INTERPRETING AND TEACHING SCIENTIFIC CONCEPTS
A STUDY OF ''ACCELERATION''

By

Frederick Reif†
Department of Physics and School of Education
University of California, Berkeley

and

Sue Allen
Group in Science and Mathematics Education
University of California, Berkeley

Cognitive Science Program

Institute of Cognitive Studies

University of California at Berkeley

March 1990

† Now at the Center for Design of Educational Computing, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213.

The production of the Berkeley Cognitive Science Report Series is supported by a grant in cognitive science to the University of California at Berkeley from the Alfred P. Sloan Foundation.

INTERPRETING AND TEACHING SCIENTIFIC CONCEPTS: A STUDY OF "ACCELERATION"

Frederick Reif
Department of Physics and School of Education
University of California, Berkeley

Sue Allen
Group in Science and Mathematics Education
University of California, Berkeley

Abstract

Interpreting a scientific concept—i.e., identifying or constructing it in any particular instance—is a complex cognitive task. We analyze the underlying knowledge required to achieve such concept interpretation accurately and efficiently. This analysis is then used to examine our detailed observations of expert scientists and novice students interpreting the physics concept "acceleration." Most of the experts interpret the concept well in expected ways; however, even some experienced scientists exhibit marked deficiencies in concept interpretation. Novice students, in spite of using a scientific concept for some months, interpret it incorrectly in many cases; their poor performance can be traced to concept knowledge which consists largely of disconnected knowledge fragments, and which can often not be properly applied even if correctly stated. Instructional experiments, which directly address such deficiencies, indicate that it is possible to improve substantially students' ability to interpret a scientific concept.

1. INTRODUCTION

Many special concepts (such as "acceleration," "potential energy," and "electric field") are used in science as the basic building blocks of theories highly successful in explaining and predicting observable phenomena. An adequate understanding of the cognitive processes required to interpret and learn such concepts is thus of interest for the following reasons:

- The cognitive processes needed to use concepts in science are complex and significantly different from those involved in using everyday concepts.
- Although science courses devote much time teaching many scientific concepts, students often have great difficulties learning to interpret these concepts appropriately. Hence there is a need to develop improved instructional methods.
- The proper interpretation of scientific concepts is an essential prerequisite for solving problems in science. Furthermore, many students' problem-solving difficulties are

primarily due to deficiencies in their understanding of the required concepts.

- There have been numerous recent investigations of students' scientific conceptions, misconceptions, and naive preconceptions (Caramazza, McCloskey, & Green, 1981; Clement, 1982; Cohen, Eylon, & Ganiel, 1983; Driver, Guesne, & Tiberghien, 1985; Gunstone, 1987; Halloun & Hestenes, 1985 a, 1985 b; Helm & Novak, 1983; McCloskey, Caramazza, & Green, 1980; McDermott, 1984; Trowbridge & McDermott, 1980, 1981; Viennot, 1979). These investigations have yielded valuable data, but have been mostly descriptive rather than analytic. Correspondingly, they provide rather little guidance about how to teach scientific concepts more effectively.

Concept interpretation. Simple concepts (e.g., "triangle," "particle") refer to entities. Other concepts, particularly important in science, are "property concepts" (e.g., "area," "acceleration") which are used to describe entities, or other properties, by associating particular values with them. For example, the concept "area" describes any surface by associating with it a numerical value. Similarly, the concept "acceleration" describes any moving particle by associating with it a particular *vector* (a value characterized jointly by a magnitude and a direction).

A property concept is only meaningful if one can "interpret" it—i.e., if one can generate or identify the value of the concept in any particular instance. The ability to interpret a scientific concept is essential for using the concept in scientific work. However, the mere task of interpreting a scientific concept is far from simple (Reif, 1987a) and, as we shall see, deficiencies of concept interpretation cause many difficulties and mistakes.

Central questions. The work described in this paper addresses the following questions:

(1) What kinds of knowledge and thought processes facilitate the accurate and efficient interpretation of scientific concepts?

(2) How do expert scientists and novice students interpret scientific concepts? How can their good or deficient performance be traced to features of their underlying knowledge and thought processes?

(3) How can answers to the preceding questions be exploited for improving the teaching of scientific concepts?

Focus on the concept "acceleration." Our work was based on the premise that detailed answers to the preceding questions could best be obtained by investigating in depth a well-chosen scientific concept. The insights obtained from such an investigation would be intrinsically valuable and would also facilitate subsequent studies of various other scientific concepts.

Accordingly, we focused our investigation on the physics concept "acceleration." This concept describes how rapidly the magnitude and direction of a particle's velocity change.

Correspondingly, the "acceleration" defined in physics is a vector and has a meaning distinctively different from that of the word "acceleration" in everyday life. The choice of this particular concept was motivated by the following considerations:

(a) Acceleration is a fundamental concept in the Newtonian theory of mechanics.

(b) It is an elementary concept encountered at the beginning of any introductory physics course.

(c) Yet its complexities, discussed in the following pages, are typical of more advanced physics concepts (e.g., "potential energy," "torque," "electric field") and of mathematical concepts (e.g., "derivative"). Thus, it causes many difficulties for students, and occasionally even for experienced physicists.

Overview of the paper. The next section describes the methods which we used to observe how expert scientists and novice students interpret the concept acceleration in various situations. Section 3, which summarizes the performance of these subjects, indicates that "expert" scientists perform well, but sometimes far from perfectly, and that students very often interpret the concept incorrectly despite some months of experience. Section 4 presents a theoretical analysis of the kinds of knowledge required to interpret a scientific concept effectively and efficiently. In the next several sections this analysis is applied to the concept acceleration, and used to examine in detail how expert scientists and novice students interpret this concept. Section 10 examines reasons why an ostensible expert performed badly. Section 11 describes the design and implementation of instruction aiming to remedy student's conceptual deficiencies. Finally, Section 12 summarizes the results of our investigation and points out some of their general instructional implications.

2. OBSERVATIONAL METHODS

This section describes the methods which we used to investigate how expert scientists and relatively inexperienced students interpret the concept acceleration. To obtain detailed information about their underlying knowledge and thought processes, all our observations were on individual subjects. The subjects were asked to think out loud, and the protocols resulting from transcriptions of the tape-recordings were later analyzed in detail.

Subjects

The expert scientists (denoted by E1, E2, E3, E4 and E5) were five long-time faculty members, all male, of the Physics Department at the University of California at Berkeley. Each had recently taught the concept of acceleration in introductory physics courses.

The novice students (denoted by N1, N2, N3, N4 and N5) were five undergraduate students, two male and three female, enrolled at the University of California in the first semester of an introductory physics course designed for prospective scientists and engineers. All students had been in this course for at least two months, had learned about acceleration during the first two weeks of the course, and had been using this concept in the course since then.

