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ABSTRACT

A self-consistent solution for the dynamics of a high voltage, capacitive r.f. sheath driven by a
sinusoidal current source is obtained, under the assumptions of time-independent, collisional ion motion
and inertialess electrons. Results are: (1) the ion current density is $1.68 \varepsilon_0(2e/M)^{1/2}\sqrt{V/s_m^{3/2}}$, where
$V$ is the d.c. self-bias voltage, $s_m$ is the sheath thickness, $e/M$ is the ion charge-to-mass ratio, and $\varepsilon_0$ is
the free space permittivity; (2) the sheath capacitance per unit area for the fundamental voltage harmonic is
$1.52 \varepsilon_0/s_m$; (3) the ratio of the d.c. to the peak value of the oscillating voltage is 0.40; (4) the
second and third voltage harmonics are respectively 19.3% and 5.3% of the fundamental; and (5) the
conductance per unit area for stochastic heating by the oscillating sheath is
$2.17 \left( \varepsilon_0/n_0/mu_e \right) \lambda_D^{2/3} / (s_m)^{1/3}$, where $n_0$ is the ion density and $\lambda_D$ is the Debye length at the plasma-
sheath edge, and $u_e = (8eT_e/nm)^{1/2}$ is the mean electron speed.
I. INTRODUCTION

Low pressure capacitive, radio frequency (r.f.) discharges are widely used for materials processing in the electronics industry. Typical discharge parameters are pressure \( p \approx 10-300 \text{ mTorr} \), r.f. frequency \( \omega/2\pi = 13.56 \text{ MHz} \), and r.f. voltage \( V_d \approx 50-500 \text{ volts} \). Almost all the applied voltage is dropped across capacitive r.f. sheaths at the discharge electrodes. In order to develop adequate models for these discharges, it is important to determine the dynamics and current-voltage characteristics of the sheaths. The sheath dynamics are strongly nonlinear. Godyak and collaborators have developed a homogeneous model of a collisionless sheath.\(^1\,^2\) Other authors have used a Child-Langmuir law for the ions within the sheath to model the sheath dynamics.\(^3\,^4\,^5\) An approximate model of the effect of the time-average electron density on the ion dynamics within the sheath has been developed.\(^6\) The self-consistent voltages for a single sheath and for a symmetrically driven discharge having two sheaths 180° out of phase have been given.\(^7\) While numerical solutions of the self-consistent dynamics can be obtained,\(^8\) they are not particularly illuminating.

In a previous study\(^9\), the solution for a collisionless sheath driven by a sinusoidal r.f. current source was found. The ion response to the average electric field \( \bar{E} \) was assumed to be collisionless; i.e., the force equation was taken to be \( M \dot{u}_i = e \bar{E} \), where \( e, M, \) and \( u_i \) are respectively the ion charge, mass and velocity. To estimate the pressure regime where this is valid, we can compare the ion mean free path \( \lambda_i \) to the ion sheath thickness \( s_m \). For example, in an argon discharge, \( \lambda_i = (300p)^{-1} \) cm. For \( s_m \) of order 1 cm, the sheath is collisionless \((\lambda_i \geq s_m)\) for \( p \leq 3 \text{ mTorr} \). Therefore, the collisionless theory is not valid for typical materials processing pressures.

In this work we give an analytical, self-consistent solution for a collisional sheath driven by a sinusoidal, r.f. current source. We obtain the time-average electric field and potential within the sheath, the nonlinear oscillation motion of the electron sheath boundary and the nonlinear oscillating sheath voltage. Finally, we determine the effective sheath capacitance and conductance.

The assumptions of the analysis are:

1. The ion motion is collisional with \( \lambda_i \) a constant within the sheath. The ions respond only to the time-average electric field. The ion sheath-plasma boundary is stationary, and ions enter the
sheath with a Bohm presheath velocity \( u_B = (eT_e/M)^{1/2} \), where \( T_e \) is the electron temperature (in volts).

(2) The electrons are inertialess and respond to the instantaneous electric field. The electron Debye length \( \lambda_D \) everywhere within the sheath is assumed to be much smaller than \( s_m \). This holds provided \( V_{rf} \gg T_e \). Since \( \lambda_D \ll s_m \), the electron density falls sharply (within a few Debye lengths) from \( n_e \approx n_i \) at the plasma side of the electron sheath boundary to \( n_e \approx 0 \) at the electrode side. The electron sheath oscillates between a maximum thickness of \( s_m \) and a minimum thickness of a few Debye lengths from the electrode surface.

