Effects of Flow Structures on Particle Clustering in Homogeneous Isotropic Turbulence

By

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Abstract

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This work experimentally examines the geometric characterization and spatial distribution of particle clusters and voids in homogeneous isotropic turbulence. Specifically, we analyze the relationship between turbulent flow structures and the spatial distribution of particles. A Voronoi cell analysis and particle number density are used to identify regions of particle clusters and voids. The turbulent flow structures analyzed are vortices, stagnation points, and fast moving jets.

A 40-cm Eaton box is used to generate a homogeneous isotropic turbulent flow with $Re_\lambda$ ranging from 157 to 319. The first set of experiments examines the geometric characterization of particle voids and clusters by injecting aluminum-oxide particles with $d_p = 0.5, 1, 3, \text{and } 5\mu m$ into four turbulence levels. In the second set of experiments aluminum-oxide particles with $d_p = 1\mu m$ and fluorescent polymer microspheres with $d_p = 12\mu m$ are injected into the Eaton box under four turbulence levels. Images of the aluminum-oxide particles are analyzed using planar Particle Image Velocimetry to identify turbulent flow structures. Simultaneously, a second camera is used to capture images through a filter of the fluorescent polymer microspheres to identify regions of particle clusters and voids. The spatial distribution of voids and clusters are compared with the distribution of turbulent flow structures.

The characteristic length scales for particle voids and clusters vary significantly between the two sets of experiments. The images of polymer microspheres, which have a greater resolution, indicate a characteristic length scale for particle clusters of 10-20$\eta$ depending on the $Re_\lambda$, which matches the results of previous studies. Similarly, we find the characteristic length scales of particle voids ranges from 10 to 100$\eta$. In both sets of experiments we find the distribution of particle voids, and to some extent particle clusters, follow a power-law fit indicating self-similarity. Results are inconclusive in linking turbulent flow structures to particle clusters and voids, which was unexpected.
# Contents

List of Figures iii

List of Tables ix

Acknowledgements xiii

Nomenclature xiv

1 Introduction 1
   1.1 Literature Review ........................................ 1
   1.1.1 Equations of Motion for Spheres in Flow Field .......... 2
   1.1.2 Flow Structures in Homogeneous Isotropic Turbulence .... 4
   1.1.3 Preferential Concentrations in Homogeneous Isotropic Turbulence .. 5
   1.1.4 Settling and Relative Velocities in Homogeneous Isotropic Turbulence .......... 11
   1.1.5 Collision Rate and the Droplet Size Distribution .......... 12
   1.2 Thesis Purpose ........................................... 15

2 Characterization of the Turbulent Flow Field 17
   2.1 Turbulence Chamber/Eaton Box .................................. 17
   2.2 Particle Image Velocimetry (PIV) ............................... 19
     2.2.1 PIV Set-up ........................................... 19
     2.2.2 Turbulence Parameters ................................. 22
   2.3 Turbulence Characterization ................................... 25

3 Geometric Characterization of Particle Clusters and Voids 40
   3.1 Experimental Parameters ..................................... 40
   3.2 Particle Detection ......................................... 44
   3.3 Voronoi Analysis ............................................ 50
   3.4 Particle Clustering Characterization ......................... 54
   3.5 Particle Void Characterization ................................ 63

4 Flow Structures and Particle Distribution 71
   4.1 Experimental Set-up ......................................... 71
   4.2 Particle Clustering and Void Geometric Characteristics ........ 80
   4.3 Flow Structure Identification ................................ 86
   4.4 Turbulent Flow Structures and Particle Distribution ........ 93
5 Conclusions and Future Work
  5.1 Conclusions ................................................. 108
  5.2 Future Work ................................................. 110

References ...................................................... 111

A Additional Particle Data ..................................... 118
# List of Figures

1.1 Sweep-out volumes geometries. ........................................... 13

2.1 Turbulence chamber used to generate homogeneous isotropic turbulence. ........ 18

2.2 Set-up of PIV system with the CW laser shown. The CW laser was interchangeable with the ND-YAG laser. ................................. 19

2.3 Schematic of PIV system set-up from top and side views. Note images are not to scale. ......................................................... 20

2.4 Cemented cylindrical and spherical lens system. ................................ 20

2.5 Particle discharge system. ................................................. 21

2.6 PIV timing diagram. ....................................................... 22

2.7 Compensated longitudinal second order structure function for varying levels of forcing amplitude. $\epsilon$ is approximated at the inertial subrange, which aligns with the plateau portion of the graph, denoted by the dashed line. ........ 27

2.8 The second-order structure function, $D_{LL}$, from the experimental results are indicated as open circles. $C_r^{2/3}$ as a solid line. Where $D_{LL}$ and $C_r^{2/3}$ overlap, indicates $r$ corresponding to the inertial subrange. ................................. 27

2.9 A comparison of the various methods to calculate $\epsilon$. The Large-Eddy PIV method with $C_s = 0.12$ is seen to match the Structure Function Fitting method. 28

2.10 Energy spectra at the various levels of turbulence. $E_{11}$ is the energy spectrum calculated from the $u$-velocity component and $E_{22}$ is the energy spectrum calculated from the $v$-velocity component. ........................................ 31

2.11 Longitudinal, $f$, and transverse, $g$, structure functions are plotted in green and blue, respectively. ......................................................... 32

2.12 Autocorrelation of $u$ and $v$-components of the fluctuating velocity field. ........ 33

2.13 $u$ and $v$-components of mean velocity field. ................................ 34

2.14 Mean velocity flow field with the streamline outlined in red. This indicates a stagnation point in the center of the flow field. ....................... 35

2.15 $u$ and $v$-components of rms velocity fluctuations. ................................ 36

2.16 PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. ....................... 37

2.17 $u_{rms}/v_{rms}$ is plotted to indicate isotropic flow when approaching unity, and a scatter plot of $u'$ and $v'$ indicates isotropy when azimuthally symmetric. .... 38

2.18 $u_{rms}/\langle u_{rms}\rangle$ and $v_{rms}/\langle v_{rms}\rangle$ are plotted to indicate homogeneity when approaching unity. ........................................ 39
3.1 PDF of \( u \) and \( v \)-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. The PIV analysis is performed with an image size: 10cm \( \times \) 10cm using a YAG laser light source and a forcing amplitude of \( A = 2V_{pp} \) with a forcing frequency of \( f = 100 \text{Hz} \). The tracer particles have an average diameter of \( d_p = 0.5, 1, 3, 5 \mu \text{m} \).

3.2 Average image pixel intensity and curve-fit used to normalize the light intensity.

3.3 Image processing to detect individual particles for a 10 \( \times \) 10cm\(^2\) image area.

3.4 Image processing to detect individual particles for a 20 \( \times \) 20mm\(^2\) image area.

3.5 The image on the left marks the particles that exceed the threshold level, and in the right image the center of each particle detected is marked. The blob contours are outlined in red.

3.6 Number of particles detected over time for 20 \( \times \) 20mm\(^2\) images.

3.7 The particles detected are marked along with the Voronoi cells associated with them.

3.8 A sample contraction coefficient map for both 20 \( \times \) 20mm\(^2\) and 10 \( \times \) 10mm\(^2\) images.

3.9 Normalized Voronoi cell area PDF for an image compared to a Poisson distribution. Voronoi cells with a normalized area greater than \( V_v \) belong to a void regions. Similarly, Voronoi cells with a normalized area less than \( V_c \) belong to a cluster region.

3.10 \( A/\langle A \rangle \) from Voronoi analysis of 20 \( \times \) 20mm\(^2\) images.

3.11 The Voronoi cells are outlined in red, and the cluster cells are shaded. Connected cluster cells shaded with matching colors indicate cluster regions.

3.12 \( \sqrt{A_c}/\eta \) from Voronoi analysis for 20 \( \times \) 20mm\(^2\) images.

3.13 \( \sqrt{A_c}/\eta \) from Voronoi analysis for 10 \( \times \) 10cm\(^2\) images.

3.14 Power-law fit to Figures A.9 and A.10 with Eq. (3.12) on both a linear and log plot. The solid line indicates the curve-fit, and the markers indicate data points.

3.15 \( P \) versus \( \sqrt{A_c} \) for each cluster. In both cases \( Re_{\lambda} = 253 \) and particles with \( d_p = 1 \mu \text{m} \) are used. These are representative examples of all test cases for their respective image areas.

3.16 Comparison of the two different void detecting techniques.

3.17 PDF of \( \sqrt{A_v}/\eta \) for 20 \( \times \) 20mm\(^2\) images. The dashed line indicates the void areas analyzed with the blurring analysis, and the solid line indicates the voids detected using the Voronoi analysis.

3.18 PDF of \( \sqrt{A_v}/\eta \) for 10 \( \times \) 10mm\(^2\) images. The dashed line indicates the void areas analyzed with the blurring analysis, and the solid line indicates the voids detected using the Voronoi analysis.
3.19 Power-law fit with Eq. (3.12) to Figures A.11 and A.13, i.e. the PDF of $A_{v/\eta}$ for $20 \times 20\text{mm}^2$ images. The two images on the left correspond to the voids found using the blurring technique, and the two plots on the right side correspond to the voids found using the Voronoi technique. The solid line indicates the curve-fit, and the markers indicate data points. 68

3.20 Power-law fit with Eq. (3.12) to Figures A.12 and A.14, i.e. the PDF of $A_{v/\eta}$ for $10 \times 10\text{cm}^2$ images. The two images on the left correspond to the voids found using the blurring technique, and the two plots on the right side correspond to voids found using the Voronoi technique. The solid line indicates the curve-fit, and the markers indicate data points. 69

3.21 $P$ versus $\sqrt{A_v}$ for each void. In both cases $Re_\lambda = 153$ and particles with $d_p = 3\mu\text{m}$ are used. These are representative examples for all test cases for their respective image areas. 70

4.1 Dual camera set-up to analyze the relationship between the spatial distribution of particles and turbulent flow structures. 72

4.2 Camera and laser set-up to study the relationship between the spatial distribution of particles and turbulent flow structures. 72

4.3 Targets used to determine the displacement between cameras. 73

4.4 Mean velocity flow field with the streamline outlined in red. In general, this indicates a stagnation point in the center of the flow field with some motion upward. 76

4.5 $u_{rms}/v_{rms}$ is plotted to indicate isotropic flow when approaching unity, and a scatter plot of $u'$ and $v'$ indicates isotropy when azimuthally symmetric. 77

4.6 PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. 78

4.7 $u_{rms}/\langle u_{rms} \rangle$ and $v_{rms}/\langle v_{rms} \rangle$ are plotted to indicate homogeneity when approaching unity. 79

4.8 Sample image of the polymer microspheres detected from the Y3.2-S1 camera. 80

4.9 Number of particles detected over time after injection. 81

4.10 $A/\langle A \rangle$ for various turbulence levels for 2.5 grams of polymer microspheres. 82

4.11 $\sqrt{A_v}/\eta$, where $-\bullet-$ indicates the 1 gram of polymer microspheres and $-\ast-$ indicates the 2.5 grams of polymer microspheres. 83

4.12 $\sqrt{A_v}/\eta$, where $-\bullet-$ indicates the 1 gram of polymer microspheres and $-\ast-$ indicates the 2.5 grams of polymer microspheres. 84

4.13 Power-law fit to Figures A.22b and A.23b for 1 gram of polymer microspheres. The solid line indicates the curve-fit and the markers indicate data points. 85

4.14 Representative test cases for $P$ versus $\sqrt{A}$ for clusters and voids. In both cases $Re_\lambda = 285$ with 1 gram of polymer microspheres. 86

4.15 $Q$ and $u'$ for a single test case where $Re_\lambda = 243$. 88

4.16 $Q$ and $u'$ for a single test case where $Re_\lambda = 285$. 89
4.17 Detected zones in the flow field where $Re_\lambda = 243$: green denotes the convergence zones, blue the eddy zones, and yellow the stream zones. The red lines are streamlines of the flow field. .......................................................... 90
4.18 Detected zones in the flow field where $Re_\lambda = 285$: green denotes the convergence zones, blue the eddy zones, and yellow the stream zones. The red lines are streamlines of the flow field. .......................................................... 91
4.19 Comparison between the binary image, detected void and cluster regions, and particle number density. .......................................................... 92
4.20 PDF of the normalized particle number density in each flow zone for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres. .......................................................... 96
4.21 Normalized particle number density versus the normalized strain rate and enstrophy for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres. .......................................................... 98
4.22 Particle number density versus the second invariant of the velocity gradient tensor, $Q$, for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres. .......................................................... 99
4.23 Regions of particle cluster/void regions are detected in the normalized enstrophy and strain rate fields, and the average values are calculated within these regions. .......................................................... 100
4.24 Particle number density versus horizontal and vertical local velocities for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres. .......................................................... 102
4.25 Particle number density versus horizontal and vertical local velocities for only the regions of the flow field classified as a stream zones for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres. .......................................................... 103
4.26 Streamlines analyzed for particle response behavior. .......................................................... 105
4.27 Depicted is the velocity flow field marked with red streamlines. The regions which the user has chosen to investigate the particle number density are outlined in black polygons. Again, convergence zones are highlighted in green, eddy zones in blue, and stream zones in yellow. The particle number density identifies, which points are used to calculate the average particle number density for each area selected by the user. .......................................................... 106
4.28 PDF of the average normalized particle number density near various streamlines for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres. .......................................................... 107

A.1 PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. The PIV analysis is performed with particles with $d_p = 0.5\mu m$ using a YAG laser light source. The forcing amplitude is $A = 2, 4, 6$, and $7V_{pp}$ and the forcing frequency is $f = 100Hz$. This is the same information given in Figure 2.16, but on a logarithmic scale. .......................................................... 120
A.2 PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. The PIV analysis is performed with $d_p = 0.5, 1, 3, 5\mu m$ particles using a YAG laser light source and a forcing amplitude of $A = 4V_{pp}$ and a forcing frequency of $f = 100Hz$. .......................................................... 121
A.3 PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. The PIV analysis is performed with $d_p = 0.5, 1, 3, 5\mu m$ particles using a YAG laser light source and a forcing amplitude of $A = 6V_{pp}$ and a forcing frequency of $f = 100Hz$. 122
A.4 PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. The PIV analysis is performed with $d_p = 0.5, 1, 3, 5\mu m$ particles using a YAG laser light source and a forcing amplitude of $A = 7V_{pp}$ and a forcing frequency of $f = 100Hz$. 123
A.5 PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. The PIV analysis is performed with $d_p = 0.5, 1, 3, 5\mu m$ particles using a continuous laser light source and a forcing amplitude of $A = 2V_{pp}$ and a forcing frequency of $f = 100Hz$. 124
A.6 $A/\langle A\rangle$ from Voronoi analysis of $10 \times 10cm^2$ images. 126
A.7 $A_v$ from Voronoi analysis for $20 \times 20mm^2$ images. 127
A.8 $A_v$ from Voronoi analysis for $10 \times 10cm^2$ images. 128
A.9 $A_c/\eta^2$ from Voronoi analysis for $20 \times 20mm^2$ images. 129
A.10 $A_c/\eta^2$ from Voronoi analysis of $10 \times 10cm^2$ images. 130
A.11 $A_v/\eta^2$ for $20 \times 20mm^2$ images with the Voronoi analysis. 131
A.12 $A_v/\eta^2$ for $10 \times 10cm^2$ images with the Voronoi analysis. 132
A.13 $A_v/\eta^2$ for $20 \times 20mm^2$ images with the Gaussian Blur analysis. 133
A.14 $A_v/\eta^2$ for $10 \times 10cm^2$ images with the Gaussian Blur analysis. 134
A.15 $A_v$ for $20 \times 20mm^2$ images with the Voronoi analysis. 135
A.16 $A_v$ for $10 \times 10cm^2$ images with the Voronoi analysis. 136
A.17 $A_v$ for $20 \times 20mm^2$ images with the Gaussian Blur analysis. 137
A.18 $A_v$ for $10 \times 10cm^2$ images with the Gaussian Blur analysis. 138
A.19 PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. 139
A.20 $A/\langle A\rangle$ for various turbulence levels for 1 gram of polymer microspheres. 140
A.21 RDF corresponding to data in Figure 4.10 and A.20, where the solid line indicates 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres and a dashed line indicates 2.5 grams ($\Phi_v \approx 3.7e - 5$) of polymer microspheres. 141
A.22 $A_v/\eta^2$ and $\frac{\Delta \epsilon}{\eta^2}$ corresponding to data in Figure 4.11, where the solid line indicates 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres and a dashed line indicates 2.5 grams ($\Phi_v \approx 3.7e - 5$) of polymer microspheres. 141
A.23 $A_v/\eta^2$ and $\frac{\Delta \epsilon}{\eta^2}$ corresponding to data in Figure 4.12, where the solid line indicates 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres and a dashed line indicates 2.5 grams ($\Phi_v \approx 3.7e - 5$) of polymer microspheres. 142
A.24 The overall breakdown of the total image into regions of particle clusters, particle voids, and neither for images containing 2.5 grams ($\Phi_v \approx 3.7e - 5$) of polymer microspheres. 142
A.25 The breakdown of how particle clusters and voids are distributed among the flow regions for images containing 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres. ................................................................. 143
A.26 The average normalized particle number density for each flow zone for images containing 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres. ............................. 143
A.27 PDF of the normalized particle number density in each flow zone for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres. Here we present the same data as in Figure 4.20, but on a log-log plot. ......................................................... 146
A.28 PDF of the normalized particle number density in each flow zone for 1 gram ($\Phi_v \approx 1.5e^{-5}$) of polymer microspheres. ................................................................. 147
A.29 PDF of the normalized particle number density in each flow zone for 1 gram ($\Phi_v \approx 1.5e^{-5}$) of polymer microspheres. Here we present the same data as in Figure A.28, but on a log-log plot. ......................................................... 148
A.30 Particle number density versus the normalized strain rate and enstrophy for 1 gram ($\Phi_v \approx 1.5e^{-5}$) of polymer microspheres. ................................................................. 149
A.31 Particle number density versus the second invariant of the velocity gradient tensor, $Q$, for 1 gram ($\Phi_v \approx 1.5e^{-5}$) of polymer microspheres. ............................. 150
A.32 Particle number density versus horizontal and vertical local velocities for 1 gram ($\Phi_v \approx 1.5e^{-5}$) of polymer microspheres. ................................................................. 151
A.33 Particle number density versus horizontal and vertical local velocities for only the regions of the flow field classified as a stream zones for 1 gram ($\Phi_v \approx 1.5e^{-5}$) of polymer microspheres. ................................................................. 152
A.34 The average normalized enstrophy and strain rate values for areas identified as particle cluster and void regions for images containing 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres. ................................................................. 153
LIST OF TABLES

1.1 Summary of experiments focused on particle clustering in particle-laden homogeneous isotropic turbulent flow. .......................................................... 9

2.1 Turbulence parameters calculated for $d_p = 0.5\mu m$ particles under various forcing amplitudes with a forcing frequency of $f = 100Hz$, where the Large-Eddy PIV technique with the corrected Smagorinsky constant is used to calculate $\epsilon$. A YAG laser is used for illumination. ...................................................... 26

3.1 The relaxation time for various particle sizes. ......................................................... 40

3.2 Stokes number, $St_\eta$, for various forcing amplitudes and particles sizes. In all cases the forcing frequency is $f = 100Hz$. .......................................................... 41

3.3 Particle Reynolds number, $Re_p$, for various forcing amplitudes and particles sizes. In all cases the forcing frequency is $f = 100Hz$. ...................................................... 41

3.4 Average number of particles detected in the $20 \times 20mm^2$ images. In all test cases $f = 100Hz$. .......................................................... 49

3.5 Average $\Phi_v$ for the $20 \times 20mm^2$ images. In all test cases, $f = 100Hz$. ....... 50

3.6 $V_v$ and $V_c$ for various test cases for $20 \times 20mm^2$ images. .......................... 53

3.7 Average peak value in the PDF of $\sqrt{A_c}/\eta$ in Figures 3.12 and 3.13, which indicates characteristic cluster length scales. ...................................................... 58

3.8 Values of $d_p$, $\eta$, and image resolution from previous studies on characteristic cluster length scales. .......................................................... 60

4.1 Turbulence parameters calculated under various forcing amplitudes with a forcing frequency of $f = 100Hz$, where the Large-Eddy PIV technique with the corrected Smagorinsky constant is used to calculate $\epsilon$. A YAG laser is used for illumination. ...................................................... 74

4.2 $Re_p$, $d_p/\eta$, $V_{air}/\bar{u}'$ and for various turbulence levels. ..................................... 74

4.3 Relaxation time, $\tau_p$, and Stokes number, $St_\eta$, for various forcing amplitudes for the polymer microspheres. In all cases the forcing frequency is $f = 100Hz$. 75

4.4 $V_v$ and $V_c$ for various turbulence levels for 2.5 grams of polymer microspheres. 82

4.5 Range of characteristic cluster length scales indicated in Figure 4.11. ................. 84

4.6 Distribution of the three flow zones among the flow domain for 2.5 grams $(\Phi_v \approx 3.7e - 5)$ of particles. .......................................................... 87

4.7 Breakdown of the image into regions of particle clusters, particle voids, and neither for 2.5 grams $(\Phi_v \approx 3.7e - 5)$ of polymer microspheres. ......................... 94

4.8 Breakdown of how cluster Voronoi cells are distributed among the flow zones for 2.5 grams $(\Phi_v \approx 3.7e - 5)$ of polymer microspheres. ......................... 94

4.9 Breakdown of how the void Voronoi cells are distributed among the flow zones for 2.5 grams $(\Phi_v \approx 3.7e - 5)$ of polymer microspheres. ......................... 94
<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.10</td>
<td>The average normalized particle number density in each flow zone for 2.5 grams ($\Phi_v \approx 3.7e - 5$) of polymer microspheres.</td>
<td>95</td>
</tr>
<tr>
<td>4.11</td>
<td>The standard deviation of the particle number density in each flow zone for 2.5 grams ($\Phi_v \approx 3.7e - 5$) of polymer microspheres.</td>
<td>95</td>
</tr>
<tr>
<td>4.12</td>
<td>Average normalized enstrophy and scaler-strain values for regions of particle clusters for 2.5 grams ($\Phi_v \approx 3.7e - 5$) of polymer microspheres.</td>
<td>100</td>
</tr>
<tr>
<td>4.13</td>
<td>Average normalized enstrophy and scaler-strain values for regions of particle voids for 2.5 grams ($\Phi_v \approx 3.7e - 5$) of polymer microspheres.</td>
<td>101</td>
</tr>
<tr>
<td>4.14</td>
<td>Average normalized particle number density near various streamlines for 2.5 grams ($\Phi_v \approx 3.7e - 5$) of polymer microspheres.</td>
<td>106</td>
</tr>
<tr>
<td>4.15</td>
<td>Standard deviation of the normalized particle number density near various streamlines for 2.5 grams ($\Phi_v \approx 3.7e - 5$) of polymer microspheres.</td>
<td>106</td>
</tr>
<tr>
<td>A.1</td>
<td>Turbulence parameters with YAG laser from $d_p = 0.5\mu m$ particles using uncorrected Smagorinsky constant for $f = 100\text{Hz}$</td>
<td>118</td>
</tr>
<tr>
<td>A.2</td>
<td>Turbulence parameters with continuous laser calculated with $d_p = 0.5\mu m$ particles for $A = 2V_{pp}$</td>
<td>118</td>
</tr>
<tr>
<td>A.3</td>
<td>Turbulence parameters with YAG laser calculated with $d_p = 5\mu m$ particles for $f = 100\text{Hz}$</td>
<td>118</td>
</tr>
<tr>
<td>A.4</td>
<td>Turbulence parameters with YAG laser calculated with $d_p = 3\mu m$ particles for $f = 100\text{Hz}$</td>
<td>119</td>
</tr>
<tr>
<td>A.5</td>
<td>Turbulence parameters with YAG laser calculated with $d_p = 1\mu m$ particles for $f = 100\text{Hz}$</td>
<td>119</td>
</tr>
<tr>
<td>A.6</td>
<td>$V_v$ and $V_c$ for various test cases for $10 \times 10\text{cm}^2$ images.</td>
<td>125</td>
</tr>
<tr>
<td>A.7</td>
<td>Parameters for aluminum-oxide particles with $d_p = 1\mu m$ for various turbulence levels in the set-up and conditions described in Chapter 4.</td>
<td>138</td>
</tr>
<tr>
<td>A.8</td>
<td>$V_v$ and $V_c$ for various turbulence levels for 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres.</td>
<td>140</td>
</tr>
<tr>
<td>A.9</td>
<td>Distribution of the three flow zones among the flow domain for 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres.</td>
<td>144</td>
</tr>
<tr>
<td>A.10</td>
<td>Breakdown of the image into regions of particle clusters, particle voids, and neither for 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres.</td>
<td>144</td>
</tr>
<tr>
<td>A.11</td>
<td>Breakdown of how cluster Voronoi cells are distributed among the flow zones for 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres.</td>
<td>144</td>
</tr>
<tr>
<td>A.12</td>
<td>Breakdown of how the void Voronoi cells are distributed among the flow zones for 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres.</td>
<td>144</td>
</tr>
<tr>
<td>A.13</td>
<td>The average normalized particle number density in each flow zone for 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres.</td>
<td>145</td>
</tr>
<tr>
<td>A.14</td>
<td>The standard deviation of the particle number density in each flow zone for 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres.</td>
<td>145</td>
</tr>
<tr>
<td>A.15</td>
<td>Average normalized enstrophy and scaler-strain values for regions of particle clusters for 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres.</td>
<td>145</td>
</tr>
</tbody>
</table>
A.16 Average normalized enstrophy and scaler-strain values for regions of particle voids for 1 gram ($\Phi_v \approx 1.5e^{-5}$) of polymer microspheres. . . . . . . . . . . 145
This thesis is dedicated to my parents for all their patience, encouragement, and love.
I am extremely grateful to my Ph.D. adviser, Professor Ömer Savaş, for his support, kindness, guidance, and patience. It has been an absolute privilege to learn from and work alongside him.

