Similarity of limiting currents in planar and cylindrical crossed-field diodes

V. P. Gopinath, J. P. Verboncoeur and C. K. Birdsall
EECS Department, University of California,
Berkeley, CA 94720

June 25 1995

Abstract

Limiting currents in planar crossed-field diodes have been studied for over 70 years [1]- [3] for $B < B_{Hull}$ and more recently for $B > B_H$ [4]. Recent simulations of cylindrical crossed-field diodes for anode/cathode radius ratios of 2 and 5 indicate that the limiting current curve in cylindrical diodes follows the planar theory and simulations very closely [5]. Newer simulations for larger radius ratios (10, 20) show somewhat larger limits. A possible explanation for this behavior is also examined.

1 Background

1.1 Space Charge Limited Current in Cylindrical Systems

For zero applied magnetic field, we need to solve the one dimensional Poisson equation in cylindrical coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial V(r)}{\partial r}) = -\frac{\rho(r)}{\epsilon} \tag{1}$$

where $V(r)$ is the local potential and $\rho(r)$ is the local charge density. The kinetic energy gained by a charged particle of mass m, charge e and velocity v, accelerated through a time-independent potential drop V is given by (particle starting at $V = 0 = v$)

$$\frac{1}{2}mv^2 = eV. \tag{2}$$

The current density $J(r)$ inside the system can be written in terms of the charge density and velocity as

$$J(r) = \rho(r)v(r). \tag{3}$$

It should be noted here, that, unlike the planar case, $J(r) = I/(2\pi rl)$, is not a constant across the system. Equation (1) can be rewritten as

$$\frac{d^2}{dr^2}V(r) + \frac{1}{r}\frac{\partial V(r)}{\partial r} = \frac{I}{2\pi rl\epsilon}\sqrt{\frac{m}{2eV(r)}}. \tag{4}$$

Langmuir and Blodgett [7] noticed the similarity between equation (4) and the planar case and found a solution for the J_{LB}, the space charge limited current density, as

$$J_{LB} = 2.335 \times 10^{-6}\frac{V_p^{3/2}}{r_c r_a \beta^2}(\frac{\mathrm{A}}{\mathrm{m}^2}) \tag{5}$$

where V_p is the anode potential, r_a is the anode radius and r_c is the cathode radius. The Langmuir-Blodgett factor β^2 varies with r and is tabulated for different radius ratios in [2], [7] and [8]

1.2 Hull Cutoff in Cylindrical Systems

The magnetic field B_H required such that an electron emitted from rest from a cathode will barely graze the anode is given by [2]

$$B_H = \sqrt{\frac{8m}{e}}\frac{V_p}{r_a}\frac{1}{(1 - r_c^2/r_a^2)}. \tag{6}$$

This equation reduces to the form given by Hull [1] for the special case where $(r_c^2/r_a^2) \ll 1$.

1.3 Limiting Current Theory for Planar Systems

The critical current density that can be transmitted (J_c) at any value of magnetic field between 0 and B_H, for a planar crossed-field diode has been worked out by Lau *et. al.* [4] and Pollack [6]. The relationship between normalized value of the transmitted current density (J_c/J_{CL}) and the normalized magnetic field (B/B_H) is given by

$$\frac{J_c}{J_{CL}} = \frac{9}{4}\frac{y^2}{(1+y^2)^{3/2}[h(y)]^2} \tag{7}$$

where

$$y = \frac{B/B_H}{\sqrt{1-(B/B_H)^2}} \tag{8}$$

and $x = h(y)$ is obtained by inverting the function

$$y = f(x) = \begin{cases} \frac{1}{x^2}(\sin^{-1}(x) - x\sqrt{1-x^2}) & 0 < y < \frac{\pi}{2} \\ \frac{1}{x^2}(\frac{\pi}{2} + \cos^{-1}(x) - x\sqrt{1-x^2}) & \frac{\pi}{2} < y < \infty \end{cases} \tag{9}$$

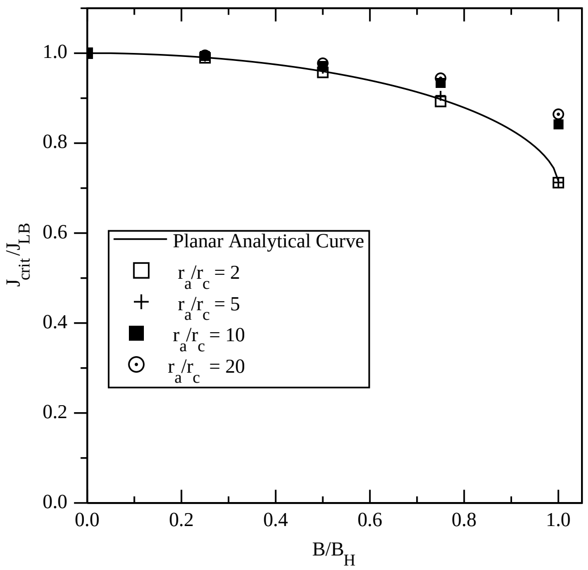

Figure 1: Comparison of Cylindrical Simulations with planar theory

Equation (9) shows that the value of the normalized limiting current varies continuously from 1, at zero magnetic field to $9/4\pi$ near Hull cutoff, and is the solid line in Figure 1.

