Copyright © 1964, by the author(s).
All rights reserved.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission.
LOW TEMPERATURE PLASMA EXPERIMENT
FOR STUDY OF HIGH TEMPERATURE PLASMA INSTABILITIES

by

Charles K. Birdsall**
Jack A. Byers
Tao Yuan Chang

* The research reported herein is made possible through support received from the Departments of Army, Navy, and Air Force under Contract AF-AFOSR 139-66 and the Department of Air Force, Aeronautical Systems Division under Contract AF 33(615)-1078.


May 19, 1964
INTRODUCTION TO THE EXPERIMENT

Experiments on thermal plasmas with kilo electron volt energies and with densities like $10^{14}$ cm$^{-3}$ are very difficult and very expensive in time and equipment. Experiments at lower energies and densities on steady-state plasmas can be considerably less expensive. Fortunately, there is the possibility of performing experiments on low energy plasmas which simulate some of the behavior at high energies, especially with respect to instabilities, such as growth of flutes. Because of the importance of understanding and preventing instabilities, experiments at low energies have become very attractive. The purpose of this report is to show that there is strong analogy between the high and low energy regimes using a particular set of parameters.

The low temperature plasma will use positive ions of atoms with low ionization potential, such as the alkalis (Li, Na, K, Cs) or Ba, to be obtained from surface ionization on high work function metal or insulator plates. The electrons are to be obtained from thermionic emission, generally from the same emitter used for the ions. The plasma temperature will be about that of the plates, roughly 2000$^\circ$ K, or 0.2 eV. Thus the plasma source will consist of one or more hot plates, with the alkali atoms obtained from an oven (molecular beam mode) or from the background gas (vapor pressure mode). Such techniques are well known with extensive journal publication (see, for example, Rynn, 1964); these are not discussed here.

The plasma configurations of most interest to us at present are those in mirror fields. Thus, the experimental design and objectives are tied to mirror fields and the modification of mirror fields.
Sec. I presents some ideas on the choice of parameters for the low temperature plasma and comparison with a hot plasma. The key is to show in the ranges chosen that the collision effects do not dominate over the flute growth in mirror fields. In the model using two source plates (one at each $B_{\text{max}}$, "in" the mirrors) see Fig. 1, line tying, or shorting out of $E$ in the central region, due to conduction to the end plates, can be reduced by a large resistivity $\eta_{||}$; however, this is in conflict with good communication along field lines for flute growth of long wavelength along $B_0$. In the new model proposed, using a central source plate, see Fig. 2, line tying is not possible for $r > r_{\text{plate}}$, so that there is no line tying problem, no conflict with large $\lambda_{||}$ flute growth; furthermore, the spatial distribution of the plasma is much improved. This idea was suggested by T. Y. Chang.

Sec. II presents briefly some of the initial experiments proposed.

I. CHOICE OF PARAMETERS: COMPARISON WITH A HOT PLASMA

The purpose of this effort is to simulate a hot plasma with a cold plasma. It is most important to make the analogy as close as possible through careful choice of parameters. The analogy in this case means the equivalence of all important characteristic quantities of the plasma when the units of mass, length, and time are scaled according to a definite set of rules. The scaling is presented in Sec. A and B, to follow. Secs. C and D discuss line tying and the central plate source. Sec. E discusses finite gyroradius stabilization effects.

A. RANGES OF $n$, $T_i$, $e$, $B$, $m_+$ IN TERMS OF CHARACTERISTIC LENGTHS $D$ AND $r_g$

In Figs. 3 and 4 the fundamental plasma quantities are plotted as functions of experimental parameters. (The constants and formulas used follow these figures.) The regions for hot, thermonuclear plasmas and cold alkali-metal plasmas are indicated on both diagrams as regions I and II, respectively (somewhat arbitrarily chosen). Region I is for
Fig. 1. Two-source plate low temperature experiment.
Fig. 2. Central source plate low temperature experiment.
Fig. 3. Region I is a typical high temperature plasma with energies of 20 to 200 kiloelectron volts. Region II is the analogous low temperature plasma, with energies of 0.1 to 0.2 ev. $D$ is the Debye shielding length. $N_D$ is the number of particles in a Debye sphere. $f_p$ is the electron plasma frequency.
N, D, and \( f_p \) as functions of density, \( n \), and temperature, \( T \), for typical high temperature plasmas, Region I, and typical low plasmas, Region II.

