A New Model to Construct Ice Stream Surface Elevation Profiles
and Calculate Contributions to Sea-Level Rise

by

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Abstract

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Sea-level rise is a problem that affects regions worldwide – from the marshlands of the San Francisco Bay Area to the farmlands in coastal Bangladesh. Three-dimensional ice sheet models are the principle tools to evaluate mass loss from ice sheets that contribute to sea-level rise. We recognize that given the current limitations in representing the full extent of dynamical processes that affect ice sheet mass loss in 3-D ice sheet models, we cannot make reliable forecasts of sea-level rise from melting polar land ice. Thus, we take a completely different approach to gaining insight about the potential effects of climate change-induced perturbations on ice sheets.

We build a flowline model that resolves the fast-flowing portions of ice sheets (i.e., ice streams). We express the dynamics along the flowline with \((a)\) vertical shear deformation, \((b)\) horizontal shear deformation, and \((c)\) basal slip. Knowledge accumulated from prior force balance analyses performed on some polar ice streams allows us to form relations between \((a)\) and \((c)\), and between \((a)\) and \((c)\) combined and \((b)\). Based on these relationships, we numerically construct surface elevation profiles along flowlines centered on ten select ice streams in Greenland and Antarctica, by prescribing three climate change-induced perturbations: grounding line retreat, ice stream widening, and surface mass balance increase. Comparing these constructed profiles to the current observed ones allows us to quantify the effect of these perturbations on the various characteristics that these ten ice streams possess.

Pine Island Glacier, which flows over a long overdeepening, will lose more than half of its stored ice volume that is contributable to sea-level rise before it reaches a possible steady state. Recovery Ice Stream, with its slippery base, long stretch of streaming-flow, and longest flowline among those we examined, loses the most mass (812 km\(^3\)/km width). Jutulstraumen, which has little room to widen and a short stretch of streaming-flow, experiences more mass gain due to surface mass balance increase than mass loss due to grounding line retreat and widening. The broad range of ice streams and their diverse responses to prescribed perturbations is a convincing message that an accurate assessment of the contribution of ice sheets to future sea-level rise can only be obtained by raising the resolution of models to resolve the fast-flowing features and looking at their mass changes individually over time.
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Introduction

I. Climate Warming and Ice Sheet Mass Balance Studies – A History

Even though World Wars and a Cold War scarred much of the twentieth century landscape, one could argue that they provided for a fertile ground for multiple nations to tackle cooperatively, the challenge of the twenty first century: climate change.

Despite the development of non carbon-emitting cars, regions most susceptible to climate warming remain the less developed, financially less resourceful nations of the world. While the Netherlands builds dikes to combat rising sea-level at a rate of $1.3 billion a year, Bangladesh continues to witness the encroaching coastline onto their nation’s farmland with their government disbursing $3 million to NGOs from a recently created trust fund to tackle climate change. The governments of Greece and Portugal get bailed out with internationally gathered funds of $257 billion, but will the endangered farmlands of Bangladesh and coastal resorts of the Maldives ever get bailed out with the same kind of crisis frenzy? This is the kind of question we will have to face in the twenty first century and provide answers to through action in the coming decades.

Humans have increased the injection of carbon dioxide and other greenhouse gases into the atmosphere since the onset of the Industrial Revolution. This was at first not realized due to theoretical and observational limitations, but at the end of the nineteenth century a Swedish scientist named Svante Arrhenius first hypothesized an enhanced greenhouse effect due to anthropogenic increases of greenhouse gases into the atmosphere. This idea has taken long, just until the recent decade, to be corroborated. And corroboration has been made possible due to a long string of observational and theoretical advances made during the past century or so. An important advance came in the mid-twentieth century when Charles David Keeling started making direct measurements of carbon dioxide atop Mauna Loa on the island of Molokai in Hawai‘i. Technologies using carbon isotopes (14C, or radiocarbon) to distinguish the mark of fossil fuel-derived carbon dioxide in the atmosphere from other sources of carbon dioxide in the atmosphere, along with concerted efforts to calculate amounts of fossil fuel-derived carbon dioxide emitted into the atmosphere, allowed for budgeting studies of how much of the carbon dioxide released by humans from fossil fuel combustion was being retained in the atmosphere. Since the 1950’s, analyses of bubbles of air trapped in numerous ice cores taken from the polar regions have demonstrated that carbon dioxide increases in the atmosphere have indeed taken place from the onset of the Industrial era. These records of carbon dioxide levels from ice cores are consistent with the direct atmospheric measurements of carbon dioxide (ref.) – such as those taken from atop Mauna Loa – that exist since the late 1950’s, confirming our assurance in what we know about how atmospheric levels of carbon dioxide have fluctuated from 800,000 years.
ago to the current day. The final piece of the puzzle, which establishes that the warming observed in the atmosphere cannot be explained by natural influences alone and has to be explained by anthropogenic increases of greenhouse gases into the atmosphere, came in 2007 when the Intergovernmental Panel on Climate Change released some modeling results in the Fourth Assessment Report on climate change.

Scientists and some other members of the public would not be so concerned about the future of the Earth’s climate, if all science has taught us is that the Earth is capable of warming as a result of humans injecting more greenhouse gases into the atmosphere. The issue is threefold: (i) the magnitude of the warming, (ii) the numerous consequences of that warming on our daily lives, and (iii) the regional heterogeneity of those consequences on various parts of the globe.

Compared to 1980-1999 levels, the latest Climate Change Assessment Report by the IPCC predicts a 1.8 to 4.0 degrees Celsius rise is global average surface temperatures by the last decade of the twenty first century. The spread is due to various scenarios by which we could emit greenhouse gases into the atmosphere this century. This magnitude of warming is said to have various consequences on the Earth system, both inside and outside the sphere of human activity.

The large consequences of greatest concern are: more drought in already arid regions, more rainfall in already wet regions, diminishing of snowpack in areas where surrounding communities depend heavily on the melting of the snowpack in the spring and summer time for their water supply, less snowfall and hence shorter seasons for winter sports, and sea-level rise.

The third part of the issue concerns which parts of the world are most affected by these consequences of the warming. A large fraction of the arid region of the world lies in Africa. A recent study found vast stores of groundwater in the continent, which are said to sustain water abstraction through inter-annual climate variability. But resources to deal with increasing lack of water – a possible consequence of global warming – remain insufficient. Meanwhile, the Southwest region of the United States, which also lies in an arid region of the world, will be able to better deal with increasingly little rainfall, because of better developed infrastructures of irrigation to channel water from less water-deprived areas to the areas of concern.

Increased rainfall in South Asia (e.g., Dhaka, Bangladesh currently receives 200cm of rain per year) will be felt intensely because of less warning systems and escape routes in case of flooding. Conversely, Mobile, Alabama – the wettest city in the contiguous 48 states of the United States with 170cm of rain per year – will probably be able to adapt better to more severe rainfall events because the National Weather Service will give abundant warning before predicted events, and the susceptible citizens will have more resources to evacuate the area more quickly.

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8 ibid: p.11: Figure SPM.4.
9 Solomon et al., 2007, Technical Summary.
10 Meehl et al., 2007, Global Climate Projections, p.750.
11 Kundzewicz et al., 2007, Freshwater resources and their management (AR4, WGII), p.175.
13 Nicholls et al., 2007, Coastal systems and low-lying areas (AR4, WGII), p.323: Table 6.2.
14 Kundzewicz et al., 2007, p.187.
15 MacDonald et al., 2012, Quantitative maps of groundwater resources in Africa.
16 See report: Improving Drought Preparedness in the West by the Western Governors’ Association for initiatives being taken by governments in the western United States to tackle future water shortage concerns (www.westgov.org/initiatives/water/383-drought).
Finally, the Netherlands and Bangladesh are both nations whose lands rest on low elevation and are susceptible to sea-level rise. While the Netherlands spends $1.3 billion a year to newly build or reinforce old dykes to keep their land from being inundated, Bangladesh can spend only $3 million for similar purposes. This seems to be an insufficient amount considering the large number of people living in the coastal areas, which have become increasingly vulnerable due to rising sea-levels.

The recent globally averaged rise in sea-level has been 3.1mm/year. Half of this amount is attributable to thermal expansion of seawater, and the other half to melting of land ice. Of the 1.8mm of sea-level rise that land ice-melting is responsible for, 60% comes from glaciers and ice caps. The remaining 40% is ascribed to polar ice sheets. A study shows that glaciers and ice caps will continue to be the dominant contributor to sea-level rise from the land ice-melting sector at least until the end of this century. Beyond then, when there is little or no glacier or ice cap left to melt, polar ice sheet ice will become an increasingly larger player in raising sea-level.