2 Discussion of Results

1d3v (just r, v_r, v_θ, v_z) PIC simulations in cylindrical coordinates were done using XPDC1 [9]. The cathode (electron emitter) was the inner conductor and anode was the outer conductor, held at a positive voltage with respect to the cathode. Runs were conducted for r_a/r_c ratios of 2, 5, 10 and 20 in order to study the behavior in small and large radius ratio regimes. For all these cases, the cathode radius was held at 0.01 m. The cathode was modeled as an electron emitter emitting particles with initial energy $1/2eV$, at a given current density. For all these runs, the anode was held at a DC potential of $10^4\,V$. Theoretical values of J_{LB} and B_H from equations (5) and (6) respectively were used as starting points for the simulations. Numerous runs were conducted to locate accurately the value of the injected current density above which the system exhibited oscillatory/turbulent behavior. The magnitude of this current density for zero magnetic field was considered to be J_{CL} (planar) and J_{LB} (cylindrical) for the particular radius ratio and was used as the normalizing factor. Similarly,

⊙

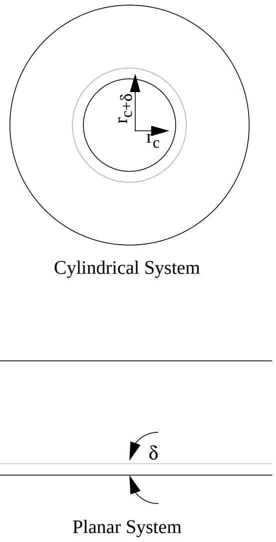

Figure 2: Possible Explanation of the results

B_H was considered to be the maximum value of the magnetic field above which no laminar flow was present. In all these runs, the simulated values of J_{LB} were within 2% the theoretically calculated values. The simulated value of B_H differed from the theoretical value by less than 0.1%. The results of simulations for all four radius ratios is given in Figure 1. For r_a/r_c ratios of 2 and 5, J_c/J_{LB} near cutoff was very close to $9/4\pi$ as predicted in the planar model. Therefore, equation (9), represented by the solid line, was numerically calculated, and was used to hunt for the critical current in a manner similar to the search for J_{LB}. For the simulated points, the cylindrical results track the planar theory very closely for r_a/r_c ratios of 2 and 5. However, the transmitted current limit for larger radius ratios increases slightly over the planar theory at higher magnetic fields, transmitting 86% of J_{LB} at B_H as opposed to 71.6% predicted by the planar theory.

Figure 2 is a representation of a cylindrical and planar cross field diode. It should be noted here that for both these cases, the charge density inside the structure drops from a very large value to a small constant value within a very short distance from the cathode as can be seen in Figure 3 from simulation for $r_a/r_c = 2$. This surface is represented by the dashed line in Figure 2 and is located at a distance δ from the cathode. Since δ is $\ll r_c$, an electron present in this region "perceives" the structure to be planar. Further, from planar theory, it can be shown that an electron emitted from the cathode spends a relatively large amount of time in this region. If τ_δ be the time spent in the high charge

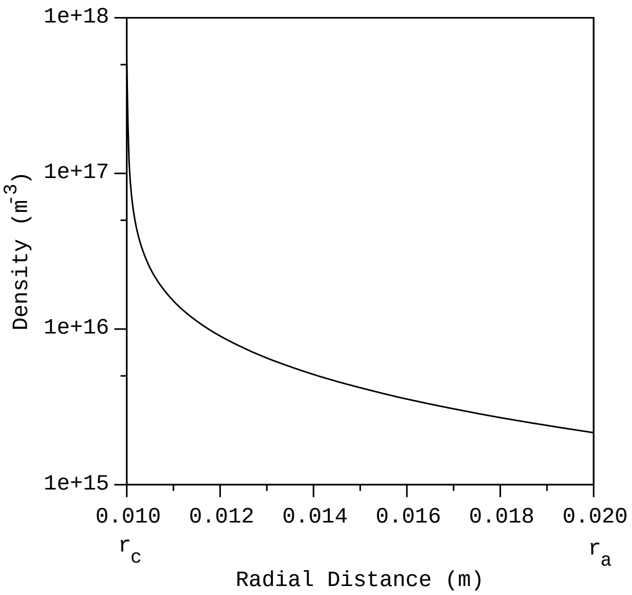

Figure 3: Spatial Variation of Charge Density (Simulation)