II. BASIC EQUATIONS

The structure of the r.f. sheath is shown in Fig. 1. Ions crossing the ion sheath boundary at \( x = 0 \) accelerate within the sheath and strike the electrode with 50-500 volt energies. Since the ion flux \( n_i u_i \) is conserved and \( u_i \) increases as ions transit the sheath, \( n_i \) drops. This is sketched as the heavy solid line in Fig. 1. The ion particle and momentum conservation equations are respectively

\[
\begin{align*}
  n_i u_i &= n_0 u_B, \\
  u_i &= \mu_i \frac{\varepsilon}{\pi M u_i} \bar{E},
\end{align*}
\]

where \( n_0 \) is the plasma density at \( x = 0 \) and the mobility \( \mu_i \) is itself a function of \( u_i \); \( \bar{E} \), \( n_i \) and \( u_i \) are functions of \( x \). The factor \( 2/\pi \) in (2) accounts for the time averaging of the ion velocity over the distribution of mean free paths to obtain the mean velocity \( u_i \). The Maxwell equation for the instantaneous electric field \( E(x, t) \) within the sheath is

\[
\frac{\partial E}{\partial x} = \frac{e}{\varepsilon_0} n_i(x), \quad s(t) < x;
\]

\[= 0, \quad s(t) > x. \quad (3)\]

Here, \( s(t) \) is the distance from the ion sheath boundary at \( x = 0 \) to the electron sheath edge; the electron sheath thickness is \( s_m - s(t) \). The instantaneous potential \( \Phi(x, t) \) is determined from the equation

\[
\frac{\partial \Phi}{\partial x} = -E. \quad (4)
\]
Time-averaging (3) and (4) over an r.f. cycle, we obtain the equations for the time-average electric field \( \overline{E}(x) \) and potential \( \overline{\Phi}(x) \):

\[
\frac{d\overline{E}}{dx} = \frac{e}{\varepsilon_0} (n_i(x) - \overline{n}_e(x)),
\]

\[
\frac{d\overline{\Phi}}{dx} = -\overline{E},
\]

where \( \overline{n}_e(x) \) is the time-average electron density within the sheath. We can determine \( \overline{E}, \overline{\Phi} \) and \( \overline{n}_e \) from \( s(t) \). For example, we note that \( n_e(x, t) = 0 \) during the part of the r.f. cycle when \( s(t) < x \); otherwise, \( n_e(x, t) = n_i(x) \). We therefore have

\[
\overline{n}_e(x) = \left[ 1 - \frac{2\phi}{2\pi} \right] n_i(x)
\]

where \( 2\phi(x) \) is the phase interval during which \( s(t) < x \). Qualitatively, we sketch \( \overline{n}_e \) as the dashed line in Fig. 1. For \( x \) near zero, \( s(t) < x \) during only a small part of the r.f. cycle; therefore \( 2\phi = 0 \) and \( \overline{n}_e = n_i(x) \). For \( x \) near \( s \), \( s(t) < x \) during most of the r.f. cycle; therefore \( 2\phi = 2\pi \) and \( \overline{n}_e = 0 \).

To determine the time averages quantitatively, we assume that a sinusoidal r.f. current density passes through the sheath:

\[
J_{rf}(t) = -J_0 \sin \omega t.
\]

Equating this displacement current to the conduction current at the electron sheath boundary, we obtain the equation for the electron sheath motion:

\[
-e n_i(s) \frac{ds}{dt} = -J_0 \sin \omega t.
\]

III. SOLUTION

We integrate (3) to obtain
\[ E = \frac{e}{\varepsilon_0} \int_s^x n_i(\xi) d\xi, \quad s(t) < x, \quad (10) \]

\[ = 0, \quad s(t) > x. \]

We integrate (9) to obtain

\[ \frac{e}{\varepsilon_0} \int_0^s n_i(\xi) d\xi = \frac{J_0}{\varepsilon_0 \omega} (1 - \cos \omega t), \quad (11) \]

where we have chosen the integration constant so that \( s(t) = 0 \) at \( \omega t = 0 \). From (10) and (11), we obtain

\[ E(x, \omega t) = \frac{e}{\varepsilon_0} \int_0^{s(x)} n_i(\xi) d\xi - \frac{J_0}{\varepsilon_0 \omega} (1 - \cos \omega t), \quad s(t) < x; \quad (12) \]

\[ = 0, \quad s(t) > x. \]

