An improved immersed boundary method for atmospheric boundary layer simulations over complex terrain

by Jingyi Bao

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Abstract

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Accurately simulating flow over complex terrain has long been a challenging scientific problem. As computational power increases, atmospheric simulations are pushed toward higher resolutions with enough computational resources to better resolve complex topography. Turbulence and flow dynamics which decades ago could not be resolved (e.g. drainage flow over a steep mountain) can now be studied using computational tools. The Weather Research and Forecasting (WRF) model is a numerical weather prediction and atmospheric research model that can be used in a wide range of grid resolutions from mesoscale weather prediction (km scale) to large-eddy simulation for atmospheric boundary layer studies (~100 m and finer scales). The WRF model uses terrain-following coordinates, which is adequate for mesoscale models where the details of the terrain are not well resolved and the terrain slope is not very steep. With increased resolution, resolved terrain slopes become steeper, and the native terrain-following coordinates used in WRF result in numerical errors and instability. To eliminate these numerical errors and instability, but still be able to use WRF’s grid nesting strategy to include weather effects for complex terrain simulation, an immersed boundary method was implemented into WRF by Lundquist et al. (2010, 2012). The immersed boundary method uses a non-conforming grid where the terrain surface is immersed into the grid. The immersed boundary conditions are enforced by adding an additional forcing term to the Navier-Stoke equations for the points near the immersed boundary. The original immersed boundary method in WRF-IBM uses a no-slip boundary condition, which is suitable for urban simulations with fine resolution (1 m). The no-slip boundary condition is not appropriate for atmospheric simulations at 100 m scale, where the surface layer and the underlying topography are not extremely well resolved.

This dissertation focuses on the development of improved log-law boundary conditions for WRF-IBM to enable atmospheric simulations over complex terrain at horizontal resolutions on the order of 100 m. First an existing velocity-reconstruction immersed boundary method (VR-IBM) is implemented into WRF. The VR-IBM is tested extensively for flat terrain, an idealized hill, and flow over Askervein Hill. At very fine resolutions, VR-IBM performs well, but further tests show some limitations at intermediate resolutions, where VR-IBM for example develops spatial
oscillations. Other limitations are found for VR-IBM and another shear-stress reconstruction (SR-IBM) approach depending on resolution and grid aspect ratios (see also Arthur et al., 2019). Next, a new hybrid log-law boundary condition for IBM (HYBRID-IBM) is developed and implemented into WRF. This hybrid method is evaluated against VR-IBM and SR-IBM for a wide range of idealized cases (both flat terrain and idealized 3D hill at different slopes and resolutions) and real cases, including Askervein hill (intermediate resolutions) and Bolund hill (steep slope, fine resolution). Sensitivity tests of the three IBM approaches are performed to examine the effects of the location where terrain intersects with the grid, the grid aspect ratio, the grid resolution, and the terrain slope.

Based on our knowledge, this work is the first time a log-law boundary condition has been used with the immersed boundary method and WRF to enable simulations of atmospheric flow at intermediate resolutions. This work also represents the first time that several IBM methods, including the VR-IBM, SR-IBM, and HYBRID-IBM have been validated using a large number and variety of test cases. Specifically, these methods have been tested not only in comparison to theory (flat terrain), but also to observations (Askervein and Bolund Hills) and through direct comparison to WRF results using the native terrain-following coordinate (for the flat terrain, idealized hill and Askervein Hill cases). Furthermore, this dissertation makes a first attempt toward a robust evaluation of the different IBM approaches, using test cases with different resolutions, different terrain slopes, different aspect ratios and different grid setups, to demonstrate the robustness and sensitivity of implementation for the different methods. In almost all test cases, the HYBRID-IBM approach outperforms the other methods. This series of test cases provides rigorous validation of all three IBM methods and thus conclusions about the advantages and disadvantages of different methods can be made.
To my parents, my husband and my daughter
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Chapter 1

Introduction

1.1 Overview

As computational power increases, mesoscale models, such as the Weather Research and Forecasting model model, are being used at higher grid resolutions for complex terrain simulations. Terrain-following coordinates, often used by WRF and other mesoscale atmospheric models, have historically worked well on coarse grids for mesoscale simulations, where the details of the topography are not well captured and the terrain slope is relatively small. With fine resolution simulations over complex terrain, the resolved terrain is much steeper; this results in highly skewed grids near the surface due to the terrain-following coordinates. An immersed boundary method (IBM) was recently implemented into WRF (Lundquist et al., 2010, 2012), to alleviate numerical errors associated with steep terrain slopes. The WRF-IBM uses a Cartesian grid with the terrain boundary “immersed” within the grid. Boundary conditions are set through interpolation procedures for grid cells intersected by the immersed surface. Lundquist et al. (2010) first developed a WRF-IBM model with no-slip boundary conditions and coupled it to WRF’s atmospheric physics parameterizations including a radiation model and land-surface model. Lundquist et al. (2010, 2012) validated this WRF-IBM model with no-slip boundary conditions with idealized flow simulations over mountainous terrain and over urban terrain, comparing to data from the Joint Urban 2003 field campaign in Oklahoma City. While the use of no-slip boundary conditions is an approximation sometimes used for urban simulations with grid resolutions of about 1 m, it is inappropriate for coarser resolutions (e.g. 100 m), where the topography and the surface layer are not as well resolved.

This dissertation extends the WRF-IBM model to allow for new momentum flux boundary conditions based on Monin-Obukhov similarity theory at the terrain surface. This will enable simulations over complex terrain where a no-slip condition is not appropriate. The flux boundary condition (also known as the “log law”) implementation specifies the stress at the wall in accordance with surface similarity theory.

There are several methods combining log-law boundary condition with the immersed boundary method in the recent literature. One approach is to reconstruct the velocity field near the immersed
surface (VR-IBM) to enforce a log law profile (Senocak et al., 2004; Choi et al., 2007; Senocak et al., 2015; Umphrey et al., 2017). Senocak et al. (2015) tested flow over the Stanford Dragon and Bunny at 0.5 m resolution. Choi et al. (2007) validated flow over a cylinder with mm scale resolution. We also implemented this VR-IBM method into WRF (Bao et al., 2018) and validated it through a neutral atmospheric boundary layer over flat terrain, simulations over Askervein Hill (at 5 m resolution) and simulations of flow over Bolund Hill (2 m resolution). A second approach reconstructs the shear stress terms near the immersed surface (SR-IBM) to account for the constant stress log layer (Chester et al., 2007; Diebold et al., 2013; Cheng and Porté-Agel, 2013, 2015; Fang and Porté-Agel, 2016; Ma and Liu, 2017). Ma and Liu (2017) implemented the method of Chester et al. (2007) into the WRF model and validated it for flow over Bolund Hill (at 1 m resolution). We note that these existing methods in the literature which combine log-law boundary conditions with the immersed boundary method work well at fine resolutions. In this work, we evaluate the ability of existing methods and implement a new IBM boundary condition, which reconstructs both velocity and shear stress near the surface. This new method improves existing methods at intermediate resolutions (e.g. 100 m for hilly terrain) which extends the applicability of IBM to more reasonable resolutions and thus a greater range of applications. This chapter provides a detailed background of techniques and tools used to model flows over complex terrain, followed by an overview of the rest of the dissertation.

1.2 Background

This section reviews the immersed boundary method as a technique to deal with steep complex topography. The original development of the immersed boundary method in computational fluid mechanics and its introduction into the WRF model will be presented. An attempt at combining these two tools is also reviewed here. Additionally, the WRF model and the the wall modeling technique for complex terrain, known as Monin-Obukov similarity theory, are also described.

1.2.1 Immersed boundary method

In an immersed boundary (IB) method, the topography is immersed into the grid (see Figure 1.1). The Navier-Stokes (NS) equations are discretized on the background Cartesian, non-conforming grid and the boundary conditions are imposed through modification of the NS equations. The modification can take the form of an additional forcing function in the original NS equations to account for the influence of the boundary. Another approach is a cut-cell approach, where fluxes are prescribed on the faces of cells cut by the boundary, as described further below.

1.2.1.1 Forcing method

Applying a forcing term in the NS equations leads to two categories of forcing method, namely continuous forcing and direct forcing.
1.2.1.2 Continuous forcing approach

The continuous forcing approach for rigid boundaries is called a virtual boundary method, used by Goldstein et al. (1993). The main idea for this method is to apply a force on the fluid so that the fluid will be at rest on the surface (no-slip condition). Since the body force is not known a priori, it must be calculated using a feedback loop, where the velocity on the boundary is used to determine the desired force.

\[
F(s,t) = \alpha \int_0^t u(s,t) d\tau + \beta u(s,t)
\]  

(1.1)

In this equation, \( u \) is the fluid velocity at surface points. The forcing term acts as a damping term, where \( \alpha \) is the spring constant and \( \beta \) is the damping coefficient. When the forcing term is added to the NS equations, it will act as a damping term to damp the velocity to zero. The drawback of the method is that the coefficients \( \alpha \) and \( \beta \) must be chosen carefully for different flow types and in highly unsteady and turbulent flow, the solution will keep oscillating (Goldstein et al., 1993).
1.2.1.3 Direct forcing approach

Another approach was developed by Mohd-Yusof (Mohd-Yusof, 1997), called a direct forcing method. In the direct forcing approach, the governing equations are discretized on a Cartesian grid neglecting the immersed boundary. Then, cells near the immersed boundary (IB) are adjusted to account for its presence. This method is similar to setting a velocity boundary condition. Fadlun et al. (2000) used both feedback forcing and direct forcing approach and found that the two forcing methods produced similar results, however, the direct forcing method has some advantages. Unlike with the continuous forcing method, which oscillates and has limitations on choosing constants, the boundary condition can be satisfied exactly at each time step with direct forcing. For these reasons, the immersed boundary method chosen in this work is the direct forcing method (Lundquist et al., 2010).

1.2.1.4 Cut-cell method

Cut-cell methods or embedded boundary methods propose a different approach from direct forcing methods for accounting for the boundary (Udaykumar, 1995; Udaykumar et al., 1999, 2001; Ye et al., 1999). For example, Ye et al. (1999) performed simulations of convection-dominated flows on a non-staggered grid with a cut-cell method. In this method, cells in the Cartesian grid cut by the IB are tagged. For the cells cut by the IB, if the cell centers lie in the fluid, they are reshaped by discarding the portion that is located within the solid. If the cell centers lie in the solid (under the immersed boundary), they will be absorbed by neighboring cells with cell centers in the fluid. Then conservation laws are applied at the cut cells conformed to the boundary. Compared to the ghost cell direct forcing method, the cut-cell method is more challenging in implementation, involving calculation of convective and diffusive flux integrals on each of the cut cell faces. Extending the cut-cell method to 3D is also extremely difficult (Kirkpatrick et al., 2003), because of more complicated 3D geometry.

1.2.2 WRF

The Weather Research and Forecasting (WRF) model is a community model used for a variety of purposes including mesoscale weather forecasting and microscale weather research. WRF solves the non-hydrostatic compressible Navier-Stokes equations, including a complete set of atmospheric physics parameterizations (e.g. radiation models and surface physics models). Uniform grid spacing is used in the horizontal directions and a terrain-following pressure coordinate system is used as the vertical coordinate. An explicit Runge-Kutta method is used to advance the time step and a C-staggered grid is used in space (Skamarock et al., 2008).

Several options for lateral boundary conditions are available to WRF users including periodic, open, symmetric and real meteorology reanalysis data. The first three boundary conditions are normally used in idealized simulations, while the last one is often used in real simulations with real external meteorology data like North American Mesoscale (NAM) analysis data. One or two way grid nesting is available, allowing the capability to nest from the mesoscale to the microscale.
In one way nesting, the fine resolution receives boundary conditions interpolated from the coarse
domain. In two way nesting, the solution from the fine domain will affect the solution on the coarse
domain at each time step.

In the vertical direction, the top boundary condition is isobaric with zero $w$ velocity. Additionally, a Rayleigh damping layer can be applied to all three velocities or only the vertical velocity. At the bottom boundary, there are two options for the bottom boundary condition. One is that the free slip boundary condition can be satisfied using the following set of equations:

$$\eta_{surf} = 0$$  \hspace{1cm} (1.2)

$$w_x = \nabla h$$  \hspace{1cm} (1.3)

Equation 1.2 sets the contravariant coordinate velocity to zero and Equation 1.3 sets the kinematic boundary condition for WRF and forces no flow through the surface. In Equation 1.2, $\eta$ defines the normalized terrain-following vertical coordinate, which is zero at the top and one at the bottom. In Equation 1.3, $\nabla h$ is the horizontal velocity vector and $h$ is the terrain height. In the case of a rough terrain surface with real topography, similarity theory is used to account for the shear stress at the wall using the aerodynamic drag formula as follows.

$$\tau_{13} = C_d |U|U$$  \hspace{1cm} (1.4)

$$\tau_{23} = C_d |U|V$$  \hspace{1cm} (1.5)

Here $C_d$ is the bulk drag coefficient. The value of $C_d$ depends on the height at which $(U, V)$ is defined in this calculation. $|U|$ denotes the magnitude of the horizontal velocities, which could be calculated using $U$ and $V$. $U$ is the velocity in $x$ direction and $V$ is the velocity in the $y$ direction. When the formulation in equation 1.4 and equation 1.5 is used, it is assumed that the flow is dominated by the horizontal velocity components.

When a surface layer scheme is used, it is usually an equilibrium model. The surface layer scheme calculates a value of friction velocity based on a roughness length depending on the land use and vegetation type, the atmospheric velocity profile and the atmospheric stability condition. The drag coefficient is defined as $C_d = (u^* / |U|)^2$. The calculated stress would then correspond to the stress required for an instantaneous log law (modified appropriately for stability). There are five surface layer schemes in WRF. Each is based on an instantaneous log law and similarity theory. There are slight differences in how the length scales for momentum and heat, or the roughness height are treated. If a surface layer scheme is not used, the drag coefficient $C_d$ is specified as constant value in the namelist as an input.

### 1.2.3 Immersed boundary method in WRF (WRF-IBM)

The original IBM approach implemented in WRF, called WRF-IBM, uses a ghost cell immersed boundary method with direct forcing implemented into the Advanced Research WRF (ARW) dynamic solver (Lundquist et al., 2010). In this method, the velocity or scalar values at ghost cells are modified near the boundary to enforce the boundary conditions. An illustration
of this method for a no-slip boundary condition is shown in Figure 1.2. The first step is to find
the nearest ghost points below the terrain, and then to find the image points according to the ghost
points. Third, the velocities or the scalar values are interpolated to the image points. The last step
is to assign the ghost cell value according to the boundary condition and the interpolated image
point value.

There are several advantages to this ghost-cell method. One is that it is straightforward for
staggered grids, where different velocity components are located at different places. The boundary
condition can be satisfied exactly at each time step with direct forcing. Compared to continuous
forcing methods, it does not have limitations on choosing constants that might lead to oscillating
solutions. (Section 1.2.1.2). Compared to the cut-cell method, it is easier to implement because
it does not require modification of grid cell volumes and fluxes (Section 1.2.1.4). One drawback
of this method is that it is not strictly locally conservative (Mittal and Iaccarino, 2005). Sev-
eral interpolation methods have been implemented, including linear and inverse distance weighted
interpolation (Lundquist et al., 2010). In trilinear interpolation, the value of the image point is

Figure 1.2: Illustration of WRF-IBM no-slip boundary condition.
calculated using the following function (Equation 1.6).

\[ \psi = c_1 + c_2 x + c_3 y + c_4 z + c_5 x y + c_6 x z + c_7 y z + c_8 x y z \]  

(1.6)

Here \( \psi \) is the interpolated value of the image point. \( x, y, \) and \( z \) are the coordinate locations of the image point and \( c_i \) are the weighted coefficients. Eight neighboring points are chosen to define the interpolation region, and they can be either computational nodes near the image points or boundary nodes. The constants \( c_i \) can be determined by solving a system of equations (Equation 1.7) for each image point, where the rank is equal to the number of neighbors:

\[ c = A^{-1} \psi \]  

(1.7)

The matrix \( A \) and the vector \( \psi \) are dependent on the neighbors chosen and the boundary conditions. If the neighbors are all computational nodes, \( \psi \) takes the value at the calculated nodes. If the neighbors have boundary points, for a Dirichlet no-slip boundary condition, we use the same exact equation above. For a Neumann boundary condition, the gradient interpolation function is used in Equation 1.7. The same above equation is used, but one line is replaced in the matrix term \( A \) for the gradient interpolation function. After the constants are calculated, the value of image points can be found using Equation 1.7 (Lundquist et al., 2010).

For the inverse distance weighting method, the value of the image point is calculated by a weighted average of the surrounding neighbors:

\[ \psi = \frac{\sum_{i=1}^{n} c_n \psi_n}{\sum_{i=1}^{n} c_n} \]  

(1.8)

The weighting coefficient \( c_n \), is a function of the radial distance from the interpolation point, given by

\[ c_n = \left( \frac{R_{max} - R_n}{R_{max} R_n} \right)^p \]  

(1.9)

where \( R_{max} \) is the maximum radius from the group of the interpolation points, and \( R_n \) and \( c_n \) are the radial distance and weighting coefficient for the \( n^{th} \) neighbor.

A few atmospheric physics packages, which provide surface forcing to the WRF model, have also been modified to couple with the immersed boundary, including the radiative transfer model, surface layer scheme and land-surface model (Lundquist et al., 2012).

### 1.2.4 Monin-Obukhov similarity theory

#### 1.2.4.1 Obukhov length scale

The Obukhov length scale was proposed by Obukhov (1946), based on the relationship between surface heat flux and momentum flux, which defines the Obukhov length as follows:

\[ L = -\frac{\left( \frac{u_*}{\frac{\rho}{c_p}} \right)^2}{k \frac{\chi}{T_0} \frac{a}{\epsilon_p \rho}} \]  

(1.10)
where $g$ is the gravitational acceleration, $T_0$ is the surface temperature, $u_*$ is the friction velocity, $q$ is the kinetic heat flux, $c_p$ is the specific heat, and $\rho$ is the air density. When it comes to the case of a moist atmosphere, virtual potential temperature is the more accurate variable to use than dry surface temperature.

The basic interpretation of the Obukhov length is that it is a characteristic scale for the thickness of the dynamic sublayer (Monin and Obukhov, 1954), a layer where the influence of stratification is negligible. The Obukhov length is proportional to this thickness but not identical. Recently, the definition has been interpreted as the ratio of buoyancy and shear effects.

The surface layer is traditionally defined as the layer above the roughness sublayer, which is only the bottom 10% of the planetary boundary layer. We assume that the surface layer is sufficiently thin so that the vertical divergence of flux across the surface layer is so small that it can be neglected. Thus, the surface layer is also called a constant flux layer, and the vertical fluxes are assumed to be constant, including heat flux and momentum flux.

\[
\overline{w'T'} = \frac{q}{c_p \rho} = \text{const} \quad (1.11)
\]

\[
\overline{u'w'} = \frac{\tau}{\rho} = u_*^2 = \text{const} \quad (1.12)
\]

Equation 1.11 describes the constant heat flux in the surface layer, where $T'$ is the temperature perturbation, $w'$ is the fluctuation of the vertical velocity, $q$ is surface heat flux, $\rho$ is the density of air, and $c_p$ is the specific heat of air. Equation 1.12 describes the constant momentum flux in the surface layer, where $u'$ is the $u$ velocity perturbation, $\tau$ is the shear stress, and $u_*$ is the friction velocity. The overbar denotes temporal or spatial averaging.

### 1.2.4.2 Basic Monin-Obukhov similarity theory

Monin-Obukhov similarity theory uses the Buckingham $\Pi$ theorem of dimensional analysis to propose the wind and temperature profile in the surface layer. It is based on the theory of a logarithmic wind profile (Prandtl, 1925), the Obukhov length scale, as well as experimental work. Using the Buckingham $\Pi$ theorem, Monin and Obukhov (1954) argue that the reasonable dimensionless groups for the wind and temperature gradients at the surface layer, are $kz/u_*$ and $z/T_*$ with $T_* = -w'T'/u_*$. Both gradients are only a function of the parameters $g, T_0, u_*, q, c_p, \rho$, and the height $z$, so the only existing dimensionless group is $z/L$, where $L$ is the Obukhov length scale.

In accordance with the dimensionless parameter $z/L$, the wind speed and temperature profile can be written as follows:

\[
\frac{kz \, \partial u}{u_* \, \partial z} = \psi_1 \left( \frac{z}{L} \right) \quad (1.13)
\]

\[
\frac{z \, \partial T}{T_* \, \partial z} = \psi_2 \left( \frac{z}{L} \right) \quad (1.14)
\]

Both $\psi_1$ and $\psi_2$ are called universal functions, and in general, they are determined using experimental or field data. The dimensionless group $z/L$ is always treated as a stability parameter. For
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strong stable stratification, $z/L \gg 1$ and for strong unstable cases, $z/L \ll -1$. When the atmosphere is neutral, $z/L$ is equal to 0. The temperature and velocity gradient profile becomes $z/T^* = 1$ and $kz/u^* = 1$.

1.2.5 Atmospheric applications of WRF-IBM with log-law boundary conditions

Improved simulations for flow over complex terrain will have a wide range of atmospheric applications including in the fields of wind energy, air pollution modeling and mountain weather modeling. Simulations of flow over very steep terrain such as steep mountains and urban areas have typically been performed with computational fluid dynamics (CFD) codes which are capable of handling complex terrain geometries with body-fitted grids. Most atmospheric physics including surface physics and radiation physics are neglected. Furthermore, traditional CFD codes often use idealized lateral boundary conditions, which means the boundaries are forced with simple and idealized velocity profiles without including any regional weather effects.