Debye length

\[
D \equiv \frac{v_T}{\omega_p} = \left( \frac{kT}{4\pi ne^2} \right)^{1/2} = 6.90 \left( \frac{T}{n} \right)^{1/2} = 740\sqrt{\frac{V_T}{n}} \text{ cm.}
\]

\( V_T \) in electron-volts, using \( eV_T = kT \).

Number in a Debye sphere

\[
N_D \equiv n \frac{4}{3} \pi D^3.
\]

Plasma frequency

\[
f_p \equiv \frac{\omega_p}{2\pi} = \left( \frac{n e^2}{\pi m_e} \right)^{1/2} = 8.97 \times 10^3 \ n^{1/2}.
\]
Fig. 4. Gyroradii of single electrons and ions for various temperatures and magnetic fields. Also noted are ion and electron cyclotron frequencies as a function of field.
Gyroradii as a function of magnetic field for various ions at low and high temperatures; the ion cyclotron frequencies are also noted.

Cyclotron frequency

\[ f_{ce} = \frac{1}{2\pi} \frac{eB}{m_e c} = 2.78 \text{ Mc/ gauss} \]

\[ f_{ci} = \frac{1}{2\pi} \frac{eB}{m_i c} = 1.51 \text{ kc/ gauss for protons} \]

Masses

\( m_D = 2m \text{ proton} \quad f_{c_d} = 760 \text{ c/ gauss} \)

\( m_{Li} = 7m \text{ proton} \quad f_{c_{Le}} = 216 \text{ c/ gauss} \)

\( m_{Na} = 23m \text{ proton} \quad f_{c_{Na}} = 64 \text{ c/ gauss} \)

\( m_{K} = 39m \text{ proton} \quad f_{c_{K}} = 39 \text{ c/ gauss} \)

\( m_{Rb} = 85m \text{ proton} \quad f_{c_{Rb}} = 18 \text{ c/ gauss} \)

\( m_{Cs} = 133m \text{ proton} \quad f_{c_{Cs}} = 11.3 \text{ c/ gauss} \)

\( m_{Ba} = 137m \text{ proton} \quad f_{c_{Ba}} = 11 \text{ c/ gauss} \)

Gyroradius

\[ r_g = \frac{1}{\omega_c} \equiv \frac{\sqrt{2kT/m}}{eB_0/mc} \equiv \frac{\sqrt{2eV_T/m}}{eB_0/mc} = 3.37 \frac{\sqrt{V_T}}{B_0} \text{ cm for electrons} \]

\( r_{gi} = 3.37\sqrt{V_T/B_0} \sqrt{mi/me} \text{ cm} \)

\( r_{gLi} = 38\sqrt{V_T/B_0} \text{ cm} \)

\( r_{gCs} = 165\sqrt{V_T/B} \text{ cm} \).
20 to 200 keV deuterons with densities of $10^{12}$ to $10^{16}$ cm$^{-3}$, in 20 to 200 kilogauss fields; region II is for 0.1 to 0.2 eV ions in 200 to 2000 gauss fields. The choice of parameters can be made conveniently with these diagrams.

B. LENGTH-TIME SCALING FOR FLUTE GROWTH, DIFFUSION VELOCITIES, COLLISION RATES

We present here the parameters of a cold lithium plasma (0.2 eV) and its comparison with a hot deuterium plasma (20 keV). From Fig. 4 we see that the cyclotron frequencies compare as $f_{D^+} \approx 300 f_{Li^+}$. If we demand the same time scaling for plasma frequencies, then, from Fig. 3, we must choose $n_{Li} \approx 10^9$ cm$^{-3}$. We use this as an initial choice of $n$. Note that this choice gives the same Debye length, $D^* \approx 10^{-2}$ cm, for both plasmas; this is of minor concern as the plasma diameter in both models will generally be centimeters, many times $D$. Also, the number in a Debye sphere, $n_D^* = \frac{4}{3} \pi D^3$, is about $10^{10}$ (hot plasma) as against $10^4$ (cold plasma), interesting only as both should be large to qualify as a thermal plasma. On the other hand, the gyroradii, from Fig. 4, scale as $r_{D^+} \approx 4r_{Li^+}$, a scaling difference that will be taken into account. Note that $n \approx 10^9$ is a vacuum of $2 \times 10^{-8}$ torr; hence $n < 10^9$ requires an excellent vacuum, making for some difficulty.