Preceded by conceptual and mainly laboratory-based insights into the flow of glacier ice, the first systematic observations of glacier flow were made in the first half of the nineteenth century by Louis Agassiz in the European Alps. Mountain glaciers in mid-latitudes were much more the object of glacial research throughout the twentieth century until satellites were introduced to enable remote sensing of the less accessible polar ice sheets. In 1972, the Landsat satellite brought back imagery of Antarctica, and in 1975, GEOS-C was the first satellite to provide elevation data with altimetry, covering the southern tip of Greenland at 65 degrees N. Since then, numerous satellites have been flown to provide us with data regarding ice elevations and ice flow velocity through radar and laser altimetry. Such data have been complemented by airborne altimetry and ground-based radar echo sounding to gain information on ice elevations and ice thicknesses.

One area that extensive satellite-based measurements have played an enormous role in developing is of ice sheet-scale mass balance studies.

The first alarm bell to urge communities to actively engage in studies – both monitoring and modeling – was rung in 1978 when J.H. Mercer postulated the possibility of a rapid disintegration of the marine-based, unstable West Antarctic Ice Sheet, in response to man-made climate warming. In contrast to the previous proponents of the issue, who predicted a much slower reaction of the polar ice sheets to climate warming, Mercer laid eyes on the vulnerabilities of the ice shelves of West Antarctica that buttress much of the ice sheet’s ice – much of which is grounded below sea-level – to both warming of sea and atmosphere. He warned that the breakup of the smaller ice shelves rimming the Antarctica Peninsula would be a signal that larger, more significant Ross and Ronne-Filchner Ice Shelves further south would be the next to disintegrate. The disintegration of these ice shelves would result in the rapid outflow of the inland ice that had been being buttressed.

This paper was ensued by studies that shine light on the possibility and likelihood of catastrophic collapse of the inherently unstable West Antarctic ice sheet. In 1981, Hughes brought forth the scenario whereby Pine Island and Thwaites Glaciers, which flow into Pine Island Bay in the Amundsen Sea, are not buttressed by pinned ice shelves and yet drain large

parts of the WAIS interior and hence play the major role in the collapse of the WAIS as its “weak underbelly.”

In more recent years, Bamber and colleagues computed balance velocities across the entire Antarctic ice sheet using primarily three datasets: those of surface elevation, ice thickness, and surface mass balance. The surface elevation dataset was compiled from radar altimetry data taken from the ERS-1 satellite and was augmented with data taken from the ground. The mean surface mass balance dataset was a compilation of both passive microwave satellite and in situ measurements. This amalgam of extensive data allowed us to see that ice flow in the Antarctic ice sheet was not simply a two-piece puzzle of slow-flowing sheet-flow in the interior Antarctic plateau and fast-flowing stream-flow near the coastal margins but rather a complex situation where flow-speeds exhibiting intermediate speeds penetrate considerably inland in some places more than others and that ice flow in the continent should be viewed as occupying someplace along the gradation rather than solely one end or the other of this continuum of flow states.

Thomas et al. showed how the contrasting strengths of satellite altimetry (extensive coverage, lower resolution) and airborne altimetry (limited coverage, high resolution) can together give us useful insights into the present and future of ice sheet change. Flight lines flown near the coastal region of the six glaciers flowing in the Amundsen Sea captured the picture that thinning in that area (averaging 1.0m/yr) accounted for roughly two thirds of the mass loss from the entire catchment area, 15% of which is accounted for by the surveyed area.

In 2006, Rignot and Kanagaratnam presented three sets of data from satellites and consequently computed the dynamic factor of Greenland ice mass loss during the past decade. They showed that dynamics account for two thirds of the mass loss with the other third being due to enhanced runoff minus accumulation. This was an update from a previous study that reported a fifty-fifty partitioning between dynamic and climatic contributions to ice mass loss, a conclusion made from numerous airborne laser altimetry measurements that surveyed a large portion but not the entire extent of the Greenland Ice Sheet.

Another important contribution made from satellites has been the finding of regional contrasts in surface elevation change in Antarctica. Between 1992 and 2003, the East Antarctic interior showed a rather spatially homogeneous thickening, likely due to snowfall increase, of ~1.8cm/yr. Meanwhile, West Antarctica showed a more complex picture, with regions of strong thickening (~16cm/yr) and regions of rapid thinning (~15cm/yr) existing within 1200km of each other.

Satellite gravity surveys have made measurements to produce independent assessments of the state of polar ice mass change. Greenland lost 248 km³/yr of ice between 2002 and 2006 (ref. 26). The Antarctic ice sheet lost 152 km³/yr of ice between 2002 and 2005 (ref. 27).

In spite of all the observational evidence of the nature described above, the largest international body of authority regarding climate change has not been able to produce the most convincing forecasts of sea-level rise for the rest of this century. The IPCC, in the last report (AR4), predicted a 0.18-0.59 m rise in sea-level by 2090-2099 compared to 1980-1999. But

23 Thomas et al., 2004, *Accelerated Sea-Level Rise from West Antarctica*.
25 Davis et al., 2005, *Snowfall-Driven Growth in East Antarctic Ice Sheet Mitigates Recent Sea-Level Rise*.
many believe this is an underestimation because it has not taken into account some dynamical contributions from ice sheets. The models used to produce these predictions account for thermal expansion, changes in non-polar glaciers and ice caps, and changes in ice sheets due to surface mass balance and some ice dynamics. Some of the dynamics accounted for include the effect of ice thickening towards the interior of the Greenland ice sheet, which steepens the ice surface slope, allowing for more ice flux into the ablation zone\textsuperscript{28}. This leads to higher surface elevations and hence less melting in the ablation zone, which contributes to a dampening role of dynamics to ice sheet mass loss. A similar effect is seen in Antarctica, where the increased transfer of ice is across the grounding line instead of the equilibrium line; hence the increasing ice sheet-wide mass of Antarctica is damped by the dynamical role. In both cases, the mechanisms incorporated in the ice sheet models do not explain the enhanced loss of ice due to changes in dynamics such as has been shown with observations in Greenland by Rignot and Kanagaratnam\textsuperscript{29}.

\footnotesize
\begin{itemize}
  \item \textsuperscript{28} Huybrechts and De Wolde, 1999, \textit{The Dynamic Response of the Greenland and Antarctic Ice Sheets to Multiple-Century Climatic Warming}.
  \item \textsuperscript{29} Rignot and Kanagaratnam, 2006.
\end{itemize}

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II. Overview of Project

As sea-level rise from melting polar ice seems to be unignorable in the current century and more so in the coming centuries, we focus our energy on understanding it. Rather than attempt to make pinpoint forecasts of how much polar ice might contribute to global sea-level at which year, we, using the data currently made widely available (either on the Web (e.g., ice sheet surface topography), or in the literature (i.e., force balance studies)), try to open up a different perspective to viewing how parts of the polar ice sheets might respond to climate change-induced perturbations. This is done in the hope that we can aid to narrow down which parts of the ice sheets need deeper scrutiny, leading to both more theoretical considerations and data collection in those areas. Ultimately, this would lead to more precise pinpoint predictions of the nature described above.

Our first principle to accomplishing this task is a “break-it-down and add-it-up” approach. We focus on what are known to be the key sections of the ice sheets in terms of affecting sea-level rise: the fast-flowing channels of ice near the coasts. We align flowlines, along which we base our mass balance calculations, through the center of ten of these fast-flowing features in Greenland and Antarctica. The mass balance calculations yield profiles of surface elevation along the flowlines. Any mass changes, detected from these individual flowline profiles, can then be integrated to get an idea of ice sheet-wide mass change.

The second pillar is pinning down the situation at the base of each flowline regarding slipperiness, which allows us to calculate the movement of ice due to basal slip.

We try to test the significance of the specificities of the slipperiness profiles (i.e., values of slipperiness plotted against distance for each ice stream), by seeing how the ice stream responds to perturbations that are caused by climate warming. These perturbations are: (i) increase in snowfall, or surface mass balance, (ii) retreat of the grounding line, and (iii) expansion of the ice stream width. Some features we look at are: an ice stream with a low surface slope, an ice stream flowline with a short or long stretch of streaming flow; an ice stream that finds its downstream end filling an overdeepening of the bed topography; an ice stream that flows through a well-defined topographic trough with rock walls confining it on either side.

The overall structure of the dissertation is as follows: the first chapter presents the ice stream flowline model we developed and the numerical experiments that are performed with it. This core chapter is followed by a chapter with materials that support the core chapter. The dissertation ends with a summary of the project and some closing words.
Acknowledgements

I am deeply grateful to Dr. Johnny Sanders for enabling me to get first-hand experience of glaciological fieldwork by taking me as an assistant on his field campaign to the Canadian Rockies: West Washmawapta Glacier. Dr. Nicole Schlegel, another former lab-mate, guided my introduction to managing large datasets and applying those data to usage in MATLAB modeling. The entire technical and administrative staff crew at the UC Berkeley Geography Department enabled me to carry out my daily duties at the Department with ease. I would also like to thank the University of California, Berkeley for support.