On the other hand, WRF, as a mesoscale atmospheric model, includes full atmospheric physics parameterizations and real lateral boundary conditions provided by actual meteorological data. The vertical coordinates used by WRF are pressure based and terrain-following, which limits applications to highly complex terrain. The WRF-IBM capability we present here can extend WRF’s applicability to highly complex terrain simulations using WRF’s native atmospheric physics parameterizations, real lateral boundary conditions and grid nesting strategy Wiersema et al. (2018).

WRF-IBM previously used no-slip boundary conditions and was validated for urban flow simulations from the Joint Urban 2003 field campaign in Oklahoma City with 1 m resolution. This no-slip boundary condition works appropriately for urban simulations, where the resolutions used are fine enough to fully resolve the surface layer. When it comes to mountainous flow studies, with intermediate resolutions used (10 m to 100 m), the surface layer is not well resolved and the no-slip boundary condition is no longer appropriate. The WRF-IBM with log-law boundary conditions can extend the model into the field of wind energy and mountain weather studies, where such intermediate resolutions are used. This will be the first time that WRF is able to simulate at the 10 m scale for flow over very steep complex terrain. Once the WRF-IBM with log-law boundary condition is validated and combined with the work from Wiersema et al. (2018) on nesting WRF-IBM within terrain-following WRF simulations, simulations of complex terrain with log law boundary condition can be nested into larger mesoscale domains with correct lateral boundary conditions from regional weather data. These simulation over complex terrain with real lateral boundary conditions and atmospheric physics then can be used to study small scale flow dynamics and turbulence over steep terrain, for example nocturnal drainage flows (Arthur et al., 2018), flow interactions with wind turbine wakes, scalar transport of air pollutants, and wildfire modeling among others. Thus the availability of WRF-IBM with log-law boundary conditions will have large impacts in the atmospheric modeling community.
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1.3 Overview

This dissertation extends the existing WRF-IBM model to implement appropriate flux boundary conditions (also known as wall models) based on Monin-Obukhov (M-O) similarity theory. This will enable simulations over complex terrain at resolutions where a no-slip boundary condition is not appropriate. The appropriate IBM boundary conditions will be developed for intermediate scales (e.g. 10-100 m), where the surface layer is not necessarily resolved and requires parameterization of surface fluxes of momentum.

In Chapter 2, we implement the log-law IBM boundary condition from Senocak et al. (2004) into WRF. This method was selected for initial full implementation and testing from among the existing IBM options we implemented and explored in Bao et al. (2016) (as described further in Chapter 3). The method from Senocak et al. (2004) is most straightforward in directly setting the velocity boundary condition to satisfy the log velocity profile near the surface. This velocity reconstruction IBM method (VR-IBM) reconstructs the first velocity point above the surface assuming there is a logarithmic profile near the surface. The implementation of this log law boundary condition with IBM is validated for ideal and real test cases. First, comparisons are made to similarity theory and standard WRF results for the flat terrain case. Then, simulations of flow over the moderately-sloped Askervein Hill are used to demonstrate agreement between the IBM and terrain-following WRF results, as well as agreement with observations. Finally, Bolund Hill simulations show that WRF-IBM can handle steep topography (where standard WRF fails) and compares well to observations. Overall, the new WRF-IBM boundary condition shows improved performance compared to previous no slip boundary condition at fine resolution (1-2 m scale), though the lee side representation of the flow can be potentially further improved.

Many IBM methods are originally developed for engineering applications where resolution is quite fine compared to the variability of the topography. Thus the IBM methods found in the literature to work well were tested on flows with very fine resolutions (1-2 m scale). Diebold et al. (2013) and Ma and Liu (2017) validated the SR-IBM for flow over Bolund hill at 1 m resolution. Similarly, in Chapter 2, we validate the VR-IBM method for flow over Askervein hill at 5 m resolution and flow over Bolund hill over 1 m resolution. However, all of the various IBM implementations tested introduced inaccuracies over sloped terrain at intermediate resolutions (100 m scale) which are more typically used to atmospheric models (Fang and Porté-Agel, 2016; Bao et al., 2016). In Chapter 3, we develop a new IBM log-law boundary condition, which reconstructs both velocity terms and shear stress terms near the immersed boundary in accordance with M-O similarity theory. As such, it combines features of other approaches which specify either velocities or shear stresses on the boundary and this new method is thus called HYBRID-IBM. This new hybrid method is first validated in Chapter 3 for a neutral atmospheric boundary layer over flat terrain. It is simulated using the native terrain-following coordinates in WRF, the new hybrid IBM implementation, and the VR-IBM method from Chapter 2. Four different resolutions are tested including an intermediate resolution of 128 m. Next, simulations over a large idealized hill are performed for terrain-following WRF, the new IBM implementation and the VR-IBM method at intermediate resolutions (100 m). This test case is chosen to have a relatively moderate slopes (15 degree) and large domain (6 km in the horizontal directions), which enable direct comparison between
native WRF simulations and IBM implementations at intermediate resolution. Finally, simulations over Askervein Hill are performed at both intermediate resolution (30 m) and fine resolution (5 m), comparing the hybrid method to the VR-IBM and standard WRF simulations. Results are also compared to field observations. Our results indicate that the HYBRID-IBM method improves model performance over the VR-IBM at intermediate resolutions.

In Chapter 4, we validate the newly implemented HYBRID-IBM method against a wider range of test cases to confirm and quantify the improvements seen with our new method, especially at intermediate resolutions. First, we test the hybrid method for flow over flat terrain where the location of the immersed boundary is adjusted so that it intersects the grid in different ways. With an IBM approach, it is desirable that the velocity profiles generated are the same for flow over flat terrain no matter where the IB surface intersects the grid. Next, the HYBRID-IBM method is tested for a range of different grid aspect ratios. Native WRF’s performance is known to have a dependence on aspect ratio, so it is important to understand the HYBRID-IBM’s dependence on aspect ratio. We find that HYBRID-IBM works well at most aspect ratios except when the aspect ratio is less than or equal to 1 (coarser vertical resolution than horizontal resolution). This kind of grid cell is known to have poor performance in large-eddy simulation (Mirocha et al., 2013a). The sensitivities of the HYBRID-IBM in sloped terrain simulations are also examined using HYBRID-IBM at various resolutions over a moderately-sloped hill using a constant aspect ratio of 5. Then a sensitivity study of HYBRID-IBM over various slopes from 0° (flat terrain), 7°, 15° and 22° using a constant aspect ratio of 5 is performed. The differences between terrain-following WRF and the IBM solutions are found to increase with increasing terrain slopes. HYBRID-IBM shows the smallest differences from WRF among the three IBM methods. Finally simulations over Bolund Hill are performed and results are compared to field observations. The simulation over Bolund hill is performed at high resolution (2 m), thus it does not show significant differences compared to VR-IBM in Chapter 2, but confirms the capability of HYBRID-IBM over very steep terrain which cannot be accommodated using the terrain-following coordinates in native WRF.

In Chapter 5, a brief summary of the dissertation is included together with comments about future research directions.

1.4 Summary of contributions

In summary, the main contributions of this research are:

- Implementation of the velocity reconstruction immersed boundary method (VR-IBM) method into WRF and validation with a variety of test cases, including flow over flat terrain, and flow over Askervein Hill and Bolund Hill at fine resolutions.

- Investigation of the performance of the VR-IBM and shear stress reconstruction immersed boundary method (SR-IBM) in WRF, and evidence that these existing methods have poor performance at intermediate resolutions and in the lee of hills.
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- Development and testing of a new hybrid log-law boundary condition (HYBRID-IBM) in WRF, designed to work for fine and intermediate-resolution atmospheric simulations.

- Validation of the new HYBRID-IBM at intermediate resolution for flow over flat terrain, over a moderately-sloped idealized hill, and over Askervein Hill.

- More detailed validation of the HYBRID-IBM approach over a wide range of test cases, including flat terrain test cases where the immersed surface intersects the grid at different locations, flat terrain test cases with different grid aspect ratios, and sensitivity tests for grid resolution and different hill slopes at intermediate resolutions.

- Evidence that the new HYBRID-IBM works well for very steep terrain including at intermediate resolutions, thus enabling WRF-IBM simulations over complex terrain that cannot be accommodated by the native terrain-following coordinate system in WRF.
Chapter 2

Implementation of a velocity-reconstruction immersed boundary method

2.1 Abstract

The Weather and Research Forecasting (WRF) model is increasingly being used for higher resolution atmospheric simulations over complex terrain. With increased resolution, resolved terrain slopes become steeper, and the native terrain-following coordinates used in WRF result in numerical errors and instability. The immersed boundary method (IBM) uses a non-conformal grid with the terrain surface represented through interpolated forcing terms. The WRF-IBM implementation of Lundquist et al. (2010, 2012) eliminates the limitations of WRF’s terrain-following coordinate and was previously validated with a no-slip boundary condition for urban simulations and idealized terrain. This chapter describes the implementation of a log-law boundary condition into WRF-IBM to extend its applicability to general atmospheric complex terrain simulations. The implementation of the improved WRF-IBM boundary condition is validated for neutral flow over flat terrain and the complex terrain cases of Askervein Hill and Bolund Hill. First, comparisons are made to similarity theory and standard WRF results for the flat terrain case. Then, simulations of flow over the moderately-sloped Askervein Hill are used to demonstrate agreement between the IBM and terrain-following WRF results, as well as agreement with observations. Finally, Bolund Hill simulations show that WRF-IBM can handle steep topography (standard WRF fails) and compares well to observations. Overall, the new WRF-IBM boundary condition shows improved performance, though the lee side representation of the flow can be potentially further improved.

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2.2 Introduction

Improved simulations of atmospheric flow over complex terrain would benefit a wide range of atmospheric applications, allowing for improvements in the fields of wind energy, wildfire prediction, and air pollution modeling, among others. With this aim, mesoscale weather prediction models, such as the Weather Research and Forecasting (WRF) model, are increasingly being used for high-resolution simulations, including those resolutions required for large-eddy simulation (LES). While most LES models make use of idealized lateral boundary conditions, LES simulations within the WRF model can use the available grid nesting framework, which employs multiple telescoping grids of increasing resolution, to provide downscaled lateral boundary conditions (Wang et al., 2013; Taylor et al., 2016; Muñoz-Esparza et al., 2017). Additionally, LES simulations performed in the WRF model can make use of an extensive suite of atmospheric physics parameterizations, including an emerging set of parameterizations that are scale-aware (Talbot et al., 2012; Powers et al., 2017).

While it is highly desirable to perform LES-scale simulations in models such as WRF, the use of terrain-following coordinates in WRF and most other mesoscale atmospheric models has limited LES to flow over terrain with shallow slopes (Catalano and Moeng, 2010; Taylor et al., 2016). Terrain-following coordinates work well at the resolutions used in mesoscale simulation, but numerical errors can arise at fine resolutions where steep terrain slopes can be captured, resulting in a highly-skewed grid with large numerical errors (Mahrer, 1984; Schär et al., 2002; Zängl, 2002; Klemp et al., 2003; Zängl et al., 2004). These numerical errors contaminate the simulation, often leading modelers to apply smoothing filters to the terrain or increase numerical damping constants in an effort to enable the simulation to run (see e.g Marjanovic et al., 2014).

To address the limitations of terrain-following coordinates in WRF, an immersed boundary method (IBM) has previously been implemented into the WRF model (Lundquist et al., 2010, 2012; Ma and Liu, 2017). Immersed boundary methods use non-conforming grids, often Cartesian, with the terrain boundary immersed within the grid, thereby alleviating numerical errors associated with the terrain-following grid transformation. When IBM is used, boundary conditions at the terrain surface are set through interpolation procedures for grid cells near the immersed surface, rather than at a grid interface aligning with the terrain.

Lundquist et al. (2010) first developed a WRF-IBM implementation and tested it for mountainous and urban terrain, including coupling the IBM to atmospheric physics parameterizations (e.g. the radiation and land-surface models). Lundquist et al. (2012) showed that WRF-IBM performed comparably to a code using body-fitted coordinates for urban flow simulations from the Joint Urban 2003 field campaign in Oklahoma City. A limitation of these IBM simulations was the use of a no-slip boundary condition at the immersed surface. The no-slip boundary condition simplified the implementation and validation of IBM and is commonly used in urban simulations, but is inappropriate for general atmospheric flows, where the surface layer is not resolved (Moeng, 1984; Garratt, 1994).

Here, we extend WRF-IBM to include a boundary condition which parameterizes surface stresses in the unresolved surface layer using Monin-Obukhov (M-O) similarity theory. We initially consider only neutral stability, in which case M-O theory simplifies to the commonly known
CHAPTER 2. IMPLEMENTATION OF A VELOCITY-RECONSTRUCTION IMMERSED BOUNDARY METHOD

“log law”:

\[ U = \frac{u^*}{\kappa} \ln \left( \frac{z}{z_0} \right), \]  

(2.1)

where \( U \) is the wind speed, \( u^* \) is the friction velocity, \( \kappa \) is the von Karman constant, \( z \) is the height above the surface, and \( z_0 \) is the surface roughness length. Methods for combining the log law with the immersed boundary method currently exist in the literature (Senocak et al., 2004; Choi et al., 2007; Roman et al., 2009; Jafari et al., 2011; Cheng and Porté-Agel, 2013; Anderson, 2013; Anderson and Chamecki, 2014; Diebold et al., 2013; Cheng and Porté-Agel, 2015; Fang and Porté-Agel, 2016; Umphrey et al., 2017), and notably Ma and Liu (2017) implemented the method of Chester et al. (2007) into the WRF model. While the method in Ma and Liu (2017) modifies stress terms near the immersed boundary, the method presented here works by modifying velocity terms.

We begin with a detailed description of the improved WRF-IBM implementation where a log-law boundary condition is used to parameterize surface stress. The implementation in WRF is described in detail and the method is compared to existing methods in the literature. Then, a neutral atmospheric boundary layer over flat terrain is simulated using both the native terrain-following coordinates in WRF and the new IBM implementation. Next, simulations over Askervein Hill are performed and results are compared to field observations. This test case is chosen for its relatively shallow terrain slopes, which enable direct comparison between WRF simulations using the native terrain-following coordinate and those with the new IBM implementation. Finally, simulations of flow over Bolund Hill are performed and WRF-IBM results are compared to field data. This case includes steep terrain slopes, including a near-vertical escarpment on the west face of the hill, which presents difficulty for simulations using native WRF terrain-following coordinates.

This work is one piece of several ongoing efforts to enable simulations spanning the meso- to micro-scale using the WRF framework. This log-law boundary condition implementation enables high-resolution simulations over complex terrain, contributing to the overall purpose to develop a model which can include meso-scale input and atmospheric physics parameterizations in fine scale complex terrain simulations. To include real atmospheric physics for WRF-IBM, Lundquist et al. (2010) coupled the WRF-IBM to the MM5 short-wave radiation scheme, the RRTM long wave radiation scheme, the MM5 surface layer model and the Noah land surface model. Arthur et al. (2018) coupled the topographic shading effect with WRF-IBM and validated it with idealized simulations and field data. Daniels et al. (2016) developed and tested a new vertical-refinement capability allowing the grid on each domain to be optimized and thus improving WRF’s multiscale simulation capabilities. Mirocha and Lundquist (2017) further assessed the effects of vertical refinement and grid aspect ratios at fine resolution in WRF using large-eddy simulation. Finally, Wiersema et al. (2016) described work on nesting WRF-IBM within terrain-following WRF simulations. With these combined efforts, including the implementation of the log-law boundary condition for WRF-IBM detailed in this chapter, WRF-IBM will enable complex terrain simulations at a new level: allowing steep terrain to be represented with much finer resolution and simultaneously including meso-scale forcing and the full atmospherics capabilities in WRF.
2.3 Background and Methods

2.3.1 Immersed boundary method

The description of the immersed boundary method here parallels that of Bao et al. (2016) as follows in this subsection. The influence of the immersed boundary acts as an additional body force \( F_b \) term in the momentum equation,

\[
\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} = -\frac{1}{\rho} \nabla P + \nabla \cdot \mathbf{\tau} + F_b
\]  

(2.2)

where \( \mathbf{U} \) is the simulated velocity field, \( \rho \) is the density, \( P \) is pressure, and \( \mathbf{\tau} \) is the turbulent stress term. Molecular viscosity effects are neglected as in WRF. The body force is zero at computational nodes away from the immersed boundary, but takes a non-zero value close to the boundary. Mohd-Yusof (1997) proposed a direct forcing approach, by indirectly incorporating \( F_b \) into the discretized form of the momentum equations. On the Cartesian grid, the governing equations are first discretized neglecting the immersed boundary. Then, cells near the IB are adjusted to account for the boundary presence. Full details of the overall (no-slip) IBM implementation in WRF are given in Lundquist et al. (2010, 2012).

The implementations of the log law with the immersed boundary method in the literature can be divided into three categories. Chester et al. (2007) developed a shear stress reconstruction IBM (SR-IBM), which modifies several layers of shear stress in the vicinity of the immersed boundary. Shear stress values on the immersed surface are calculated according to the log law. The shear stress values within a defined distance, external to the immersed boundary, are then set to the surface stress value. Stress values interior to the immersed boundary are linearly extrapolated from the surface value and a value outside the band of nodes being reconstructed. Velocity is set to zero interior to the immersed boundary. Diebold et al. (2013) used this SR-IBM with large-eddy simulation to simulate flow over Bolund Hill. The results obtained are among the best results of the modeling efforts for the Bolund case summarized in Bechmann et al. (2011). Cheng and Porté-Agel (2013, 2015) used this SR-IBM to investigate a turbulent boundary layer flow over a two-dimensional cube and uniform arrays of cubes. The simulations were validated with wind tunnel experimental data with good agreement. Ma and Liu (2017) followed this SR-IBM method and implemented it into WRF, testing it with large-eddy simulations over Bolund Hill compared with observations. Simulations with fine vertical resolution and the Lagrangian-averaged scale-dependent Smagorinsky model showed improvement over standard Smagorinsky for the mean speed-up error and for capturing the recirculation zone. Fang and Porté-Agel (2016) performed an intercomparison of terrain-following coordinates and the SR-IBM in large-eddy simulation for flow over an idealized and moderately sloped 3D hill. They found that the SR-IBM results predicted a larger recirculation zone in the lee side of the hill compared to benchmark simulations. The SR-IBM method was also tested in WRF by (Bao et al., 2016), where similar issues were found in the recirculation zone.

Another approach was taken by Anderson (2013) who used a canopy-like drag model to characterize surface stress. In this method, a canopy stress model is used to impose a momentum sink
for cells near the surface. Cells where the extra drag is applied are determined based on surface
geometry and can include single or multiple points, interior and exterior to the immersed surface.
Nodes internal to the immersed boundary are set to have zero velocity. Validation cases performed
by Anderson (2013) included flow over a single cube and multiple cubes. Results using very fine 1
m resolution showed good agreement with benchmark data from previous studies. The canopy-like
method was also tested in WRF by (Bao et al., 2016), where the model predicted smaller velocities
and a larger recirculation zone in the lee of a hill, compared to benchmark simulations and other
immersed boundary options.

In a third approach, Fadlun et al. (2000) and Senocak et al. (2004) used velocity reconstruction
near the surface assuming a linear or a log profile, respectively. Fadlun et al. (2000) first used a
linear interpolation between a point in the fluid (the second point from the boundary) and a no-slip
boundary condition at the surface to reconstruct the velocity at a layer of fluid points outside of the
immersed surface (the first point from the boundary). For atmospheric flows, the fine resolution
required to use a linear interpolation scheme (to resolve the viscous sublayer) would not be prac-
tical. Senocak et al. (2004) extended this method by using a log-law reconstruction scheme. The
tangential flow is reconstructed at the first fluid node using the log law. Choi et al. (2007) extended
this to reconstruction of a power law between the surface and a reference velocity at a fluid node
\( u(z) = U_{ref} \left( \frac{z}{z_{ref}} \right)^k \). A small value of \( k \) can approximate a logarithmic distribution while a choice
of \( k = 1 \) can represent a linear distribution for a no-slip condition. In Senocak et al. (2004), the
implementation is one dimensional (normal to the immersed surface) and the viscosity near the
surface is modified based on a RANS/LES hybrid approach. Senocak et al. (2015) and Umphrey
et al. (2017) extended the implementation to 3D and included the heat flux with application to
atmospheric katabatic flow.

Here, we implement the method from Senocak et al. (2004) into WRF, with some minor dif-
fferences. It is the first time the VR-IBM method from Senocak et al. (2004) is implemented into
WRF. This method was selected for this study from among the three IBM options implemented
and explored in detail in Bao et al. (2016) (and in Chapter 3). The method from Senocak et al.
(2004) is the most straightforward in directly setting the velocity boundary condition to satisfy the
log velocity profile near the surface. Details of the differences between the method from Senocak
et al. (2004) and our method implemented into WRF are provided in Section 2.32.3.3. Notably,
WRF’s pressure-based terrain-following coordinate system presents challenges in an IBM imple-
mentation which are unique compared to traditional incompressible computational fluid dynamics
codes. We compare our results to simulations using a simple log-law condition with WRF’s native
terrain-following coordinate (see Section 2.32.3.2). This allows the surface stress in both WRF
and WRF-IBM to be parameterized with a surface roughness length scale \( z_0 \), facilitating the com-
parison of results from simulations using terrain-following WRF and IBM coordinates.