For a cesium plasma, the scaling is enough different to warrant noting. Let region II still be defined by fields of 200 to 2000 gauss. Then $f_{D^+} \approx 6000 f_{Cs^+}$; however, to scale $f_c$ by this same ratio would require $n = 10^6$ cm$^{-3}$. As $n_o \approx 10^6$ cm$^{-3}$ is a pressure of $2 \times 10^{-11}$, this scaling is not attractive. In order to have density like $n = 10^9$, and keep $f_c$ and $f_p$ time scaling, it is necessary to move region II, for Cs to 2000 to 20,000 gauss, that is, ten times the field needed for Li.

For the plasma temperature, we assign a value of 0.2 eV, or $2,320^\circ$K, which is about the maximum practical temperature for the hot plates. For the magnetic field, we use 2,000 gauss. The diameter of the plasma will be a fraction of the inner diameter of the available
coils, which is nine inches. The shape and the length of the plasma depends on the experiment and will not be assigned at the stage. However, the radius of curvature \( R \), which is needed in the calculation of flute growth rate, is arbitrarily assumed to be 50 cm. All of the characteristic quantities of the plasma can now be calculated from these parameters using some reasonable assumptions concerning density gradients. These quantities are presented in the first column in Table I.

\[ N_D \] is the number of particles in a Debye sphere, \( 4/3 \pi n D^3 \).

\( \tau_{\text{flute}} \) is the (e-folding) growth time of a flute (Rosenbluth and Longmire, 1957).

\[
\tau_{\text{flute}} = \left[ \frac{R \kappa}{\left( v_{||}^2 + 1/2 v_{\perp}^2 \right)} \right]^{1/2} = \left[ \frac{m_1 R \kappa}{k T} \right]^{1/2} \text{ sec.} \quad (1)
\]

The reduced wavelength is \( \kappa = (\text{plasma radius})/m \); measured azimuthally, \( \kappa \) is arbitrarily assumed to be \( R/100 \), roughly an \( m = 5 \) mode on a 5 cm diameter plasma.

\( \nu_D \) is the classical diffusion velocity (Spitzer 1962).

\[
\nu_D = -D_i \frac{\nabla n}{n} = - \left( \frac{n k T}{B^2} \right) \frac{\nabla n}{n} = \frac{1.78 \times 10^{-3} Z \ln \Lambda}{B^2 T^{1/2}} \nabla n, \text{ cm/sec.} \quad (2)
\]

\( \nabla n \) is arbitrarily assumed to be equal to \( -(100n/R) \).

\( \nu_{D_{\text{t}}} \) is the Bohm diffusion velocity (Spitzer 1962)

\[
\nu_{D_{\text{t}}} = - \frac{5.4 \times 10^2 T}{B} \frac{\nabla n}{n} \text{ cm/sec} \quad (3)
\]

\( \nabla n/n \) is arbitrarily assumed to be \( -(100/R) \).

\( \nu_R \) is the curvature drift velocity (Spitzer 1962)

\[
\nu_R = \frac{v_{||}^2}{R \omega_c} = \frac{c k T}{e BR} \text{ cm/sec} \quad (4)
\]

\[
\left( \frac{k T}{e BR} \text{ m/sec} \right) *
\]