We must time average (12) to obtain \( \overline{E} \). Figure 2 shows a sketch of \( s(t) \) vs \( \omega t \). We note that \( s(t) = x \) for \( \omega t = \pm \phi \), and that \( s(t) < x \) for \( -\phi < \omega t < \phi \). The time average is then

\[ \overline{E} = \frac{1}{2\pi} \int_0^{\phi} E(x, \omega t) d(\omega t). \quad (13) \]

Inserting (12) into (13), we find

\[ \overline{E}(x) = \frac{e}{\varepsilon_0} \int_0^{\phi} n_i(\xi) d\xi - \frac{J_0}{\varepsilon_0 \omega \pi} (\sin \phi - \phi). \quad (14) \]

Inserting (11) with \( s = x, \omega t = \phi \) into (14) we obtain

\[ \overline{E}(x) = \frac{J_0}{\varepsilon_0 \omega \pi} (\sin \phi - \phi \cos \phi). \quad (15) \]

Using (6),

\[ \frac{d \Phi}{dx} = -\frac{J_0}{\varepsilon_0 \omega \pi} (\sin \phi - \phi \cos \phi). \quad (16) \]

Solving (1) and (2) for \( n_i \), we obtain
\[ n_i = n_0 \mu_B (2e \lambda_i \vec{E} / \pi MN)^{1/2} . \]  
\[ (17) \]

Inserting (17) into (9) with \( s = x, \omega t = \phi \), we obtain
\[ \frac{d\phi}{dx} = \frac{\mu_B}{\pi_0} \left[ \frac{\pi M}{2e \lambda_i} \right]^{1/2} \frac{1}{E^{1/2} \sin \phi} , \]
\[ (18) \]
where
\[ \pi_0 = \bar{J}_D(\epsilon \omega n_0) \]
\[ (19) \]
is an effective oscillation amplitude.

Equations (15) and (18) are the fundamental equations of the self-consistent r.f. sheath. Inserting (15) into (18) and integrating, we obtain

\[ x / \pi_0 = H \int_0^\phi (\sin \zeta - \zeta \cos \zeta)^{1/2} \sin \zeta d\zeta , \]
\[ (20) \]
where
\[ H = \left[ \frac{2 \lambda_i \pi_0}{\pi^2 \lambda_D^2} \right]^{1/2} \]
\[ (21) \]
and \( \lambda_D = (\epsilon_0 T_e / e \pi n_0)^{1/2} \) is the electron Debye length at \( x = 0 \). In (20), we have used the boundary condition that \( x = 0 \) at \( \phi = 0 \). Setting \( x = s(t) \) and \( \phi = \omega t \) in (20), we obtain the nonlinear oscillation motion of the electron sheath, which is shown in Fig. 3. Setting \( s = s_m \) at \( \phi = \pi \) in (20), we obtain the ion sheath thickness

\[ s_m = 1.95 H \pi_0 . \]
\[ (22) \]

Using (21) in (22) and solving for \( \pi_0 \), we obtain

\[ \pi_0 = 1.09 \lambda_D^{2/3} s_m^{2/3} / \lambda_i^{1/3} . \]
\[ (23) \]
The time-average potential is found by integrating (16), which yields

\[ \overline{\Phi} = \frac{J_0}{\pi \epsilon_0 \omega 0} \int_0^\phi (\sin \zeta - \zeta \cos \zeta) \frac{dx}{d\zeta} d\zeta . \]
\[ (24) \]
Using (18) and (19) in (24), we obtain
The total d.c. voltage across the sheath is related to the d.c. ion current and the ion sheath thickness by:

\[
J_i = K \varepsilon_0 \left( \frac{2e}{M} \right)^{1/2} \frac{\bar{V}^{3/2}}{s_m^{2}}.
\]  

(26)

where \(J_i = en_0u_b\) is the d.c. ion current and \(\bar{V} = -\Phi(\phi = \pi)\) is the voltage across the sheath. Setting \(\phi = \pi\) and \(\Phi = -\bar{V}\) in (25) and evaluating the integral, we obtain

\[
\frac{\bar{V}}{T_s} = 3.15 \frac{H}{\pi} \frac{3_m^2}{\lambda^2}.
\]  

(27)

Using (21) and (23) in (27) and the definitions for \(\lambda_D\) and \(J_i\), we obtain (26) with

\[
K_c = 1.68 \left( \lambda_i/s_m \right)^{1/2}.
\]  

(28)

In contrast, the self-consistent result is \(K_f = 0.82\) for collisionless ion motion in the sheath. We see that the current density scales as the inverse \(5/2\) power of the sheath thickness, in contrast to the (collisionless) Child law scaling as the inverse square power. We also note that for a fixed voltage and current, ion collisions in the sheath lead to a reduction in the sheath thickness.