### 2.3.2 WRF’s surface boundary condition

In WRF, surface stresses are set using

\[
\tau_{i3}^s = -C_d |U| U_i \tag{2.3}
\]
where \( i = 1 \) or 2. In this equation, \( \tau_{i3} \) denotes the surface shear stress in the \( x \) and \( y \) directions, \( C_d \) is the drag coefficient, and \( |U| \) denotes the wind speed. There are several ways to obtain \( C_d \) in WRF. It can be specified directly as an input in the namelist, or M-O theory can be used to calculate \( u_* \) in the land surface model in WRF. Additionally, we added an option which calculates the drag coefficient based on a specified surface roughness parameter \( z_0 \) and the log law. In this formulation, the coefficient of drag is given by

\[
C_d = \left( \frac{\kappa}{\ln \frac{z}{z_0}} \right)^2
\]

Surface stresses are imposed in the vertical direction in native WRF coordinates, rather than in the surface normal direction. This is adequate for the mesoscale resolutions for which WRF is often used, where terrain slopes are typically very shallow. At microscale resolutions, terrain can become very steep (30-60 degrees or steeper), and this approximation may introduce error in the surface stress. In addition to setting the shear stress at the surface, WRF enforces no flow through the lower boundary by diagnosing the vertical velocity \( w_s \) at the surface using equation (2.5), where \( \mathbf{V}_h \) is the horizontal velocity vector and \( h \) is the terrain height.

\[
w_s = \mathbf{V}_h \cdot \nabla h
\]

2.3.3 Velocity-reconstruction IBM implementation

Here the implementation of the velocity reconstruction immersed boundary method (VR-IBM) in WRF follows that of Senocak et al. (2004), where the velocity is reconstructed at the first fluid
node assuming there is a logarithmic profile near the surface, as represented by the open blue circle $(U_1)$ in Figure 2.1. The use of log law models in both VR-IBM and native WRF is not accurate for flow over complex terrain and in flow separation regions, however, better options for surface momentum flux models are not yet available (Piomelli and Balaras, 2002; Piomelli, 2008). On the staggered grid used by WRF, cut cells are first determined for each velocity variable for which a boundary condition will be imposed. (The $U_1$ illustrated is the $U$-component of the velocity. The same reconstruction procedures are required for each velocity component ($U$, $V$ and $W$) on the staggered grid.) Then the reconstruction velocity point $(U_1)$, which is the first fluid point above the immersed boundary, is identified. A surface normal line is projected from the immersed boundary through the reconstruction point $(U_1)$ until it hits the cell face above. Where the surface normal line intersects the cell face above is denoted the image point, $U_{img}$ (blue open circle in Figure 2.1). $U_1$ and $U_{img}$ are thus normal to the immersed boundary. Neighboring fluid nodes (green squares) are used to interpolate the value at $U_{img}$ for all $U$, $V$ and $W$ velocities. $U_{img}$ is then projected in the surface normal ($U_{img}^n$) and surface tangential ($U_{img}^t$) directions. Assuming that multiple nodes reside within the logarithmic layer, and that $u_*$ is relatively constant within this region, the tangential velocity at $U_1^t$ can then be calculated using (2.6) based on the interpolated velocity at $U_{img}^t$:

$$U_1^t = U_{img}^t \ln \frac{d_1}{\delta_0} \frac{\ln \frac{d_{img}}{\delta_0}}{\ln \frac{d_{img}}{\delta_0}}$$

(2.6)

where $d_1$ and $d_{img}$ are the perpendicular distances from the boundary to the $U_1$ point and the $U_{img}$ point, respectively.

The normal component is reconstructed at $U_1$ with linear interpolation (assuming no flow normal to the boundary) between the normal velocity at $U_{img}^n$ and the zero normal velocity at the surface, assuming there is no flow through the immersed boundary:

$$U_1^n = U_{img}^n \frac{d_1}{d_{img}}$$

(2.7)

Once the surface normal and surface tangential velocities at $U_1$ are individually reconstructed, they are rotated back to set the boundary condition value $U_1$. Velocities below the immersed boundary are allowed to evolve without any special treatment.

The slight difference between our method and Senocak et al. (2004) is with respect to setting the eddy viscosity. Senocak et al. (2004) modify the near-surface eddy viscosity based on a hybrid RANS/LES approach (see also Senocak et al., 2015; Umphrey et al., 2017). Prandtl’s mixing length hypothesis is adopted near the surface and a dynamic Smagorinsky model is used away from the surface. We tested this approach and found no significant effect on the near-surface velocities. We therefore retained WRF’s native eddy viscosity parameterization to simplify comparisons between native WRF and the VR-IBM implementation.
2.4 Validation

2.4.1 Neutral boundary layer simulation over flat terrain

For initial validation of the new IBM method in WRF, simulations are carried out for neutral atmospheric boundary layer flow over flat terrain, where the mean velocity profile in the surface layer should follow the log law. The following section details the domain setup and results for simulations using the native WRF and WRF-IBM grids. Comparisons are made between the simulation results when using the native WRF boundary condition and both the no-slip and velocity-reconstruction IBM techniques (NS-IBM and VR-IBM).

2.4.1.1 Simulation setup

The neutral boundary layer simulation setup in WRF is similar to the standard setups used in Chow et al. (2005) and Mirocha et al. (2013a,b). Flow is driven by a pressure gradient which would balance a geostrophic wind of \((U_g, V_g) = (10, 0) \, \text{m s}^{-1}\), with the Coriolis parameter \(f\) set to be \(10^{-4} \, \text{s}^{-1}\). Periodic lateral boundary conditions are used with a horizontal resolution of \(\Delta x = \Delta y = 33 \, \text{m}\), with an overall domain size of 3300 m in each horizontal direction. 132 vertical grid points are used in both the terrain-following and IBM cases, with the terrain surface at 0 m and the domain top at 1500 m. With terrain-following coordinates, the minimum vertical grid spacing \(\Delta z_{\text{min}}\) is 10.7 m and the maximum \(\Delta z_{\text{max}}\) is 12.19 m. When IBM is used, the flat terrain must be immersed within the domain, therefore the 132 vertical levels span from -50 to 1500 m, with \(\Delta z_{\text{min}} = 11.7 \, \text{m}\) and \(\Delta z_{\text{max}} = 12.57 \, \text{m}\). Rayleigh damping is applied over the top 300 m of the domain. The Smagorinsky turbulence closure is used in this case. The initial conditions are perturbed, such that the flow becomes turbulent after a few hours of spin up time.

All cases are initialized with a neutral and dry sounding with a uniform \(10 \, \text{m s}^{-1}\) wind in the \(x\)-direction. The surface is set to have a constant surface roughness of \(z_0 = 0.1 \, \text{m}\). Inertial oscillations are present, due to imbalances between the pressure gradient and the Coriolis forcing, but significantly decrease in amplitude over several inertial periods. The inertial oscillations have a period of \(2\pi / f\), which is approximately 17.5 hours for the prescribed Coriolis parameter in this model. Total integration time for this case is 72 hours, and the results shown here are time-averaged over the last 24 hours when the inertial oscillations have damped substantially.

2.4.1.2 Results

Figure 2.2 shows the time evolution of the inertial oscillations using the domain averaged \(U\) and \(V\) velocities. Inertial oscillations are sufficiently damped after 48 hours so that the solution can be considered to have reached a steady state condition and time averaging is appropriate. It can be seen here that while all three methods have similar oscillation periods, the NS-IBM results in a different domain-averaged velocity. The cause of this can be seen in Figure 2.3, which shows the time and planar-averaged \(U\) and \(V\) velocities for simulations using the original WRF terrain-following coordinate and the two IBM methods. Results are time averaged over hours 48-72 of simulation time, with data at 15 min intervals. As seen in the figure, the VR-IBM method does an
excellent job in recreating the original WRF solution for this case. The no-slip boundary condition, shown for contrast, results in slow velocities and additional stress near the surface.

This initial validation for idealized neutral boundary layer flow verifies that our VR-IBM implementation can recreate the native WRF surface stress boundary condition and the velocity profile over flat terrain. Additionally, the VR-IBM method has the flexibility to handle complex terrain, as demonstrated in the following sections.
2.4.2 Askervein Hill simulation

2.4.2.1 Field campaign description

The Askervein Hill field campaign, which took place in 1982 and 1983, studied flow over a low amplitude and moderately sloped hill near the west coast of South Uist in Scotland (Taylor, 1983). Askervein is a 116 m high hill and elliptical in plan view, with a 2 km major axis along a general NW-SE line and a 1 km minor axis. During the field campaign, there were more than 50 towers, which were placed in three arrays (A, AA, and B lines in Figure 2.4), as well as an “up-stream” reference site (RS). These tower measurements generated a detailed dataset of the surface wind field. The dominant wind directions during the campaign were from the southwest, nearly perpendicular to the major axis of the hill. A detailed description of the field campaign including the instrumentation and measurements are given in Taylor (1983) and Taylor and Teunissen (1985, 1987). We select Askervein Hill to evaluate the implementation of the log-law boundary condition for WRF-IBM because the small terrain slope makes it possible to compare to a reference simulation using standard WRF with terrain-following coordinates.

2.4.2.2 Simulation setup

Topographic data for Askervein are provided by Walmsley and Taylor (1996) at 25 m resolution. The incoming wind direction is 210°, and the terrain is rotated 60 degrees in the clockwise direction, so that the incoming winds align with the x-axis of the domain in the simulation (i.e. the incoming wind direction is set to 270° in the simulation). A one-way nested grid setup is used, where the parent domain has flat terrain and uses periodic lateral boundary conditions, and the nested domain contains Askervein Hill and is forced at its boundaries by the parent domain. This
setup is similar to simulations in Golaz et al. (2009) where flow over Akservein hill is simulated in the COAMPS mesoscale model. The nesting is one-way so that the parent domain is not influenced by the terrain-induced flow in the nested domain but simply provides the boundary forcing for the inner domain. The boundary layer in the parent domain is neutral and driven by a geostrophic wind speed of \( \sim 18 \text{ m s}^{-1} \), which was previously used in Golaz et al. (2009) to obtain agreement with observations at the reference site. The horizontal grid in the parent domain contains 303 \( \times \) 303 points with 20 m spacing to cover a 6060 m square domain, while the nested domain is 801 \( \times \) 701 points with 5 m spacing. The total domain size for the nested domain is 4000 m by 3500 m. For the terrain-following WRF case, both the parent and nested grids share the same 70 vertical levels with spacing of \( \Delta z = 1 \text{ m} \) in the lowest 10 m of the domain and stretching above with a 1.1 stretching ratio until the vertical resolution is approximately 100 m, after which the grid remains uniform up to the 2010 m domain height. For the IBM case, there are 72 vertical levels because at least two grid points are required below the surface. The same vertical grid spacing and stretching are applied, therefore the two grids match relatively well in the flat topography areas of the domain.

The flow on the outer domain is allowed to spin up for 18 hours to allow inertial oscillations in the neutral boundary layer to dampen. At 18 hours, the nested domain is spawned and nested boundary conditions are used. The parent and nested domain are run concurrently for an additional 1.5 hours. Results presented here are averaged over the last 30 minutes of the simulation after a 1 hour spin up time for the inner domain. Figure 2.4 shows the parent and nested domain with the topography of Askervein Hill and the field data transects. We use a constant roughness value of \( z_0 = 0.03 \text{ m} \), as given in the field report. A separate roughness length is not used over the water. A land surface model is not used, so there are no heat or moisture fluxes included for the short time period of the simulation. The Smagorinsky turbulence closure is used in this case.

\[ \Delta S = \frac{S(z) - S_{RS}(z)}{S_{RS}(z)} \]

2.4.2.3 Comparison with observations

Figure 2.5 shows the simulated time- and planar-averaged velocity profile from the outer domain compared to observation data at the reference site. The simulation results are time-averaged over the last 30 mins of the outer domain with an output frequency of 1 mins. The RMSE between the wind speed observations and WRF results is 0.8 m s\(^{-1}\), while the RMSE is 0.4 m s\(^{-1}\) for VR-IBM. Reasonable agreement with observations is achieved in both cases, indicating that the outer domain provides the necessary forcing to the inner domain. Note that small differences between WRF and VR-IBM are seen here due to the coarser vertical resolution used in this case, compared to the flat terrain case presented above. Differences between the velocity profiles and observations are comparable to those in several previous studies of Askervein Hill (Castro et al., 2003; Chow and Street, 2009; Golaz et al., 2009).

Observations along lines A, AA, and B are compared with averaged velocities from the two simulations. Comparison of the wind flow over Askervein Hill is performed in terms of fractional speedup, \( \Delta S \) which is defined as

\[ \Delta S = \frac{S(z) - S_{RS}(z)}{S_{RS}(z)} \]
where $S$ is the horizontal wind speed and $S_{RS}$ is the wind speed at the reference site. $\Delta S$ provides a measure of the influence of the terrain on the wind field based on the upwind undisturbed inflow. The simulation results are time averaged over the last 30 mins with 1 mins output frequency.

The 10 m wind speedup along lines A, AA and B is shown in Figure 6. Along lines A and AA, the maximum speedup occurs at the top of the hill. The standard WRF model underpredicts the speedup at the hill top especially along line A, which passes over the peak of Askervein hill. The greatest difference between WRF and VR-IBM again occurs in the lee of the hill, which relates to the discrepancies in the prediction of flow separation and recirculation. Both of WRF and VR-IBM successfully predict the flow deceleration, while the location, size and strength are different. The standard WRF model tends to overestimate the flow deceleration along line AA and line A. The poor prediction of lee side flow separation could be due to the grid resolution chosen (Lopes et al., 2007), turbulence closure scheme (Chow and Street, 2009; Golaz et al., 2009), and/or the use of the log law model (Lopes et al., 2007). Lopes et al. (2007) noticed the maximum difference in speedup along line A can be as big as 0.4 on the lee side of the hill between their coarse resolution ($\Delta X = 50$ m) and their fine resolution ($\Delta X = 25$ m). They argued that the coarser resolution led to an unrealistic subgrid structure introduced by the turbulence model. The maximum difference
in speedup on the lee side of the hill along line A between our VR-IBM and observations is as small as 0.14. Chow and Street (2009) reported that the TKE 1.5 model failed to produce the flow deceleration on the lee side of the Askervein hill, while the dynamic reconstruction model did a much better job. Chow and Street (2009) calculated the RMSE along line A as 0.22 when using the TKE 1.5 turbulence closure model and 0.09 with the dynamic reconstruction model. The RMSE for the speedup along line A here is 0.11 for our VR-IBM and 0.25 for WRF. The RMSE along line A for VR-IBM is comparable to the dynamic reconstruction model of Chow and Street (2009), which is a more sophisticated turbulence model. The RMSE for the speedup along line AA is 0.09 for VR-IBM and 0.18 for WRF; along line B it is 0.13 for VR-IBM and 0.24 for WRF. In the VR-IBM case, grid distortion and the associated truncation error are avoided on the lee side of the hill, and the RMSE of the speedup along all three lines is improved over standard WRF when compared to observations. Golaz et al. (2009) also noticed a difference in speedup ratio (approximately 0.2) on the lee side deceleration zone of Askervein hill when using different mixing lengths for the Smagorinsky turbulence closure. Furthermore, the use of a log law model in both VR-IBM and WRF, which is certainly not accurate in flow separation regions, could be another reason for poor prediction of the strength and size of the flow deceleration on the lee side of Askervein hill (Lopes et al., 2007; Piomelli and Balaras, 2002; Piomelli, 2008).

Next, a direct comparison of the time-averaged velocity profiles between standard WRF and VR-IBM is shown in Figure 2.7. The velocity profiles are averaged for the last 30 minutes (at 1 minute intervals) and are shown at several locations along line A. It can be seen that the VR-IBM results are in good agreement with standard WRF, and that the differences between WRF and VR-IBM decrease with height. The difference at the domain top can be seen as due to the difference in bulk inflow velocity, which is about 1 m s\(^{-1}\). The maximum difference of 2.53 m s\(^{-1}\) is found near the surface, in the lee side of the hill (the 5th profile). Considering the difference in bulk inflow velocity, which is about 1 m s\(^{-1}\), the difference between WRF and VR-IBM due to the surface representation is about 1.5 m s\(^{-1}\). As discussed in Bao et al. (2016), the VR-IBM method has some shortcomings in the representation of the near-surface velocity field when compared to WRF, particularly in the lee of the hill. This may be due to sensitivity of the flow to interpolation errors in the flow separation region. The simulation of the lee side of the hill is always very challenging due in part to intermittent flow separation, and results can be influenced by different parameters such as grid resolution and turbulence closure schemes (Chow and Street, 2009; Ma and Liu, 2017). For example, Chow and Street (2009) found differences of up to 5 m s\(^{-1}\) in the lee of Askervein Hill for simulations using different closure models. As another example, Marjanovic et al. (2014) compared WRF simulations to observations over a different complex terrain site using several closure models; the root mean squared errors between the WRF results and observations improved from 5 to 3.5 m s\(^{-1}\) simply by choosing a different closure model. Furthermore, WRF uses a vertical gradient and IBM uses a surface-normal gradient so there is no guarantee that the WRF result is better than the IBM result over sloped terrain (Lundquist et al., 2010). Given the relatively straightforward ease of implementation of VR-IBM and the points discussed above, the RMSE differences between WRF and VR-IBM seem acceptable, yet remain the subject of further work, as discussed in Bao et al. (2016).
Figure 2.6: (a) Wind speed-up at 10 m AGL over Askevein Hill for WRF and VR-IBM, compared with field observations with error bars (squares), (a) along line A, (b) along line AA, and (c) along line B.
Figure 2.7: Time-averaged $U$ profiles at various locations along line A over Askervein Hill for terrain-following WRF and VR-IBM in m s$^{-1}$.

2.4.3 Bolund Experiment simulation

2.4.3.1 Field campaign description

The Bolund Hill field campaign, conducted in the winter of 2007-2008 over Bolund Hill, was designed to study atmospheric flow over steep terrain. A description of the field campaign can be found in Berg et al. (2011). The hill is 130 m long (West-East direction) and 75 m wide (North-South direction) with a maximum height of 12 m, as shown in figure 2.8. The hill is covered with grass and is surrounded by water with a long uniform fetch over the sea in the westward direction, which is the origin of the incoming flow for the case simulated here. The geometry of the hill makes atmospheric flow simulations challenging because the western (windward) face of the hill is a steep 90-degree slope and the lee side of the hill creates a recirculation zone with flow separation. As shown in Røkenes and Krogstad (2009), the details of the crest geometry representation can strongly affect the flow recirculation.

During the field campaign, 38 anemometers were deployed over the hill at 10 meteorological tower locations as shown in figure 2.8 (Bechmann et al., 2011), including 26 sonic anemometers, 12 cup anemometers, and two lidars. Mast M9 is located east of the hill and not shown. Instruments were placed at multiple heights on the observational towers, frequently located at 2, 5, and 9 m AGL, though each tower did not have instruments at all three heights, and some towers used alternate heights. At each tower, turbulent wind velocities and fluxes were measured. The simulation presented here is Case 3 as listed in Bechmann et al. (2011), which details results of a blind simulation intercomparison project. The mean wind speed during the selected measurement period approached 10 m s$^{-1}$ at about 20 m height, with a wind direction from the southwest at 242 degrees. The observation data are averaged over 10 minutes.

Results using NS-IBM and VR-IBM are included in this section. Standard terrain-following WRF simulations fail because of the nearly 90-degree steep slope, which leads to large grid distor-
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Figure 2.8: Tower locations over Bolund Hill. The dashed line is along the 242 degree wind direction.

2.4.3.2 Simulation setup

As in the previous simulation, a domain containing Bolund Hill is nested within an outer domain on which the flow is a fully-developed boundary layer over flat terrain. Flow is driven in the parent domain with a uniform pressure gradient chosen to produce a velocity profile which matches observations (see Fig. 2.10). Boundary conditions are passed from the flat domain to the nested Bolund Hill domain at each outer domain time step, as in the Askervein Hill case above. The outer domain is periodic, with $\Delta x = \Delta y = 6$ m, with an overall domain size of 912 m in the $x$ direction and 504 m in the $y$ direction. The domain top is located at 250 m, and the vertical domain dimension is 252.25 m for IBM cases to allow for points below the terrain. 132 vertical grid points are used in both of the IBM cases. The minimum vertical grid spacing is $\Delta z_{\text{min}} = 0.75$ m and the maximum is $\Delta z_{\text{max}} = 2$ m. For the inner domain, which includes Bolund Hill, $\Delta x = \Delta y = 2$ m, with an overall domain size of 804 m in the $x$ direction and 444 m in the $y$ direction. The vertical domain height and grid are the same as on the outer domain. The Smagorinsky turbulence closure is used. A uniform roughness length of $z_0 = 0.3$ mm (water) is used for the entire outer domain. Roughness lengths of $z_0 = 0.3$ mm over water and $z_0 = 15$ mm over the hill are used for the nested domain following Diebold et al. (2013). Simulations are carried out for a wind direction of 242 degrees, to match conditions observed during the field campaign at the reference site (5 m agl). The parent domain is spun up for 18 hours. The nested domain which contains Bolund hill is forced at its boundaries by the parent domain and run for another for 1.5 hr; the first hour is spin up, and the last 0.5 hr is used to time average the results. The setup of the Bolund simulation is shown in Figure 2.9.

