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A PHYSICAL POLY-SILICON THIN FILM TRANSISTOR (TFT) MODEL FOR CIRCUIT SIMULATION

by

Chester Li

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ELECTRONICS RESEARCH LABORATORY

College of Engineering
University of California, Berkeley
94720
Abstract

This report presents a poly-silicon thin film transistors model for circuit simulations. The drain current model includes the effects of hot carrier, drain induced barrier lowering (DIBL), channel length modulation (CLM), and gate induced drain leakage (GIDL). The capacitance model is linked to the drain current and its derivatives. This model has been implemented in SPICE. Simulation and experimental results are compared.
# Table of Contents

Chapter 1: Introduction  
Chapter 2: Drain Current Model  
Chapter 3: Intrinsic Capacitance Model  
Chapter 4: Spice Implementation  
Chapter 5: Simulation Results  
Appendix A: Equations Implemented in SPICE  
Appendix B: Parameter Extraction  
Reference  
Acknowledgments
Chapter 1: Introduction

Poly-silicon Thin Film Transistors (TFTs) are widely used in active matrix liquid crystal displays (LCDs) to drive the pixels and decode the image signals. A common substrate material for TFTs is poly-silicon deposited on a glass substrate (fig 1.1). The typical operating bias in the small size LCD environment is about 12V. The characteristics of the TFTs are severely affected by imperfections at the poly-silicon surface. Moreover, the TFTs operate with a floating substrate. As a result, the characteristics of a TFT cannot be modeled accurately by the common bulk MOSFET model in SPICE.

![Cross sections of a bulk NMOSFET and a NTFT](image)

This report presents a poly-TFT drain current model with an accompanying intrinsic capacitance model. The formulation of the drain current model is similar to BSIM3 [2]. The capacitance model is capacitance based and linked to the drain current. They have been implemented in SPICE. Ring oscillator simulation results will be compared to experimental data. The data used in this study are measured from a LCD wafer with p-substrate and top gate structure. The gate oxide thickness is 76nm. The effective channel length ranges from 3.3μm to 5.3μm.

Chapter 2 presents the drain current model. Chapter 3 discusses the intrinsic capacitance model. Chapter 4 describes the model implementation in SPICE. Finally, chapter 5 compares the simulation results and experimental data. Equations implemented in SPICE and parameters extraction procedures are summarized in the appendixes.
Chapter 2: Drain Current Model

2.1 Overview

This chapter discusses the TFT drain current model. The model is separated into subthreshold and strong inversion regions. The strong inversion region is further divided into the linear and saturation regions. This physical model describes the hot carrier, drain induced barrier lowering (DIBL), channel length modulation [1], thermal generation, and gate induced rain leakage (GIDL) [4] effects. A parabolic smoothing function [1] is used to ensure continuity of the first order derivative between different regions of operation. Four parameters, $V_{gtranl}$, $V_{gtranh}$, $V_{dtranl}$, and $V_{dtranh}$ are used to define the transition regions between different bias regions. The transition regions are defined around $V_T$ and $V_{dsat}$. $V_T$ is the threshold voltage. $V_{dsat}$ is the saturation voltage. Figure 2.1a illustrate the boundary values.

Section 2.2 and 2.3 discuss the model in the strong inversion and subthreshold region. Section 2.4 describes the transition region between the strong inversion and subthreshold region. Section 2.5 compares the model with the measured data. Appendix A summaries the drain current equations implemented in SPICE. The parameters and their meanings are summarized in chapter 4. The following symbols are defined for model derivation (see figure 2.1a). T is temperature.

\[
\begin{align*}
V_{dh} &= V_{dsat} + V_{dtranh} \\
V_{gh} &= V_T + V_{gtranh} \\
V_{gl} &= V_T - V_{gtranh} \\
V_d &= V_{dsat} - V_{dtranl} \\

\end{align*}
\]
\[ V_T = V_{TO} - bT \quad \text{and} \quad V_{dsat} = \left( \frac{1}{V_{gs} - V_T} + \frac{1}{E_{sat} L_{eff}} \right)^{-1} \]

2.2 Strong Inversion Region Model

2.2.1 Linear Region \((V_{gs} > V_{gh}, 0 < V_{ds} < V_{dl})\)

Following Huang's [2] approach, the drain current in the linear region is:

\[ I_d = \frac{W_{eff}}{L_{eff}} C_{ox} \mu_{eff} \left( V_{gs} - V_T - \frac{V_{ds}}{2} \right) \frac{V_{ds}}{1 + \frac{V_{ds}}{E_{sat} L_{eff}}} \]

\[ E_{sat} = \frac{2 \nu_{sat}}{\mu_{eff}} \quad \text{and} \quad C_{ox} = \frac{\varepsilon_{ox}}{T_{ox}} \]

Poly-silicon TFTs have many interface traps at the Si-SiO₂ interface, especially at or near the grain boundaries. As a result, the electrons (or holes in p-channel TFT) have to hop over the barrier formed at the grain boundaries along the channel during electrical conduction (figure 2.2.1a). \( \ln(\mu) \), the logarithm of mobility, is inversely proportional to the barrier height \((\phi_b)\). Since \( \phi_b \) is modulated by the gate bias \((V_{gs})\), mobility has the form of \( \alpha \exp(-\beta(V_{gs}-V_T)) \). \( \alpha, \beta \), and \( V_T \) are function of temperature. \( \alpha \) and \( \beta \) are \( \mu_0(kT/q)^{-\mu_1} \) and \(-q\mu_2 \exp(\mu_3 T)/kTC_{ox} \) [1]. This expression describes the mobility well when \( V_{gs} \) is much bigger than \( V_T \). An additional \( \mu_4 \), the minimum mobility at low \( V_{gs} \), is added to the model. Therefore, the effective mobility is modeled as:

\[ \mu_{eff} = \mu_0 \left( \frac{kT}{q} \right)^{-\mu_1} \exp \left( \frac{-q\mu_2 \exp(\mu_3 T)}{kTC_{ox}(V_{gs} - V_T)} \right) + \mu_4 \]

\( \mu_0, \mu_1, \mu_2, \mu_3, \) and \( \mu_4 \) are fitting parameters. \( V_T \) is the threshold voltage, which is approximated as a linear function of temperature [4].

\[ V_T = V_{TO} - bT \]

Figure 2.2.1b shows the temperature dependence of \( V_T \). \( V_T \) decreases when temperature increases, which has the same trend as bulk MOSFET [4]. It indicates that the change in fermi level with temperature is the dominant mechanism. Figure 2.2.1c plots the mobility of a p-channel (PTFT) and a n-channel (NTFT) poly-TFT at \( V_{ds} = 0.1V \) at room temperature. When \( V_{gs} \) increases, \( \phi_b \) at the grain boundaries drops. Therefore, mobility increases as \( V_{gs} \) increases. The
mobility of poly-TFTs is lower than that for bulk MOSFET. Therefore the current drive is also smaller. Since the conduction is limited by excitation over the barrier, the mobility rises as temperature increases. Figure 2.2.1d plots the mobility of a NTFT at different temperatures.

Figure 2.2.1a: Band diagram of a poly-TFT channel and electron conduction of a n-channel device

Figure 2.2.1b: Threshold voltage dependence on temperature
Figure 2.2.1c: Mobility of a PTFT and an NTFT at room temperature

Figure 2.2.1d: Mobility of a NTFT at different temperatures. Lines are model and symbols are data.
2.2.2 Saturation Region ($V_{gs} > V_{gb}, V_{ds} > V_{dh}$)

Similar to BSIM3 [2J approach, the DIBL, hot carrier, and channel length modulation effect of drain current in the saturation region are modeled. The drain model is expressed as:

$$I_d = I_{dsat} \left( 1 + \frac{V_{ds} - V_{dsat}}{V_A} \right) f_{hc}$$

$$V_A = \left( \frac{1}{V_{ACLM}} + \frac{1}{V_{ADIBL}} \right)^{-1}$$

$$V_{ACLM} = \frac{(E_{sat} L_{eff} + V_{gs} - V_T) (V_{ds} - V_{dsat})}{E_{sat} \ell}$$

$$V_{ADIBL} = \frac{E_{sat} L_{eff} + V_{gs} - V_T}{\theta \left( 1 + 2E_{sat} L_{eff} / (V_{gs} - V_T) \right)}$$

$$f_{hc} = 1 + s_1 (V_{ds} - V_{dsat}) \exp \left( \frac{-s_2}{V_{ds} - V_{dsat}} \right)$$

$$E_{sat} = 2 v_{sat} / \mu_{eff}$$

$I_{dsat}$ is the drain current at $V_{ds}=V_{dsat}$ using the linear $I_d$ equation. $V_{ACLM}$ models the channel length modulation effect. $\ell$ is a fitting parameter. $V_{ADIBL}$ models the DIBL effect. The hot carrier effect, which is caused by the high electric field at the drain, is modeled by $f_{hc}$. $\theta$ is the DIBL effect coefficient. $s_1$ and $s_2$ are fitting parameters for hot carrier effect.

2.2.3 Transition Region in Strong Inversion Region ($V_{gs} > V_{gb}, V_{dl} < V_{ds} < V_{dh}$)

To improve the convergence property of the model, the first order derivative is made continuous by using a parabolic smoothing function from BSIM3 [2]. The basic concept is illustrated by figure 2.2.3a. $I_{dl}$ is the current in the linear region at $V_{dl}$ and the applied $V_{gs}$. $I_{dh}$ is the current in the saturation region at $V_{dh}$ and the applied $V_{gs}$. $L_1$ is the tangent to the linear region at $V_{ds}=V_{dl}$. $L_2$ is the tangent to the saturation region at $V_{ds}=V_{dh}$. The intersect of $L_1$ and $L_2$ is ($V_{dp}, I_{dp}$). Once ($V_{dh}, I_{dh}$), ($V_{dp}, I_{dp}$), and ($V_{dl}, I_{dl}$) are determined, the points between $V_{dh}$ and $V_{dl}$ can be computed using the parabolic smoothing function with first order derivative continuity. The expression for $V_{dp}, I_{dp}, I_{d}$, and the first order derivatives are shown below.
\[
V_{ds} = (1-t)^2 V_{dl} + 2t(1-t) V_{dp} + t^2 V_{dh} \quad \text{and} \quad I_d = (1-t)^2 I_{dl} + 2t(1-t) I_{dp} + t^2 I_{dh}
\]

\[
V_{dp} = \frac{I_{dh} - I_{dl} - (g_{dsh} V_{dh} - g_{dsl} V_{dl})}{g_{dsl} - g_{dsh}}
\]

\[
I_{dp} = g_{dsl} (V_{dp} - V_{dl}) + I_{dl}
\]

\[
g_{ds} = \frac{t(I_{dh} - I_{dp}) + (1-t)(I_{dp} - I_{dl})}{t(V_{dh} - V_{dp}) + (1-t)(V_{dp} - V_{dl})}
\]

\[
g_m = g_{ml} + \frac{I_d - I_{dl}}{I_{dh} - I_{dl}} \frac{V_{dh} - V_{dl}}{V_{ds} - V_{dl}} (g_{mh} - g_{ml})
\]

\[
g_{dsh} = \frac{\partial I_d}{\partial V_{ds}} \bigg|_{V_{ds}, V_{gs}} \quad \text{and} \quad g_{dsl} = \frac{\partial I_d}{\partial V_{ds}} \bigg|_{V_{dl}, V_{gs}}
\]

\[
g_{mh} = \frac{\partial I_d}{\partial V_{gs}} \bigg|_{V_{dl}, V_{gs}} \quad \text{and} \quad g_{ml} = \frac{\partial I_d}{\partial V_{gs}} \bigg|_{V_{dl}, V_{gs}}
\]

t can be expressed as a function of \(V_{ds}, V_{dp}, \) and \(V_{dh}\) as:

\[
t = \frac{(V_{dl} - V_{dp}) + \sqrt{(V_{dl} - V_{dp})^2 - (V_{dl} - V_{ds})(V_{dl} - 2V_{dp} + V_{dh})}}{V_{dl} - 2V_{dp} + V_{dh}}
\]