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**NOMENCLATURE**

**ACRONYMS**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>DNS</td>
<td>Direct Numerical Simulations</td>
</tr>
<tr>
<td>PDA</td>
<td>Phase Doppler Anemometry</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle Image Velocimetry</td>
</tr>
<tr>
<td>PTV</td>
<td>Particle Tracking Velocimetry</td>
</tr>
<tr>
<td>RDF</td>
<td>Radial Distribution Function</td>
</tr>
<tr>
<td>ROI</td>
<td>Region of Interest</td>
</tr>
<tr>
<td>TKE</td>
<td>Turbulent Kinetic Energy</td>
</tr>
</tbody>
</table>

**VARIABLES**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Area/Signal amplitude</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Area of clusters</td>
</tr>
<tr>
<td>$A_v$</td>
<td>Area of voids</td>
</tr>
<tr>
<td>$&lt; C &gt;$</td>
<td>Normalized particle number density</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Universal constant</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Smagorinsky constant</td>
</tr>
<tr>
<td>$D$</td>
<td>Vortex tube diameter</td>
</tr>
<tr>
<td>$D_{LL}$</td>
<td>Second order longitudinal structure function</td>
</tr>
<tr>
<td>$E(k)$</td>
<td>Energy spectrum function</td>
</tr>
<tr>
<td>$E_{coal}$</td>
<td>Coalescence efficiency</td>
</tr>
<tr>
<td>$E_{coll}$</td>
<td>Collision efficiency</td>
</tr>
<tr>
<td>$I$</td>
<td>Pixel intensity</td>
</tr>
<tr>
<td>$I$</td>
<td>Natural log of pixel intensity</td>
</tr>
<tr>
<td>$K$</td>
<td>Collision kernel</td>
</tr>
<tr>
<td>$L$</td>
<td>Characteristic length/Vortex tube length</td>
</tr>
<tr>
<td>$L_e$</td>
<td>Integral length scale</td>
</tr>
<tr>
<td>$M$</td>
<td>Number of particles</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of particle pairs</td>
</tr>
<tr>
<td>$P$</td>
<td>Perimeter</td>
</tr>
<tr>
<td>$N_c$</td>
<td>Collision rate</td>
</tr>
<tr>
<td>$P(W,r)$</td>
<td>Particle pair probability distribution</td>
</tr>
<tr>
<td>$Q$</td>
<td>Second invariant of velocity gradient tensor</td>
</tr>
</tbody>
</table>
R_{ij} \quad \text{Velocity correlation tensor}

Re \quad \text{Reynolds number}

S \quad \text{Strain rate tensor}

S_f \quad \text{Settling parameter}

St \quad \text{Stokes number}

T \quad \text{Thresholding level}

T_e \quad \text{Large eddy turnover time}

U_o \quad \text{Characteristic velocity}

V_{pp} \quad \text{Peak-to-Peak voltage}

V_{air} \quad \text{Terminal fall velocity}

V_p \quad \text{Particle velocity}

W \quad \text{Relative particle velocity}

a \quad \text{Fluid acceleration}

a_p \quad \text{Particle radius}

d_p \quad \text{Particle diameter}

f \quad \text{Frequency/First order longitudinal structure function/Lens focal length}

f(m) \quad \text{Number of droplets with mass, m}

g \quad \text{Gravitational acceleration/First order transversal structure function}

g(r) \quad \text{Radial distribution function}

k \quad \text{Wavenumber}

m_f \quad \text{Mass of displaced fluid}

m_p \quad \text{Mass of particle}

n \quad \text{Particle number density}

q^2 \quad \text{Turbulent kinetic energy}

r \quad \text{Radius}

r_{IR} \quad \text{Inertial length scale}

s \quad \text{Normalized scaler-strain}

u \quad \text{Velocity of far field fluid}

u' \quad \text{Velocity fluctuations}

u_k \quad \text{Kolmogorov velocity scale}

u_{rms} \quad \text{RMS of the velocity fluctuations}

\Delta \quad \text{Resolved length scale}

\Phi_{i,j} \quad \text{Energy spectral density}

\Phi_v \quad \text{Volume fraction}

\Omega \quad (1/s) \quad \text{Vorticity tensor}

\epsilon \quad \text{Energy dissipation rate}

\eta \quad \text{Kolmogorov length scale}

\lambda \quad \text{Wavelength/Mean number of particles in a window/Taylor microscale}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( \mu )</td>
<td>Dynamic viscosity</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density of fluid</td>
</tr>
<tr>
<td>( \rho_d )</td>
<td>Density of particle/droplet</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>( \tau_{ij} )</td>
<td>Sub-grid scale stress</td>
</tr>
<tr>
<td>( \tau_p )</td>
<td>Particle relaxation time</td>
</tr>
<tr>
<td>( \tau_\eta )</td>
<td>Kolmogorov time scale</td>
</tr>
<tr>
<td>( \omega \ (1/s^2) )</td>
<td>Normalized enstrophy</td>
</tr>
<tr>
<td>( \mathcal{V} )</td>
<td>Volume of fluid domain</td>
</tr>
<tr>
<td>( \mathcal{V}_p )</td>
<td>Volume of a single particle</td>
</tr>
<tr>
<td>( \forall )</td>
<td>Sweep-out volume</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

Cloud droplets initially grow due to condensation to approximately 10µm in diameter, after which growth due to collision and coalescence are the main mechanisms for droplet growth. Gravitational effects dominate for droplets with diameters greater than 50µm, in which they overpass smaller droplets due to an increase in their settling velocity. The collision and coalescence process for droplets with diameters ranging from 10 to 50µm, known as the size-gap, is still not fully understood. Nevertheless, atmospheric turbulence is known to increase the droplet growth rate in this range by enhancing the relative velocity between droplets and the formation of droplet clustering, thus, increasing the droplet collision rate (Jonas, 1996; Shaw, 2003).

The motivation for this thesis is to further understand the growth of droplets from 10 to 50µm in warm clouds by analyzing particle concentration characteristics within homogeneous isotropic turbulence. Specifically, we are interested in the relationship between flow structures and the distribution of particles, with the goal of extending this work to analyzing the distribution of droplets in future studies. Although the growth of cloud droplets is our motivation, we note the applications for understating particle dynamics in turbulent flows range a variety of topics beyond cloud physics. For instance, Cuzzi et al. (2001) and Pan et al. (2011) studied the relationship between turbulent flows and particle inertia with respect to collisional growth of protoplanetary dust in the formation of planetesimals. Zamanisky et al. (2016) analyzed the effects of particle clustering in turbulent flow on the heat transfer process from heated inertial particles, and De Lillo et al. (2014) studied the effects of turbulent flows on the movement and clustering of phytoplankton.

1.1 LITERATURE REVIEW

This section briefly introduces five main topics related to droplet growth in homogeneous isotropic turbulence:

1. particle/droplet motion
2. flow structures in homogeneous isotropic turbulence
3. preferential concentration of particles/droplets in homogeneous isotropic turbulence
4. increased settling and relative velocities of particles/droplets in turbulent flows
5. droplet collision and the growth rate of the droplet size distribution.
In these sections, the basic equations along with significant research done in these areas are highlighted. We begin by introducing the particle/droplet motion and flow structures in homogeneous isotropic turbulence to lay the foundation for understanding the governing equations for preferential concentration of particles/droplets. Preferential concentration is significant because it affects the particle/droplet settling velocities, relative velocities, and collision rates. Therefore, a brief overview of these topics is given for the reader to gain an understanding of how increased preferential concentration leads to an increase in droplet collision, and subsequently affects droplet growth.

1.1.1 Equations of Motion for Spheres in Flow Field

The Eulerian-Lagrangian approach is widely used in particle-laden flows, in which the motion of the particles is described in a Lagrangian reference frame and the carrier fluid is treated in an Eulerian reference frame (Hwang, 2004). The Navier-Stokes equations are used to describe the motion of the carrier fluid, and the motion of a spherical particle in a non-uniform, unsteady, incompressible flow field is given by Maxey & Riley (1983)

\[ m_p \frac{dV_{p_i}}{dt} = m_p g_i - m_f g_i + m_f \frac{Du_i}{Dt} - \frac{1}{2} m_f \frac{d}{dt} \left( V_{p_i}(t) - u_i - \frac{1}{10} a_p^2 \nabla^2 u_i \right) - 6 \pi a_p \mu \left( V_{p_i} - u_i - \frac{1}{6} a_p^2 \nabla^2 u_i \right) - 6 \pi a_p^2 \mu \int_0^t \left( \frac{\frac{d}{dt} \left( V_{p_i} - u_i - \frac{1}{6} a_p^2 \nabla^2 u_i \right)}{(\pi \nu (t - t'))^{0.5}} \right) dt', \]

where \( a_p \) is the radius of the sphere, \( m_p \) is the mass of the sphere, \( m_f \) is the mass of the displaced fluid, \( V_{p_i} \) is the velocity of the sphere, \( u_i \) is the far field fluid velocity, \( \nu \) is the kinematic viscosity of the fluid, \( g_i \) is the gravitational constant, and \( \mu \) is the dynamic viscosity of the fluid. Note that \( \frac{D}{Dt} \) is the material derivative of the carrier fluid, and \( \frac{d}{dt} \) is the material derivative with respect to the fluid particle. In Eq. (1.1) \( i = 1, 2, \) and 3 indicates the \( x_1, x_2, \) and \( x_3 \) directions in index notation. Forces due to the flow field result from both the undisturbed flow field and the flow established by the sphere. The first three terms on the right hand side of Eq. (1.1) represent the gravitational force, buoyancy force, and stresses generated by fluid acceleration, respectively. The final three terms on the right hand side result from the flow field established by the sphere: the added mass, Stokes drag, and the Basset force, respectively (Maxey & Riley, 1983). The final expression in these terms is known as the Faxén correction term. The Basset force represents the augmentation of the drag force due to the diffusion of vorticity generated by an accelerating particle (Hwang, 2004).
Chapter 1: Introduction

The equation of motion of the particle is simplified by assuming that the diameter of the particle, \(d_p\), is much smaller than the characteristic length scale of the flow. Thus, the Faxén correction term scales as \(a_p^2 \nabla^2 u_i \vert_{Y(t)} \sim O\left(\frac{a_p^2 U_o}{L^2}\right)\) and can be neglected (Bocanegra Evans, 2013), where \(U_o\) and \(L\) are the characteristic velocity and length scales of the flow. To further evaluate Eq. (1.1), we introduce the following order arguments, where we assume the velocity of the fluid and particle are of the same order of magnitude

\[
V_p \sim U_o \quad u_i \sim U_o \quad t \sim \frac{L}{U_o} \quad m_f \sim a^3_p \rho \quad m_p \sim a^3_p \rho_d \quad ,
\]

where \(\rho\) and \(\rho_d\) are the fluid and particle densities, respectively. Thus, the order of each term in Eq. (1.1) after the Faxén terms have been neglected is

\[
\frac{\rho_d a^3_p U^2_o}{L} \quad \text{(particle inertia)} \quad \rho a^3_p g \quad \text{(gravity)} \quad \rho a^3_p g \quad \text{(buoyancy)} \quad \rho a^3_p g \quad \text{(shear stress)} \quad \rho a^3_p g \quad \text{(added mass)} \quad - a_p \mu U_o \quad \text{(Stokes Drag)} \quad - \left(\frac{a^2_p U_o}{L^2}\right)^{0.5} \quad \text{(Basset force)} \quad .
\]

Scaling the terms of Eq. (1.2) with the Stokes drag term results in

\[
\frac{\rho_d a^2 U_o}{\rho L \nu} \quad \sim \frac{\rho a^2 g}{\rho \nu U_o} \quad - \frac{a^2 g}{\rho \nu U_o} \quad + \frac{a^2 U_o}{L \nu} \quad - \frac{a^2 U_o}{L \nu} \quad - \frac{1}{\rho a^2 U_o} \quad - \left(\frac{a^2 U_o}{L \nu}\right)^{0.5} \quad \text{(Basset force)} \quad .
\]

In the derivation of Eq. (1.1) we assume a low particle Reynolds number \(\left(\frac{a_p U_o}{\nu} \ll 1\right)\) and a low “shear Reynolds number condition” \(\left(\frac{a^2 U_o}{\nu L} \ll 1\right)\). A low particle Reynolds number allows for Stoke’s drag to be used. The shear Reynolds number condition results from assuming the velocity gradients \((U_o/L)\) in the undisturbed flow field are small, and allows us to neglect the added mass, fluid acceleration, and Basset terms. When \(\rho_d \gg \rho\) the buoyancy term is negligible compared to the gravitational term. The gravitational force is assumed comparable to the particle’s inertia and drag force when the settling parameter, \(S_f\), is near unity. The settling parameter is defined as

\[
S_f = \frac{\rho_d a^2 g}{\rho \nu U_o} \quad .
\]

Thus, Eq. (1.1) reduces to
\[
\frac{dV_p}{dt} = \frac{1}{\tau_p} (V_p - u_i) - g_i ,
\]
where \( \tau_p \) is the relaxation time, i.e. the time for a particle to respond to changes in the flow field,
\[
\tau_p = \frac{\rho d_p^2}{18 \mu} .
\]

1.1.2 Flow Structures in Homogeneous Isotropic Turbulence

In order to study the effects of homogeneous isotropic turbulence on particle dynamics, it is important to discuss the flow structures that are typically found in that flow field. Hunt & Wray (1990) introduced four classification zones for a turbulent flow field: eddy, shear (or rotational), convergence, and stream zones. Squires & Eaton (1991b) used these four classification zones to study the effects of flow structures on particle concentration using Direct Numerical Simulation (DNS). The eddy, shear, convergence, and stream zones signify regions of vortices, shear layers, stagnation points, and jets, respectively (Rouson & Eaton, 2001). These regions are identified through the pressure field, velocity field, and second invariant of the velocity gradient tensor. The second invariant of the velocity gradient tensor, \( \partial u_i/\partial x_j \), is defined as
\[
Q = \frac{1}{2} (\| S^2 \| - \| \Omega^2 \|) ,
\]
where \( S \) is the strain rate tensor and \( \Omega \) is the vorticity tensor. Eddy and shear zones are both characterized by \( Q < \frac{1}{2} Q_{rms} \), but are differentiated by their pressure levels. Eddy zones are identified as regions of low pressure, \( p < \frac{1}{2} p_{rms} \), whereas shear zones are regions of moderate pressure levels, \( \frac{1}{2} p_{rms} \leq p \leq p_{rms} \). Convergence zones are regions with high pressure, \( p > p_{rms} \), and high invariant \( Q > Q_{rms} \). Finally, stream zones are identified as regions having “weak deformation but substantial speed” (Hunt & Wray, 1990). Therefore, stream zones are identified as regions where \( -\frac{1}{2} Q_{rms} < Q < Q_{rms} \) and \( u'^2 > u_{rms}^2 \), where \( u_{rms} \) is the rms of the velocity fluctuations. Roughly 50% of the flow field fell into one of these four categories for both Hunt & Wray (1990) and Squires & Eaton (1991b), and the remainder of the flow field was unclassified.

A number of studies have further examined coherent structures in eddy zones. DNS models, most notably by Siggia (1981), Kerr (1985), Ruetsch & Maxey (1991), She et al. (1991), Vincent & Meneguzzi (1991), Jiménez et al. (1993), and Moisy & Jiménez (2004), have shown that regions of intense vorticity in homogeneous turbulent flows appear as tube-like structures, i.e. “worms”. These vortex tubes are found to have diameters ranging from \( \eta \leq D \leq \lambda \) with an average diameter of \( D \approx 6\eta \) and an average tube length of \( L \approx 2.5\lambda \).
where \( \eta \) is the Kolmogorov length scale and \( \lambda \) is the Taylor microscale. In comparison, moderate levels of vorticity tend to appear more sheet-like. Ruetsch & Maxey (1991) define moderate levels of vorticity as \( 5 \bar{\omega}^2 < \omega^2 < 9.5 \bar{\omega}^2 \) and high levels of vorticity as \( \omega^2 > 9.5 \bar{\omega}^2 \), where \( \omega^2 \) is the enstrophy of the flow field and \( \bar{\omega}^2 \) is the spatial average. Comparatively, Jiménez et al. (1993) defined high vorticity regions as those where vorticity is greater than a threshold which covers 1% of the flow field.

More recently, the velocity gradient tensor has been used to determine vortex structures in the flow field. Chakraborty et al. (2005) and Chen et al. (2015) evaluated four common methods based on the velocity gradient tensor to identify regions containing vortex structures: the Q-criteria, \( \lambda_2 \)-criteria, \( \Delta \)-criteria, and \( \lambda_{ci} \)-criteria, where the Q-criteria matches that of Hunt & Wray (1990). The \( \lambda_2 \)-criteria was introduced by Jeong & Hussain (1995) in an aim to combine the pressure and Q conditions in identifying eddy regions set by Hunt & Wray (1990). Both Chakraborty et al. (2005) and Chen et al. (2015) found the four methods agree well when a non-zero threshold is used.

Using DNS modeling, Squires & Eaton (1991b) and Wang & Maxey (1993) found that particles tend to avoid eddy and shearing zones, and concentrate in convergence zones and streaming zones between eddies. In particular, particles cluster in long sheets aligned with gravity (Wang & Maxey, 1993). Ruetsch & Maxey (1991) more specifically studied the effects of vortex tubes on the scalar gradients. They found that although “scalar gradient sheets” wrap around vortex tubes, these are not the regions of greatest intensity in the scalar gradient. Rather, regions of greatest changes appear as flat sheets away from the regions of high vorticity, and typically near sheets of moderate vorticity.

### 1.1.3 Preferential Concentrations in Homogeneous Isotropic Turbulence

When particles cluster in an incompressible flow field, they tend to avoid regions of high vorticity and concentrate in regions of high strain, as previously mentioned. This was originally presented by Maxey (1987) by integrating Eq. (1.5), resulting in

\[
V_p(t) = (V_p(0) - u(0))e^{-t/\tau_p} + u(t) - \frac{du}{dt}\tau_p + \frac{du}{dt}\tau_p e^{-t/\tau_p}|_{t=0} + \tau_p \int_{0}^{t} e^{(t' - t)/\tau_p} \frac{d^2u}{dt'^2} dt',
\]

where we have further simplified the integration of Eq. (1.5) presented by Maxey (1987) by neglecting gravity. The reader is directed to the original work of Maxey (1987) and Bocanegra Evans (2013) for a detailed analysis of this integration. Restricting our scope to \( t \gg \tau_p \), Eq. (1.8) reduces to

\[
V_p = u - \tau_p \frac{du}{dt}.
\]
Taking the divergence of Eq. (1.9) and accounting for the incompressibility of the flow field, \( \nabla \cdot u = 0 \), yields

\[
\nabla \cdot V_p = -\tau_p \nabla \cdot (u \cdot \nabla u) \quad .
\]

(1.10)

We see here that if the gravity term had been retained in solving for Eq. (1.8), it would have gone to zero when taking the divergence of Eq. (1.9). The right hand side of Eq. (1.10) is more easily expressed in index notation as

\[
\nabla \cdot (u \cdot \nabla u) = \frac{\partial}{\partial x_i} \left( u_j \frac{\partial u_i}{\partial x_j} \right) = \frac{\partial u_j}{\partial x_i} \frac{\partial u_i}{\partial x_j} = \frac{1}{4} \left[ \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 - \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)^2 \right] ,
\]

thus

\[
- \nabla \cdot V_p = \frac{\tau_p}{4} \left( S^2 - \Omega^2 \right) \quad .
\]

(1.12)

Similarly, we can express the change in the particle number density, \( n \), as

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n V_p) = 0 \quad ,
\]

(1.13)

or

\[
\frac{\partial n}{\partial t} + n \nabla \cdot V_p + (V_p \cdot \nabla) n = 0 \quad ,
\]

(1.14)

which can be rearranged as

\[
\frac{1}{n} \frac{Dn}{Dt} = -\nabla \cdot V_p \quad .
\]

(1.15)

Therefore, comparing Eq. (1.12) and (1.15) we can conclude

\[
\frac{1}{n} \frac{Dn}{Dt} = \frac{\tau_p}{4} \left( S^2 - \Omega^2 \right) \quad .
\]

(1.16)

When the final term in Eq. (1.15) is positive, the number density of particles increases, indicating that particles move from regions of high vorticity to high strain when clustering. Regions of high vorticity act as a centrifuge ejecting heavy particles \( (\rho_d \gg \rho) \). Heavy particles have greater inertia, i.e. greater \( \tau_p \), making it more difficult to follow the carrier fluid and
causing the particles to cluster in regions of high strain. This is known as preferential concentration \cite{sundaram1997}. Goto & Vassilicos \cite{goto2006, goto2008} state that this is not a sufficient argument alone to describe particle clustering because it does not explain the multi-scale phenomenon of particle clusters at higher levels of turbulence. Rather, they suggest that particles cluster with eddies of various scales, in which the Stokes number, $St$, defined as the ratio of $\tau_p$ and the time scale of the eddy, approaches unity. This multi-scale phenomenon of clustering has not been studied in much detail because in the majority of studies, the turbulence intensity is not sufficient to separate the scales of turbulence well. Goto & Vassilicos \cite{goto2008} have also proposed the sweep-stick mechanism. By assuming $St$ to be small, Eq. (1.9) can be approximated as

$$V_p \approx u - \tau_p a.$$ (1.17)

Particles located where $a \approx 0$ tend to move together with velocity $u$, whereas particles located in the flow field where $a \neq 0$ move away with relative velocity, $a \tau_p$. In this case, as long as $\tau_p$ is less than the time scale of the points in the carrier flow where $a \approx 0$, particles will stick to these regions \cite{sumbekova2016}. Falkovich et al. \cite{falkovich2002} introduced the sling-shot effect in which droplets are ejected due to a curved streamline from excessive centrifugal acceleration. Thus, particles deviate from the underlying flow field as a jet of particles contributing to an increased rate in particle collisions.

Experimental studies of particle dispersion in homogeneous isotropic turbulence have been performed using both grid turbulence in wind tunnels and turbulence boxes. A turbulence box is constructed by replacing the 8 corners of a cube with either synthetic jet actuators or fans. Recently, advances have been made to create a isocahedron shaped box to allow for greater homogeneity. Experiments on particle dispersion performed using wind tunnels include Aliseda et al. \cite{aliseda2002}, Saw et al. \cite{saw2008}, Monchaux et al. \cite{monchaux2010}, and Saw et al. \cite{saw2012b}. Those performed using a turbulence box include Wood et al. \cite{wood2005}, Salazar et al. \cite{salazar2008}, Bocanegra Evans \cite{bocanegra2013}, and Yavuz \cite{yavuz2016}. A number of experiments performed on clustering in particle-laden homogenous isotropic turbulent flows have been laid out in Table 1.1. DNS models have also been used to study particle concentration in homogeneous isotropic turbulence including Squires & Eaton \cite{squires1991a, squires1991b}, Wang & Maxey \cite{wang1993}, Sundaram & Collins \cite{sundaram1997}, Reade & Collins \cite{reade2000}, Yoshimoto & Goto \cite{yoshimoto2007}, Saw et al. \cite{saw2012a}, Saw et al. \cite{saw2012b}, and Baker et al. \cite{baker2017}. In all listed experimental and numerical studies, the two main assumptions are: $\rho_d/\rho \gg 1$ and $d_p \ll \eta$.