2.4.3.3 Comparison with observations

The inflow wind profiles of velocity are shown in Figure 2.10, where the simulations are compared with measurements at tower M0, which is about 150 m upstream of the hill. The red solid
Figure 2.9: Horizontal slice of instantaneous $U$-velocity at 5 m elevation for (a) outer domain and (b) inner domain. The dashed line indicates the inner nest location. The black contours are the Bolund Hill terrain at 1.2 m intervals.

The line is the theoretical log profile obtained using observation points:

$$u(z_{agl}) = \frac{u_*}{k} \ln \left( \frac{z_{agl}}{z_0} \right)$$

(2.9)

where $z_0 = 0.3$ mm is the roughness length, $u_* = 0.45$ m s$^{-1}$ is the friction velocity measured during the field and $z_{agl}$ is the elevation above ground level. Red circles are additionally shown, which are observations taken on the tower at 2 m, 5 m, 9 m, and 15 m. Time-averaged results are shown for the inner and outer domain at the tower location for both IBM methods. Simulation results from the outer domain are additionally planar-averaged, which make the profiles appear smoother than those on the inner domain. It can be seen from the figure that the VR-IBM method produces a velocity profile more similar to the log law than the NS-IBM method, although both IBM methods produce slower wind speeds than the log law at low elevations and faster at higher elevations.

Figure 2.11 shows time-averaged wind vectors at instrument locations placed 2 m, 5 m and 9 m above the terrain. Observations are shown as red vectors while the VR-IBM and NS-IBM simulations are shown in blue and black, respectively. Qualitatively, it can be seen that the VR-IBM method improves agreement with observations over the NS-IBM method. For example, in the top panel of figure 2.11, the wind direction of the NS-IBM simulation is incorrect on the windward face of the hill, and the wind speed is too small at almost all locations, but especially in the lee of
CHAPTER 2. IMPLEMENTATION OF A VELOCITY-RECONSTRUCTION IMMERSED BOUNDARY METHOD

Figure 2.10: Averaged wind speed profiles at M0 for NS-IBM and VR-IBM on both outer (Domain 1) and inner (Domain 2) nests compared to observations and the idealized log law. Results are time and planar-averaged on the outer domain, but only time-averaged on the inner domain at the location M0.

<table>
<thead>
<tr>
<th>Wind speed RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
</tr>
<tr>
<td>NS-IBM</td>
</tr>
<tr>
<td>VR-IBM</td>
</tr>
</tbody>
</table>

Table 2.1: RMSE (m s$^{-1}$) for 2, 5 and 9 m wind vectors compared with observations from the Bolund experiment at all 9 meteorological towers.

the hill. Flow separation and reattachment is difficult to predict in CFD models (including WRF). Piomelli and Balaras (2002) tested an equilibrium log law model and Piomelli (2008) extended the work to three different surface flux models for large eddy simulation (equilibrium model, zonal model and hybrid model). They found that the accuracy of the surface flux models was highly dependent on the grid resolution and turbulence closure, and that there was no single method which was clearly better than others in representing flow separation and reattachment. To quantify simulation error, the RMSE for the 9 measurement stations are listed in Table 2.1 for both NS-IBM and VR-IBM. The RMSE for wind speed decreases with height, with the maximum RMSE at the 2 m wind vector for both NS-IBM and VR-IBM. The VR-IBM method has significantly lower RMSE compared with observations than the NS-IBM results. The maximum differences between simulations and measurements exist in the 2 m field.

In Figure 2.12, the ratio of LES wind speed to measured wind speed is shown as a function of elevation. It is clear that the largest mismatch occurs at the lowest points (2 m). There are many factors that can influence the mismatch near the surface, perhaps including the need for a more
Figure 2.11: Wind vectors for NS-IBM (black), VR-IBM (blue), and observations (red) at tower locations shown in Figure 2.8 for three different heights: 2, 5, and 9 m AGL ((a) panel to (c) panel).
accurate subgrid model to parameterize turbulence near the surface, and a finer grid resolution. In general, the VR-IBM performs better than NS-IBM, leading to a mean ratio close to 1 and a smaller standard deviation than NS-IBM. The results are comparable to Diebold et al. (2013) who reported a mean value of 0.96 with a standard deviation of 0.18.

Figure 2.13 (left) shows a scatter plot of VR-IBM and NS-IBM results vs. observation data. In this plot, the wind speeds are normalized by $u_*$, which is the friction velocity at the reference point, M0, the station far upstream of the hill. The VR-IBM method shows better agreement between the field data than NS-IBM (see $R^2$ values in the figure). Figure 2.13 (right) shows the ratio of LES wind speeds to the measured wind speeds as a function of measurement location. As mentioned in Diebold et al. (2013), there are certain locations of the flow field which are hard to predict. NS-IBM tends to underestimate the wind speed at most of these locations. The maximum underestimations occur at two lee side locations (M4 and M8). While VR-IBM reduces the under-prediction problem at M4 and M8, VR-IBM overestimates the wind speed at M2 and M6, which are just behind escarpment. When using the Smagorinsky closure, Diebold et al. (2013) and Ma and Liu (2017) noted that the model overestimates the wind speed in the lee of the hill, which is consistent with our findings. Diebold et al. (2013) and Ma and Liu (2017) both concluded that for the Bolund case, the choice of the turbulence model is important to accurately predict the near surface wind, though the use of similarity theory likely contributes to errors in the lee of the hill as well. Other turbulence models will be examined with VR-IBM in future work.

To investigate discrepancies between simulations and measurements over the hill (not including the first tower M0), we follow Bechmann et al. (2011) and quantify changes in the wind field as changes in speed-up. Speed-up for this case is defined by

$$\Delta S_m = \frac{(U/\bar{u}_*)_{z_{a gl}} - (U_0/\bar{u}_0)_{z_{a gl}}}{(U_0/\bar{u}_0)_{z_{a gl}}}$$

(2.10)

where $U$ is the mean wind speed at the sensor location and $U_0$ is the mean wind speed at the tower M0 (slightly different from the speed-up definition in the Askervein experiment which is not normalized by $u_*$). The comparison is made for two different elevations, 2 m and 5 m above ground level, as shown in Figure 2.14. Overall agreement is good, though the NS-IBM speedup decreases too much on the lee side of hill. Both of the IBM methods show large velocity speed up on the top of the hill compared to the observations. This may be partly due to the grid resolution we use (2 m), which is coarser than Diebold et al. (2013) and Ma and Liu (2017) (1 m). As shown in Røkenes and Krogstad (2009), the slight difference in crest geometry representation can strongly affect the flow recirculation and speedup.

To quantify model performance, statistical performance metrics are calculated for both NS-IBM and VR-IBM for all sonic and cup anemometer observations for wind speed in Table 2.2. These metrics are used in Lundquist et al. (2012) and Chang and Hanna (2004) and prove useful for quantifying the performance of NS-IBM and VR-IBM models at predicting wind speed. The selected metrics are FAC2 (the fraction of predictions within a factor of 2 of observations), FB (fractional bias), MG (geometric mean bias), and NMSE (the normalized mean square error),
Figure 2.12: Ratio ($R_{se}$) of LES wind speed to field data vs. elevation for (a) VR-IBM and (b) NS-IBM. The colors correspond to different tower locations as defined in Fig. 2.13 (b). The black dashed line indicates a ratio of 1.
Figure 2.13: (a) Scatter plot of normalized wind speed ($U/u_*$) comparing field data with LES results. (b) Ratio of LES wind speed from VR-IBM (triangles) and NS-IBM (circles) to field data for different tower locations.

defined as

$$\text{FAC}_x = \text{fraction of data satisfying } \frac{1}{x} \leq \frac{C_p}{C_o} \leq x$$ \hspace{1cm} (2.11)

$$\text{FB} = \frac{\bar{C}_o - \bar{C}_p}{0.5(\bar{C}_o + \bar{C}_p)}$$ \hspace{1cm} (2.12)

$$\text{MG} = \exp \left( \ln C_o - \ln C_p \right)$$ \hspace{1cm} (2.13)

$$\text{NMSE} = \frac{(C_o - C_p)^2}{C_o C_p}$$ \hspace{1cm} (2.14)

where $C_p$ is the predicted time-averaged value, $C_o$ is the observed time-averaged value, and the overbar denotes averaging over the dataset. A perfect model would have the values of $\text{FAC}_x = 1.0$, $\text{FB} = 0$, $\text{MG} = 1.0$, and $\text{NMSE} = 0$, while a good model as defined in Chang and Hanna (2004) has metrics of $\text{FAC}_2 > 0.5$, $-0.3 < \text{FB} < 0.3$, $0.7 < \text{MG} < 1.3$ and $\text{NMSE} < 4$. Based on these criteria, the VR-IBM performance is quite satisfactory, with significant improvement over NS-IBM results, as shown in Table 2.2 and earlier in Figure 2.11.

The scaled average angle (SAA) difference is suggested in Calhoun et al. (2004) for evaluating the wind direction prediction and is defined as

$$\text{SAA} = \frac{\sum |U_p||\Phi_p - \Phi_o|}{N|U_p|}$$ \hspace{1cm} (2.15)
Figure 2.14: Speed-up ratio over Bolund Hill along incoming wind direction of $242^\circ$ (M1, M2, M3 and M4) showing observations, NS-IBM and VR-IBM results at 5 m (a) and 2 m (b), with topography (c).
### Statistical performance metrics

<table>
<thead>
<tr>
<th>Model name</th>
<th>FAC2</th>
<th>FB</th>
<th>MG</th>
<th>NMSE</th>
<th>SAA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect model</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NS-IBM</td>
<td>0.95</td>
<td>0.2849</td>
<td>1.5016</td>
<td>0.1838</td>
<td>48.4158</td>
</tr>
<tr>
<td>VR-IBM</td>
<td>1</td>
<td>-0.1373</td>
<td>0.9096</td>
<td>0.0798</td>
<td>26.5495</td>
</tr>
<tr>
<td>Good model or acceptable model</td>
<td>[0.5,1]</td>
<td>[-0.3,0.3]</td>
<td>[0.7,1.3]</td>
<td>[0.4]</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2: Statistical performance metrics for VR-IBM and NS-IBM compared to observations at towers M1-M8 (see text for details).

<table>
<thead>
<tr>
<th>Model name</th>
<th>Error at 5 m</th>
<th>Error at 2 m</th>
<th>Average error</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS-IBM-2m</td>
<td>15.9</td>
<td>20.9</td>
<td>18.4</td>
</tr>
<tr>
<td>VR-IBM-2m</td>
<td>10.2</td>
<td>14.0</td>
<td>12.1</td>
</tr>
<tr>
<td>Ma and Liu (2017)-2m</td>
<td>11.4</td>
<td>16.2</td>
<td>13.8</td>
</tr>
<tr>
<td>Ma and Liu (2017)-1m</td>
<td></td>
<td>9.4</td>
<td></td>
</tr>
<tr>
<td>Diebold et al. (2013)-2m</td>
<td>14.5</td>
<td>9.2</td>
<td>11.9</td>
</tr>
<tr>
<td>Diebold et al. (2013)-1m</td>
<td>4.7</td>
<td>5.6</td>
<td>5.1</td>
</tr>
<tr>
<td>Bechmann et al. (2011)</td>
<td></td>
<td>16.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3: Comparison between the speed up percent errors from the present study (NS-IBM and VR-IBM), results from Ma and Liu (2017), Diebold et al. (2013), and from the average of models taking part in the blind test (Bechmann et al., 2011).

where $|U_p|$ is predicted wind speed, $\Phi_p$ is the predicted wind direction, $\Phi_o$ is the observed wind direction, and N is the number of points being averaged. SAA applies heavier weights to wind direction differences for large wind speeds compared to small wind speeds.

The speed-up errors obtained with VR-IBM are comparable to the modeling efforts summarized by Bechmann et al. (2011); Diebold et al. (2013); Ma and Liu (2017) in Table 2.3. VR-IBM compares quite favorably to the results (2 m grid resolution) from Diebold et al. (2013) and Ma and Liu (2017), which both used a more sophisticated turbulence model. Both Diebold et al. (2013) and Ma and Liu (2017) showed that increasing grid resolution (1 m horizontal spacing, compared to the 2 m spacing used here) can decrease the speedup error further.

### 2.5 Conclusions

We have developed an improved IBM for the WRF model with a log-law boundary condition using velocity reconstruction (VR-IBM) which is capable of handling steep terrain in atmospheric flow simulations. This feature will enable the use of WRF-IBM in nested meso- to micro-scale simulations where the topography is well resolved and therefore can be quite steep on the finest grids, causing WRF with its traditional terrain-following coordinates to fail. The VR-IRM imple-
CHAPTER 2. IMPLEMENTATION OF A VELOCITY-RECONSTRUCTION IMMERSED BOUNDARY METHOD

Implementation was first validated by simulating a neutral atmospheric boundary layer flow over flat terrain and comparing the solution to results achieved using the native terrain-following coordinate WRF. This ensures that our implementation of VR-IBM performs well over flat terrain, where detailed comparison with theory is possible.

Next, the method was validated for flow over Askervein Hill, which has a slope moderate enough for WRF using its native terrain-following coordinates. Comparison of the Askervein simulations reveals reasonable agreement between VR-IBM and native WRF simulations and compared with observations. The RMSE for fractional wind speedup are comparable to previous LES studies (see e.g. Chow and Street, 2009). The maximum differences occur on the lee side of the hill, where models are quite sensitive to intermittent flow separation (Chow and Street, 2009; Castro et al., 2003; Golaz et al., 2009). Overall, the performance of the VR-IBM method is well within the expected range, thus showing the model's suitability for complex flow situations. Differences between WRF and VR-IBM can be due to truncation errors in the WRF representation due to steep terrain slopes, errors in WRF because derivatives are taken in the vertical rather than slope-normal direction, and interpolation errors in the VR-IBM approach discussed by Lundquist et al. (2012) and Bao et al. (2016).

Finally, the VR-IBM is further tested against experimental data from the Bolund Hill field campaign, a test case which is too steep for the native WRF coordinates. In general, the VR-IBM method shows good agreement with field measurements with improvement in predicting the points closest to the hill crest and lee side over the NS-IBM results. The points on the lee side of the steep slopes near the ground are the most difficult to predict accurately, as found in previous LES simulations (Bechmann et al., 2011; Diebold et al., 2013). Nevertheless, the results compare quite well to other LES approaches (Diebold et al., 2013; Bechmann et al., 2011; Ma and Liu, 2017) and provide further confidence in the implementation of VR-IBM.

Compared to other work, this WRF-IBM implementation has been validated against a large number of test cases, not only to field observations (Bolund Hill), but also to theory (flat terrain), and directly compared to native WRF (flat terrain and Askervein Hill). Test cases are also chosen at different slopes, to demonstrate the robustness of the implementation (the Askervein Hill case with moderate slopes, and the Bolund Hill case with steep slopes). This series of test cases provides rigorous validation of the implementation of the velocity reconstruction method into WRF-IBM by ensuring that the VR-IBM implementation matches results from WRF with its native coordinate system for moderately sloped terrain and that it has the capability to handle steeply sloped terrain. WRF-IBM requires adequate grid resolution to resolve the immersed boundary to limit interpolation errors which lead to flow differences. On the other hand, native WRF has errors due to the skewed grid cells over steep terrain and its use of vertical derivatives instead of surface-normal derivatives to set fluxes. The Bolund test case illustrates the ability of the WRF-IBM implementation to handle very steep complex terrain which cannot be represented in native WRF at all, gaining further confidence in using WRF-IBM by comparing not just with field data but with other numerical studies which have used a variety of IBM or conformal coordinate systems. We therefore have a robust new tool in WRF which can be used for different applications, such as wind energy, complex terrain, urban dispersion studies, etc. Future work will consider additional complex terrain applications of VR-IBM and further improvement of the immersed boundary conditions to
CHAPTER 2. IMPLEMENTATION OF A VELOCITY-RECONSTRUCTION IMMERSED BOUNDARY METHOD

consider the findings of Bao et al. (2016).
Chapter 3

Development and testing of a new hybrid immersed boundary method

3.1 Introduction

The Weather Research and Forecasting (WRF) model is a numerical weather prediction and atmospheric research model. It can be used for a wide range of grid resolutions from mesoscale weather prediction (km scale) to large-eddy simulations for atmospheric flow studies (100 m and finer scales). Recently, atmospheric simulations have been pushed towards higher resolutions to better understand and capture the turbulence and dynamics of flow over complex terrain. Terrain-following coordinates, used by WRF and other mesoscale atmospheric models, have historically worked well on the coarse grids used in mesoscale simulations, where the details of the topography are not well captured and the terrain slope is thus relatively small. When it comes to fine resolution simulations over complex terrain, the details of the steep terrain are well resolved at fine resolutions, but highly skewed terrain-following grids near the surface create large numerical errors which often lead to model instability.

An immersed boundary method (IBM), which uses a non-conforming grid and imposes boundary conditions along the grid cells near the immersed topography, was previously implemented into the WRF model (Lundquist et al., 2010, 2012). This IBM can reduce numerical errors associated with steep terrain slopes in terrain-following coordinates, and thus provides an alternative to terrain-following coordinates used in WRF model for simulations over complex and steep terrain. The WRF-IBM model uses a background grid that does not conform to the terrain; instead the terrain boundary is immersed within the grid. Boundary conditions are set through the addition of a forcing term in the governing equation, whose magnitude is determined using interpolation procedures for grid cells intersected by the immersed surface.

Lundquist et al. (2010) first developed a WRF-IBM model with a no-slip boundary condition and coupled it to a subset of WRF’s atmospheric physics parameterizations including radiation models and land-surface models. Lundquist et al. (2010, 2012) validated this WRF-IBM model using a no-slip boundary condition for flow over mountainous terrain and urban terrain, including
comparisons to the Joint Urban 2003 field campaign in Oklahoma City. While the use of no-slip boundary conditions is sometimes used for high-resolution urban simulations (e.g., for grid resolutions smaller than 1 m), it is generally inappropriate for large-eddy simulations of atmospheric flows, where the surface layer is not well resolved. This is especially true for the coarser resolutions (10-100 m scale) used in most large-eddy simulations of the atmosphere.

Bao et al. (2018), included here as Chapter 2, extended the WRF-IBM implementation to include a surface stress parameterization which models surface stresses in the unresolved surface layer using similarity theory based on the log-law. This implementation of the WRF-IBM is referred to here as the velocity-reconstruction log-law boundary condition (VR-IBM). Only neutral stability cases are considered and extension of this work to Monin-Obukhov similarity theory, which accounts for additional stability states, is left for future work. This VR-IBM method is validated against a wide range of cases in Bao et al. (2018) (Chapter 2), including turbulent flow over flat terrain at 20 m resolution, flow over Askervein Hill at 5 m resolution, and flow over Bolund hill with 2 m resolution. Good agreement was achieved when comparing with not only theoretical results (flat terrain case) but also with observations (Askervein Hill and Bolund Hill) at these resolutions.

Ma and Liu (2017) implemented a different log-law boundary condition for IBM into WRF, which reconstructs the shear stress near the surface based on similarity theory (SR-IBM). This method was first developed by Chester et al. (2007), which modifies the shear stress at band grid points which are within a defined distance above the immersed boundary. Diebold et al. (2013) and Ma and Liu (2017) validated the SR-IBM for flow over Bolund hill at 1 m resolution.

IBM approaches originally developed for engineering applications have used very fine resolutions in previous tests (e.g., mm scale resolution) (Peskin, 1972; Iaccarino and Verzicco, 2003). In previous atmospheric studies, the VR-IBM and SR-IBM method worked well for flow with very fine resolution (m scale), as mentioned above. When tested at coarser resolutions (e.g., 100 m) more typically used in atmospheric models, these IBM implementations introduced inaccuracies over sloped terrain (Fang and Porté-Agel, 2016; Bao et al., 2016).

In this work, a new hybrid IBM boundary condition is developed, which reconstructs both velocity terms and shear stress terms near the immersed boundary in accordance with similarity theory. A detailed description of this improved new IBM method is included in the background section, along with descriptions of the WRF log law boundary condition and existing IBM approaches in literature. Then a neutral atmospheric boundary layer over flat terrain is simulated using the native terrain-following coordinates in WRF, the new hybrid IBM implementation and the VR-IBM method implemented in WRF by Bao et al. (2018) (Chapter 2) at four different resolutions ranging from 16 to 128 m. Next, simulations over a large idealized hill are performed for terrain-following WRF, the new IBM implementation and the VR-IBM method at intermediate resolutions (100 m). The test case is chosen for its relatively moderate slopes (15 degree) and larger domain size (6 km in horizontal direction), which enable direct comparison between native WRF simulation and IBM implementations at intermediate resolutions. Finally, simulations with HYBRID-IBM are performed over Askervein Hill at both an intermediate resolution (30 m) and a fine resolution (5 m); VR-IBM was previously been validated at 5 m resolution in Bao et al. (2018) (Chapter 2). Results are compared to field observations and to native WRF results.
The new HYBRID-IBM implementation contributes to some efforts to create a mesoscale to microscale modeling framework using WRF. Previously, Daniels et al. (2016) and Mirocha and Lundquist (2017) implemented and tested a vertical nesting scheme to allow flexible grid nesting strategies across different resolutions while maintaining desirable grid aspect ratios. Wiersema et al. (2016) used vertical nesting with the VR-IBM approach to perform seamless multi-scale simulations which span from mesoscale weather simulations to microscale simulations of Oklahoma City where flow through buildings in the central business district is resolved. Arthur et al. (2018) expanded the physics parameterizations available for use with the IBM implementation by coupling the topographic shading algorithm with IBM and testing the implementation over the steep terrain of Granite Mountain, the site of the MATERHORN observational field campaign. A remaining challenge has been working with intermediate resolutions over complex terrain which are too fine (too steep) for terrain following coordinates and too coarse for existing IBM implementations. Now, the work presented here, a new hybrid IBM boundary condition, will enable simulations over steep complex terrain at intermediate grid resolutions.