density region, τ_T be the transit time across the structure and d be the length of the diode, for the planar case, it can be shown that since

$$v(x) = \sqrt{\frac{2eV}{m}}(\frac{x}{d})^{\frac{2}{3}} \tag{10}$$

and the time required to traverse a distance y is

$$\tau_y = \int_0^y \frac{dx}{v} \tag{11}$$

that

$$\frac{\tau_\delta}{\tau_T} = (\frac{\delta}{d})^{\frac{1}{3}}. \tag{12}$$

It can be seen here that for $\delta/d = 1/64$, the electron will spend 25% of its lifetime in the small region. Therefore, for systems where δ/d is larger i.e. smaller radius ratios, the cathode region behaves like a region of $r_a/r_c = 1$, i.e. planar.

3 Conclusions

Simulations of transmitted current in cylindrical cross field diodes show close agreement with **planar** analytical theory for smaller radius ratios. We have attempted to explain this behavior by pointing out that the region of maximum significance for transmitted current, in smaller r_a/r_c ratio cylindrical systems, is essentially planar. This effect is further accentuated by the comparatively large amount of time spent by the electron in this region.

4 Acknowledgments

We wish to thank Y. Y. Lau and Peggy Christenson for the many stimulating discussions on cross-field devices. We would also like to acknowledge helpful discussions with D. Chernin, J. Swiegle and B. H. Vanderberg. This work was supported in part by AFOSR under grant F49620-92-J0487 and ONR under Contract FD-N00014-90-J-1198

References

[1] A. W. Hull, "The Effect of Uniform Magnetic Fields on the Motion of Electrons Between Coaxial Cylinders," *Phys. Rev* 18, 31 (1921).

[2] William G. Dow, *Fundamentals of Engineering Electronics*, John Wiley and Sons, New York, 1944

[3] C. K. Birdsall and W. B. Bridges, *Electron Dynamics of Diode regions*, Academic Press, (1966).

[4] Y. Y. Lau, P. J. Christenson and D. Chernin "Limiting Current in an Crossed-Field Gap," *Phys. Fl. I.5* 4486 (1993).

[5] V. P. Gopinath J. P. Verboncoeur and C. K. Birdsall, "Simulation of Transmitted Current in a Cylindrical Cross-Field Diode," presented at IEEE ICOPS'95, Madison, Wisconsin.

[6] M. A. Pollack, *Noise Transport in the Crossed-Field Diode*, Ph.D. thesis, University of California, Berkeley, CA, Series No. 60, Issue No. 485 (1962).

[7] I. Langmuir and K. B. Blodgett, "Currents Limited by Space Charge Between Coaxial Cylinders," Phys. Rev., Ser 2. vol 22, pp. 347-357, October 1923

[8] Karl R. Spangenberg, *Vacuum Tubes*, McGraw-Hill, New York, 1948

[9] J. P. Verboncoeur, V. Vahedi, M. V. Alves and C. K. Birdsall, *PDP1, PDC1 and PDS1: Plasma Device one-dimensional Bounded Electrostatic Codes*, Plasma Theory and Simulation Group,University of California, Berkeley, CA 94720. Available via *http://ptsg.eecs.berkeley.edu.*

```
#  1D planar hull diode
#       (r2/r1) = 10,
# Table V on page 178 of Spangenberg gives beta2 as 0.9782 => JLB = 2387.037
# Bhull for this geometry is 6.8135e-3 Tesla
#

-nsp---nc---nc2p---dt[s]------r0[m]----r1[m]---height[m]--epsilonr--Bz[Tesla]-
  1   500   6e6   6e-13      0.01    0.1       0.01       1.0     5.109e-3

-rhoback[C/m^3]-backj[Amp/m^2]---dde---extR[Ohm]--extL[H]---extC[F]--q0[C]-
     0.0           0.0            0.0     0.0         0.0      1.0      0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
   0          v       -10e3           0.0              0.0       1e7       0.0

--secondary----e_collisional----i_collisional----reflux---nfft--
      0            0                  0              0       512

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
         0.0             0.2           2                5e-2          .026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----

--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
     1.0e-19         0.0            0.0            10.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
      1.0e-20       12.0            50.0           100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
      1.0e-20       13.6            60.0           110.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----

---achrgx[m^2]--bchrgx[m^2*V^1/2]-------ascat[m^2]--bscat[m^2*V^1/2]---
    3.0e-19          0.0                  2.0e-19         0.0

SPECIES 1

----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19   9.11e-31     2.175e3       0.0            0.0

-v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]--vtperp[m/s]--
  4.2e4        0.0        0.0       0.0         0.0           0.0      0.0

---nbin----Emin[eV]----Emax[eV]---max-np---
    100       0            15e3      100000
```

Figure 4: XPDC1 Input File $r_a/r_c = 10$