Materials

The materials used in the study consisted of two special tests, labeled A and B, asking subjects to interpret the concept acceleration in various situations.

Test A

Test A consisted of 13 problems, each asking several questions about the value of the acceleration of a moving object in some *specific* situation.

Situations. The problem situations involved motions of objects along straight lines or curves (e.g., circles, ellipses, or spirals); with constant, increasing, or decreasing speed; and with zero or non-zero velocity at the instant of interest. For example, questions were asked about the acceleration at a particular instant in the following situations illustrated in Figure 1.

(a) Sled moving up along a hill with decreasing speed (Problem A1), or at the instant when it is at its highest point B along this hill (Problem A8).

(b) Swinging pendulum bob at its highest point A where its speed is zero (Problem A5), at an intermediate point B while descending with increasing speed (Problem A10), or at its lowest point C where its speed is maximum (Problem A7).

(c) Car moving around a horizontal racetrack with straight and elliptical sections, moving with constant speed (Problem A3) or decreasing speed (Problem A6).

(d) Car moving along a horizontal spiral road with constant speed (Problem A9).

Questions. The questions in the problems required qualitative rather than quantitative responses. Typical questions were the following: Is the acceleration zero or non-zero? If it is non-zero, indicate its direction by drawing an appropriate arrow. Some questions asked subjects also to *compare* the magnitudes or directions of two accelerations. For example, Problem A9, about the spiral road illustrated in Figure 1d, asked also whether the magnitude of the acceleration of the car at the point B is larger than, smaller than, or equal to that at the point A. Each question asked subjects to indicate their degree of confidence in their answer by using a three-point scale (certain, halfway certain, mostly guessing). Appendix 1 summarizes all the 13 problems of Test A.

Test B

Test B consisted of 6 problems, each asking several *general* questions about the possible values of the acceleration in various *types* of situations. The answers thus required some general reasoning beyond simple concept interpretation.

Insert Figure 1 about here

Types of Situations. The types of situations included motion with constant speed (Problem B1), motion with increasing speed (Problem B4), motion along a straight path (Problem B5), and motion along a curved path (Problem B2).

Questions. The questions asked whether certain values of acceleration are possible or not. Thus typical questions were the following:

(a) Can the magnitude of the acceleration be zero? Can it be non-zero?

(b) Can the direction of the acceleration be along the velocity? Opposite to it? Perpendicular to it? Neither parallel nor perpendicular to it?

(c) If you claim that something is possible, give a specific example illustrating this possibility. If you claim that something is not possible, explain why not.

Again each question asked subjects to indicate their degree of confidence in their answer on a three-point scale (certain, halfway certain, mostly guessing). Appendix 2 summarizes all the 6 problems of Test B.

Procedures

Each subject was asked to respond to each test in two successive passes.

Pass 1. The aim of the first pass was to elucidate the subject's spontaneously invoked knowledge and thinking, with minimum intrusiveness or interference by the experimenter. Accordingly, the subject was simply asked to answer all the questions while talking out loud. The experimenter reminded the subject to keep talking and, if necessary, answered requests for clarification of a question. However, the experimenter did not intervene in any other way.

Pass 2. The aim of the second pass was to probe the subjects' underlying knowledge and thought processes under conditions where the risk of affecting subjects' thinking would no longer matter. Accordingly, each subject was asked to reconsider each question, to explain his or her previous reasoning, and to change his or her previous answers if so desired. However, the experimenter did not give any feedback regarding the correctness of answers or reasoning.

3. OVERALL PERFORMANCE OF EXPERTS AND NOVICES

Figure 2 summarizes the overall performance of the expert scientists and novice students on the test problems requiring interpretation of the concept acceleration.

As is apparent from the figure, the subjects' performance on the second pass through these problems was only slightly better than on the first pass. The expert scientists performed well, although not perfectly. For example, on the first pass almost all the experts answered correctly at least 10 of the 13 problems on Test A. The one exception was E2 who answered correctly only 7 of these problems and later also refused to participate in Test B. This

----
Insert Figure 2 about here
----

particular expert was anomalous and will be discussed separately in Section 10.

The performance of the novice students was very much poorer. None of them was able to answer correctly, on either pass, more than 3 of the 13 problems of Test A, nor more than 3 of the 6 problems of Test B. This performance seems particularly poor since all these students had been using the concept acceleration for at least two months in their ongoing physics course.

For both novices and experts, the questions causing the greatest difficulty were those asking about the acceleration of a particle moving along a curved path. Indeed, these questions are more complex since the acceleration may then be due to both a changing magnitude and a changing direction of the velocity. However, gross measures of performance, like the number of correct problems, reveal little about the underlying reasons for the observed performance. Hence, we devote the next section to a brief analysis of the kinds of knowledge required to interpret a scientific concept effectively and efficiently. This analysis will provide a framework for examining in greater detail the observed concept interpretations of the expert scientists and novice students.

4. ANALYSIS OF SCIENTIFIC CONCEPT INTERPRETATION

The task of interpreting a scientific concept requires the availability in memory of pertinent knowledge about this concept; some of this knowledge then needs to be retrieved and applied in any particular instance of interest. Correspondingly, deficiencies of concept interpretation may arise because of deficiencies in the available concept knowledge or because of deficiencies in its retrieval or application.

Ideally good concept interpretation. A theoretical model about "ideally good" concept interpretation should specify the knowledge features required to interpret a scientific concept in a manner that is both effective (unambiguous, precise, consistent, and general) and cognitively efficient (fast and requiring little effort). The formulation of such an ideal model is of interest for several reasons:

(a) Even a simple model, not necessarily unique or optimal, can be useful to identify essential knowledge needed to achieve good concept interpretation.

(b) It can serve as a standard of comparison for analyzing the observed concept interpretations of expert scientists or novice students.

(c) It can provide a target of instruction for teaching scientific concepts.

The model briefly outlined in the following paragraphs has been suggested partly by theoretical analysis and partly by observations of scientists. Of course, its utility is to be judged solely by how well it explains or predicts empirical observations of successful and deficient concept interpretations.

Knowledge facilitating good concept interpretation. According to our model, effective and efficient interpretation of a scientific concept requires that a person possesses the kinds of knowledge indicated in Figure 3 and exemplified in the next few sections for the concept acceleration.

Insert Figure 3 about here

The main concept knowledge in Figure 3 consists partly of general knowledge and partly of case-specific knowledge. The general knowledge is designed to permit unambiguous, precise, and consistent interpretation of the concept in all cases. Such knowledge includes a formal definition of the concept, useful knowledge derivable from this definition, and other knowledge allowing determination of the concept by exploiting relationships independent of the definition. Deliberate use of such general knowledge allows reliably correct concept interpretation in all situations, but may be slow and require considerable mental effort. Hence the main concept knowledge includes also a repertoire of specific knowledge about the concept in various special cases. This knowledge can be used to determine the concept quickly and relatively effortlessly in situations which match the known cases. Such concept interpretation is thus somewhat similar to everyday modes of concept identification (Rosch, 1978).