3.2 Background

3.2.1 Immersed boundary method

The immersed boundary method was originally developed for engineering flow applications with complex geometries; for example, Peskin (1972) originally developed IBM to simulate blood flow over the valve of the heart. The influence of the immersed boundary acts as an additional body force term in the momentum equation

$$\frac{\partial U}{\partial t} + U \cdot \nabla U = -\frac{1}{\rho} \nabla P + \nabla \cdot \tau + F_b, \quad (3.1)$$

where $U$ is the simulated velocity field, $\rho$ is the density, $P$ is pressure, and $\tau$ is the turbulent stress term. The body force is zero when it is far away from the immersed boundary and takes a non-zero value when it is near the cut cells of the immersed boundary. The immersed boundary method used in this work adapts this direct forcing approach proposed by Mohd-Yusof (1997). In this direct forcing approach, the governing equations are first discretized without accounting for the immersed boundary. Then velocities and components of the shear stress tensor are directly modified at the cells near the immersed boundary, accounting for the boundary presence.

There are three categories of log-law boundary condition implementations for the immersed boundary method which have been proposed in the recent literature. Chester et al. (2007) developed a shear stress reconstruction IBM (SR-IBM), which modifies the shear stress at a band of grid points within a set distance above the immersed boundary. Shear stress is additionally extrapolated down to the first grid below the immersed boundary. Velocity is set to zero interior to the immersed boundary. Diebold et al. (2013) validated this SR-IBM implementation with comparisons of a large-eddy simulation to observations of flow over Bolund Hill. Ma and Liu (2017) first implemented this SR-IBM into WRF, testing it with large-eddy simulations of flow over Bolund Hill.
and making comparisons to observations. Their simulations, using fine vertical resolution and the Lagrangian-averaged scale-dependent turbulence model, showed improvement over simulations using the constant coefficient Smagorinsky turbulence closure for the mean speed-up error and for capturing the correct flow properties in the recirculation zone.

A second IBM implementation is presented in Anderson (2013), who used a canopy-like drag model to characterize surface stress. A canopy stress model is used to impose a momentum sink for cells near the surface. Velocities within the immersed boundary are set to zero. Validation cases performed by Anderson (2013) included flow over a single cube and an array of cubes. Results using very fine 1 m resolution showed good agreement with benchmark data from previous studies.

In a third approach, Fadlun et al. (2000) and Senocak et al. (2004) used velocity reconstruction near the surface, which sets velocities at grid nodes in the flow field near the immersed boundary, assuming a linear or a log profile, respectively. Fadlun et al. (2000) first used a linear interpolation between a point in the fluid (the second point from the boundary) and a no-slip boundary condition at the surface to reconstruct the velocity at a layer of fluid points outside of the immersed surface (the first point from the boundary). For atmospheric flows, the fine resolution required to use a linear interpolation scheme (to resolve the viscous sublayer) would not be practical. Senocak et al. (2004) extended this method by using a log-law reconstruction scheme, where the tangential flow is reconstructed at the first fluid node using the log law. The implementation appeared in one dimension (i.e. for flow over flat terrain) and was not generalized to three dimensional terrain. Senocak et al. (2015) and Umphrey et al. (2017) extended the implementation to three dimensional terrain and included the heat flux with application to atmospheric katabatic flow. Bao et al. (2018) (Chapter 2) followed this VR-IBM method and implemented it into WRF, validating against a large number of test cases, not only to field observations (Bolund Hill), but also to theory (flat terrain), and directly compared to native WRF (flat terrain and Askervein Hill). Test cases are also chosen at different slopes, to demonstrate the robustness of the implementation (the Askervein Hill case with moderate slopes, and the Bolund Hill case with steep slopes), providing rigorous validation of the implementation of the VR-IBM into WRF by ensuring that the VR-IBM implementation matches results from WRF with its native coordinate system for moderately sloped terrain and that it has the capability to handle steeply sloped terrain. But we do notice that errors arise with the VR-IBM implementation, and are particularly evident in the lee of the hill and with increasing grid resolution.

In this work, we implement a new log law IBM method into WRF by creating a new hybrid method, based on combining both the SR-IBM and VR-IBM methods. Details of the new HYBRID-IBM approach are given in Section 3.3.1.

### 3.2.2 Native WRF surface boundary condition

A description of the WRF surface boundary condition is included in Bao et al. (2018). In general, surface stresses are set using the following equation

\[ \tau_{i3}^s = C_d |U| U_i, \]  

(3.2)
where \( i = 1 \) or \( 2 \). In this equation, \( \tau_{s}\zeta \) denotes the surface shear stress in the \( x \) and \( y \) directions, \( C_d \) is the drag coefficient, \( |U| \) denotes the horizontal wind speed, and \( U_i \) denotes the \( U \) or \( V \) velocity. \( C_d \) can be specified directly in the WRF namelist as a constant value, or if the surface layer scheme is turned on, the \( u_* \) value calculated in the surface layer scheme is used to determine the drag coefficient. Additionally, we added a new equilibrium stress model option which calculates the drag coefficient based on a specified surface roughness parameter \( \zeta_0 \) and the log law. In this formulation, the coefficient of drag is given by

\[
C_d = \left( \frac{\kappa \ln \frac{\zeta}{\zeta_0}}{\ln \frac{d}{\zeta_0}} \right)^2.
\] (3.3)

It should be noted that WRF specifies its stresses in the vertical direction instead of in the surface normal direction when the native terrain-following coordinates are used, which will introduce errors with fine resolution simulations over complex terrain when the surface slope is steep. In addition to setting surface stresses, WRF also sets a kinematic boundary condition for the velocity given by (3.4) and (3.5), which specifies no flow in the surface normal direction.

\[
\dot{\eta} = 0 \quad \text{(3.4)}
\]
\[
w_s = u_s \frac{\Delta h}{\Delta x} + v_s \frac{\Delta h}{\Delta y} \quad \text{(3.5)}
\]

Here, \( \dot{\eta} \) is the velocity of the vertical coordinate, \( h \) is the terrain height, and \( (u, v, w)_s \) are the Cartesian components of velocity at the surface.

### 3.2.3 Velocity-reconstruction IBM implementation

The implementation of the velocity reconstruction immersed boundary method (VR-IBM) in WRF is detailed in Bao et al. (2018) (Chapter 2), where the velocity is reconstructed at the first fluid node (\( U_1 \) in Figure 3.1) assuming there is a logarithmic profile near the surface (between \( U_1 \) and \( U_{img} \)).

On the staggered grid used by WRF, each velocity component (\( U, V \) and \( W \)) is reconstructed individually using same procedure, as illustrated in figure 3.1. The reconstruction velocity point (\( U_1 \)), which is the first fluid point above the immersed boundary, is identified. Then an interpolation point, \( U_{img} \), is found by projecting a surface normal line from the immersed boundary through the reconstruction point (\( U_1 \)) until it intersects the cell face above. \( U_1 \) and \( U_{img} \) are perpendicular to the immersed boundary. All \( U, V \) and \( W \) velocity values are then interpolated at \( U_{img} \) using the neighboring fluid nodes (green squares). \( U_{img} \) is then decomposed into the surface normal (\( U_{img}^n \)) and surface tangential (\( U_{img}^t \)) components. Next, the tangential velocity at \( U_1^t \) can be calculated using (3.6) based on the log law profile using the tangential velocity at \( U_{img}^t \):

\[
U_1^t = U_{img}^t \frac{\ln \frac{d_1}{\zeta_0}}{\ln \frac{d_{img}}{\zeta_0}} \quad \text{(3.6)}
\]
where \( d_1 \) and \( d_{img} \) are the perpendicular distances from the boundary to the \( U_1 \) point and the \( U_{img} \) point, respectively.

The normal component is reconstructed at \( U_1 \) with linear interpolation between the normal velocity at \( U_{img} \) and the zero normal velocity at the surface, assuming there is no flow through the immersed boundary. Once the normal and the tangential velocities at \( U_1 \) are calculated, they are rotated back to set the boundary condition \( U_1 \) value in Cartesian coordinates.

### 3.3 New Immersed Boundary Method

#### 3.3.1 Hybrid IBM Implementation

While several IBM implementations using the log-law have been proposed in the literature, as seen in the work included here, there are errors in the flow field predicted compared to the native WRF solution for flow over sloping terrain, especially at intermediate resolutions (Bao et al., 2016).

A new IBM implementation (HYBRID-IBM), capable of matching the WRF solution for flat and sloped terrain at intermediate resolution, is developed here. Of the IBM implementations, this method is most similar to WRF’s log-law boundary condition, as it requires both reconstruction of the velocity and stress fields in the vicinity of the immersed boundary. As mentioned in section 3.2.2, WRF implements its boundary condition by setting the shear stress according to M-O theory, as well as a kinematic boundary condition for velocity.

The new IBM algorithm is illustrated in Figure 3.2, where the treatment of the velocity field is shown in the left panel and treatment of the shear stress components is shown in the right panel. Like the velocity reconstruction methods of Senocak et al. (2004) and Bao et al. (2018) (Chapter 2), we use the log-law (for neutral stratification) to enforce a logarithmic velocity profile near the
surface. However, unlike the existing method, where velocity is reconstructed at a fluid node, velocity in the new method is reconstructed at a ghost point ($U_g$ and $W_g$ in the left panel of Figure 3.2) interior to the terrain. On the staggered grid used by WRF, the same reconstruction procedures are required for both the $U$ and $V$ velocities. A no slip boundary condition is applied for $W$ velocity, meaning there is no flow in or out of terrain surface.

First, ghost cells, $U_g$ (blue open circle in Figure 3.2), are identified below the immersed boundary at each cut cell. Then two interpolation points, $U_{i1}$ and $U_{i2}$ (blue stars in figure 3.2), are found based on projecting the surface normal vector outward from $U_g$ a distance of $0.5\Delta z$ and $1.5\Delta z$ from the immersed surface. All three points $U_g, U_{i1},$ and $U_{i2}$ are aligned in the surface normal direction to the immersed boundary. Neighboring fluid nodes (green squares) are used to interpolate the value at $U_{i2}$ for all $U, V$ and $W$ velocities. $U_{i2}$ is then projected in the surface normal ($U_{n1}^g$) and surface tangential ($U_{t1}^g$) directions. The log-law equation 3.7 is used to calculate the tangential velocity at $U_{i1}$.

$$U_{t1}^g = U_{i2}^g \ln \frac{0.5\delta z}{z_0} \ln \frac{1.5\delta z}{z_0}$$ (3.7)

The normal component is reconstructed at $U_{i1}$ with linear interpolation between the normal velocity at $U_{i2}^n$ and the zero normal velocity at the surface, assuming there is no flow through the immersed boundary:

$$U_{i1}^n = U_{i2}^n \frac{0.5\delta z}{1.5\delta z}$$ (3.8)

Once the surface normal and surface tangential velocities at $U_{i1}$ are individually reconstructed, linear extrapolation, using the values of $U_{i1}$ and $U_{i2}$, is used to calculate $U_g$. Once the normal and
tangential velocities at $U_g$ are individually reconstructed, they are rotated back to set the boundary condition value $U_g$.

The right panel of figure 3.2 illustrates the shear stress reconstruction implemented in this HYBRID-IBM method. Three layers of shear stress are reconstructed near the immersed boundary (blue open circles labeled as $\tau_{g-1}$, $\tau_g$ and $\tau_{g+1}$). The shear stress reconstruction part is similar to the method presented in Chester et al. (2007).

First, ghost cells ($\tau_g$) of shear stress are identified below the immersed boundary. $\tau_{g+1}$ is the point above the $\tau_g$ and $\tau_{g-1}$ is the point below the $\tau_g$. For the reconstruction point above the immersed boundary ($\tau_{g+1}$), assuming it is within the constant stress layer, the tangential component of shear stress at $\tau_{g+1}$ is reconstructed using drag law equation:

$$\tau_{i3} = C_d |U| U_i$$

where $i = 1$ or 2. In this equation, $\tau_{i3}^s$ denotes the shear stress in the $x$ and $y$ directions, $|U|$ denotes the wind speed at $\tau_{g+1}$ point, $U_i$ denotes the $U$ or $V$ velocities at $\tau_{g+1}$ location, and $C_d$ is the drag coefficient. $C_d$ is calculated using the same equation 3.3 as WRF, based on a specified surface roughness parameter $z_0$ and the log law. Other components of the shear stress tensor are assumed to be zero. The stress value at the reconstruction point is then set by rotating the stress tensor back to Cartesian grid coordinates.

For shear stress points below the immersed boundary, once $\tau_g$ is located, a surface normal line is projected from the ghost point through the immersed boundary until it hits the cell face above. Where the surface normal line intersects the cell face above is denoted the image point, $\tau_i$ (blue star in the right panel of Figure 3.2). $\tau_g$ and $\tau_i$ are thus normal to the immersed boundary. Neighboring fluid nodes (green squares) are used to interpolate the shear stress value at $\tau_i$ for all stress component. Then, surface shear stress ($\tau_{13}$ and $\tau_{23}$) is calculated using Equation 3.9. $\tau_i$ and surface shear stress are then projected in the surface normal and surface tangential directions. Linear extrapolation is then used to calculate $\tau_g$ using surface shear stress and $\tau_i$. $\tau_{g-1}$ is calculated using the same reconstruction procedures as $\tau_g$.

### 3.4 Validation of the Hybrid IBM

In this section, we examine the performance of the new implementation of the log-law boundary conditions (HYBRID-IBM) at the immersed boundary using three test cases. The first is neutral atmospheric boundary layer flow over flat terrain, the second is flow over an idealized hill (the slope of which is around 15 degrees), and the third is the Askervein experiment with comparisons made between observations and simulations. We additionally present results using the native terrain-following coordinates in WRF and using the VR-IBM approach.

The WRF model is run in an idealized mode, such that atmospheric processes other than turbulence are not parameterized. This allows us to verify the new surface boundary condition at the immersed boundary without adding the complexity of additional atmospheric processes.
3.4.1 Turbulent flat terrain simulations

3.4.1.1 Simulation Setup

Simulations are carried out for a neutral atmospheric boundary layer over flat terrain located at a height of 0 m using the two IBM methods (VR-IBM and HYBRID-IBM) and the native terrain-following coordinates in WRF. For each method, we test four different horizontal grid resolutions, 128 m, 64 m, 32 m and 16 m, with the same total domain size (3200 m) in each horizontal direction. Because many previous studies of IBM approaches were conducted at fine resolution, this varying grid resolution study is meant to evaluate each method’s performance at intermediate resolutions more typical for LES studies in the atmospheric boundary layer. The simulation setup is similar to the standard neutral flow setup used in Chow et al. (2005) and Bao et al. (2018) (Chapter 2). All cases are initialized with a neutral and dry sounding with a 10 m s\(^{-1}\) wind in the x-direction. The flow is driven by a geostrophic wind of \((U_g, V_g) = (10, 0)\) m s\(^{-1}\), with a Coriolis parameter \(f = 10^{-4}\) s\(^{-1}\). The domain top is located at 1.5 km, and 71 vertical grid points are used in the terrain-following case, while 73 vertical grid points are used in the IBM cases to allow for points below the terrain. A constant vertical grid spacing of \(\Delta z = 8\) m is used near surface to a height of \(z = 100\) m for both terrain-following WRF and IBM cases. Then a 1.05 grid stretching ratio is used until \(z = 500\) m. Above that, a constant \(\Delta z\) is used. Rayleigh damping is applied to the top 500 m of the domain. The Smagorinsky turbulence closure is used for simplicity, and the initial velocity is seeded with perturbations such that the solution becomes turbulent after 6 hours of spin-up time.

The surface is set to have a constant roughness, with a roughness length scale of \(z_0 = 0.1\) m. Periodic boundary conditions are used for the lateral boundaries. The total integration time for this case is 48 hours. The velocity profiles are averaged over the last 24 hours, with data sampled at 15 min intervals. Inertial oscillations are significantly damped over the last 24 hours and velocity profiles reach quasi-steady state over the last 24 hours.

3.4.1.2 Results

Figure 3.3 shows the time and planar-averaged wind speed profiles for terrain-following WRF and the two IBM implementations at four different horizontal resolutions (128, 64, 32, 16 m). As seen in the figure, the surface wind speed converges for both the WRF and HYBRID-IBM methods, while the surface wind speed depends strongly on the horizontal grid resolution for VR-IBM method.

Figure 3.3 shows the same time and planar-averaged wind speed profile for WRF and the two IBM methods at 16 m and at 128 m horizontal resolutions. Note that the difference in wind speed profile between the WRF and VR-IBM solutions is smaller when the horizontal resolution is 16 m, as compared to the 128 m case, while the HYBRID-IBM method does a better job in recreating the original WRF solution at both the intermediate and fine resolutions. The VR-IBM results (mean profiles) are highly sensitive to the grid resolution and are better at recreating the original WRF solution at finer resolutions.
CHAPTER 3. DEVELOPMENT AND TESTING OF A NEW HYBRID IMMERSED BOUNDARY METHOD

Figure 3.3: vertical averaged velocity for WRF (left), VRM (middle) and HYBRID (right) method at 128, 64, 32, 16 m

Figure 3.4: vertical averaged velocity at 128 (left) and 16 (right) resolution
CHAPTER 3. DEVELOPMENT AND TESTING OF A NEW HYBRID IMMERSED BOUNDARY METHOD

3.4.2 Idealized hill simulations at intermediate resolution

3.4.2.1 Simulation Setup

Next, the WRF-IBM boundary conditions are validated for flow over an idealized hill. The hill setup is similar to the setup used in Lundquist et al. (2012). The hill terrain is defined using the following Witch of Agnesi function,

\[ h(x, y) = \frac{h_p}{1 + \left( \frac{x}{L_h} \right)^2 + \left( \frac{y}{L_h} \right)^2} \]

(3.10)

where \( h_p \) is the peak height, which is 350 m in our setup. \( h(x, y) \) is the hill height at specific \( x, y \) location. \( L_h \) is the horizontal length scale of hill, which is 800 m in our case. With this terrain, the hill has a roughly 15 degree maximum slope, which is relatively shallow. Therefore the simulation results from WRF are still reasonable using terrain-following coordinates, allowing use of the WRF results for validation of the IBM methods.

In this idealized hill test case, the domain height is located at 4 km, and 76 vertical grid points are used for the terrain-following grid, while 78 vertical points are used for the IBM grid to allow for points below the terrain. In both setups a constant vertical grid spacing of \( \Delta z = 20 \) m is used near the surface until it reaches \( z = 460 \) m, which is approximately 100 m above the hill top. Then a 1.05 grid stretching ratio is used until \( z = 2 \) km, and then \( \Delta z \) remains constant. Rayleigh damping is applied over the top 2 km. The grid spacing in the horizontal is \( (\Delta x, \Delta y) = (100, 100) \) m, and the simulation uses 61 grid points in each horizontal direction, which creates a total total domain size of 6000 m in each horizontal direction. This relatively coarse grid resolution is chosen to validate the performance of HYBRID-IBM and VR-IBM at intermediate resolutions.

As with the flat terrain case, the flow is driven by a geostrophic wind of \( (U_g, V_g) = (10, 0) \) m s\(^{-1}\) with a Coriolis parameter \( f = 10^{-4} \) s\(^{-1}\). The valley case is initialized with a neutral, dry sounding with a 10 m/s wind in the \( x \)-direction. The surface is set to have a constant surface roughness of \( z_0 = 0.1 \) m. The Smagorinsky turbulence closure is used, and periodic boundary conditions are used for the lateral boundaries. The total integration time for this case is 2 days, and the profiles shown are averaged over the last 24 hours.

3.4.2.2 Results

A direct comparison of the time-averaged velocity profile between standard WRF and the two IBM methods (HYBRID-IBM and VR-IBM) is shown in Figure 3.5. The velocity profiles are averaged for the last 24 hours at 15-min output intervals and are shown at several locations along the center line of the hill. The main differences between the HYBRID-IBM and WRF results are found on the lee side of the hill, in particular near the seventh profile. Larger differences are found between VR-IBM and WRF at this intermediate resolution of 100 m. The lee region of flow over a hill is notoriously difficult to predict and is a further challenge because the log law may not hold in this region. Nonetheless, because all the methods are designed using the same log law condition, we expect agreement among the methods and find that the HYBRID-IBM approach outperforms the VR-IBM approach at this resolution.
To quantify the differences between WRF, VR-IBM and HYBRID-IBM, the maximum absolute differences are calculated using WRF as the reference for the lowest 200 m of the 11 profiles in Fig 3.5; the values are listed in Table 3.1. The biggest difference between VR-IBM and WRF is 2.74 m s\(^{-1}\) and is found near the surface, in the lee side of the hill at \(x = 3300\) m. The biggest difference between HYBRID-IBM and WRF is at the same \(x\) location, but smaller, with a value of 1.95 m s\(^{-1}\). The differences between VR-IBM and WRF exceed 2 m s\(^{-1}\) at three locations: at the hill top (\(x = 3000\) m), in the lee side (\(x = 3300\) m), and at one uphill location (\(x = 1500\) m). The differences between HYBRID-IBM and WRF never exceed 2 m s\(^{-1}\) at any of the profiles evaluated here.