Figure 2.2.3a: Scheme to connect the saturation and linear region
2.3 Subthreshold Region Model (Vgs < V_T - V_{tranl})

The diffusion current, thermal generation current, and GIDL current are modeled in the subthreshold region. Since the interface traps density is high in TFT, the subthreshold swing (~380mV/decade) of the diffusion current is higher than that in the normal MOSFET (~100mV/decade). The subthreshold drain current is expressed as:

\[ I_d = I_{diff} + I_{gidl} + I_{thermal} \]

\[ I_{diff} = W_{eff} I_{do} \left( 1 - \exp \left( -\frac{V_{ds}}{kT/q} \right) \right) \exp \left( \frac{V_{gs} - V_T - V_{off}}{nkT/q} \right) \]

\[ I_{gidl} = W_{eff} A_{gidl} (V_{dg} - V_i) \exp \left( -\frac{B_{gidl}}{V_{dg} - V_i} \right) \quad V_{dg} = V_{ds} - V_{gs} \]

\[ I_{thermal} = W_{eff} I_{thermal0} \exp \left( -\frac{E_a}{kT/q} \right) \]

V_{off} is the offset voltage for I_{diff}. n is the subthreshold slope. I_{do} is a function of oxide thickness and substrate doping [2]. It is treated as a fitting parameter in this model. A_{gidl}, B_{gidl}, and V_i are fitting parameters for GIDL current. I_{gidl} is set to zero when (V_{dg} - V_i) is less than zero. This equation is same as in [3], except that [3] fixes V_i to 1.2V, where V_i here is kept as a parameter because of the abundance of interface traps between the conduction band and valance band. E_a is the activation energy for the thermal current generation. I_{thermal0} is a fitting parameter for I_{thermal}.

2.4 Transition Region between Strong Inversion and Subthreshold Region (V_{gl} < V_{gs} < V_{gh})

To improve the convergence property of the model, the first order derivative is made continuous by using the same scheme as in section 2.2.3. A parabolic smoothing function in the linear-linear scale is used. The basic concept is illustrated by figure 2.4a.
Figure 2.4a: Scheme to connect the subthreshold and strong inversion region

$I_{dl}$ is the current in the subthreshold region at $V_{gl}$ and the applied $V_{ds}$. $I_{dh}$ is the current in the strong inversion region at $V_{gh}$ and the applied $V_{ds}$. $L_1$ is the tangent to the subthreshold region at $V_{gs}=V_{gl}$. $L_2$ is the tangent to the strong inversion region at $V_{gs}=V_{gh}$. The intersect of $L_1$ and $L_2$ is $(V_{gp}, I_{dp})$. Once $(V_{gh}, I_{dh})$, $(V_{gp}, I_{dp})$, and $(V_{gl}, I_{dl})$ are determined, the points between $V_{gh}$ and $V_{gl}$ can be computed using the parabolic smoothing function with first order derivative continuity. The expression for $V_{gp}$, $I_{dp}$, $I_d$, and the first order derivatives are shown below.

$$V_{gs} = (1-t)^2V_{gl} + 2t(1-t)V_{gp} + t^2V_{gh}$$

$$I_d = (1-t)^2I_{dl} + 2t(1-t)I_{dp} + t^2I_{dh}$$

$$V_{gp} = \frac{I_{dh} - I_{dl} - (g_{mh}V_{gh} - g_{ml}V_{gl})}{g_{ml} - g_{mh}}$$

$$I_{dp} = g_{ml}(V_{gp} - V_{gl}) + I_{dl}$$

$$g_m = \frac{t(I_{dh} - I_{dp}) + (1-t)(I_{dp} - I_{dl})}{t(V_{gh} - V_{gp}) + (1-t)(V_{gp} - V_{gl})}$$

$$g_{ds} = g_{dsl} + \frac{I_d - I_{dl} \cdot V_{gh} - V_{gl}}{I_{dh} - I_{dl} \cdot V_{gs} - V_{gl}}(g_{dsh} - g_{dsl})$$
\[ g_{dsh} = \left. \frac{\partial I_d}{\partial V_{ds}} \right|_{V_{ds}, V_{gh}} \quad g_{dsi} = \left. \frac{\partial I_d}{\partial V_{ds}} \right|_{V_{ds}, V_{gl}} \]

\[ g_{mh} = \left. \frac{\partial I_d}{\partial V_{gs}} \right|_{V_{ds}, V_{gh}} \quad g_{mi} = \left. \frac{\partial I_d}{\partial V_{gs}} \right|_{V_{ds}, V_{gl}} \]

t can be expressed as a function of \( V_{gs} \), \( V_{gp} \), and \( V_{gh} \) as:

\[
t = \frac{(V_{gl} - V_{gp}) + \sqrt{(V_{gl} - V_{gp})^2 - (V_{gl} - V_{gs})(V_{gl} - 2V_{gp} + V_{gh})}}{V_{gl} - 2V_{gp} + V_{gh}}
\]
2.5 Verification

Figure 2.5a: $I_d V_{ds}$ of a 20/5.3 NTFT with $T_{ox}=76$nm

Figure 2.5b: $I_d V_{ds}$ of a 20/3.3 NTFT with $T_{ox}=76$nm. The model curve is generated using the same set of parameters extracted from the 20/5.3 device. $\Delta L=1.7\mu m$ (extracted by capacitance method)
Figure 2.5c: \( I_dV_{ds} \) of a 20/4.42 PTFT with \( T_{ox}=76\)nm.

Figure 2.5d: \( I_dV_{ds} \) of a 20/2.42 PTFT with \( T_{ox}=76\)nm. The model curve is generated using the same set of parameters extracted from the 20/4.42 device. \( \Delta L=2.58\)\( \mu m \) (extracted by capacitance method)
Figure 2.5e: $I_dV_{gs}$ of a 20/5.3 NTFT with $T_{ox}=76$nm

Figure 2.5f: $I_dV_{gs}$ of a 20/3.3 NTFT with $T_{ox}=76$nm
Chapter 3: Intrinsic Capacitance Model

3.1 Overview

A capacitance-base model is developed to model $C_{gs}$ and $C_{gd}$. The model is separated into strong inversion and subthreshold regions. The strong inversion region model is developed from charge equations and linked to the drain current model. The subthreshold region model is empirical. The subthreshold region, strong inversion linear region, and strong inversion saturation region are linked by a linear function to model the gradual change in capacitance when the TFT is switched from one bias region to another. Four parameters are used to define the transition region between different bias regions. They are: $V_{dtranhc}$, $V_{dtranlc}$, $V_{gtranhc}$, and $V_{gtranlc}$. The transition regions are defined around $V_T$ and $V_{dsat}$. $V_T$ is the threshold voltage. $V_{dsat}$ is the saturation voltage, which is $[1/(V_{gs}-V_T) + 1/(E_{sat}L_{eff})]^{-1}$. The boundaries are illustrated in figure 3.1a. Two additional parameter, $A_{cgs}$ and $A_{cgd}$, are added to model $C_{gs}$ and $C_{gd}$ in the GIDL dominant region.

![Figure 3.1a: Definition of the boundaries values](image)

Section 3.2 and 3.3 describe the strong inversion and subthreshold model. Section 3.4 verifies the model with measured data. The equations implemented in SPICE is summarized in appendix A. The parameters and their meanings are summarized in chapter 4. The following symbols are defined for the model derivation (see figure 3.1a).

$$V_{dl} = V_{dsat} - V_{dtranhc}$$
$$V_{gh} = V_T + V_{gtranhc}$$
$$V_{dl} = V_{dsat} - V_{dtranlc}$$
$$V_{gh} = V_T + V_{gtranlc}$$
$$V_{gl} = V_T - V_{gtranlc}$$

14
\[ V_{gst} = V_{gs} - V_T \quad V_{gstd} = V_{gst} - V_{ds} \quad V_{gstdsat} = V_{gst} - V_{dsat} \]

\[ g_{ds} = \frac{\partial I_d}{\partial V_{ds}} \quad g_m = \frac{\partial I_d}{\partial V_{gs}} \]

3.2 Strong Inversion Region Model

3.2.1 \(C_{gd}\) in the Linear Region \((0 < V_{ds} < V_{dsat}, V_{gs} > V_{gh})\)

![Cross section of a NTFT in linear region](image)

Figure 3.2.1a: Cross section of a NTFT in linear region

Figure 3.2.1a shows the cross section of a NTFT in the linear region \((V_{ds}<V_{dsat}\) and \(V_{gs}>V_T\)). The inversion charge density gradually drops from \(C_{ox}(V_{gs} - V_T - V_s)\) at the source \((y_s)\) to \(C_{ox}(V_{gs} - V_T - V_d)\) at the drain \((y_d)\). The charge at the gate is:

\[ Q_g = -W_{eff} \int_{y_s}^{y_d} Q_n(y) dy + Q_{bulk} \]

\(Q_{bulk}\) is the depletion charge in the substrate. \(Q_n(y)\), the inversion charge, is:

\[ Q_n(y) = C_{ox}[V_{gs} - V_T - V(y)] \]

\(Q_g\) can also be re-written as:

\[ Q_g = -W_{eff} \int_{V_s}^{V_d} Q_n(V) \frac{1}{dV/dy} dV + Q_{bulk} \]

