho) \sigma y} (e^{\sigma \tilde{s}} - 1) \right) dF_s(\tilde{s}) dF_s(s) \right] = 0
\]

\[
e^{\mu - \rho \alpha \sigma^2 - (1 - \rho) \sigma_y - r_f} - 1 + e^{-0.5\rho^2 \sigma^2 - 0.5(1 - \rho)^2 \sigma^2_{\gamma} - \rho \alpha (1 - \rho) \sigma^2_{\gamma} - r_f}
\]

\[
\eta(\lambda - 1) E \left[ \int_{s}^{\infty} \int_{\tilde{s}}^{\infty} \left( e^{(1 - \rho \alpha \sigma) s - (1 - \rho) \sigma y} e^{\sigma s} - e^{(1 - \rho \alpha \sigma) \tilde{s} - (1 - \rho) \sigma y} e^{\sigma \tilde{s}} + e^{-\rho \alpha \sigma \tilde{s} - (1 - \rho) \sigma y} dF_s(\tilde{s}) dF_s(s) \right) \right] = 0
\]

and if I approximate \( e^x \approx 1 + x \) for \( x \) small (to approximate \( e^{(1 - \rho \alpha \sigma) s - (1 - \rho) \sigma y} \) more accurately than \( e^{-\rho \alpha \sigma s - (1 - \rho) \sigma y} (e^{\sigma s} - 1) \) if \( 1 - \rho \alpha \sigma < 1 \) (note that \( \rho < 1 \) and \( \alpha < 1 \)), to approximate \( e^{(1 - \rho \alpha \sigma) s - (1 - \rho) \sigma y} \) more accurately than \( e^{(1 - \rho \alpha \sigma) s} \) and \( e^{-\rho \alpha \sigma \tilde{s} - (1 - \rho) \sigma y} \) if \( r \) and \( y \) are positively correlated (although the result is not affected by this choice), and the term \( e^{-0.5\rho^2 \sigma^2 - 0.5(1 - \rho)^2 \sigma^2_{\gamma} - \rho \alpha (1 - \rho) \sigma^2_{\gamma} - r_f} \) is taken into the integral before the approximation) I end up with

\[
\mu - \rho \alpha \sigma^2 - (1 - \rho) \sigma_y - r_f + \eta(\lambda - 1) E \left[ \int_{s}^{\infty} (s - \tilde{s}) dF_s(s) \right] = 0.
\]

This first-order condition results in the portfolio share in the text.

I now illustrate the agent’s first-order risk aversion in the presence of background risk without relying on the approximation above. To simplify the exposition, suppose the agent’s consumption is \( \alpha \tilde{r} + y \) with \( \tilde{r} \sim F_\tilde{r} \) and \( y \sim F_y \) independently distributed, his risk premium for investing into the risky return \( r \) is then given by

\[
\pi = \log(\alpha E[r] + y) + E[\eta(\lambda - 1) \int_{y}^{\infty} (\log(\alpha E[r] + y) - \log(\alpha E[r] + \tilde{y})) dF_y(\tilde{y})]
\]

\[
- E[\log(\alpha \tilde{r} + y) + \eta(\lambda - 1) \int_{r}^{\infty} (\log(\alpha \tilde{r} + y) - \log(\alpha \tilde{r} + \tilde{y})) dF_y(\tilde{y}) dF_r(\tilde{r})]
\]

and its marginal value for an additional increment of risk is

\[
\frac{\partial \pi}{\partial \alpha} = E[\frac{E[r]}{\alpha E[r] + y} + \eta(\lambda - 1) \int_{y}^{\infty} (\frac{E[r]}{\alpha E[r] + y} - \frac{E[r]}{\alpha E[r] + \tilde{y}}) dF_y(\tilde{y})]
\]

\[
- E[\frac{r}{\alpha \tilde{r} + y} + \eta(\lambda - 1) \int_{r}^{\infty} (\frac{r}{\alpha \tilde{r} + y} - \frac{\tilde{r}}{\alpha \tilde{r} + \tilde{y}}) dF_y(\tilde{y}) dF_r(\tilde{r})].
\]

Now, what happens if return risk becomes small, i.e., \( \alpha \to 0 \)

\[
\left. \frac{\partial \pi}{\partial \alpha} \right|_{\alpha=0} = \eta(\lambda - 1) E \left[ \int_{y}^{\infty} (\frac{E[r]}{y} - \frac{E[r]}{\tilde{y}}) dF_y(\tilde{y}) \right] - E[\eta(\lambda - 1) \int_{y}^{\infty} (\frac{r}{y} - \frac{\tilde{r}}{\tilde{y}}) dF_y(\tilde{y}) dF_r(\tilde{r})].
\]
For the standard agent $\frac{\partial z}{\partial \alpha}|_{\alpha=0} = 0$, which implies that he is second-order risk averse. For the news-utility agent, the first integral is necessarily positive and dominates the second integral that can be negative or positive as in the second integral the positive effect of $r$ enters on top of the negative effect of $y$.

I Derivation of the Inattentive Life-Cycle Portfolio Choice Model

I.1 The monotone-personal equilibrium

The agent adjusts his portfolio share and consumes a fraction $\rho_t$ out of his wealth if he looks up his portfolio and a fraction $\rho^{in}_t$ out of his wealth if he stays inattentive. I follow a guess and verify solution procedure. Suppose he last looked up his portfolio in $t-i$, i.e., he knows $W_{t-i}$. And suppose he will look up his portfolio in period $t+j_1$. Suppose he is inattentive in period $t$, then his inattentive consumption in periods $t-i+1$ to $t+j_1-1$ is given by (for $k = 0, \ldots, i+j_1-1$)

$$C^{in}_{t-i+k} = (W_{t-i} - C_{t-i})(R^d)^k \rho^{in}_{t-i+k}$$

and his consumption when he looks up his portfolio in period $t+j_1$ is given by

$$C_{t+j_1} = W_{t+j_1}\rho_{t+j_1} = (W_{t-i} - C_{t-i} - \sum_{k=1}^{i+j_1-1} \frac{C^{in}_{t-i+k}}{\binom{R^d}{k}})((R^f)^{i+j_1} + \alpha_{t-i}(\prod_{j=1}^{i+j_1} R_{t-i+j} - (R^f)^{i+j_1}))\rho_{t+j_1}$$

$$= W_{t-i}(1 - \rho_{t-i})(1 - \sum_{j=1}^{i+j_1-1} \rho^{in}_{t-i+j})(\prod_{j=1}^{i+j_1} R_{t-i+j} - (R^f)^{i+j_1})\rho_{t+j_1}.$$ 

Now, suppose the agent looks up his portfolio in period $t$ and then chooses $C_t$ and $\alpha_t$ knowing that he will look up his portfolio in period $t+j_1$ next time. I first explain the optimal choice of $C_t$. First, the agent considers marginal consumption and contemporaneous marginal news utility given by

$$u'(C_t)(1 + \eta F_t(r_t) \ldots F_{t-i+1} + \eta \lambda (1 - F_t(r_t) \ldots F_{t-i+1})).$$

To understand these terms note that the agent takes his beliefs as given, his admissible consumption function $C_t$ is increasing in $r_t + \ldots + r_{t-i+1}$ such that $F_t^{t-i}(C_t) = F_t(r_t) \ldots F_{t-i+1}$, and

$$\frac{\partial (\eta \int_{-\infty}^{C_t} (u(C_t) - u(x))dF_t^{t-i}(x) + \eta \lambda \int_{-\infty}^{\infty} (u(C_t) - u(x))dF_t^{t-i}(x))}{\partial C_t} = u'(C_t)(\eta F_t^{t-i}(C_t) + \eta \lambda (1 - F_t^{t-i}(C_t))).$$