Joint use of general knowledge and case-specific knowledge permits concept interpretation which is both reliably accurate and cognitively efficient. In particular, if one encounters a familiar situation recognized to match a known case, one may quickly interpret the concept by using available case-specific knowledge; if need be, the conclusions may then be checked against general knowledge. On the other hand, if one encounters an unfamiliar situation, runs into inconsistencies, or needs to make widely applicable inferences, it is best to resort to general knowledge about the concept; case-specific knowledge may then provide useful checkpoints for such general arguments (Reif, 1987a).

To be effectively usable, each part of the main concept knowledge must be accompanied by "application knowledge" specifying when and how it is to be applied. In particular, "relevance knowledge," which specifies *when* it can validly and usefully be applied, facilitates appropriate retrieval. Procedural "implementation knowledge" then specifies *how* it is to be applied.

The usability of all the preceding concept knowledge is also significantly affected by its form. Specifically, it is enhanced by good description (e.g., by helpful symbolism) and by coherent structure.

Finally, a person should have appropriate confidence in his or her concept knowledge. In particular, unwarranted underconfidence may lead to failure to apply available knowledge, while unwarranted overconfidence may lead to incorrect application of such knowledge. Of course, all the preceding kinds of knowledge about a particular scientific concept must be used in conjunction with background knowledge about other prerequisite concepts, mathematics, and the physical world. In the next several sections these important aspects of concept knowledge, summarized in Figure 3, will be exemplified in the case of the concept "acceleration" and will provide a framework for analyzing our observations of experts scientists and novice students.

5. MAIN CONCEPT KNOWLEDGE

Definitional Knowledge

Definition of acceleration

In physics the concept "acceleration" is defined by the statement, "the acceleration of a particle is the rate of change of its velocity with time," or equivalently by the mathematical statement $\boldsymbol{a} = d\boldsymbol{v}/dt$ (where $\boldsymbol{a}$ is the acceleration and $d\boldsymbol{v}/dt$ is the derivative of the velocity $\boldsymbol{v}$ with respect to the time t). Specification of the definition by such declarative statements has the advantage of being compact, general, and precise. Indeed, resort to this definition would, in principle, be sufficient to answer all questions on Tests A and B. However, as discussed in Section 6, the declarative definition does *not* explicate the detailed procedural knowledge specifying what one must actually do to interpret the concept in any particular instance.

Note that the preceding definition implies that the acceleration $\boldsymbol{a}$, like the velocity $\boldsymbol{v}$, is a *vector* specified jointly by a magnitude and a direction.

Expert scientists

As is apparent from Figure 4, the expert scientists rarely invoked explicitly the definition of acceleration when trying to answer the questions about that concept in Test A. However, they interpreted the definition correctly whenever they invoked it.

Novice students

Figure 4 shows that the students also rarely invoked explicitly the definition of acceleration. However, unlike the expert scientists, the students were often unable to use the invoked definition to interpret the concept, especially in situations more complex than motion along straight lines. For example, in Problem A3 dealing with the car moving along an elliptical path, N2 invoked the definition of acceleration as "rate of change of velocity." But then the

----
Insert Figure 4 about here
----

student interpreted velocity as "speed" (i.e., as the *magnitude* of the velocity, irrespective of its direction) and thus arrived at the erroneous conclusion that the acceleration should be zero.

Misinterpretations of the definition of acceleration are not too surprising. As we shall discuss more fully in Section 6, the proper interpretation of this seemingly simple definition is remarkably complex.

Derivable Knowledge

Derivable knowledge about components of acceleration

The definition of a scientific concept may allow derivation of other general statements. These may often provide easier starting points for interpreting the concept.

For example, acceleration, like any other vector, can be described in terms of its components along particular directions. As indicated in Figure 5, one of these directions can conveniently be chosen tangent to the particle's path and directed along its velocity, the other perpendicular to the path and toward its inward side. The definition of acceleration then allows one to derive the following general results: The tangential component of the acceleration describes how rapidly the particle's speed v (i.e., the *magnitude* of its velocity $\boldsymbol{v}$) changes with time. This component is equal to dv/dt, the rate of change of the particle's speed. The perpendicular component of the acceleration describes how rapidly the *direction* of the particle's velocity changes with time. This component is equal to v^2/r, where r is the radius of curvature of the path. The acceleration $\boldsymbol{a}$ is then the vector sum of these two component vectors.

Expert scientists

Figure 4 shows that the expert scientists frequently used their knowledge about the components of the acceleration to answer the questions about this concept. Among the 19 problems of Tests A and B, a component description could be helpful in 15, those which involved motions more complex than along straight lines. On the first pass, three of the experts used their component knowledge in more than 50% of these problems.

Application of this knowledge about the components of acceleration facilitated correct and efficient concept interpretation. For example, expert E1, who used this component knowledge most frequently, exhibited the best performance; conversely, expert E3, who used this component knowledge least frequently, exhibited the worst performance. The experts who used component knowledge most often also resorted least frequently to less direct arguments about forces.

Novice students

As Figure 4 indicates, students rarely invoked knowledge about the components of acceleration. Furthermore, their knowledge about these was often deficient. For example, N5 talked about the two acceleration components in Problem A6, saying: "There's probably a

----
Insert Figure 5 about here
----

tangential acceleration.... And because it's going on a circle or around an ellipse...there's acceleration toward the center." However, this student did not know how to combine these components to find the resultant acceleration.

Some students invoked acceleration components, but lacked specific knowledge to use them properly. For example, in problems A7 and A12, N3 tried to combine a component of acceleration toward the center with a downward component due to gravity. Similar mistakes were made by a student in our instructional experiments who, when dealing with the motion of a particle around a vertical circle, considered properly the acceleration component toward the center, but discarded the tangential component in favor of a downward component "due to gravity."

Some students, although lacking precise knowledge about the components of acceleration, had intuitive notions about the superposition of different effects. The following is an example of such a basically correct notion: "Since constant-speed motion around a circle produces an acceleration toward the center, and since changing-speed motion along a straight line produces an acceleration parallel to the path, non-constant speed motion around a circle should produce a combination of both (i.e., an acceleration directed partly toward the center and partly along the circular path).

Independent Knowledge

Independent knowledge about forces

The motion of a particle depends on its interaction with other particles. This relationship is expressed quantitatively by Newton's famous mechanics principle $m\boldsymbol{a} = \boldsymbol{F}$, which relates the acceleration $\boldsymbol{a}$ of a particle, of mass m, to the total force $\boldsymbol{F}$ exerted on this particle by all other particles. This principle allows one to determine the acceleration, and thus to make inferences about motion, from independent knowledge about forces. This is why Newton's principle provides enormous predictive power about motion.