*Eqs. in ( ) are in MKS units.
<table>
<thead>
<tr>
<th>Quantity</th>
<th>Cold Li Plasma</th>
<th>Scaling Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass ( M_i )</td>
<td>7 amu</td>
<td>2 amu</td>
</tr>
<tr>
<td>density ( n )</td>
<td>( 10^9 \text{ cm}^{-3} )</td>
<td></td>
</tr>
<tr>
<td>energy ( kT )</td>
<td>0.2 eV</td>
<td>10^{14} \text{ cm}^{-3}</td>
</tr>
<tr>
<td>field ( B )</td>
<td>2 kG</td>
<td>20 keV</td>
</tr>
<tr>
<td>radius of curvature ( R )</td>
<td>50 cm</td>
<td>100 kG</td>
</tr>
<tr>
<td>ion gyroradius ( r_{ci} )</td>
<td>0.1 cm</td>
<td>100 cm</td>
</tr>
<tr>
<td>Debye length ( D )</td>
<td>( 6 \times 10^{-4} \text{ cm} )</td>
<td>0.3 cm</td>
</tr>
<tr>
<td>0.1 ( R/D )</td>
<td>500</td>
<td>5 \times 10^{-3} cm</td>
</tr>
<tr>
<td>( D/r_{ce} )</td>
<td>17</td>
<td>( 5 \times 10^{-2} \text{ cm} )</td>
</tr>
<tr>
<td>( D_{/Debye} )</td>
<td>( 5 \times 10^3 )</td>
<td>1 \times 10^{-3} cm</td>
</tr>
<tr>
<td>electron plasma freq. ( f_{pe} )</td>
<td>( 3 \times 10^8 \text{ sec}^{-1} )</td>
<td>9 \times 10^{11} \text{ cm}^{-1}</td>
</tr>
<tr>
<td>electron cyclotron freq. ( f_{ce} )</td>
<td>( 6 \times 10^9 \text{ sec}^{-1} )</td>
<td>1.2 \times 10^{-3} \text{ cm}</td>
</tr>
<tr>
<td>ion cyclotron freq. ( f_{ci} )</td>
<td>( 4 \times 10^5 \text{ sec}^{-1} )</td>
<td>2 \times 10^{10} \text{ cm}^{-1}</td>
</tr>
<tr>
<td>flute growth time ( \tau_{flute} )</td>
<td>( 3 \times 10^{-5} \text{ sec} )</td>
<td>2 \times 10^{10} \text{ cm}^{-1}</td>
</tr>
<tr>
<td>resistive diffusion velocity ( V_{diff} )</td>
<td>0.2 cm/sec</td>
<td>1.2 \times 10^{12} \text{ cm}^{-1}</td>
</tr>
<tr>
<td>Bohm diffusion velocity ( V_{Bohm} )</td>
<td>( 10^3 \text{ cm/sec} )</td>
<td>6 \times 10^{15} \text{ cm}^{-1}</td>
</tr>
<tr>
<td>curvature drift vel. ( V_j )</td>
<td>( 2 \times 10^{-2} \text{ cm/sec} )</td>
<td>2 \times 10^{15} \text{ cm}^{-1}</td>
</tr>
<tr>
<td>ion thermal vel. ( V_i )</td>
<td>( 2.4 \times 10^5 \text{ cm/sec} )</td>
<td>1.4 \times 10^{15} \text{ cm}^{-1}</td>
</tr>
<tr>
<td>Alfven vel. ( V_{A} )</td>
<td>( 5.2 \times 10^9 )</td>
<td>3 \times 10^{15} \text{ cm}^{-1}</td>
</tr>
<tr>
<td>dielectric constant ( K_{\perp} )</td>
<td>34</td>
<td>1.6 \times 10^{15} \text{ cm}^{-1}</td>
</tr>
<tr>
<td>( \ln \Lambda )</td>
<td>10.4</td>
<td>410</td>
</tr>
<tr>
<td>collision time ( \tau_{ei} )</td>
<td>( 5.8 \times 10^{-6} \text{ sec} )</td>
<td>10.4</td>
</tr>
<tr>
<td>( \tau_{ee} )</td>
<td>( 2.8 \times 10^{-6} \text{ sec} )</td>
<td>1.9 \times 10^{-6} \text{ T}</td>
</tr>
<tr>
<td>( \tau_{ii} )</td>
<td>( 3.2 \times 10^{-4} \text{ sec} )</td>
<td>9.3 \times 10^{-9} \text{ T}</td>
</tr>
<tr>
<td>( \tau_{in} )</td>
<td>( \sim 1 \text{ sec} )</td>
<td>1.1 \times 10^{-6} \text{ T}</td>
</tr>
<tr>
<td>relaxation time</td>
<td>( 5.1 \times 10^{-2} \text{ sec} )</td>
<td>( \sim 300 \text{ T} )</td>
</tr>
<tr>
<td>time constant ( \tau_{\perp} )</td>
<td>( 5.4 \times 10^{-14} \text{ sec} )</td>
<td>1.7 \times 10^{-4} \text{ T}</td>
</tr>
<tr>
<td>pressure ratio ( \beta )</td>
<td>( 2 \times 10^{-9} )</td>
<td>( \sim 100 \text{ sec} )</td>
</tr>
<tr>
<td>energy ratio ( eV_e/kT )</td>
<td>( 2 \times 10^{-3} )</td>
<td>2 \times 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>Hot D Plasma</td>
<td></td>
</tr>
</tbody>
</table>
\( v_i \) is the ion thermal velocity,

\( K_\perp \) is the low frequency transverse dielectric constant of the plasma,

\[
K_\perp = \frac{\epsilon_1}{\epsilon_0} = 1 + \frac{4\pi \rho_m c^2}{B^2} \approx 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2}
\]

\[
(= 1 + \frac{\rho_m}{\epsilon_0 B^2})
\]  

\( K_\perp \gg 1 \) is required in the hydromagnetic approximation; i.e., dropping the 1 is equivalent to ignoring displacement current.