Professor John Chiang provided crucial constructive comments; I was thereby able to improve the section in the Introduction of this dissertation on the state of knowledge about ice sheet mass balance. Professor Chiang furthermore suggested I do the controlled experiments in Chapter 2 so as to make the conclusions drawn from the perturbation experiments in Chapter 1 more understandable to the reader.

Finally, Professor Kurt Cuffey gave me exquisite guidance throughout the six years I have been a graduate student at UC Berkeley. Only he could have given me the carefully thought out guidance at every twist and turn, to bring me where I am right now. I thank him greatly.
1. A New Model to Construct Ice Stream Surface Elevation Profiles and Calculate Contributions to Sea-Level Rise
A New Model to Construct Ice Stream Surface Elevation Profiles and Calculate Contributions to Sea-Level Rise

Abstract
To contribute to the communal effort to better specify the contribution from ice sheets to sea-level rise, we devise a new flowline model that resolves the fast-flowing features within the ice sheets and specializes in constructing surface elevation profiles to calculate ice volume changes due to climate change-induced perturbations. There are three types of motions – internal deformation produced by both basal shear stress and lateral shear stress, and basal slip – that are taken into consideration in this model, which allows us to extract the specificities of the ice streams in Antarctica and Greenland. The model allows us to constrain the magnitude of basal slipperiness along an ice stream flowline as long as datasets of surface and bed elevation, surface mass balance, and ice stream width are available. We choose the flowlines of Pine Island Glacier, Bindschadler Ice Stream, Whillans Ice Stream, Shirase Glacier, Mellor Glacier, Lambert Glacier, Jutulstraumen, Recovery Ice Stream, Jakobshavn Isbrae, and North-East Greenland Ice Stream, as these seem to encompass a wide range of behavioral traits observed in the large polar ice streams today. At this point, we are able to apply the perturbations – ice stream widening, surface mass balance increase, and grounding line retreat – to examine what kind of characteristics are vulnerable to what kind of perturbation. We find that an ice stream that overlies an extensive overdeepening, like Pine Island Glacier, allows for a lot of ice loss due to grounding line retreat. An ice stream that has low slopes and whose grounding line does not currently sit on the downstream end of an overdeepening of the bed topography, like Whillans Ice stream, seems to be less vulnerable to grounding line retreat. An ice stream like Recovery that has a long stretch of streaming-flow and a long flowline is prone to losing much ice, because features such as its abundant potential for widening, due to lack of lateral constraint, are magnified. Conversely, an ice stream such as Jutulstraumen with rock walls limiting its sideward expansion and a short stretch of streaming flow, in addition to an overdeepening that does not extend far inland, has little concern of losing much ice.

I. Introduction
Climate warming, and sea-level rise resulting from it, is likely to cause significant damage to the societal infrastructure, especially systems of food production and the built environment in coastal regions. Notwithstanding the predicted regional impacts worldwide – from ecological degradation in the San Francisco Bay Area (Stralberg et al., 2011) to diminishing of farmlands in South Asia (Parry, 1992, p.80; Inman, 2009) – there is much uncertainty about how this phenomenon will progress over time, in the decades and centuries ahead. The most recent IPCC assessment report (Meehl et al., 2007) forecast a 0.18 to 0.59m rise of sea-level by the end of this
century, due to thermal expansion, changes in non-polar glaciers and ice caps, and changes in ice sheets due to surface mass balance and its ordinary effects on ice dynamics. Taking into account additional losses from enhanced ice flow, Pfeffer et al. (2008) suggested a possibly much higher rise in sea-level – 0.8 to 2m – by 2100. Using more realistic constraints, Cuffey and Paterson (2010, p. 609) suggested a plausible range of 0.30 to 1.25m by century’s end. This is roughly equivalent to 25% to 45% of the area around Miami, Florida, becoming submerged underwater.\(^1\) Forecasts with such large ranges are probably not satisfying to geophysicists, but they are essential for delimiting scenarios that can be analyzed by planners, engineers, and policy makers.

The magnitude of potential sea-level rise is large. Of the three currently existing ice sheets, the East and West Antarctic Ice Sheets store 52 and five meters equivalent of sea-level, respectively, while the Greenland Ice Sheet stores seven (Cuffey and Paterson, 2010, p. 577).

The only tools that can, in principle, make a rigorous prediction of ice sheet contributions are three-dimensional ice sheet models. However, the ones currently available are inadequate in their representation of the actual situation and hence are not reliable. For example, Huybrechts and DeWolde (1999) predicted that surface mass balance changes will dominate over ice dynamic changes in Greenland, but this conclusion was negated less than a decade later by observations showing rapid losses of ice from accelerating ice streams (Rignot and Kanagaratnam, 2006). The need to better represent ice streams in the 3D models is recognized. However, this effort faces profound challenges that suggest it will be a very long time before any rigorous predictions can be made. The need to know more about (a) the processes that control subglacial water pressure, (b) the material (bedrock or sediment) underlying the ice sheets, (c) how (a) and (b), among other variables, affect change in basal slip over time, and (d) the basal melt rates of ice shelves in contact with warming ocean waters, are a few of such challenges (Cuffey and Paterson, 2010, p.603-604). Although some recent studies provide useful insight into these problems (e.g., Rignot and Jacobs (2002) for (d)), the most fundamental problems remain intractable.

Although rigorous predictions are not currently possible, we can still gain insight into how these systems behave. Such insights are essential for designing the range of scenarios that society must be prepared for, or seek to avoid through changes of the energy system. Our study looks specifically at the behavior of ice streams, which is the largest challenge for 3D models. Ice streams are diverse, ranging from one such as Whillans Ice Stream which is wide, is characterized by low driving stresses, and flows at around 500m/yr, to one such as Jakobshavn Isbrae which is narrow, has high driving stresses, and flows at 2300m/yr. Another unique one is Pine Island Glacier which is fast-flowing (2700m/yr), relatively short and has a very wide drainage basin; the configuration of its streaming-flow relative to its drainage basin contrasts to the case with Jutulstraumen, which has a short stretch of streaming-flow but whose flowline upstream of the streaming-flow continues many hundreds of kilometers more inland.

For our analysis we developed a unified framework (a form of flowline model) that can accommodate all the different types of ice streams, in a rough fashion. We use our framework to examine the sensitivity of ice volume to imposed changes in grounding line position, surface mass balance, and ice stream width. Our model development was guided by the following points of emphasis: (I) focus on the ice stream flowline so that the fast-flowing features, which wield a

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\(^1\) This is based on NOAA’s “Sea Level Rise and Coastal Flooding Impacts Viewer” (csc.noaa.gov/digitalcoast/tools/slrviewer). The areal extent considered here is the coastal area of Miami-Dade County where data is available, and the comparison is made between a simulated 1-ft rise versus a 4-ft rise in sea-level.
lot of influence on the entire mass balance, can be resolved, (II) account for spatial variations in the slipperiness of the ice stream beds, and (III) include the role that ice stream width, hence lateral drag, plays in determining velocity. A primary limitation of our method is that it applies only to steady states; although transient behavior can be incorporated in future work, it is not the focus here.

II. The Model

1. Relevant Background Theory

In a steady state, a column of ice within a flowing glacier maintains constant thickness due to the following balance:

\[ \dot{b} = \frac{dQ}{dx} = \frac{d(uH)}{dx}, \]

where, \( \dot{b} \) is specific mass balance, \( Q \) is flux, \( u \) is depth-averaged ice flow velocity, \( H \) is ice thickness, and \( x \) is distance in the direction of ice flow. This equation means that any loss/gain of ice in the vertical direction, due to melting and precipitation, is compensated by an excess of flow into/out of the column in the direction that the stream is flowing. In this study, we have equated mass balance to surface mass balance because, in the systems we model (polar ice streams inland of grounding lines), the surface term far exceeds englacial and basal ones. Transverse compression and extension has been omitted, as its effect was judged to be negligible in most cases (see Chapter 2, Section 2D).