The particle number density is compared to a Poisson distribution in some manner in all studies. Fessler et al. \cite{fessler1994}, Aliseda et al. \cite{aliseda2002}, and Wood et al. \cite{wood2005} use a box counting method, in which the particle number density is found by discretizing the image into windows and counting the number of particles per window. Two parameters are used to quantify clustering, $D_1$ and $D_2$. The first parameter was introduced by Fessler et al. \cite{fessler1994}
Chapter 1: Introduction

\[ D_1 = \frac{\sigma - \sigma_{\text{binom}}}{\lambda}, \quad (1.18) \]

where \( \lambda \) is the mean number of particles per window, \( \sigma \) is the standard deviation of the number of particles per window, and \( \sigma_{\text{binom}} \) is the standard deviation following a binomial distribution. The second parameter introduced by Wang & Maxey (1993) is

\[ D_2 = \sum_{n=1}^{N_p} (P(n) - P_{\text{binom}}(n))^2, \quad (1.19) \]

where “\( P(n) \) is the probability of finding \( n \) particles in a box” (Aliseda et al., 2002), and \( P_{\text{binom}}(n) \) is the number of particles found in a box following a binomial distribution. Similarly, Monchaux et al. (2010) quantified clustering by comparing the probability density function (PDF) of the Voronoi area for a particle-laden flow to a Poisson distribution. This is similar to the box counting method, but does not assume a prescribed area. This method is further discussed in Chapter 3.
<table>
<thead>
<tr>
<th>Researcher</th>
<th>Flow Facility</th>
<th>$Re_\lambda$</th>
<th>Measurements</th>
<th>Cluster Characterization</th>
<th>Instrumentation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aliseda et al. (2002)</td>
<td>wind tunnel</td>
<td>48-88</td>
<td>length scale &amp; settling velocity of clusters</td>
<td>particle number density</td>
<td>PDA &amp; flow visualization</td>
</tr>
<tr>
<td>Fessler et al. (1994)</td>
<td>downward channel flow</td>
<td>$Re_h \sim 13800$</td>
<td>length scale of clusters</td>
<td>particle number density</td>
<td>flow visualization</td>
</tr>
<tr>
<td>Monchaux et al. (2010)</td>
<td>wind tunnel</td>
<td>72-114</td>
<td>length scale of clusters &amp; voids</td>
<td>Voronoi analysis</td>
<td>flow visualization</td>
</tr>
<tr>
<td>Salazar et al. (2008)</td>
<td>Eaton box</td>
<td>110-149</td>
<td>length scale of clusters</td>
<td>RDF</td>
<td>3D holographic imaging</td>
</tr>
<tr>
<td>Saw (2008)</td>
<td>wind tunnel</td>
<td>440-660</td>
<td>length scale of clusters</td>
<td>RDF</td>
<td>1D PDA</td>
</tr>
<tr>
<td>Sumbekova (2016)</td>
<td>wind tunnel</td>
<td>172-447</td>
<td>length scale and dynamics of clusters &amp; particle settling velocity</td>
<td>Voronoi analysis</td>
<td>PTV and flow visualization</td>
</tr>
<tr>
<td>Wood et al. (2005)</td>
<td>Eaton box</td>
<td>228-233</td>
<td>length scale of clusters</td>
<td>RDF &amp; particle number density</td>
<td>flow visualization</td>
</tr>
<tr>
<td>Yavuz (2016)</td>
<td>Eaton box</td>
<td>155-388</td>
<td>length scale of clusters &amp; settling velocity</td>
<td>RDF</td>
<td>PTV &amp; PDA</td>
</tr>
</tbody>
</table>

Table 1.1: Summary of experiments focused on particle clustering in particle-laden homogeneous isotropic turbulent flow.

Alternatively, Sundaram & Collins (1997), Wood et al. (2005), Saw et al. (2008), Saw (2008), Salazar et al. (2008), and Yavuz (2016) have characterized particle clustering using the radial distribution function (RDF), $g(r)$. The RDF is the probability of finding a neighboring particle to a reference particle within a spherical shell between a radius, $r_i$, and $r_i + dr$ (Reade & Collins, 2000). McQuarrie (2000) defined $g(r)$ as “the factor that multiplies the bulk density $n$ to give a local density $n(r) = ng(r)$”. From this definition, we see that as $r \to \infty$ the local density should approach that of the bulk density, and thus, $g(\infty) \to 1$. Therefore, $g(r) \geq 1$ represents particle clustering at a scale $r$. For a monodisperse particle-laden flow the 2D RDF is calculated as
\[ g(r) = \frac{N_i / \Delta A_i}{N/A} \]  
where \( N_i \) is the number of particle pairs within an area, \( \Delta A_i \). \( N \) is the total number of particle pairs in area, \( A \), which is calculated by

\[ N = \frac{M(M - 1)}{2} \]  

where \( M \) is the total number of particles. Wood et al. (2005) performed these calculations using 2D planar images. Aliseda et al. (2002) and Saw (2008) calculated a 1D RDF by estimating time of arrival of each droplet using a phase Doppler anemometry (PDA) system. By using a PDA system, they were able to calculate the RDF for particular Stokes numbers. Salazar et al. (2008) used a 3D holographic particle image velocimetry (PIV) technique and Yavuz (2016) used 3D particle tracking velocimetry (PTV) in order to calculate the RDF with the 3D location of particles.

In general, the findings from these studies agree that particles cluster most efficiently when \( St_\eta \approx 1 \), where the Stokes number is defined as the ratio between the relaxation time and the Kolmogorov time scale, \( \tau_\eta \).

\[ St_\eta = \frac{\tau_p}{\tau_\eta} \]  

Additionally, it is generally concluded that clustering increases with increasing turbulence intensity, and particle clusters tend toward a length of approximately \( 10\eta \). Squires & Eaton (1991b) and Wang & Maxey (1993) found that the probability of finding a void region doubled when comparing a particle-laden flow with \( St \approx 1 \) to a Poisson distribution. When \( St \ll 1 \), particles have little inertia, and thus, continue to follow the turbulent flow field rather than cluster. However, when \( St \gg 1 \) the particles again become more randomly distributed because they have greater inertia, and do not respond to the turbulent velocity fluctuations (Aliseda et al., 2002).

In the work by Sumbekova (2016), however, they found that the length scales associated with clusters and voids have little dependency on the \( St \), but rather are dependent on the Reynolds number and particle volume fraction, \( \Phi_v \),

\[ \Phi_v = \frac{MV_p}{V} \]  

where \( V_p \) is the volume of a single particle and \( V \) is the total volume of the fluid domain. Both Aliseda et al. (2002) and Sumbekova (2016) found that increasing the particle concentration leads to decreasing cluster sizes. However, Aliseda et al. (2002) also found that an increase
in $\Phi_n$ leads to an increase in the number and density of particle clusters. Aliseda et al. (2002) and Sumbekova (2016) attribute the effects of $\Phi_n$ on particle concentration to particle interactions.

With recent increased efforts to incorporate preferential concentration in numerical models, attention has also been given to modeling the RDF of particles based on the Stokes and Reynolds numbers, including the work by Reade & Collins (2000), Chun et al. (2005), and Saw et al. (2012a). A power-law fit is the most common form to describe the RDF of particle concentration, such that

$$g(r) \propto c \left( \frac{R}{r} \right)^b,$$

where both $c$ and $b$ are possible functions of $St_\eta$ and $Re_\lambda$. Incorporating preferential concentration in numerical models is desired because it has implications on the particle collision rate, settling velocities of particles, and turbulence modulation. The first two of these will be discussed in the subsequent sections.

### 1.1.4 Settling and Relative Velocities in Homogeneous Isotropic Turbulence

Wang & Maxey (1993) and Yang & Lei (1998) studied the effects of homogeneous isotropic turbulence on particle settling velocities, or terminal velocities, using DNS modeling. To do so, Wang & Maxey (1993) defined the turbulent settling speed difference as

$$\Delta V = \frac{V_p - V_{air}}{u'},$$

where $V_{air}$ is the terminal settling velocity of a particle in still-air, i.e. for a sphere

$$V_{air} = \frac{\rho_d d_p^2 g}{18 \mu}.$$

Positive $\Delta V$ indicates an enhanced settling velocity. Experimental work by Aliseda et al. (2002), Zhou & Cheng (2009), Good et al. (2014), Sumbekova (2016), and Yavuz (2016) used the same parameter to study the effects of turbulence on the settling velocity of particles. Aliseda et al. (2002) and Sumbekova (2016) used a PDA system to measure the change in the settling velocity of droplets in a wind tunnel. In comparison, Yavuz (2016) and Good et al. (2014) used particle tracking techniques to measure the settling velocities of droplets in a turbulence box. Finally, Zhou & Cheng (2009) used a “shadow processing” PIV technique to study the effects of local flow structures on the enhanced settling velocities, where turbulence was generated by an oscillating grid.
In general, it has been found that turbulence enhances the settling velocity of particles, and is maximized when \( St \approx 1 \) and \( 1 < \frac{V_{\text{air}}}{u_\eta} < 3 \), where \( u_\eta \) is the Kolmogorov velocity. Wang & Maxey (1993) attribute the increase in settling velocity to the concentration of particles on the peripheral, downflow sides of vortices. Particles typically encounter vortices from above, thus, causing them to be swept downward. This is known as preferential sweeping. The inertia of the particles causes them to cluster to the periphery of vortices, as previously discussed. Aliseda et al. (2002) has also suggested that the motion of the clusters themselves must be accounted for with regards to the increased settling velocities of particles located within the clusters, which is supported by the results of Sumbekova (2016).

Yang & Lei (1998) argued that although the importance of small scales in turbulence are significant with respect to enhanced settling velocities via particle clustering, the large scales of turbulence impact the settling velocities through the drag force. In support of this, Wang & Maxey (1993) and Good et al. (2014) found numerical evidence of reduced settling velocities of particles in turbulent flow when non-linear drag forces were used. The settling velocity can also be reduced by the loitering effect. This effect occurs when the settling particle travels through the carrier fluid without modification of its trajectory because its settling velocity is too great to be modified significantly by the carrier fluid. Thus, the particle settles against the general motion of the carrier fluid (Nielsen, 1993; Sumbekova, 2016).

In general, the relative velocity between particles is also found to increase with increasing turbulence levels. In the limiting case where \( St \rightarrow 0 \), i.e. when the particles faithfully follow the turbulent fluctuations of the carrier fluid, the relative velocity between particles increases with increasing turbulence levels because the turbulent velocity fluctuations also increase. As \( St \) increases, the finite inertia of the particles complicates the relative velocity because the particles no longer respond perfectly to the carrier fluid and begin to cluster, as previously discussed. Sundaram & Collins (1997) and Wang et al. (2000) studied the change in the relative velocity for various flow conditions using DNS modeling. Both studies concluded that the maximum relative velocity between particles occurs when \( \tau_p \approx T_e \), where \( T_e \) is the large eddy turnover time. When \( \tau_p < T_e \), the relative velocity increases with increasing \( St \) because the inertia of the particles increases, and thus, the velocity of the particles become less correlated with the carrier fluid and surrounding particles. However, this also leads to particles clustering, meaning more particles are subject to similar features in the carrier fluid. When \( \tau_p > T_e \), the particles begin to respond more slowly to the turbulent velocity fluctuations, and thus, the relative velocity between particles decreases.

### 1.1.5 Collision Rate and the Droplet Size Distribution

Increases in preferential concentration and relative velocities between droplets due to turbulent fluctuations increases the collision rate of particles. A collision occurs when one particle with a greater relative velocity overtakes another in the sweep-out volume. The sweep-out volume can be described as either cylindrical or spherical as indicated in Figure 1.1.
Chapter 1: Introduction

A cylindrical sweep-out volume is defined as $\forall_{cyl} = \pi (r_1 + r_2)^2 |W|$, and a spherical sweep-out volume is defined as $\forall_{sph} = 2\pi (r_1 + r_2)^2 |W_r|$, where $|W|$ is the total relative velocity and $|W_r|$ is the radial component of the relative velocity between particles. The collision rate using a cylindrical sweep-out volume is

$$N_c = \pi (r_1 + r_2)^2 n_1 n_2 \int |W| P(W, r) dW ,$$  \hspace{1cm} (1.27)

where $n_i$ is the particle number density with a radius $r_i$, and $P(W, r)$ is the particle pair probability distribution (Sundaram & Collins, 1997). Sundaram & Collins (1997) modified Eq. (1.27) to account for particle clustering and the relative velocities using two distribution functions: the RDF of droplets, $g(r)$, and the conditional relative velocity probability density function, $P(W | r)$.

$$N_c = \pi (r_1 + r_2)^2 n_1 n_2 g(r) \int |W| P(W | r) dW$$  \hspace{1cm} (1.28)

The average collision rate can be described as

$$\langle N_c \rangle = n_1 n_2 K(r_1, r_2) ,$$  \hspace{1cm} (1.29)

where $K(r_1, r_2) = \pi (r_1 + r_2)^2 \langle |W| \rangle g(r)$ is the collision kernel. Similarly, if a spherical sweep-out volume is used, the collision kernel is $K(r_1, r_2) = 2\pi (r_1 + r_2)^2 \langle |W| \rangle g(r)$. Wang et al. (2000) found that cylindrical collision kernels tend to over predict the collision rate up to 22% for $St \to 0$. However, this over prediction decreases with increasing Stokes number. The collision kernel can also be multiplied by the collision efficiency, $E_{coll}$, and coalescence efficiency, $E_{coal}$. The collision efficiency is the probability that two droplets
collide when located in the sweep-out volume accounting for hydrodynamic effects. The coalescence efficiency is the probability that two droplets coalesce once they have collided. Both of these coefficients are typically taken to be 1. Saffman & Turner (1956) first analyzed the collision rate due to turbulent velocity fluctuations assuming the relative velocity has a Gaussian distribution and that the particles move with the air velocity, i.e. $St = 0$, resulting in

$$N_c = \pi (r_1 + r_2)^3 n_1 n_2 \left( \frac{8\pi \epsilon}{15\nu} \right)^{1/2},$$

(1.30)

where $\epsilon$ is the energy dissipation rate. Abrahamson (1975) analyzed the collision rate for droplets, where $St \to \infty$, i.e. the droplets have a large inertia and do not respond to the carrier fluid, resulting in

$$N_c = (r_1 + r_2)^2 n_1 n_2 \left( \frac{16\pi \bar{v}_p^2}{3} \right)^{1/2},$$

(1.31)

where $\bar{v}_p$ is the variance of the particle velocity distribution.

Few experimental studies have been performed to evaluate the collision rate for droplets in turbulent air flow. Early works by Woods & Mason (1964), Woods & Mason (1965), and Jonas & Goldsmith (1972) studied the collision and coalescence efficiency. More recently, Bordás et al. (2013) measured the collision rate for droplets in a grid-generated turbulent flow using shadowgraphy and PTV. A collision is assumed to occur with PTV when the trajectories of two droplets merge. Shadowgraphy, however, assumes a collision has occurred when the eccentricity of a droplet exceeds a threshold level while accounting for aerodynamic deformation (Bordás et al., 2010). Bordás et al. (2013) concluded that although the number of collisions estimated by PTV and shadowgraphy were on the same order of magnitude, shadowgraphy over predicts and PTV under predicts the number of collisions. In general, the experimental collision rates from Bordás et al. (2013) were within a factor of 10 compared with theoretical prediction by Ayala et al. (2008), Pinsky et al. (2008), and Wang & Grabowski (2009). However, more experimental work is needed to compare with current numerical and theoretical predictions of droplet collision rates.

Recently, the majority of work on collision rates for particles in homogeneous isotropic turbulence has been done using DNS modeling including Sundaram & Collins (1997), Wang et al. (2000), Reade & Collins (2000), Zhou et al. (2001), Franklin et al. (2005), Falkovich & Pumir (2007), Dallas & Vassilicos (2011), and Onishi & Vassilicos (2014). Sundaram & Collins (1997) were the first to account for both the effects of increased relative velocities and enhanced particle concentration, as previously mentioned. They found that although particle clustering is dominant at $St_\eta \approx 1$, the maximum collision rate occurs at $St_\eta \approx 4$ because the increase in relative velocities between particles is scaled by $T_e$. Wang et al.
(2000), similarly, found for low levels of turbulence that the maximum collision rate occurs at \( \tau_\eta > t > T_e \), due to a high overlap in the effects of clustering and increased relative velocities. However, as the turbulence level increases, there are two peaks in the collision rate. The first peak occurs at \( \tau_p/\tau_\eta \sim 1 \) due to clustering effects, and the second peak at \( \tau_p/T_e \sim 1 \) due to the increase in relative velocities between particles. They also concluded that the effects from particle clustering were dominant compared to the effects of increased relative velocities. Franklin et al. (2005) found that at low \( Re \), the effects of preferential concentration on the collision rate dominated. As \( Re \) increased, the effects from preferential concentration and increased relative velocities on the collision rate were equal.

The motivation for studying the collision rate between droplets is to ultimately analyze the growth rate of the droplet size distribution, \( f(m) \). Studies examining the growth rate of the droplet size distribution with increases in the collision rate include Xue et al. (2008), Franklin (2008), Pinsky et al. (2008), Wang & Grabowski (2009), Dallas & Vassilicos (2011), and Onishi & Vassilicos (2014). The time rate of change in the droplet size distribution due to droplet collision is typically modeled using the Smoluchowski equation (Pinsky & Khain, 1997)

\[
\frac{\partial f(m_1, t)}{\partial t} = \int_0^{m_1} K(m_1 - m_2, m_2) f(m_1 - m_2) f(m_2) dm_2 - f(m_1) \int_0^\infty f(m_2) K(m_1, m_2) dm_2,
\]

where \( f(m) \) is the number of droplets with a mass \([m, m + dm]\) per unit volume and \( K \) is the collision kernel. The first term on the right hand side represents the increase in particles with a mass \( m_1 \), due to the collision and coalescence of particles with a mass \( m_2 \) and \( m_1 - m_2 \). The second term on the right hand side represents the decrease in the number of particles with a mass, \( m_1 \), because of its collision and coalescence with particles with a mass, \( m_2 \). Onishi & Vassilicos (2014) calculated the time rate of change in the droplet size spectrum by applying a 2D form of Eq. (1.30) to the Smoluchowski equation, and found that it matched the rate of change in the droplet size spectrum using a Langrangian DNS model. As expected, the previously listed studies find that increases in turbulence levels lead to a greater growth rate in the droplet size distribution, especially at smaller droplet sizes. Again more experimental studies are needed in this field. We also suggest that possible studies on the collision and coalescence between more than simply two particles/droplets may be of interest to future researchers, which has yet to be explored.

1.2 Thesis Purpose

The focus of the current thesis is to analyze the geometric properties of voids and clusters, and their spatial distribution in the flow field. Specifically, we aim to analyze if particle voids and clusters are spatially distributed based on the presence of specific turbulent flow
structures, complementing the numerical work of Squires & Eaton (1991b) and Wang & Maxey (1993). Through this analysis, we comment on the following questions:

1. What are the characteristic length scales used to describe clusters and voids?
2. Are regions of particle voids most highly correlated to eddy zones?
3. Do particles tend to cluster in the stream zones, and in particular stream zones in the downward direction?
4. How are particles typically distributed in convergence zones with converging/diverging streamlines?
5. Do we observe an increase in correlation between particle distribution and turbulent flow structures with increasing $Re_\lambda$, and thus, increasing $St_\eta$?

Planar PIV measurements are used to calculate the turbulence parameters, to verify that the flow field is homogeneous and isotropic, and to identify flow structures. Cluster and void geometric characteristics are analyzed using a Voronoi area PDF. This method has been chosen because it analyzes particle voids in addition to particle clusters, and identifies particle clusters and voids in the flow field itself rather than in terms of a global analysis. Thus, allowing us to study the relationship between the distribution of particles with respect to turbulent flow structures.
CHAPTER 2

CHARACTERIZATION OF THE TURBULENT FLOW FIELD

In this chapter we characterize the carrier flow field. We begin by introducing the turbulence chamber used to generate homogeneous isotropic turbulence. This is followed by a description of the 2D particle image velocimetry (PIV) system, which is used to measure the turbulence parameters describing the flow field. Finally we analyze our PIV measurements, specifically, the turbulence parameters are presented and the flow is shown to be isotropic and homogeneous in the region of interest (ROI).

2.1 Turbulence Chamber/Eaton Box

The turbulence chamber used in these experiments, shown in Figure 2.1, is based on the work of Hwang & Eaton (2004). The motivation for using this design is that the turbulent kinetic energy (TKE) does not decay over time because kinetic energy is continuously injected into the flow, without introducing mass flux. Thus, the energy dissipation, $\epsilon$, does not decay as seen in the grid generated turbulence in wind tunnels. This design has been utilized by Wood et al. (2005), De Jong et al. (2009), Bocanegra Evans (2013), Good et al. (2014), Bertens et al. (2015), and Yavuz (2016).

The turbulence chamber is a $40 \times 40 \times 40\text{cm}^3$ acrylic box with the 8 corners replaced by synthetic jet actuators with a frequency response of 40Hz to 10kHz. The jet actuators are driven by in-phase sine waves generated using a PCI-2517 board (Measurement Computing, 16-bit resolution analog input/output board), which drives two amplifiers. Forcing frequencies of 80 to 110Hz and peak-to-peak voltage amplitudes of 2, 4, 6, and $7V_{pp}$ are used to drive the synthetic jet actuators, speakers, in our case. In front of each synthetic jet actuator is a perforated metal screen with a spacing of 0.47cm, holes with diameters of 0.38cm, and a solidity of 0.11.
Figure 2.1: Turbulence chamber used to generate homogeneous isotropic turbulence.
2.2 \textbf{Particle Image Velocimetry (PIV)}

2.2.1 PIV Set-up

The general set-up of the PIV system is shown in Figure 2.2. An IDT Y3.2-S1 camera with a $1024 \times 1024$ pixel array is used with a 50mm Canon lens ($f:1.4$). The ROI is located at the center of the turbulence box and is $10 \times 10\text{cm}^2$. A laser sheet at the center of the turbulence box is produced by passing the laser beams through a spherical ($f = 400\text{mm}$) and cylindrical ($f = -9.7\text{mm}$) lens system. The laser sheet is approximately 1.5mm thick. The set-up of the lens, mirror, laser, and camera is displayed in Figures 2.2 and 2.3. The cylindrical and spherical lenses are cemented together, which is shown in Figure 2.4.

Figure 2.2: Set-up of PIV system with the CW laser shown. The CW laser was interchangable with the ND-YAG laser.
Two sets of PIV measurements are analyzed to describe the turbulent flow field produced in the turbulence chamber. The first set of PIV measurements are performed with a continuous wave Krypton ion laser at a frame rate of 3,750Hz for 10,000 frames with an exposure of 200µs in single-trigger mode. The second set of PIV measurements are performed with an ND-YAG laser ($\lambda = 532$nm) in double-trigger mode. The camera is synced to the lasers, and the pulse delay for each image pair ranges from $\Delta t = 30\mu s$ to $\Delta t = 200\mu s$ depending on the turbulence level, in which the pulses are synchronized using a CIO-CTR10 counter.
board (10 channel counter, 0.2\(\mu\)s resolution). The time between image pairs is 30ms, and 498 images are taken. For each test case five sets of PIV measurements are performed for a total of 1,245 image pairs per test case.

The first set of PIV measurements with the continuous laser are used in characterizing the flow for forcing frequencies \(f = 80, 90, 100,\) and \(110\)Hz, and a forcing amplitude of \(A = 2V_{pp}\). The time resolution of the camera is not sufficient to analyze images of higher forcing amplitudes. The second set of PIV images, taken with the YAG laser, are used to characterize the flow subject to a forcing frequency of \(f = 100\)Hz and forcing amplitudes of \(A = 2, 4, 6,\) and \(7V_{pp}\).

Aluminum-oxide particles \((\rho_{Al_2O_3} = 3750 \ \text{kg/m}^3)\) with a mean diameter of \(d_p = 0.5\mu\text{m}\) are used because these particles have a low Stokes number, and thus, follow the flow field faithfully. Therefore, the PIV results from these particles are used to analyze the turbulence chamber. Particles are initially loaded to a two valve particle discharge system, which is shown in Figure 2.5. Pressurized air (60 psi) is then supplied to the system by opening a solenoid valve, and subsequently particles are released by opening a pneumatic valve. To activate each valve, a 5 Volt DC signal from the PCI-2517 board is sent to a digital relay board (CIO-ERB08), which then sends a 24 Volt DC signal to the valve. A timing diagram of the process is seen in Figure 2.6. Note the camera is also triggered with a 5 Volt DC signal from the PCI-2517 board for the PIV measurements taken with the continuous laser.

![Figure 2.5: Particle discharge system.](image-url)
The PIV processing code, WALPT5, developed in-house is used to evaluate the velocity flow field from the image pairs. The window and step sizes used are 32 pixel $\times$ 32 pixel and 16 pixel $\times$ 16 pixel, respectively (50% overlap), and a Langrangian parcel tracking mode with two passes is used for PIV analysis.

2.2.2 TURBULENCE PARAMETERS

A planar PIV system is used to characterize the turbulent flow field created by the synthetic jet actuators. From the measured velocity field we calculate: the energy dissipation rate, Kolmogorov length scale, Kolmogorov time scale, Taylor microscale, structure functions, and energy spectrum.

De Jong et al. (2009) compared multiple methods to calculate the energy dissipation rate including: the direct method, the structure function fitting, and the Large-Eddy PIV technique. Assuming the flow is isotropic and homogeneous, the energy dissipation rate can be directly calculated for a 1D measurement as

$$\epsilon = 15\nu \left\langle \frac{\partial u_1}{\partial x_1} \right\rangle^2 = \frac{15\nu u'^2}{\lambda^2}, \quad (2.1)$$

or for 2D measurements as

$$\epsilon = 4\nu \left[ \left\langle \frac{\partial u_1^2}{\partial x_1} \right\rangle + \left\langle \frac{\partial u_2^2}{\partial x_2} \right\rangle + \left\langle \frac{\partial u_1}{\partial x_1} \frac{\partial u_2}{\partial x_2} \right\rangle + \frac{3}{4} \left\langle \left( \frac{\partial u_2}{\partial x_1} \frac{\partial u_2}{\partial x_1} \right)^2 \right\rangle \right] \quad (2.2)$$

if the spatial resolution is sufficient. Note that in this thesis, angle brackets denote an ensemble average and overbar denotes a spatial average. However, for the current study
the resolution is not sufficiently resolved to directly calculate the energy dissipation rate. Therefore, $\epsilon$ is estimated using the structure function fitting and Large-Eddy PIV techniques for the current work.