\( \tau_{ei} \) is an equivalent electron-ion collision time defined by

\[
\tau_{ei} = \frac{m_e c^2}{\eta n e^2} \left( \frac{m_e}{\eta n e^2} \right) \text{ sec.}
\]

where \( \eta \) is the resistivity of the plasma, (Spitzer 1962)

\[
\eta = 6.53 \times 10^3 \frac{\ln \Lambda}{T^{3/2}} \text{ ohm-cm}
\]

\( \tau_{ee} \) is the electron self collision time.

\( \tau_{ii} \) is the ion self collision time (Spitzer 1962)

\[
\tau_{ii} = \frac{11.4 A^{1/2} T^{3/2}}{n Z^4 \ln \Lambda} \text{ sec.}
\]

\( \tau_{in} \) is the ion neutral collision time,

\[
\tau_{in} = \frac{1}{v_i n_o \sigma_{in}} \text{ sec.}
\]

\( n_o \) is assumed to be \( 10^9 \text{ cm}^{-3} \) for both cases; \( p_o \cong 2 \times 10^{-8} \text{ torr} \).
\( \epsilon_1/\sigma_1 \) is the transverse relaxation time of the plasma.

\( \sigma_1 \) is the transverse conductivity (Rose and Clark 1961)

\[
\sigma_1 = \frac{1}{\eta} \frac{\nu_1^2}{\nu_1^2 + \omega_{ce}^2}
\]

For collision frequency \( \nu_1 \), we use an equivalent value defined by

\[
\nu_1 = \frac{ne^2}{m_e c^2} \eta_1 = \left( \frac{ne^2}{m_e} \right) \sec^{-1}
\]

Since \( \omega_{ce}^2 \gg \nu_1^2 \), \( \sigma_1 \) becomes

\[
\sigma_1 = \frac{n^2 e^2}{B^2 c^2} \eta_1 \left( = \frac{n^2 e^2}{B^2} \eta_1 = \frac{\omega_{pe}}{\omega_{ce}^2} \epsilon_0 \eta_1 \right)
\]

where (Spitzer 1962) gives

\[
\eta_1 = 1.29 \times 10^4 \frac{Z \ln A}{T^{3/2}} \text{ ohm-cm}
\]

eV_e/kT is the ratio of the external potential energy to the thermal energy. \( V_e \) is defined as the potential across a distance equal to \( r_{ci} \) which produces an \( (E \times B)/B^2 \) drift equaling \( v_R \).

The second column is obtained from the first column simply by converting the unit length from 1 cm to 0.5 cm, and the unit time from 1 sec to 300 sec. The third column gives the quantities for a hot deuterium plasma with \( n = 10^{14} \text{ cm}^{-3} \), \( kT = 20 \text{ keV} \), \( B = 100 \text{ kg} \) and \( R = 100 \text{ cm} \).

Direct comparison of column two and column three reveals that the lithium plasma is a close analogy to the hot deuterium plasma, that is, using \( L = 1 \), \( T = 1 \) reproduces much of the third column. The main
differences between the two plasmas are that in the lithium plasma lengths are smaller by a factor of two, and time rates are slower by a factor of 300. Both of these features are of great advantage to performing experiments at low temperatures.

There are, however, several nonscaling quantities. The first one is the classical diffusion velocity \( v_{Dn} \), which is greatly enhanced (scales \( 10^4 \) too fast) in the cold plasma case. However, it is to be noted that in both cases \( v_{Dn} \gg v_{Dt} \). So, if Bohm diffusion (which scales well) is present in both cases, then we can ignore the classical diffusion, at least at \( n = 10^9 \); raising \( n \) to \( 10^{12} \) would make \( v_{Dn} > v_{Dt} \), which is not desirable, \( (v_{Dn} \propto n, v_{Dt} \propto \nabla n/n) \).