Huang [2] shows that:
\[ \frac{dV}{dy} = \frac{I_d}{W_{\text{eff}} \mu_{\text{eff}} Q_n(V) - \frac{I_d}{E_{\text{sat}}}} \]

Therefore, \( Q_g \) is:

\[ Q_g = -W_{\text{eff}} \int_{V_s}^{V_d} Q_n(V) \left[ W_{\text{eff}} \mu_{\text{eff}} Q_n(V) - \frac{I_d}{E_{\text{sat}}} \right] I_d dV + Q_{\text{bulk}} \]

\( C_{gd} \) is defined as \( \frac{\partial Q_g}{\partial V_d} \). Since \( Q_{\text{bulk}} \) is not a strong function of \( V_{ds} \), \( C_{gd} \) is:

\[ C_{gd} = \frac{g_{ds}}{I_d^2} W_{\text{eff}}^2 \mu_{\text{eff}} \int_{V_s}^{V_d} Q_n^2(V) dV - \frac{W_{\text{eff}}^2 \mu_{\text{eff}}}{I_d} Q_n^2(V_d) + \frac{W_{\text{eff}}}{E_{\text{sat}}} Q_n(V_d) \]

\[ \int_{V_s}^{V_d} Q_n^2(V) dV = \frac{1}{3} C_{ox}^2 \left( V_{gs}^3 - V_{gsid}^3 \right) \]

\[ E_{\text{sat}} = \frac{2v_{\text{sat}}}{\mu_{\text{eff}}} \]

### 3.2.2 \( C_{gd} \) in the Saturation Region (\( V_{ds} > V_{dh}, V_{gs} > V_{gh} \))

![Cross section of a NTFT in saturation region](image)

Figure 3.2.2a: Cross section of a NTFT in saturation region
Figure 3.2.2a shows the cross section of a NTFT in the saturation region ($V_{ds}>V_{dsat}$ and $V_{gs}>V_T$). The inversion charge density drops from $C_{ox}(V_{gs}-V_T-V_s)$ at the source ($y_s$) to $C_{ox}(V_{gs}-V_T-V_{dsat})$ at $y_{dsat}$, where $V(y_{dsat})=V_{dsat}$. Then the charge density stays constant at $C_{ox}(V_{gs}-V_T-V_{dsat})$ from $y_{dsat}$ to the drain ($y_d$), which is known as the velocity saturation region [4]. The gate charge is:

$$Q_g = Q_{lin} + Q_{sat} + Q_{bulk}$$

$$Q_{lin} = -W_{eff} \int_{y_s}^{y_{dsat}} Q_n(y) dy \quad Q_{sat} = -W_{eff} \int_{y_{dsat}}^{y_d} Q_n(y) dy$$

$Q_{lin}$ is the charge from $y_s$ to $y_{dsat}$. $Q_{sat}$ is the charge in the velocity saturation region. Assuming that the gradual channel approximation still holds from $y_s$ to $y_{dsat}$, $Q_{lin}$ can be expressed as a function of voltage bias in the same way as section 3.2.1.

$$Q_{lin} = -W_{eff} \int_{V_s}^{V_{dsat}} Q_n(V) \left[ W_{eff} \mu_{eff} Q_n(V) - \frac{I_d}{E_{sat}} \right] / I_d dV$$

$$\frac{\partial Q_{lin}}{\partial V_d} = \frac{1}{3} W_{eff} \mu_{eff} \frac{g_{ds}}{I_d} C_{ox}^2 \left( V_{gst}^3 - V_{gstdsat}^3 \right)$$

$Q_{sat}$ is approximated as:

$$Q_{sat} = -W_{eff} C_{ox} (V_{gs} - V_T - V_{dsat}) \Delta L$$

$$\frac{\partial Q_{sat}}{\partial V_d} = -W_{eff} C_{ox} (V_{gs} - V_T - V_{dsat}) \frac{\partial \Delta L}{\partial V_d}$$

Ko [5] shows that:

$$\Delta L = \ell \ln \left( \frac{(V_d - V_{dsat})/\ell + E_m}{E_{sat}} \right)$$

$$E_m = \left[ \left( \frac{V_d - V_{dsat}}{\ell} \right)^2 + E_{sat}^2 \right]^{1/2}$$
Therefore, \( \frac{\partial \Delta L}{\partial V_d} \) is:

\[
\frac{\partial \Delta L}{\partial V_d} = \frac{1}{(V_d - V_{dsat}) / \ell + E_m \left(1 + \frac{V_d - V_{dsat}}{\ell E_m}\right)}
\]

\( C_{gd} \) is defined as \( \frac{\partial Q_g}{\partial V_d} \). Since \( Q_{bulk} \) is not a strong function of \( V_d \), \( C_{gd} \) is:

\[
C_{gd} = \frac{\partial Q_{lin}}{\partial V_d} + \frac{\partial Q_{sat}}{\partial V_d}
\]

### 3.2.3 \( C_{gs} \) in the Linear Region (\( 0 < V_{ds} < V_{db}, V_{gs} > V_{gh} \))

From charge conservation, one can conclude that \( \frac{\partial Q_g}{\partial V_g} + \frac{\partial Q_g}{\partial V_s} + \frac{\partial Q_g}{\partial V_d} = 0 \). Since \( C_{gd} \) is

\[
\frac{\partial Q_g}{\partial V_d} \text{ and } \frac{\partial Q_g}{\partial V_s} \text{ is } \frac{\partial Q_g}{\partial V_d}, \ C_{gs} \text{ can be expressed as :}
\]

\[
C_{gs} = \frac{\partial Q_g}{\partial V_g} - C_{gd}
\]

Using the same approach as in section 3.2.1 and performing the integration, \( Q_g \) becomes:

\[
Q_g = \frac{1}{3} \frac{C_{ox}^2 W_{ef}^2 \mu_{ef}}{I_d} (V_{gst}^3 - V_{gstd}^3) - \frac{1}{2} \frac{W_{ef} C_{ox}}{E_{sat}} (V_{gstd}^2 - V_{gst}^2) + Q_{bulk}
\]

Therefore, \( \frac{\partial Q_g}{\partial V_g} \) is:

\[
\frac{\partial Q_g}{\partial V_g} = \frac{1}{3} \frac{C_{ox}^2 W_{ef}^2}{I_d} \left[ \left( \frac{\partial \mu_{ef}}{\partial V_g} - \frac{\mu_{ef}}{I_d} g_m \right) (V_{gst}^3 - V_{gstd}^3) + 3 \mu_{ef} (V_{gstd}^3 - V_{gst}^3) \right]
\]

\[
- \frac{1}{2} W_{ef} C_{ox} \left[ \frac{\partial (1/E_{sat})}{\partial V_g} (V_{gstd}^2 - V_{gst}^2) + \frac{2}{E_{sat}} (V_{gstd}^2 - V_{gst}^2) \right]
\]
To compute $C_{gs}$, the above equation and the $C_{gd}$ equation from section 3.2.1 will be substituted into $C_{gs} = -\frac{\partial Q_g}{\partial V_g} - C_{gd}$.

### 3.2.4 $C_{gs}$ in the Saturation Region ($V_{ds} > V_{dh}$, $V_{gs} > V_{gh}$)

Following the same approach in section 3.2.3 and 3.2.2 (figure 3.2.2a), $Q_g$ is expressed as:

$$Q_g = Q_{bulk} + Q_{lin} + Q_{sat}$$

$$Q_{lin} = \frac{1}{3} \frac{\mu_{eff}}{I_d} W_{eff}^2 C_{ox}^2 \left( V_{gstdsat}^3 - V_{gst}^3 \right) - \frac{1}{2} \frac{W_{eff} C_{ox}}{E_{sat}} \left( V_{gstdsat}^2 - V_{gst}^2 \right)$$

$$Q_{sat} = -W_{eff} C_{ox} V_{gstdsat} \Delta L$$

$Q_{sat}$ is the charge in the velocity saturation region. $Q_{lin}$ is the charge from $y_s$ to $y_{sat}$ (figure 3.2). $Q_{bulk}$ is the depletion charge. Since $Q_{bulk}$ is a weak function of $V_g$, $\frac{\partial Q_g}{\partial V_g}$ is:

$$\frac{\partial Q_g}{\partial V_g} = \frac{\partial Q_{lin}}{\partial V_g} + \frac{\partial Q_{sat}}{\partial V_g}$$

$$\frac{\partial Q_{lin}}{\partial V_g} = \frac{1}{3} \frac{C_{ox}^2 W_{eff}}{I_d} \left[ \left( \frac{\mu_{eff}}{V_{gstdsat}} - \mu_{eff} I_d g_m \right) V_{gstdsat}^3 - V_{gst}^3 \right] + 3 \mu_{eff} \left( V_{gstdsat}^2 \left( 1 - \frac{\partial V_{dsat}}{\partial V_g} \right) - V_{gst}^2 \right)$$

$$- \frac{1}{2} W_{eff} C_{ox} \left[ \frac{\partial (1/E_{sat})}{\partial V_g} \right] \left( V_{gstdsat}^2 - V_{gst}^2 \right) + \frac{2}{E_{sat}} \left( V_{gstdsat} \left( 1 - \frac{\partial V_{dsat}}{\partial V_g} \right) - V_{gst}^2 \right)$$

$$\frac{\partial Q_{sat}}{\partial V_g} = -W_{eff} C_{ox} V_{gstdsat} \frac{\partial \Delta L}{\partial V_g}$$

$$\frac{\partial V_{dsat}}{\partial V_g} = V_{dsat}^2 \left( \frac{1}{V_{gst}^2} - \frac{1}{L} \frac{\partial (1/E_{sat})}{\partial V_g} \right)$$
\[
\frac{\partial \Delta L}{\partial V_g} = \frac{\ell E_{\text{sat}}}{(V_d - V_{\text{dsat}}) / \ell + E_m} \left[ -1 \frac{\partial V_{\text{dsat}}}{\partial V_g} + \frac{\partial E_m}{\partial V_g} \frac{V_{\text{dsat}} - V_d}{E_{\text{sat}}} + \left( \frac{V_{\text{dsat}} - V_d}{E_{\text{sat}}} + E_m \right) \frac{\partial}{\partial V_g} \frac{1}{E_{\text{sat}}} \right]
\]

\[
\frac{\partial E_m}{\partial V_g} = \frac{1}{E_m} \left[ -\frac{\partial V_{\text{dsat}}}{\partial V_g} \frac{V_{\text{dsat}} - V_d}{\ell^2} + E_{\text{sat}} \frac{\partial E_{\text{sat}}}{\partial V_g} \right]
\]

To compute \( C_{gs} \), the above equations and the \( C_{gd} \) equations from section 3.2.2 will be substituted into \( C_{gs} = -\frac{\partial Q_g}{\partial V_g} - C_{gd} \).

