CHUA'S CIRCUIT WITH CURRENT-OFFSET
CHUA DIODE

by
Michael Peter Kennedy

Memorandum No. UCB/ERL M92/47
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Abstract
This circuit uses a current source to force a current offset in the piecewise-linear characteristic of a Chua diode. The modified Chua diode consists of a negative resistance converter, two ideal diodes, and a current source.

1 Circuit description
Arrays of Chua's circuits have been shown by simulation to exhibit interesting wave propagation effects [PMPVC92]. This work is motivated by the need for identical Chua's circuits each of which can be altered in a similar manner by a single control signal.

We define Chua's diode [Ken92b] as any two-terminal nonlinear resistor which is described by an active but eventually passive [Chu69] v-i characteristic. This class of active nonlinear resistors was first identified by Leon Chua in the 1960s as the fundamental building blocks for nonlinear oscillation. Since then, he has developed comprehensive synthesis procedures for these elements [Chu69].

The v-i characteristic of a voltage-controlled nonlinear resistor may be offset along the i-axis by adding a current source in parallel with the element [CDK87]. This is illustrated in Figs. 1 and 2.

Chua's circuit using a current-offset Chua diode is shown in Fig. 3.

The desired nonlinear resistor characteristic is obtained by connecting in parallel a negative impedance converter \( N_{R1} \), two ideal diodes \( N_{R2} \) and \( N_{R3} \) [Ken92a], and a Howland current source \( N_{R4} \) [Ana92].

A negative resistance converter \( N_{R1} (A_1, R_1, R_2, \text{and } R_3) \) is used to produce the underlying negative resistance. Connected in parallel with the negative impedance converter are two ideal diodes [CDK87] \( (A_2, R_4, D_1) \) and \( (A_3, R_8, D_2) \) with dc offsets set by \( V^+, R_5, \) and \( R_6, \) and \( V^-, R_9, \) and \( R_{10} \) respectively. These connect a positive resistance \( R_7 \) in parallel with the negative resistor \( N_{R1} \) whenever the voltage \( v_R \) exceeds the breakpoints \( BP^- \) and \( BP^+ \) in magnitude. \( N_{R4} \) supplies the desired dc offset current \( I_{OS} \).

This architecture allows one to control independently the slopes, breakpoints, and current offset of the Chua diode's v-i characteristic. In addition, several circuits may be adjusted simultaneously by applying the same voltage \( V_{OS} \) to each circuit.

2 Example
We consider the case \( BP^+ = BP^- = 1V \), and \( \alpha = 10, \beta = 39 \).

Using a dual 15V supply to power the op amps gives \( V^+ = 15V \) and \( V^- = -15V \).

A complete list of components is given in Table 1. In addition, the power supplies should be decoupled by connecting two 100 nF capacitors from pins 4 and 11 of the 14-pin quad op amp package to ground as close to the package as possible.

- Resistor pairs \( R_5, R_6 \) and \( R_9, R_{10} \) act as voltage dividers to set the breakpoints of the v-i characteristic at

\[-BP^- = -\frac{1.00}{1.00 + 14.0}15V = -1V, \quad BP^+ = \frac{1.00}{1.00 + 14.0}15V = 1V\]
Figure 1: Chua diode (a) and its v-i characteristic (b).

Figure 2: (a) A current source $I_{OS}$ in parallel with the nonlinear resistor produces a current offset in the v-i characteristic (b).