The second nonscaling quantity is the Alfven velocity, large by \( 2 \times 10^3 \). Actually the phase velocity of all electromagnetic waves are too large by a factor of 600 for the cold plasma. This comes from the fact that the speed of light is the same in both cases. Note that the acoustic waves whose phase velocities are proportional to the ion thermal velocity are all scaled properly.

The third nonscaling quantity is the transverse relaxation time, down by \( 10^4 \). The hot plasma is a good dielectric for \( E_\perp \); the cold plasma is more like a lossy dielectric. Hence, \( E_\perp (dc) \) will be shorted out after about 1/20 sec in the cold plasma. Of course using \( E_\perp (ac), f > 50 \) cps will allow field penetration.

The nonscaling of \( N_n \) is no problem; we ask only that \( N_n \gg 1 \), as it is. The nonscaling of \( \tau_{ei}' \), \( \tau_{ee}' \), \( \tau_{ii}' \) too small by about \( 10^4 \) is of consequence only as related to instability growth times, line tying et. al., as discussed in the sections following.

The use of \( n = 10^9 \) in the vapor pressure mode asks for pressure of \( 2 \times 10^{-8} \) torr. If the molecular beam mode is used, with the beam smaller than that of the plate, \( r_{beam} < r_{plate} \); then, the plasma density for \( r_{beam} < r < r_{plate} \) is not zero but dependent on the pressure. Also, the plate electron emission generally will be uniform so that
in this outer region, at least at the plate, \( n_e > n_1 \). Thus the plasma will be blanketed by a conducting layer, quite undesirable. Because of the necessity of vacuum better than, say \( 2 \times 10^{-9} \) torr for the beam mode, we will use the pressure mode initially.

C. LINE TYING, FLUTE GROWTH IN TWO PLATE SYSTEMS

The main instability expected in plasmas in mirror type fields is the flute instability caused by the charge-separation particle drift velocities. The flexibility of the proposed experiment with respect to density of particles, \( n \), and ion gyro-radius \( r_{gi} \) make it ideal for studying flute growth over a wide change of these parameters. In particular, the theory of flute stabilization due to finite ion gyroradius depends on both \( n \) and \( r_{gi} \); any experiment which can easily continuously change these parameters is inherently capable of fully testing the theory.

The flutes manifest themselves by an interchange of plasma tubes extending along the lines of magnetic force. In open-ended systems (as in actual mirror-machine devices), the lines of force are free to move (and the plasma with them) while if the lines of force end in a conducting surface which is in contact with the plasma (as in the two metal plate version of the proposed device) the plasma may be prevented from making an interchange. This effect can be explained either by an MHD theory which says that the lines of force frozen into both plasma and end conducting plates will prevent the interchange, or by a particle theory which says the conducting surface short circuits the lines of force and provides a charge drainage mechanism which can dominate over the charge being provided by the charge separation drifts. The fluid theory requires a high plasma conductivity so that the plasma remains "frozen" onto the field lines; equivalently, the particle view requires a low enough ion-electron collision frequency so that the charge can freely drain along field lines which in this model, lead to the conducting end plates.

Expected flute growth times can be much shorter than resistive diffusion times (see Table I). However, in order for the flutes to grow
in the proposed two end-plate systems, the ion-electron collisions must be frequent enough to prevent the conduction to the end from dominating the charge separation. A crude estimate can be made by comparing the growth rate of an unimpeded flute (which represents the rate of growth of the charge due to the charge separation currents) to the charge decay rate due to end conduction. Note the competing effects: for uniform flute growth over the entire length of the plasma column, the communication along $B_0$ should be good; for avoiding the short circuit at a metal plate of sheath, the opposite should be so. Thus, the two plate system which depends on plasma parameters to avoid the short circuit is a compromise at best, allowing fluting only in the central region where the curvature of the magnetic lines of force is approximately constant.