### 3.2.5 Connection Scheme between Linear and Saturation Region \((V_{gs}>V_{gh}, V_{dl}<V_{ds}<V_{dh})\)

To reduce the discontinuity between the linear and saturation region, a linear function is used to connect the two regions. A straight line is drawn between \( C_{gd}(C_{gs}) \) at \( V_{dh} \) to \( C_{gd}(C_{gs}) \) at \( V_{dl} \). Figure 3.2.5a below illustrates this idea. The linear function used is

\[
C_{gd} = a_{cgd} V_{ds} + b_{cgd}
\]

\[
a_{cgd} = \frac{C_{gdh} - C_{gdl}}{V_{dh} - V_{dl}} \quad b_{cgd} = C_{gdh} - a_{cgd} V_{dh}
\]

\[
C_{gs} = a_{cgs} V_{ds} + b_{cgs}
\]

\[
a_{cgs} = \frac{C_{gs} - C_{gsl}}{V_{dh} - V_{dl}} \quad b_{cgs} = C_{gsh} - a_{cgs} V_{dh}
\]
$C_{gs}$ and $C_{gdh}$ are computed using the equations in the saturation region at the applied $V_{gs}$ and $V_{dh}$. $C_{gs}$ and $C_{gdl}$ are computed using the equations in the linear region at the applied $V_{gs}$ and $V_{dl}$.

### 3.3 Subthreshold Region Model

#### 3.3.1 $C_{gd}$ Model ($V_{gs} < V_{gh}$)

Figure 3.3.1a shows the $C_{gd}$ and $C_{gs}$ data in a $V_g$ sweep. Following the behavior of a bulk MOSFET, $C_{gd}$ gradually decreases to 0 when the TFT is switched from the strong inversion to the weak inversion region. Figure 3.3.1b shows the scheme of the empirical model from the strong inversion to weak inversion region. A linear function, $C_{gd} = \frac{C_{gdh}}{V_{gtran}} (V_{gs} - V_{gl})$, is used to modeled this region. The boundaries of the region are $V_{gh}$ and $V_{gl}$. $V_{gtran}$ is $V_{gtranIC} + V_{gtranNC}$ (figure 3.3.1b). $C_{gdh}$ is computed using the equations in the strong inversion region at the applied $V_{ds}$ and $V_{gh}$.
However, the RC coupling between the drain and substrate makes $C_{gd}$ increases in the GIDL [3] dominant region (figure 3.3.1a). This does not happen in bulk MOSFET because of the presence of the bulk contact. The coupling efficiency depends on the junction leakage. $A_{cgd}$ describes this coupling efficiency. Therefore, $C_{gd}$ at $V_{gs}<V_{gl}$ is modeled as:

$$C_{gd} = \left( \frac{1}{W_{eff}L_{eff}C_{ox}} + \frac{1}{A_{cgd}I_D} \right)^{-1}$$

The maximum $C_{gd}$ possible is $W_{eff}L_{eff}C_{ox}$. The above formulation will limit $C_{gd}$ to $W_{eff}L_{eff}C_{ox}$.

3.3.2 $C_{gs}$ Model ($V_{gs} < V_{gh}$)

$C_{gs}$, similar to $C_{gd}$ (figure 3.3.1a), also gradually drops to 0 from strong inversion to weak inversion region. The same scheme from section 3.3.1 is applied to model the gradual change in $C_{gs}$ when $V_{gs}$ is between $V_{gl}$ and $V_{gh}$, i.e. $V_{gl} < V_{gs} < V_{gh}$. $V_{gtran}$ is $V_{gtranhc} + V_{gtranlc}$ (figure 3.3.2a). $C_{gsh}$ is computed using the equations in the strong inversion region at the applied $V_{ds}$ and $V_{gh}$.

$$C_{gs} = \frac{C_{gsh}}{V_{gtran}}(V_{gs} - V_{gl})$$

The RC coupling effect is higher in $C_{gs}$ (figure 3.3.1a). $A_{cgs}$ describes this coupling efficiency. $C_{gs}$ at $V_{gs} < V_{gl}$ is modeled as:

$$C_{gs} = \left( \frac{1}{W_{eff}L_{eff}C_{ox}} + \frac{1}{A_{cgs}I_D} \right)^{-1}$$

Figure 3.3.1b : Modeling scheme for $C_{gd}$ weak inversion region
The maximum $C_{gs}$ possible is $W_{eff}L_{eff}C_{ox}$. The above formulation will limit $C_{gs}$ to $W_{eff}L_{eff}C_{ox}$.

Figure 3.3.2a: Modeling scheme for subthreshold $C_{gs}$
3.4 Verification

The parasitic capacitance from the measuring equipment and the drain-source overlapped capacitance are subtracted from the data in the following graphs. Figure 3.4a and b plot the $C_{gs}$ and $C_{gd}$ vs $V_{ds}$ of a n-channel TFT. Figure 3.4c and d plot the $C_{gs}$ and $C_{gd}$ vs $V_{ds}$ of a p-channel TFT. Figure 3.4e and f plot the $C_{gs}$ and $C_{gd}$ vs $V_{gs}$ of a n-channel TFT.

![Graph showing $C_{gd}$ vs $V_{ds}$ for n-channel TFT](image)

Figure 3.4a: $C_{gd}$ vs $V_{ds}$ for NTFT with $W_{eff}/L_{eff}=20/5.38$ and $T_{ox}=76$nm
Figure 3.4b: $C_{gs}$ vs $V_{ds}$ for NTFT with $W_{eff}/L_{eff}=20/5.38$ and $T_{ox}=76\text{nm}$

Figure 3.4c: $C_{gd}$ vs $V_{ds}$ for PTFT with $W_{eff}/L_{eff}=20/4.42$ and $T_{ox}=76\text{nm}$
Figure 3.4d: $C_{gs}$ vs $V_{ds}$ for PTFT with $W_{eff}/L_{eff} = 20/4.42$ and $T_{ox} = 76$nm

Figure 3.4e: $C_{gd}$ vs $V_{gs}$ for NTFT with $W_{eff}/L_{eff} = 20/5.38$ and $T_{ox} = 76$nm
Figure 3.4f: $C_{gs}$ vs $V_{gs}$ for NTFT with $W_{eff}/L_{eff}=20/5.38$ and $T_{ox}=76$nm
Chapter 4: Spice Implementation

4.1 Overview

The special features of SPICE implementation of the model and the model parameters names are discussed in this chapter. Section 4.2 lists the model parameters, their meanings and their default values. Section 4.3 describes special features in the implementation of the model.

4.2 Model Parameters

Table 4.2 lists all the parameters used in this model. Symbol is the symbol used in the equations summarized in appendix A. Spice Name is the spice model parameter name.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Spice Name</th>
<th>Meaning</th>
<th>Unit</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>temp</td>
<td>operating temperature</td>
<td>K</td>
<td>300</td>
</tr>
<tr>
<td>b</td>
<td>vto or vt0</td>
<td>temperature dependence of VTO (V_T = V_TO + bT)</td>
<td>V/K</td>
<td>0.01</td>
</tr>
<tr>
<td>V_TO</td>
<td>vto or vt0</td>
<td>threshold voltage at T=0K</td>
<td>V</td>
<td>4.5</td>
</tr>
<tr>
<td>μ0</td>
<td>u0 or uo</td>
<td>mobility parameter in pre-exponential term</td>
<td>cm²/Vs</td>
<td>50.5</td>
</tr>
<tr>
<td>μ1</td>
<td>u1</td>
<td>mobility parameter for temperature dependence</td>
<td>m/s²</td>
<td>0.134</td>
</tr>
<tr>
<td>μ2</td>
<td>u2</td>
<td>mobility parameter in exponential term</td>
<td>m/s</td>
<td>1750</td>
</tr>
<tr>
<td>μ3</td>
<td>u3</td>
<td>mobility parameter for temperature dependence</td>
<td>m/s</td>
<td>0.00308</td>
</tr>
<tr>
<td>μ4</td>
<td>u4</td>
<td>mobility parameter for low gate bias fitting</td>
<td>cm²/Vs</td>
<td>2</td>
</tr>
<tr>
<td>v_{sat}</td>
<td>vmax</td>
<td>maximum drift velocity</td>
<td>m</td>
<td>10⁵</td>
</tr>
<tr>
<td>ℓ</td>
<td>l2</td>
<td>channel length modulation parameter</td>
<td>m</td>
<td>10⁻¹⁰</td>
</tr>
<tr>
<td>θ</td>
<td>phita</td>
<td>DIBL effect parameter</td>
<td>V</td>
<td>0.05</td>
</tr>
<tr>
<td>s₁</td>
<td>s1</td>
<td>hot carrier pre-exponential parameter</td>
<td>V⁻¹</td>
<td>1.2</td>
</tr>
<tr>
<td>s₂</td>
<td>s2</td>
<td>hot carrier exponential parameter</td>
<td>V</td>
<td>30</td>
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<tr>
<td>n</td>
<td>subslope</td>
<td>diffusion current subthreshold slope</td>
<td>V</td>
<td>6.5</td>
</tr>
<tr>
<td>V_{off}</td>
<td>voff</td>
<td>offset voltage of diffusion current</td>
<td>V</td>
<td>0</td>
</tr>
<tr>
<td>i_{do}</td>
<td>ido</td>
<td>diffusion current pre-exponential factor per unit width</td>
<td>A/m</td>
<td>0.000625</td>
</tr>
<tr>
<td>A_{gidl}</td>
<td>gidla</td>
<td>GIDL effect pre-exponential parameter per unit width</td>
<td>A/V/m</td>
<td>0.00187</td>
</tr>
<tr>
<td>B_{gidl}</td>
<td>gidlb</td>
<td>GIDL effect exponential parameter</td>
<td>V</td>
<td>90</td>
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<tr>
<td>V_i</td>
<td>gidlv</td>
<td>GIDL effect offset voltage</td>
<td>V</td>
<td>1.12</td>
</tr>
<tr>
<td>I_{thermal0}</td>
<td>thermal</td>
<td>thermal current pre-exponential parameter / unit width</td>
<td>A/m</td>
<td>62.5n</td>
</tr>
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<td>E_a</td>
<td>ea</td>
<td>activation energy for I_{thermal}</td>
<td>eV</td>
<td>0.5</td>
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<tr>
<td>L_{diff}</td>
<td>ld</td>
<td>lateral diffusion for channel length</td>
<td>m</td>
<td>0</td>
</tr>
<tr>
<td>W_{diff}</td>
<td>lw</td>
<td>lateral diffusion for channel width</td>
<td>m</td>
<td>0</td>
</tr>
<tr>
<td>T_{ox}</td>
<td>tox</td>
<td>gate oxide thickness</td>
<td>m</td>
<td>85</td>
</tr>
<tr>
<td>cmod</td>
<td>cmod</td>
<td>capacitance model selection flag</td>
<td>V</td>
<td>1</td>
</tr>
<tr>
<td>A_{cgs}</td>
<td>acgs</td>
<td>subthreshold Cgs modeling parameter</td>
<td>F/A</td>
<td>10⁻⁸</td>
</tr>
<tr>
<td>A_{cgd}</td>
<td>acgd</td>
<td>subthreshold Cgd modeling parameter</td>
<td>F/A</td>
<td>10⁻⁸</td>
</tr>
<tr>
<td>V_{granl}</td>
<td>vgtranl</td>
<td>transition parameter for I_{d} in V_{gs} domain</td>
<td>V</td>
<td>1.5</td>
</tr>
<tr>
<td>V_{granh}</td>
<td>vgtranh</td>
<td>transition parameter for I_{d} in V_{gs} domain</td>
<td>V</td>
<td>0.5</td>
</tr>
<tr>
<td>V_{draln}</td>
<td>vdranl</td>
<td>transition parameter for I_{d} in V_{gs} domain</td>
<td>V</td>
<td>0.1</td>
</tr>
<tr>
<td>V_{draln}</td>
<td>vgtranhc</td>
<td>transition parameter for cap model in the V_{gs} domain</td>
<td>V</td>
<td>1</td>
</tr>
<tr>
<td>V_{granc}</td>
<td>vgtranlc</td>
<td>transition parameter for cap model in the V_{gs} domain</td>
<td>V</td>
<td>1.5</td>
</tr>
<tr>
<td>V_{dralnc}</td>
<td>vdranhc</td>
<td>transition parameter for cap model in the V_{ds} domain</td>
<td>V</td>
<td>0.5</td>
</tr>
<tr>
<td>V_{dralnc}</td>
<td>vgtranlc</td>
<td>transition parameter for cap model in the V_{ds} domain</td>
<td>V</td>
<td>0.5</td>
</tr>
</tbody>
</table>
4.3: Implementation of Model