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Figure 3: Realization of Chua's circuit using a current-offset Chua diode. The basic nonlinear resistor is constructed by connecting a negative resistance converter $N_{R_1}$ in parallel with two parallel-connected ideal diodes with dc offsets ($N_{R_1}$ and $N_{R_2}$) in series with a linear resistor $R_7$. The offset current is provided by a current source $N_{R_4}$. 
<table>
<thead>
<tr>
<th>Element</th>
<th>Description</th>
<th>Value</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{R_1}$</td>
<td>Negative resistance converter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_1$</td>
<td>$\frac{1}{4}$ AD713K quad BiFET op amp</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_1$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>150 $\Omega$</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>$R_2$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>150 $\Omega$</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>$R_3$</td>
<td>20-turn cermet trimmer</td>
<td>1.00 k$\Omega$</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>$R_7$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>2.32 k$\Omega$</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>$N_{R_2}$</td>
<td>Series ideal diode and voltage source</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_2$</td>
<td>$\frac{1}{4}$ AD713K quad BiFET op amp</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_1$</td>
<td>1N4148 silicon switching diode</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_4$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>2.20 k$\Omega$</td>
<td>$\pm 5%$</td>
</tr>
<tr>
<td>$R_5$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>14.0 k$\Omega$</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>$R_6$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>1.00 k$\Omega$</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>$N_{R_3}$</td>
<td>Series ideal diode and voltage source</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_3$</td>
<td>$\frac{1}{4}$ AD713K quad BiFET op amp</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_2$</td>
<td>1N4148 silicon switching diode</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_8$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>2.20 k$\Omega$</td>
<td>$\pm 5%$</td>
</tr>
<tr>
<td>$R_9$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>14.0 k$\Omega$</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>$R_{10}$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>1.0 k$\Omega$</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>$N_{R_4}$</td>
<td>Current source</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_4$</td>
<td>$\frac{1}{4}$ AD713K quad BiFET op amp</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{11}$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>1.00 M$\Omega$</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>$R_{12}$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>2.00 k$\Omega$</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>$R_{13}$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>100 $\Omega$</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>$R_{14}$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>1.00 M$\Omega$</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>$R_{15}$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>2.00 k$\Omega$</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>$C_1$</td>
<td>Capacitor</td>
<td>39 nF</td>
<td>$\pm 2%$</td>
</tr>
<tr>
<td>$R$</td>
<td>$\frac{1}{4}$ W Resistor</td>
<td>1.00 k$\Omega$</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Capacitor</td>
<td>390 nF</td>
<td>$\pm 2%$</td>
</tr>
<tr>
<td>$L$</td>
<td>TOKO type 10RB fixed inductor</td>
<td>10 mH</td>
<td>$\pm 10%$</td>
</tr>
</tbody>
</table>

Table 1: Complete component list for Chua's circuit with current offset in the Chua diode characteristic.
• The slope of the v-i characteristic over its central region (for the special case \( R_1 = R_2 \) and \( R_{11}/R_{12} = R_{14}/R_{15} \)) is given by

\[
m_1 \approx -\frac{R}{R_3} + \frac{R}{R_{11} + R_{12}},
\]

where \( R_{11} + R_{12} \) is the approximate output impedance of the current source in its linear region. With \( R = 1.00 \, \text{k}\Omega, \, R_{11} = 1.00 \, \text{M}\Omega \) and \( R_{12} = 2.00 \, \text{k}\Omega \), a desired value of \(-8/7\) for \( m_1 \) forces us to choose

\[
R_3 \approx 874\, \Omega.
\]

Set \( R_3 \) to 874 \( \Omega \) by adjusting the 1 k\Omega cermet trimmer.

• The slope \( m_0 \) outside the breakpoints is determined from

\[
m_0 = m_1 + \frac{R}{R_7}
\]

Selecting \( m_0 = -5/7 \) gives

\[
R_7 = 2.333\, \text{k}\Omega.
\]

We use the standard value 2.32 \( \text{k}\Omega \) instead.

• With \( R_{11}/R_{12} = R_{14}/R_{15} \), the current offset is given by

\[
I_{OS} = \frac{R_{12}}{R_{11}R_{13}}V_{OS}.
\]

For this set of resistor values,

\[
I_{OS} = 0.02V_{OS} \, \text{mA},
\]

where \( V_{OS} \) is measured in Volts. For example, \( V_{OS} = 5 \, \text{V} \) gives an offset of \( I_{OS} = 0.1 \, \text{mA} \). Over the linear region of \( A_4 \),

\[
i_{R4} \approx I_{OS}
\]

• The normalized circuit parameters are given by:

\[
\alpha = \frac{C_2}{C_1} = 10
\]

and

\[
\beta = \frac{C_2R^2}{L} = 39.
\]

3 SPICE simulation

The SPICE deck in Fig. 4 may be used to simulate the behavior of the current-offset Chua’s circuit. The AD713 op amp [Ana90] is modeled by Analog Devices’ AD713 SPICE macro-model [Ana91]. Node numbers are as in Fig. 3. The power rails are 100 and 200.

The v-i characteristic may be verified by means of the SPICE deck shown in Fig. 5. The characteristics for \( V_{OS} = 0 \, \text{V} \) and \( V_{OS} = 10 \, \text{V} \) are shown in Figs. 6(a) and (b) respectively.

Acknowledgements

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Figure 4: SPICE deck to simulate the transient response of Chua's circuit with current-offset Chua diode. Node numbers are as in Fig. 3. The op amps are modeled by the AD713 macro-model from Analog Devices.
Figure 5: SPICE deck to plot the v-i characteristic of the current-offset Chua diode in Fig. 3. As before, the op amps are modeled by the AD713 macro-model from Analog Devices.
Figure 6: Simulated v-i characteristic of current-offset Chua diode on the right of Fig. 3. (a) $V_{OS} = 0 \, \text{V} -$ no offset; (b) $V_{OS} = 10 \, \text{V} -$ 0.2 mA offset.

References