IV. SHEATH CAPACITANCE

The instantaneous electric field within the sheath is given by (12). Substituting (11) with \(s = x\) and \(\omega t = \phi\) into (12), we obtain

\[
E(x,t) = \frac{\bar{J}_0}{\varepsilon_0 \omega} (\cos \omega t - \cos \phi), \quad s(t) < x,
\]  

\[
= 0, \quad s(t) > x.
\]  

(29)

Integrating with respect to \(x\), we obtain the instantaneous voltage from the plasma to the electrode across the sheath

\[
V(t) = \int_s^{s_m} E(x,t)dx.
\]  

(30)

Changing variables from \(x\) to \(\phi\) and using (29), we obtain
\[ V(t) = \frac{J_0}{\varepsilon_0 \omega} \int_{\phi}^{\pi} (\cos \omega t - \cos \phi) \frac{d\phi}{d\phi} d\phi. \]  

(31)

Using (15) and (18) to evaluate \( dx/d\phi \) in (31) we obtain, for \( 0 < \omega t < \pi \),

\[ V(t) = (en_0 \delta^2 / \varepsilon_0) H \int_{\phi}^{\pi} (\cos \omega t - \cos \phi)(\sin \phi - \phi \cos \phi)^{1/2} \sin \phi d\phi. \]

(32)

\( V(t) \) is an even, periodic function of \( \omega t \) with period \( 2\pi \). For \( -\pi < \omega t < 0 \), we find that \( V(t) \) is given by the right hand side of (32) with \( \omega t \) replaced by \( -\omega t \). A plot of \( V \) versus \( \omega t \) is given in Fig. 4.

The peak value of \( V(t) \) occurs at \( \omega t = 0 \):

\[ V(0) = 2.50 H (en_0 \delta^2 / \varepsilon_0). \]

(33)

Expanding \( V(t) \) in a Fourier series

\[ V(t) = \sum_{k=0}^{\infty} V_k \cos (k \omega t), \]

we obtain

\[ V_0 = \bar{V} = 1.00 H (en_0 \delta^2 / \varepsilon_0), \]

\[ V_1 = 1.28 H (en_0 \delta^2 / \varepsilon_0), \]

\[ V_2 = 0.25 H (en_0 \delta^2 / \varepsilon_0), \]

\[ V_3 = -0.34 H (en_0 \delta^2 / \varepsilon_0). \]

(34)

The second harmonic is 19.3% of the fundamental, and the third harmonic is 5.3% of the fundamental.

The ratio of the d.c. value to the peak value of the voltage is \( \bar{V}/V(0) = 0.40 \). Defining the effective capacitance per unit area from the relation

\[ -J_0 \sin \omega t = C_\varepsilon' \frac{d}{dt} (V_1 \cos \omega t), \]

we obtain

\[ C_\varepsilon' = 1.52 \varepsilon_0 / \delta_m, \]

(35)

where \( \delta_m \) is the ion sheath thickness given by (22). In contrast, the coefficient in (35) is 1.23 for collisionless ion motion in the sheath.9
For a symmetrically-driven, parallel plate r.f. discharge (equal area plates) there are two r.f.
sheaths in series. We let \( V_{ap}(t) \) be the voltage on plate \( a \) with respect to the plasma and \( V_{bp}(t) \) be the
voltage on plate \( b \) with respect to the plasma. By symmetry, \( V_{bp}(\omega t) = V_{ap}(\omega t - \pi) \). The series voltage
across both sheaths is \( V_{ab} = V_{ap} - V_{bp} \). Using (32), we obtain, for \( 0 < \omega t < \pi \),
\[
V_{ab} = (en_0s_0^2/\varepsilon_0) \left[ \int_0^\pi (\cos \omega t - \cos \phi)(\sin \phi - \phi \cos \phi)^{1/2} \sin \phi \, d\phi + \right. \\
+ \left. \int_{\pi-\omega t}^{\pi} (\cos \omega t + \cos \phi)(\sin \phi - \phi \cos \phi)^{1/2} \sin \phi \, d\phi \right].
\]
The peak-to-peak value of \( V_{ab} \) is \( 2V(0) \), with \( V(0) \) given in (33). Expanding \( V_{ab} \) in a Fourier series,
we obtain \( V_{ab1} = -2V_1 \) and \( V_{ab3} = -2V_3 \). All even harmonics, including the d.c. value, are zero, as
expected for a symmetrically-driven discharge. The third harmonic is 5.3% of the fundamental, and the
higher harmonics are much smaller. Thus, to a very good approximation, a sinusoidal sheath current
leads to a linear response; i.e., a sinusoidal voltage across the discharge. Defining the effective capaci-
tance per unit area of the series combination of the two sheaths from the relation
\[
J_{ab}(t) = C_{sym}' \frac{d}{dt} V_{ab1}(t),
\]
we obtain \( C_{sym}' = 0.76 \varepsilon_0 s_m \).