4.3.1: Cgd and Cgs Model Implementation

Cgd and Cgs are proportional to $1/I_d^2$ in the strong inversion region. When $V_{ds}$ is small, $I_d$ is very small. This may cause $C_{gd}$ to become unreasonably big and inaccurate. Therefore $C_{gd}$ and $C_{gd}$ are set to $0.5C_{ox}$ when $V_{ds}<0.1V$ to prevent this situation.

Two capacitance models are implemented. They are (1) the one described in chapter 3, and (2) a simplified version of chapter 3 for speed consideration. The flag $cmod$ is used to specified which model to use. When $cmod$ is 1, (1) will be used. When $cmod$ is 2, (2) will be used. (1) and (2) are identical in the subthreshold region and strong inversion linear region. The difference is in the saturation region. $C_{gd}$ and $C_{gs}$ of (2) in the saturation region are set to 0 and $2/3C_{ox}L_{eff}W_{eff}$ (see fig 4.3.1a).

![Figure 4.3.1.a: Difference between the two capacitance model (cmod=1 and 2)](image)
Chapter 5: Simulation Results

5.1 Overview

This chapter compares the results of a 33-stage ring oscillator simulation with measured data. The drawn size of all p-channel TFT is 16μm/7μm. The drawn size of all n-channel TFT is 9μm/7μm. The oxide thickness is 76nm. Section 5.2 compares and discusses the simulation results and measured data. The data will be presented in both tables and graphs.

5.2 Simulation Results and Measured Data

Table 5.2a tabulates the simulation results and measured data. Figure 5.2a and 5.2b plots the results in the table. A positive error means the simulation overestimate the data. A negative error means the simulation underestimate the data.

<table>
<thead>
<tr>
<th>Vcc (V)</th>
<th>Freq from Simulation (MHz)</th>
<th>Freq from Measured data (MHz)</th>
<th>% Error</th>
<th>Power from Simulation (mW)</th>
<th>Power from Measured Data (mW)</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.62</td>
<td>0.277</td>
<td>124</td>
<td>0.075</td>
<td>0.033</td>
<td>127</td>
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<td>7</td>
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<td>0.504</td>
<td>65</td>
<td>0.145</td>
<td>0.084</td>
<td>73</td>
</tr>
<tr>
<td>8</td>
<td>1.13</td>
<td>0.788</td>
<td>43</td>
<td>0.255</td>
<td>0.171</td>
<td>49</td>
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<tr>
<td>9</td>
<td>1.48</td>
<td>1.11</td>
<td>33</td>
<td>0.42</td>
<td>0.318</td>
<td>32</td>
</tr>
<tr>
<td>10</td>
<td>1.8</td>
<td>1.52</td>
<td>18</td>
<td>0.65</td>
<td>0.548</td>
<td>19</td>
</tr>
<tr>
<td>11</td>
<td>2.15</td>
<td>1.92</td>
<td>12</td>
<td>0.935</td>
<td>0.854</td>
<td>9.5</td>
</tr>
<tr>
<td>12</td>
<td>2.36</td>
<td>2.42</td>
<td>-2</td>
<td>1.38</td>
<td>1.31</td>
<td>5</td>
</tr>
<tr>
<td>13</td>
<td>2.78</td>
<td>2.99</td>
<td>-7</td>
<td>1.82</td>
<td>1.92</td>
<td>-5</td>
</tr>
<tr>
<td>14</td>
<td>3.2</td>
<td>3.66</td>
<td>-12</td>
<td>2.45</td>
<td>2.82</td>
<td>-13</td>
</tr>
<tr>
<td>15</td>
<td>3.5</td>
<td>4.33</td>
<td>-19</td>
<td>3.15</td>
<td>4.02</td>
<td>-21</td>
</tr>
</tbody>
</table>
Figure 5.2a: Compare simulated and measured frequency of a 33-stage ring oscillator.

Figure 5.2b: Compare simulated and measured power of a 33-stage ring oscillator.
The simulation results in the higher $V_{cc}$ region is more accurate than those in the lower $V_{cc}$ region. In the lower $V_{cc}$ region, the simulations overestimate the frequency and power, because the capacitance model underestimates the capacitance data by a large margin. Therefore the percentage error is big. In the higher $V_{cc}$ region, the simulations underestimate the frequency and power, because the drain current model underestimates the drain current data by a small margin. Therefore the percentage error is small.
Appendix A: Equations Implemented in SPICE

This appendix summaries all equations implemented in spice3e1. Section 1 lists the drain current model equations. Section 2 lists the capacitance model equations. \( g_m \) is \( \partial I_d / \partial V_{gs} \). \( g_{ds} \) is \( \partial I_d / \partial V_{ds} \).

Section 1: Drain Current Equations
First of all, we need to define the boundaries of different regions (Figure 2.1a) and some common symbols.

\[
V_T = V_{To} - bT \quad V_{gst} = V_{gs} - V_T \quad C_{ox} = \varepsilon_{ox} / T_{ox}
\]

\[
V_{dsat} = \left( \frac{1}{V_{gst}} + \frac{1}{E_{sat} L_{eff}} \right)^{-1} \quad V_{dss} = V_{ds} - V_{dsat} \quad E_{sat} = \frac{2v_{sat}}{\mu_{eff}}
\]

\[
L_{eff} = L - 2L_{diff} \quad W_{eff} = W - 2W_{diff}
\]

\[
\mu_{eff} = \mu_0 \left( \frac{kT}{q} \right)^{-\mu_1} \exp \left( \frac{-q\mu_2 \exp(\mu_3 T)}{kT_{Cox} V_{gst}} \right) + \mu_4
\]

\[
V_{dl} = V_{dsat} - V_{dtranh} \quad V_{dh} = V_{dsat} + V_{dtranh}
\]

\[
V_{gl} = V_T - V_{gtranh} \quad V_{gh} = V_T + V_{gtranh}
\]

1.1 Strong Inversion Linear Region: \( V_{gs} > V_{gh} \) and \( 0 < V_{ds} < V_{dl} \)

\[
I_d = \frac{W_{eff}}{L_{eff}} C_{ox} \mu_{eff} \left( V_{gst} - \frac{V_{ds}}{2} \right) \frac{V_{ds}}{1 + \frac{V_{ds}}{E_{sat} L_{eff}}}
\]

\[
g_{ds} = \frac{W_{eff}}{L_{eff}} \frac{\mu_{eff} C_{ox}}{1 + \frac{V_{ds}}{E_{sat} L_{eff}}} \left[ \left( V_{gst} - V_{ds} \right) - \frac{\left( V_{gst} - V_{ds}/2 \right) V_{cs}}{E_{sat} L_{eff} \left( 1 + \frac{V_{ds}}{E_{sat} L_{eff}} \right)} \right]
\]
\[ g_m = \frac{W_{\text{eff}}}{L_{\text{eff}}} C_{\text{ox}} \left[ 1 + \frac{1}{V_{ds}} \right] \frac{1}{E_{\text{sat}} L_{\text{eff}}} \left[ \left( V_{gst} - V_{ds}/2 \right) V_{ds} \frac{\partial \mu_{\text{eff}}}{\partial V_{gs}} + \mu_{\text{eff}} V_{ds} - \frac{\mu_{\text{eff}} V_{ds}^2}{L_{\text{eff}} + \frac{V_{ds}}{E_{\text{sat}}}} \frac{\partial (1/E_{\text{sat}})}{\partial V_{gs}} \right] \]

\[ \frac{\partial V_{\text{gs}}}{} = \frac{1}{2V_{\text{sat}}} \frac{\partial \mu_{\text{eff}}}{\partial V_{gs}} \]

\[ \frac{\partial \mu_{\text{eff}}}{\partial V_{gs}} = \frac{(\mu_{\text{eff}} - \mu_d) \mu_2 \exp(\mu_3 T)}{kTC_{\text{ox}} V_{gst}^2} \]