In all our test problems the information specified about the motion of a particle is sufficient to determine its acceleration. Of course, one may also try to find the acceleration by using Newton's principle and the independent information about forces. (Indeed, this may be appealing because it provides a causal explanation of the acceleration.) However, in our test problems the available information about forces is often inadequate to determine the acceleration. For example, in Problem A5 the magnitude of the force exerted by the string of the pendulum is not known; indeed, this magnitude can only be inferred from a knowledge about the acceleration of the pendulum bob.

Expert scientists

Figure 4 indicates that the expert scientists fairly often invoked forces to determine the acceleration, sometimes in as many as half of the problems of Test A. However, in many of these problems they invoked forces as an alternative argument or as a check, having first reached a solution by using knowledge about the components of the acceleration.

The expert E3, whose overall performance was the worst, invoked forces most often; conversely, expert E1, whose overall performance was best, invoked forces least often and relied mostly on his knowledge of acceleration components.

The experts usually implemented their force arguments correctly, but sometimes exhibited deficiencies. For example, in Problem A10 about the descending pendulum bob, E3 concluded incorrectly, on the first pass, that the acceleration of the bob is directed below the tangent to the path. (His difficulty was due to lack of information about the force exerted by the string of the pendulum.) Even on the second pass this expert did not exploit his knowledge about the definition of acceleration or about its components. But he finally succeeded with his force arguments by laborious reasoning about a hypothetical situation in which gravity is "switched off."

Novice students

Figure 4 shows that most students invoked forces more frequently than knowledge about acceleration components. However, deficiencies in their knowledge about forces often led them to confusions and incorrect results. The following are some examples.

Faulty identification of relevant forces. In Problem A1 the student N2, considering the motion of the sled moving with decreasing speed up along the hill, first concluded correctly that the acceleration should be directed down along the hill. However, she then considered the forces on the sled and drew a "free-body diagram" which included the gravitational force on the sled, but omitted the force exerted on the sled by the hill. Thus she arrived at the wrong conclusion that the acceleration of the sled should be vertically downward and changed her answer accordingly. She maintained this conclusion even on the second pass through this problem.

Salience of gravity for non-horizontal motion. In many cases of non-horizontal motion, students invoked gravity and, without further considerations, inferred therefrom a downward acceleration. For example, several students claimed incorrectly, without considering the influence of the string, that the acceleration of a pendulum bob is vertically downward because of gravity. Thus, student N5, in problem A2 about the pendulum in a horizontal position, said: "Acceleration is straight down, that’s the way gravity works, and I’m sure about that." Although with somewhat less certainty, N5 also believed that the acceleration should be vertically downward for other positions of the pendulum (in problems A7 and A10).

Poorly understood relation between acceleration and force. The distinctive differences between the concepts "acceleration" and "force" were sometimes not clear to students. For example, N5, considering the car moving along the curved track in problem A3, said: "That doesn’t really explain why there’s acceleration towards the middle. I don’t really see it as an acceleration, I see it as more of some sort of tug toward the middle, some sort of pull. I don’t really see it as acceleration, but I call it that because that’s what they tell me to, and I don’t know why else."

Case-specific Knowledge

As mentioned previously, a repertoire of knowledge about special cases of a concept can often be used quickly, and with relatively little mental processing, to identify the concept in situations that match these cases.

Case-specific knowledge about acceleration

The following are examples of knowledge about specific cases of acceleration:

(a) If a particle moves along a straight line with increasing speed, its acceleration is directed along its velocity; if it moves along the line with decreasing speed, its acceleration is directed opposite to its velocity.

(b) If a particle moves with a constant speed v around a circle of radius r, its acceleration is directed toward the center and has a magnitude v^2/r.

(c) If a particle moves along a curved path, its acceleration is directed toward the inside of the path, and is exactly perpendicular to the path if the particle’s speed is constant.

Expert scientists

The expert scientists often used such specific knowledge about the acceleration in familiar cases. However, a few experts lacked some of this case-specific knowledge. For example, E3 did not seem to know that, if a particle moves with constant speed along a curved path, its acceleration is perpendicular to the path. As a result, he was unsure about the exact direction of the acceleration in problem A3 dealing with the car moving along an elliptical path. It was only on the second pass through this problem that he inferred the correct answer by reasoning that the elliptical path might be locally approximated by a circular arc.

Novice students

The students interpreted the concept acceleration most frequently by invoking various case-specific knowledge elements in their repertoire. They rarely resorted to other knowledge, unlike the expert scientists who also often used more general knowledge, such as knowledge about acceleration components. Students' knowledge repertoires about acceleration, like that of the expert scientists, included the familiar cases of straight-line motion and uniform circular motion. However, they also included many incorrect knowledge elements which often led students to wrong answers.

Invocation of incorrect knowledge elements. The following are common examples of such incorrect knowledge elements (indicated by statements in italics). Many of these are incorrect because, as discussed further in the next section, students often apply knowledge without heeding any restrictive validity conditions.

If a particle moves around a circle, its acceleration is directed toward the center. This knowledge element is incorrect unless the particle moves with constant speed. The indiscriminate application of this knowledge element led several students to claim that the acceleration of the particle in Problem A13 is directed toward the center of its circular path, even though the particle moves with increasing speed. Similarly, N2, when considering the moving pendulum in Problem A2, said explicitly: "You will always have acceleration in a circular path...and it's always directed toward the center of the circle."

A particle moving with constant speed has zero acceleration. Also, *A particle moving with increasing speed has an acceleration directed along its velocity*. These knowledge elements were often applied by students to motion along curved paths, although they are only correct for motion along a straight line.

If a particle's velocity is zero, its acceleration is zero. This knowledge element is incorrect except in the very special case when a particle's velocity is zero and remains so. Yet, all five students invoked this knowledge element with great confidence in each of the two problems A5 and A8 (pendulum bob at its highest point and sled at the highest point of a hill) where the velocity of a particle is equal to zero at some instant only. The great plausibility of this knowledge element is probably due to a lack of discrimination between velocity and change of velocity (i.e., failure to realize that the velocity may be zero even if its change is not), and from students' imprecise use of the word "motion" (discussed in Section 7.)

If gravity is present, the acceleration is directed downward. This statement is incorrect except in the very special case of a particle acted on solely by the gravitational force due to the earth. Yet some students applied this knowledge element inappropriately to the pendulum bob which is also acted on by the force due to the string. The invocation of such incorrect knowledge elements frequently led students not only to wrong answers, but also to various inconsistencies and paradoxes of the kind discussed in Section 7.