The energy dissipation rate is alternatively estimated using the second order, longitudinal structure function, $D_{LL}$, as

$$
\epsilon = \left[ \frac{D_{LL}}{C_2} \right]^{3/2} \left( \frac{1}{r_{IR}} \right),
$$

(2.3)

where $C_2$ is the universal constant and $r_{IR}$ is the length scale in the inertial subrange. As $Re \to \infty$, $C_2 = 2.12$ (Pope, 2000), which is a valid approximation for the levels of turbulence seen in this study (De Jong et al., 2009). The second order structure function is defined as

$$
D_{LL} = \left[ u_i(x + r) - u_i(x) \right] \left[ u_j(x + r) - u_j(x) \right],
$$

(2.4)

which reduces to

$$
D_{LL} = 2u' \Delta^2 \left( 1 - f_{11} \right),
$$

(2.5)

where $f_{11}$ is the first order longitudinal structure function in the direction of $u'$ (Batchelor, 1953; Pope, 2000).

Finally, the energy dissipation rate also can be calculated using the Large-Eddy PIV technique. This assumes that the velocity field from PIV measurements show a low-pass filtered velocity field with the “resolved” length scale, $\Delta$, equal to the PIV interrogation cell size (Sheng et al., 2000; De Jong et al., 2009). Thus, using the velocity field from the PIV measurements, $\epsilon$ is calculated as

$$
\epsilon (x_1, x_2) = -2 \langle \tau_{ij} \bar{S}_{ij} \rangle.
$$

(2.6)

The sub-grid scale (SGS) stress, $\tau_{ij}$, is typically modeled using the Smagorinsky method, which is calculated as

$$
\tau_{ij} = -C_s^2 \Delta^2 \bar{S}_{ij},
$$

(2.7)

where $C_s = 0.12$ is the corrected Smagorinsky constant, and

$$
|\bar{S}_{ij}| = \sqrt{2 \bar{S}_{ij} \bar{S}_{ij}}.
$$

(2.8)
Chapter 2: Characterization of the Turbulent Flow Field

Evoking the continuity equation and the assumption of isotropic flow, all three-dimensions are accounted for in the evaluation of $\epsilon$, where $\bar{S}_{ij}\bar{S}_{ij}$ reduces to

$$\bar{S}_{ij}\bar{S}_{ij} = 2 \left( \frac{\partial U_1}{\partial x_1} \right)^2 + 2 \left( \frac{\partial U_2}{\partial x_2} \right)^2 + 2 \left( \frac{\partial U_1}{\partial x_1} \frac{\partial U_2}{\partial x_2} \right) + \frac{3}{2} \left( \frac{\partial U_1}{\partial x_2} + \frac{\partial U_2}{\partial x_1} \right)^2 , \quad (2.9)$$

where $U_1$ and $U_2$ are the filtered velocity components calculated from the PIV analysis. For a more extensive discussion on calculating $\epsilon$ from PIV measurements the reader is directed to the work of Sheng et al. (2000), De Jong et al. (2009), Xu & Chen (2013), and Bertens et al. (2015). The Kolmogorov length scale, Kolmogorov time scale, and Taylor microscale are calculated from the spatially averaged energy dissipation rate as

$$\eta = \left( \frac{\nu^3}{\bar{\epsilon}} \right)^{\frac{1}{4}} , \quad (2.10)$$

$$\tau_\eta = \left( \frac{\nu}{\bar{\epsilon}} \right)^{\frac{1}{2}} , \quad (2.11)$$

$$\lambda = \left( \frac{15\nu u'^2}{\bar{\epsilon}} \right)^{\frac{1}{2}} . \quad (2.12)$$

By assuming isotropic flow we can define the turbulent kinetic energy as (Hwang & Eaton, 2004)

$$q^2(x_1, x_2) = 3 \frac{u'^2_1 + u'^2_2}{2} = 3 \left( \frac{u'^2_1 + u'^2_2}{2} \right) \quad (2.13)$$

from which the integral length scale and large-eddy turnover time can be approximated, respectively, as

$$L_E = \left( \frac{\bar{q}^2/3}{\bar{\epsilon}} \right)^{3/2} \quad (2.14)$$

and

$$T_E = \frac{L_E}{u'} = \frac{\bar{u}^2}{\bar{\epsilon}} . \quad (2.15)$$

The Taylor microscale Reynolds number is defined as
\[ Re_\lambda = \frac{\bar{u}' \lambda}{\nu} = \sqrt{\frac{15}{\bar{\epsilon} \nu}} u'^2. \]  

(2.16)

The first order structure functions are calculated from the \(u\)-component of the fluctuating velocity field as

\[ f_{11} = \frac{u'_p(x)u'_p(x+r)}{u'^2_p}, \quad (2.17) \]

\[ g_{11} = \frac{u'_n(x)u'_n(x+r)}{u'^2_n}, \quad (2.18) \]

where \(u'_p\) is the velocity component parallel to the \(x\)-direction, and \(u'_n\) is the velocity component normal to the \(x\)-direction. Note that the first order structure functions can also be calculated from the \(v\)-component of the velocity field. Finally, the 2D radial energy spectrum function, \(E(k)\), is calculated from the PIV measurements following Liu et al. (1994, 1999) by integrating the 2D energy spectral density, \(\Phi_{ij}(k)\), in rings with a radius \(k\)

\[ \Phi_{ij}(k) = \left( \frac{1}{2\pi} \right)^2 \int R_{ij}(r)e^{-ik \cdot r}dr, \]

(2.19)

\[ E(k) = \frac{1}{2} \int_0^{2\pi} \Phi_{ij}(k)kdk \theta, \quad (2.20) \]

where \(R_{ij}\) is the velocity correlation tensor.

### 2.3 Turbulence Characterization

The averaged turbulence parameters for forcing amplitudes of 2, 4, 6, and 7\(V_{pp}\), and a forcing frequency of 100Hz are presented in Table 2.1, which are calculated with 1,245 independent PIV measurements. The images used in Table 2.1 are taken with an ND-YAG laser illumination source and using particles with \(d_p = 0.5\mu m\), as previously described. In Table 2.1, \(\epsilon\) is calculated with the Large-Eddy PIV technique following the work of Sheng et al. (2000) and De Jong et al. (2009).
Chapter 2: Characterization of the Turbulent Flow Field

Table 2.1: Turbulence parameters calculated for $d_p = 0.5\mu m$ particles under various forcing amplitudes with a forcing frequency of $f = 100$Hz, where the Large-Eddy PIV technique with the corrected Smagorinsky constant is used to calculate $\epsilon$. A YAG laser is used for illumination.

<table>
<thead>
<tr>
<th>A ($V_{pp}$)</th>
<th>$\bar{\epsilon}$ (m$^2$/s$^3$)</th>
<th>$\tau_\eta$ (ms)</th>
<th>$\lambda$ (mm)</th>
<th>$R\epsilon_{\lambda}$</th>
<th>$\eta$ (µm)</th>
<th>$u'$ (m/s)</th>
<th>$v'$ (m/s)</th>
<th>$q^2$ (m$^2$/s$^2$)</th>
<th>$T_e$ (ms)</th>
<th>$L_e$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.55</td>
<td>2.42</td>
<td>4.71</td>
<td>157</td>
<td>190</td>
<td>0.50</td>
<td>0.51</td>
<td>0.77</td>
<td>101</td>
<td>51</td>
</tr>
<tr>
<td>4</td>
<td>11.95</td>
<td>1.12</td>
<td>3.54</td>
<td>193</td>
<td>129</td>
<td>0.82</td>
<td>0.84</td>
<td>2.06</td>
<td>58</td>
<td>47</td>
</tr>
<tr>
<td>6</td>
<td>53.46</td>
<td>0.53</td>
<td>2.79</td>
<td>253</td>
<td>89</td>
<td>1.36</td>
<td>1.43</td>
<td>5.84</td>
<td>37</td>
<td>51</td>
</tr>
<tr>
<td>7</td>
<td>116.60</td>
<td>0.36</td>
<td>2.43</td>
<td>285</td>
<td>73</td>
<td>1.75</td>
<td>1.85</td>
<td>9.79</td>
<td>28</td>
<td>51</td>
</tr>
</tbody>
</table>

Estimation of $\epsilon$ from the second order structure function for various forcing amplitudes is presented in Figure 2.7. Following Salazar et al. (2008) and De Jong et al. (2009), $\epsilon$ is selected from Figure 2.7 at the plateau region. This plateau region corresponds to where $D_{LL}$ overlaps with $Cr^{2/3}$ in Figure 2.8, where $C$ is a constant chosen for the curves to intersect. This follows from the “second hypothesis of similarity” by Kolmogorov (1941), in which $D_{LL} \propto r^{2/3}$ in the inertial subrange. Thus, we use $Cr^{2/3}$ to find where $r$ corresponds to the inertial subrange for analyzing $\epsilon$ from $D_{LL}$, which we estimate to be $r \approx 15 – 30$mm. We see agreement between the $\epsilon$ determined from the Large-Eddy PIV technique, which are listed in Table 2.1, and the second order structure function fit.

Using the standard Smagorinsky constant, $C_s = 0.17$, rather than the corrected Smagorinsky constant, $C_s = 0.12$, results in over estimating $\epsilon$ compared to the second order structure function. This agrees with the findings from De Jong et al. (2009). A comparison of $\epsilon$ calculated from the second-order structure function fitting technique and the Large-Eddy PIV technique with the modified and standard Smagorinsky constant is presented in Figure 2.9. From Figure 2.9 we can again conclude that $\epsilon$ calculated using the Large-Eddy PIV technique with the corrected Smagorinsky constant and the second-order structure function fitting technique are in agreement, as previously stated. For comparison, the turbulence parameters with $\epsilon$ calculated using the Large-Eddy PIV technique with the standard Smagorinsky constant are presented in Table A.1 in Appendix A.

From Table 2.1, we observe that the forcing amplitude is the determining parameter in adjusting the turbulence levels. We note that a short-coming in both experimental and DNS studies to date, including the current study, which are attempting to understand the growth of droplets in warm, cumulus clouds is the inability to model $Re_{\lambda}$ and $\epsilon$ typical of cumulus clouds, $Re_{\lambda} \sim 10^3 – 10^4$ and $\epsilon \sim 10^{-2}$ m$^2$/s$^3$ (Shaw, 2003; Onishi & Vassilicos, 2014).
Figure 2.7: Compensated longitudinal second order structure function for varying levels of forcing amplitude. $\epsilon$ is approximated at the inertial subrange, which aligns with the plateau portion of the graph, denoted by the dashed line.

Figure 2.8: The second-order structure function, $D_{LL}$, from the experimental results are indicated as open circles. $C_r^{2/3}$ as a solid line. Where $D_{LL}$ and $C_r^{2/3}$ overlap, indicates $r$ corresponding to the inertial subrange.
Chapter 2: Characterization of the Turbulent Flow Field

Figure 2.9: A comparison of the various methods to calculate $\epsilon$. The Large-Eddy PIV method with $C_s = 0.12$ is seen to match the Structure Function Fitting method.

The turbulence parameters from the PIV analysis on images taken with the continuous laser are presented in Table A.2 in Appendix A. The turbulence parameters for a forcing amplitude of $A = 2V_{pp}$ and a forcing frequency of $f = 100$Hz in Tables 2.1 and A.1 are similar. We find a 4% and 2% difference in $\bar{u}'$ and $\bar{v}'$ values, respectively, and an 11% difference between $\epsilon$ values.

From Table 2.1, we note how the increase in turbulence intensity also increases the ratio (separation) in the length and time scales of the flow. The large-eddy turnover time compared to the Kolmogorov time scale steadily increases from $T_\epsilon/\tau_\eta = 42$ to 78 with $Re_\lambda$ increasing from 157 to 285. Similarly, $L_\epsilon/\lambda$ increases from 11 to 21 with $Re_\lambda$ from 157 to 285. The change in $L_\epsilon/\lambda$ is largely due to a decrease in $\lambda$ with increasing $Re_\lambda$, as $L_\epsilon$ does not seem to change. We see similar results of $L_\epsilon$ remaining unchanged for various levels of turbulence in a turbulence chamber in the work of De Jong et al. (2009) and Yavuz (2016). This suggests that the particulars of the turbulence box fixes the length scales characteristic of the large-eddies. The increase in the separation of length and time scales is significant because the majority of particle clustering studies up to this point have been limited to low $Re_\lambda$ flows as noted by Goto & Vassilicos (2006). Thus, the multiscale nature of clustering has not been analyzed in much depth.
Figures 2.10-2.18 assess various aspects of the flow field with PIV measurements. Figure 2.10 presents the ensemble averaged energy spectra, where the energy spectrum calculated from the \(u\)-velocity component is plotted in blue, and from the \(v\)-velocity component in green. The shape of the energy spectra match that of Liu et al. (1999) and Hwang & Eaton (2004) in both the high and low frequencies. The flaring out of the energy spectra curves seen at the higher wave numbers is attributed to measurement uncertainty by Liu et al. (1999), Hwang & Eaton (2004), and De Jong et al. (2009). In the inertial range of Figure 2.10, we find \(E(k) \propto \epsilon^{2/3}k^{-5/3}\). The inertial subrange range found in Figure 2.10 matches the predicted values from Figure 2.8 of approximately 15 – 30mm. The ensemble averaged longitudinal and transverse structure functions are shown in Figure 2.11, where \(f_{11}\) and \(g_{11}\) are calculated from the \(u\)-components of the velocity fluctuations. Similarly, \(f_{22}\) and \(g_{22}\) are calculated from the \(v\)-components. We see that the general shape of the structure functions matches what we would expect for homogeneous isotropic turbulence. However, the longitudinal structure function does not reach zero for any case suggesting the resolution of the image is not high enough to fully resolve the integral length scale, although it has been approximated through Eq. (2.14). Figure 2.12 presents the autocorrelation of the \(u\) and \(v\)-components of the velocity fluctuations for the various levels of turbulence. The azimuthal symmetry of these plots indicates the flow is isotropic.

The \(u\) and \(v\)-components of the mean velocity are presented in Figure 2.13, and the mean velocity field is presented in Figure 2.14 with several streamlines marked in red. In both figures we find the mean velocity generally behaves as a stagnation flow. It seems that the jets of fluid diverge from the center and form convergence zones on each side of the domain, thus, in the center we have a zero mean flow. Each plot of \(U_{\text{mean}}\) in Figure 2.13 indicates a strong flow moving to the right on the right side of the image and a strong flow moving to the left on the left side of the image. This indicates a mean flow away from the center, which is shown in Figure 2.14. We observe similar behavior in \(V_{\text{mean}}\) in Figure 2.13, in which there is a strong upward flow in the top portion of the domain and a strong downward flow in the bottom portion of the domain. The regions of strong flow correspond to the convergence zones seen in Figure 2.14.

The \(u\) and \(v\)-components of the rms fluctuating velocity are presented in Figures 2.15 and 2.16. In both figures we find that the \(u_{\text{rms}}\) field is greatest in the vertical center plane, i.e. where the horizontal mean velocity is lowest. Similarly, we find the \(v_{\text{rms}}\) field is greatest in the horizontal center plane, where the vertical mean velocity is lowest. Thus, the regions with the greatest mean flow have reduced levels of velocity fluctuations. Note Figure 2.16 is also presented on a logarithmic scale in Figure A.1 in Appendix A.

In Figure 2.17 we present further evidence that the flow field is isotropic near the center. The \(u_{\text{rms}}/v_{\text{rms}}\) plots indicate isotropy when approaching unity (Hwang & Eaton, 2004; Bocanegra Evans, 2013), which we see in Figure 2.17 near the center of the image. However, because \(u_{\text{rms}}\) is confined more vertically and \(v_{\text{rms}}\) is confined more horizontally we find that the top and bottom of the \(u_{\text{rms}}/v_{\text{rms}}\) plots tend to be greater than 1, and the sides tend to
be less than 1. On the right hand side of Figure 2.17 is the scatter plot of the fluctuating velocity components. In isotropic turbulence, we expect $u'v' = 0$. Thus we expect to see an azimuthally symmetric scatter plot for isotropic turbulence, which is roughly seen in Figure 2.17.

Therefore, we argue that the flow is fairly isotropic around the center of the flow field based on the following observations:

- The profile of the longitudinal and transverse structure functions in Figure 2.11 indicates the flow is isotropic.
- Figures 2.13 and 2.14 indicate there is a zero mean flow in the center of the turbulence box.
- The scatter plot of $u'$ and $v'$ in Figure 2.17 is azimuthally symmetric, which indicates $u'v' = 0$.
- $u_{rms}/v_{rms} \approx 1$ in the center of Figure 2.17, which is seen in Table 2.1 in which $\bar{u}'$ and $\bar{v}'$ are nearly identical in each case. This behavior of $u'/v' \approx 1$ is also seen in Figure 2.16 for the center of the image area.

We approximate the region in which we have isotropic turbulence as approximately a 40mm $\times$ 40mm area in the center of the flow field. The length scale associated with the region of isotropic flow corresponds to the integral length scale in Table 2.1.

Finally, in Figure 2.18 we present $u_{rms}/\langle u_{rms} \rangle$ and $v_{rms}/\langle v_{rms} \rangle$, which indicates a homogeneous flow field when approaching unity (Hwang & Eaton, 2004). The majority of the area seen in the $u_{rms}/\langle u_{rms} \rangle$ and $v_{rms}/\langle v_{rms} \rangle$ fields fluctuates between 1.4 and 0.8, similar to Hwang & Eaton (2004), with the center area approaching unity. However, away from the center the flow becomes less homogeneous following the argument made for the general shape of $u_{rms}$ and $v_{rms}$.
Chapter 2: Characterization of the Turbulent Flow Field

Figure 2.10: Energy spectra at the various levels of turbulence. $E_{11}$ is the energy spectrum calculated from the $u$-velocity component and $E_{22}$ is the energy spectrum calculated from the $v$-velocity component.
Figure 2.11: Longitudinal, \( f \), and transverse, \( g \), structure functions are plotted in green and blue, respectively.
Figure 2.12: Autocorrelation of $u$ and $v$-components of the fluctuating velocity field.
Figure 2.13: $u$ and $v$-components of mean velocity field.
Figure 2.14: Mean velocity flow field with the streamline outlined in red. This indicates a stagnation point in the center of the flow field.
Figure 2.15: \( u \) and \( v \)-components of rms velocity fluctuations.
Figure 2.16: PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field.
Figure 2.17: $u_{rms}/v_{rms}$ is plotted to indicate isotropic flow when approaching unity, and a scatter plot of $u'$ and $v'$ indicates isotropy when azimuthally symmetric.
Figure 2.18: $u_{rms}/\langle u_{rms} \rangle$ and $v_{rms}/\langle v_{rms} \rangle$ are plotted to indicate homogeneity when approaching unity.
Chapter 3

Geometric Characterization of Particle Clusters and Voids

In this chapter we discuss the geometric properties of particle clusters and voids, which have been identified using a Voronoi area method. We begin by analyzing the significant particle parameters in this experiment with regards to preferential concentration, followed by a description of the particle detection process and Voronoi area method. Finally, the characteristic length scales associated with cluster and void areas are analyzed.

3.1 Experimental Parameters

The geometric features of particle clusters and voids are analyzed for four particle sizes subject to the four flow conditions described in Table 2.1. In this experiment particles are discharged into the turbulence chamber and images are taken in the manner described in Chapter 2.2, but with aluminum-oxide particles with \(d_p = 0.5, 1, 3, \text{ and } 5 \mu m\) injected into the flow. Two sets of images are taken for each test case. The first set have an image area of \(10 \times 10 \text{cm}^2\) using the same camera and lens settings as described in Section 2.2. The second set have an image area of \(20 \times 20 \text{mm}^2\) using an 85mm Nikon lens (f1:1.4) with a 55mm extension tube. The images with an area of \(10 \times 10 \text{cm}^2\) have a resolution of 97 µm/pixel, whereas those with an image area of \(20 \times 20 \text{mm}^2\) have a resolution of 19 µm/pixel.

The relaxation times associated with each particle size is presented in Table 3.1, and the Stokes number for each test case based on \(\tau_\eta\) from Table 2.1 is given in Table 3.2. Although the separation of length and time scales is large in this study, we note that the relaxation time of the particles is low. Therefore, the particles will most likely only resonate with eddies with times scales near \(\tau_\eta\) as suggested by Goto & Vassilicos (2006).

<table>
<thead>
<tr>
<th>(d_p \text{ (µm)})</th>
<th>0.5</th>
<th>1</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau_p \text{ (ms)})</td>
<td>0.003</td>
<td>0.014</td>
<td>0.125</td>
<td>0.347</td>
</tr>
</tbody>
</table>

Table 3.1: The relaxation time for various particle sizes.
Table 3.2: Stokes number, $St_\eta$, for various forcing amplitudes and particles sizes. In all cases the forcing frequency is $f = 100\text{Hz}$.

Although the particles used do not offer large relaxation times, they satisfy the assumption used in deriving Eq. (1.5): $d_p \ll \eta$, $\rho_d \gg \rho$, and $Re_p < 1$. The particle Reynolds number for each test case is presented in Table 3.3, where $Re_p$ is calculated following Wood et al. (2005),

$$Re_p = \frac{\bar{u}'d_p}{\nu}.$$  \hspace{1cm} (3.1)

Table 3.3: Particle Reynolds number, $Re_p$, for various forcing amplitudes and particles sizes. In all cases the forcing frequency is $f = 100\text{Hz}$.

As $Re_\lambda$ increases, $Re_p$ also increases because $u'$ increases. Therefore, at high values of $Re_\lambda$, the particles must be chosen to have a sufficiently small diameter to satisfy the criteria $d_p \ll \eta$ and $Re_p < 1$. We acknowledge that the particle diameters are taken as their advertised values, and that the size distribution and shape of the particles have not been analyzed.

One point of further concern is if the turbulence parameters presented in Table 2.1 are valid when particles larger than $d_p = 0.5\mu\text{m}$ are injected into the flow field, i.e. if turbulence modulation is occurring. Turbulence modulation typically begins to occur when $\Phi_v > 10^{-6}$. The regime where $\Phi_v < 10^{-6}$ is known as one-way coupling (Elghobashi, 1994; Sundaram & Collins, 1999). In this regime the carrier fluid affects the dynamics of the particles, but the particles do not affect the carrier flow field. As the size and number of particles increases,
the particles begin to influence the turbulence structures (Elghobashi, 1994; Sundaram & Collins, 1999). This is known as two-way coupling, and is defined as the regime where $10^{-6} \leq \Phi_v \leq 10^{-3}$. Finally, when $\Phi_v > 10^{-3}$ particle-particle interaction becomes dominant and particle clusters also begin to modulate the carrier flow. This is known as four-way coupling.

It is difficult, however, to discern if turbulence modulation is occurring because we are not able to accurately predict the number of particles in an image. Therefore, PIV analysis is performed on the $10 \times 10\text{cm}^2$ images with the various sized particles analyzed as tracer particles. The turbulence parameters from this analysis are presented in Tables A.3-A.5 in Appendix A. In general, the $\overline{u}$ and $\overline{v}$ values in Tables 2.1 and A.3-A.5 are in agreement. However, to analyze more accurately the behavior of the fluctuating velocity components, Figure 3.1 presents the PDF of the rms of the velocity fluctuations at three points in the flow field. The flow field analyzed in Figure 3.1 is subject to a forcing amplitude of $A = 2V_{pp}$ and forcing frequency of $f = 100\text{Hz}$. Figures A.2-A.4 in Appendix A present the same data as Figure 3.1, but with higher levels of turbulence. Similarly, Figure A.5 presents the same data as Figure 3.1, but with the images taken with a continuous laser illumination source. Here we acknowledge the obvious contradiction in performing PIV analysis on images with particles that do not respond to the flow field faithfully, and are thus clustering. In general, however, the PIV analysis from particles of various sizes indicate similar fluctuating velocity components in each region of the flow field, despite the fact that larger particles do not follow the flow field as faithfully. Figure A.2 is the exception, in which the particles with $d_p = 0.5\mu\text{m}$ tend to have slower velocity fluctuations away from the center of the image.

This is reflected in the lower $\overline{u}$ and $\overline{v}$ values for a forcing amplitude of $4V_{pp}$ when comparing Tables 2.1 and A.3-A.5. In Figures 2.16 and A.2-A.5, we also note similar features previously discussed in Section 2.3 mainly:

- The $u_{rms}$ and $v_{rms}$ values in the center of the flow field closely align, indicating isotropic flow.
- The $u_{rms}$ values are skewed to the lower velocities in the center-right of the domain compared to the center-top of the domain. Similarly, the $v_{rms}$ values are skewed to the lower velocities in the center-top of the domain. This reflects the increased difficulty in the fluid to fluctuate where the mean flow is greatest.