1.2 Strong Inversion Saturation Region: \( V_{gs} > V_{gh} \) and \( V_{ds} > V_{dh} \)

\[ I_d = I_{\text{dsat}} \left( 1 + \frac{V_{\text{dss}}}{V_A} \right) f_{\text{kink}} \]

\[ V_A = \left( \frac{1}{V_{\text{ACHM}}} + \frac{1}{V_{\text{ADIBL}}} \right)^{-1} \]

\[ V_{\text{ACHM}} = \frac{\left( E_{\text{sat}} L_{\text{eff}} + V_{gst} \right) V_{\text{dss}}}{E_{\text{sat}} \ell} \quad V_{\text{ADIBL}} = \frac{E_{\text{sat}} L_{\text{eff}} + V_{gst}}{\theta \left( 1 + \frac{2E_{\text{sat}} L_{\text{eff}}}{V_{gst}} \right)} \]

\[ f_{\text{kink}} = 1 + s_1 V_{\text{dss}} \exp \left( \frac{-s_2}{V_{\text{dss}}} \right) \]

\[ I_{\text{dsat}} = \frac{W_{\text{eff}}}{L_{\text{eff}}} C_{\text{ox}} \mu_{\text{eff}} \left( V_{\text{gst}} - \frac{V_{\text{dsat}}}{2} \right) \frac{V_{\text{dsat}}}{1 + \frac{V_{\text{dsat}}}{E_{\text{sat}} L_{\text{eff}}}} \]

\[ g_{\text{ds}} = I_{\text{dsat}} s_1 \left( 1 + \frac{V_{\text{dss}}}{V_A} \right) \left( 1 + \frac{s_2}{V_{\text{dss}}} \right) \exp \left( \frac{-s_2}{V_{\text{dss}}} \right) + I_{\text{dsat}} \left( 1 - \frac{V_{\text{dss}}}{V_A} \frac{\partial V_A}{\partial V_{ds}} \right) f_{\text{kink}} \]

\[ \frac{\partial V_A}{\partial V_{ds}} = \frac{V_A^2 E_{\text{sat}} \ell}{(E_{\text{sat}} L_{\text{eff}} + V_{gst}) V_{\text{dss}}^2} \]

35
\[
g_m = \frac{\partial I_{dsat}}{\partial V_{gs}} \left( 1 + \frac{V_{dss}}{V_A} \right) f_{kink} - I_{dsat} \left( \frac{\partial V_{dsat}}{\partial V_{gs}} \frac{1}{V_A} + \frac{V_{dss}}{V_A^2} \frac{\partial V_A}{\partial V_{gs}} \right) f_{kink} + I_{dsat} \left( 1 + \frac{V_{dss}}{V_A} \right) \frac{\partial f_{kink}}{\partial V_{gs}}  
\]

\[
\frac{\partial V_{dsat}}{\partial V_{gs}} = V_{dsat}^2 \left( \frac{1}{V_A^2} - \frac{1}{2V_{sat}L_{eff}} \frac{\partial \mu_{eff}}{\partial V_{gs}} \right) 
\]

\[
\frac{\partial f_{kink}}{\partial V_{gs}} = -\left( 1 + \frac{s_2}{V_{dss}} \right) s_1 \exp \left( \frac{s_2}{V_{dss}} \right) \frac{\partial V_{dsat}}{\partial V_{gs}} 
\]

\[
\frac{\partial I_{dsat}}{\partial V_{gs}} = \frac{E_{sat}^\ell}{V_{dss} \left( V_{gs} + V_{sat}L_{eff} \right)} \left[ \frac{V_{gst} - V_{dsat}/2) V_{dsat} \frac{\partial \mu_{eff}}{\partial V_{gs}} + \mu_{eff} V_{dsat}^2 \left( V_{gst} - V_{dsat}/2 \right) \frac{\partial \left( 1/E_{sat} \right)}{\partial V_{gs}} \right] 
\]

\[
\frac{\partial V_A}{\partial V_{gs}} = -V_A^2 \left( \frac{\partial \left( 1/V_{ACHM} \right)}{\partial V_{gs}} + \frac{\partial \left( 1/V_{ADIBL} \right)}{\partial V_{gs}} \right) 
\]

\[
\frac{\partial \left( 1/V_{ACHM} \right)}{\partial V_{gs}} = E_{sat}^\ell \left( 1 + \frac{L_{eff}}{V_{dss} \frac{\partial E_{sat}}{\partial V_{gs}}} \right) \left[ \frac{1}{V_{dss}} \frac{\partial V_{dsat}}{\partial V_{gs}} - \frac{1 + L_{eff}}{E_{sat}L_{eff} + V_{gst}} \frac{\partial E_{sat}}{\partial V_{gs}} \right] 
\]

\[
\frac{\partial E_{sat}}{\partial V_{gs}} = -2V_{sat} \frac{\partial \mu_{eff}}{\partial V_{gs}} 
\]

\[
\frac{\partial \left( 1/V_{ADIBL} \right)}{\partial V_{gs}} = 2\theta \left( \frac{L_{eff} \frac{\partial E_{sat}}{V_{gst}} - E_{sat}L_{eff}}{V_{gst} \frac{\partial V_{dsat}}{V_{gst}} - V_{dss}^2} \right) \left( 1 + \frac{2E_{sat}L_{eff}}{V_{gst}} \right) \left( L_{eff} \frac{\partial E_{sat}}{\partial V_{gs}} + 1 \right) 
\]

\[
\frac{\partial \left( 1/V_{ADIBL} \right)}{\partial V_{gs}} = \frac{2\theta}{E_{sat}L_{eff} + V_{gst}} - \frac{\theta \left( 1 + \frac{2E_{sat}L_{eff}}{V_{gst}} \right) L_{eff} \frac{\partial E_{sat}}{\partial V_{gs}} + 1 \right)}{(E_{sat}L_{eff} + V_{gst})^2} 
\]
1.3 Subthreshold Region: $V_{gs} < V_{gl}$

\[
E_{sat} = \frac{2V_{sat}}{\mu_{eff}} \\
\mu_{eff} = \mu_{o} \left( \frac{kT}{q} \right)^{-\mu_{1}} \\
V_{dsat} = \left( \frac{1}{V_{gran}} + \frac{1}{E_{sat} L_{eff}} \right) \\
I_{d} = I_{diff} + I_{thermal} + I_{gil}\]

\[
I_{diff} = W_{eff} I_{do} \left( 1 - \exp \left( \frac{-V_{ds}}{kT/q} \right) \right) \exp \left( \frac{V_{gs} - V_{T} - V_{off}}{(kT/q)n} \right) \\
I_{gil} = W_{eff} A_{gil} (V_{dg} - V_{i}) \exp \left( \frac{-B_{gil}}{V_{dg} - V_{i}} \right) \\
I_{thermal} = W_{eff} I_{thermal0} \exp \left( \frac{E_{a}}{kT/q} \right)
\]

If $(V_{dg} - V_{i}) > 0$, then

\[
g_{ds} = \frac{W_{eff} I_{do}}{kT/q} \exp \left( \frac{-V_{ds}}{kT/q} \right) \exp \left( \frac{q(V_{gs} - V_{T} - V_{off})}{kTn} \right) + A_{gil} \left( 1 + \frac{B_{gil}}{V_{dg} - V_{i}} \right) \exp \left( \frac{-B_{gil}}{V_{dg} - V_{i}} \right) \\
g_{m} = \frac{W_{eff} I_{do}}{(kT/q)n} \left( 1 - \exp \left( \frac{-V_{ds}}{kT/q} \right) \right) \exp \left( \frac{V_{gs} - V_{T} - V_{off}}{(kT/q)n} \right) - A_{gil} \left( 1 + \frac{B_{gil}}{V_{dg} - V_{i}} \right) \exp \left( \frac{-B_{gil}}{V_{dg} - V_{i}} \right)
\]

else

\[
g_{ds} = \frac{W_{eff} I_{do}}{kT/q} \exp \left( \frac{-V_{ds}}{kT/q} \right) \exp \left( \frac{q(V_{gs} - V_{T} - V_{off})}{kTn} \right) \\
g_{m} = \frac{W_{eff} I_{do}}{(kT/q)n} \left( 1 - \exp \left( \frac{-V_{ds}}{kT/q} \right) \right) \exp \left( \frac{V_{gs} - V_{T} - V_{off}}{(kT/q)n} \right)
\]

1.4 Transition between Linear and Saturation Region: $V_{gs} > V_{gh}$ and $V_{dl} < V_{ds} < V_{dh}$

\[
V_{ds} = V_{dl} (1-t)^2 + 2V_{dp} t (1-t) + V_{dht}^2 \\
I_{d} = I_{dl} (1-t)^2 + 2I_{dp} t (1-t) + I_{dht}^2
\]

37
\[ I_{dl} = I_{d|V_{gs}, V_{ds}} \quad I_{dh} = I_{d|V_{gh}, V_{ds}} \]
\[ g_{dsl} = g_{ds|V_{gs}, V_{ds}} \quad g_{dsh} = g_{ds|V_{gh}, V_{ds}} \]
\[ g_{ml} = g_{m|V_{gs}, V_{ds}} \quad g_{mh} = g_{m|V_{gh}, V_{ds}} \]
\[ t = \frac{(V_{dl} - V_{dp}) + \sqrt{(V_{dp} - V_{dl})^2 - (V_{dl} - 2V_{dp} + V_{dh})(V_{dl} - V_{ds})}}{V_{dh} - 2V_{dp} + V_{dl}} \]
\[ V_{dp} = \frac{I_{dh} - I_{dl} - (g_{dsh}V_{dh} - g_{dsl}V_{dl})}{-g_{dsh} + g_{dsl}} \]
\[ I_{dp} = g_{dsl}(V_{dp} - V_{dl}) + I_{dl} \]
\[ g_{ds} = \frac{t(I_{dh} - I_{dp}) + (1-t)(I_{dp} - I_{dl})}{t(V_{dh} - V_{dp}) + (1-t)(V_{dp} - V_{dl})} \]
\[ g_{m} = g_{ml} + I_{d} - I_{dl} \frac{V_{dh} - V_{dl}}{V_{dh} - I_{dl} V_{ds} - V_{dl}} (g_{mh} - g_{ml}) \]

1.5 Transition between Strong Inversion and Subthreshold Region: \( V_{gl} < V_{gs} < V_{gh} \)
\[ V_{gs} = V_{gl}(1-t)^2 + 2V_{gp}(1-t) + V_{ght}t^2 \]
\[ \ln(I_{d}) = \ln(I_{dl})(1-t)^2 + 2\ln(I_{dp})(1-t) + \ln(I_{dh})t^2 \]
\[ I_{dl} = I_{d|V_{gl}, V_{ds}} \quad I_{dh} = I_{d|V_{gh}, V_{ds}} \]
\[ g_{dsl} = g_{ds|V_{gl}, V_{ds}} \quad g_{dsh} = g_{ds|V_{gh}, V_{ds}} \]
\[ g_{ml} = g_{m|V_{gl}, V_{ds}} \quad g_{mh} = g_{m|V_{gh}, V_{ds}} \]
\[ t = \frac{(V_{gl} - V_{gp}) + \sqrt{(V_{gp} - V_{gl})^2 - (V_{gl} - 2V_{gp} + V_{gh})(V_{gl} - V_{gs})}}{V_{gh} - 2V_{gp} + V_{gl}} \]
\[ V_{gp} = \frac{I_{dh} - I_{dl} - (g_{mh}V_{dh} - g_{ml}V_{dl})}{-g_{mh} + g_{ml}} \]
\[ I_{dp} = g_{ml}(V_{gp} - V_{gl}) + I_{dl} \]

\[ g_m = \frac{t(I_{dh} - I_{dp}) + (1 - t)(I_{dp} - I_{dl})}{t(V_{gh} - V_{gp}) + (1 - t)(V_{gp} - V_{gl})} \]

\[ g_{ds} = g_{dsl} + \frac{I_d - I_{dl}}{I_{dh} - I_{dl}} \frac{V_{gh} - V_{gl}}{V_{gst} - V_{gl}} (g_{dsh} - g_{dsl}) \]
Section 2: Capacitance Model Equations