Invocation of particular episodic knowledge. A few students also invoked episodic knowledge about particular cases remembered from past encounters. For example, one student said "I know from my bike that the tighter the curve, the greater the force on you"; he then used this recalled knowledge to conclude correctly that the acceleration is larger if a path is more curved. Similarly, another student recalled that he felt pushed forward when sitting in a car slowing down on a straight road; but this correct remembrance led him to the wrong conclusion that the acceleration of a slowing car should be directed along the velocity of the car.

6. APPLICATION KNOWLEDGE

As indicated in Figure 3, it is important to have appropriate application knowledge specifying when and how to use any of the main knowledge about a concept. We now examine the nature of this application knowledge possessed by experts and novices dealing with the concept acceleration.

Relevance Knowledge about Validity and Utility

Relevance knowledge about acceleration

Knowing when some knowledge is relevant involves knowing when it can validly be applied and when (if valid) it can *usefully* be applied.

Validity knowledge. Any knowledge about the acceleration must be accompanied by validity conditions. For example, the definition of acceleration is validly applicable to any *particle*, but not to an extended object (such as a rope) whose position cannot be adequately described by that of a single point. As another example, the knowledge that "a particle moving around a circle has an acceleration directed toward the center" is valid only if the speed of the particle is constant.

Utility knowledge. Knowledge about acceleration can be used more effectively if it is also accompanied by utility conditions. For example, the definition of acceleration can usefully be applied to find the acceleration whenever one knows how the particle moves near the time of interest. Similarly, the knowledge about the components of acceleration is especially useful to find the acceleration in complex situations (e.g., those involving motion along curved paths).

Expert scientists

The expert scientists used their concept knowledge about acceleration appropriately and almost never violated any validity conditions. Most of the expert scientists seemed well aware of the utility of their knowledge about the acceleration components, as is apparent from their frequent invocation of this knowledge. However, it is interesting to note that they invoked the definition of acceleration quite rarely, even though it might sometimes have been useful to do so; indeed, they might thereby have avoided some difficulties or cumbersome arguments.

Novice students

Frequent violations of validity. As already discussed in the last section, students made very many mistakes because they invoked knowledge which violated validity conditions. In many cases, such restrictive validity conditions were probably not attached to students' remembered knowledge. Alternatively, they may have been defective or might not have been heeded when the knowledge was retrieved.

Extensions of validity conditions. Sometimes students attempted to extend their knowledge beyond the bounds of previously known validity. For example, some students tried to extend their knowledge about circular motion to motion along ellipses or spirals. Such a generalization is possible, if correctly implemented, since any curve can be locally approximated by a circular arc. Thus N2, considering a curved path in Problem B2, argued correctly that "if you make it (the curved path) act like a circle, then you can apply v^2/r." However, the same student, considering the elliptical path in Problem A3, generalized incorrectly that the acceleration should be directed toward the center of the ellipse (rather than perpendicular to the elliptical path).

Validity and case-specific knowledge. Unlike more general knowledge, case-specific knowledge requires many more restrictive validity conditions to delimit the domain of applicability for each case. Hence students' great reliance on case-specific knowledge makes them particularly error-prone since they often fail to store all the needed validity conditions in memory, or fail to heed them properly when applying their knowledge.

Implementation Knowledge

Any declarative statement is scientifically meaningless without accompanying procedural "implementation knowledge" specifying what one must actually do to determine its truth or falsity. Similarly, any scientific concept is uninterpretable without accompanying implementation knowledge specifying what one must do to identify or construct the concept in any particular instance. Such implementation knowledge not only leads to "operational definitions" designed to ensure that scientific concepts are well-specified, but also explicates the cognitive complexities underlying some seemingly simple statements.

Implementation knowledge about acceleration

As mentioned in Section 5, acceleration is defined by the statement "acceleration is the rate of change of velocity with time," or equivalently by the mathematical statement that $\boldsymbol{a} = dv/dt$. The implementation knowledge, specifying how to apply this definition in order to interpret the acceleration, can be specified by the following procedure illustrated in Figure 6 for a particle moving along any path:

(a) Identify the velocity $\boldsymbol{v}$ of the particle at the time t of interest.

(b) Identify the velocity $\boldsymbol{v}'$ of the particle at a slightly later time t'.

(c) Find the small velocity change $\Delta\boldsymbol{v} = \boldsymbol{v}' - \boldsymbol{v}$ of the particle during the short time interval $\Delta t = t' - t$. (Graphically, draw from the same point the arrows representing $\boldsymbol{v}$ and $\boldsymbol{v}'$. The arrow from the tip of $\boldsymbol{v}$ to that of $\boldsymbol{v}'$ represents then $\Delta\boldsymbol{v}$.)

(d) Find the ratio $\Delta\boldsymbol{v}/\Delta t$. (This is indicated by an arrow having the same direction as $\Delta\boldsymbol{v}$, but a different magnitude which, unlike $\Delta\boldsymbol{v}$, is ordinarily not small.)

(e) Repeat the preceding calculation, with the time t' chosen progressively closer to the original time t, until the ratio $\Delta\boldsymbol{v}/\Delta t$ approaches a limiting value which no longer changes. This value is called the "acceleration $\boldsymbol{a}$ of the particle at the time t of interest." (Thus $\boldsymbol{a} = d\boldsymbol{v}/dt$ if $d\boldsymbol{v}$ and dt denote the infinitesimally small differences $\Delta\boldsymbol{v}$ and Δt.)

Insert Figure 6 about here

This procedural specification of the definition provides an explicit and detailed specification of the concept acceleration. It also makes apparent the complexities hidden by the seemingly simple definitional statement $\boldsymbol{a} = d\boldsymbol{v}/dt$. Indeed, further knowledge is required to implement each of the major steps in the preceding procedure. For instance, adequate knowledge is needed to implement step *c* (i.e., to subtract two velocities vectorially so as to find the velocity change $\Delta\boldsymbol{v}$. Substantial implementation knowledge is also needed in step *e* to find the limiting value of the ratio $\Delta\boldsymbol{v}/\Delta t$ when Δt becomes sufficiently small.

Adequate implementation knowledge is also required to apply knowledge about the components of the acceleration or to apply case-specific knowledge about this concept.

Implementation knowledge includes also the ability to make appropriate discriminations so as to avoid errors and confusions (e.g., to discriminate between vectors and numbers, between velocity and speed, etc.).

Expert scientists

The expert scientists implemented reliably and correctly the knowledge which they invoked about the definition of acceleration or about its components. For example, although the overall performance of E2 was inferior to that of all the other experts, he invoked the definition of acceleration in Problem A9 (dealing with the car moving along the spiral road) and then executed very well the procedure implementing this definition.

The experts also made all needed discriminations and thus avoided the confusions commonly observed among novice students.

Novice students

Even when novice students invoked explicitly the definition of acceleration, they were often incapable of implementing it correctly. The definition thus remained merely a superficial verbal statement.