The growth rate for a freely growing flute, based on linear theory, assuming a sharp density gradient, is $\exp t/\tau_f$, where $\tau_{\text{flute}}$ was given earlier in Eq. (1). This latter time should be short compared with the charge drainage time to the end plates. The charge decay time can be given, approximately, by an $R-C$ time constant of an equivalent circuit composed of two lines of force separated by $\lambda/2$ and short circuited a distance $l$ ($= 2R$) away. Or, for the $m = 1$ mode, the approximate model of Fig. 5 holds, with $\tau_{RC} = R_{\parallel} C_{\perp}$. Thus,

$$R_{\parallel} \approx \eta_{\parallel} \frac{l}{W(d/m)}$$

$$C_{\perp} \approx \epsilon_0 K_{\perp} \frac{l W}{(d/m)}$$

so

$$\tau_{RC} \approx \epsilon_0 \eta_{\parallel} K_{\perp} \left( \frac{l m}{d} \right)^2$$

The result is, using $l = 2R_0 (d, R_0$ in cm $v_i$ in cm/sec)

$$\frac{\tau_{RC}}{\tau_f} \approx \left[ \left( \frac{m}{d} \right)^5 R_0^3 \right]^{1/2} 4 \epsilon_{\perp} \eta_{\parallel} v_i.$$
Fig. 5. (a) R-C for line tying to end plate calculation, $m = 1$;  
(b) Equivalent network; $\sigma_\perp$ is neglected as it is small.
This ratio should be large for line tying to be ineffective. The values are, using Eq. (7)

$$\eta_{||} = 0.62 \text{ ohm cm}$$ \hspace{1cm} (18)

and using Eq. (5),

$$\epsilon_\perp = \epsilon_0 K = 34 \epsilon_0 = 3 \times 10^{-12} \text{ farad/cm}$$ \hspace{1cm} (19)

so

$$\tau_{RC} \approx 7.4 \times 10^{-12} \left( \frac{m}{d} R_o \right)^2.$$ \hspace{1cm} (20)

This is very short unless $R_o$ is made very large - a long skinny column; also as $\epsilon_\perp \sim (n m_i)$ is larger at higher densities, larger masses, these also increase $\tau_{RC}$.

Finally,

$$\frac{\tau_{RC}}{\tau_f} \approx 2.4 \times 10^{-6} \left[ \frac{m}{d} \right]^5 R_o^3 \]^{1/2}.$$ \hspace{1cm} (21)

For $(\tau_{RC}/\tau_f) > 1$, then $(m/d)^5 R_o^3 > 10^{11}$. Using $R = 50$ cm, then $m > 16 d$.

Hence only the large $m$ modes, shortest wavelengths, will be seen to grow, if at all, for the implied wavelength is less than $r_{gi}$. By going to Cs (increase in $m_i$ by 19) and density of $10^{12}$, $(\tau_{RC}/\tau_f)$ increases by $10^3 \sqrt{19}$ and the result is $m > 0.6 d$ cm which is more attractive. Unfortunately, increasing $n$ to $10^{12}$ increases the classical diffusion velocity, $v_D e_\perp$, to twice the Bohm diffusion velocity, $v_{Dt}$, destroying the analogy with a collisionless plasma.
The conclusion is that line tying will be effective for \( n \approx 10^9 \) for a Li plasma except for absurdly large \( m \) numbers. For a Cs plasma at higher density, \( n \approx 10^{12} \), \( m > 3 \) should be seen except that classical diffusion may wash out flute growth.

It is thus obviously desirable to find ways of obviating line tying. One such method is given in the next section. Another (thanks to S. A. Colgate) is to use insulating plates which, with high work functions like metal plates, create ion-electron pairs from neutrals; this type of source is not known to us.

An additional concern with the two plate system, is that the mirror reflection point of all particles emitted at the plates, following adiabatic motion, is at or axially beyond the plates; no particles are mirrored in the central region between plates. Thus any particles diffusing to \( r > r_{\text{plate}} \) are immediately lost out the ends; it is not possible to have a sheath outside the plates which thus would not be bothered by line tying to the plate.

D. CENTRAL PLATE SOURCE

The preliminary concept of the experiment is to use a single hot disk instead of two. The hot disk is to be located at the center of the machine and supported on the axis from the ends. Ions and electrons are emitted from the disc in both directions, and are reflected back by the magnetic mirrors surfaces associated with the particle energies, not just at \( B_{\text{max}} \) as with two plates. The surface of the plasma may extend outside the hot disc without loss of all the particles as is not true for the two plate experiment; the extension is made by decreasing \( B_0 \). The plasma in the surface layer is not only trapped by the mirrors but also is detached from the hot plate. The flutes can be excited deliberately in the surface layer, and there is no problem of line tying to the plate. Of course, for \( r < r_{\text{plate}} \) there can still be internal line tying through the \( R_{\parallel} C_{\perp} \) mechanism discussed in the previous section; the stabilized central core will act as a good conductor and as
such, will maintain stability until $B_0$ is decreased to expand the plasma. The $m = 1$ mode will be inhibited in the plate; however, using a small diameter plate will allow growth of $m = 1$ until the edge of the plasma reaches the edge of the plate.