From Figures 3.1 and A.2-A.5 we conclude that the TKE has not been modified significantly, and thus, we proceed in the current work using the parameters listed in Table 2.1 to characterize the four levels of turbulence.
Chapter 3: Geometric Characterization of Particle Clusters and Voids
43

(a) Center of the image

(b) Center of the image

(c) Center-top of the image

(d) Center-top of the image

(e) Center-right of the image

(f) Center-right of the image

Figure 3.1: PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. The PIV analysis is performed with an image size: 10cm×10cm using a YAG laser light source and a forcing amplitude of $A = 2V_{pp}$ with a forcing frequency of $f = 100$Hz. The tracer particles have an average diameter of $d_p = 0.5, 1, 3, 5\mu m$. 
Chapter 3: Geometric Characterization of Particle Clusters and Voids

As previously stated, the variables connected with preferential concentration are $St_\eta$, $Re_\lambda$, and $\Phi_v$. $Re_\lambda$ is varied through the forcing amplitude of the speakers, and $St_\eta$ is varied through both the forcing amplitude and particle diameters. The volume fraction, however, is more difficult to control in this system. First, it is difficult to add a consistent number of particles to the discharge system. Additionally, once the particles are discharged they typically remain in suspension for a significant amount of time. Thus, as images are taken, a build-up of suspended particles forms in the turbulence chamber. This chapter, therefore, does not attempt to characterize the cluster and void properties to any one parameter, but rather give an overall analysis of what trends exist with regards to cluster and void areas.

3.2 Particle Detection

To detect particles, the background is initially subtracted from the original image and the pixel intensity is normalized to account for varying light intensity from right to left, in the direction of illumination. This is done by averaging the pixel intensity for each column and then applying a 9th order Lagrangian polynomial curve-fit to this, as seen in Figure 3.2. The 9th order Lagrangian polynomial has been chosen by inspection. The pixel intensity of each column is normalized by the curve-fit and scaled to 8-bit depth; from 0 to 255.

![Figure 3.2: Average image pixel intensity and curve-fit used to normalize the light intensity.](image)

This image processing steps are shown in Figure 3.3 for a $10 \times 10 \text{cm}^2$ image, where we present the original image, the processed image, and the binary image of the final particles detected. The $10 \times 10 \text{cm}^2$ images are initially cropped to a $768 \times 768$ pixel array before being processed, in order to obtain the clearest picture possible. The final size of a $768 \times 768$ pixel array for the $10 \times 10 \text{cm}^2$ images was also chosen by inspection. For comparison we present the same image processing technique for a $20 \times 20 \text{mm}^2$ image in Figure 3.4. Note that the $20 \times 20 \text{mm}^2$
images are not cropped as the light intensity is distributed fairly evenly because of the small image area.

(a) Raw image  
(b) Background subtracted and normalized  
(c) Binary image

Figure 3.3: Image processing to detect individual particles for a $10 \times 10\text{cm}^2$ image area.
The pixel threshold level, $s$, is determined for each set of images based on the threshold entropy algorithm (Kapur et al., 1985; Sezgin et al., 2004). This method was chosen because the pixel histogram of the images is not clearly bimodal. The pixel threshold is chosen such that the value of $s$ maximizes the quantity $T$ defined as
\[
T = \sum_{i=0}^{s} p_i \ln \left( \frac{p_i}{P_s} \right) + \sum_{i=s+1}^{255} - \frac{p_i}{1 - P_s} \ln \left( \frac{p_i}{1 - P_s} \right),
\]

where \(p_i\) are the PDF of the pixel intensity levels from \(i = 0\) to 255 for grey-scale, and \(P_s\) is the cumulative distribution function (CDF) of the pixel intensity levels. A collection of connected pixels which exceed the threshold level are termed a blob. This is shown in Figure 3.5a, in which the particles that exceed the threshold level are marked with blue points, and the blobs are outlined with red contours. The point of maximum pixel intensity within each blob is chosen as the general location of a particle, unless the blob exceeds a maximum threshold size. When the blob area exceeds this threshold, individual particles are detected within the blob using the particle mask correlation technique following the work of Takehara & Etoh (1998) and Lei et al. (2012). Following this technique, a mask image of a nine-point 2D Gaussian distribution, \(I_m\), is cross-correlated with the pixel intensity, \(I\), of each point within the blob. The cross-correlation coefficient is calculated at each point in the domain \((x, y)\) as

\[
R(x, y) = \frac{\sum_{i=x-1}^{x+1} \sum_{j=y-1}^{y+1} (I(i, j) - \bar{I})(I_m(i, j) - \bar{I}_m)}{\sqrt{\sum_{i=x-1}^{x+1} \sum_{j=y-1}^{y+1} (I(i, j) - \bar{I})^2} \sqrt{\sum_{i=x-1}^{x+1} \sum_{j=y-1}^{y+1} (I_m(i, j) - \bar{I}_m)^2}},
\]

where the overbar again denotes a spatial average. The peak points of the cross-correlation indicate the general location of particles. We have chosen this method because it handles overlapping particles well. No particles on the edges of the images are included following Ouellette et al. (2006). The reader is directed to the work of Ouellette et al. (2006) and Kreizer et al. (2010) for further information on particle detection techniques.

The centers of the particles, \((x_o, y_o)\), are then further refined using a second order Gaussian interpolation scheme following the work of Sholl & Savas (1997). The scheme is laid out below, in which, coordinates \((x, y)\) are the general particle locations, as previously described.

\[
x_o = x + \Delta x \quad y_o = y + \Delta y
\]

\(\Delta x\) and \(\Delta y\) are calculated as

\[
\Delta x = \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{12}^2} \quad \Delta y = \frac{a_{11}b_2 - a_{12}b_1}{a_{11}a_{22} - a_{12}^2}
\]

\[
a_{11} = \frac{\partial^2 I}{\partial x^2} = I_{i+1,j} - 2I_{i,j} + I_{i-1,j}
\]

(3.5)
\[ a_{12} = \frac{\partial^2 I}{\partial x \partial y} = \frac{1}{4} (I_{i+1,j+1} - I_{i-1,j+1} - I_{i+1,j-1} + I_{i-1,j-1}) \] (3.7)

\[ a_{22} = \frac{\partial^2 I}{\partial y^2} = I_{i,j+1} - 2I_{i,j} + I_{i,j-1} \] (3.8)

\[ -b_1 = \frac{\partial I}{\partial x} = \frac{1}{2} (I_{i+1,j} - I_{i-1,j}) \] (3.9)

\[ -b_2 = \frac{\partial I}{\partial y} = \frac{1}{2} (I_{i,j+1} - I_{i,j-1}) \] , (3.10)

where \( I_{i,j} \) is the natural log of the pixel intensity at \((x, y)\). An example of the final particle detection is shown in Figure 3.5b. Table 3.4 presents the average number of particles detected for 1,000 uncorrelated images for the various test cases in the 20 \( \times \) 20 mm\(^2\) images, and Figure 3.6 presents the number of particles detected over time for the same test cases in Table 3.4. We note that the particle concentration almost consistently decreases over time.

Figure 3.5: The image on the left marks the particles that exceed the threshold level, and in the right image the center of each particle detected is marked. The blob contours are outlined in red.
Table 3.4: Average number of particles detected in the $20 \times 20\text{mm}^2$ images. In all test cases $f = 100\text{Hz.}$

<table>
<thead>
<tr>
<th>$d_p$</th>
<th>$0.5\mu\text{m}$</th>
<th>$1\mu\text{m}$</th>
<th>$3\mu\text{m}$</th>
<th>$5\mu\text{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2V_{pp}$</td>
<td>1.27e4</td>
<td>1.84e4</td>
<td>1.35e4</td>
<td>1.07e4</td>
</tr>
<tr>
<td>$4V_{pp}$</td>
<td>9.50e3</td>
<td>1.21e4</td>
<td>1.09e4</td>
<td>7.49e3</td>
</tr>
<tr>
<td>$6V_{pp}$</td>
<td>7.18e3</td>
<td>1.21e4</td>
<td>1.03e4</td>
<td>1.04e4</td>
</tr>
<tr>
<td>$7V_{pp}$</td>
<td>6.71e3</td>
<td>9.49e3</td>
<td>1.10e4</td>
<td>5.98e3</td>
</tr>
</tbody>
</table>

Figure 3.6: Number of particles detected over time for $20 \times 20\text{mm}^2$ images.
Table 3.5: Average $\Phi_v$ for the $20 \times 20 \text{mm}^2$ images. In all test cases, $f = 100\text{Hz}$.

<table>
<thead>
<tr>
<th>$V_{pp}$</th>
<th>$\Phi_v$ (Average)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.1e-9 2.4e-8 4.7e-7 1.7e-6</td>
</tr>
<tr>
<td>4</td>
<td>1.5e-9 1.6e-8 3.8e-7 1.2e-6</td>
</tr>
<tr>
<td>6</td>
<td>1.1e-9 1.6e-8 3.6e-7 1.7e-6</td>
</tr>
<tr>
<td>7</td>
<td>1.0e-9 1.2e-8 3.9e-7 9.8e-7</td>
</tr>
</tbody>
</table>

Although it is difficult to control $\Phi_v$, we can estimate it from the values found in Table 3.4 for the $20 \times 20 \text{mm}^2$ images. The total volume is estimated as $V = 20\text{mm} \times 20\text{mm} \times 1\text{mm}$, where 1mm is approximately the thickness of the laser sheet. The estimated values for $\Phi_v$ are listed in Table 3.5. From Table 3.5 we note that by adjusting $d_p$ we alter both $St_\eta$ and $\Phi_v$, making it difficult to discern the effects of any one parameter. We also note that because we are using smaller particles compared to the majority of studies exploring particle-laden turbulent flows, e.g. Monchaux et al. (2010) and Sumbekova (2016), we are able introduce a larger number of particles before we modulate the flow field.

### 3.3 Voronoi Analysis

A Voronoi diagram is constructed such that each Voronoi cell is a convex polygon indicating the regions closest to each particular particle. Figure 3.7 presents a sample image of the flow field discretized into Voronoi cells. Voronoi cells located at the border of the image are not included. The vertices of the Voronoi cells are determined using the `voronoin` function in Matlab. The area of the cells is calculated by discretizing each cell into triangles and summing the area of triangles.
Figure 3.7: The particles detected are marked along with the Voronoi cells associated with them.

Although the pixel intensity is normalized to allow for particles to be more evenly detected using the pixel intensity threshold, it does not account for the fact that particles appear more numerous in the regions of the flow which are most clearly in focus. This biases the clusters to be located in the center of the image where the flow is more in focus, and the voids to be in the periphery of the images. To account for the spatial bias in Voronoi cell areas, we have applied the methodology of Sumbekova (2016) in using a contraction coefficient. The contraction coefficient is calculated by normalizing a spatial map of the average Voronoi cell area for 1,000 images with the average Voronoi cell area in the center of the domain. Example of these contraction coefficient maps for a 20 × 20mm$^2$ and 10 × 10cm$^2$ image are shown in Figure 3.8. The area of each Voronoi cell is, therefore, divided by the contraction coefficient based on the location of the center of gravity for each Voronoi cell. We note that the contraction coefficient at the sides of the images are skewed because the Voronoi cells at the borders are neglected. In the 20 × 20mm$^2$ images we observe that the overall the Voronoi cell area is uniformly distributed, with the exception of a horizontal strip near the center of the image caused by a lens aberration. The Voronoi cell area of the 10 × 10cm$^2$ images, however, are less uniformly distributed because the light source varies greater for larger areas.
Figure 3.8: A sample contraction coefficient map for both $20 \times 20\text{mm}^2$ and $10 \times 10\text{mm}^2$ images.

The Voronoi cell area CDF is then calculated, and its derivative is taken to calculate the PDF. The cell area PDF for the Poisson distribution is calculated following Ferenc & Néda (2007), in which

$$
PDF = \frac{343}{15} \sqrt{\frac{7}{2\pi}} \left( \frac{A}{\langle A \rangle} \right)^{5/2} \exp \left( -\frac{7}{2} \frac{A}{\langle A \rangle} \right), \tag{3.11}
$$

where $1/\langle A \rangle$ is representative of the bulk particle number density. Following the methodology of Monchaux et al. (2012) the normalized Voronoi cell area PDF is compared to the PDF for a Poisson distribution. As seen in Figure 3.9, the two curves intersect twice. Cell areas larger than $V_v$ indicate cells belonging to a void region. Likewise, cell areas smaller than $V_c$ indicate cells belonging to a cluster.
Figure 3.9: Normalized Voronoi cell area PDF for an image compared to a Poisson distribution. Voronoi cells with a normalized area greater than $V_v$ belong to a void region. Similarly, Voronoi cells with a normalized area less than $V_c$ belong to a cluster region.

The Voronoi area PDF for various turbulence levels for the $20 \times 20\text{mm}^2$ images are presented in Figure 3.10, and Table 3.6 presents $V_c$ and $V_v$ for each test case shown in Figure 3.10. From Table 3.6 we find on average $\langle V_c \rangle \approx 0.49$ and $\langle V_v \rangle \approx 1.77$ for the $20 \times 20\text{mm}^2$ images.

<table>
<thead>
<tr>
<th>$d_p$</th>
<th>0.5$\mu$m</th>
<th>1$\mu$m</th>
<th>3$\mu$m</th>
<th>5$\mu$m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2V_{pp}$</td>
<td>$V_v = 1.65$</td>
<td>$V_v = 1.82$</td>
<td>$V_v = 1.79$</td>
<td>$V_v = 1.74$</td>
</tr>
<tr>
<td></td>
<td>$V_c = 0.49$</td>
<td>$V_c = 0.53$</td>
<td>$V_c = 0.51$</td>
<td>$V_c = 0.47$</td>
</tr>
<tr>
<td>$4V_{pp}$</td>
<td>$V_v = 1.64$</td>
<td>$V_v = 1.76$</td>
<td>$V_v = 1.82$</td>
<td>$V_v = 1.75$</td>
</tr>
<tr>
<td></td>
<td>$V_c = 0.45$</td>
<td>$V_c = 0.50$</td>
<td>$V_c = 0.49$</td>
<td>$V_c = 0.46$</td>
</tr>
<tr>
<td>$6V_{pp}$</td>
<td>$V_v = 1.76$</td>
<td>$V_v = 1.78$</td>
<td>$V_v = 1.85$</td>
<td>$V_v = 1.77$</td>
</tr>
<tr>
<td></td>
<td>$V_c = 0.47$</td>
<td>$V_c = 0.54$</td>
<td>$V_c = 0.50$</td>
<td>$V_c = 0.49$</td>
</tr>
<tr>
<td>$7V_{pp}$</td>
<td>$V_v = 1.82$</td>
<td>$V_v = 1.78$</td>
<td>$V_v = 1.89$</td>
<td>$V_v = 1.78$</td>
</tr>
<tr>
<td></td>
<td>$V_c = 0.47$</td>
<td>$V_c = 0.49$</td>
<td>$V_c = 0.50$</td>
<td>$V_c = 0.48$</td>
</tr>
</tbody>
</table>

Table 3.6: $V_v$ and $V_c$ for various test cases for $20 \times 20\text{mm}^2$ images.

Similarly, the Voronoi area PDF for the $10 \times 10\text{cm}^2$ images is presented in Figure A.6, and Table A.6 presents the corresponding $V_c$ and $V_v$ values. For the $10 \times 10\text{cm}^2$ images, on
average, $\langle V_c \rangle \approx 0.54$ and $\langle V_v \rangle \approx 1.84$. In comparison, Monchaux et al. (2010) and Sumbekova (2016) consistently found $V_c \approx 0.6$ and $V_v \approx 2.1$. 

![Figure 3.10: $A/\langle A \rangle$ from Voronoi analysis of 20 $\times$ 20mm$^2$ images.](image)

### 3.4 Particle Clustering Characterization

Again following Monchaux et al. (2010), clusters are defined as a collection of connected cluster cells, as shown in Figure 3.11. The Voronoi cells are outlined in red in Figure 3.11, with the cluster cells shaded. In Figure 3.11, connected cells shaded with matching colors indicate cluster regions, where the color of each cluster region is randomly selected. We
have also set a condition that clusters are required to contain at least three clusters cells. Qualitatively, we note that our $20 \times 20 \text{mm}^2$ images match to some extent with those by Wood et al. (2005) and Monchaux et al. (2010). However, our $10 \times 10 \text{cm}^2$ images appear to contain a significantly higher quantity of particles compared to any previous experimental studies. We do note a significant deviation between the particle clusters in our work from Yang & Shy (2005) who tended to find the particles cluster in sheet-like formations around regions of intense vorticity. In studies using DNS modeling such as Wang & Maxey (1993) and Reade & Collins (2000), the sheet-like behavior of the cluster regions is even more exaggerated.

The PDF of $\sqrt{A_c/\eta}$ for images with an area of $20 \times 20 \text{mm}^2$ and $10 \times 10 \text{cm}^2$ are presented in Figures 3.12 and 3.13, respectively. The same data in Figures 3.12 and 3.13 are also presented in terms of $A_c/\eta^2$ and $A_c$ in Figures A.7-A.10 in Appendix A. Each curve is an ensemble average of 1,000 uncorrelated images. In general, $\sqrt{A_c/\eta}$ monotonically increases with increasing $Re_\lambda$, which also corresponds to an increase in $St_\eta$. However, this increase in $\sqrt{A_c/\eta}$ with increasing $Re_\lambda$ seems to result from the decrease in $\eta$ with increasing $Re_\lambda$. This observation comes from examining the collapse of curves in Figures A.7 and A.8, with the exception that the large cluster sizes increase monotonically with increasing $Re_\lambda$ for the $20 \times 20 \text{mm}^2$ images. This final point may be explained by the decrease in $\tau_\eta$ with increasing $Re_\lambda$, which resonates with the particle response time thereby increasing the number of large cluster sizes with increasing $Re_\lambda$. We also observe in Figures 3.12 and 3.13 that the peaks in the PDF indicating the characteristic cluster length scales do not seem to shift with
increasing particle diameters, which corresponds to an increase in \(St_\eta\) and \(\Phi_v\).

Figure 3.12: \(\sqrt{A_c/\eta}\) from Voronoi analysis for 20 \(\times\) 20mm\(^2\) images.
The characteristic cluster length scales for each turbulence level for both the 20 × 20mm$^2$ and 10 × 10cm$^2$ images are outlined in Table 3.7. In comparison, Aliseda et al. (2002), Wood et al. (2005), and Obligado et al. (2014) found that the typical cluster length scale is $\sim 10\eta$. Aliseda et al. (2002) and Wood et al. (2005) used a box counting technique to analyze clustering, and Obligado et al. (2014) used a Voronoi analysis. We note that their results more closely agree with the results from our 10 × 10cm$^2$ images.
Table 3.7: Average peak value in the PDF of $\sqrt{A_c/\eta}$ in Figures 3.12 and 3.13, which indicates characteristic cluster length scales.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>ROI</th>
<th>$20 \times 20\text{mm}^2$</th>
<th>$10 \times 10\text{cm}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>157</td>
<td>0.9</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td>193</td>
<td>1.5</td>
<td>5.7</td>
<td></td>
</tr>
<tr>
<td>253</td>
<td>2.2</td>
<td>8.4</td>
<td></td>
</tr>
<tr>
<td>285</td>
<td>2.8</td>
<td>9.8</td>
<td></td>
</tr>
</tbody>
</table>

As previously mentioned, Goto & Vassilicos (2008) have suggested that the spectrum describing the geometry of the cluster and void areas may be attributed to the cascade of eddies at different length scales following the energy spectrum. In support of this, Moisy & Jiménez (2004) found that flow structures of both intense vorticity and strain follow a spectrum with a power-law fit of $P(V) \propto V^{-2}$, where $P(V)$ is the probability of a structure with a volume $V$. Therefore, attention has been recently given in analyzing the shape of the PDF of the area of clusters and voids to determine if the size distribution mimics the distribution of flow structures. Here we apply a power-law fit of the form

$$y = c \left(\frac{A_c}{\eta^2}\right)^b \rightarrow \log_{10}(y) = b \log_{10}\left(\frac{A_c}{\eta^2}\right) + \log_{10}(c)$$

(3.12)

to Figures A.9 and A.10 to the right side of the peak, where $y$ is the probability of having a cluster of the size $A_c/\eta^2$. We have chosen to analyze $A_c/\eta^2$ rather than $\sqrt{A_c/\eta}$ following Goto & Vassilicos (2006). In Figure 3.14 we present a representative case of the power-law fit for both the $20 \times 20\text{mm}^2$ and $10 \times 10\text{cm}^2$ images, both in linear and logarithmic scales. Applying a power-law fit following Eq. (3.12) to the $20 \times 20\text{mm}^2$ and $10 \times 10\text{cm}^2$ images, results in exponential values of $b \approx -2.4$ to $-3.0$ in both cases. The exponential value is slightly higher than that found by Monchaux et al. (2010) and Sumbekova (2016) who applied a power-law fit to the PDF curve of $A_c$, and found an exponential value of $b \approx -2$.

Thus, the cluster size distribution seems to follow similar behavior to the distribution of flow structures found by Moisy & Jiménez (2004). This suggests that turbulent flow structures may influence the geometric properties of clusters. However, from Figure 3.14 we note the power-law fit only matches well for a limited range of scales. This suggests that the probability of having a large cluster area decreases more rapidly than a power-law would indicate.
Additionally, the portion of the PDF which follows the power-law scaling indicates at what scales the cluster structures exhibit *self-similarity*, i.e. objects that when magnified display repeating structural elements at smaller scales. From Figure 3.14 we observe that for each case the PDF only follows a power-law fit near the peak of the curve, thus, indicating that the cluster areas are self-similar only for the most common cluster areas. However, we also note the level of resemblance between the PDF for cluster sizes from the $20 \times 20\text{mm}^2$ and $10 \times 10\text{cm}^2$ images, which also indicates a level of self-similarity in the cluster areas. The observations previously made raise several questions:

1. Why are the characteristic cluster length scales associated with the $20 \times 20\text{mm}^2$ and $10 \times 10\text{cm}^2$ images for the same particle diameter and turbulence levels different?
2. Why is there a difference between the cluster length scales found by Aliseda et al. (2002), Wood et al. (2005), and Obligado et al. (2014) compared to the $20 \times 20\text{mm}^2$ images in this experiment?

3. Qualitatively, why do the particle clusters appear more evenly dispersed and broken-up compared to the long-sheet like structures seen by Wang & Maxey (1993), Reade & Collins (2000), and Yang & Shy (2005)?

4. Why does the probability of large clusters decrease more rapidly than a power-law fit would suggest, indicating that the self-similarity argument does not apply to the largest clusters?

With regard to the first point, we tentatively suggest that it is most likely the difference in resolution between the two image sets that leads to the different characteristic cluster length scales. As previously stated, the $10 \times 10\text{cm}^2$ images have a resolution of $97\mu\text{m/pixel}$, and the $20 \times 20\text{mm}^2$ images have a resolution of $19\mu\text{m/pixel}$. In comparison, the largest particle size has a diameter of $d_p = 5\mu\text{m}$. Therefore, it is probable that each pixel representing a single particle in the $10 \times 10\text{cm}^2$ images is actually composed of many particles, some of which become more clearly defined in the $20 \times 20\text{mm}^2$ images. However, we observe that the characteristic cluster length scales from the $10 \times 10\text{cm}^2$ images matches with previously listed studies unlike the $20 \times 20\text{mm}^2$ images, which indicates that perhaps the two images sizes are in fact detecting different sized clusters. For comparison, Table 3.8 lists $d_p$, $\eta$, and the image resolution for previously listed studies which have also suggested characteristic cluster length scales.

<table>
<thead>
<tr>
<th></th>
<th>$d_p$ (µm)</th>
<th>$\eta$ (µm)</th>
<th>image resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aliseda et al. (2002)</td>
<td>10</td>
<td>210 – 273</td>
<td>34µm/pixel</td>
</tr>
<tr>
<td>Wood et al. (2005)</td>
<td>20, 25, 50</td>
<td>170 &amp; 190</td>
<td>40µm/pixel</td>
</tr>
<tr>
<td>Monchaux et al. (2010)</td>
<td>50</td>
<td>450 &amp; 790</td>
<td>97µm/pixel</td>
</tr>
<tr>
<td>Obligado et al. (2014)</td>
<td>50</td>
<td>140 – 280</td>
<td>97µm/pixel</td>
</tr>
<tr>
<td>Sumbekova (2016)</td>
<td>50</td>
<td>114, 162, 250, &amp; 431</td>
<td>97µm/pixel</td>
</tr>
</tbody>
</table>

Table 3.8: Values of $d_p$, $\eta$, and image resolution from previous studies on characteristic cluster length scales.

The difference in the characteristic cluster length scale between $20 \times 20\text{mm}^2$ images and those from the previously listed studies may result from:

1. The significantly smaller particle sizes used in the current experiment

2. Higher particle concentration in the current experiment, which may also lead to turbulence modulation
3. The possibility that the $20 \times 20\text{mm}^2$ images in the current study are in fact detecting different cluster sizes compared to the previous studies.