Second, the agent takes into account that he will experience prospective news utility over all consumption in periods $t+1, \ldots, T$. Inattentive consumption in periods $t+1$ to $t+j_1-1$ is as above given by (for $k = 1, \ldots, j_1-1$)

$$C^{in}_{t+k} = (W_t - C_t)(R^d)^k \rho^{in}_{t+k}.$$
and thus proportional to \( W_t - C_t \). Attentive consumption in period \( t + j_1 \) is given by

\[
C_{t+j_1} = W_{t+j_1} \rho_{t+j_1} = (W_t - C_t - \sum_{k=1}^{j_1-1} \frac{C_{t+1}^{in} \cdot (R^f)^t}{(R^f)^k})((R^f)^{j_1} + \alpha_i(\prod_{j=1}^{j_1} R_{t+j} - (R^f)^{j_1})) \rho_{t+j_1}
\]

\[
= (W_t - C_t)(1 - \sum_{j=1}^{j_1-1} \rho_{t+j}^i) \cdot ((R^f)^{j_1} + \alpha_i(\prod_{j=1}^{j_1} R_{t+j} - (R^f)^{j_1})) \rho_{t+j_1}
\]

and thus proportional to \( W_t - C_t \). As in the derivation in Section 19.2, prospective marginal news utility is

\[
\frac{\partial (\gamma \sum_{k=1}^{j_1-1} \beta^k n(F_{t+j-i}^{t-i}(W_t - C_t) \rho_{t+j}^i)) + \gamma \sum_{j=j_1}^{T-t-j_1} \beta^{j-i} n(F_{C_{t+j}}^{t-i})}{\partial C_t}
\]

\[
= \frac{\partial \log(W_t - C_t)}{\partial C_t} \gamma \sum_{j=1}^{T-t-j_1} \beta^j (\eta F_t(r_t) \ldots F_t(r_{t-i+1}) + \eta \lambda (1 - F_t(r_t) \ldots F_t(r_{t-i+1}))).
\]

To understand this derivation note that the agent takes his beliefs as given, future consumption is increasing in today’s return realization, and the only terms that realize and thus do not cancel out of the news-utility terms are \( W_t - C_t \) such that \( F_{W_t-C_t}^{t-i}(W_t - C_t) = F_t(r_t) \ldots F_t(r_{t-i+1}) \). As an example consider the derivative of prospective news-utility in period \( t + 1 \)

\[
\frac{\partial \gamma \sum_{k=1}^{j_1-1} \beta^k n(F_{t+j-i}^{t-i}(W_t - C_t) \rho_{t+j}^i)) + \gamma \sum_{j=j_1}^{T-t-j_1} \beta^{j-i} n(F_{C_{t+j}}^{t-i})}{\partial C_t}
\]

\[
= -u'(W_t - C_t) \gamma \beta (\eta F_{W_t-C_t}^{t-i}(W_t - C_t) + \eta \lambda (1 - F_{W_t-C_t}^{t-i}(W_t - C_t))).
\]

Third, thanks to log utility, the agent’s continuation utility is not affected by expected news utility as \( \log(W_t - C_t) \) cancels out of these terms. However, expected consumption utility matters. Consumption utility beyond period \( t + j_1 \) can be iterated back to \( t + j_1 \) wealth which can be iterated back to \( W_t \) as \( \log(W_t - C_t - \sum_{k=1}^{j_1-1} \rho_{t+j}^k) = \log((W_t - C_t)(1 - \sum_{k=1}^{j_1-1} \rho_{t+j+k}^i)) \) such that

\[
\frac{\partial \sum_{j=1}^{T-t-j_1} \beta^j E_t[\log(C_{t+j})]}{\partial C_t} = -u'(W_t - C_t) \sum_{\tau=1}^{T-t-j_1} \beta^\tau
\]

Putting the three pieces together, optimal consumption if the agent looks up his portfolio in period \( t \) is determined by the following first-order condition

\[
u'(C_t) (1 + \eta F_t(r_t) \ldots F_t(r_{t-i+1}) + \eta \lambda (1 - F_t(r_t) \ldots F_t(r_{t-i+1})))
\]

\[\ldots \gamma u'(W_t - C_t) \sum_{j=1}^{T-t} \beta^j (\eta F_t(r_t) \ldots F_t(r_{t-i+1}) + \eta \lambda (1 - F_t(r_t) \ldots F_t(r_{t-i+1}))) - u'(W_t - C_t) \sum_{\tau=1}^{T-t} \beta^\tau = 0.\]
In turn, the solution guess can be verified
\[
\frac{C_t}{W_t} = \rho_t = \frac{1}{1 + \sum_{t=1}^{T-t} \beta_t^{1+\gamma} \eta F_t(r_t...F_t(r_{t-i+1}) + \eta \lambda (1-F_t(r_t...F_t(r_{t-i+1}))}
\]

The optimal portfolio share depends on prospective news utility in period \( t \) for all the consumption levels in periods \( t + j_1 \) to \( T \) that are all proportional to \( W_t + j_1 \) (that is determined by \( \alpha_t \)). Moreover, the agent’s consumption and news utility in period \( t + j_1 \) matters. However, consumption in inattentive periods \( t \) to \( t + j_1 - 1 \) depends only on \( W_t - C_t \), which is not affected by \( \alpha_t \). Thus, the relevant terms in the maximization problem for the portfolio share are given by
\[
\gamma \beta_{j_1} \sum_{\tau=0}^{T-t-j_1} \beta_t^{\tau} n(F_{C_{t+j_1+\tau}}^{t-j-i}) + \beta_{j_1} E_t[n(C_{t+j_1}, F_{C_{t+j_1}}^{t+j_1})] + \gamma \sum_{\tau=0}^{T-t-j_1} \beta_t^{\tau} n(F_{C_{t+j_1+\tau}}^{t+j_1})] + \beta_{j_1} \sum_{\tau=0}^{T-t-j_1} \beta_t^{\tau} \log(C_{t+j_1+\tau})
\]

The derivative of the first term is
\[
\frac{\partial \gamma \beta_{j_1} \sum_{\tau=0}^{T-t-j_1} \beta_t^{\tau} n(F_{C_{t+j_1+\tau}}^{t-j-i})}{\partial \alpha_t} = j_1(\mu - \rho_t - \alpha_t \sigma^2) \gamma \beta_{j_1} T-t-j_1 \beta_t^{\tau} (\eta F_t(r_t...F_t(r_{t-i+1}) + \eta \lambda (1-F_t(r_t...F_t(r_{t-i+1}))].
\]

To illustrate where the derivative comes from
\[
\log(C_{t+j_1}) = \log(W_{t+j_1} + \rho_{t+j_1}) = \log((W_t - C_t) (1 - \sum_{j=1}^{j_1-1} \rho_{t+j}^{in} ((R^f)_j + \alpha_t (\prod_{j=1}^{j_1} R_{t+j} - (R^f)_j))) \rho_{t+j_1})
\]

thus the only term determined by \( \alpha_t \) is \( \log((R^f)_j + \alpha_t (\prod_{j=1}^{j_1} R_{t+j} - (R^f)_j)) \) and the only terms that are different from the agent’s prior beliefs are \((W_t - C_t)\) (and the agent takes his beliefs as given and future consumption is increasing in today’s return as in the derivation of the consumption share above). Moreover,
\[
C_{t+j_1+1}^{in} = (W_t - C_t) (1 - \sum_{j=1}^{j_1-1} \rho_{t+j}^{in} ((R^f)_j + \alpha_t (\prod_{j=1}^{j_1} R_{t+j} - (R^f)_j))) (1 - \rho_{t+j_1}) R^d \rho_{t+j_1}^{in}
\]

thus the only term determined by \( \alpha_t \) are \( \log((R^f)_j + \alpha_t (\prod_{j=1}^{j_1} R_{t+j} - (R^f)_j)) \) and the only terms that are different from the agent’s prior beliefs are \((W_t - C_t)\). Thus, in the derivation the term \( j_1(\mu - \rho_t - \alpha_t \sigma^2) \) is left as the agent does not consider his beliefs in the optimization and the integrals are determined by \( F_{W_t-C_t}^{t-1}(W_t - C_t)\).