Ice velocity is driven by gravitational forces whose magnitude, per unit horizontal area, equal the driving stress, \( \tau_d \):

\[ \tau_d = \rho g H \alpha, \]

where \( \rho \) is ice density (assumed to be 917\,kg/m\(^3\)), \( g \) is gravitational acceleration, and \( \alpha \) is ice surface slope. In our model, we assume that the driving stress is balanced by a combination of basal shear stress, \( \tau_b \) and wall drag, \( \tau_w \):

\[ \tau_d = \tau_b + \tau_w. \] (2)

Thus we assume that the contribution of longitudinal stress gradients to the balance of forces can be neglected. This is probably a good assumption for ice streams of all sorts (see Table 8.3 of Cuffey and Paterson (2010)), though not for ice shelves. The two stresses that balance the driving stress in eq.(2) are both associated with deformation in the ice stream as it shears against its boundaries.

It is convenient to define a number, \( f \), as the proportion of the driving stress supported by the base:

\[ f = \frac{\tau_b}{\tau_d} \]

2. (i) Motion in the Vertical Plane

In accordance with conventional description, the depth-averaged velocity due to internal deformation in the vertical plane is:

\[ u_{v,d} = KH\tau_b^3 \]

where \( A \) is ice softness, and the exponent in the ice flow law commonly known as \( n \) is set to 3. We parameterize the velocity from basal slip as:
\[ u_{v,b} = \gamma \tau_b^3, \] (4)

We call \( \gamma \) [m s\(^{-1}\) Pa\(^{-3}\)] slipperiness, and we envision it as embodying both hydrological and geological properties at the bed that reduce the basal drag associated with a given rate of basal slip.

\( \gamma \) takes whatever value is necessary to make the equation true; in other words, eq.(4) is a definition of \( \gamma \).

Motivated by observations that ice streams, or sections of ice streams, with low basal shear stress-to-driving stress-ratio (i.e., \( f \) values) tend to have high basal sliding velocities, we develop this component of the model by searching for some systematic relationship between \( f \) and \( \gamma \). Previous authors gathered measurements of ice stream width, ice thickness, and flow velocity of sections (or “boxes”) for a number of ice streams. The ice streams we consider here are: Whillans, Kamb, Bindschadler, MacAyeal, Recovery, Rutford, and North-East Greenland Ice Streams, Pine Island and Shirase Glaciers, and Jutulstraumen. They then used these data to perform force balance analyses that yield \( f \) values for those boxes.

Next, we outline how we calculated the corresponding \( \gamma \) values for these boxes. We begin by dividing eq.(4) by eq.(3). Rearranging the result gives:

\[ \gamma = KH \left( \frac{u_{v,b}}{u_{v,d}} \right). \] (5)

Using a representative value of ice thickness for each box, the basal drag value calculated from the force balance study, and a uniform ice softness value of \( 1.6 \times 10^{-24} \text{s}^{-1} \text{Pa}^{-3} \), we calculate deformational velocity from eq.(3). Then, we subtract the deformational velocity from observed surface velocity to determine basal velocity. Plugging in the deformational velocity and basal velocity values, along with ice thickness and softness into eq.(5) yields a representative \( \gamma \) value for the box. We repeat this procedure for all the boxes.

The value for \( f \) depends in part on cross-sectional geometry of an ice stream. Call this geometrical effect \( f_1 \). Then we define a parameter \( f_2 \) representing the effects of hydrological and geological properties of the bed, according to:

\[ f = f_1 f_2. \] (6)

Hence, \( f_2 \) is the effect of slipperiness on the ability of the base to support driving stress.

We calculate \( f_1 \) for the boxes involved using the analysis by Nye (1965). \( f_1 \) is a function of the ratio of an ice stream’s half-width (\( Y \)) to center thickness (\( H \)). Table 1 shows the values. For each box, we calculated \( f_2 \) using eq.(6).

Table 1. Values of \( f_1 \) as a function of ice stream half-width to center thickness ratio, in the case that the ice stream cross section is a rectangle (Source: Nye, 1965, Table IV).

<table>
<thead>
<tr>
<th>( Y/H )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>( \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1 )</td>
<td>0</td>
<td>0.558</td>
<td>0.789</td>
<td>0.884</td>
<td>1</td>
</tr>
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Table 2 includes the necessary numbers leading to the computation of \( \gamma \). We plotted \( f_2 \) against the logarithm of \( \gamma \) in Figure 1 and find that indeed there is a general tendency for places with little support from the base due to hydrological and/or geological causes (i.e., low \( f_2 \)) to be experiencing high proportion of slip (i.e., high \( \gamma \)). The values for Whillans Ice Stream typify this kind of regime. At the other end are Shirase Glacier and Jutulstraumen, two glaciers with high driving stresses, mostly supported by the base. These high driving stresses contribute to a large deformational velocity, and the basal velocity in comparison is small. We believe that once subglacial water and/or deformable sediment enter into these systems in large amounts, the
controlling factor in eq. (4) becomes $\gamma$ rather than $\tau_b$. With a large $\gamma$, less of the driving stress is supported by the base (and typically the driving stress decreases as well).

2. (ii) Motion in the Horizontal Plane

In the limiting case of zero basal drag, internal deformation in the horizontal plane depends on gravitational force and ice stream width according to:

$$u_h = K \left( \frac{\tau_w Y}{H} \right)^3 Y,$$

where $\tau_w$, the lateral drag can be expressed as:

$$\tau_w = (1 - f) \rho g H \alpha$$

See Cuffey and Paterson (2010, p.339) for a detailed derivation of eq.(7).

2. (iii) Putting the three types of motion together

To determine the value $u$ in eq.(1) requires identifying the relative importance of the three components of possible ice motion at a given point along an ice stream flowline. While the factor $\gamma$ determines the partitioning between basal slip and vertical shearing, we need to introduce a new parameter (call it $\varphi$) to specify where the motion lies on the spectrum between pure control by basal drag and pure control by lateral drag. We adopt the simplest mathematical relationship, a linear mixture:

$$u = \varphi (u_{v,d} + u_{v,b}) + (1 - \varphi) u_h,$$

We hypothesize that $\varphi$ should depend strongly on $f$. We determined the best $f$-$\varphi$ relationship by calculating values of $f$ based on some hypothetical $f$-$\varphi$ relationships and comparing them to the corresponding literature values. We hypothesized three relationships: one whereby $\varphi$ is 0% while $f$ is less than 50% and $\varphi$ is 100% when $f$ is more than 50% (a step function), another whereby the two variables are in a direct linear relationship, and a third which is intermediate between the previous two scenarios and can be expressed mathematically as:

$$\varphi = \frac{1 + erf(f - 0.5) \times 6}{2}.$$

We picked six ice streams (Pine Island Glacier, Whillans Ice Stream, Bindschadler Ice Stream, Shirase Glacier, Jutulstraumen, and Recovery Ice Stream), for which $f$ values from previously published force balance studies by other authors are available for segments (or “boxes”) of the ice stream. We drew flowlines that extend from grounding line to ice divide and pass through the boxes used in the force balance analyses, for these six ice streams. Calibrating an $f$ value for each grid cell along such flowline – for each of the three hypothetical $f$-$\varphi$ relationships – in order to reproduce the current surface elevation profile allows us to compare our $f$ values to literature $f$ values where our flowline overlaps with the boxes in the aforementioned force balance analyses (see Chapter 2, Figures A1-A6). Based on these comparisons, we found that each scenario worked well with different ice streams and that there was no one scenario that was overwhelmingly favored over the others. Hence we settled on employing the intermediate scenario – expressed mathematically as in eq.(9) and graphically as in Figure 2 – as the most appropriate $f$-$\varphi$ relationship to use for all further calculations.
3. Slipperiness Profiles

In preparation for the perturbation experiments we conduct in the following section, we selected ten polar ice streams from all three ice sheets to examine: Pine Island Glacier, Bindschadler Ice Stream, Whillans Ice Stream, Shirase Glacier, Mellor Glacier, Lambert Glacier, Jutulstrauden, Recovery Ice Stream, Jakobshavn Isbrae, and North-East Greenland Ice Stream. These ice streams represent a comprehensive spectrum of the behavior currently seen in ice streams. The areal and volumetric capacities of these ice streams are large too, indicating their potential effect on global sea-level. For each ice stream, we delineated a flowline starting from the grounding line, following the center of the ice stream, and ending at the ice divide of the drainage basin.

Using eqs.(1), (3), (4), (7), (8), (9), and the logarithmic relationship shown in Figure 1, we optimized $f_2$ and $\gamma$ simultaneously by matching calculated surface elevation to observed current surface elevation for each grid cell along each flowline. The ranges of $f_1$ and $f_2$, and hence $f$ are from 0 to 1, and $f_2$ is tuned to the nearest one hundredth. This process required datasets of bed elevation, surface mass balance, and ice stream width (measured from ice velocity maps). We plotted $\gamma$ against distance from grounding line, and we show such slipperiness profile for all ten ice stream flowlines in Figure 3.