V. SHEATH CONDUCTANCE

The r.f. conductance of the sheath is due to stochastic heating of the electrons by the oscillating
sheath. An electron that is reflected from a moving sheath experiences a change of energy. If the
sheath moves toward the electron, then the energy increases; if the sheath moves away, then the energy
decreases. For an oscillating sheath, some electrons gain energy and others lose energy. However,
averaging over an oscillation period, the net effect is an energy gain, corresponding to a dissipation in
the sheath. This mechanism also has been called "Fermi acceleration"\(^{10-14} \) or "wave riding"\(^{4-5,15} \).

If \( u \) is the parallel velocity (along \( z \)) of an incident electron at the electron sheath edge \( s(t) \) and
\( u_s(t) \) is the sheath velocity, then the reflected electron has a velocity \( u_r = -u + 2u_s \). We let \( f_s(u_s,t) \) be
the electron velocity distribution at \( s \), normalized so that
\[ \int_{-\infty}^{\infty} f_s(u, t) \, du = n_i(s(t)) = n_s(t). \]

The electron flux \( \Gamma_s \) incident on the sheath is

\[ \Gamma_s = \int_{0}^{\infty} u f_s(u, t) \, du. \tag{36} \]

To determine the power transferred to the electrons, we note that in a time interval \( dt \) and for a speed interval \( du \), the number of electrons per unit area that collide with the sheath is given by \((u - u_0) f_s(u, t) \, du \, dt\). This results in a power transfer \( dS \) per unit area

\[ dS = \frac{1}{2} m (u_r^2 - u^2)(u - u_r) f_s(u, t) \, du. \tag{37} \]

Using \( u_r = -u + 2u_0 \) and integrating over all incident velocities, we obtain

\[ S = -2m \int_{u_s}^{\infty} u_r (u - u_r)^2 f_s(u, t) \, du. \tag{38} \]

To determine \( f_s \), we first note that the sheath is oscillating because the electrons in the plasma are oscillating in response to a time-varying electric field. If the velocity distribution function within the plasma in the absence of the electric field is a Maxwellian \( g_0(u) \), then the distribution within the plasma is \( f_0(u, t) = g_0(u - u_0) \), where \( u_0(t) \) is the time-varying oscillation velocity of the plasma electrons. Because \( n_s < n_0 \), not all electrons having \( u > 0 \) at \( x = 0 \) collide with the sheath at \( s \). Many electrons are reflected within the region \( 0 < x < s \) where the ion density drops from \( n_0 \) to \( n_s \). This reflection is produced by a weak electric field whose value is such that \( n_e = n_i \) at all times. The transformation of \( f_0 \) across this region to obtain \( f_s \) is complicated. However, the essential features to determine the stochastic heating are seen if we approximate

\[ f_s = \frac{n_s}{n_0} g_0(u - u_0), \quad u > 0. \tag{39} \]

Inserting (39) into (38) and transforming to a new variable \( u' = u - u_0 \), we obtain
\[ S(t) = -2m \int_{u_s - u_0}^{\infty} u_s n_s [u'^2 - 2u'(u_s - u_0) + (u_s - u_0)^2] g_0(u') \, du'. \]  

(40)

Assuming that \(|u_s - u_0|\) is much less than the characteristic electron thermal velocity, we can take the lower limit of the integral in (40) to be zero. From (9) we note that

\[ n_s u_s = \bar{n}_0 \bar{u}_0 \sin \phi, \]  

(41)

and differentiating (20) with respect to \(\phi = \omega \tau\), we obtain

\[ u_s = \bar{u}_0 H (\sin \phi - \phi \cos \phi)^{1/2} \sin \phi. \]  