In the continuation value, the only term affected by today’s portfolio share are the future consumption terms in periods \( t + j_1, ..., T \) as in all news-utility terms (except period \( t + j_1 \)) the portfolio share of period \( t \) will cancel and the portfolio share does not matter for all inattentive consumption up until period \( t + j_1 \). Thus, the derivative of the continuation value is
\[
\frac{\beta_{j_1} E_t[n(C_{t+j_1}, F_{C_{t+j_1}}^{t+j_1})] + \gamma \sum_{\tau=1}^{T-t-j_1} \beta_t^{\tau} n(F_{C_{t+j_1+\tau}}^{t+j_1})] + \beta_{j_1} \sum_{\tau=0}^{T-t-j_1} \beta_t^{\tau} \log(C_{t+j_1+\tau})}{\partial \alpha_t}
\]
\[= \beta^j_i (1 + \gamma \sum_{t=1}^{T-t-j_1} \beta^t) \sqrt{j_1} \sigma E[\eta(\lambda - 1) \int_s^{\infty} (s-\bar{s})dF(\bar{s})] + \beta^j_i \sum_{t=0}^{T-t-j_1} \beta^t j_i (\mu - r^f - \alpha_t \sigma^2).\]

Altogether, the optimal portfolio share is given by

\[\alpha_t = \frac{\mu - r^f + \beta^j_i \sum_{t=0}^{T-t-j_1} \beta^t \sqrt{j_1} \sigma E[\eta(\lambda - 1) \int_s^{\infty} (s-\bar{s})dF(\bar{s})]}{1 + \gamma \sigma^2}.\]

The sufficient condition for an optimum is satisfied only if \(\alpha_t > 0\); if \(\alpha_t < 0\), the agent would choose \(\alpha_t = 0\) instead. If the agent does not look up his portfolio in period \(t\), his consumption is determined by the following first-order condition

\[u'(C^\text{in}_{t-i})(T-t-j_1) - \sum_{\tau=0}^{T-t-j_1} \beta^{j+\tau} u'(W_{t-i} - C_{t-i}) = \sum_{k=1}^{i+j_1-1} \frac{C^\text{in}_{i-i+k}}{(R^d)^k} - \frac{1}{(R^d)^i} = 0.\]

The term concerning consumption in period \(t\) is self-explanatory. The terms concerning consumption from periods \(t - i\) to \(t + j_1 - 1\) drop out as they are determined by the solution guess \(C^\text{in}_{t-i} = (W_{t-i} - C_{t-i})(R^d)^{\rho^\text{in}_{t}}\). The terms concerning consumption from period \(t + j_1\) on are all proportional to \(\log(W_{t+j_1})\) which equals \(\log(W_{t-i} - C_{t-i} - \sum_{k=1}^{i+j_1-1} \frac{C^\text{in}_{i-i+k}}{(R^d)^k})\) plus the log returns from period \(t - i + 1\) to period \(t + j_1\), which, however, drop out by taking the derivative with respect to \(C^\text{in}_{t-i}\). Accordingly,

\[\frac{1}{C^\text{in}_{t-i}} \sum_{\tau=0}^{T-t-j_1} \beta^{j+\tau} \frac{1}{W_{t-i} - C_{t-i} - \sum_{k=1}^{i+j_1-1} \frac{C^\text{in}_{i-i+k}}{(R^d)^k}} = 0 \Rightarrow \frac{1}{\rho^\text{in}_{t}} = \sum_{\tau=0}^{T-t-j_1} \beta^{j+\tau} \frac{1}{1 - \sum_{k=1}^{i+j_1-1} \rho^\text{in}_{t-i+k}}.\]

And the solution guess \((C^\text{in}_{t-i}) = (W_{t-i} - C_{t-i})(R^d)^{\rho^\text{in}_{t}}\) can be verified. The agent is not going to deviate from his inattentive consumption path in period \(t - i + j\) for \(j = 1, \ldots, i - 1\) so long as \(u'(C^\text{in}_{t-i-j})(1 + \eta) < (\beta R^d)^{i-j} u'(C^\text{in}_{t-i})(1 + \gamma \eta \lambda)\). As \(u'(C^\text{in}_{t-i-j}) \approx (\beta R^d)^{i-j} u'(C^\text{in}_{t-i-1})\), this condition is roughly equivalent to \(\gamma \lambda > 1\).

### I.2 The monotone-precommitted equilibrium

Suppose the agent has the ability to pick an optimal history-dependent consumption path for each possible future contingency in period zero when he does not experience any news utility. Thus, in period zero the agent chooses optimal consumption in period \(t\) in each possible contingency jointly with his beliefs, which of course coincide with the agent’s optimal state-contingent plan. For instance, consider the joint optimization over consumption and beliefs for \(C(Y^*)\) when

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income $Y^*$ has been realized

$$\frac{\partial}{\partial C(Y^*)} \left\{ \int \int \mu(u(C(Y)) - u(C(Y')))dF_Y(Y')dF_Y(Y) \right\}$$

$$= \frac{\partial}{\partial C(Y^*)} \int \eta \int_{-\infty}^{Y} \{(u(C(Y)) - u(C(Y')))dF_Y(Y') + \eta \lambda \int_{Y}^{\infty} (u(C(Y)) - u(C(Y')))dF_Y(Y') \}dF_Y(Y)$$

$$= u'(C(Y^*)) (\eta F_Y(Y^*) + \eta \lambda (1 - F_Y(Y^*)) - u'(C(Y^*)) (\eta (1 - F_Y(Y^*)) + \eta \lambda F_Y(Y^*))$$

$$= u'(C(Y^*)) \eta (\lambda - 1) (1 - 2F_Y(Y^*)) \text{ with } \eta (\lambda - 1) (1 - 2F_Y(Y^*)) > 0 \text{ for } F_Y(Y^*) < 0.5.$$ 

From the above consideration it can be easily inferred that the optimal precommitted portfolio function, if the agent would look up his portfolio his share will be

$$\alpha_c^c = \mu - r^c + \frac{\frac{\beta_1}{(1 + \gamma)^{T-1}} \beta_\tau}{1 + \gamma \eta} \frac{\sigma^2}{F_Y(s_1 \cdots s_{T-1})} \left[ \frac{\eta \lambda + 1}{ \eta (\lambda - 1) (2F_Y(r_i) \cdots F_Y(r_{T-1}) - 1) - 1} \right].$$

Moreover, the optimal precommitted attentive and inattentive consumption shares are

$$\rho_t^c = \frac{1}{1 + \sum_{T-t}^{T} \beta_\tau (1 + \gamma \eta (\lambda - 1) (2F_Y(r_i) \cdots F_Y(r_{T-1}) - 1) - 1)}$$

and

$$\rho_t^{cin} = \frac{1}{1 + \sum_{T-t}^{T} \beta_\tau (1 - \sum_{k=1}^{T-t} \rho_{i-k}^{cin}) - \sum_{k=1}^{T-t} \rho_{i-k}^{cin}.}$$

Thus, the optimal precommitted portfolio share is always lower and the gap increases in good states.