III. Perturbation Experiments

To illustrate possible applications of our model, we imagine what types of changes would be conceivable in the next few centuries, with the climate warming 5 degrees Celsius. There are three changes we consider pertinent to ice stream mass balance for such a scenario. (1) One is a climatic effect: that of increasing precipitation in Antarctica. Under conditions of temperatures well below freezing currently seen in Antarctica, a warming is expected to predominantly result in increased snowfall. (2) A second change is grounding line retreat. Warming ocean waters are expected to melt away the basal ice of ice shelves near the grounding line of many polar ice streams. A thinning of the ice near the grounding line leads to local acceleration of the ice, which leads to further thinning. Such thinning will result in ungrounding of the ice, a process that will propagate upstream, especially in cases where the bed topography has a negative slope. It is hard to determine where exactly the grounding line will retreat to, but a study shows that a grounding line that retreats from the downstream end of an overdeepening will not obtain a steady state until it crosses the overdeepening and the bedslope is down-glacier (Schoof, 2007). (3) A third change is widening of the ice streams. Ice streams currently flowing through a trough and filling the width of it probably do not have much potential to expand laterally. On the other hand, an ice stream that is flowing through a valley-like structure but is not taking up the full width of it, or an ice stream that is not constrained by rock structure on its sides is susceptible to widening in the coming decades as flow accelerates and grounding lines retreat.

To examine such potential changes, we applied the following perturbations: (1) Surface mass balance increase: Meehl et al. (2007) present a range of 5.5-9.0% increase in Antarctic snowfall per degree Celsius rise in temperature, so we implement a 50% increase in surface mass balance uniformly around Antarctica as an upper-end estimate. We do not apply this perturbation to the two Greenland ice streams, as here warming climate will increase ablation at low elevations while having uncertain effects on snowfall at mid-range elevations.
(2) Grounding line retreat: In the eight cases (all except Whillans and Bindschadler Ice Streams) where the current grounding line is at the downstream end of, or in the middle of, an overdeepening of the bed topography, we prescribe a new grounding line position that is landward of the bottom of the overdeepening and the bedslope is sufficiently large and facing down-glacier. For Whillans and Bindschadler Ice Streams, the grounding line is prescribed to retreat a distance worth 6% of the current flowline length, to illustrate the sensitivity.

(3) Widening: Based on observations of bed topographic cross sections at 2-7 locations for each ice stream (these locations were sampled at intervals of a few tens of kilometers), we determined first whether the current ice stream is flowing through a topographic trough. If it is, we designate as the potential for widening the space between the current lateral boundary of the streaming flow and the rock wall, if any. If there is no topographic constraint, we investigate the sensitivity by assigning a widening potential of 50% of the current ice stream width for each side lacking a constraint. See Figures B1-B9 (Chapter 2) for these cross-sectional figures and widening potential evaluations.

When applying these perturbations, we assumed that bed topography is fixed and that the longitudinal extent of the stream-flow regime is fixed. Stream-flow status is assigned to areas where surface velocity is more than ~100m/yr. In areas of smaller surface velocity, ice flow is calculated as the sum of vertical shearing and basal slip (i.e., $\varphi = 1$ and $u_h = 0$ in eq.(8)).

Surface elevation profiles that are simulated as a result of the climate change-induced perturbations are shown in Figure 4. They are compared to the current configuration for each ice stream flowline. The change in the surface elevation profile is translated into numbers and tabulated in Table 3. It is expressed in three different parameters for each ice stream flowline. See caption for details.

Pine Island – Pine Island Glacier has the shortest flowline length (350km) of the ten we studied, but a large extent of it lies upon a topographic overdeepening, which, according to the model suggested by Schoof (2007), leads us to apply a grounding line retreat for over half the flowline length. This perturbation alone deprives Pine Island Glacier of 80% of its originally stored ice that is contributable to sea-level rise. However, it is not easy to know where a plausible new grounding line position may be. It could retreat a further 100km upstream, or even further, initiating the entire disintegration of the West Antarctic ice sheet as first speculated by Hughes (1981). Pine Island Glacier also has high $\gamma$ values for 30km upstream of the stream-flow/sheet-flow boundary, so there is no acute rise in surface elevations due to high surface slopes in the lower end of the sheet-flow regime.

Whillans – Whillans Ice Stream has notably the smallest ice mass loss due to grounding line retreat both in absolute value and as a fraction of its own volume stored. Part of this is because the current grounding line is not lying at the end of a notable overdeepening and there is no potential for a large-scale retreat like we have seen for PIG (we applied a 6% grounding line retreat). However, we bring to the reader’s attention that the surface elevation difference at the new grounding line position before and after the retreat is a mere 25m, as opposed to 130m if the grounding line of PIG were forced to retreat for a distance 6% of its flowline length. This small drop in surface elevation is attributed to the low surface slope (~0.04%) for much of the downstream portion of Whillans Ice Stream. The post-retreat surface elevation profile continues to closely follow the pre-retreat profile from the grounding line position upwards until it is only 9m lower than the current elevation at the ice divide. The dependence of surface slope on
sensitivity to a grounding line perturbation is illustrated with controlled experiments, the results of which are shown in Section 2E of Chapter 2.

Shirase – This glacier is characterized by low \( \gamma \) values throughout the flowline, but especially upstream of the stream-flow/sheet-flow boundary. (For example, compared to the corresponding area on Recovery Ice Stream, the \( \gamma \) value is two orders of magnitude smaller.) This results in high surface slopes just upstream of the new grounding line position, and the surface elevation drop compared to the current elevation is recovered by 97.5% by the time the profile gets to the divide. (This compares to only 87.3% of drop being recovered at the divide in the case of Recovery.)

Mellor and Lambert – These two glaciers are distinct in their incapacity to expand sideways, due to rock walls closely lining the current breadth of much of the entire length of currently streaming flow. Hence, there is very little change in surface elevation profile due to widening alone that, in essence the difference between current profile and after applying the three perturbations is explained by the volume loss due to grounding line retreat being partly compensated by the surface mass balance increase.

Jutulstraumen – Jutulstraumen is similar to Mellor and Lambert in having little room to expand sideways in the topographic trough that it is currently lying in. It also has a shorter stream-flowing length (35km vs. 131km and 150km), making its potential for mass loss due to widening even less. Another difference from Mellor and Lambert is that the overdeepening in which the glacier’s downstream end lies extends less upstream, so there is less potential mass loss resulting from grounding line retreat. Hence, as a result of little mass loss from both widening and grounding line retreat, the mass gain from surface mass balance increase is the dominating factor in the three perturbations, and the overall volume change is an increase.

Recovery – Recovery Ice Stream has the longest flowline of the ten we studied, spanning 1,430km from grounding line to divide. This alone leads to a large loss of ice as volume changes are integrated over distance (see Experiment (d) of Chapter 2, Section 2E). As a fraction of the entire flowline length, this ice stream also has a long stretch of the profile that can be characterized as stream-flow (38.4%). The effect of a long stretch of streaming-flow on the mass balance of an ice stream in response to a perturbation is illustrated in Experiment (c) of Section 2E, of Chapter 2. Recovery Ice Stream also has some of the highest slipperiness values outside of West Antarctica.

Potential for lateral widening also contributes to this ice stream experiencing the greatest mass loss (Table 3). The stream-flow region of Recovery Ice Stream does not lie in a well-defined topographic trough. In cases where there is no visible lateral constraint within a reasonable range, we generously allow a widening of 50% of the current ice stream width on each side, resulting in a doubling of the current ice stream width if there is no constraint on either side. Such extensive widening, again integrated over the long length of Recovery’s stream-flow region, results in much mass loss.
IV. Conclusions

We have taken an approach drastically different from 3-D ice sheet models, with the ultimate goal of calculating contributions from polar ice sheets to sea-level rise in the coming decades and centuries in mind. The main goal of this study was to gain insight into how different parts of ice sheets respond differently to plausible climate change-induced perturbations. The flowline-based ice stream dynamics model we made is unique in two key areas: (i) we determined how to best distribute the motion of ice on the spectrum between complete control by basal drag and complete control by lateral drag (\(f-\varphi\) relationship; Figure 2), and (ii) we determined the ratio between vertical shearing and basal slip (\(f_2-\gamma\) relationship; Figure 1), which allows for a quantification of basal slipperiness for an entire flowline. Both of these components relied heavily on force balance analyses performed by previous authors to come to fruition. Using this model we constructed surface elevation profiles along an ice stream flowline and compared them to the current surface elevation profile to evaluate the ice stream’s response to perturbations.