First of all, we need to define the boundaries of different regions (Figure 3.1a) and some common symbols.

\[ V_T = V_{T_0} - bT \]
\[ V_{\text{dsat}} = \left( \frac{1}{V_{\text{gst}}} + \frac{1}{E_{\text{sat}}/L_{\text{eff}}} \right)^{-1} \]
\[ C_{\text{ox}} = \varepsilon_{\text{ox}} / T_{\text{ox}} \]

\[ V_{\text{gst}} = V_{\text{gs}} - V_T \]
\[ V_{\text{gstd}} = V_{\text{gs}} - V_{\text{ds}} \]
\[ V_{\text{gstdsat}} = V_{\text{gst}} - V_{\text{dsat}} \]

\[ V_{\text{dl}} = V_{\text{dsat}} - V_{\text{dtransc}} \]
\[ V_{\text{dh}} = V_{\text{dsat}} + V_{\text{dtransc}} \]

\[ V_{\text{gl}} = V_T - V_{\text{gtransc}} \]
\[ V_{\text{gh}} = V_T + V_{\text{gtransc}} \]

2.1 \( C_{gd} \) in Strong Inversion Linear Region: \( V_{\text{gs}} > V_{\text{gh}} \) and \( 0 < V_{\text{ds}} < V_{\text{dl}} \)

\[ C_{gd} = \frac{g_{ds}}{I_d^2} W_{\text{eff}}^2 \mu_{\text{eff}} \int_{V_s}^{V_d} Q_n^2(V) dV - \frac{W_{\text{eff}}^2 \mu_{\text{eff}}}{I_d} Q_n^2(V_{\text{ds}}) + \frac{W_{\text{eff}}}{E_{\text{sat}}} Q_n(V_{\text{ds}}) \]

\[ \int_{V_s}^{V_d} Q_n^2(V) dV = \frac{1}{3} C_{\text{ox}} \left( V_{\text{gst}}^3 - V_{\text{gstd}}^3 \right) \]

2.2 \( C_{gd} \) in the Strong Inversion Saturation Region: \( V_{\text{gs}} > V_{\text{gh}} \) and \( V_{\text{ds}} > V_{\text{dh}} \)

If \( \text{cmod} = 1 \), then

\[ C_{gd} = \frac{\partial Q_{\text{lin}}}{\partial V_d} + \frac{\partial Q_{\text{sat}}}{\partial V_d} \]

\[ \frac{\partial Q_{\text{lin}}}{\partial V_d} = \frac{1}{3} W_{\text{eff}}^2 C_{\text{ox}}^2 \mu_{\text{eff}} \frac{g_{ds}}{I_d^2} \left( V_{\text{gst}}^3 - V_{\text{gstdsat}}^3 \right) \]

\[ \frac{\partial Q_{\text{sat}}}{\partial V_d} = -W_{\text{eff}} C_{\text{ox}} V_{\text{gstdsat}} \frac{\partial \Delta L}{\partial V_d} \]

\[ \frac{\partial \Delta L}{\partial V_d} = \frac{1}{(V_{\text{ds}} - V_{\text{dsat}}/\ell + E_{\text{m}}) \left( 1 + \frac{V_{\text{ds}} - V_{\text{dsat}}}{\ell E_{\text{m}}} \right)} \]
\[ E_m = \left( \frac{V_{ds} - V_{dsat}}{E_{sat}} \right)^2 + E_{sat}^2 \right)^{1/2} \]

else if cmod=2, then

\[ C_{gd} = 0 \]

2.3 \textit{C}_{gd} \text{ in the Subthreshold Region: } V_{gs}\lessgtr V_{gh}

If \( V_{gs}\geq V_{gl} \), then

\[ C_{gd} = \frac{C_{gdh}}{V_{gtran}} (V_{gs} - V_{gl}) \quad V_{gtran} = V_{gtranlc} + V_{gtranhc} \]

else,

\[ C_{gd} = \left( \frac{1}{W_{eff}L_{eff}C_{ox}} + \frac{1}{A_{cgd}I_d} \right)^{-1} \]

\( C_{gdh} \) is the \( C_{gd} \) computed at \( V_{gs}=V_{gh} \) and the applied \( V_{ds} \). If \( V_{ds}>V_{dsat} \), then the equations in section 2.2 are used. Otherwise, the equations in section 2.1 are applied.

2.4 \textit{C}_{gd} \text{ in the Strong Inversion Transition Region: } V_{gs}\geq V_{gh} \text{ and } V_{dl}\lessgtr V_{ds}\lessgtr V_{dh}

\[ C_{gd} = a_{cgd} V_{ds} + b_{cgd} \quad a_{cgd} = \frac{C_{gdh} - C_{gdl}}{V_{dh} - V_{dl}} \quad b_{cgd} = C_{gdh} - a_{cgd} V_{dh} \]

\( C_{gdh} \) is the \( C_{gd} \) computed at \( V_{ds}=V_{dh} \) and the applied \( V_{gs} \) using equations in section 2.1. \( C_{gdh} \) is the \( C_{gd} \) computed at \( V_{ds}=V_{dh} \) and the applied \( V_{gs} \) using equations in section 2.2.

2.5 \textit{C}_{gs} \text{ in the Strong Inversion Linear Region: } V_{gs}\geq V_{gsgs} \text{ and } 0\lessgtr V_{ds}\lessgtr V_{dl}

\[ C_{gs} = -\frac{\partial Q_g}{\partial V_g} - C_{gd} \]

\[ \frac{\partial Q_g}{\partial V_g} = \frac{1}{3} \frac{C_{ox}^2 W_{eff}^2}{I_d} \left( \frac{\partial \mu_{eff}}{\partial V_{gs}} - \mu_{eff} g_m \right) \left( V_{gstd}^3 - V_{gst}^3 \right) + 3 \mu_{eff} \left( V_{gstd}^2 - V_{gst}^2 \right) \]

\[ C_{gs} = \frac{1}{2} W_{eff} C_{ox} \left[ \frac{\partial (1/E_{sat})}{\partial V_{gs}} (V_{gstd} - V_{gst})^2 + \frac{2}{E_{sat}} (V_{gstd} - V_{gst}) \right] \]
$C_{gd}$ is the $C_{gd}$ computed at the applied $V_{gs}$ and $V_{ds}$.

2.6 $C_{gs}$ in the Strong Inversion Saturation Region: $V_{gs}>V_{gh}$ and $V_{ds}>V_{dh}$

If $cmod=1$, then

\[
C_{gs} = -\frac{\partial Q_g}{\partial V_g} - C_{gd}
\]

\[
\frac{\partial Q_g}{\partial V_g} = \frac{\partial Q_{lin}}{\partial V_g} + \frac{\partial Q_{sat}}{\partial V_g}
\]

\[
\frac{\partial Q_{lin}}{\partial V_g} = \frac{1}{3} \frac{C_{ox}^2}{I_d} \left[ \left( \frac{\partial \mu_{eff}}{\partial V_{gs}} - \frac{\mu_{eff}}{I_d} \right) g_m \left( V_{gstdsat}^3 - V_{gst}^3 \right) + 3 \mu_{eff} \left( V_{gstdsat}^2 \left( 1 - \frac{\partial V_{dsat}}{\partial V_{gs}} \right) \right) - V_{gst}^2 \right]
\]

\[
-\frac{1}{2} \frac{W_{eff} C_{ox}}{V_{gs}} \left[ \frac{\partial (1/E_{sat})}{\partial V_{gs}} \left( V_{gstdsat}^2 - V_{gst}^2 \right) + 2 \frac{E_{sat}}{V_{gstdsat}} \left( 1 - \frac{\partial V_{dsat}}{\partial V_{gs}} \right) \right]
\]

\[
\frac{\partial Q_{sat}}{\partial V_{gs}} = -W_{eff} C_{ox} \left( 1 - \frac{\partial V_{dsat}}{\partial V_{gs}} \right) \Delta L - W_{eff} C_{ox} V_{gstdsat} \frac{\partial \Delta L}{\partial V_{gs}}
\]

\[
\Delta L = \ell \ln \left( \frac{V_{ds} - V_{dsat}}{V_{dsat} - V_{gst}} / \ell + E_m \right)
\]

\[
\frac{\partial \Delta L}{\partial V_{gs}} = \frac{\ell E_m}{(V_{ds} - V_{dsat}) / \ell + E_m} \left[ -\frac{1}{\ell} \frac{\partial V_{dsat}}{\partial V_{gs}} + \frac{E_m}{E_{sat}} \right] + \left( \frac{V_{ds} - V_{dsat}}{\ell} + E_m \right) \frac{\partial (1/E_{sat})}{\partial V_{gs}}
\]

\[
\frac{\partial E_m}{\partial V_{gs}} = \frac{1}{E_m} \left[ -\frac{\partial V_{dsat}}{\partial V_{gs}} \frac{V_{ds} - V_{dsat}}{\ell} + E_{sat} \frac{\partial E_{sat}}{\partial V_{gs}} \right]
\]

else if $cmod=2$, then

\[
C_{gs} = \frac{2}{3} C_{ox} L_{eff} W_{eff}
\]

2.7 $C_{gs}$ in the Subthreshold Region: $V_{gs}<V_{gh}$

If $V_{gs}>V_{gl}$, then
\[ C_{gs} = \frac{C_{gsh}}{V_{gtr}^{\text{eff}}} (V_{gs} - V_{gl}) \]

\[ V_{gtr} = V_{gtr^{\text{nc}}} + V_{gtr^{\text{nc}}} \]

\[ C_{gs} = \left( \frac{1}{W_{\text{eff}} L_{\text{eff}} C_{ox}} + \frac{1}{A_{cgs} I_{d}} \right)^{-1} \]

\( C_{gsh} \) is the \( C_{gs} \) computed at \( V_{gs} = V_{gh} \) and the applied \( V_{ds} \). If \( V_{ds} > V_{dsat} \), then the equations in section 2.6 are used. Otherwise, the equations in section 2.5 are applied.