### I.3 Signals about the market

Instantaneous utility is either prospective news utility over the realization of $R$ or prospective news utility over the signal $\tilde{R}$. Prospective news utility over the signal is given by

$$\gamma \beta \int_{-\infty}^{\infty} \mu(e^{log(R)}y) - u(e^{x-y})dF_{r+e}(x)dF_{e}(y),$$

because the agent separates uncertainty that has been realized in period one, represented by $\tilde{R}$ and $F_{r+e}(x)$, from uncertainty that has not been realized, represented by $F_{e}(y)$. The agent’s expected news utility from looking up the return conditional on the signal $\tilde{R}$ is given by

$$\gamma \beta \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(e^{log(R)}y) - u(e^{x-y})dF_{r}(x)dF_{e}(y).$$

Thus, the agent expects more favorable news utility over the return when having received a favorable signal. As can be easily seen, expected news utility from checking the return is always less than news utility from knowing merely the signal. The reason is simple, the agent expects to experience news utility, which is negative on average, over both the signal $\tilde{R}$ and the error $e$. Thus, he always prefers to not look up his return when prospective news utility in period one is concerned.
But, the difference between the two is smaller when the agent received a more favorable signal. The reason is that the expected news disutility from $\varepsilon$ is less high up on the utility curve, i.e., when $\hat{R}$ is high.

### I.4 Derivation of the Standard Portfolio-Choice Model

The agent lives for $t = \{1, \ldots, T\}$ periods and is endowed with initial wealth $W_1$. Each period the agent optimally decides how much to consume $C_t$ out of his wealth $X_t$ and how to invest $A_t = X_t - C_t$. The agent has access to a risk-free investment with return $R^f$ and a risky investment with i.i.d. return $R_t$. The risky investment’s share is denoted by $\alpha_t$ such that the portfolio return in period $t$ is given by $R_t = R^f + \alpha_t (R_t - R^f)$. Additionally, the agent receives labor income in each period $t$ up until retirement in period $T - R$ given by $Y_t = P_t N_t^T = P_{t-1} G_t e^{i^p}$ with $X_t^p \sim N(0, \sigma^2_Y)$. Accordingly, the agent’s maximization problem in each period $t$ is given by

$$
\max_{C_t} \{u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^{T-t} \beta^\tau n(F_{C_{t+\tau}}^{t-1}) + E_t \left[ \sum_{\tau=1}^{T-t} \beta^\tau U_{t+\tau} \right] \}
$$

subject to the budget constraint

$$
X_t = (X_{t-1} - C_{t-1}) R_t^p + Y_t = A_{t-1} (R_f + \alpha_{t-1} (R_t - R_f)) + P_{t-1} G_t e^{i^p}.
$$

I solve the model by numerical backward induction. The maximization problem in any period $t$ is characterized by the following first-order condition

$$
u'(C_t) = \frac{\Psi_t' + \gamma \Phi_t' (\eta F_{C_t}^{t-1}(C_t) + \eta \lambda (1 - F_{C_t}^{t-1}(C_t)))}{1 + \eta F_{A_t}^{t-1}(A_t) + \eta \lambda (1 - F_{A_t}^{t-1}(A_t))}
$$

with

$$
\Phi_t' = \beta E_t [R^p_{t+1} \frac{\partial C_{t+1}}{\partial X_{t+1}} u'(C_{t+1}) + R^p_{t+1} (1 - \frac{\partial C_{t+1}}{\partial X_{t+1}}) \Phi_{t+1}']
$$

and

$$
\Psi_t' = \beta E_t [R^p_{t+1} \frac{\partial C_{t+1}}{\partial X_{t+1}} u'(C_{t+1}) + \eta (\lambda - 1) \int_{C_{t+1}}^{\infty} (R^p_{t+1} \frac{\partial C_{t+1}}{\partial X_{t+1}} u'(C_{t+1}) - c) dF_t
$$

$$
+ \eta (\lambda - 1) \int_{A_{t+1}}^{\infty} (R^p_{t+1} \frac{\partial A_{t+1}}{\partial X_{t+1}} \Phi_{t+1}' - x) dF_t + R^p_{t+1} (1 - \frac{\partial C_{t+1}}{\partial X_{t+1}}) \Phi_{t+1}] + \eta (\lambda - 1) \int_{A_{t+1}}^{\infty} (R^p_{t+1} \frac{\partial A_t}{\partial X_{t+1}} \Phi_t' - x) dF_t + R^p_{t+1} (1 - \frac{\partial C_{t+1}}{\partial X_{t+1}}) \Phi_{t+1}]
$$

Note that, I denote $\Psi_t = \beta E_t [\sum_{\tau=0}^{T-t} \beta^\tau U_{t+\tau}]$, $\Phi_t = \beta E_t [\sum_{\tau=0}^{T-t} \beta^\tau u(C_{t+1+\tau})]$, $\Psi_t' = \frac{\partial \Psi_t}{\partial A_{t+1}}$, and $\Phi_t' = \frac{\partial \Phi_t}{\partial A_{t+1}}$. In turn, the optimal portfolio share can be determined by the following first-order condition

$$
\gamma \frac{\partial \Phi_t}{\partial \alpha_t} (\eta F_{A_t}^{t-1}(A_t) + \eta \lambda (1 - F_{A_t}^{t-1}(A_t))) + \frac{\partial \Psi_t}{\partial \alpha_t} = 0
$$

or equivalently by maximizing $\gamma \sum_{\tau=1}^{T-t} \beta^\tau n(F_{C_{t+\tau}}^{t-1}) + \Psi_t$ (maximizing the transformed function...
\( u^{-1}(\cdot) \) yields more robust results). More details on the numerical backward induction solution of a news-utility life-cycle model is provided in Pagel (2012b).

**J Proofs**

**J.1 Proof of Proposition 11**

The news-utility agent’s optimal portfolio share is

\[
\alpha = \frac{\mu - r^f + E[\eta(\lambda - 1) \int_r^\infty (r - \bar{r}) dF_r(\bar{r})]}{\sigma^2}
\]

and the second-order condition is

\[-\alpha \sigma^2 < 0.
\]

It can be easily seen that \( \frac{\partial \alpha}{\partial \eta} < 0 \) and \( \frac{\partial \alpha}{\partial \lambda} < 0 \). As \( E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s}) dF_s(\bar{s})] < 0 \) even for \( \sigma = 0 \) (note that \( s \sim N(0,1) \)), the portfolio share is first-order decreasing in \( \sigma \) (the second-order approximation can be found in the text). Now, let me redefine \( \mu \triangleq h \mu, \sigma \triangleq \sqrt{h} \sigma, \) and \( r^f \triangleq hr^f \). The optimal portfolio share is given by

\[
\alpha = \frac{\mu - r^f + \sqrt{h} \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s}) dF_s(\bar{s})]}{\sigma^2}.
\]

1. Samuelson’s colleague and time diversification: As can be easily seen, as \( \lim \sqrt{h} \rightarrow \infty \) as \( h \rightarrow 0 \), there exists some \( h \) such that \( \mu - r^f > -\sqrt{h} \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s}) dF_s(\bar{s})] \) and thus \( \alpha = 0 \). As \( \alpha > 0 \) only if

\[
\frac{\mu - r^f}{\sigma} > -E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s}) dF_s(\bar{s})] > 0.
\]

In contrast, \( \alpha^* > 0 \) whenever \( \mu > r^f \). On the other hand, \( \alpha > 0 \) for some \( h \) as if \( h \rightarrow \infty \) then \( \lim \sqrt{h} \rightarrow 0 \) and \( \alpha \rightarrow \alpha^* \). Furthermore, it can be easily seen that \( \frac{\partial \alpha}{\partial h} > 0 \).

2. Inattentive: Normalized expected utility is \( \frac{EU}{h} = (r^f + \alpha(\mu - \frac{\sigma^2}{2} - r^f) + \alpha(1 - \alpha) \frac{\sigma^2}{2}) + \sqrt{h} \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s}) dF_s(\bar{s})] \) which is increasing in \( h \) whereas it is constant for the standard agent.