For example, in problem A5 dealing with the pendulum bob at its highest point, N2 said: "Well, the velocity is zero so that acceleration has to be zero. Because acceleration equals the change in velocity over the change in time, and...the derivative of velocity over time...is zero." Here the student concluded incorrectly that the acceleration is zero after invoking explicitly the definition of acceleration and also its mathematical statement *dv/dt*. But the student implemented incorrectly both the definition of acceleration and the concept "derivative" learned in previous mathematics classes.

Students were frequently led to faulty interpretations by failing to make appropriate discriminations. For example, we already mentioned how students sometimes did not discriminate velocity from speed (i.e., from the *magnitude* of the velocity without attention to its direction), or how they sometimes failed to discriminate a *change* of velocity from the velocity itself.

Failures to make appropriate discriminations, together with failures to heed restrictive validity conditions, led to many troubling inconsistencies and seeming inconsistencies of the kind discussed in the next section.

7. FORM OF KNOWLEDGE

The ability to interpret a concept depends significantly on the form of the concept knowledge (i.e., on its description and structure). Accordingly, this section examines the descriptive precision, terminology, and coherence of experts' and novices' knowledge about acceleration.

Knowledge Description

The knowledge about a concept may be expressed with different terminologies (i.e., different words or symbols) and with varying degrees of precision. However, words used in science must always be connected to their referents with sufficient precision to avoid ambiguities and confusions. Furthermore, judiciously chosen words may facilitate important discriminations, while other words may make such discriminations more difficult.

Expert scientists

In most cases the expert scientists used special words (like "velocity," "speed," "acceleration," "magnitude") consistently with well-specified technical meanings and without confusion. Occasionally, some experts did use words loosely (e.g., by saying "velocity" instead of "speed"; but they did not confuse the underlying concepts and made correct inferences despite their sloppy wording.

Novice students

Imprecise use of words. The students often used words imprecisely and failed to make important discriminations. As a result, they frequently became confused and misinterpreted the concept acceleration in some of the ways discussed in the previous sections. For example, we already mentioned the common lack of discrimination between the words "velocity" and "speed."

Sometimes students even failed to discriminate adequately between the words "velocity" and "acceleration." For example, N3 in Problem A7 said: "If there was acceleration, then I would put it in that direction...toward its maximum height. ...But I'm not sure if that arrow is representing acceleration, maybe it's just the velocity vector, I may be confusing the two very much." Trowbridge and McDermott (1981) also observed that students frequently confused acceleration and velocity in simpler cases of motion along straight lines.

As another example, the word "deceleration" was sometimes used by students without a precisely defined meaning. (Indeed, that word is not a technical term defined in physics.) The resulting difficulties can be illustrated in the case of N5 who was trying to cope with problem A1 about the sled slowing down while moving up along a hill. In the first pass through this problem, N5 answered the question about the direction of the acceleration by drawing an arrow *up* along the hill. In the second pass, she showed her confusion by saying: "It's slowing down, so is acceleration the other way and deceleration this way? ...Deceleration, acceleration, it's the same thing as far as direction goes. And I'm not sure about the direction."

Difficulties caused by terminology. The scientific term "acceleration" has a meaning different from that of the same word used in everyday life, since it describes changes in the direction as well as in the magnitude of the velocity. Hence it is not surprising that students sometimes failed to make the appropriate discrimination and interpreted acceleration without attention to directional changes of the velocity.

It is perhaps more interesting that students were sometimes led to confusions by words which, although seemingly innocuous, were misinterpreted or had misleading connotations. The following are some examples.

Motion. The words *motion* or *move* are vague since they can refer either to velocity or to acceleration. The use of these intuitively appealing words can easily lead to unwarranted inferences.

Thus many students used the following argument: If the velocity of a particle is zero, then the particle does not move; and if it does not move, its acceleration must be zero. The use of the word *move* seemed to make this argument obviously valid to many students. Indeed, four out of the five students confidently made this argument in Problems A5 and A8, thus claiming incorrectly that the acceleration is zero. For example, one student said, "If it is not moving, it can't have any acceleration." Another said, "It has zero velocity when it reaches the top. No motion, so no acceleration." Still another said "Must be potential energy and not acceleration, since it has no motion."

Zero. Zero is a special value of a quantity, but is sometimes interpreted by students as the non-existence of this quantity. The difference can be expressed by the phrase "the velocity of the particle *is* zero" versus the phrase "the particle *has* no velocity." The second phrase suggests the non-existence of a velocity, and the corresponding inference that something non-existent cannot change. Hence it contributes to the erroneous conclusion that zero velocity implies zero acceleration.

Acceleration at an instant. The acceleration of a particle at a particular instant (or at the particular point where the particle is at that instant) has an unambiguous meaning, but involves implicitly a comparison of the velocity at this instant with that at a slightly later one. Unfortunately, the wording "acceleration at an instant" suggests to some students that the acceleration involves only one instant, and thus leads them to the conclusion that the acceleration must be zero since velocity cannot change instantaneously.

For example, N1, considering the pendulum at its highest point in Problem A5, said: "It wouldn't be moving as it was right at point A. So the acceleration would be zero...because it's not moving. But if you took it, like, in the area of point A, then that wouldn't be true. ...At first, I wasn't sure if I should count it at the very instant or not." Thus the student concluded erroneously that the acceleration of the pendulum bob is zero.

Coherence of the Knowledge Structure

A person's knowledge is coherent to the degree that the various elements of this knowledge are interconnected by implementable inferences among them. For example, concept knowledge about acceleration is more coherent if various case-specific knowledge elements about acceleration can be inferred from each other or from the basic definition of acceleration.

The coherence of concept knowledge can greatly increase the ease of remembering it, of regenerating it if partially forgotten, of resolving inconsistencies and paradoxes, and of extending the knowledge by generalization or further learning.

It is highly desirable that any new concept knowledge be coherent within itself as well as with pre-existing knowledge. For example, knowledge about a scientific concept is most useful if (a) the case-specific knowledge is consistent with formal definitional knowledge and testable against it, and (b) if it is connected to everyday intuitive knowledge, but properly discriminated from it.

Expert scientists

As already mentioned, the expert scientists possessed general knowledge about the definition of acceleration and especially about the properties of its components. This knowledge was regarded as primary and was often reverted to as a source for inferences when answers obtained by other means (e.g., by force arguments) seemed dubious or incomplete.

Experts' case-specific knowledge was consistent with such formal knowledge. As a result, experts' concept interpretations rarely led to any inconsistencies or paradoxes.

Novice students

By contrast, students' knowledge about acceleration was quite incoherent, seemingly consisting of various knowledge elements only weakly interconnected by possible inferences among them. Thus their knowledge exhibited some of the fragmentation typical of the naive scientific knowledge described elsewhere (diSessa, 1988). The following observations provide evidence of the incoherence of students' concept knowledge and indicate the consequences of such incoherence.