The perturbation experiments show that grounding line retreat will not lead to much ice loss for an ice stream section with low slope, like we saw in the case of Whillans Ice Stream. In contrast, the case study for Pine Island Glacier shows that an ice stream overlying an overdeepening that extends over a large extent of its flowline will experience much ice mass loss. An ice stream basin filled by an ice stream with a long segment of streaming-flow is sensitive, leaving the potential for a lot of mass loss, as any type of perturbation along the streaming-flow region, such as widening, will be amplified as the effect is integrated along the entire stretch of the streaming-flow region. This, we saw, resulted in a lot of mass loss for Recovery Ice Stream. Contrary to much potential for widening in the case of Recovery Ice Stream, Mellor and Lambert Glaciers, which are currently flowing in tightly constrained topographic structures sideways, have little potential of mass loss from potential widening. Jutulstraumen demonstrates how the insensitivity of a short length of streaming-flow regime, results in even less mass loss due to potential widening than the two Amery Ice Shelf glaciers, even though it is in a similar environment where it has little room to expand on its sides.

The new model extracts the various features of the polar ice stream basins and portrays the picture that there will be heterogeneity within the ice sheets as to the response to ongoing and upcoming climate change. This reinforces the view that we need to look at spatial resolutions that resolve the individual ice streams that lie in the ice sheets, in order to make accurate and meaningful projections of contributions from ice sheets to future sea-level rise.
Figure 1. $f_2$ plotted against corresponding $\gamma$ for boxes used in previously published force balance analyses for ice streams in Greenland and Antarctica.
Figure 2. The $f$-$\phi$ relationship.
Figure 3. Slipperiness as a function of distance, normalized between the grounding line and ice divide.
Figure 4. (previous page) Elevation profiles of ice stream flowlines before and after climate change-induced perturbations.
Table 2. Values used to calculate $f_2$ and $\gamma$. $u_s$ is observed surface velocity, $u_d$ deforming velocity in the vertical plane, and $u_b$ is basal slip. The references for the ice stream data are as follows, and the naming of the boxes used follows the way it is done in the respective references: Whillans, Bindschadler, Kamb, and MacAyeal Ice Streams (Joughin et al., 2004); Pine Island Glacier (Payne et al., 2004); Shirase Glacier (Pattyn and Derauw, 2002); Recovery and Rutford Ice Streams (Joughin et al., 2006); North-East Greenland Ice Stream (Joughin et al., 2001); and Jutulstraumen (Rolstad et al., 2000).

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Table 3. Volume change, as a result of one or all applicable climate change-induced perturbations.

$\Delta v$ (km$^3$) = volume change per width; $\Delta v/v$ (%) = volume change as a percentage of current volume stored that is contributable to sea level rise; $\Delta v/L$ (m) = average change in the surface elevation along the flowline.

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<td>-16</td>
<td>-362</td>
<td>-5</td>
</tr>
<tr>
<td>PIG</td>
<td>-173</td>
<td>-80</td>
<td>-500</td>
<td>-43</td>
</tr>
<tr>
<td>Jutulstraumen</td>
<td>-62</td>
<td>-5</td>
<td>-98</td>
<td>-2</td>
</tr>
<tr>
<td>Bindschadler</td>
<td>-57</td>
<td>-10</td>
<td>-80</td>
<td>-89</td>
</tr>
<tr>
<td>Whillans</td>
<td>-15</td>
<td>-3</td>
<td>-19</td>
<td>-93</td>
</tr>
<tr>
<td>NEGIS</td>
<td>-431</td>
<td>-22</td>
<td>-461</td>
<td>-61</td>
</tr>
<tr>
<td>Jakobshavn</td>
<td>-189</td>
<td>-16</td>
<td>-365</td>
<td>0</td>
</tr>
</tbody>
</table>
2. Supplementary Material to “A New Model to Construct Ice Stream Surface Elevation Profiles and Calculate Contributions to Sea-Level Rise”
2A. Comparing calculated $f$ values to previously published values.

Figures A1-A6 show comparisons between $f$ values published in previous studies and $f$ values calculated based on the following relationships between $f$ and $\varphi$: a step function, a linear function, and one that is intermediate between the former two (see Chapter 1, eq.(9) for mathematical expression of this intermediate function). The naming of the boxes or segments along the ice stream at which the previously published $f$ values were determined follows the naming in the respective publications: Pine Island Glacier (Payne et al., 2004); Whillans and Bindschadler Ice Streams (Joughin et al., 2004); Shirase Glacier (Pattyn and Derauw, 2002); Jutulstraumen (Rolstad et al., 2000); and Recovery Ice Stream (Joughin et al., 2006).

For each figure, panel (a) shows the $f$ values that were calculated when smoothing was applied to the bed and surface elevation profiles of the ice stream flowline. Panel (b) shows the $f$ values that were calculated with less smoothing applied.
Figure A1. $f$ values: comparison with literature for Pine Island Glacier
Figure A2. $f$ values: comparison with literature for Bindschadler Ice Stream
Figure A3. $f$ values: comparison with literature for Whillans Ice Stream
Figure A4. $f$ values: comparison with literature for Shirase Glacier
Figure A5. \( f \) values: comparison with literature for Jutulstraumen
Figure A6. $f$ values: comparison with literature for Recovery Ice Stream
2B. Evaluation of widening potential

Cross sectional surface and bed elevation profiles are shown in Figures B1-B9 (surface elevation: blue line; bed elevation: green line). In each case, the lower-most cross section was taken at or near the grounding line (named “cs1”), and from there cross sections go upstream in increasing order (i.e., “cs2,” “cs3,” …) taken every few tens of kilometers. Where the bed elevation drops below sea-level at any point within the current extent of streaming flow, sea-level is marked with a bold light blue line for reference.

Current lateral extent of streaming flow is denoted at each cross section with two vertical black lines unless otherwise noted. Where widening was seen to be possible, the new lateral boundary position is marked with a downward-facing arrow. If there is no constraint for the new position, a sideward-facing arrow is marked in the direction of expansion. Cases where there was deemed to be no widening potential are marked with an “x” above the current margin.

The following summarizes the key for Figures B1-B9.

Key for Figures B1-B9:
Surface elevation:  
Bed elevation:  
Sea-level:  
Current lateral boundary of streaming flow:  
No potential for widening:  
New position after widening:  
No constraint for new position:  or  

Figure B1. Cross sections for Bindschadler Ice Stream: (a) cross section 1 at grounding line

(b) cross section at 350m surface-elevation-contour (cs2)

(c) cross section at 800m surface-elevation-contour (cs3)

(d) cross section at 1100m surface-elevation-contour (cs4)
Figure B2. Cross sections for Whillans Ice Stream: (a) cross section at grounding line (cs1)

(b) cross section at 150m surface-elevation contour (cs2)

(c) cross section at 300m surface-elevation contour (cs3)

(d) cross section at 1000m surface-elevation contour (cs4)
Figure B3. Cross sections for Pine Island Glacier: (a) cross section at grounding line (cs1) (b) cross section at 450m surface-elevation-contour (cs2) (c) cross section at 600m surface-elevation-contour (cs3) (d) cross section at 650m surface-elevation-contour (cs4) (e) cross section at 725m surface-elevation-contour (cs5)
(cont.) Figure B3: (f) cross section at 750m surface-elevation-contour (cs6)

(g) cross section at 800m surface-elevation-contour (cs7)
Figure B4. Cross sections for Shirase Glacier: (a) cross section at grounding line (cs1)

(b) cross section at 1200m surface-elevation-contour (cs2)

(c) cross section at 1600m surface-elevation-contour (cs3)
Figure B5. Cross sections for Mellor Glacier: (a) cross section at grounding line (cs1)

(b) cross section at 600m surface-elevation-contour (cs2)

(c) cross section at 1100m surface-elevation-contour (cs3)

(d) cross section at 1300m surface-elevation-contour (cs4)
Figure B6. Cross sections for Lambert Glacier: (a) cross section at grounding line (cs1), (b) cross section at 900m surface-elevation-contour (cs2), (c) cross section at 1200m surface-elevation-contour (cs3), (d) cross section at 1500m surface-elevation-contour (cs4).
Figure B7. Cross sections for Jutulstraumen: (a) cross section at grounding line (cs1)

(b) cross section at 700m surface-elevation-contour (cs2)
Figure B8. Cross sections for Recovery Ice Stream:
(a) cross section at 200m surface-elevation-contour (cs1)
(b) cross section at 800m surface-elevation-contour (cs2)
(c) cross section at 1000m surface-elevation-contour (cs3)
(d) cross section at 1400m surface-elevation-contour (cs4)
(e) cross section at 1900m surface-elevation-contour (cs5)
Figure B9. Cross sections for North-East Greenland Ice Stream: (a) cross section at grounding line (cs1)

(b) cross section at 1100m surface-elevation-contour (cs2)

(c) cross section at 1700m surface-elevation-contour (cs3)

(d) cross section at 2300m surface-elevation-contour (cs4)

(e) cross section at 2700m surface-elevation-contour (cs5)
2C. Adjustment of slipperiness values.