**J.2 Proof of Lemma 3**

This proof can be immediately inferred from the derivation of the approximate portfolio share found in Appendix H. The comparative statics hold strictly if \( \alpha^* \in (0,1) \).
J.3  Proof of Proposition 12

If the consumption function derived in Appendix I is admissible then the equilibrium exists and is unique as the equilibrium solution is obtained by maximizing the agent’s objective function, which is globally concave, and there is a finite period that uniquely determines the equilibrium. As the consumption share in attentive periods is constant consumption, $C_t^{in}$ is necessarily increasing in $W_t-i - C_t-i$ and thus $r_t-i + ... + r_{t-i-j_0+1}$ (as $\frac{\partial p_{t-i}}{\partial (r_{t-i+1} + ... + r_{t-i-j_0+1})} < 0$ which is shown below). For attentive periods, $\sigma_t^*$ is implicitly defined by the log monotone consumption function restriction

$$\frac{\partial \log(C_t)}{\partial (r_{t+1} + ... + r_{t-i})} > 0$$

as

$$\log(C_t) = \log(W_t) + \log(p_t)$$

$$= \log(W_t-i - C_t-i - \sum_{k=1}^{i-1} \frac{C_{t-i+k}^{in}}{(R^d)^k}) + \log(R_t^p) + \log(1 + \sum_{\tau=1}^{T-t} \beta^{\tau} \frac{1 + \gamma(\eta F_t(r_t) ... F_t(r_{t-i+1}) + \eta \lambda (1-F_t(r_t) ... F_t(r_{t-i+1})))}{1 + \eta F_t(r_t) ... F_t(r_{t-i+1}) + \eta \lambda (1-F_t(r_t) ... F_t(r_{t-i+1}))})$$

as

$$\frac{\partial \log(R_t^p)}{\partial (r_{t+1} + ... + r_{t-i+1})} = \frac{\partial \log((R_t^p) + \alpha_{t-1}(R_t ... R_{t-i+1} - (R_t^p)))}{\partial (R_t ... R_{t-i+1})} \frac{\partial (R_t ... R_{t-i+1})}{\partial (r_{t+1} + ... + r_{t-i+1})} \approx \frac{\partial R_t ... R_{t-i+1}}{\partial (r_{t+1} + ... + r_{t-i+1})}$$

the restrictions are equivalent to

$$\frac{\partial \rho_t}{\partial (F_t(r_t) ... F_t(r_{t-i+1}))} \frac{\partial (F_t(r_t) ... F_t(r_{t-i+1}))}{\partial (r_t + ... + r_{t-i+1})} > -\alpha_{t-1} \frac{\partial R_t ... R_{t-i+1}}{\partial (r_{t+1} + ... + r_{t-i+1})}$$

with

$$\frac{\partial \rho_t}{\partial (F_t(r_t) ... F_t(r_{t-i+1}))} = -\frac{(1+\gamma(\eta F_t(r_t) ... F_t(r_{t-i+1}) + \eta \lambda (1-F_t(r_t) ... F_t(r_{t-i+1}))) \beta^{\tau}}{1 + \sum_{\tau=1}^{T-t} \beta^{\tau} \frac{1 + \gamma(\eta F_t(r_t) ... F_t(r_{t-i+1}) + \eta \lambda (1-F_t(r_t) ... F_t(r_{t-i+1})))}{1 + \eta F_t(r_t) ... F_t(r_{t-i+1}) + \eta \lambda (1-F_t(r_t) ... F_t(r_{t-i+1}))})^2 < 0.$$ 

Increasing $\sigma$ unambiguously decreases $\frac{\partial \rho_t}{\partial (r_{t+1} + ... + r_{t-i+1})} > 0$. Thus, there exists a condition $\sigma \geq \sigma_t^*$ for all $t$ which ensures that an admissible consumption function exists. If $\sigma < \sigma_t^*$ for some $t$ the agent would optimally choose a flat section that spans the part his consumption function is decreasing. In that situation, the admissible consumption function requirement is weakly satisfied and the model’s equilibrium is not affected qualitatively or quantitatively.

J.4  Proof of Corollary 2

The proof of Corollary 2 in the dynamic model is very similar to the proof of Proposition 11 in the static model. Please refer to Appendix I for the derivation of the dynamic portfolio share; redefining $\mu \triangleq h\mu$, $\sigma \triangleq \sqrt{h}\sigma$, $r^f \triangleq hr^f$, and $\beta \triangleq \beta^h$, the optimal portfolio share (in any period $t$ as the agent has to look up his portfolio every period) if $0 \leq \alpha_t \leq 1$ can be rewritten as

$$\alpha_t = \frac{\mu - r^f + \sqrt{h}\sigma}{\frac{1 + r^f \frac{1-\beta^h}{\beta^h}}{1 + \gamma(\eta F_t(r_t) ... F_t(r_{t-i+1}) + \eta \lambda (1-F_t(r_t) ... F_t(r_{t-i+1})))}}$$

$$\frac{1 + \gamma(\eta F_t(r_t) ... F_t(r_{t-i+1}) + \eta \lambda (1-F_t(r_t) ... F_t(r_{t-i+1}))) \beta^{\tau}}{1 + \sum_{\tau=1}^{T-t} \beta^{\tau} \frac{1 + \gamma(\eta F_t(r_t) ... F_t(r_{t-i+1}) + \eta \lambda (1-F_t(r_t) ... F_t(r_{t-i+1})))}{1 + \eta F_t(r_t) ... F_t(r_{t-i+1}) + \eta \lambda (1-F_t(r_t) ... F_t(r_{t-i+1}))})^2 < 0.$$
• Samuelson’s colleague and time diversification: As can be easily seen, as \( \lim \sqrt{\frac{h}{r}} \to \infty \) as \( h \to 0 \) whereas \( \lim \frac{1 + \gamma \sum_{t=1}^{T} \beta^{th} \beta}{\sum_{t=0}^{T} \beta} \to \frac{1 + \gamma (T+1)}{T+1} \), there exists some \( h \) such that
\[
\mu - r^f > -\frac{\sqrt{h}}{h} \sigma \frac{1 + \gamma \sum_{t=1}^{T} \beta^{th}}{1 + \gamma \sum_{t=0}^{T} \beta} E_s[\eta(\lambda - 1)_{t} \beta^s(s-\hat{s})dF(\hat{s})] \quad \text{and thus } \alpha_t = 0 \text{ for any } t. \]

In contrast, \( \alpha^* > 0 \) whenever \( \mu > r^f \). On the other hand, \( \alpha_t > 0 \) for some \( h \) as if \( h \to \infty \) then \( \lim \sqrt{\frac{h}{r}} \to 0 \) and \( \alpha_t \to \alpha^* \). Furthermore, it can be seen that \( \frac{\partial \alpha_t}{\partial r} > 0 \) since
\[
\frac{\partial}{\partial h} \left( \frac{\sqrt{h} 1 + \gamma \sum_{t=1}^{T} \beta^{th}}{h} \right) = \frac{1}{2 \sqrt{h}} - \frac{\sqrt{h}}{h^2} + \frac{\gamma \sum_{t=0}^{T-1} \beta^{th}}{h^2} > 0
\]
if \( \beta \approx 1 \). The first term is necessarily negative as \( \frac{\sqrt{h} - \sqrt{h}}{h^2} < 0 \Rightarrow \frac{1}{2 \sqrt{h}} - \frac{\sqrt{h}}{h^2} < 0 \Rightarrow \frac{1}{2} h < h \) which is multiplied by \( \frac{\gamma \sum_{t=0}^{T-1} \beta^{th}}{h^2} > 0 \). The second term is positive though as
\[
\frac{\sqrt{h}}{h} \gamma \sum_{t=1}^{T} \log(\beta) k \beta^{th} \sum_{t=0}^{T-1} \beta^{th} - \sum_{t=0}^{T-1} \log(\beta) \tau \beta^{th} (1 + \gamma \sum_{t=0}^{T-1} \beta^{th}) < 0
\]
and the denominator is positive while the numerator equals \( (\gamma - 1) \sum_{t=1}^{T} \log(\beta) k \beta^{th} \), which is positive if \( \beta < 1 \). However, if \( \beta \approx 1 \) the first negative term will necessarily dominate the second positive term as its numerator is \( (\gamma - 1) \sum_{t=1}^{T-1} \log(\beta) \tau \beta^{th} \approx 0 \). Moreover, \( \frac{\gamma \sum_{t=0}^{T-1} \beta^{th}}{h^2} \) is decreasing in \( T - t \) if \( \gamma < 1 \) such that \( \alpha_t \) is decreasing in \( T - t \).