Unperceived inconsistencies. Some of the knowledge elements invoked by students were inadvertently inconsistent with each other. For example, N1 claimed that the acceleration of a particle moving around a circle is directed along its velocity; yet she also claimed that this acceleration is not zero because the velocity changes its direction. Actually, the definition of acceleration implies that the acceleration must be directed *perpendicular* to the velocity precisely *because* the direction of the velocity changes.

Similarly, N2 said in Problem A2, "You will always have acceleration in a circular path. It's not the change in velocity, it's just the velocity squared over r—and it's always directed toward the center of the circle." She neither seemed to recognize that this statement is false unless the speed is constant, nor that the acceleration is v^2/r toward the center precisely *because* the velocity changes.

Difficulty of regenerating knowledge. The incoherence of students' concept knowledge made it difficult for them to remember such knowledge or to regenerate it if it had been forgotten. For example, N1 early in Test A said, "I was trying to remember, I'm fairly sure that...the direction of the acceleration around a circle is the tangent.... I was just not sure for a minute, but I couldn't think of anything else that it would be, so I put the tangent there anyway." Note that, although the student could not remember the correct result, she made no attempt to regenerate it from the definition of acceleration; instead, she was satisfied to make a guess which was, in fact, wrong.

Frequent paradoxes. Because of the incoherence of their knowledge, students often retrieved knowledge elements which they recognized as inconsistent.

A typical such paradox was encountered by N2 when considering the car moving along the elliptical path of Problem A3: "I'm trying to figure out the acceleration, and there shouldn't be any because there's no change in velocity because it's always constant.... But then, with a circle, um, acceleration equals velocity squared over r, I think. Um, so there is an acceleration." (The error was in claiming that the velocity, rather than the speed, is constant.) Unable to resolve the paradox, the student opted for the incorrect answer that the acceleration is zero.

The following are other examples of paradoxes encountered by students. In each case the paradox is due to an error in one of the student's conflicting knowledge elements.

- Pendulum in problem A2: Circular motion implies that the acceleration is directed toward the center; but increasing speed implies that the acceleration is directed along the velocity.
- Sled on hill in problem A1: Decreasing speed implies that the acceleration is opposite to the velocity; but the presence of gravity implies that the acceleration is downward.
- Pendulum at highest point in problem A5: Zero velocity implies that the acceleration is zero; but the change in the direction of the velocity implies that the acceleration is not zero.

Difficulty of resolving paradoxes. Students were usually unable to resolve such paradoxes because they could not make inferences from the definition of acceleration or from other more fundamental knowledge. In the words of N5, who could not resolve the last of the above-mentioned paradoxes, "It gets harder and harder to think the longer I think about it." Thus the students resolved more than half of their paradoxes (13 out of 25) by merely making an arbitrary choice that revealed little or no reasoning. (As one student said: "I'm stuck. Well, I put it in the direction it's moving.") As might be expected from such random choices, only about half of these were correct.

Students dealt with most of the other paradoxes by ad-hoc reasoning, or by merely retaining their initial choice rather than dealing with the complications of examining an alternative. Only in three of these 25 paradoxes did students revert to the definition of acceleration, and were then indeed able to resolve these paradoxes successfully.

Difficulty of making general inferences. An incoherent collection of knowledge elements makes it difficult to make general inferences. For example, Test B asked general questions such as the following: If a particle moves along a curved path, can its acceleration be neither parallel nor perpendicular to its velocity? Students often tried to answer such questions by resorting to their knowledge about special cases of the acceleration. But unless such cases are enumerated exhaustively, one cannot make a general argument proving impossibility. Thus students often argued unconvincingly that something was impossible merely because they could not think of a case where it would be possible.

Self-awareness of incoherence. Some students were quite aware of the incoherence of their own concept knowledge. For example, one student made the following unsolicited comments at the end of the observational session, "It's not that I have competing ideas, but I didn't find a uniform way of putting all the pieces together. So the motivation is to try to use all the words from each situation, but not always all the words apply with the same magnitude to each situation. So you have to be leery of what is going on and what does and does not apply."

8. CONFIDENCE LEVEL

Persons' confidence in their knowledge can significantly affect their appropriate use of that knowledge. For example, it can affect whether they are likely to retrieve such knowledge, to check it, or to retain it in the face of conflicting information. Ideally, a person should be confident about knowledge which is correct and effectively usable, and not confident about knowledge of dubious merits. Either unwarranted confidence, or unwarranted lack of confidence, can lead to deficient concept interpretations.

Comparison of experts and novices

As is apparent from Figure 7, the expert scientists not only interpreted the concept acceleration correctly most of the time, but were also quite confident in the correctness of their interpretations. Although they were not always completely confident, they never merely guessed. Furthermore, the correctness of any of their answers was highly consistent with their confidence in it (i.e., the experts were usually correct when they were confident).

By contrast, the novice students not only interpreted the concept acceleration incorrectly in most problems, as indicated in Figure 7, they were seldom confident about their answers; most of the time they were uncertain or guessing. Furthermore, the correctness of any of their answers was poorly consistent with their confidence in it: students were often wrong when they were confident, and were sometimes correct when they were merely guessing.

Unwarranted high confidence. Even some expert scientists exhibited occasionally unjustified overconfidence in their answers. For example, when E5 faced Problem A7 about the pendulum at its lowest point, he asserted confidently (for interesting reasons discussed in

----
Insert Figure 7 about here
----

the next section) that the acceleration of the pendulum bob is zero. He did not bother to check his answer. Even when asked to explain his reasoning more fully on the second pass, he confidently repeated his wrong argument and persisted in his wrong answer.

Students were much more prone to be highly confident of incorrect knowledge. For example, several students confidently, but incorrectly, believed that the acceleration of a particle moving around a circle is directed toward the center, whether or not the speed is constant.

A particularly striking example of students' unwarranted confidence was their notion (mentioned in Section 5) that a particle with zero velocity must have zero acceleration. Students' great confidence in this incorrect knowledge was manifested by the fact that all five students invoked this knowledge on the first pass in both of the relevant problems (A5 and A8) where the velocity of a particle is zero at a particular instant. Furthermore, two of these students were very sure about their wrong answers in both problems. For example, N5 said explicitly in Problem A5, "Acceleration at A is zero because it's not moving. I am sure of that." and said again in Problem A8, "Acceleration is zero, I'm sure." Two other students were entirely sure in Problem A8 about the sled at its highest point, and somewhat less sure in Problem A5 about the pendulum at its highest point. Only N3 felt somewhat uncertain in both these problems.