Basal slipperiness along a flowline will probably change as a result of any of the perturbations imposed (ice stream widening, grounding line retreat, and surface mass balance increase). Therefore, we made some plausible adjustments to the slipperiness profile after applying any of the perturbations. Here, we outline this procedure, taking the grounding line retreat experiment on Pine Island Glacier as an example.

We assume that $\gamma$ can be expressed as:

$$\gamma = \frac{k}{N}, \quad (C1)$$

where $k$ is a parameter describing the geological properties and $N$ is the effective pressure, both defined for each grid cell along the ice stream flowline. Because $N = P_i - P_w$, where $P_i$ is ice overburden pressure and $P_w$ subglacial water pressure, we calculate $N$ along the flowline by assigning a plausible $P_w$ value for each grid cell. First, we define parameter:

$$f_w = \frac{P_w}{P_i}, \quad (C2)$$

Then, we assign a function for $f_w$ against distance, based on the assumption that $P_w = P_i$ at grounding line and that the ratio of $P_w$ to $P_i$ should not become much less than one over the entire length of the flowline. Hence, a mathematical expression for such a relationship is:

$$f_w = 1 - \frac{cx}{(x+d)} \quad (c,d: \text{const.})$$

Figure C1 shows the $f_w$ profile for Pine Island Glacier. $c = 0.02$, and $d = 1$ are used for this and all other flowlines.

We recalculate the $f_w$ profile after the perturbation is applied. First, we plot $f_w$ against the thickness of ice above flotation level (before the perturbation is applied) for each grid cell (see Figure C2). We interpolate the $f_w$ values with respect to ice thickness above flotation and, we assign a new $f_w$ value to each grid cell based on the interpolated relationship and the newly calculated ice thickness above flotation. The new $f_w$ profile is shown in relation to the original $f_w$ profile in Figure C3.

Finally, assuming the geological properties ($k$) at each grid cell stays the same, we use equations (C1) and (C2) to calculate the slightly altered slipperiness value ($\gamma$) at each grid cell. Figure C4 shows the new slipperiness profile in relation to the original.
Figure C1. Original profile of $f_w$ from grounding line to divide for Pine Island Glacier
Figure C2. Relationship between ice thickness above flotation and $f_w$ for Pine Island Glacier.
Figure C3. Profiles of $f_w$ before and after adjustment due to grounding line retreat at Pine Island Glacier
Figure C4. Profiles of $\gamma$ before and after grounding line retreat at Pine Island Glacier
2D. Transverse compression.

In order to elucidate the significance of transverse compression, we begin by rewriting eq.(1) (Chapter 1) in two-dimensional form:

\[
\dot{b} = \frac{dQ}{dx} + \frac{dQ}{dy} = \frac{d(uH)}{dx} + \frac{d(vH)}{dy}
\]  

(D1)

where \(v\) is ice velocity in the direction transverse to the flowline. Using values representative of the width of the ice stream for \(\dot{b}\), \(u\), and \(H\), integrating eq.(D1) across the width of the ice stream yields:

\[
\dot{b}W = W \frac{d}{dx}(uH) + H(v_1 + v_2)
\]

(D2)

where \(v_1\) and \(v_2\) are velocities going into the column of ice from the two lateral boundaries. We evaluate the effect of including transverse compression in the model by integrating eq.(D2) from point \(x\) to the ice divide:

\[
\int_x^{\text{divide}} \dot{b}W \, dx = uHW + \int_x^{\text{divide}} H(v_1 + v_2) \, dx
\]  

(D3)

and using eq.(D3) to calculate ice flow velocity at each grid cell:

\[
u = \frac{1}{HW} \left\{ \int_x^{\text{divide}} \dot{b}W \, dx - \int_x^{\text{divide}} H(v_1 + v_2) \, dx \right\}
\]

(D4)

We applied this analysis on Mellor Glacier, which receives the influence of three major tributaries along its flowline, including Lambert Glacier and Fisher Glacier with which it converges near the grounding line. The \(\gamma\) values computed after accounting for these influences deviate slightly from those before, as shown in Figure D1. When volume change is computed in response to the three perturbations with and without consideration of transverse compression, there is no change in the values at the level of significant figures given in Table 3 of Chapter 1. We assume that there will be no significant change in the results given in this study by incorporating transverse compression in the other glaciers either.
Figure D1. $\gamma$ values with and without consideration of transverse compression
2E. Controlled experiments.

In order to clarify the underlying principles by which the numerical model works, we conducted experiments using controlled characteristics of the glacier. We group the experiments into five sets in order to answer the following questions:

a) How different are the sensitivities to perturbations with regards to mass loss between a glacier with small surface slope and large surface slope?

b) What happens when you widen an ice stream?

c) What happens if you lengthen the stream-flow segment?

d) How different are the sensitivities to perturbations with regards to mass loss between a glacier with a long flowline and a short flowline?

e) What determines the magnitude of surface slope upstream of the stream-flow/sheet-flow boundary?
a) **How different are the sensitivities to perturbations with regards to mass loss between a glacier with small surface slope and large surface slope?**

**Model Glaciers:**  
The two glaciers used in this set of experiments differ in their surface elevation profiles in the following way:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Glacier 1</strong></td>
<td>Steep near the grounding line; convex up</td>
<td></td>
</tr>
<tr>
<td><strong>Glacier 2</strong></td>
<td>Small slope near the grounding line; concave up</td>
<td></td>
</tr>
</tbody>
</table>

The two glaciers are given the same conditions for all other attributes:  
- bed elevation is uniformly -826m from grounding line to ice divide  
- surface balance is 100mm/yr (water equivalent) from grounding line to ice divide  
- flowline length from grounding line to ice divide is 800km  
- segment of stream-flow stretches 400km from the grounding line upstream  
- width of the stream-flow segment is uniformly 40km

**Perturbations:**  
(1) **Widening**  
The width of the stream-flow segment of each glacier is uniformly widened to 80km. The resulting surface elevation profiles for the two glaciers are shown in Figure E1, along with their initial surface profiles. Table E1 shows volume change in three ways.

**Table E1. Volume changes for Glaciers 1 and 2 in widening experiment**

<table>
<thead>
<tr>
<th></th>
<th>( \Delta v ) (km(^2))</th>
<th>( \Delta v/v ) (%)</th>
<th>( \Delta v/L ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glacier 1</td>
<td>-47</td>
<td>-3.2</td>
<td>-59.3</td>
</tr>
<tr>
<td>Glacier 2</td>
<td>-164</td>
<td>-24</td>
<td>-206</td>
</tr>
</tbody>
</table>

(2) **Grounding line retreat**  
The grounding line of each glacier was made to retreat by ~6% of the flowline length (i.e., 46km). The resulting surface elevation profiles for the two glaciers are shown in Figure E2, along with their initial surface profiles. Table E2 shows volume change in three ways.

**Table E2. Volume changes for Glaciers 1 and 2 in grounding line retreat experiment**

<table>
<thead>
<tr>
<th></th>
<th>( \Delta v ) (km(^2))</th>
<th>( \Delta v/v ) (%)</th>
<th>( \Delta v/L ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glacier 1</td>
<td>-396</td>
<td>-26.3</td>
<td>-494</td>
</tr>
<tr>
<td>Glacier 2</td>
<td>-41.4</td>
<td>-6.00</td>
<td>-51.8</td>
</tr>
</tbody>
</table>
b) What happens when you widen an ice stream?

Generally, widening of the ice stream will cause a speedup of the local ice flow velocity, which is expressed as:

\[ u = (\rho g \alpha)^3 \{ \varphi (fH)^3 (KH + \gamma) + (1 - \varphi)(1 - f)^3 KY^4 \} \]

Speaking in terms of the numerical model, the slope of the ice surface at a certain grid cell in the stream-flow segment of the flowline is computed in the following way:

\[ \alpha = \left[ \frac{Q_b}{(\rho g)^3 \{ \varphi (fH)^3 (KH + \gamma) + (1 - \varphi)(1 - f)^3 KY^4 \} HW} \right]^{1/3} \]

Hence, a speedup of local ice velocity causes a decrease in the ice surface slope. The local ice thickness decreases as a result of the modified ice surface. Changes such as this cause further modifications in the local slope, hence surface elevation profile, but the overall effect of widening an ice stream is that the surface elevation along the flowline is lowered.
c) What happens if you lengthen the stream-flow segment?