• Inattention: Expected utility for any period \( t \), i.e., \( E_{t-1}[U_t] \), is given by
\[
E_{t-1}[U_t] = E_{t-1}[\log(W_t \rho_t)] + \alpha_{t-1}(1 + \gamma \sum_{j=1}^{T} \beta^{j}) \sqrt{h} \sigma E_s[\eta(\lambda - 1) \int_{s}^{\infty} (s-\hat{s})dF(\hat{s})].
\]

If \( \beta \approx 1 \), the change in the \( h \) terms in \( \beta^{th} \) will be close to zero (as shown in the previous proof), in which case \( \frac{E_{t-1}[U_t]}{h} \) with \( E_{t-1}[\log(W_t \rho_t)] = 0 \) is given by
\[
\frac{E_{t-1}[U_t]}{h} = E_{t-1}[i \alpha_{t-1}(1 + \gamma \sum_{j=1}^{T} \beta^{j}) \sqrt{h} \sigma E_s[\eta(\lambda - 1) \int_{s}^{\infty} (s-\hat{s})dF(\hat{s})]]
\]
which is increasing in \( h \) while the standard agent’s normalized expected utility is constant in \( h \).

### J.5 Proof of Proposition 13

I start with proving that there exists some \( \bar{h} \) such that, if \( h > \bar{h} \), the news-utility agent will be attentive in every period. I pick \( t \) such that \( T - t \) is large and I can simplify the exposition by
replacing $\sum_{k=1}^{T-1} \beta^k$ with $\frac{\beta}{1-\beta}$. If the agent is attentive every period, his value function is given by

$$\beta E_{t-1}[V_t(W_t)] = E_{t-1}[\frac{\beta}{1-\beta} \log(W_t) + \psi_t^{-1}(\alpha_{t-1})]$$

$$\psi_t^{-1}(\alpha_{t-1}) = \beta E_{t-1}[\log(\rho_t) + \alpha_{t-1}(1 + \gamma \frac{\beta}{1-\beta})\sigma E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s})dF(\bar{s})]$$

$$+ \frac{\beta}{1-\beta} \log(1 - \rho_t) + \frac{\beta}{1-\beta} (r^f + \alpha_t(r_{t+1} - r^f) + \alpha_t(1 - \alpha_{t-1}) \frac{\sigma^2}{2}) + \psi_{t+1}'(\alpha_t).$$

Now, the agent compares the expected utility from being inattentive to the expected utility from being attentive

$$E_{t-1}[\log(C_t^i\bar{n}) + \frac{\beta}{1-\beta} \log(W_{t+1}) + \psi_t^{-1}(\alpha_{t-1})] = E_{t-1}[\log(W_{t-1} - C_{t-1}) + \log(R^d) + \log(\rho_t^i)]$$

$$+ \frac{\beta}{1-\beta} \log(W_{t-1} - C_{t-1} - \frac{C_t^i \bar{n}}{R^d}) + \frac{\beta}{1-\beta} (2r^f + \alpha_{t-1}(r_t + r_{t+1} - 2r^f) + \alpha_{t-1}(1 - \alpha_{t-1}) \sigma^2) + \psi_{t+1}'(\alpha_{t-1})]$$

$$\Rightarrow E_{t-1}[\log(C_t^i) + \alpha_{t-1}(1 + \gamma \frac{\beta}{1-\beta})\sigma E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s})dF(\bar{s})] + \frac{\beta}{1-\beta} \log(W_{t+1}) + \psi_{t+1}'(\alpha_t)]$$

$$\Rightarrow E_{t-1}[r^d + \log(\rho_t^i) + \frac{\beta}{1-\beta} \log(W_{t-1} - C_{t-1}) + \frac{\beta}{1-\beta} \log(1 - \rho_t^i)]$$

$$\Rightarrow E_{t-1}[r^d + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1}) \frac{\sigma^2}{2} + \log(\rho_t) + \alpha_{t-1}(1 + \gamma \frac{\beta}{1-\beta})\sigma E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s})dF(\bar{s})]]$$

$$\Rightarrow E_{t-1}[r^f + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1}) \frac{\sigma^2}{2} + \log(\rho_t) + \alpha_{t-1}(1 + \gamma \frac{\beta}{1-\beta})\sigma E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s})dF(\bar{s})]]$$
which finally results in the following comparison

$$E_{t-1}[r^d + \log(p_{t+1}^m) + \beta \log(1 - p_{t+1}^m)] > E_{t-1}[r^f + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1}) \frac{\sigma^2}{2}]$$

As $T - t$ is large, for an average period $t - 1$ it holds that $E_{t-1}[\alpha_t] \approx \alpha_{t-1}$ such that

$$E_{t-1}[r^d + \log(p_{t+1}^m) + \beta \log(1 - p_{t+1}^m) + \psi_{t+1}^{L-1}(\alpha_{t-1})] > E_{t-1}[r^f + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1}) \frac{\sigma^2}{2}]$$

$$+ \log(p_t) + \alpha_{t-1}(1 + \gamma \frac{\beta}{1 - \beta}) \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \hat{s})dF(\hat{s})] + \beta \log(1 - p_t) + \psi_{t+1}^{L+1}(\alpha_t).$$

The agent’s continuation utilities are given by

$$\psi_{t+1}^{L-1}(\alpha_{t-1}) = \beta E_{t-1}[\log(p_{t+1}) + \sqrt{2} \alpha_{t-1}(1 + \gamma \frac{\beta}{1 - \beta}) \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \hat{s})dF(\hat{s})]]$$

$$+ \frac{\beta}{1 - \beta} \log(1 - p_{t+1}) + \frac{\beta}{1 - \beta}(r^f + \alpha_{t+1}(r_{t+2} - r^f) + \alpha_{t+1}(1 - \alpha_{t+1}) \frac{\sigma^2}{2}) + \psi_{t+2}^{L+1}(\alpha_{t+1}).$$

The agent’s behavior from period $t + 2$ on is not going to be affected by his period $t$ (in)attentiveness (as his period $t + 1$ self can be forced to look up the portfolio by his period $t$ self). Moreover, as $T - t$ is large, for an average period $t - 1$ it holds that $E_{t-1}[\alpha_t] \approx \alpha_{t-1}$ such that

$$E_{t-1}[\psi_{t+1}^{L-1}(\alpha_{t-1}) - \psi_{t+1}^{L+1}(\alpha_t)] = E_{t-1}[(\sqrt{2} - 1) \alpha_{t-1}(1 + \gamma \frac{\beta}{1 - \beta}) \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \hat{s})dF(\hat{s})]]$$

$$\Rightarrow E_{t-1}[r^d + \log(p_{t+1}^m) + \frac{\beta}{1 - \beta} \log(1 - p_{t+1}^m) + (\sqrt{2} - 2) \alpha_{t-1}(1 + \gamma \frac{\beta}{1 - \beta}) \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \hat{s})dF(\hat{s})]]$$

$$> E_{t-1}[r^f + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1}) \frac{\sigma^2}{2} + \log(p_t) + \frac{\beta}{1 - \beta} \log(1 - p_t)],$$

which finally results in the following comparison

$$E_{t-1}[\log(p_{t+1}^m)] > E_{t-1}[r^f + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1}) \frac{\sigma^2}{2} - r^d]$$