The HYBRID-IBM approach clearly improves the VR-IBM results for flow over an idealized hill (15 degree) at 100 m resolution. The VR-IBM is originally derived from engineering IBM applications and has previously been validated for relatively fine resolutions (1 m to 5 m) in atmospheric simulations. With these previous VR-IBM implementations, some set the velocity value to
Table 3.1: Maximum absolute difference (m s\(^{-1}\)) for VR-IBM and HYBRID-IBM results compared with WRF results at the 11 profiles shown in Fig. 3.5.

<table>
<thead>
<tr>
<th>MODEL</th>
<th>x=0.3 km</th>
<th>0.9</th>
<th>1.5</th>
<th>2.1</th>
<th>2.7</th>
<th>3</th>
<th>3.3</th>
<th>3.9</th>
<th>4.5</th>
<th>5.1</th>
<th>5.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>VRM</td>
<td>0.85</td>
<td>0.58</td>
<td>2.76</td>
<td>0.57</td>
<td>0.98</td>
<td>2.07</td>
<td>2.74</td>
<td>0.64</td>
<td>0.52</td>
<td>0.86</td>
<td>0.79</td>
</tr>
<tr>
<td>HYBRID</td>
<td>0.70</td>
<td>0.57</td>
<td>0.43</td>
<td>0.08</td>
<td>1.05</td>
<td>0.68</td>
<td>1.95</td>
<td>0.56</td>
<td>0.48</td>
<td>0.66</td>
<td>0.30</td>
</tr>
</tbody>
</table>

zero beneath the surface (Umphrey et al., 2017), while some do not additionally treat the velocity under the surface (Bao et al., 2018) (Chapter 2). Some methods additionally reconstruct the eddy viscosity near the surface (Senocak et al., 2004), and others do not (Bao et al., 2018) (Chapter 2). When it comes to coarser resolutions, the velocity beneath the terrain (especially the first grid point below the surface) needs to be well controlled. To fully understand the effects of all these surface options, Arthur et al. (2019) tested the influence of all these surface and sub-surface velocity treatment options. Arthur et al. (2019) found that setting the velocity beneath the terrain can greatly affect the velocity profile. Setting zero velocities beneath the terrain adds additional drag near the surface and can cause the velocity near the surface to be underestimated. In the HYBRID-IBM method, we reconstruct the ghost point velocity below the terrain as well as the shear stress near the surface. This method is most similar to WRF’s boundary condition, as it requires both reconstruction of the velocity and stress fields in the vicinity of the immersed boundary.

### 3.5 Application of the HYBRID-IBM to Askervein Hill

The Askervein Hill field campaign was conducted in 1982 and 1983 to study flow over an isolated and moderately sloped hill. Askervein hill is located along the west coast of South Uist Island, Scotland, and is a 116 m high hill, with a main 2 km axis along NW-SE direction (before rotation) and a 1 km axis along NE-SW direction. During the field campaign, both velocity and turbulence data were collected along lines A, AA and B (See Figure 3.6), providing a unique dataset to validate numerical simulations. The details data can be found in two technical reports including Taylor (1983); Taylor and Teunissen (1985). Since Askervein hill is relatively isolated, simulations for flow over Askervein hill can be set up using a constant inflow condition and neutral atmospheric stability without adding any complex atmospheric physics and external forcing.

Because of the simple semi-idealized setup along with the observation data available, Askervein hill has been chosen to validate many modeling studies (Lopes et al., 2007; Chow and Street, 2009; Golaz et al., 2009). Bao et al. (2018) (Chapter 2) validated the VR-IBM over Askervein hill at 5 m resolution with 800x700 grid points. Great agreement was achieved at this fine 5 m resolution when comparing to observations except in predicting the size of the lee side recirculating zone. However, Askervein simulations are typically performed at much coarser resolutions in other work. Chow and Street (2009) used 35 m horizontal resolution with 163x163 grid points in the horizontal and Golaz et al. (2009) used 30 m horizontal resolution with 175x175 grid points.
Here we validate our HYBRID-IBM method over Askervein hill using this coarser intermediate resolution (30 m) as is common in the literature. The performance of VR-IBM over Askervein hill at this intermediate resolution is also tested. Results are compared to observations and to the terrain-following native WRF solution, which is possible given the moderate slopes of Askervein hill.

3.5.1 Simulation setup

The following description of the Askervein field campaign follows the description given by (Bao et al., 2018) (Chapter 2) Topographic data for Askervein are provided by Walmsley and Taylor (1996) at 25 m resolution. A one-way nested grid setup is used, where the parent domain has flat terrain and uses periodic lateral boundary conditions, and the nested domain contains Askervein Hill and is forced at its boundaries by the parent domain. This setup is similar to simulations in Golaz et al. (2009) where flow over Askervein hill is simulated in the COAMPS model. The boundary layer in the parent domain is neutral and driven by a geostrophic wind speed of $\sim 18 \text{ m s}^{-1}$, which was previously used in Golaz et al. (2009); Bao et al. (2018) (Chapter 2) to obtain agreement with observations at the reference site. The horizontal grid in the parent domain contains $135 \times 135$ points with 90 m spacing to cover a 12060 m square domain, while the nested domain is $175 \times 175$ points with 30 m spacing. The total domain size for the nested domain is 5220 m by 5220 m. For the terrain-following WRF case, both the parent and nested grids share the same 81 vertical levels with a spacing of $\Delta z = 6.66 \text{ m}$ in the lowest 100 m of the domain and stretched above with a 1.05 stretching ratio until the vertical resolution is approximately 100 m, after which the grid remains uniform up to the 4000 m domain height. For the IBM case, there are 83 vertical levels because at least two grid points are required below the surface. The same vertical grid spacing and stretching are applied, therefore the two grids match relatively well in the flat topography areas of the domain. The flow on the outer domain is allowed to spin up for 24 hours to allow inertial oscillations in the neutral boundary layer to damp. At 24 hours, the nested domain is spawned and nested boundary conditions are used. The parent and nested domain are run concurrently for an additional hour. Results presented here are averaged over the last 40 minutes of the simulation after a 20-min spin up time for the inner domain. Figure 3.6 shows the parent and nested domain with the topography of Askervein Hill and the field data transects. We use a roughness value of $z_0 = 0.03 \text{ m}$ over land and $z_0 = 0.0015 \text{ m}$ over water. A land surface model is not used, so there are no heat or moisture fluxes included for the short time period of the simulation. The Smagorinsky turbulence closure is used in this case.

3.5.2 Results

Figure 3.7 shows the time and planar averaged velocity profile from the outer domain and compares to observations at the reference site. This can be seen as the inflow condition for the nested domain. The results are averaged over the last 40 mins of the outer domain results with 1 min output frequency. Both the HYBRID-IBM and WRF results match the observations within the error bars. The RMS difference between WRF and HYBRID-IBM is as small as 0.1 m s$^{-1}$. 

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VR-IBM does not produce accurate inflow conditions as compared to observations. The overall velocity profiles are too slow especially near the surface. The same feature was seen previously in Figure 3.3 for flow over flat terrain at 128 m resolution (the Askervein outer domain is at 90 m resolution).

Despite the differences in inflow conditions, the influence of the hill on the wind field can be quantified using a fractional speed up ratio, $\Delta S$, which is defined as:

$$\Delta S = \frac{S(z) - S_{RS}(z)}{S_{RS}(z)}$$

(3.11)

where $S$ is the horizontal wind speed and $S_{RS}$ is the horizontal wind speed at reference site.

Figure 3.8 shows the speedup along line AA (over the center of the hill) and line A (which crosses the hill top). The maximum speedup should occur at the hill top and followed by a slow down of the velocities on the lee side of the hill. Both the WRF and HYBRID-IBM results successfully produce the speedup and slow down along line AA. Both WRF and HYBRID-IBM slightly underpredict the speedup at the hill top along line A, which passes the peak of Askervein Hill. The same problem was found in Bao et al. (2018) (Chapter 2), where WRF failed to predict the maximum speedup along line A. In general both WRF and HYBRID-IBM successfully produce the slow down on the lee side recirculation zone of Askervein Hill, though the strength is slightly different. Standard WRF overestimates the flow deceleration along both line AA and line A, as previously shown in Bao et al. (2018) (Chapter 2). VR-IBM generally overestimates the speedup along line A and line AA at this coarse resolution, compared to what was seen in the 5 m resolution simulation (Bao et al., 2018) (Chapter 2) in Figure 2.6. Another drawback of VR-IBM is the jaggedness of the speedup profile both for line AA (see around x = -350, -250, -150, 200 m) and line A (see around x = -400 and -300 m). Oscillations were also seen in (DeLeon et al., 2018) when VR-IBM was used to simulate flow over Askervein at around 12-25 m resolution. When IBM is used, the immersed surface will intersect with the grid at arbitrary locations. Thus it is important that the IBM method have no sensitivity to where the surface hits the grid. The oscillations in the VR-IBM result at this
coarse resolution may be due to the fact that the VR-IBM method will produce different results when the immersed surface intersects the grid differently. These oscillations are not apparent at the fine resolution. The VR-IBM’s sensitivities to where the surface intersects with the grid are confirmed in Arthur et al. (2019). Detailed sensitivity tests on different aspects including where the surface intersects with the grid will be described in Chapter 4. Next, a direct comparison of the time-averaged velocity profiles between standard WRF, VR-IBM and HYBRID-IBM is shown in Figure 3.8 for line A and Figure 3.9 for line AA. The velocity profiles are averaged for the last 30 min of the simulation with 1 min output intervals. The velocity profiles are shown at several locations along lines A and AA. It can be seen that in general, the HYBRID-IBM results are in good agreement with standard WRF. The maximum differences between HYBRID-IBM and WRF along A and AA are both located at the sixth profile (in the lee side). The VR-IBM has a bigger difference compared to WRF. As has been discussed in (Bao et al., 2018) (Chapter 2), simulating the lee side of the hill is always very challenging. This may be due to sensitivity of the flow to interpolation errors in the flow separation region and may also be influenced by different parameters such as grid resolution and turbulence closure schemes (Chow and Street, 2009; Ma and Liu, 2017).
3.6 Summary and conclusion

This chapter describes a new hybrid log-law IBM (HYBRID-IBM) method based on a combined velocity reconstruction and shear stress reconstruction approach. This new method has been implemented and tested in WRF. The hybrid IBM method is designed to work for atmospheric flow over complex terrain at intermediate resolutions (100 m). Previous work (Bao et al. (2018) (Chapter 2)) extended IBM to incorporate a velocity-reconstruction log-law boundary condition at the surface (Bao et al., 2018) (Chapter 2). VR-IBM works well at very fine resolutions (e.g. 1-5 m horizontal spacing) relative to the topography, but its performance becomes worse at intermediate resolutions (e.g. 100 m) which are more practical for atmospheric simulations (for example as seen in the jaggedness of the profiles in the Askervein simulations shown in this chapter). This new HYBRID-IBM method based on a combined velocity reconstruction and shear stress reconstruction approach was developed and validated in this chapter, with an aim to improve the performance of IBM with a log law boundary condition for atmospheric simulations at intermediate resolutions.

The HYBRID-IBM approach is first validated by simulating neutral atmospheric boundary layer flow over flat terrain at four different resolutions (16, 32, 64, 128 m); two of these resolutions (64 m and 128 m) are considered to be intermediate resolutions. Solutions are compared to results achieved using the native terrain-following coordinate WRF. At 16 m resolution, both VR-IBM and HYBRID-IBM perform well over flat terrain compared to WRF. At 128 m, VR-IBM has poor performance while HYBRID-IBM still matches WRF very well.
Next we validated the HYBRID-IBM by simulating flow over a moderately sloped idealized hill (15 degree) at 100 m resolution. Direct comparison of velocity profiles along the center line of the hill showed that the maximum error between VR-IBM and WRF is 2.7 m s\(^{-1}\), while the maximum difference between HYBRID-IBM and WRF never exceeds 2 m s\(^{-1}\). Note that some difference between the results is due to the use of different grids in the IBM and WRF setups and also the fact that WRF computes vertical gradients at the surface rather than the surface-normal gradients used in IBM.

Finally the HYBRID-IBM was validated for flow over Askervein Hill, which was previously used by Bao et al. (2018) (Chapter 2) to validate VR-IBM’s performance, but at 5 m resolution. Comparison of the Askervein simulations reveals good agreement between HYBRID-IBM and native WRF simulations and compared with observations. The maximum differences occur on the lee side of the hill, where the models are quite sensitive to intermittent flow separation (Chow and Street, 2009; Castro et al., 2003; Golaz et al., 2009). Overall, the performance of the HYBRID-IBM method is well within the expected range, thus showing the model’s suitability for complex

Figure 3.9: Askervein velocity profiles along line AA at 30 m resolution.
flow situations at intermediate resolutions. The VR-IBM results have several drawbacks. One is oscillations seen along the speedup profile along line A and line AA in Figure 3.8. This might be due to VR-IBM’s sensitivity to where the immersed surface interacts the grid, which will be tested in detail in Chapter 4. VR-IBM also generally overestimated the speedup ratio along line A and line AA at intermediate resolutions which was not seen at fine resolutions (Bao et al., 2018) (Chapter 2).

These three test cases provide confidence in HYBRID-IBM’s performance at intermediate resolutions. More detailed test cases including sensitivity tests of the influence of immersed surface interacting with different location of grid, aspect ratio, grid resolution and steepness of slope are included in Chapter 4. A real case of atmospheric flow over steep terrain (Bolund hill) will also be included in Chapter 4 to show the capability of HYBRID-IBM in handling steep terrain.
Chapter 4

Sensitivity studies of the HYBRID-IBM approach

4.1 Introduction

4.1.1 Background

As computational power increases, the Weather Research and Forecasting (WRF) model has been pushed towards finer resolution large-eddy simulation (LES) for atmospheric flow to investigate fine scale flow dynamics and turbulence over complex terrain. WRF, which is designed to run at km-scale horizontal resolutions, uses terrain-following coordinates (Skamarock et al., 2008) which work well when the terrain slope is relatively shallow. When terrain-following WRF is used at finer resolutions for flow over complex terrain, the grid cells near the surface become highly skewed, leading to large numerical errors and model instability (Mahrer, 1984; Schär et al., 2002; Zängl, 2002; Klemp et al., 2003; Zängl et al., 2004).

To eliminate this limitation of the terrain-following coordinates used by WRF for complex terrain simulations, an immersed boundary method (IBM) was implemented into WRF first by Lundquist et al. (2010, 2012) and later by Ma and Liu (2017). Instead of using terrain-following coordinates, the IBM uses a non-conforming Cartesian grid and the terrain surface is immersed into the grid. WRF-IBM can therefore alleviate the numerical errors associated with the skewed grids for complex terrain simulations using terrain-following coordinates and extend WRF’s application to finer-scale simulations. WRF-IBM can be coupled with atmospheric physics packages included in WRF and can thus enable real-case simulations of atmospheric flow over steep complex terrain at finer resolutions. This chapter further examines the newly-developed hybrid IBM approach described in Chapter 3.

IBM was first implemented into WRF by Lundquist et al. (2010) including coupling to atmospheric physics parameterizations (e.g. the radiation and land-surface models) with no-slip boundary conditions (NS-IBM). The NS-IBM was validated for the urban flow simulations from the Joint Urban 2003 field campaign in Oklahoma city in Lundquist et al. (2012), comparing well with observations and other CFD codes. The no-slip boundary condition is sometimes used for
urban simulations, where the resolution is relatively fine (less than 1 m) and the surface layer is relatively well resolved. When it comes to coarser atmospheric simulations (100 m horizontal resolution), the no-slip boundary condition is no longer appropriate (Moeng, 1984; Garratt, 1994).

Bao et al. (2018) (Chapter 2) and Ma and Liu (2017) extended WRF-IBM to include a log-law boundary condition for atmospheric simulations with two different approaches. Several previous efforts developed log-law boundary conditions for IBM for engineering applications (Chester et al., 2007; Fadlun et al., 2000; Choi et al., 2007). Chester et al. (2007) first developed a shear stress reconstruction method (SR-IBM) for small-scale engineering flows, where the shear stress within a constant distance (normally 1.1Δz) above the surface is reconstructed based on Monin-Obukhov (M-O) similarity theory and then extrapolated down to the points below the surface. Diebold et al. (2013) extended this method with an application to atmospheric flow to simulate flow over Bolund Hill at 1 m resolution. Ma and Liu (2017) implemented this method into WRF and tested it for large-eddy simulation over Bolund hill at 1 m resolution as well. Ma and Liu (2017) compared the results from SR-IBM with observations and concluded that with fine vertical resolutions and the Lagrangian-averaged scale-dependent Smagorinsky model, the SR-IBM generally captured the speedup ratio except for some locations in the lee side recirculation zone. Fadlun et al. (2000) developed a velocity reconstruction method (VR-IBM), which reconstructed the velocity at the first grid point above the surface assuming a linear profile. Senocak et al. (2004) extended this method with a log-law reconstruction method based on Monin-Obukhov similarity theory and Senocak et al. (2015) extended it for three dimensional topography. DeLeon et al. (2018) further extended the VR-IBM’s application in atmospheric flow for flow over Askervein Hill at 11 m to 22 m resolution. Bao et al. (2018) (see Chapter 2) implemented the VR-IBM method into WRF and tested it with flow over flat terrain at 20 m, flow over Askervein Hill at 5 m and flow over Bolund Hill at 2 m. VR-IBM showed promising results at these resolutions compared to theoretical profiles (flat terrain) and observations (Askervein and Bolund Hill). The detailed methodology of the two methods is discussed in Section 4.1.2.

Both the SR-IBM and VR-IBM approaches have primarily been validated with meter scale atmospheric simulations for which they show reasonable results. Most atmospheric simulations are performed at much coarser resolutions, even for large-eddy simulation, perhaps at 100 m scale to effectively include a larger topographic area within the computational limits. There is evidence that the performance of SR-IBM and VR-IBM decreases with coarser grid resolution. DeLeon et al. (2018) performed a grid resolution test for flow over Askervein Hill at three different resolutions (11.3 m, 15.1 m and 22.8 m) and noticed the performance of the VR-IBM method decreased with coarse resolution, especially on the lee side.

Arthur et al. (2019) tested both VR-IBM and SR-IBM in a wide range of test cases to fully understand the performance of these two methods at intermediate resolutions, especially the “grey zone” or terra incognita region of atmospheric simulations. Arthur et al. (2019) concluded that the VR-IBM demonstrates some sensitivity to where the immersed surface intersects with the grid. They found that SR-IBM tends to overestimate the drag near the surface especially on the lee side, which leads to underestimation of the velocity near the surface. Both VR-IBM and SR-IBM display some sensitivity to the grid aspect ratio (α); VR-IBM tends to show good performance with low aspect ratios (α <= 4) and SR-IBM shows good performance with high aspect ratios.
(α >= 4).

To improve the performance of the IBM with log-law boundary conditions at intermediate resolutions for atmospheric simulations, in Chapter 3 we developed a new hybrid IBM log-law approach coupled with WRF (HYBRID-IBM). This method combines elements of the VR-IBM and SR-IBM approaches and reconstructs both the velocity below the surface and a few layers of the shear stress across the immersed surface. The details of the method will be summarized in Section 4.1.2 (see Chapter 3 for full discussion). HYBRID-IBM is most similar to WRF’s native terrain-following boundary condition (Section 4.1.2), as it requires both reconstruction of the velocity and stress fields in the vicinity of the immersed boundary. HYBRID-IBM was validated in Chapter 3 for several intermediate resolution test cases, including flow over flat terrain at different resolutions (including two intermediate resolution cases of 64 m and 128 m), flow over a moderately sloped idealized hill at 100 m and flow over Askervein Hill at 30 m, a typical resolution used in previous Askervein studies (Chow and Street, 2009; Golaz et al., 2009). The new HYBRID-IBM approach showed good performance in these cases.

In this chapter, a detailed sensitivity analysis of the HYBRID-IBM is performed to better understand its performance. First, given that the VR-IBM displays sensitivity to where the surface intersects with the grid (Arthur et al., 2019), we examine the sensitivity of the HYBRID-IBM approach to the immersed surface location within the grid (Section 4.2.1.1). Second, given the poor performance of SR-IBM in the lee side of hills, including too much drag on the lee side (Fang and Porté-Agel, 2016), we further examine the performance of HYBRID-IBM in the lee side (Section 4.2.2.1). Third, both VR-IBM and SR-IBM show sensitivity to the grid aspect ratio, so we investigate and provide guidelines for the grid aspect ratio to be used with HYBRID-IBM (Section 4.2.1.2). Next, we explore the performance of HYBRID-IBM across different grid resolutions (from fine to intermediate) and across different slopes (from shallow to moderate) and quantify differences with native WRF simulations (Section 4.2.2.2). Finally, we demonstrate the capability of HYBRID-IBM to accurately handle very steep terrain with simulation of flow over Bolund Hill (Section 4.2.3).