(42)

Averaging (40) over \(\phi = \omega \tau\) and noting that (41) and (42) are odd functions of \(\phi\), the first and third terms in (40) average to zero and we obtain

\[ \bar{S} = 4m \Gamma_s \bar{n}_0^{-1} \langle (u_s - u_0) u_s n_s \rangle \phi. \]  

(43)

Noting that \(|u_0| \ll |u_s|\) for \(H \gg 1\), inserting (41) and (42) into (43) and averaging, we obtain

\[ \bar{S} = 0.49 \bar{H} m n_0 u_s \bar{u}_0^2, \]  

(44)

where for a Maxwellian distribution \(g_0\), the incident flux is

\[ \Gamma_s = \frac{1}{4} n_0 u_s, \]  

(45)

and

\[ u_s = \left[ \frac{8eT_s}{nm} \right]^{1/2} \]  

(46)

is the mean electron speed.

The sheath conductance \(G_s'\) per unit area is defined through the relation

\[ \bar{S} = \frac{1}{2} \frac{\bar{J}_0^2}{G_s'}. \]  

(47)

where \(\bar{J}_0 = e n_0 \bar{u}_0\). Equating (44) and (47), we obtain

\[ G_s' = \frac{1.02}{H} \left[ \frac{e^2 n_0}{mu_s} \right]. \]  

(48)
We note using (21) and (23) that

\[
H = 0.47 \left( \frac{\lambda_i}{s_m} \right)^{1/3} \left( \frac{s_m}{\lambda_D} \right)^{2/3}.
\]

We then obtain

\[
G_s' = 2.17 \left( \frac{s_m}{\lambda_i} \right)^{1/3} \left( \frac{e^2 n_0}{m u_e} \right) \left( \frac{\lambda_D}{s_m} \right)^{2/3}.
\]

This effective surface conductance per unit area represents a powerful electron heating mechanism in a capacitive r.f. discharge. The quantity \(2.17 (s_m/\lambda_i)^{1/3}\) in (50) is replaced by the coefficient 2.98 for collisionless ion motion in the sheath\(^9\).

As an example, we choose \(\bar{V} = 400\) V, \(p = 47\) mTorr, \(T_e = 3\) eV, \(J_i = 0.5\) mA/cm\(^2\), \(\omega = 2\pi \times 13.56\) MHz, and \(M = 40\) amu (i.e., argon). Then we obtain \(\lambda_i = 0.070\) cm, \(u_B = 2.7 \times 10^5\) cm/s, \(n_0 = 1.2 \times 10^{10}\) cm\(^{-3}\), \(H = 4.1\), \(\bar{f}_0 = 10.8\) mA/cm\(^2\), \(\bar{s}_0 = 6.8 \times 10^{-2}\) cm, \(\lambda_D = 1.2 \times 10^{-2}\) cm, \(s_m = 0.54\) cm, \(G_s' = 0.25\) pF/cm\(^2\), \(\bar{u}_0 = 5.8 \times 10^6\) cm/s, \(u_e = 1.2 \times 10^8\) cm/s, \(S = 8.3 \times 10^{-3}\) W/cm\(^2\), and \(G_s' = 7.1 \times 10^{-3}\) W/cm\(^2\). The d.c. ion power flux incident on the electrode is \(S_i = J_i \bar{V} = 0.2\) W/cm\(^2\).

For a homogeneous (uniform ion density) sheath, \(n_s = n_0\) and \(u_s = u_0\). Then the integral in (40) vanishes and there is no stochastic heating; \(G_s' \to \infty\). We can understand this physically as follows: In the accelerated frame moving with the plasma, the electron sheath edge at \(s(t)\) is stationary; therefore, no energy is transferred to electrons that collide with the sheath. Thus the non-homogeneous nature of the self-consistent ion density within the r.f. sheath is an essential feature of the stochastic heating mechanism.

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Fig. 1. Structure of high voltage, capacitive r.f. sheath.
Fig. 2. Sketch of $s(t)$ versus $\omega t$, showing the definition of the phase $\phi(x)$. 
Fig. 3. Normalized position versus phase for the self-consistent r.f. sheath.
Fig. 4. Normalized time-varying sheath voltage versus $\omega t$. 

\[
\frac{V}{H} = \frac{\epsilon_0}{e^2 n_0 \tilde{s}_0^2}
\]