$>$ 0 in expectation and increasing with $h$

$$+ \log(p_t) + \frac{\beta}{1 - \beta} \log(1 - p_t) + (2 - \sqrt{2}) \alpha_{t-1}(1 + \gamma \frac{\beta}{1 - \beta}) \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \hat{s})dF(\hat{s})]]$$

< 0 and increasing with $\sqrt{h}$

In turn, if I increase $h$, I increase the positive return part, which becomes quantitatively relatively
more important than the negative news-utility part. Increasing \( h \) implies that the difference between
the positive return part and the negative news-utility part will at some point exceed the difference
in consumption utilities \( E_{t-1}[\log(\rho_t^{in}) + \frac{\beta}{1-\beta} \log(1-\rho_t^{in})] - E_{t-1}[\log(\rho_t) + \frac{\beta}{1-\beta} \log(1-\rho_t)] \), which
is positive because \( \log(\rho_t^{in}) + \frac{\beta}{1-\beta} \log(1-\rho_t^{in}) \) is maximized for \( \rho_t^{in} = \frac{1}{1 + \frac{\beta}{\gamma + \eta}} \), which corresponds
the standard agent’s portfolio share (plus given that \( E_{t-1}[\log(\rho_t)] < \log(E_{t-1}[\rho_t]) \) as \( \log(\cdot) \) is a
concave function). The consumption utilities are given by
\( (\rho_t = \frac{1}{1 + \frac{\beta}{1+\gamma}}, \text{\( \rho_t^{in} = \frac{1}{1 + \frac{\beta}{\gamma + \eta}} \)} \) (with
\( \tilde{\eta} \in (\eta, \eta \lambda) \) and as the agent is inattentive for just one period \( \rho_t^{in} = \frac{1}{1 + \frac{\beta}{\gamma + \eta}} \))

\[
\log(\rho_t^{in}) + \frac{\beta}{1-\beta} \log(1-\rho_t^{in}) = \log\left(\frac{1}{1 + \frac{\beta}{\gamma + \eta}}\right) + \frac{\beta}{1-\beta} \log\left(\frac{\beta}{1+\gamma}\right) = \frac{\beta}{1-\beta} \log\left(\frac{\beta}{1+\gamma}\right) - (1 + \frac{\beta}{1-\beta}) \log(1 + \frac{\beta}{1-\beta})
\]

and

\[
E_{t-1}[\log(\rho_t)] + \frac{\beta}{1-\beta} \log(1-\rho_t)] = E_{t-1}[\log\left(\frac{1}{1 + \frac{\beta}{\gamma + \eta}}\right) + \frac{\beta}{1-\beta} \log\left(\frac{\beta}{1+\gamma}\right) + (1 + \frac{\beta}{1-\beta}) \log(1 + \frac{\beta}{1-\beta})]
\]

\[
= E_{t-1}[\frac{\beta}{1-\beta} \log\left(\frac{\beta}{1+\gamma}\right) - (1 + \frac{\beta}{1-\beta}) \log(1 + \frac{\beta}{1+\gamma})] - \log(1 + \frac{\beta}{1-\beta}) - E_{t-1}[\log\left(\frac{1}{1 + \frac{\beta}{\gamma + \eta}}\right) - (1 + \frac{\beta}{1-\beta}) \log(1 + \frac{\beta}{1+\gamma})] + (1 + \frac{\beta}{1-\beta}) \log(1 + \frac{\beta}{1+\gamma})]
\]

And their difference is given by

\[
-(1 + \frac{\beta}{1-\beta}) \log(1 + \frac{\beta}{1-\beta}) - E_{t-1}[\log(1 + \frac{\beta}{1+\gamma})] - \log(1 + \frac{\beta}{1-\beta}) - (1 + \frac{\beta}{1-\beta}) \log(1 + \frac{\beta}{1+\gamma})]
\]

which is decreasing in \( \frac{\beta}{1-\beta} \), i.e., \( \frac{\partial(\cdot)}{\partial \frac{\beta}{1-\beta}} < 0 \), unless \( \gamma \) is too small, in which case it is increasing, i.e.,

\( \frac{\partial(\cdot)}{\partial \frac{\beta}{1-\beta}} > 0 \), because

\[
\frac{\partial(\cdot)}{\partial \frac{\beta}{1-\beta}} = -\log(1 + \frac{\beta}{1-\beta}) - \log(1 + \frac{\beta}{1+\gamma}) - (1 + \frac{\beta}{1-\beta}) \log(1 + \frac{\beta}{1+\gamma}) - \log(1 + \frac{\beta}{1-\beta}) - (1 + \frac{\beta}{1-\beta}) \log(1 + \frac{\beta}{1+\gamma})]
\]

and note that

\[
\frac{\partial(\log(1 + \frac{\beta}{1-\beta}) - \log(1 + \frac{\beta}{1+\gamma}))}{\partial \frac{\beta}{1-\beta}} = \frac{1}{1 + \frac{\beta}{1-\beta}} - \frac{1}{1 + \frac{\beta}{1+\gamma}} + \frac{\beta}{1-\beta} > 0
\]

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Thus, an increase in $h$ decreases $\frac{\beta}{1-\beta}$ which increases the difference in consumption utilities (unless $\gamma$ is small such that $\frac{\partial(\cdot)}{\partial \beta} > 0$). If $\frac{\partial(\cdot)}{\partial \beta} < 0$, however, the increase in the difference in consumption utilities due to the increase in $h$ will be less than the rate at which the difference between the return and news-utility part increases if $h$ becomes large. The reason is that an increase in $h$ will result in a decrease in $\frac{\beta}{1-\beta}$ given by $\frac{\beta^{k+1}\log\beta}{(1-\beta)^2}$, which goes to zero as $h \rightarrow \infty$. Thus, I conclude that the agent will find it optimal to be inattentive in every period for $h > \bar{h}$.

In turn, I can prove that the agent will be inattentive for at least one period if $h < \bar{h}$. If I decrease $h$, I decrease the positive return part, which becomes quantitatively less important (as it is proportional to $h$) relative to the negative news-utility part (as it is proportional to $\sqrt{h}$). Moreover, the difference in consumption utilities speaks towards not looking up the portfolio too, i.e., $E_t[-1]log(\rho_i) + \frac{\beta}{1-\beta}log(1-\rho_i) - E_{t-1}[log(\rho_i) + \frac{\beta}{1-\beta}log(1-\rho_i)] > 0$ as shown above. The intuition for this additional reason to not look up the portfolio is that inattentive consumption is not subject to a self-control problem while attentive consumption is. Furthermore, $h$ affects the prospective news utility term via $1+\gamma\frac{\beta}{1-\beta}$. However, as $\frac{\beta}{1-\beta}$ increases if $h$ decreases this will only make the agent more likely to be inattentive. Thus, I conclude that the agent will find it optimal to be inattentive for at least one period if $h < \bar{h}$. It cannot be argued that the agent would behave differently than what is assumed from period $t + 1$ on unless he finds it optimal to do so from the perspective of period $t$ because the agent can restrict the funds in the checking account and determine whether or not his period $t + 1$ self is attentive.