The sensitivity of glaciers to perturbations with respect to volume change was inspected from the perspective of the length of the glacier’s segment of stream-flow. The model glacier used in this set of experiments has the same physical attributes as Glacier 2 used in Experiment (a) except for the length of the stream-flow segment, which was varied between 200km, 400km, and 600km. The three perturbations were applied individually and all at once to this model glacier. Volume change results for this set of experiments are given in Table E3.

Table E3. Volume change of ice streams with varying stream-flow segment lengths

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Stream-flow segment length (km)</th>
<th>( \Delta V ) (km(^3))</th>
<th>( \Delta V/V ) (%)</th>
<th>( \Delta V/L ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Widening only</td>
<td>200</td>
<td>-88</td>
<td>-12.7</td>
<td>-110</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>-164</td>
<td>-23.8</td>
<td>-206</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>-181</td>
<td>-26.2</td>
<td>-227</td>
</tr>
<tr>
<td>Grounding line retreat only</td>
<td>200</td>
<td>-39</td>
<td>-5.6</td>
<td>-48.4</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>-41.4</td>
<td>-6.0</td>
<td>-51.8</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>-41.7</td>
<td>-6.0</td>
<td>-52.1</td>
</tr>
<tr>
<td>Surface balance increase only</td>
<td>200</td>
<td>66.0</td>
<td>9.6</td>
<td>82.5</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>68.3</td>
<td>9.9</td>
<td>85.4</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>68.5</td>
<td>9.9</td>
<td>85.6</td>
</tr>
<tr>
<td>All three perturbations</td>
<td>200</td>
<td>-49</td>
<td>-7.1</td>
<td>-61.3</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>-133</td>
<td>-19.3</td>
<td>-167</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>-151</td>
<td>-21.9</td>
<td>-189</td>
</tr>
</tbody>
</table>
d) How different are the sensitivities to perturbations with regards to mass loss between a glacier with a long flowline and a short flowline?

This set of experiments aims to elucidate the effect that flowline length has on volume change in response to perturbations. Perturbations are applied to two model glaciers, one of which is Glacier 2 (see Experiment (a)). The second glacier is a miniature version of Glacier 2, and it is called Glacier 3. The ratio of stream-flow segment length to flowline length is the same for Glaciers 2 and 3. Here are the major differences between Glaciers 2 and 3:

<table>
<thead>
<tr>
<th></th>
<th>Glacier 2</th>
<th>Glacier 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flowline length</td>
<td>800km</td>
<td>400km</td>
</tr>
<tr>
<td>Stream-flow segment length</td>
<td>400km</td>
<td>200km</td>
</tr>
</tbody>
</table>

The following are characteristics that the two glaciers share:
- bed elevation is uniformly -826m from grounding line to ice divide
- stream-flow segment width is uniformly 40km
- surface balance is uniformly 100mm/yr (water equivalent)
- surface slope is small near the grounding line and increases upstream

The elevation profiles of the two glaciers are shown on the same scale in Figure E3. The three perturbations were applied individually and all at once to these model glaciers. Volume change results are given in Table E4.

Table E4. Results of volume change of ice streams with different flowline lengths

<table>
<thead>
<tr>
<th>Experiment</th>
<th>parameter</th>
<th>Glacier 2</th>
<th>Glacier 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grounding line retreat</td>
<td>$\Delta v$ (km$^2$)</td>
<td>-41.4</td>
<td>-18.0</td>
</tr>
<tr>
<td></td>
<td>$\Delta v/L$ (%)</td>
<td>-6.0</td>
<td>-5.8</td>
</tr>
<tr>
<td></td>
<td>$\Delta v/L$ (m)</td>
<td>-51.8</td>
<td>-45.0</td>
</tr>
<tr>
<td>Widening</td>
<td>$\Delta v$ (km$^2$)</td>
<td>-164</td>
<td>-51.5</td>
</tr>
<tr>
<td></td>
<td>$\Delta v/L$ (%)</td>
<td>-23.8</td>
<td>-16.5</td>
</tr>
<tr>
<td></td>
<td>$\Delta v/L$ (m)</td>
<td>-206</td>
<td>-129</td>
</tr>
<tr>
<td>Surface balance increase</td>
<td>$\Delta v$ (km$^2$)</td>
<td>68.5</td>
<td>30.0</td>
</tr>
<tr>
<td></td>
<td>$\Delta v/L$ (%)</td>
<td>9.9</td>
<td>9.6</td>
</tr>
<tr>
<td></td>
<td>$\Delta v/L$ (m)</td>
<td>85.4</td>
<td>75.0</td>
</tr>
<tr>
<td>All 3 perturbations</td>
<td>$\Delta v$ (km$^2$)</td>
<td>-133</td>
<td>-37.5</td>
</tr>
<tr>
<td></td>
<td>$\Delta v/L$ (%)</td>
<td>-19.3</td>
<td>-12.0</td>
</tr>
<tr>
<td></td>
<td>$\Delta v/L$ (m)</td>
<td>-167</td>
<td>-93.8</td>
</tr>
</tbody>
</table>
e) What determines the magnitude of surface slope upstream of the stream-flow/sheet-flow boundary?

The primary factor that determines how much the slope increases abruptly when crossing the stream-flow/sheet-flow boundary upwards seems to be the contribution of horizontal shear to total velocity just downstream of the boundary. If it is significantly large compared to the two velocities in the vertical plane, then there is a large drop in the magnitude of the denominator in the equation to determine slope, when transitioning from stream-flow to sheet-flow. Hence, there is a large increase in the slope value.

The magnitude of $\varphi$ just downstream of the boundary is not a very meaningful yardstick to look at. If $\varphi$ is very small downstream of the boundary, one may think that the sudden increase in $\varphi$ (to 1) by transitioning across the boundary may compensate for the loss of the horizontal shear term, but a small $\varphi$ means a large $(1-\varphi)$, hence the horizontal shear might have been very large just downstream of the boundary. Hence, here again it is useful to look at the relative magnitude of the vertical term and the horizontal term just before the boundary to explain the magnitude of the jump in slope at the boundary.
Figure E1. (a) Glacier 1: Surface profiles, before and after widening

Figure E1. (b) Glacier 2: Surface profiles, before and after widening
Figure E2. (a) Glacier 1: Surface elevation profiles before and after grounding line retreat

Figure E2. (b) Glacier 2: Surface elevation profiles before and after grounding line retreat
Figure E3. Surface elevation profiles of Glaciers 2 and 3

- Blue line: surface elevation: Glacier 2
- Green line: surface elevation: Glacier 3
- Red line: bed elevation
Summary and Closing words

The advancement of satellite-based data acquisition in remote regions of the world and the concern for global warming and sea-level rise resulting from it, paved the way for this study. Previous authors have built 3-D ice sheet models to model Greenland and Antarctica and their responses to climate warming, but the performances of these models have been proved to be inadequate in light of recent observations and hence beckon for newer and independent perspectives for tackling this problem.

We built a model of ice stream dynamics, centered on a one-dimensional flowline. We used this model to investigate the sensitivity of ice sheets at the level of ice streams, to climate change-induced perturbations. We used bed and surface elevation, ice surface velocity and surface mass balance datasets to calibrate basal slipperiness along the flowlines of ten glaciers and ice streams in Antarctica and Greenland. Our simulation results showed that Pine Island Glacier, which flows through an extensive overdeepening, will lose 80% or more of its currently stored ice that is contributable to sea-level rise, through grounding line retreat. Recovery Ice Stream, which has much potential for widening and has a long stretch of streaming flow, will lose by far the most ice in response to the three imposed perturbations, among the ten ice streams we studied. Jakobshavn Isbrae, Lambert and Mellor Glaciers, and Jutulstraumen are constrained well on their sides by rock walls. Comparing Jutulstraumen to Mellor and Lambert Glaciers, Jutulstraumen has a shorter stretch of streaming flow and hence experiences less mass loss due to the widening perturbation than the latter two. We found that the perturbation experiments provided very different responses for the ten ice streams we studied, and hence they provided strong reason to model ice sheets at the ice stream level in order to accurately assess sea-level rise potential from ice sheets in the future.

Future work will have to zoom in even more on the locations of interest. An area of particular relevance is sticky spots, and how the supply of water and till beneath fast-flowing ice changes over time. Ultimately, we will need to know better the controls on grounding line position, which will necessitate examination from both upstream (glaciological and sedimentological factors) and downstream (thermodynamics and dynamics of ocean waters) of the grounding line. Filling in the gap of knowledge in these areas will undoubtedly lead to the ability of ice sheet models to better predict future contribution of ice sheets to sea-level rise.
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Siddiqi, S., 2011: *Bangladesh’s Communities Adapt, Innovate to Survive Climate Change.* (asiafoundation.org/in-asia/2011/04/20/bangladesh’s-communities-adapt-innovate-to-survive-climate-change/)