### 2.8 \( C_{gs} \) in the Strong Inversion Transition Region: \( V_{gs} > V_{gh} \) and \( V_{dl} < V_{ds} < V_{dh} \)

\[ C_{gs} = a_{cgs} V_{ds} + b_{cgs} \]

\[ a_{cgs} = \frac{C_{gsh} - C_{gsl}}{V_{dh} - V_{dl}} \]

\[ b_{cgs} = C_{gsh} - a_{cgs} V_{dh} \]

\( C_{gsl} \) is the \( C_{gs} \) computed at \( V_{ds} = V_{dl} \) and the applied \( V_{gs} \) using equations in section 2.5. \( C_{gsh} \) is the \( C_{gs} \) computed at \( V_{ds} = V_{dh} \) and the applied \( V_{gs} \) using equations in section 2.6.
Appendix B: Parameter Extraction

This section discusses the parameter extraction procedures used in this project. A spreadsheet program, EXCEL 4.0, is used to visually fit the model with the measured data for both the drain current and capacitance model parameters locally. Temperatures are in unit of Kelvin. Section 1 discusses the drain current parameters extraction. Section 2 discusses the capacitance parameter extraction.

Section 1: Drain Current Model Extraction

To extract the drain current parameters with temperature dependence, the following measurements are needed at different temperatures (e.g. 300K, 325K, 350K, and 375K) are needed. If temperature dependence is ignored, only one set of data is necessary.

1) $C_g V_{gs}$ data with both drain and source grounded (e.g. $V_{gs} = -3V$ to 12V)
2) $I_d V_{ds}$ data with several $V_{gs}$ bias bigger than $V_T$ (e.g. $V_{ds} = 0V$ to 12V and $V_{gs} = 3V$, 6V, 9V, and 12V)
3) $I_d V_{gs}$ data with different $V_{ds}$ bias (e.g. $V_{gs} = -5V$ to 12V and $V_{ds} = 0.1V$ and 5V)

The gate oxide thickness and process lateral diffusion length must be extracted first. Section 1.1 to 1.5 describes the extraction of the parameters in different regions. Section 1.6 discusses the order of extraction.

1.1: Threshold Voltage ($V_{TO}$ and $b$)

We use the equation $Q_n = C_{ox}(V_{gs} - V_T)$ to define $V_T$. First of all, $C_g V_{gs}$ data are measured with both the drain and source grounded. Then the parasitic capacitance is subtracted from $C_g$. $Q_n$ is computed by integrating the $C_g V_{gs}$ curve using the relation $Q_n(V_{gs}) = \int_{-\infty}^{V_{gs}} C_g dV$.

A straight line will be fitted to $Q_n$ and the x-intercept is $V_T$ (figure b1).
If the temperature dependence is ignored, $V_{TO}$ is $V_T$ and $b$ is 0. To extract the temperature dependence, $b$, $V_T$'s at several temperature are measured. Then a linear fit is used to determine $V_{TO}$ and $b$, where $V_T = V_{TO} - bT$.

1.2: Mobility ($\mu_0$, $\mu_1$, $\mu_2$, $\mu_3$, and $\mu_4$)

At a particular temperature, $\mu_{eff}$ is modeled with an expression in the form of

$$A \exp\left(-\frac{B}{V_{gs} - V_T}\right) + \mu_4,$$

where $A$ and $B$ are function of temperature, oxide thickness, and mobility parameters. An estimate of $A$, $B$, and $\mu_4$ are obtained by fitting the $\mu_{eff}$ at low drain bias (e.g. 0.1V) using the $I_dV_{gs}$ data. When $A$ increases and $B$ decreases, $\mu_{eff}$ increases. $\mu_4$ determines $\mu_{eff}$ at $V_{gs}$ close to $V_T$. $\mu_{eff}$ increases when $\mu_4$ increases. Since the objective is to fit the drain current in the strong inversion linear region, the estimated $A$ and $B$ are further optimized by fitting the $I_dV_{ds}$ data in the strong inversion linear region (figure b2) with $V_{gs} > V_T$. Using the IV curves at different temperatures, different sets of $A$ and $B$ are found. Then we can compute $\mu_0$, $\mu_1$, $\mu_2$, $\mu_3$, and $\mu_4$. 

Figure b1: $V_T$ extraction procedure
1.3: Saturation Velocity ($v_{sat}$)

$v_{sat}$ affects the magnitude of the drain current when $V_{ds}$ is near $V_{dsat}$ and the location of $V_{dsat}$. When $v_{sat}$ increases, $V_{dsat}$ and $I_d$ increase. $v_{sat}$ is extracted by visually fitting the drain current near $V_{dsat}$ and the location of $V_{dsat}$ (figure b3).

1.4: DIBL, Channel Length Modulation, and Hot Carrier Effects ($\ell$, $\theta$, $s_1$, and $s_2$)

$\ell$ and $\theta$ affects $V_A$. When $\ell$ and $\theta$ increase, $V_A$ drops and $I_d$ increases. $s_1$ and $s_2$ affect the hot carrier tail at high $V_{ds}$. When $s_1$ increases and $s_2$ decreases, the hot carrier effect will be more pronounced (figure b4). $s_1$ is set to 1.2 for NTFT and 2.2 for PTFT in this study. Only $s_2$ is varied to fit the data. However, the user can vary both $s_1$ and $s_2$ as they see fit.
1.5: Subthreshold Region \((n, V_{\text{off}}, I_{\text{do}}, A_{\text{gidi}}, B_{\text{gidi}}, V_i, E_a, \text{and } I_{\text{thermal0}})\)

\(I_{\text{do}}, n, \) and \(V_{\text{off}}\) affect the region where the diffusion current dominates. When \(n\) and \(I_{\text{do}}\) increase and \(V_{\text{off}}\) decreases, the diffusion current increases. Huang [1] shows that \(I_{\text{do}}\) is a function of substrate doping. \(I_{\text{do}}\) is treated as a model parameter in this model. \(A_{\text{gidi}}, B_{\text{gidi}},\) and \(V_i\) affects the region where the GIDL effect dominates \((V_{\text{dg}}\) is big). When \(A_{\text{gidi}}\) increases, \(B_{\text{gidi}}\) decreases, and \(V_i\) decreases, the GIDL current increases. \(I_{\text{thermal0}}\) and \(E_a\) set the minimum leakage current. The thermal generation current is not a function of bias. When \(E_a\) decreases and \(I_{\text{thermal0}}\) increases, the thermal generation current increases. To extract \(I_{\text{thermal0}}\) and \(E_a\), \(I_{\text{thermal}}\) at different temperature are extracted first. Then \(I_{\text{thermal0}}\) and \(E_a\) are extracted from the \(I_{\text{thermal}}\) found. If temperature dependence are not extracted, then \(I_{\text{thermal0}}\) is \(I_{\text{thermal}}\) and \(E_a\) is 0. Figure b5 illustrates the regions that the above parameters affect.
1.6: Order of Extraction

1) Extract threshold voltage parameter ($V_{TO}$ and $b$).
2) Extract mobility parameters in the linear region of $I_dV_d$ ($\mu_0$, $\mu_1$, $\mu_2$, $\mu_3$, and $\mu_4$).
3) Adjust $V_{sat}$ to fit the $V_{dsat}$ location and $I_d$ near $V_{dsat}$.
4) Adjust $\ell$, $\theta$, $s_1$, and $s_2$ to fit the saturation region of the $I_dV_d$ curves.
5) Extract $n$ and $V_{off}$ for diffusion current.
6) Extract $A_{gidl}$, $B_{gidl}$, $V_i$, and $I_{thermal}$ together for GIDL and thermal generation current.
7) Repeat the extraction of $I_{thermal}$ for different temperature to extract $I_{thermal0}$ and $E_a$.
8) Adjust $V_{granl}$, $V_{granh}$, $V_{dtranl}$, and $V_{dtrans}$ to improve continuity.

Section 2: Capacitance Model Extraction

For $C_{gs}$ and $C_{gd}$, only $A_{cgs}$, $A_{cgd}$, and the transition region parameters need to be extracted. The following measurements are needed.
1) $C_{gs}$ and $C_{gd}$ in a $V_{ds}$ sweep with different $V_{gs}$ bias (e.g. $V_{ds} = 0V$ to 12V and $V_{gs} = 3V$, 6V, 9V, and 12V)
2) $C_{gs}$ and $C_{gd}$ in a $V_{gs}$ sweep with different $V_{ds}$ bias (e.g. $V_{gs} = -5V$ to 12V and $V_{ds} = 3V$, 6V, 9V, and 12V)

The gate oxide thickness and process lateral diffusion length must be known. The parasitic capacitance and the overlap capacitance should be subtracted from the data. Drain current model parameters can also be slightly altered to fit the capacitance data more accurately.

2.1 $A_{cgs}$ and $A_{cgd}$

$A_{cgs}$ and $A_{cgd}$ affects $C_{gs}$ and $C_{gd}$ in the GIDL dominant region ($V_{dg}>0$, e.g. high $V_{ds}$ in accumulation region). When $A_{cgs}$ and $A_{cgd}$ increase, $C_{gs}$ and $C_{gd}$ increase. $A_{cgs}$ and $A_{cgd}$ are extracted by fitting the model with the data in the GIDL. The GIDL dominant region in 3.3.1a of chapter 3 is from between -10V and -2V. The figure is re-drawn below.
2.2 Transition Region Parameters ($V_{d_{tranl}}, V_{d_{tranhc}}, V_{g_{tranl}}, V_{g_{tranhc}}$)

To extract those transition region parameters, we can just examine the data and get a reasonable estimate. $V_{d_{tranl}}$ and $V_{d_{tranhc}}$ are the transition width from saturation to linear region on the $V_{ds}$ domain in $C_{gs}$-$V_{ds}$ and $C_{gd}$-$V_{ds}$ data. A default value of 0.5V for both $V_{d_{tranl}}$ and $V_{d_{tranhc}}$ works well for the data used in this project.

When the gate bias decreases below $V_T$, $C_{gs}$ and $C_{gd}$ gradually drop to 0. $V_{g_{tranl}}$ is the voltage below $V_T$ that $C_{gs}$ and $C_{gd}$ drop to 0. The $C_{gd}$ and $C_{gs}$ model assume a linear drop from $(V_T + V_{g_{tranhc}})$ to 0 at $(V_T - V_{g_{tranhc}})$. Therefore, $V_{g_{tranhc}}$ is the voltage above $V_T$ that the model begin the linear drop (see figure 3.3.2a of chapter 3).
Reference

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