Other benefits include reduced problems with lack of balance (potential, temperature) between two plates, and only half the heating power. This configuration is most attractive.

E. FINITE GYRORADIUS STABILIZATION EFFECTS

An extremely important consideration when calculating growth rates is the finite gyro-radius stabilization effect which must be considered whenever $(r_{gi}/\lambda)^2$ becomes comparable to $(\omega_{flut}/\omega_{ci})$ as shown by Rosenbluth, Krall, Rostoker (1961). Note that this condition can be satisfied even though $r_{gi} \ll \lambda$, i.e., this is not a scrambling effect but a charge separation due to a differential drift of ions and electrons. The differential drift is a result of the different effective $E$ fields seen by particles of different orbit size. The difference in the effective $E$ fields is larger for larger spatial gradients of $E$; the $m = 1$ mode is unaffected as the $E$ field is constant in this mode by definition.

For our choice of parameters, the $r_{gi} \gg r_{ge}$ and $r_{gi}$ is of appreciable size, so we should expect stabilization of modes with $m \geq 2$.

As an estimate, we use Post (1963) who gives a simplified stability criterion which applies in our case:

$$\left(\frac{L\alpha_i}{r_o}\right) \gtrsim 1 \quad \text{stable (} m \geq 2 \text{)} .$$

Using $L = 2\ell = 100$ cm, $\alpha_i = r_{gi} = 0.2$ cm, $r_o = d/2 = 2$ cm, we find the left hand side is 25 which easily satisfies stability.
By going to a system with smaller $l$, larger $d$ we could perhaps become marginally unstable for the low $m$ modes, but it would only be marginal with decreased growth rate and very possibly the growth would then be washed out by resistive diffusion.

II. INITIAL SIMULATION EXPERIMENTS

Numerous experiments can be conceived to be performed on low energy plasmas in simulation of flute instabilities at high energies. We have chosen two experiments that are of initial interest to us. The first is study of the ordinary mirror. The second is the study of a hollow plasma in a mirror field, with possible addition of an $E_r$ to counteract inner surface flute growth, or of $B_q$ (stuffing) to make a minimum $B$ configuration. There are other things of interest, such as heating electrons or ions, but not contemplated for initial work.

A. PLASMA IN MIRROR FIELD

The initial operation will be with the simplest configuration, a cylindrical plasma in a simple mirror field. This experiment will be essentially the shakedown operation of the source, vacuum, magnet and diagnostic methods. The second step would be to look at flute stabilization with fields of varying curvature, taking advantage of the plasma (not end) line tying between regions of opposite curvature.

B. THE HOLLOW PLASMA EXPERIMENT

One type of plasma of interest to us is a hollow plasma confined in an inside-out mirror field, as in Fig. 6. The inner surface of the plasma is subject to the flute instability due to the unfavorable curvature of the magnetic field. Two possible ways are proposed to stabilize the inner surface. The first method is to pass current along the axis of the machine to form a minimum $B$ magnetic field. The second method is to apply a centrifugal pressure on the inner surface by a radial electric field. Both variations can be investigated by an alkali-metal plasma.

Because of the short transverse relaxation time in the cold plasma, the application of a dc radial electric field will end up in a thin sheath on
Fig. 6. Hollow plasma configuration.
the plasma boundary. This can be avoided by using an audio frequency electric field whose period of oscillation is much shorter than the transverse relaxation time.

ACKNOWLEDGMENT

We are grateful to C. W. Hartman of UCLRL, Livermore for introducing us to the general idea of studying high temperature plasma instabilities with low temperature plasmas and for use of some of his early calculations. We are also indebted to S. A. Colgate for similar encouragement and fruitful discussions.

REFERENCES

1961 D. Rose and M. Clark, Plasmas and Controlled Fusion, MIT Press, and J. Wiley and Sons, N.Y.
1963 R. F. Post, "Observation of plasma instability with rotational effects in a mirror machine," University of California Radiation Laboratory 7302.