J.6 Proof of Corollary 3

Please refer to Appendix I for the derivation of the dynamic portfolio share. The expected benefit of inattention (as defined in the text) is given by

$$- (\sqrt{iE[\alpha_{t-i}]}(1+\gamma \sum_{\tau=1}^{T-t} \beta^{\tau}) + (\sqrt{j_{1}E[\alpha_{t}]} - E[\alpha_{t-i}](1+\gamma \sum_{\tau=1}^{T-t-j_{1}} \beta^{\tau})\sigma E[\eta(\lambda - 1) \int_{s}^{\infty} (s-\tilde{s})dF(\tilde{s})]).$$

and is always positive if $E[\alpha_{t-i}] \approx E[\alpha_{t}]$ (i.e., if $T-t$ is large). As can be easily inferred $\frac{1+\gamma \sum_{\tau=1}^{T-t-j_{1}} \beta^{\tau}}{\sum_{\tau=0}^{T-t-j_{1}} \beta^{\tau}}$ is converging as $T-t$ becomes large and thus increases quickly if $T-t$ is small. Therefore, $\alpha_{t}$ is decreasing more quickly, i.e., $\frac{\partial(\alpha_{t-i}E_{t-1}[\alpha_{t}])}{\partial(\alpha_{T-t})} < 0$, and the expected benefit of inattention is lower if $E[\alpha_{t-i}] > E[\alpha_{t}]$ (toward the end of life). The benefit of inattention is lower if $\alpha_{t-i} > E[\alpha_{t}]$ and (as I will show in the proof of Proposition 14) $\frac{\partial \alpha_{t}}{\partial r_{t}+...+r_{t-i+1}} > 0$. Thus, the benefit of inattention is low if $r_{t-i}+...+r_{t-i-j_{0}+1}$ is low.
J.7 Proof of Proposition 14

The agent’s optimal portfolio share is given by

$$\alpha_t = \frac{\mu - r^f + \frac{1 + \gamma^D_{t-1, j} \beta^\tau}{\sum_{\tau=0}^{T-1-j} \beta^\tau} \mathbb{E}[\eta(\lambda - 1) \int_{r_{t+1}}^{\infty} \rho_t (r_{t+1} - \bar{r}) dF_\tau(\bar{r})]}{\sigma^2}. \quad (1)$$

As can be easily seen

$$\frac{\partial \alpha_t}{\partial F_t(r_t) \ldots F_t(r_{t-i+1})} \frac{\partial F_t(r_t) \ldots F_t(r_{t-i+1})}{\partial (r_t + \ldots + r_{t-i+1})} \frac{1 + \gamma^D_{t-1, j} \beta^\tau}{\sum_{\tau=0}^{T-1-j} \beta^\tau} \mathbb{E}[\eta(\lambda - 1) \int_{r_{t+1}}^{\infty} \rho_t (r_{t+1} - \bar{r}) dF_\tau(\bar{r})] = \frac{\gamma(\lambda - 1)}{\sigma^2} \frac{\partial F_t(r_t) \ldots F_t(r_{t-i+1})}{\partial (r_t + \ldots + r_{t-i+1})} < 0.$$

And $1 + \gamma^D_{t-1, j} \beta^\tau / \sum_{\tau=0}^{T-1-j} \beta^\tau$ is decreasing in $T - t$ iff $\gamma < 1$. And $\frac{\partial \alpha_t}{\partial \alpha_t}$ is more negative if $1 + \gamma^D_{t-1, j} \beta^\tau / \sum_{\tau=0}^{T-1-j} \beta^\tau$ is high thus the degree of extensive rebalancing is higher late in life.

J.8 Proof of Corollary 4

The agent’s optimal consumption share is given by

$$\rho_t = \frac{1}{1 + \gamma^D_{t-1, j} \beta^\tau / \sum_{\tau=0}^{T-1-j} \beta^\tau} \mathbb{E}[\eta(\lambda - 1) \int_{r_{t+1}}^{\infty} \rho_t (r_{t+1} - \bar{r}) dF_\tau(\bar{r})]. \quad (2)$$

As can be easily seen, $1 + \gamma^D_{t-1, j} \beta^\tau / \sum_{\tau=0}^{T-1-j} \beta^\tau$ is increasing in $F_t(r_t) \ldots F_t(r_{t-i+1})$ and $\rho_t$ is decreasing in $F_t(r_t) \ldots F_t(r_{t-i+1})$, i.e.,

$$\frac{\partial \rho_t}{\partial F_t(r_t) \ldots F_t(r_{t-i+1})} = \frac{(1 - \gamma)(\lambda - 1) \beta^\tau / \sum_{\tau=0}^{T-1-j} \beta^\tau}{(1 + \gamma^D_{t-1, j} \beta^\tau / \sum_{\tau=0}^{T-1-j} \beta^\tau)^2} < 0 \text{ if } \gamma < 1.$$

If $\frac{\partial \rho_t}{\partial F_t(r_t) \ldots F_t(r_{t-i+1})} < 0$, it follows that $\frac{\partial \rho_t}{\partial \rho_t} < 0$ such that if $\gamma = 0$ (in which case $\frac{\partial \alpha_t}{\partial \alpha_t} = 0$) and $\frac{\partial ((R^f)^i + \alpha^D_{t-1, j} (R_t \ldots R_{t-i+1} - (R^f)^i))}{\partial \alpha_t} = \alpha_t (1 - \rho_t) \alpha_t - ((R^f)^i + \alpha^D_{t-1, j} (R_t \ldots R_{t-i+1} - (R^f)^i)) \frac{\partial \rho_t}{\partial \rho_t} \alpha_t > \alpha_t (1 - \rho_t) \alpha_t$. If $\gamma > 0$, then there exist some threshold $\tilde{\gamma}$ such that the increasing variation in $1 - \rho_t$ outweighs the decreasing variation in $\alpha_t$.

J.9 Proof of Proposition 15

Comparing the precommitted-monotone and personal-monotone portfolio share it can be easily seen that they are not the same for any period $t \in \{1, \ldots, T - 1\}$.
1. The precommitted consumption share for attentive consumption is

\[ \rho_c^t = \frac{1}{1 + \sum_{\tau=1}^{T-t} \beta^\tau \frac{(1+\eta(\lambda-1)(2F_r(r_i)\ldots F_r(r_{t-i+1})-1))}{1+\eta(\lambda-1)(2F_r(r_i)\ldots F_r(r_{t-i+1})-1))}} \]

which is lower than the personal-monotone share if \( \gamma < 1 \) as \( \eta(\lambda-1)(2F_r(r_i) - 1) < \eta F_r(r_i) + \eta \lambda (1 - F_r(r_i)) \) and the difference increases if \( F_r(r_i)\ldots F_r(r_{t-i+1}) \) increases.

2. The precommitted portfolio share is

\[ \alpha_c^t = \frac{\mu - r_f + \frac{1+\gamma\sum_{\tau=1}^{T-t-j} \beta^\tau E[\eta(\lambda-1)(2F_r(r_i)\ldots F_r(r_{t-i+1})-1)]}{1+\eta(\lambda-1)(2F_r(r_i)\ldots F_r(r_{t-i+1})-1)}}{\sigma^2} \]

which is lower than the personal-monotone share as \( \eta(\lambda-1)(2F_r(r_i) - 1) < \eta F_r(r_i) + \eta \lambda (1 - F_r(r_i)) \) and the difference increases if \( F_r(r_i)\ldots F_r(r_{t-i+1}) \) increases.

3. \( \gamma \) does not necessarily imply an increase in \( \rho_c^t \) because \( \eta(\lambda-1)(2F_r(r_i)\ldots F_r(r_{t-i+1})-1) \) can be negative. Thus, attentive consumption is higher only due to the differences in returns and not due to the time inconsistency any more. Thus, the cost of less consumption utility in period \( t \), is (as defined in the text)

\[ E[\log(\rho_c^{tin})] - E[\log(\rho_c^t)] \]

is lower as \( \rho_c^{tin} = \rho_c^{tin} \) but \( E[\log(\rho_c^t)] < E[\log(\rho_c^t)] \).

4. As precommitted marginal news utility is always lower and the gap increases in good states there is larger variation in it, i.e., it varies from \( \{-\eta(\lambda-1), \eta(\lambda-1)\} \) which is larger than the variation in non-precommitted marginal news \( \{\eta, \eta \lambda\} \), as \( 2\eta(\lambda-1) > \eta(\lambda-1) \). Thus, the variation in \( \alpha_c \) (extensive rebalancing) is more pronounced on the pre-committed path. More formally it can be easily seen that

\[ \frac{\partial \alpha_c^t}{\partial F_r(r_i)\ldots F_r(r_{t-i+1})} < 0 \]