### 4.1.2 Method

#### 4.1.3 WRF’s boundary condition

As described in detail in both in Chapter 2 and Chapter 3, when a log-law boundary condition is applied, WRF calculates its surface shear stress using the following equation:

$$\tau_{si}^3 = -C_d |U_1| U_1^i$$

(4.1)

where $|U_1|$ is the wind speed at the first grid point above the surface and $C_d$ is the drag coefficient, which can be calculated based on the surface roughness $z_0$ and the distance from the first grid point to the surface, $z_1$ (the distance from $U_1$ to surface in left plot in Figure 4.1) as given by

$$C_d = \left( \frac{\kappa}{\ln \frac{z_1}{z_0}} \right)^2$$

(4.2)
4.1.4 VR-IBM

The details of the VR-IBM approach were given in Bao et al. (2018) (Chapter 2). In general, when VR-IBM is used with a log-law boundary condition, the velocity at the first grid point (\(U_1\) in the middle plot of Figure 4.1) is reconstructed assuming there is a logarithmic profile between the first grid point above the surface (\(U_{1}\)) and the image point (\(U_{img}\) in the middle panel of Figure 4.1). The image point is found by projecting a line perpendicular to the immersed surface, passing through \(U_1\) until it hits the next grid cell face (see the middle panel of Figure 4.1). The value of \(U_{img}\) can be interpolated using surrounding fluid nodes. Because of the staggered grid used, the reconstruction procedures need to be done for \(U\), \(V\) and \(W\) points separately. For more details of this method, refer to Chapter 2 or Bao et al. (2018).

4.1.5 SR-IBM

Shear stress reconstruction was first developed by Chester et al. (2007) and implemented into WRF model by Ma and Liu (2017). Following Chester et al. (2007) and Ma and Liu (2017), the SR-IBM method we use here is the version implemented and tested in WRF by Arthur et al. (2019). The results of the SR-IBM (including for flat terrain and for an idealized hill with different resolutions) are taken from Arthur et al. (2019) as reference for comparing to the performance of the HYBRID-IBM approach, which we developed and implemented into WRF as described below and in Chapter 3. When the shear stress reconstruction method is used, the shear stress is
reconstructed for points within a fixed distance ($1.1\Delta z$) above the surface ($\tau_r$ in the right panel of Figure 4.1) and extrapolated down to the points below the surface ($\tau_g$). $\tau_r$ is calculated using Equation 4.1, where $U$ uses the interpolated velocities at $U_r$, which is $1.1\Delta z$ from the surface. $C_d$ is calculated using Equation 4.2 where $z_1$ now is the distance from $U_r$ to surface. After a layer of shear stress above the immersed surface is reconstructed, it is assumed that the shear stress above the surface follows a log-law. Then the first shear stress value below the surface ($\tau_g$) is extrapolated using the shear stress values above the immersed boundary. In the right panel of Figure 4.1, the value of $\tau_g$ is extrapolated using a second order Lagrange polynomial based on $\tau_{i1}$, $\tau_{i2}$, and $\tau_{i3}$. A more detailed description of this method is given by Arthur et al. (2019); note that the ghost point extrapolation employed therein differs from the linear method (based on 2 points, $\tau_{i1}$ and $\tau_{i2}$) used by Chester et al. (2007) and Ma and Liu (2017).

### 4.1.6 HYBRID-IBM

Some drawbacks of both the VR-IBM and SR-IBM method have become apparent for atmospheric flow simulations at intermediate resolutions, as described in Chapter 3 and Arthur et al. (2019) and demonstrated further below. A new IBM log law boundary (HYBRID-IBM) was developed in Chapter 3, aiming to improve the performance of WRF-IBM over complex terrain at intermediate resolutions. The detailed description of this HYBRID-IBM approach can be found in Chapter 3. In general, HYBRID-IBM combines elements of the VR-IBM and SR-IBM approaches to better compare to the native WRF log law boundary condition. HYBRID-IBM reconstructs the velocity below the surface assuming there is a log profile near the surface and also reconstructs a few layers of the shear stress near the surface.

The new IBM algorithm is illustrated in Figure 4.1.6, showing the treatment of the velocity reconstruction in the left panel and the treatment of the shear stress reconstruction in the right panel. The treatment of the velocity is not exactly the same as the VR-IBM method. The VR-IBM method reconstructs the velocity at the first fluid nodes above the surface ($U_1$ in the middle panel of the Figure 4.1), while the new HYBRID-IBM reconstructs the velocity at the first ghost point below the surface ($U_g$ in the right panel of Figure 4.1.6). The ghost cell reconstruction method is chosen because the VR-IBM results are sensitive to this ghost point velocity. In particular, Arthur et al. (2019) noticed that the VR-IBM produces quite different velocity profiles depending on whether the velocity beneath the terrain is set to be zero or not. In the velocity reconstruction part of the HYBRID-IBM method, the first ghost point below the surface, $U_g$ is first identified. $U_{i1}$ and $U_{i2}$ are then found by projecting a line from $U_g$, perpendicular to immersed surface, to distances $0.5\Delta z$ and $1.5\Delta z$ from surface. The $U_{i2}$ value is interpolated from surrounding fluid nodes, while $U_{i1}$ is calculated assuming there is a log profile between $U_{i2}$ and $U_{i1}$. Then the value of $U_g$ is extrapolated using the value of $U_{i1}$ and $U_{i2}$.

The shear stress reconstruction part of the HYBRID-IBM method is illustrated in the right panel of Figure 4.1.6. It is similar to the SR-IBM approach, but not exactly the same. In the shear stress reconstruction portion of HYBRID-IBM, three layers of shear stress points are reconstructed including one above the surface ($\tau_{g+1}$) and two below the surface ($\tau_g$ and $tau_{g-1}$). The shear stress above the surface is reconstructed based on the drag law formula, Equation 4.2. The shear stress
4.2 Results

4.2.1 IBM sensitivity test over flat terrain

Though WRF-IBM is designed for improving the performance of WRF in simulating flow over complex terrain, it is important to test the IBM methods in a series of flat terrain test cases to help fully understand the performance of the three IBM methods. For example, oscillations in the velocity speed up profile were found when using VR-IBM to simulate flow over Askervein hill at coarse resolutions in Chapter 3, as well as in DeLeon et al. (2018). These oscillations are most likely due to the fact that VR-IBM produces different results when the immersed surface intersects the background grid at different locations, which is not desirable. Here, we examine five test cases...
where the flat terrain intersects different locations of the grid and three test cases to explore the sensitivity of the IBM methods to grid aspect ratios. The flat terrain test cases allow us to compare the profile to theoretical log-law profiles as well as to native WRF results.

### 4.2.1.1 Varying immersed surface location

Unlike terrain-following coordinates, where the first grid points above the ground are always a similar distance to the surface, with IBM the immersed surface will cut the grid at arbitrary places. It is important that the IBM results are consistent regardless of where the immersed boundary cuts the grid. Here, to evaluate the sensitivity of the location of the immersed boundary for VR-IBM, SR-IBM and HYBRID-IBM, we set up five flat terrain cases with the same grid but with different terrain heights. The basic setup is the same as the flat terrain case in Chapter 3, from which the description follows here, except that the terrain height varies. The horizontal grid resolution (dx and dy) is 32 m with a total domain size of 3200 m in each horizontal direction. The flow is driven by a geostrophic wind of \((U_g, V_g) = 10m/s\) with Coriolis parameter \(f = 10^{-4}s^{-1}\). The domain height is located at 1500 m with Rayleigh damping applied at the top 500 m. The vertical grid spacing is constant at 8 m near the surface for bottom 100 m. Then a 1.05 stretching ratio is used until the grid spacing reaches \(z = 500\) m; after that a constant \(\Delta z\) is used. 71 vertical grid points are used for WRF and 73 vertical grid points are used for all IBM methods, allowing two points below the terrain. It is important to note that we use an aspect ratio of 4 in this case, based on the fact that IBM methods have sensitivities to aspect ratio and all three IBM method (VR-IBM, SR-IBM and HYBRID-IBM) seem to work well with an aspect ratio of 4 (see the detailed tests in the next section, Section 4.2.1.2). An aspect ratio of 4 is also known as an optimal value in WRF and other large eddy simulations (Mirocha et al., 2010; Ercolani et al., 2017). The surface roughness is set to be constant with \(z_0 = 0.1\) m and the Smagorinsky turbulence closure is used with perturbations introduced at initialization. Thus the flow is turbulent after 6 hours and becomes relatively steady after 24 hours. The simulations run for 48 hours and results are shown with the velocities averaged for the last 24 hours.

We test five different terrain heights \((h = -3.95, -2, 0, 2, 3.85\) m) as shown in Figure 4.3. These terrain heights are chosen so that they span the entire range of one grid level \((\Delta z = 8\) m). For the staggered grid used by WRF, \(h = 0\) m aligns exactly with \(w\) points. \(h = 2\) m is located between the \(w\) and \(u\) points, and \(h = -2\) m is located between the \(u\) and \(w\) points. \(h = 3.85\) m is used to test the case where the first fluid node for \(u\) is close to the immersed surface, where in this case, the first fluid node is at \(z = 4\) m, which is only 0.15 m away from immersed surface. \(h = -3.95\) m is used to test the case where the first fluid node is far away from the immersed surface; in this case, the first fluid node is located at \(z = 4\) m, which is 7.95 m away from the immersed surface.

Figure 4.4 shows the time and planar averaged wind speed profile for WRF, VR-IBM, SR-IBM and HYBRID-IBM for these five different terrain heights. Among the three IBM methods, VR-IBM has the strongest sensitivity to where the immersed surface hits the grid. The VR-IBM results only match native WRF well when \(h = 0\) m, where the terrain surface intersects exactly at a grid level. As the intersection of the terrain surface moves further away from the grid level, the VR-IBM results deviate from WRF. This behavior helps to explain the oscillations we see.
Figure 4.3: Illustration of IBM intersection locations for flat terrain grid.
in Figure 3.8 in Chapter 3 where VR-IBM was used to simulate flow over Askervein Hill with oscillations resulting in the wind speed-up profile. With a complex terrain immersed surface like that of Askervein Hill, for each location along the A or AA lines the immersed boundary will interact with the grid differently, leading to spatial oscillations in the resulting velocity profile. Both SR-IBM and HYBRID-IBM display very little sensitivity to the immersed surface location. All of their profiles match WRF well for the five different terrain heights.

4.2.1.2 Varying grid resolution and aspect ratio

In Chapter 3, we tested the IBM methods (VR-IBM, HYBRID-IBM and WRF) for flow over flat terrain at 4 different horizontal resolutions (16, 32, 64 and 128 m). As the resolution decreased, the VR-IBM performance deteriorated compared to the native WRF results. In contrast, HYBRID-
IBM generally worked well at these different resolutions. In these flat terrain cases from Chapter 3, the horizontal grid resolution was varied, while keeping the vertical resolution the same, resulting in four different grid aspect ratio (ranging $\alpha$ from 2 to 16). In previous studies, the performance of WRF was found to depend on the aspect ratio (Mirocha et al., 2010; Ercolani et al., 2017). Here, a set of test cases are used to investigate the dependence of the three IBM approaches on the grid aspect ratio. Here, the VR-IBM, SR-IBM, HYBRID-IBM and WRF are tested among a range of different aspect ratios (2, 4 and 8) using different combinations of horizontal and vertical resolutions. All the test cases with different aspect ratios and resolutions are summarized in Table 4.2.1.2. All the cases have the same setup as the previous section except for varying the grid resolution and aspect ratio and all cases have the flat terrain located at $h = 0$ m.

Figure 4.5 shows the time and planar averaged wind speed profiles for flow over flat terrain with different resolution and aspect ratio combinations. The columns show aspect ratios of 2, 4, or 8, and each row compares a different IBM approach with native WRF results. All three IBM approaches show some sensitivity to grid aspect ratio. The most obvious observation is that VR-IBM performs poorly at an aspect ratio of 8. With an aspect ratio of 4, there is overall good performance of all three methods with minimum differences to WRF. With a lower aspect ratio of 2, the VR-IBM actually performs a little bit better than the other two methods, though overall, SR-IBM and HYBRID-IBM still perform well. Another observation is that though SR-IBM and HYBRID-IBM perform reasonably well at an aspect ratio of 8, there is a “kink” in the velocity profile at the first grid point above the surface, especially with $\Delta x = 128$ m. Arthur et al. (2019) tested the same cases with a constant eddy viscosity and no resolved turbulence, and the kink is still present at the lower aspect ratio for $\Delta x = 128$ m. We believe that the kink stems from the shear stress reconstruction method, as HYBRID-IBM displays a smaller but similar kink. Based on the aspect ratio sensitivities tests explored here, a better understanding of the performance of the different IBM approaches at different resolutions and aspect ratios is achieved.

4.2.2 IBM sensitivity tests over idealized hills

Next we perform a series of sensitivity tests for flow over an idealized hill. Two particular issues are examined here, which are very important for flow over complex terrain. One is to evaluate the performance of the three IBM approaches for flow over an idealized hill at different resolutions. In Chapter 3, we performed a test case of flow over a moderately sloped hill for VR-IBM and HYBRID-IBM and concluded that the HYBRID-IBM is an improvement over VR-IBM’s per-
Figure 4.5: Velocity profile for flow over flat terrain at different aspect ratios for different methods. By columns, different aspect ratio: 2 (left), 4 (middle), 8 (right). By rows, different method: HYBRID-IBM (lines with H, Row 1), SR-IBM (lines with S, Row 2), VR-IBM (lines with V, Row 3)
performance at 100 m resolution. A series of test cases is designed here to understand the differences for the three IBM approaches compared to WRF at different corresponding grid resolutions. Another interesting question is the performance of the three IBM methods with different hill slopes. Thus a sensitivity test of the performance of the three IBM approaches over different slopes is included as well. The hill setup is the same as the setup used in Chapter 3 and in Lundquist et al. (2010), except for the varying resolutions and slopes. The hill terrain is defined using the following Witch of Agnesi function:

\[ h(x,y) = \frac{h_p}{1 + \left(\frac{x}{L_h}\right)^2 + \left(\frac{y}{L_h}\right)^2} \]  

(4.3)

where \( h_p \) is the peak height. In the varying grid resolution sensitivity tests, we set this \( h_p \) to be 350 m, which creates a hill of about 15° slope. In the varying slope tests, we vary this value accordingly to get different slopes. \( h(x,y) \) is the hill height at different \( x, y \) locations. \( L_h \) is the horizontal length scale of the hill, which is 800 m in the varying grid resolution study. In all idealized hill test cases, the total domain height is 4 km and Rayleigh damping is applied for the top 2 km. With \( \Delta z = 20 \) m near the surface, a total of 76 vertical grid points are used for WRF and 78 points are used for IBM. When \( \Delta z = 10 \) m is used, a total of 121 grid points are used for WRF and 123 grid points for IBM. In both cases, \( \Delta z \) holds constant until about \( z = 450 \) m which is about 100 m above the hill peak. Then a 1.05 stretching ratio is used until \( \Delta z = 100 \) m. After that, \( \Delta z \) remains constant to the domain top. The horizontal domain size for all cases remains the same at 6 km, as 121 grid points are used when \( dx = 50 \) m, 61 grid points are used in when \( dx = 100 \) m and 31 grid points are used when \( dx = 200 \) m.

As with the flat terrain case, the flow is driven by a geostrophic wind of \((U_g, V_g) = (10, 0) \) m s\(^{-1}\) with a Coriolis parameter \( f = 10^{-4} \) s\(^{-1}\). The valley case is initialized with a neutral, dry sounding with a 10 m/s wind in the \( x \)-direction. The surface is set to have a constant surface roughness of \( z_0 = 0.1 \) m. The Smagorinsky turbulence closure is used, and periodic boundary conditions are used for the lateral boundaries. The total integration time for this case is 2 days, and if averaging is used, the results are averaged over the last 24 hours.

### 4.2.2.1 Grid resolution study

A grid resolution study is performed to compare the performance of the three IBM approaches to native WRF for flow over the moderately sloped idealized hill at different resolutions. Three grid resolutions are used to simulate the same hill (50 m, 100 m and 200 m) with an additional fine resolution WRF case (25 m) chosen to represent the “reference solution”. Both the native WRF and WRF-IBM have numerical errors from different sources. The idea in this sensitivity study is to see how the differences between the terrain-following and IBM results change with resolution. Differences from each method are presented later in this section all relative to the 25 m WRF results. A fixed aspect ratio of 5 is chosen for \( dx = 50 \) m and \( dx = 100 \) m, which is close to the aspect ratio of 4 based on our tests in the last section. This means the vertical resolution is 10 m near the surface for \( dx = 50 \) m and 20 m near the surface for \( dx = 100 \) m. For \( dx = 200 \) m, keeping...
the same aspect ratio of 5 would require a $dz$ of 40 m, which is relatively coarse for atmospheric boundary layer simulations. Thus we keep $dz = 20$ m for the cases with $dx = 200$ m.

Figure 4.6 and Figure 4.8 show time-averaged $x$-$y$ slices at a height of 200 m. The results from WRF, VR-IBM, SR-IBM and HYBRID-IBM are presented for 50 m resolution and 100 m resolution. SR-IBM overestimates the recirculation zone both at 50 m and at 100 m resolution, which can be clearly seen in Figure 4.9 and Figure 4.7. To illustrate the differences in the lee side
Figure 4.7: Time-averaged velocity profiles over an idealized hill with the WRF, VR-IBM, SR-IBM, HYBRID-IBM approaches using 50 m resolution.
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Figure 4.8: As in Figure 4.6 but with 100 m grid resolution.
Figure 4.9: Time-averaged velocity profiles over an idealized hill with the WRF, VR-IBM, SR-IBM, HYBRID-IBM approaches using 100 m resolution.
of the hill more clearly, a direct comparison of the time-averaged velocity profile between standard WRF and the three IBM approaches is shown in Figure 4.7 for 50 m resolution and Figure 4.9 for 100 m resolution. The velocity profiles are averaged for the last 24 hours at 15-min output intervals and are shown at several locations along the center line of the hill ($y = 3$ km). The main differences between the IBM and WRF results are found on the lee side of the hill, except for the fifth profile for SR-IBM in Figure 4.7. The SR-IBM overestimates the recirculation zone at both 50 m and 100 m resolutions. The main issue for VR-IBM is the speed-up near the surface in the sixth and seventh profiles in the bottom panel of Figure 4.7. The HYBRID-IBM generates the best agreement compared to WRF for both the 50 m and 100 m resolutions.

To further quantify the performance of the three IBM methods, the results from the three IBM methods and WRF are compared to the reference solution (the 25 m resolution WRF results). We follow the procedure for calculating the absolute differences from Arthur et al. (2019). First, time averaging is applied for all test cases over the last 24 hours. Then velocities are interpolated to the grid of the reference solution. Then differences between the test cases and the reference solution are calculated. Finally the differences are averaged for the bottom 100 m above the ground (AGL) and planar averaging is also applied.

The differences from each method at each resolution are reported in Figure 4.10. In general, as the resolution becomes coarser, the absolute differences increase for all methods, as expected. The native WRF results show the smallest differences over the range of resolutions tested, with a maximum difference of 0.45 m/s at a resolution of 200 m. HYBRID-IBM shows very similar behavior to native WRF, with similar differences compared to WRF’s 25 m reference solution; the difference at 200 m is only 0.5 m/s. The differences for SR-IBM are somewhat larger with a larger slope and differences of close to 1 m/s at 200 m resolution. Finally VR-IBM shows the largest differences, with about 1.8 m/s errors at 200 m resolution and a much steeper slope to the differences.

WRF in practice works well with slopes less than about 20°. Here, we treat WRF’s solution as the reference case and compute differences from the other IBM methods, but it is important to be aware that WRF contains errors from the terrain-following coordinate system. The terrain-following coordinates used in WRF will introduce errors that increase as slope increases (as examined below), which is where IBM results will be most useful.

It is also important to be aware that we use $dz = 20$ m for $dx = 200$ m. Thus in that case, the aspect ratio is 10 instead of 5. Based on the aspect ratio sensitivity tests in Section 4.2.1.2, the VR-IBM has poor performance with large aspect ratios. The large differences for VR-IBM at 200 m may also be due to the fact that we use an aspect ratio of 10 when $dx = 200$ m.

4.2.2.2 Hill slope study

IBM is designed for flow over steep terrain where the errors from the terrain-following coordinates in native WRF become too large. Errors from the terrain-following coordinates generally increase as the terrain slope increases, though these errors are difficult to quantify. Lundquist and K. (2010) compared IBM (no-slip) with WRF results for idealized flow over different slopes and confirmed that WRF terrain-following coordinates increasingly deviate from the IBM result.
Figure 4.10: Difference between WRF, HYBRID-IBM, SR-IBM and VR-IBM at 50 m, 100 m, and 200 m compared with a reference WRF simulation at 25 m resolution. Time and space-averaged differences computed as described in the text.
at increasing terrain slopes. Simon and Chow (2010) also found that as terrain slope increases, the differences between IBM (no-slip) and native WRF increase. Given that the IBM approach is based on an interpolation technique to set values near the immersed boundary, the local slope at the cut cell should not generally affect the method’s performance. With a log-law boundary condition, however, other aspects must be considered, such as the use of vertical vs. surface-normal derivatives and the suitability of the log-law on sloped terrain in general.