4. The difference in resolution between the current experiment and the previously listed studies

If the results of the $20 \times 20\text{mm}^2$ images are more reliable, it may suggest that the differences from previous studies are attributed to the smaller particle sizes used in the current study. Note the smaller particle sizes correspond to a larger number of particles before $\Phi_v > 10^{-6}$. This would indicate that there may be a limit on the cluster size based on the particle size and number. As the diameter of the particles decrease, so does the settling velocity and the settling velocity ratio $V_{air}/u'$. If the settling velocity ratio is low enough, this implies that the tendency of a particle to travel in the direction of gravity as opposed to traveling in the direction of the local fluctuating velocities also decreases. Wang & Maxey (1993) argued that the particles tend to travel in the downward direction, and thus, the particles typically concentrate in long, vertically oriented sheets on the peripheral, downflow sides of vortices. However, if this tendency of particles to travel in the direction of gravity is reduced then particles will encounter flow structures in different directions. We propose this may be one explanation for why the particle size limits the cluster length scales, and why the particle clusters appear less sheet-like compared to Yang & Shy (2005). However, the numerical work of Baker et al. (2017) contradicts this suggestions, as they have found that fixing the settling velocity to 0 has no impact on the power-law fit to the PDF of $V_c/\eta^3$, where $V_c$ is the cluster volumes.

We also acknowledge that Aliseda et al. (2002) found a decrease in cluster size with increasing particle concentration. We note that this may suggest why we observe smaller cluster sizes. However, the $10 \times 10\text{cm}^2$ images suggest similar cluster scales to Aliseda et al. (2002), and yet seem to have a high number particle number density.

We conjecture that the difference between the PDF of the large clusters compared to the power-law fit may be attributed to the low resolution in the images, the small particle sizes, and the large number of particles. The low resolution of the images introduces error in the particle detection process, and thus, increases the level of variability in the cluster areas, especially with regards to large clusters. The small particle sizes as previously stated may not scale as faithfully with the flow structures if they do not approach the flow structures in the same general direction. This would have implications on the role that turbulent structures play in the formation of particles clusters, which we will return to in Chapter 4. It may also be easier to disperse smaller particles compared to the larger particle sizes in previous studies, thus, causing the large clusters to break-up more easily. Lastly, it is also possible that the cluster area PDF is in fact following the scaling behavior of the turbulent flow structures, but that these structures are not solely based on the energy cascade, but are being modulated themselves by the presence of particles.

Finally, we can also comment on the self-similarity of cluster structures through fractal
geometry by applying the perimeter-area method to calculate the fractal dimension, $D$, following Aliseda et al. (2002), Monchaux et al. (2010), and Obligado et al. (2014). The fractal dimension can be thought of as a ratio between the number of self-similar pieces to a scaling factor. The perimeter-area method calculates $D$ as the slope of the square-root of the area versus the perimeter of the structure. When this slope is constant the object is considered to display self-similarity. In Figure 3.15 we present two representative examples of the perimeter, $P$, of each cluster plotted against $\sqrt{A_c}$ to determine the fractal dimension for the small and large scale clusters.

For both sets images, small scale clusters follow a linear-fit with a slope of $D \approx 1.2 - 1.3$, and the large scale clusters follow a linear-fit with a slope of $D \approx 1.5 - 1.8$, as shown in Figure 3.15. We note the small scale clusters in Figure 3.15 correspond to clusters on the left side of the peaks in Figures 3.12 and 3.13. Similarly, the large scale clusters described in Figure 3.15 correspond to the clusters on the right side of the peaks in Figures 3.12 and 3.13. This behavior matches well with the findings of Aliseda et al. (2002), Monchaux et al. (2010), and Obligado et al. (2014), which tended to find that small scales follow a slope of $D \approx 1 - 1.4$ and large scales follow a slope of $D \approx 1.7 - 2$. The larger fractal dimensions indicate higher levels of roughness, and that the perimeter begins to have characteristics of a 2D shape because of this complexity.

Figure 3.15: $P$ versus $\sqrt{A_c}$ for each cluster. In both cases $Re_\lambda = 253$ and particles with $d_p = 1\mu m$ are used. These are representative examples of all test cases for their respective image areas.
3.5 PARTICLE VOID CHARACTERIZATION

Using the Voronoi analysis, void structures are identified as a collection of one or more connected void cells, i.e. $A > V_v \langle A \rangle$. The Voronoi analysis works well with the $20 \times 20 \text{mm}^2$ images, however, the individual particles are more challenging to detect for the $10 \times 10 \text{cm}^2$ images, as previously discussed. Therefore, we propose a novel method to detect voids using a pixel threshold/blob segmentation scheme, similar to the particle detection process. In this method the background is subtracted from the image, a Gaussian blur is applied, and the pixel intensity is scaled from 0 to 255. This process is depicted in Figure 3.16a-3.16c. A threshold is then selected based on the maximum pixel intensity from the void cells from the Voronoi analysis. A collection of connected pixels with lower intensity values compared to the threshold value is considered a blob. If the area of the blob is greater than or equal to $\langle A \rangle V_v$ from the Voronoi analysis then the blob is considered a void. This segmentation method works well when there is a significant deviation in the pixel intensity histogram of the blurred image, otherwise over segmentation occurs, which happens with the $20 \times 20 \text{mm}^2$ images. Therefore, we have adapted this method for the $20 \times 20 \text{mm}^2$ images. In this adapted method the following steps are taken:

1. subtract the background from the image
2. normalize the image intensity
3. detect all particle locations
4. create a binary image in which the particle coordinates and four surrounding points are set to an intensity of 255 in gray-scale
5. apply a Gaussian blur to the binary image
6. detect voids using the threshold scheme previously described.

An example of voids being detected using the blurring and Voronoi technique are shown Figures 3.16d and 3.16e, respectively. In general, they detect the same void areas, however, the Voronoi analysis has a tendency to connect more segments together.

The PDF for $\sqrt{A_v}/\eta$ for the $20 \times 20 \text{mm}^2$ and $10 \times 10 \text{cm}^2$ images are presented in Figures 3.17 and 3.18, respectively. The same data in Figures 3.17-3.18 are presented in terms of $A_v/\eta^2$ and $A_v$ in Figures A.11-A.18 in Appendix A. Each curve is an ensemble average of the results for 1,000 uncorrelated images. In general, the PDF for $\sqrt{A_v}/\eta$ between the two methods scale similarly. In all cases there is monotonic increase in $\sqrt{A_v}/\eta$ with increasing $Re_\lambda$. As before, an increase in $Re_\lambda$ also corresponds to an increase in $St_\eta$. Additionally, in Figures A.15-A.18 we find the curves collapse, similar to the PDF of $A_c$. Therefore, we conclude the variation among the curves in Figures 3.17 and 3.18 results from the decrease in $\eta$ with increasing $Re_\lambda$. 


We observe the void area PDF has a different behavior compared to the clustering area. They do not peak in the same way the clustering area PDF does. The peaks seen in the PDF for $\sqrt{A_v}/\eta$ from the Voronoi analysis is noise resulting from the cut-off size for being considered a void cell. The PDF of $\sqrt{A_v}/\eta$ indicates a decreasing probability in detecting voids with increasing area, as we would expect. In comparing the PDF of $\sqrt{A_c}/\eta$ for the $20 \times 20 \text{mm}^2$ and $10 \times 10 \text{cm}^2$ images, we again find a discrepancy in the range of void areas detected. However, we do observe more overlap in the void scales compared to the clustering scales between the two image sets. For instance, when $Re_\lambda = 253$ the voids in the $20 \times 20 \text{mm}^2$ images scale from $\sqrt{A_v}/\eta \sim 5 - 40$, whereas in the $10 \times 10 \text{cm}^2$ images they scale as $\sqrt{A_v}/\eta \sim 20 - 100$. Again the difference between the scales detected between the two image sets may be the difference in resolution. We note however, that the detection of voids requires an absence of particles, so it is less sensitive to the detection of particles. Thus, it may be that the two image sets are in fact detecting voids at different scales.
Figure 3.16: Comparison of the two different void detecting techniques.
Figure 3.17: PDF of $\sqrt{A_v/\eta}$ for $20 \times 20\text{mm}^2$ images. The dashed line indicates the void areas analyzed with the blurring analysis, and the solid line indicates the voids detected using the Voronoi analysis.
Figure 3.18: PDF of $\sqrt{A_v}/\eta$ for $10 \times 10\text{mm}^2$ images. The dashed line indicates the void areas analyzed with the blurring analysis, and the solid line indicates the voids detected using the Voronoi analysis.

As previously analyzed with particle clustering, a power-law fit following Eq. (3.12) has been applied to Figures A.11-A.14 to determine if the size distribution of voids follows the behavior of the energy spectrum. Again, the slope of the PDF for $A_v/\eta^2$ has been analyzed to compare with Goto & Vassilicos (2006). Goto & Vassilicos (2006) argued the area of voids should scale as $P(A_v) \sim A_v^{5/3}$ following the energy cascade in the inertial range. As previously stated, Moisy & Jiménez (2004) found that regions of intense vorticity tend to scale following a power-law fit with an exponential value of $-2$ supporting the hypothesis of Goto & Vassilicos (2006). Figures 3.19 and 3.20 present representative cases comparing
a power-law fit to the void distribution for both $20 \times 20\text{mm}^2$ and $10 \times 10\text{cm}^2$ images. In general, we find a closer agreement between the power-law fit and the PDF of $A_v/\eta^2$ for the void areas detected using the Voronoi analysis, compared to the blurring technique.

![Figure 3.19: Power-law fit with Eq. (3.12) to Figures A.11 and A.13, i.e. the PDF of $A_v/\eta^2$ for $20 \times 20\text{mm}^2$ images. The two images on the left correspond to the voids found using the blurring technique, and the two plots on the right side correspond to the voids found using the Voronoi technique. The solid line indicates the curve-fit, and the markers indicate data points.](image-url)
Figure 3.20: Power-law fit with Eq. (3.12) to Figures A.12 and A.14, i.e. the PDF of $A_v/\eta^2$ for $10 \times 10 \text{cm}^2$ images. The two images on the left correspond to the voids found using the blurring technique, and the two plots on the right side correspond to voids found using the Voronoi technique. The solid line indicates the curve-fit, and the markers indicate data points.

Applying a curve-fit to Figure A.11, the PDF for $A_v/\eta^2$ for $20 \times 20 \text{mm}^2$ image analyzed using the Voronoi analysis, results in a power-law exponential value between $-1.5$ and $-1.7$. Similarly, applying a curve-fit to Figure A.12, the PDF for $A_v/\eta^2$ for $10 \times 10 \text{cm}^2$ image analyzed using the Voronoi analysis, results in a power-law exponential value between $-1.5$ and $-1.8$. These findings agree well with Goto & Vassilicos (2006) who found the PDF for $A_v/\eta^2$ follows a power-law curve-fit with an exponential value between $-1.6$ and $-2.0$. However, as with particle clusters, the probability of a large void area decreases more rapidly than the power-law indicates. Applying a power-law curve-fit to Figures A.13 and A.14, i.e.
the PDF for $A_v/\eta^2$ for $20 \times 20\text{mm}^2$ image analyzed using the blurring analysis, results in a poorer fit. For the $20 \times 20\text{mm}^2$ images the curve-fit typically has a power-law exponential value ranging from $-1.39$ to $-1.53$. Similarly, for the $10 \times 10\text{cm}^2$ images the curve-fit results in an exponential value between $-1.39$ and $-1.75$.

Finally, as with particle clusters, we plot $\sqrt{A_v}$ versus $P$, the perimeter of the void areas, to comment on the self-similarity of the particle voids using fractal geometry. Example cases for the $20 \times 20\text{mm}^2$ and $10 \times 10\text{cm}^2$ images are presented in Figure 3.21 for the void areas detected with the Voronoi method. In both cases we note the region where the power-law fit matches the PDF for $A_v/\eta^2$ well, the slope of $\sqrt{A_v}/\eta$ versus $P/\eta$, $D$, is approximately $1.58 - 1.60$. In the region where the slope of the PDF of $A_v/\eta^2$ is greater than the power-law fit, $D \approx 1.83 - 1.85$. Thus, we observe similar fractal behavior as seen in Figures 3.19 and 3.20.

![Figure 3.21: $P$ versus $\sqrt{A_v}$ for each void. In both cases $Re_\lambda = 153$ and particles with $d_p = 3\mu\text{m}$ are used. These are representative examples for all test cases for their respective image areas.](image)
Chapter 4

Flow Structures and Particle Distribution

In this chapter we analyze the relationship between flow structures and particle clusters and voids. The three flow structures analyzed are: vortices, stagnation points, and jets using the zone classification system of Hunt & Wray (1990) introduced in Section 1.2. In order to analyze the relationship between flow structures and the spatial distribution of particles, we use an alternative set-up from that in Chapters 2 and 3, in which two cameras are simultaneously used to analyze the spatial distribution of particles and flow structures separately. Various optical tools are used to overlap the ROI in the two sets of images. Additionally, the geometric characterization of particle clusters and voids is examined with the new experimental set-up.

4.1 Experimental Set-up

The experimental set-up to analyze the relationship between the spatial distribution of particles and turbulent flow structures is shown in Figures 4.1 and 4.2. Aluminum-oxide particles with $d_p = 1 \mu m$ ($\rho_{Al_2O_3} = 3750 \frac{kg}{m^3}$) and fluorescent polymer microspheres with $d_p = 12 \mu m$ ($\rho_d = 1050 \frac{kg}{m^3}$) are injected into the flow using the same discharge system as previously described in Chapter 2. Aluminum-oxide particles with $d_p = 1 \mu m$ were chosen because they have a low Stokes number yet scatter light better than aluminum-oxide particle with $d_p = 0.5 \mu m$. In this experiment both 1 gram and 2.5 grams of the polymer microspheres are added to the discharge system, which corresponds to particle volume fractions of $\Phi_v \approx 1.5 e^{-5}$ and $3.7 e^{-5}$, respectively. In both cases 0.6 grams of aluminum-oxide particles are used. The fluorescent polymer microspheres emit light at a wavelength $\lambda = 602 nm$ when excited by a laser with $\lambda = 532 nm$. The IDT X3 camera (12.0 $\mu m$ sensor array) is used to capture the aluminum-oxide particles, and PIV analysis is performed on these images to assess the turbulent flow structures present. A longpass filter with a cut-off at $\lambda = 550 nm$ is used with the IDT Y3.2-S1 camera (10.8 $\mu m$ sensor array) to solely image the polymer microspheres. A polarizing beam splitter for wavelengths of $\lambda = 532 nm$ (Thorlabs PBS25-532) is used to ensure both cameras are focused on the same ROI. Both cameras have a 1024 $\times$ 1024 pixel array and are set to a $18.5 \times 18.5 mm^2$ ROI in the center of the turbulence box for a resolution of $18.02 \mu m$ per pixel. Two positive lenses with $f = 300 mm$ and $f = 200 mm$ are used to create a linear magnification of 0.67 for the X3 camera. However, because the Y3.2-S1 and X3 cameras have different sensor sizes a positive lens with $f = 400 mm$ is placed between the beam splitter and Y3.2-S1 camera in order to match the ROI of the X3 camera. Originally a focal length larger than $f = 300 mm$ was going to be used for a larger ROI, however,
Figure 4.1: Dual camera set-up to analyze the relationship between the spatial distribution of particles and turbulent flow structures.

Figure 4.2: Camera and laser set-up to study the relationship between the spatial distribution of particles and turbulent flow structures.
increasing the focal length decreases the amount of light the cameras receive and there was difficulty in supplying the X3 camera with sufficient light. For a more accurate PIV analysis, a notch filter should have been placed in front of the X3 camera to filter out the fluorescent polymer microspheres and only image the aluminum-oxide particles. However, introducing an additional optical element in front of the X3 camera would have also further reduced the amount of light received.

In order to allow more light into the cameras, the laser has been repositioned from the set-up described in Chapter 2, so that the laser is placed directly in front of the turbulence box as shown in Figure 4.2. The laser sheet is still produced with the ND-YAG laser ($\lambda = 532\text{nm}$) passing through a spherical ($f = 400\text{mm}$) and cylindrical ($f = -9.7\text{mm}$) lens system. In this experiment the X3 camera is synced to the lasers using a CIO-CTR10 counter board, and the Y3.2-S1 camera is synced to the X3 camera using a master/slave configuration. The pulse delays for each image pair ranges from $\Delta t = 10\mu s$ to $\Delta t = 40\mu s$ depending on the turbulence level.

To determine the image shift between the two cameras, images of the target shown in Figure 4.3 are cross-correlated and all microsphere positions are accordingly adjusted.

![Cross-correlation between images](image)

(a) Cross-correlation between images

![X3 camera](image)

(b) X3 camera

![Y3.2-S1 camera](image)

(c) Y3.2-S1 camera

Figure 4.3: Targets used to determine the displacement between cameras.

A PIV analysis with the modified set-up has first been performed without the polymer microspheres, from which the average turbulence parameters are presented in Table 4.1. These are calculated with 1,100 independent PIV measurements. Again, the WALPT5 code
is used to process the PIV data with a Langrangian parcel tracking mode with two passes. The PIV analysis uses a window and step size of 32 pixels \( \times \) 32 pixels and 16 pixels \( \times \) 16 pixels, respectively (50% overlap). The PIV window, therefore, ranges from \( 4.2\eta \) to \( 9.6\eta \) for \( Re\lambda = 181 \) to 319.

\[
A (V_{pp}) \quad \bar{\epsilon} \quad \tau_\eta \quad \lambda \quad Re_\lambda \quad \eta \quad \bar{u}' \quad \bar{v}' \quad \bar{q}^2 \quad T_e \quad L_e
\]

<table>
<thead>
<tr>
<th>A (V_{pp})</th>
<th>\bar{\epsilon} (m^2/s^3)</th>
<th>\tau_\eta (ms)</th>
<th>\lambda (mm)</th>
<th>Re_\lambda</th>
<th>\eta (\mu m)</th>
<th>\bar{u}' (m/s)</th>
<th>\bar{v}' (m/s)</th>
<th>\bar{q}^2 (m^2/s^2)</th>
<th>T_e (ms)</th>
<th>L_e (mm)</th>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>9.86</td>
<td>1.23</td>
<td>3.60</td>
<td>181</td>
<td>136</td>
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<td>41</td>
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<td>4</td>
<td>60.22</td>
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<td>2.65</td>
<td>243</td>
<td>86</td>
<td>1.37</td>
<td>1.31</td>
<td>5.39</td>
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<tr>
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<td>196.28</td>
<td>0.27</td>
<td>2.14</td>
<td>285</td>
<td>64</td>
<td>2.00</td>
<td>1.97</td>
<td>11.85</td>
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<td>40</td>
</tr>
<tr>
<td>7</td>
<td>263.86</td>
<td>0.24</td>
<td>2.10</td>
<td>319</td>
<td>60</td>
<td>2.27</td>
<td>2.21</td>
<td>15.08</td>
<td>19</td>
<td>43</td>
</tr>
</tbody>
</table>

Table 4.1: Turbulence parameters calculated under various forcing amplitudes with a forcing frequency of \( f = 100\text{Hz} \), where the Large-Eddy PIV technique with the corrected Smagorinsky constant is used to calculate \( \epsilon \). A YAG laser is used for illumination.

Using the polymer microspheres we again find the assumptions \( d_p \ll \eta \) and \( \rho_d \gg \rho \) are satisfied. However, the assumption that \( Re_p < 1 \) is not satisfied as seen in Table 4.2.

\[
Re_\lambda \quad 181 \quad 243 \quad 285 \quad 319
\]

\[
Re_p \quad 0.60 \quad 1.10 \quad 1.60 \quad 1.82
\]

\[
d_p/\eta \quad 0.09 \quad 0.14 \quad 0.18 \quad 0.20
\]

\[
V_{air}/\bar{u}' \quad 0.006 \quad 0.002 \quad 0.002 \quad 0.002
\]

Table 4.2: \( Re_p, d_p/\eta, V_{air}/\bar{u}' \) and for various turbulence levels.

Since the condition that \( Re_p < 1 \) has not been met except for \( Re_\lambda = 181 \), we apply a correction condition to calculate the relaxation time of the polymer microspheres as Hwang & Eaton (2004); Wood et al. (2005)

\[
\tau_p = \frac{d_p^2 \rho_d}{18 \mu} \frac{1}{1 + 0.15 Re_p^{0.687}} \quad (4.1)
\]

The relaxation time and Stokes number for each test case for the polymer microspheres is given in Table 4.3. We note that the relaxation time for the polymer microspheres is significantly greater than that of the aluminum-oxide particles used in Chapter 3. The same parameters listed in Tables 4.2 and 4.3 are presented for the aluminum-oxide particles with \( d_p = 1\mu m \) in Table A.7 in Appendix A.
Table 4.3: Relaxation time, $\tau_p$, and Stokes number, $St_\eta$, for various forcing amplitudes for the polymer microspheres. In all cases the forcing frequency is $f = 100\text{Hz}$.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_p$ (ms)</td>
<td>0.51</td>
<td>0.48</td>
<td>0.46</td>
<td>0.45</td>
</tr>
<tr>
<td>$St_\eta$</td>
<td>0.41</td>
<td>0.97</td>
<td>1.68</td>
<td>1.91</td>
</tr>
</tbody>
</table>

The mean velocity field is presented in Figure 4.4 with several streamlines marked in red. Again we expect to see zero mean flow for isotropic turbulence. In general, we find the mean velocity in the $x$ and $y$-directions are centered near 0m/s, however, there is an overall trend of a positive vertical mean velocity. It is more clearly seen that the flow is isotropic in the new ROI in Figure 4.5, in which $u_{rms}/v_{rms}$ tends to approach unity and the scatter plot of the fluctuating velocity components is roughly azimuthally symmetric. This is reflected in Figure 4.6, in which the PDF of the rms fluctuating velocity at three points in the flow is presented. We note Figure 4.6 is also presented on a log scale in Figure A.19 in Appendix A. In Figure 4.6 we find similar behavior for both the $u$ and $v$-components for all points in the flow field. We also observe the flow is homogeneous in Figure 4.7, in which $u_{rms}/\langle u_{rms} \rangle$ and $v_{rms}/\langle v_{rms} \rangle$ fields approach unity. We note that the turbulence parameters listed in Table 4.1 do not agree with those found in Table 2.1 for the same forcing amplitudes. We suspect this fact results from the change in position of the light sheet, thus, resulting in the analysis of the turbulent flow field in a different position of the turbulence box.

There are several benefits to this set-up compared to that previously used in Chapters 2 and 3, which are:

1. The resolution of imaging particles is increased by decreasing the ROI to $18.5 \times 18.5\text{mm}^2$, and increasing the size of the particles to $d_p = 12\mu\text{m}$.

2. Decreasing the ROI from $10 \times 10\text{cm}^2$ to $18.5 \times 18.5\text{mm}^2$ ensures the entire region we are investigating is a homogeneous isotropic flow. As shown in Chapter 2, only the center portion of the $10 \times 10\text{cm}^2$ images is homogeneous and isotropic, which is approximately a $50 \times 50\text{mm}^2$ region.

3. Decreasing the ROI also increases the resolution of the flow structures, however, we note this also decreases the image size to approximately $0.47L_e$.

4. Decreasing the ROI also decreases the spread of illumination previously discussed in Chapter 3.3. This alleviates the problem that particles are more focused at the center of the image causing particles to be more likely detected in the center of the image because of the light distribution.

5. Performing PIV while simultaneously analyzing particle clustering with two different cameras, eliminates the problem of using a PIV analysis on particles that do not faithfully follow the flow field.
6. By analyzing the clustering of the polymer microspheres rather than aluminum-oxide particles, we are more confident in the size distribution and spherical shape of the particles.

![Mean velocity flow field with streamline outlined in red. In general, this indicates a stagnation point in the center of the flow field with some motion upward.](image)

(a) $Re_\lambda = 181$

(b) $Re_\lambda = 243$

(c) $Re_\lambda = 285$

(d) $Re_\lambda = 319$
Figure 4.5: $u_{rms}/v_{rms}$ is plotted to indicate isotropic flow when approaching unity, and a scatter plot of $u'$ and $v'$ indicates isotropy when azimuthally symmetric.
Figure 4.6: PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field.
Figure 4.7: $\frac{u_{rms}}{\langle u_{rms} \rangle}$ and $\frac{v_{rms}}{\langle v_{rms} \rangle}$ are plotted to indicate homogeneity when approaching unity.
4.2 Particle Clustering and Void Geometric Characteristics

Figure 4.8 presents a sample raw and binary image of the polymer microspheres detected from the Y3.2-S1 camera. Similar to the experiments performed in Chapter 3, we find a significant loss of particles from particles settling and being pushed to the side walls during the experiment. Figure 4.9 presents the number of polymer microspheres detected over time for four test cases for both 1 and 2.5 grams of polymer microspheres being added to the particle discharge system.

![Sample image of the polymer microspheres detected from the Y3.2-S1 camera](image)

(a) Raw image  
(b) Binary image

Figure 4.8: Sample image of the polymer microspheres detected from the Y3.2-S1 camera.
Chapter 4: Flow Structures and Particle Distribution

(a) After the injection of 1 gram of polymer microspheres

(b) After the injection of 2.5 grams of polymer microspheres

Figure 4.9: Number of particles detected over time after injection.