Here we examine the performance of the three IBM approaches compared to native WRF over hills of different slopes: 7° (shallow), 15° (moderate) and 22° (steep). We quantify the performance of the three methods by calculating the difference between the WRF-IBM and the native WRF results. The hill terrain is defined by Equation 4.3. The moderately sloped hill (approximate maximum slope of 15°) is the same hill used in Section 4.2.2.1, with horizontal length scale, \( L_p = 800 \) m and peak height \( h_p = 350 \) m. For the shallow hill case, the horizontal length scale is \( L_p = 800 \) m, but the peak height is decreased to \( h_p = 200 \) m, giving an approximately 7° slope. For the steep hill case, the peak height is 350 m, but with a horizontal length scale \( L_p \) of 600 m, which creates a steeper hill of approximately 22° slope. In all three cases, the horizontal domain size is 6 km, with 121 grid points and \( dx = 100 \) m. The vertical grid is the same as in the previous section with the \( dx = 100 \) m cases. A total of 76 vertical grid points is used for WRF and 78 points are used for IBM, with \( \Delta z = 20 \) m near the surface, and \( \Delta z \) is constant until about \( z = 450 \) m which is about 100 m above the hill peak. Then a 1.05 stretch ratio is used until \( \Delta z = 100 \) m. After that, \( \Delta z \) remains constant to the domain top. The domain height is 4 km with 2 km damping on the top. An aspect ratio of 5 is kept in all cases in this section.

Figures 4.12 and 4.11 show \( x-y \) slices of time-averaged velocity magnitude at a height of 100 m for the steep (22°) and shallow (7°) hills for WRF, VR-IBM, SR-IBM and HYBRID-IBM. Note that a height of 100 m is chosen for this comparison because the 200 m height used above is too high to capture the recirculation zone behind the shallow hill. We note that the SR-IBM tends to overestimate the lee side recirculation both for shallow and steep slopes compared to WRF.

To quantify the difference between the three IBM approaches over different slopes, a mean absolute difference is calculated as the difference between the IBM and WRF results for each case. All results, including the reference WRF result, are taken at 100 m horizontal resolution so as to focus on differences due to the terrain slope only. For example, the mild slope case at 100 m resolution from the three IBM simulations is compared to the WRF reference result for the mild slope case at 100 m resolution. As above, first, time averaging is applied for all test cases over the last 24 hours. Then velocities are interpolated to the WRF terrain-following grid at this specific slope so that differences between the cases can be calculated. Finally the differences are averaged for the bottom 100 m above the ground (AGL) and planar averaging is also applied to generate an aggregate mean absolute difference value. This value for each method with each slope is reported in Figure 4.13. In general, as slope increases, the differences increases for all methods.

Note that as the slope increases, the native WRF solution cannot be treated as exactly correct due to errors from the terrain-following coordinate system. In general, as the slope increases, these terrain-following errors would become larger. Thus, Figure 4.13 is also a measure of the errors from the terrain-following coordinates. All IBM methods show smaller differences from the WRF result with smaller slopes. This makes sense because the grid skewing due to the terrain-
following coordinates increases with increasing slope. In addition, the difference between using the surface-normal versus the vertical direction for calculating surface fluxes and applying the log law becomes larger with steeper slopes. Among the three methods, HYBRID-IBM has significantly smaller differences compared to WRF at all slopes. VR-IBM has larger differences than SR-IBM at shallow slopes, perhaps explained by the sensitivity of VR-IBM to the intersection of the immersed terrain with the grid, as discussed in Section 4.2.2.1. The SR-IBM shows larger differences than VR-IBM at steep slopes, perhaps because of the problem with SR-IBM overestimating lee side recirculation.

4.2.3 Bolund Hill experiment using HYBRID-IBM

Though the HYBRID-IBM is designed specifically for improving the immersed boundary method at intermediate resolution, IBM approaches in general are needed for flow over very steep complex terrain. Thus the Bolund experiment is chosen to show that HYBRID-IBM is also able to handle such complex terrain. The Bolund Hill experiment was used to validated VR-IBM at 2 m resolution in Bao et al. (2018) (Chapter 2). The Bolund simulation is run at fine resolutions again here, and a small aspect ratio of 4. Thus similar results to VR-IBM are expected and the goal of this section is simply to confirm the capability of HYBRID-IBM over complex terrain.

The description of the Bolund experiment here follows that of Bao et al. (2018) (Chapter 2). The Bolund Hill field campaign was conducted in the winter of 2007-2008 and was designed to study atmospheric flow over steep terrain. The hill is 130 m long (West-East direction) and 75 m wide (North-South direction) with a maximum height of 12 m, as shown in Figure 2.8 surrounded by water with a long uniform fetch over the sea in the westward direction, which allows us to specify an upwind boundary condition. Though the setup is simple, the flow over the Bolund hill is challenging because the western (windward) face of the hill is a steep 90-degree slope.

During the field campaign, 38 anemometers were deployed over the hill at 10 meteorological tower locations as shown in Figure 2.8 (Bechmann et al., 2011), including 26 sonic anemometers, 12 cup anemometers, and two lidars. Instruments were placed at multiple heights on the observational towers, most often located at 2, 5, and 9 m AGL. At each tower, turbulent wind velocities and fluxes were measured. The simulation presented here is Case 3 as listed in Bechmann et al. (2011), which details results of a blind simulation intercomparison project. The mean wind speed during the selected measurement period was around 10 m s\(^{-1}\) and a wind direction from the south-west is specified at 242 degrees. The observation data are averaged over 10 minutes.

Results using VR-IBM and HYBRID-IBM are included in this section. Standard terrain-following WRF simulations fail because of the nearly 90-degree steep slope.

4.2.3.1 Simulation setup and results

The Bolund setup is the same as the one in Bao et al. (2018) (Chapter 2) and a similar description follows here. A domain containing Bolund Hill is nested within an outer periodic domain which has a fully-developed boundary layer over flat terrain. Flow is driven in the parent domain with a uniform pressure gradient, which creates a mean wind of 10 m s\(^{-1}\). Boundary conditions
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Figure 4.11: Time-averaged velocity magnitude for $x$-$y$ slices at $z = 100$ m for the shallow hill case for WRF and the three IBM approaches using 100 m grid resolution. The black line shows a transect along the mean inflow direction. The gray circle shows the intersection of the hill topography.
Figure 4.12: As in Figure 4.11 but for the steep slope hill case.
Figure 4.13: Difference between WRF, HYBRID-IBM, SR-IBM and VR-IBM for shallow, moderate, and steep slope hill cases compared with a reference WRF simulation at 100 m resolution at the same slope. Time and space-averaged differences computed as described in the text.
are passed from the flat domain to the nested Bolund Hill domain at each outer domain time step. The outer domain is periodic, with $\Delta x = \Delta y = 6$ m, and an overall domain size of 912 m in each horizontal direction. The domain top is located at 250 m, and the vertical domain dimension is 252.25 m for IBM cases to allow for points below the terrain. 132 vertical grid points are used in both of the IBM cases. The minimum vertical grid spacing is $\Delta z_{\text{min}} = 0.75$ m and the maximum is $\Delta z_{\text{max}} = 2$ m. For the inner domain, which includes Bolund Hill, $\Delta x = \Delta y = 2$ m, with an overall domain size of 804 m in each horizontal direction. The vertical domain height and grid are the same as on the outer domain. The Smagorinsky turbulence closure is used. A uniform roughness length of $z_0 = 0.3$ mm (water) is used for the entire domain following Diebold et al. (2013).

Simulations are carried out for a wind direction of 242 degrees, to match conditions observed during the field campaign at the reference site (5 m agl). The parent domain is spun up for 18 hours. The nested domain which contains Bolund hill is forced at its boundaries by the parent domain and run for another for 1.5 hr; the first hour is spin up, and the last 0.5 hr is used to time average the results. The setup of the Bolund simulation is shown in Figure 4.14.

To investigate discrepancies between simulations and measurements over the hill (not including the first tower M0), we quantify changes in the wind field as changes in speed-up. Speed-up for
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Figure 4.15: Speed-up ratio over Bolund Hill along incoming wind direction of 242° (M0, M1, M3, and 4) showing observations, VR-IBM and HYBRID-IBM results at 5 m (top) and 2 m (middle), with topography (bottom).

this case is defined by

$$\Delta S_m = \frac{(U/\bar{u}_s)_{agg} - (U_0/\bar{u}_{s0})_{agg}}{(U_0/\bar{u}_{s0})_{agg}}$$

(4.4)

where $U$ is the mean wind speed at the sensor location and $U_0$ is the mean wind speed at the tower M0.

The comparison is made for two different elevations, 2 m and 5 m above ground level, as shown in Figure 2.14. In general, HYBRID-IBM and VR-IBM show almost the same result at this fine resolution. Overall agreement is good, showing the capability of the HYBRID-IBM handling complex terrain.
CHAPTER 4. SENSITIVITY STUDIES OF THE HYBRID-IBM APPROACH

4.3 Summary and conclusion

This chapter describes detailed validation and sensitivity tests of the HYBRID-IBM approach. The HYBRID-IBM algorithm is a new log-law IBM implementation based on a combined velocity and shear stress reconstruction. It was developed and implemented in WRF in Chapter 3 where it was validated for three test cases including flow over flat terrain, flow over a moderately sloped hill at intermediate grid resolution, and flow over Akservein Hill at intermediate resolution. The HYBRID-IBM is designed to improve the performance of existing methods (SR-IBM and VR-IBM) at intermediate resolution for flow over complex terrain. VR-IBM works well at very fine resolutions (e.g. 1-5 m horizontal spacing) relative to the topography, but its performance becomes worse at intermediate resolutions (e.g. 100 m) which are more practical for atmospheric boundary layer simulations (for example as seen in the jaggedness of the profiles in the Askervein simulations shown in Chapter 3). SR-IBM works well at fine resolutions as well (see e.g. the validation by Ma and Liu (2017) for flow over Bolund Hill at 1 m resolution), but had not yet been fully tested for intermediate resolutions. Here, to fully validate the performance of HYBRID-IBM at intermediate resolution, a series of test cases was performed, with an aim to showing the improvement over previous IBM algorithms.

First, two flat terrain sensitivity tests were performed. One test case examined the sensitivity of the three IBM approaches to the immersed surface location with the grid. The flat terrain was placed at five different locations within one grid cell level. Both SR-IBM and HYBRID-IBM showed little sensitivity to the immersed surface location, while the VR-IBM results greatly depended on the immersed surface placement. When VR-IBM is used, it generates different results with different immersed surface locations, thus resulting in errors depending on where the immersed surface cuts the grid. Another flat terrain test case examined the sensitivity of the IBM approaches to the grid aspect ratio. The VR-IBM was found to work well at smaller grid aspect ratios, where \( \alpha < 4 \). The SR-IBM and HYBRID-IBM methods tend to work better for higher aspect ratios, where \( \alpha > 4 \).

Additional test cases examined the sensitivity of the three IBM approaches to grid resolution for flow over a moderately sloped idealized hill (15°). Understanding how errors increase with resolution is important for determining which grid resolutions are acceptable for using VR-IBM and SR-IBM and which resolutions require the HYBRID-IBM approach. We tested flow over idealized hills for WRF, VR-IBM, SR-IBM and HYBRID-IBM at three different intermediate resolutions (50 m, 100 m and 200 m). An additional WRF simulation at 25 m resolution was treated as the “reference” solution. The mean absolute differences from the WRF, SR-IBM and HYBRID-IBM results increase linearly, though the slope of the SR-IBM method is slightly bigger than the other two. The HYBRID-IBM has the smallest differences at all three resolutions among the three IBM methods and the slope of the differences is the smallest as well. The VR-IBM has the biggest differences at all three resolutions.

Another set of sensitivity tests for flow over idealized hills at different slopes was also performed. Overall, the differences between terrain-following WRF and the IBM solutions were found to increase with increasing terrain slopes. HYBRID-IBM showed the smallest differences from WRF among the three IBM methods. This comparison will be expanded in future studies to
include tests of the different slopes at different grid resolutions. Because native WRF will suffer from increasing errors from the terrain-following coordinates with increasing slopes, these tests highlight the growth of these errors in the difference plots between WRF and the IBM results. At large slopes, the differences between the surface-normal direction (used by IBM) and the vertical direction (used by native WRF) in setting the log-law boundary condition also becomes more pronounced.

Lastly, simulation of flow over Bolund Hill for HYBRID-IBM was used to demonstrate that HYBRID-IBM is capable of handling complex terrain with reasonable speedup profiles compared to observations. This simulation was performed at high resolution and did not show significant differences from previous results obtained by VR-IBM, but confirmed the capability of HYBRID-IBM over very steep terrain that cannot be treated using the terrain-following coordinates in native WRF.
Chapter 5

Summary and Recommendations

5.1 Summary

This work contributes to the field of atmospheric flow simulation over complex terrain, by enhancing the ability of a widely-used community model (WRF) to handle very steep topography. Specifically, the immersed boundary method (IBM) in WRF was improved first by implementing an existing IBM log law boundary condition (VR-IBM) into WRF with extensive validation and testing. Next, a new log-law boundary condition (HYBRID-IBM) was developed and implemented into WRF to address deficiencies of existing methods. Comparisons with ideal and real cases and sensitivity tests demonstrated the robustness of the HYBRID-IBM approach.

The need for IBM approaches arises from enhancements in computational power which now allow for mesoscale models like WRF to handle increasingly steep topography. Traditional CFD approaches, in contrast, have had the ability to handle the complex terrain through boundary-fitting gridding techniques. Lateral boundary conditions in traditional CFD codes are usually prescribed as simple idealized inflow conditions which do not include the influence of mesoscale weather effects. Traditional atmospheric mesoscale models, including WRF, are now being pushed towards fine resolution to better understanding flow dynamics and turbulence for flow over complex terrain. The grid nesting capabilities of mesoscale models allows mesoscale forcing to be included in the lateral boundary conditions of even the finest resolution nests. However, the terrain-following coordinates used by WRF and other mesoscale models struggle over steep terrain because of numerical errors that arise and lead to model instability. Thus, the immersed boundary method (IBM) was implemented into WRF by Lundquist et al. (2010, 2012), with the goal of using grid nesting strategies to nest from mesoscale to microscale simulations and include both highly complex terrain and mesoscale weather effects. The IBM implemented in WRF was originally with no-slip boundary conditions to simulate flow over urban areas. To use it to simulate atmospheric flow over hilly or mountainous terrain, which normally use coarser resolution than urban simulations, a log-law boundary condition is required for WRF-IBM. This proved to be a very challenging task due to complexities with prescribing velocity gradients at the immersed surface where interpolation errors make the result highly sensitive to the details of the IBM approach. The main contribution
of this dissertation is thus development of a robust log-law boundary condition for WRF-IBM to enable atmospheric simulations over complex terrain at intermediate resolutions.

In Chapter 2, a velocity-reconstruction immersed boundary method (VR-IBM) following (Senocak et al., 2004) was implemented into WRF. This method reconstructs the first velocity point above the surface by assuming there is a logarithmic profile near the surface. We validated this method with a range of test cases, including flow over flat terrain, flow over Askervein Hill at 5 m resolution, and flow over Bolund Hill at 2 m resolution. These test cases were designed to ensure that the IBM approach could handle both flat topography and moderate and steep terrain. This VR-IBM performs well at these fine resolution simulations, but problems arise when intermediate resolutions (∼100 m) are used (as seen in Chapters 3 and 4 and in Bao et al., 2016). Indeed, existing log-law methods in the literature (Anderson, 2013; Chester et al., 2007) have been validated at very fine resolutions (∼mm scale to 1 m scale) and the performance of these methods decreases at coarser resolutions (Bao et al., 2016).

In Chapter 3, we developed a new log-law boundary condition for WRF-IBM (HYBRID-IBM) to address these challenges at intermediate resolutions. This hybrid method reconstructs both velocity and shear stress near the surface based on Monin-Obukhov similarity theory (here considering neutral conditions). We described the detailed implementation of this method, as well as three test cases, including flow over flat terrain at four resolutions (16, 32, 64 and 128 m), flow over a moderately sloped (15 degree) idealized hill at 100 m resolution, and flow over Askervein Hill at a much coarser resolution (30 m, as more commonly used in practice). For flow over flat terrain at 64 m and 128 m resolutions, the results of HYBRID-IBM match native WRF well. In contrast, VR-IBM matches WRF at fine resolutions (16 m) but starts to deviate at 64 m and 128 m resolutions. For flow over a moderately sloped idealized hill, the HYBRID-IBM improves significantly over VR-IBM when comparing to native WRF’s results. For flow over Askervein Hill at 30 m resolution, HYBRID-IBM generates very reasonable results compared to WRF and observations. Though VR-IBM performs well at fine resolution for flow over Askervein Hill (5 m) as seen in Chapter 2, it has problems at 30 m. One is that it tend to greatly overestimate the speedup profile and also creates oscillations in the speedup profiles. Similar problems were found in DeLeon et al. (2018) when they used VR-IBM to simulate flow over Askervein Hill at 25 m resolution. Because this oscillation problem was not present in the fine resolution Askervein simulations with VR-IBM (Chapter 2), we believe it may be due to sensitivities in VR-IBM to where the immersed boundary intersects with the background grid. With fine resolution, these differences in where the immersed boundary cuts the grid would be much smaller.

Chapter 4 further explored the performance of HYBRID-IBM compared to VR-IBM and the shear-stress reconstruction (SR-IBM) approaches. First, given that the VR-IBM displays sensitivity to where the surface intersects with the grid (DeLeon et al., 2018; Arthur et al., 2019), we examined the sensitivity of the HYBRID-IBM approach to the immersed surface location within the grid. We concluded that both HYBRID-IBM and SR-IBM have little or no sensitivity to where the surface intersects with the grid, while VR-IBM displays strong sensitivity to this. Second, VR-IBM, SR-IBM and HYBRID-IBM show sensitivity to the grid aspect ratio, so we investigated and provided guidelines for the grid aspect ratio to be used with each method. VR-IBM tends to work well at lower aspect ratios (e.g. $\alpha <= 4$) while SR-IBM and HYBRID-IBM tend to work better
with high aspect ratios (e.g. $\alpha \geq 4$) Next, we explored the performance of HYBRID-IBM across different grid resolutions (from fine to intermediate) and across different slopes (from shallow to moderate) and quantified differences. We noticed that all methods show increased differences from a high-resolution WRF reference solution as grid resolutions become coarser. At steeper slopes, differences between the IBM and WRF results also increase. This is expected because of errors from the terrain-following coordinates in WRF at steeper slopes, as well as differences in representation of the log-law condition with vertical derivatives in WRF and surface-normal derivatives used with IBM. HYBRID-IBM shows the smallest differences from native WRF among the three IBM approaches. Further, we confirmed that SR-IBM tends to overestimate leeside recirculations at all resolutions and slopes, as seen in Fang and Porté-Agel (2016). HYBRID-IBM showed improved performance in the lee of the hills. Finally, we demonstrated the capability of HYBRID-IBM to accurately handle very steep terrain with fine-resolution simulations of flow over Bolund Hill (Section 4.2.3) which show good agreement with observations.

5.2 Recommendations

Recommendations for best practices in modeling flow over complex terrain at intermediate resolution ($\sim 100$ m) and future work can be made based on the findings in this dissertation.

The goal of this work was to implement a log-law boundary condition for WRF-IBM so that ultimately the grid nesting strategy in WRF could be used to simulate flow over steep complex terrain while including mesoscale influence on the finer domains. The first log-law strategy implemented in Chapter 2 was testing the VR-IBM in WRF. Based on the test cases performed for VR-IBM, it is clear that the VR-IBM approach requires relatively fine ($\sim 1$ m) resolution to simulate atmospheric flows. Furthermore, the VR-IBM has sensitivity to where the immersed surface intersects with the grid (shown in Chapters 3 and 4), thus a relatively fine resolution is required to achieve reasonable performance. Finally, the VR-IBM works best with small grid aspect ratios, where $\alpha \leq 4$, as seen in Chapter 4. In addition to VR-IBM, we also tested the SR-IBM approach of Chester et al. (2007). We found that SR-IBM in WRF tends to overestimate leeside recirculations. The SR-IBM works best for higher grid aspect ratios where $\alpha \geq 4$.

The hybrid IBM approach developed in Chapters 3 and 4 showed much more robust performance. HYBRID-IBM shows little to no sensitivity to where the flat terrain intersects with the grid. HYBRID-IBM also shows the smallest error increase as grid resolution coarsens. It also shows the smallest differences from WRF as the terrain slope is increased. HYBRID-IBM tends to work best for large grid aspect ratio, where $\alpha \geq 4$, which is conveniently ideal for atmospheric boundary layer studies. Tests of flow over Askervein Hill at 30 m resolution and Bolund hill at 2 m resolution show the capability of HYBRID-IBM in handling complex terrain. Thus, the HYBRID-IBM is highly recommended for applications to flow over complex terrain at intermediate resolutions, with $\alpha \geq 4$.

The recommendations here are based on the results of this dissertation research, and therefore may not be generally applicable, but may still serve as guidelines for similar situations. In particular, this work with IBM approaches has been done with WRF, which has its own limitations
and benefits. For example, WRF uses pressure-based terrain-following coordinates, which generally complicate the implementation of immersed boundary methods. Additional computational cost benefits can be found by improving the efficiency of the IBM algorithm as implemented in WRF.

The work presented here is intended to improve atmospheric simulations over complex terrain at intermediate grid resolutions by incorporating a robust log-law boundary condition. This will enable grid nested simulations which include large-scale weather effects on fine-scale steep terrain cases. These improvements will help us better develop theory and understanding of flow dynamics over complex terrain, with a wide range of applications such as weather prediction, atmospheric dispersion modeling, and wind energy resource modeling, among others. Future work will include coupling the work of nesting WRF-IBM within terrain-following WRF simulations from Wiersema et al. (2018). This will allow us to perform LES scale simulations for atmospheric flow over complex terrain within the WRF model, using the grid nesting framework to provide downscaled lateral boundary conditions for use with WRF-IBM.
References


CHAPTER 5. SUMMARY AND RECOMMENDATIONS


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