Particles are detected in the same manner as described in Chapter 3.2, with the exception that the pixel intensity is not initially normalized to account for the spread of illumination because the ROI has been reduced. Additionally, a condition is set such that for an image to be evaluated a minimum number of particles must be detected, in which the minimum is set based on the initial number of particles added to the system. The minimum is chosen to achieve a balance between a high particle number density and a high number of samples. For 1 gram of polymer microspheres a minimum of 200 detected particles is set, yielding 205, 100, 132, and 72 samples for $Re_\lambda = 181, 243, 285, 319$. Similarly, for 2.5 grams of polymer microspheres a minimum of 400 detected particles is set, yielding 55, 59, 40, and 63 samples for $Re_\lambda = 181, 243, 285, 319$.

The Voronoi area PDF for the four turbulence levels with 2.5 grams of polymer microspheres is presented in Figure 4.10, and Table 4.4 presents the corresponding $V_c$ and $V_v$ values. The same data for 1 gram of polymer microspheres is presented in Figure A.20 and Table A.8. In addition, the RDF for the data presented in Figure 4.10 is presented in Figure A.21 for comparison purposes for future researchers.
Figure 4.10: $A/\langle A \rangle$ for various turbulence levels for 2.5 grams of polymer microspheres.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_v$</td>
<td>2.03</td>
<td>1.99</td>
<td>2.03</td>
<td>2.15</td>
</tr>
<tr>
<td>$V_c$</td>
<td>0.64</td>
<td>0.60</td>
<td>0.66</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Table 4.4: $V_v$ and $V_c$ for various turbulence levels for 2.5 grams of polymer microspheres.

The PDF of $\sqrt{A_c}/\eta$ and $\sqrt{A_v}/\eta$ are presented in Figures 4.11 and 4.12, respectively, where clusters and voids are detected using a Voronoi analysis. The curves in Figures 4.11 and 4.12 behave similarly irrespective of whether 1 gram or 2.5 grams of polymer microspheres are used. These data are also presented in Figures A.22 and A.23 in Appendix A following the format of Chapter 3. We find the general behavior in Figures 4.11 and 4.12 match those of Figures 3.12-3.13 and 3.17-3.18 for clusters and voids, respectively. We also observe $\sqrt{A_c}/\eta$ and $\sqrt{A_v}/\eta$ increase monotonically with increasing $Re_\lambda$. However, again this seems to result from the decreasing $\eta$ values with increasing $Re_\lambda$ based on the collapse of curves in Figures A.22a and A.23a. The characteristic cluster and void length scales in this set-up are consistently greater than those found for both cases in Chapter 3, where the range of characteristic cluster length scales are outlined in Table 4.5. These results agree more closely with those of Aliseda et al. (2002), Wood et al. (2005), and Obligado et al. (2014), which as previously discussed, found a typical cluster length scale of $\sim 10\eta$. 
Figure 4.11: $\sqrt{A_c}/\eta$, where $\bullet$ indicates the 1 gram of polymer microspheres and $\cdots\star$ indicates the 2.5 grams of polymer microspheres.
Figure 4.12: $\frac{\sqrt{A_v}}{\eta}$, where $-\bullet$ indicates the 1 gram of polymer microspheres and $--\ast$ indicates the 2.5 grams of polymer microspheres.

Table 4.5: Range of characteristic cluster length scales indicated in Figure 4.11.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{A_c}/\eta$</td>
<td>7-9</td>
<td>11-14</td>
<td>15-18</td>
<td>15-20</td>
</tr>
</tbody>
</table>

Not only are the characteristic cluster and void length scales presented in Figures 4.11 and 4.12 larger than those in Figures 3.12-3.13 and 3.17-3.18, but the actual cluster and void sizes are also larger when comparing Figures A.22a and A.23a with A.7-A.8 and A.15-A.18, respectively. Therefore, it is not simply a decrease in $\eta$ to result in larger $\sqrt{A_c}/\eta$ and $\sqrt{A_v}/\eta$ values in the current set-up, but the size of clusters and voids are simply larger. Three main
differences in the current set-up that may be responsible for the increase in cluster and void length scales are the lower particle volume fraction, increased imaging resolution of particles, and increased particle diameters.

Again we have applied a power-law fit following Equation 3.12, as shown in Figure 4.13. The exponential value in applying a power-law fit to $A_c/\eta^2$ is approximately $b \approx -2$ for each case, which matches the values found by Monchaux et al. (2010) and Sumbekova (2016). The power-law fit matches the PDF of $A_v/\eta^2$ curve near the peak as before, but does not match the larger scales as closely. In contrast, the power-law fit for the PDF of $A_v/\eta^2$ matches the curve well for all scales. The exponential value in applying a power-law fit to the PDF of $A_v/\eta^2$ varies from $b \approx -1.1$ to $-1.3$, which does not match the power-law fit used to describe the behavior of intense vorticity scales by Moisy & Jiménez (2004).

![Figure 4.13: Power-law fit to Figures A.22b and A.23b for 1 gram of polymer microspheres. The solid line indicates the curve-fit and the markers indicate data points.](image)

Finally, we again present two representative examples of the perimeter plotted against the square root of area for particle clusters and voids in Figure 4.14 to analyze self-similarity. We find in both cases the slope of a linear-fit is nearly constant with $D \approx 1.3 - 1.5$, thus, indicating self-similarity for all scales.
Figure 4.14: Representative test cases for $P$ versus $\sqrt{A_c/\eta}$ for clusters and voids. In both cases $Re_\lambda = 285$ with 1 gram of polymer microspheres.

### 4.3 Flow Structure Identification

In this analysis, the stream zones are defined as regions where $-0.5Q_{rms} < Q(x_1, x_2) < Q_{rms}$ and $u'^2(x_1, x_2) > u_{rms}^2$, and convergence zones are defined where $Q(x_1, x_2) > Q_{rms}$. For 2D flow measurements the second invariant of the velocity gradient tensor is calculated as

$$Q(x_1, x_2) = -\frac{\partial u_1}{\partial x_1} \frac{\partial u_2}{\partial x_2} + \frac{\partial u_1}{\partial x_2} \frac{\partial u_2}{\partial x_1} + \frac{1}{2} \left( \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} \right)^2. \quad (4.2)$$

In this study we have not taken pressure measurements, and therefore, the differentiation between eddy and shear zones based on pressure is not possible following Hunt & Wray (1990). Thus, we have simply identified eddy zones where $Q(x_1, x_2) < -0.5Q_{rms}$. We note a further shortcoming in the current work is the 2D nature of the analysis, and thus, we are unable to examine the particle number density against the 3D flow field structures.

Figures 4.15 and 4.16 present two examples of $u'$ and $Q$ fields overlaid with the velocity vectors, which correspond to Figures 4.17 and 4.18, respectively. Figures 4.17 and 4.18 present the velocity field with the three flow zones identified and various streamlines marked in red. In Figures 4.17 and 4.18 the green regions are convergence zones, blue regions are eddy zones, and yellow regions are stream zones. To compare the results from the Voronoi analysis with the flow field structures, the flow field information has also been scaled to match that of the image using bilinear interpolation.
The two most common types of streamlines found in eddy zones are: closed or spiral-like streamlines such as at points (2)-(4) in Figure 4.17 and (9)-(10) in Figure 4.18, or streamlines with high level of curvature but no spiral like formation as at points (12)-(13) in Figure 4.18. In the convergence zones we consistently observe converging and diverging streamlines as expected, and note that they are typically located near eddy zones. There are three flow behaviors that seem to be responsible for the presence of convergence zones:

- The flow is being diverted away from a stagnation point resulting from two impinging jets such as at point (1) in Figure 4.17.
- Two jets of fluid are converging together such as at point (6) in Figure 4.17. Typically we observe this behavior when one or both jets of fluid is experiencing rotation from a nearby eddy zone.
- A jet of fluid encounters an eddy zone, but only a portion of the stream is rotated by the eddy, and thus, the streamlines begin to diverge. We see an example of this at point (7) in Figure 4.18. However, we note in this same convergence zone two jets are also converging at point (8).

Typically 70 – 75% of the flow field is characterized as belonging to one of the three zones. The breakdown of the overall flow domain into each flow zone is summarized in Table 4.6.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy</td>
<td>6.00%</td>
<td>6.41%</td>
<td>6.54%</td>
<td>6.77%</td>
</tr>
<tr>
<td>Convergence</td>
<td>11.09%</td>
<td>10.56%</td>
<td>10.54%</td>
<td>11.92%</td>
</tr>
<tr>
<td>Stream</td>
<td>53.29%</td>
<td>58.82%</td>
<td>54.72%</td>
<td>55.72%</td>
</tr>
<tr>
<td>Amorphous</td>
<td>29.62%</td>
<td>24.21%</td>
<td>28.20%</td>
<td>25.59%</td>
</tr>
</tbody>
</table>

Table 4.6: Distribution of the three flow zones among the flow domain for 2.5 grams ($\Phi_v \approx 3.7e - 5$) of particles.
Figure 4.15: $Q$ and $u'$ for a single test case where $Re_{\lambda} = 243$. 

(a) $Q$-criteria ($1/s^2$)

(b) $u'$ field (m/s)
Figure 4.16: $Q$ and $u'$ for a single test case where $Re_\lambda = 285$. 

(a) $Q$-criteria (1/s^2)

(b) $u'$ field (m/s)
Figure 4.17: Detected zones in the flow field where $Re_\lambda = 243$: green denotes the convergence zones, blue the eddy zones, and yellow the stream zones. The red lines are streamlines of the flow field.
Figure 4.18: Detected zones in the flow field where $Re_\lambda = 285$: green denotes the convergence zones, blue the eddy zones, and yellow the stream zones. The red lines are streamlines of the flow field.

In addition to comparing the location of turbulent flow structures to particle voids and clusters identified by the Voronoi analysis, we also compare the difference in the particle number density with various turbulent flow structures. The average particle number density is calculated within 32 pixel $\times$ 32 pixel interrogation windows with 50% overlap, following the same methodology as the PIV analysis. The average particle number density in each window is then normalized by the overall spatial average particle number density for each
frame. It is important that the particle number density is normalized because the number of particles over time declines quickly. Figure 4.19 presents an example binary image, the void and cluster Voronoi cells highlighted, and the normalized particle number density map. The average number of particles detected per window for 1 and 2.5 grams of polymer microspheres are 0.26 and 0.51, respectively.

Figure 4.19: Comparison between the binary image, detected void and cluster regions, and particle number density.
4.4 **Turbulent Flow Structures and Particle Distribution**

The percentage breakdown of the total image into regions of particle clusters, particle voids, and regions classified neither particle clusters or voids is presented in Table 4.7, and Figure A.24 in Appendix A. We expect a significant increase in particle clustering at $St_\eta = 0.97$ and $1.68$. However, the percentage of the flow field described to be a region of particle clusters and voids does not seem to alter much with respect to increasing $Re_\lambda$, and subsequently increasing $St_\eta$.

In analyzing the distribution of particles with respect to local turbulent flow structures, we begin by addressing the question, “How are particle voids and clusters distributed among the three flow zones?” To address this question we analyze what portion of the particle void/cluster regions fall in an eddy zone, convergence zone, stream zone, or an amorphous region not classified. The breakdown of how particle clusters and voids are distributed among the flow zones is presented in Tables 4.8 and 4.9, respectively, and Figure A.25. Furthermore, the average normalized particle number density in the three flow zones is presented in Table 4.10 and Figure A.26, and the standard deviation of the average particle number density in each zone is given in Table 4.11. The corresponding the probability density function of the normalized particle number density in the three flow zones is presented in Figure 4.20, and is presented on a log-scale in Figure A.27. We note all results presented in this section are for 2.5 grams of polymer microspheres, but these same results are presented for 1 gram of polymer microspheres in Tables A.9-A.16 and Figure A.28-A.33 in Appendix A.

Based on the work of Squires & Eaton (1991b) and Wang & Maxey (1993) we expect the regions of particle clustering to be composed of a greater portion of stream and convergence zones compared to the overall image. Similarly, we expect the regions of particle voids to be composed of a greater portion of eddy regions compared to the overall image. Furthermore, we hypothesize with increasing $Re_\lambda$, and thereby increasing $St_\eta$, the percentage of void and cluster regions corresponding to eddy, convergence, and stream zones should be more pronounced because the relaxation time has increased for the particles.

However, we find from Table 4.6 that regions of particle clusters are composed of a greater portion of stream and eddy zones compared to the overall image, and a lower portion of convergence and amorphous zones from the overall image. The breakdown of the particle void regions among the flow zones is similar to that of the particle cluster regions, which is unexpected. It is also composed of a greater portion of stream and eddy zones compared to the overall image, and a lower portion of convergence and amorphous zones. In comparison, Table 4.10 shows that the average normalized particle number density in eddy and stream zones are slightly greater than 1, but the convergence zones are slightly less than 1. The fact that the standard deviations in Table 4.11 are greater than the mean values found in Table 4.10 indicates the non-conclusiveness of these results. In Figure 4.20 we observe the curves collapse on themselves suggesting there is little influence of turbulent flow structures on the
Chapter 4: Flow Structures and Particle Distribution

spatial distribution of particles, which again is unexpected. However, this does reflect the fact that the average normalized particle number density remained near 1 for all flow zones in Table 4.10. These findings are also inconsistent with those of Squires & Eaton (1991b) which found convergence zones to have the greatest particle concentration, and eddy zones to have the lowest particle concentration. Additionally, this is inconsistent with the theoretical work of Maxey (1987) who suggested particles cluster in regions of high strain and low vorticity as discussed in Chapter 1.1.3. Finally, in Figure 4.20 we observe an increasing spread in the PDF with increasing $Re_\lambda$ and a decrease in a bi-modal behavior.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>8.12%</td>
<td>7.07%</td>
<td>10.24%</td>
<td>11.55%</td>
</tr>
<tr>
<td>Void</td>
<td>31.07%</td>
<td>32.28%</td>
<td>32.35%</td>
<td>29.91%</td>
</tr>
<tr>
<td>Amorphous</td>
<td>60.81%</td>
<td>60.65%</td>
<td>57.41%</td>
<td>58.54%</td>
</tr>
</tbody>
</table>

Table 4.7: Breakdown of the image into regions of particle clusters, particle voids, and neither for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy</td>
<td>7.27%</td>
<td>9.27%</td>
<td>8.56%</td>
<td>9.50%</td>
</tr>
<tr>
<td>Convergence</td>
<td>9.70%</td>
<td>7.65%</td>
<td>7.30%</td>
<td>10.76%</td>
</tr>
<tr>
<td>Stream</td>
<td>59.56%</td>
<td>67.28%</td>
<td>61.03%</td>
<td>61.65%</td>
</tr>
<tr>
<td>Amorphous</td>
<td>23.47%</td>
<td>15.80%</td>
<td>23.11%</td>
<td>18.09%</td>
</tr>
</tbody>
</table>

Table 4.8: Breakdown of how cluster Voronoi cells are distributed among the flow zones for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy</td>
<td>8.56%</td>
<td>9.58%</td>
<td>7.83%</td>
<td>10.14%</td>
</tr>
<tr>
<td>Convergence</td>
<td>8.36%</td>
<td>6.64%</td>
<td>6.48%</td>
<td>9.13%</td>
</tr>
<tr>
<td>Stream</td>
<td>58.60%</td>
<td>65.97%</td>
<td>63.98%</td>
<td>63.26%</td>
</tr>
<tr>
<td>Amorphous</td>
<td>24.48%</td>
<td>17.81%</td>
<td>21.71%</td>
<td>17.47%</td>
</tr>
</tbody>
</table>

Table 4.9: Breakdown of how the void Voronoi cells are distributed among the flow zones for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.
Table 4.10: The average normalized particle number density in each flow zone for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy</td>
<td>1.03</td>
<td>1.02</td>
<td>1.00</td>
<td>1.02</td>
</tr>
<tr>
<td>Convergence</td>
<td>0.96</td>
<td>0.99</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>Stream</td>
<td>1.03</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Table 4.11: The standard deviation of the particle number density in each flow zone for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy</td>
<td>1.64</td>
<td>1.53</td>
<td>1.49</td>
<td>1.40</td>
</tr>
<tr>
<td>Convergence</td>
<td>1.56</td>
<td>1.50</td>
<td>1.52</td>
<td>1.35</td>
</tr>
<tr>
<td>Stream</td>
<td>1.62</td>
<td>1.49</td>
<td>1.52</td>
<td>1.38</td>
</tr>
</tbody>
</table>
Figure 4.20: PDF of the normalized particle number density in each flow zone for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.

(a) $Re_\lambda = 181$

(b) $Re_\lambda = 243$

(c) $Re_\lambda = 285$

(d) $Re_\lambda = 319$
Following Squires & Eaton (1991b) and Wang & Maxey (1993) we have also analyzed the normalized enstrophy field

\[ \omega = \frac{(\Omega_z \cdot \Omega_z)^{1/2}}{\langle (\Omega_z \cdot \Omega_z)^{1/2} \rangle}, \]

and normalized strain rate

\[ s = \frac{(S_{ij} \cdot S_{ij})^{1/2}}{\langle (S_{ij} \cdot S_{ij})^{1/2} \rangle}, \]

versus the normalized particle number density, \( \langle C \rangle \), as shown in Figure 4.21. In order to construct Figure 4.21 we have followed the same methodology as Wang & Maxey (1993). The enstrophy values, \( \omega_{ij} \), for each window of the PIV analysis are divided into bins \( 0.1(m-1) \leq \omega_{ij} < 0.1m \), where \( m = 1,2,3,... \) etc. The average normalized particle number density for each bin is then calculated from the normalized particle number density corresponding to the windows found to belong to each bin. This gives an indication of the global trends of particle clusters and voids in regions of high/low strain and vorticity. A similar plot is presented in Figure 4.22, in which second invariant of the velocity gradient tensor is plotted versus the average normalized particle number density following the same methodology to produce Figure 4.21.
Figure 4.21: Normalized particle number density versus the normalized strain rate and enstrophy for 2.5 grams ($\Phi_v \approx 3.7e - 5$) of polymer microspheres.
Chapter 4: Flow Structures and Particle Distribution

Figure 4.22: Particle number density versus the second invariant of the velocity gradient tensor, $Q$, for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.

Additionally, we have examined these same trends by analyzing the average normalized enstrophy and strain rate values in the regions defined to be particle clusters/voids following the Voronoi analysis, as shown in Figure 4.23. This analysis is presented in Tables 4.12 and 4.13, and in Figure A.34.
Chapter 4: Flow Structures and Particle Distribution

- (a) Cluster regions in the rate strain field
- (b) Void regions in the strain rate field
- (c) Cluster regions in the enstrophy field
- (d) Void regions in the enstrophy field

Figure 4.23: Regions of particle cluster/void regions are detected in the normalized enstrophy and strain rate fields, and the average values are calculated within these regions.

<table>
<thead>
<tr>
<th>$Re_{\lambda}$</th>
<th>$\omega$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>181</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>243</td>
<td>0.92</td>
<td>0.94</td>
</tr>
<tr>
<td>285</td>
<td>0.95</td>
<td>0.92</td>
</tr>
<tr>
<td>319</td>
<td>0.95</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 4.12: Average normalized enstrophy and scaler-strain values for regions of particle clusters for 2.5 grams ($\Phi_v \approx 3.7e - 5$) of polymer microspheres.
We observe in Figure 4.21 that the normalized particle number density only begins to diverge from the average when $\omega$ and $s$ are approximately greater than 5. This does not follow the behavior of either Squires & Eaton (1991b) or Wang & Maxey (1993), which saw decreasing particle concentration with increasing $\omega$. We also note the $Re_\lambda$ does not seem to affect the behavior in particle concentration versus $\omega$ and $s$. The behavior in Figure 4.21 suggests that either a certain strength in local vorticity or strain rate are required to affect the particle distribution or that points of larger vorticity and strain rate are rare events with less data points resulting in $< C >$ diverging from the overall average. This same behavior is also noted in Figure 4.22, in which the normalized particle number density only deviates from the average at more extreme values of $Q$.

Tables 4.12 and 4.13 indicate that the average normalized enstrophy and strain rate in the regions of particle clusters and voids are lower than the overall average. This is not reflective of what was found in Tables 4.8-4.10, which indicate that clusters and voids are composed of a larger portion of eddy regions than the overall average. We suspect that there are peak values in the normalized strain rate and enstrophy fields, but which take a small area raising the overall averages causing the majority of these fields to fall under their overall average. We do note, however, that there seems to be an upward trend in the average enstrophy and strain rate values for regions of particle clusters and voids with increasing $Re_\lambda$ most clearly seen in Figure A.34.

To further compare with the results from Wang & Maxey (1993), we also present the normalized particle number density versus the horizontal, $u_1$, and vertical, $u_2$, velocities of the carrier fluid in Figure 4.24. Wang & Maxey (1993) showed no preference for particle concentration with respect to the horizontal velocity, but did find a greater concentration of particles in the downward direction following gravity. The same methodology to produce Figure 4.21 has been used to produce Figure 4.24. However, to address the question, “Do particle clusters found in stream zones tend to occur when the velocity of the local carrier fluid is in the downward direction?” we have also presented the normalized particle number density versus the horizontal and vertical velocities of the carrier fluid for only the regions of the flow field classified as a stream zone. These results are presented in Figure 4.25.
Figure 4.24: Particle number density versus horizontal and vertical local velocities for 2.5 grams ($\Phi_v \approx 3.7e-5$) of polymer microspheres.
Figure 4.25: Particle number density versus horizontal and vertical local velocities for only the regions of the flow field classified as a stream zones for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.

From Figure 4.24 we again find the particle number density only deviates from the average at the most extreme levels of $|u_1|$ and $|u_2|$, where gravity is in the direction of $-u_2$. However, this indicates that in Figures 4.21-4.22 and 4.24-4.25 where the normalized particle number density diverges from the overall average is not from requiring a certain strength in vorticity, strain, or velocity. Rather these points indicate a rarer event with less data points to move the particle number density point to the overall average. If this was not the case then we would see the particle number density diverge at $|2|m/s for all cases in Figure 4.24, as in Figure 4.24a. Thus, points in Figures 4.21-4.22 and 4.24-4.25 in which $< C > = 0$
are suspected to result from bins in which either no strain rate or enstrophy values fall into, or one window in the PIV analysis falls into that bin with a corresponding particle number density of 0. In Figures 4.24-4.25 we also do not observe the asymmetric behavior in the particle concentration increasing in the direction of gravity for the vertical velocity, as expected.

Eaton & Fessler (1994) suggested that particles with greater inertia have more difficulty following instantaneous curved streamlines leading to preferential concentration. Therefore, we have also examined the spatial distribution of particles near certain streamline behaviors, in order to more specifically focus on particular flow structures. Our analysis explores the average normalized particle number density near the four streamline behaviors depicted in Figure 4.26. In this analysis the velocity flow field for each image pair is displayed, and the user is prompted to select where the streamlines are to be displayed. The user is then again prompted to outline the areas surrounding the streamlines they wish to analyze, as shown in Figure 4.27, and the particle number density is calculated in the selected region. This allows us to more specifically comment on the following questions:

1. Do particles have a difficult time following curved streamlines?
2. Are particles more likely to cluster near a stagnation point or in a jet of fluid?
3. How does the particle number density alter near stagnation points versus the converging of two jets?

One of the main drawbacks to this analysis is that streamlines are not Galilean invariant, and thus, are dependent on the coordinate system used. In this analysis the streamlines are defined with respect to the fixed reference frame. The average normalized particle number density near the various streamline patterns are displayed in Table 4.14 and Figure 4.28. Additionally, the standard deviation of the particle number density near the various streamline patterns is presented in Table 4.15.
Figure 4.26: Streamlines analyzed for particle response behavior.
Figure 4.27: Depicted is the velocity flow field marked with red streamlines. The regions which the user has chosen to investigate the particle number density are outlined in black polygons. Again, convergence zones are highlighted in green, eddy zones in blue, and stream zones in yellow. The particle number density identifies, which points are used to calculate the average particle number density for each area selected by the user.

<table>
<thead>
<tr>
<th></th>
<th>$Re_{\lambda}$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spiral/Closed</td>
<td>1.06 1.05 0.97</td>
<td>0.98</td>
<td>0.09</td>
<td>0.09</td>
<td>0.20</td>
</tr>
<tr>
<td>Stagnation Point</td>
<td>1.05 1.05 1.02</td>
<td>1.00</td>
<td>0.10</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>Two Converging Jets</td>
<td>1.07 1.06 1.01</td>
<td>0.93</td>
<td>0.13</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td>High Velocity Jet</td>
<td>1.04 1.07 0.97</td>
<td>0.97</td>
<td>0.14</td>
<td>0.13</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Table 4.14: Average normalized particle number density near various streamlines for 2.5 grams ($\Phi_v \approx 3.7e - 5$) of polymer microspheres.

<table>
<thead>
<tr>
<th></th>
<th>$Re_{\lambda}$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spiral/Closed</td>
<td>0.13 0.09 0.09</td>
<td>0.20</td>
<td>0.09</td>
<td>0.09</td>
<td>0.20</td>
</tr>
<tr>
<td>Stagnation Point</td>
<td>0.16 0.07 0.10</td>
<td>0.17</td>
<td>0.10</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>Two Converging Jets</td>
<td>0.14 0.08 0.09</td>
<td>0.13</td>
<td>0.13</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td>High Velocity Jet</td>
<td>0.13 0.07 0.13</td>
<td>0.14</td>
<td>0.13</td>
<td>0.07</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Table 4.15: Standard deviation of the normalized particle number density near various streamlines for 2.5 grams ($\Phi_v \approx 3.7e - 5$) of polymer microspheres.
Figure 4.28: PDF of the average normalized particle number density near various streamlines for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.

The results from Table 4.14 and Figure 4.28 suggest that there is little difference in the normalized particle number density among these four flow features. Again the fact that the standard deviations in Table 4.15 is greater than the mean values found in Table 4.14 indicates the non-conclusiveness of these results. What is most visible from Figure 4.28 is the spread in the PDF of the particle number density near jets of fluid, however, this is most likely because it is much more numerous feature.
5.1 Conclusions

The current work aims at examining the geometric characterization and spatial distribution of particle clusters and voids in homogeneous isotropic turbulence. Specifically, we present the first known experimental measurements analyzing the relationship between turbulent flow structures and the spatial distribution of particle voids and clusters in homogenous isotropic turbulence, following the numerical work of Squires & Eaton (1991b) and Wang & Maxey (1993). Returning to the original questions presented in Chapter 1.2 we derive the following conclusions:

1. What are the characteristic length scales used to describe clusters and voids?

   This was examined in Chapter 3 with image sizes of $10 \times 10\text{cm}^2$ and $20 \times 20\text{mm}^2$, four particle sizes, and four turbulence levels. It was re-addressed in Chapter 4 with an image size of $20 \times 20\text{mm}^2$, but slightly different turbulence levels and larger particle sizes. The results from these different cases did not match. However, the characteristic cluster sizes found in Chapter 4 of $\sim 10 - 20\eta$ most closely match with those from Aliseda et al. (2002), Wood et al. (2005), and Obligado et al. (2014). An interesting observation is that for each data set of varying $Re_\lambda$ and $d_p$ in Chapter 3 and the varying $Re_\lambda$ data in Chapter 4, the dimensional cluster and void size distributions collapse on to a single curve. However, the dimensional cluster and void sizes found with either ROI in Chapter 3 and the data in Chapter 4 all vary from each other.

2. Are regions of particle voids most highly correlated to eddy zones?

   Results in Table 4.8 suggest that particle voids are composed of a greater portion of eddy and stream zones than the overall image. The results from Table 4.10, however, suggest this is not the case and that regions of particle voids occur most commonly in convergence zones. We note, though, the results from Table 4.10 are inconclusive because the standard deviation is significantly greater than the average, and because Figure 4.20 suggests there is little difference in the particle number density between different flow zones. These findings do not match those of Maxey (1987), Squires & Eaton (1991b), and Wang & Maxey (1993).

3. Do particles tend to cluster in the stream zones, and in particular stream zones in the downward direction?

   Tables 4.8 and 4.10 suggest that regions of particle clusters are composed of a greater
portion of stream zones than the overall image. However, Tables 4.8 and 4.10 also suggest that particle clusters are often found in eddy zones, which is not reflective of the previous studies mentioned. Again, we find these results inconclusive based on the standard deviations presented in Table 4.11 and the PDF presented in Figure 4.20. From Figure 4.25 we also do not observe any preference for streams with a vertical velocity in the direction of gravity as expected.

4. How are particles typically distributed in convergence zones with converging/diverging streamlines?

Table 4.10 indicates that the convergence zones typically have a lower particle number density compared to the overall average, suggesting a region of particle voids. In Figure 4.20, however, the particle number density PDF curves collapse on each other for the three flow zones, which indicates there is little difference in how particles are spatially distributed with respect to the flow zones. These findings再次 contradict the work of Squires & Eaton (1991b) and the theoretical prediction of Maxey (1987) that regions of particle clusters should occur where the strain rate is high, i.e. convergence zones.

5. Do we observe an increase in correlation between particle distribution and turbulent flow structures with increasing $Re_\lambda$, and thus, increasing $St_\eta$?

Based on previous work we expected that increasing the $Re_\lambda$, and thereby increasing the $St_\eta$, would cause particles to cluster more. Thus, we predicted that with increasing $Re_\lambda$ the relationships between turbulent flow structures and particle clusters/voids should become more pronounced. Tables 4.8-4.10, 4.12-4.13, and 4.14 suggest no evidence of this. However, in Figure 4.20 we do see an increasing overall spread in the particle number density PDF with increasing $Re_\lambda$. We also note a weak trend from Table 4.7 that the percentage of the flow field described as a cluster region increases with increasing $Re_\lambda$.

We suggest several possible explanations for these unexpected findings are:

1. A 2D rather than 3D analysis is performed.

2. The PIV analysis is preformed on images containing both aluminum-oxide and fluorescent polymer microspheres. The fluorescent polymer microspheres do not faithfully follow the flow field, which adds error in our PIV analysis. Again this was because the X3 camera would not receive sufficient light if a notch filter were added to solely image the aluminum-oxide particles. We suggest as a future project repeating the current experiment with the X3 camera replaced, and filtering out the polymer microspheres in the PIV analysis.

3. The particle concentration is not sufficient to analyze the relationship between particle clusters/voids and turbulent structures.
5.2 FUTURE WORK

The current study has inspired the following possible experimental investigations:

1. Perform a 3D analysis between turbulent flow structures and the spatial distribution of particles, similar to the current work. We suggest using a PTV system to analyze the trajectory of the particles near various flow structures. Using a PTV analysis would also allow for analyzing particle cluster dynamics, in order to compare with the work of Aliseda et al. (2002). The continuous laser is needed to perform PTV measurements because the smallest time delay between images pairs using the ND-YAG laser is 31ms, which is too great to track particles in this work. However, we note that the maximum frame rate of the Y3.2-S1 and X3 cameras is limited, and is only sufficient to capture images at the lowest level of turbulence in this work without the particles appearing streaky. The main challenge in this experiment would be measuring the turbulent flow structures in 3D.

2. Measure the rate of change in the droplet size distribution for droplets in homogeneous isotropic turbulence for varying turbulence levels using a PDA system with the droplets beginning with an average size of $d_p \approx 10 \mu m$. Additionally, analyze if the spatial distribution of droplets shifts with the droplet size spectrum using a Voronoi analysis and RDF to analyze the spatial distribution of droplets. In order to achieve both monodisperse droplets and a sufficient number for performing the experiment, we suggest using a vibrating orifice aerosol generator based on the work of Berghund & Liu (1973) and Duan et al. (2016). The main challenge in this experiment will be droplets settling too quickly and possibly evaporating.

3. Analyze the relationship between the spatial distribution of particles and flow structures in turbulent boundary layers and jets using a similar experimental set-up to that presented in Chapter 4. Simultaneous 2D PIV measurements and images of fluorescent polymer microsphere would provide instantaneous measurements on the spatial distribution of particles versus turbulent flow structures. The instantaneous measurements of the particle-laden turbulent jet could be compared to the phase-averaged measurements of Longmire & Eaton (1992). Similarly, the relationship between particle dissipation and coherent structures in boundary layer flows could be compared to the work by Rashidi et al. (1990) and Kaftori et al. (1995).
REFERENCES


## Appendix A

### Additional Particle Data

<table>
<thead>
<tr>
<th>A ($V_{pp}$)</th>
<th>$\bar{\epsilon}$ (m$^2$/s$^3$)</th>
<th>$\tau_\eta$ (ms)</th>
<th>$\lambda$ (mm)</th>
<th>$Re_\lambda$</th>
<th>$\eta$ (µm)</th>
<th>$T_e$ (ms)</th>
<th>$L_e$ (mm)</th>
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<tr>
<td>2</td>
<td>5</td>
<td>1.64</td>
<td>3.18</td>
<td>106</td>
<td>157</td>
<td>46</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>0.79</td>
<td>2.50</td>
<td>136</td>
<td>109</td>
<td>29</td>
<td>24</td>
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<td>6</td>
<td>107</td>
<td>0.37</td>
<td>1.97</td>
<td>178</td>
<td>75</td>
<td>18</td>
<td>25</td>
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<td>7</td>
<td>234</td>
<td>0.25</td>
<td>1.72</td>
<td>201</td>
<td>61</td>
<td>14</td>
<td>25</td>
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Table A.1: turbulence parameters with YAG laser from $d_p = 0.5µm$ particles using uncorrected Smagorinsky constant for $f = 100Hz$

<table>
<thead>
<tr>
<th>f (Hz)</th>
<th>$\bar{\epsilon}$ (m$^2$/s$^3$)</th>
<th>$\tau_\eta$ (ms)</th>
<th>$\lambda$ (mm)</th>
<th>$Re_\lambda$</th>
<th>$\eta$ (µm)</th>
<th>$u'$ (m/s)</th>
<th>$v'$ (m/s)</th>
<th>$q^2$ (m$^2$/s$^2$)</th>
<th>$T_e$ (ms)</th>
<th>$L_e$ (mm)</th>
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<tr>
<td>80</td>
<td>2.52</td>
<td>2.44</td>
<td>4.59</td>
<td>149</td>
<td>191</td>
<td>0.48</td>
<td>0.50</td>
<td>0.73</td>
<td>99</td>
<td>48</td>
</tr>
<tr>
<td>90</td>
<td>2.28</td>
<td>2.56</td>
<td>4.74</td>
<td>151</td>
<td>196</td>
<td>0.48</td>
<td>0.48</td>
<td>0.69</td>
<td>101</td>
<td>48</td>
</tr>
<tr>
<td>100</td>
<td>2.25</td>
<td>2.58</td>
<td>4.92</td>
<td>162</td>
<td>197</td>
<td>0.49</td>
<td>0.50</td>
<td>0.74</td>
<td>111</td>
<td>55</td>
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<tr>
<td>110</td>
<td>2.48</td>
<td>2.45</td>
<td>4.87</td>
<td>166</td>
<td>192</td>
<td>0.51</td>
<td>0.50</td>
<td>0.77</td>
<td>102</td>
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Table A.2: turbulence parameters with continuous laser calculated with $d_p = 0.5µm$ particles for $A = 2V_{pp}$

<table>
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<tr>
<th>A ($V_{pp}$)</th>
<th>$\bar{\epsilon}$ (m$^2$/s$^3$)</th>
<th>$\tau_\eta$ (ms)</th>
<th>$\lambda$ (mm)</th>
<th>$Re_\lambda$</th>
<th>$\eta$ (µm)</th>
<th>$u'$ (m/s)</th>
<th>$v'$ (m/s)</th>
<th>$q^2$ (m$^2$/s$^2$)</th>
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<tr>
<td>2</td>
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<td>4.15</td>
<td>152</td>
<td>171</td>
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<td>0.90</td>
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<tr>
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<td>3.18</td>
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<td>1.00</td>
<td>2.97</td>
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<td>71</td>
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<td>263</td>
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<td>1.49</td>
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<td>139</td>
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<td>9.96</td>
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Table A.3: turbulence parameters with YAG laser calculated with $d_p = 5µm$ particles for $f = 100Hz$
### Appendix A: Additional Particle Data

#### Table A.4: turbulence parameters with YAG laser calculated with \( d_p = 3\mu m \) particles for \( f = 100Hz \)

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<th>( V_{pp} )</th>
<th>( \dot{\epsilon} ) (m(^2)/s(^3))</th>
<th>( \tau_\eta ) (ms)</th>
<th>( \lambda ) (mm)</th>
<th>( Re_\lambda )</th>
<th>( \eta ) (( \mu m ))</th>
<th>( u' ) (m/s)</th>
<th>( v' ) (m/s)</th>
<th>( q^2 ) (m(^2)/s(^2))</th>
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<td>1.86</td>
<td>9.94</td>
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</table>

#### Table A.5: turbulence parameters with YAG laser calculated with \( d_p = 1\mu m \) particles for \( f = 100Hz \)

<table>
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<th>( V_{pp} )</th>
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<th>( \lambda ) (mm)</th>
<th>( Re_\lambda )</th>
<th>( \eta ) (( \mu m ))</th>
<th>( u' ) (m/s)</th>
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<td>82</td>
<td>1.81</td>
<td>1.89</td>
<td>10.31</td>
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Figure A.1: PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. The PIV analysis is performed with particles with $d_p = 0.5\mu m$ using a YAG laser light source. The forcing amplitude is $A = 2, 4, 6,$ and $7V_{pp}$ and the forcing frequency is $f = 100Hz$. This is the same information given in Figure 2.16, but on a logarithmic scale.
Figure A.2: PDF of \( u \) and \( v \)-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. The PIV analysis is performed with \( d_p = 0.5, 1, 3, 5 \mu m \) particles using a YAG laser light source and a forcing amplitude of \( A = 4V_{pp} \) and a forcing frequency of \( f = 100Hz \).
Figure A.3: PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. The PIV analysis is performed with $d_p = 0.5, 1, 3, 5 \mu m$ particles using a YAG laser light source and a forcing amplitude of $A = 6V_{pp}$ and a forcing frequency of $f = 100Hz$. 
Figure A.4: PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. The PIV analysis is performed with $d_p = 0.5, 1, 3, 5 \mu m$ particles using a YAG laser light source and a forcing amplitude of $A = 7V_{pp}$ and a forcing frequency of $f = 100Hz$. 
Figure A.5: PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field. The PIV analysis is performed with $d_p = 0.5, 1, 3, 5\, \mu m$ particles using a continuous laser light source and a forcing amplitude of $A = 2V_{pp}$ and a forcing frequency of $f = 100Hz$. 
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<th>3µm</th>
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<td>4$V_{pp}$</td>
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<td>$V_v = 1.95$</td>
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<td>$V_c = 0.56$</td>
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<td>$V_v = 1.81$</td>
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<td>$V_v = 1.82$</td>
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<tr>
<td></td>
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</tr>
<tr>
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<td>$V_c = 0.53$</td>
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</tr>
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Table A.6: $V_v$ and $V_c$ for various test cases for 10 $\times$ 10cm$^2$ images.
Appendix A: Additional Particle Data

Figure A.6: $A/\langle A \rangle$ from Voronoi analysis of $10 \times 10$ cm$^2$ images.

(a) $d_p = 0.5 \mu$m  
(b) $d_p = 1 \mu$m  
(c) $d_p = 3 \mu$m  
(d) $d_p = 5 \mu$m
Appendix A: Additional Particle Data

Figure A.7: $A_c$ from Voronoi analysis for $20 \times 20$mm$^2$ images.
Figure A.8: $A_c$ from Voronoi analysis for $10 \times 10$ cm$^2$ images.
Appendix A: Additional Particle Data

Figure A.9: $A_c/\eta^2$ from Voronoi analysis for $20 \times 20 \text{mm}^2$ images.
Appendix A: Additional Particle Data

Figure A.10: $A_c/\eta^2$ from Voronoi analysis of $10 \times 10 \text{cm}^2$ images.
Appendix A: Additional Particle Data

Figure A.11: $A_v/\eta^2$ for $20 \times 20$mm$^2$ images with the Voronoi analysis.

(a) $d_p = 0.5\mu m$

(b) $d_p = 1\mu m$

(c) $d_p = 3\mu m$

(d) $d_p = 5\mu m$
Figure A.12: $A_v/\eta^2$ for 10 × 10cm$^2$ images with the Voronoi analysis.
Figure A.13: $A_v/\eta^2$ for $20 \times 20\text{mm}^2$ images with the Gaussian Blur analysis.
Figure A.14: $A_v/\eta^2$ for 10 × 10cm$^2$ images with the Gaussian Blur analysis.
Appendix A: Additional Particle Data

Figure A.15: $A_v$ for $20 \times 20\text{mm}^2$ images with the Voronoi analysis.
Figure A.16: $A_v$ for $10 \times 10\text{cm}^2$ images with the Voronoi analysis.
Figure A.17: $A_v$ for $20 \times 20\text{mm}^2$ images with the Gaussian Blur analysis.
Figure A.18: $A_v$ for $10 \times 10\text{cm}^2$ images with the Gaussian Blur analysis.

Table A.7: Parameters for aluminum-oxide particles with $d_p = 1\mu\text{m}$ for various turbulence levels in the set-up and conditions described in Chapter 4.
Appendix A: Additional Particle Data

(a) $Re_\lambda = 181$

(b) $Re_\lambda = 181$

(c) $Re_\lambda = 243$

(d) $Re_\lambda = 243$

(e) $Re_\lambda = 285$

(f) $Re_\lambda = 285$

(g) $Re_\lambda = 319$

(h) $Re_\lambda = 319$

Figure A.19: PDF of $u$ and $v$-components of the rms velocity fluctuations at the center, center-top, and center-right points in the flow field.
Figure A.20: $A/\langle A \rangle$ for various turbulence levels for 1 gram of polymer microspheres.

<table>
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<th>$Re_\lambda$</th>
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<th>285</th>
<th>319</th>
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</thead>
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<tr>
<td>$V_v$</td>
<td>1.97</td>
<td>1.94</td>
<td>2.07</td>
<td>2.03</td>
</tr>
<tr>
<td>$V_c$</td>
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<td>0.58</td>
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Table A.8: $V_v$ and $V_c$ for various turbulence levels for 1 gram ($\Phi_v \approx 1.5e-5$) of polymer microspheres.
Appendix A: Additional Particle Data

Figure A.21: RDF corresponding to data in Figure 4.10 and A.20, where the solid line indicates 1 gram ($\Phi_v \approx 1.5e^{-5}$) of polymer microspheres and a dashed line indicates 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.

Figure A.22: $A_c$ and $\frac{A_c}{\bar{\eta}^2}$ corresponding to data in Figure 4.11, where the solid line indicates 1 gram ($\Phi_v \approx 1.5e^{-5}$) of polymer microspheres and a dashed line indicates 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.
Figure A.23: $A_v$ and $\frac{A_v}{\eta^2}$ corresponding to data in Figure 4.12, where the solid line indicates 1 gram ($\Phi_v \approx 1.5e^{-5}$) of polymer microspheres and a dashed line indicates 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.

Figure A.24: The overall breakdown of the total image into regions of particle clusters, particle voids, and neither for images containing 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.
Figure A.25: The breakdown of how particle clusters and voids are distributed among the flow regions for images containing 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.

Figure A.26: The average normalized particle number density for each flow zone for images containing 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres.
### Table A.9: Distribution of the three flow zones among the flow domain for 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
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<th>243</th>
<th>285</th>
<th>319</th>
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</thead>
<tbody>
<tr>
<td>Eddy</td>
<td>6.16%</td>
<td>7.06%</td>
<td>6.90%</td>
<td>7.31%</td>
</tr>
<tr>
<td>Convergence</td>
<td>10.25%</td>
<td>10.66%</td>
<td>11.88%</td>
<td>10.50%</td>
</tr>
<tr>
<td>Stream</td>
<td>55.55%</td>
<td>52.52%</td>
<td>55.12%</td>
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</tr>
<tr>
<td>Amorphous</td>
<td>28.04%</td>
<td>29.76%</td>
<td>26.10%</td>
<td>28.24%</td>
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</table>

### Table A.10: Breakdown of the image into regions of particle clusters, particle voids, and neither for 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>4.78%</td>
<td>4.89%</td>
<td>5.69%</td>
<td>6.65%</td>
</tr>
<tr>
<td>Void</td>
<td>36.00%</td>
<td>37.72%</td>
<td>35.41%</td>
<td>35.83%</td>
</tr>
<tr>
<td>Amorphous</td>
<td>59.22%</td>
<td>57.39%</td>
<td>58.90%</td>
<td>57.52%</td>
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</table>

### Table A.11: Breakdown of how cluster Voronoi cells are distributed among the flow zones for 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres.

<table>
<thead>
<tr>
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<th>243</th>
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<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy</td>
<td>9.90%</td>
<td>10.40%</td>
<td>9.23%</td>
<td>10.63%</td>
</tr>
<tr>
<td>Convergence</td>
<td>6.83%</td>
<td>8.93%</td>
<td>8.48%</td>
<td>10.79%</td>
</tr>
<tr>
<td>Stream</td>
<td>61.49%</td>
<td>56.89%</td>
<td>63.29%</td>
<td>56.69%</td>
</tr>
<tr>
<td>Amorphous</td>
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<td>23.78%</td>
<td>19.00%</td>
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</tr>
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### Table A.12: Breakdown of how the void Voronoi cells are distributed among the flow zones for 1 gram ($\Phi_v \approx 1.5e - 5$) of polymer microspheres.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy</td>
<td>10.78%</td>
<td>10.44%</td>
<td>10.94%</td>
<td>11.03%</td>
</tr>
<tr>
<td>Convergence</td>
<td>6.52%</td>
<td>7.60%</td>
<td>7.90%</td>
<td>9.34%</td>
</tr>
<tr>
<td>Stream</td>
<td>61.92%</td>
<td>60.44%</td>
<td>62.10%</td>
<td>59.55%</td>
</tr>
<tr>
<td>Amorphous</td>
<td>20.78%</td>
<td>21.52%</td>
<td>18.97%</td>
<td>20.08%</td>
</tr>
</tbody>
</table>
### Appendix A: Additional Particle Data

#### Table A.13: The average normalized particle number density in each flow zone for 1 gram ($\Phi_v \approx 1.5\times10^{-5}$) of polymer microspheres.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy</td>
<td>1.02</td>
<td>0.97</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>Convergence</td>
<td>0.97</td>
<td>0.96</td>
<td>0.95</td>
<td>0.96</td>
</tr>
<tr>
<td>Stream</td>
<td>1.01</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
</tr>
</tbody>
</table>

#### Table A.14: The standard deviation of the particle number density in each flow zone for 1 gram ($\Phi_v \approx 1.5\times10^{-5}$) of polymer microspheres.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>181</th>
<th>243</th>
<th>285</th>
<th>319</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy</td>
<td>2.11</td>
<td>2.05</td>
<td>2.02</td>
<td>2.04</td>
</tr>
<tr>
<td>Convergence</td>
<td>2.05</td>
<td>2.06</td>
<td>1.98</td>
<td>2.03</td>
</tr>
<tr>
<td>Stream</td>
<td>2.09</td>
<td>2.11</td>
<td>2.05</td>
<td>2.09</td>
</tr>
</tbody>
</table>

#### Table A.15: Average normalized enstrophy and scaler-strain values for regions of particle clusters for 1 gram ($\Phi_v \approx 1.5\times10^{-5}$) of polymer microspheres.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>$\omega$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>181</td>
<td>0.96</td>
<td>0.91</td>
</tr>
<tr>
<td>243</td>
<td>0.93</td>
<td>0.90</td>
</tr>
<tr>
<td>285</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>319</td>
<td>0.98</td>
<td>0.94</td>
</tr>
</tbody>
</table>

#### Table A.16: Average normalized enstrophy and scaler-strain values for regions of particle voids for 1 gram ($\Phi_v \approx 1.5\times10^{-5}$) of polymer microspheres.

<table>
<thead>
<tr>
<th>$Re_\lambda$</th>
<th>$\omega$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>181</td>
<td>0.93</td>
<td>0.92</td>
</tr>
<tr>
<td>243</td>
<td>0.96</td>
<td>0.93</td>
</tr>
<tr>
<td>285</td>
<td>0.96</td>
<td>0.93</td>
</tr>
<tr>
<td>319</td>
<td>0.96</td>
<td>0.93</td>
</tr>
</tbody>
</table>
Figure A.27: PDF of the normalized particle number density in each flow zone for 2.5 grams ($\Phi_v \approx 3.7e^{-5}$) of polymer microspheres. Here we present the same data as in Figure 4.20, but on a log-log plot.
Figure A.28: PDF of the normalized particle number density in each flow zone for 1 gram ($\Phi_v \approx 1.5e-5$) of polymer microspheres.
Figure A.29: PDF of the normalized particle number density in each flow zone for 1 gram ($\Phi_v \approx 1.5e-5$) of polymer microspheres. Here we present the same data as in Figure A.28, but on a log-log plot.
Figure A.30: Particle number density versus the normalized strain rate and enstrophy for 1 gram ($\Phi_v \approx 1.5e^{-5}$) of polymer microspheres.

(a) $Re_\lambda = 181$

(b) $Re_\lambda = 243$

(c) $Re_\lambda = 285$

(d) $Re_\lambda = 319$
Figure A.31: Particle number density versus the second invariant of the velocity gradient tensor, $Q$, for 1 gram ($\Phi_v \approx 1.5e^{-5}$) of polymer microspheres.
Appendix A: Additional Particle Data

Figure A.32: Particle number density versus horizontal and vertical local velocities for 1 gram ($\Phi_v \approx 1.5e^{-5}$) of polymer microspheres.

(a) $Re_\lambda = 181$

(b) $Re_\lambda = 243$

(c) $Re_\lambda = 285$

(d) $Re_\lambda = 319$
Appendix A: Additional Particle Data

Figure A.33: Particle number density versus horizontal and vertical local velocities for only the regions of the flow field classified as a stream zones for 1 gram ($\Phi_v \approx 1.5e-5$) of polymer microspheres.
Figure A.34: The average normalized enstrophy and strain rate values for areas identified as particle cluster and void regions for images containing 2.5 grams ($\Phi_v \approx 3.7e-5$) of polymer microspheres.