The preceding example illustrates well the effects of unwarranted confidence in some knowledge: (a) Such knowledge is very frequently invoked. As pointed out previously, all the students, on every relevant problem, invoked their incorrect knowledge about the acceleration for zero velocity. (b) Such knowledge is rarely questioned or checked against other criteria, so that wrong conclusions are not detected. For example, only two students changed their answers to one of the problems (A8) during their second pass. (c) Such knowledge may be maintained even in the face of explicit counter-evidence, so that wrong conclusions persist. For example, N1 initially argued correctly that the acceleration of the pendulum at its highest point should be non-zero because the direction of its velocity changes; however, she quickly changed her answer when she recalled her notion that the acceleration must be zero when the velocity is zero. Similarly, N5 was certain on the first pass that both the pendulum of Problem A5 and the sled of Problem A8 should have zero accelerations. Although much more thought on the second pass convinced her to correct her answer in Problem A8, it was not sufficient to correct her answer in Problem A5.

Unwarranted low confidence. Unwarranted lack of confidence in some knowledge can also have deleterious consequences. Such knowledge may not be believed even if correct, and may be rejected in favor of incorrect conclusions.

For example, N4 initially claimed that the acceleration of the car is zero at both points of the spiral road of Problem A9. On the second pass through this problem, N4 recalled his other knowledge and proceeded to argue correctly, with beautifully justified reasoning, that the acceleration of the car at point A should be less than at point B, and that the accelerations should be directed perpendicular to the spiral path. But he was reluctant to accept these correct conclusions and retained instead his original wrong answers, saying, "Once again, centripetal acceleration confuses the issue. Um, I’m not going to change my answers, because it would probably just confuse things.... I’m not certain and I don’t want to go through and make a mess."

Self-awareness of low confidence. Students sometimes expressed explicitly their lack of confidence in their knowledge. Thus N3, considering Problem A5 about the pendulum at its highest point, said, "Uh boy, I hate this stuff when you’re partially sure about one part and you’re not sure if what you know pertains to this question or not." Similarly N4, confronting Problem A6 about the elliptical racetrack, said: "When you think about these, it makes you...more shaky."

9. RETRIEVAL OF AVAILABLE KNOWLEDGE

Even if correct concept knowledge is available, its retrieval may be faulty because of response to inappropriate cues or because of a mindset producing fixation on inappropriate information. Instances of such faulty retrieval occurred among both expert scientists and novice students.

For example, when considering problem A10 about the pendulum at an intermediate position, E5 invoked explicitly his knowledge about acceleration components and gave a perfect answer. However, in Problem A7 about the pendulum at its lowest point, E5 recalled his knowledge about "simple harmonic motion" of a particle oscillating around an equilibrium position. He thus concluded that the acceleration of the pendulum bob should be zero; he reiterated this wrong answer even on his second pass through this problem. His misleading fixation on simple harmonic motion thus prevented him from retrieving his appropriate knowledge about acceleration components, and to ignore completely the acceleration component perpendicular to the path.

Similarly, situations involving motion in a vertical plane caused many students to retrieve knowledge about gravity, and thereby to ignore their knowledge about acceleration, although the latter could be applied more advantageously. We already mentioned examples where students were thus led to claim erroneously that the acceleration of a pendulum bob is always downward, or to ruin a perfectly correct argument about acceleration by introducing an irrelevant component due to gravity.

10. LIMITED EXPERTISE OF "EXPERTS"

Figure 2 indicates that the expert scientists' interpretation of the concept acceleration was good, although not perfect. Expert E2 was particularly interesting by virtue of his rather poor performance. By many criteria, this individual would be deemed an expert about acceleration. He was an experienced research physicist and had taught acceleration in several introductory physics courses during the preceding years. Nevertheless, on the first pass he answered incorrectly five of the 13 problems of set A, and used faulty reasoning in at least one more. On the second pass, he corrected only one of his wrong answers. In tackling the 13 problems of Test A, he engaged in quite unnecessary lengthy calculations in four of these problems, yet arrived at incorrect answers in three of these. Finally, he became too flustered to attempt Test B.

Need for caution in identifying experts. The preceding observations suggest the need to specify with care the criteria used to define experts. For example, it is important to distinguish between "nominal experts" (individuals so labeled on the basis of extrinsic criteria such as academic degrees, titles, or professional positions) and "genuine experts" (individuals who actually perform very well within some specified domain). The distinction is significant because nominal expertise does not necessarily imply good performance.

Correspondingly, care is needed in interpreting cognitive research on "experts." Indeed, in some comparative studies of novices and experts, the latter have been chosen on the basis of nominal criteria rather than of observed superior performance. The resulting data must be interpreted with caution since the observations of such experts may provide misleading information about the thought processes leading to good performance.

However, there is also cognitive interest in studying "deficient experts" (i.e., those who do *not* perform well) in order to elucidate the following question: What deficiencies of the underlying knowledge and thought processes are responsible for the fact that someone does not perform well despite years of experience?

Reasons for deficient expertise of E2. Let us briefly examine the preceding question in the case of E2.

The main concept knowledge of E2 seemed quite good. For example, he knew the definition of acceleration and could use it successfully; he could apply knowledge about the components of the acceleration; and he often resorted systematically to his knowledge about forces.

However, he had unusual notions about what knowledge is particularly relevant: (a) He seemed to believe that formal mathematical knowledge is particularly useful in physics arguments. (b) He had great confidence in such formal knowledge rather than in more qualitative descriptions. (c) Moreover, he attached primary importance to Newton's mechanics principle $ma = F$ and used it in routinized ways even when other and simpler formal knowledge, like the definition of acceleration, would have been more useful.

For example, he stated explicitly, "I fall back on tried and true things... Well, not so much formulas, as let's start with Newton's second law, and uh, take that as far as we can. So that is what I do." In Problem A13 he said, "I wouldn't have confidence in, uh, the qualitative answer." Later he added, "When I get tired, my confidence in my physical insight solutions goes down, and I fall back on tried and true things. I am usually willing to trade time for, if not certainty, then at least confidence. My tendency is not to make quick answers because I have learned that my quick answers are not, uh, as accurate as I would like."

The preceding characteristics of E2's knowledge had marked consequences on his performance: he often did large amounts of unnecessary labor and mathematical computation when the acceleration could have been determined by much simpler means, and he frequently obtained wrong results despite all such labor.

For example, in Problem A7 about the pendulum at its lowest point, he began on the first pass to invoke a force argument which would have been sufficient to conclude that the acceleration must be vertical. However, instead of pursuing this argument, he went on to a mathematical calculation, spent about eight minutes dealing with differential equations, concluded erroneously that the acceleration is tangent to the path, and felt uncertain about this result. On his second pass through this problem, he changed his answer to claim that the acceleration is zero—a result even more obviously incorrect than his previous answer had been.

As another example, in Problem A8 about the sled at the highest point along the hill, E2 started from Newton's principle $m\boldsymbol{a} = \boldsymbol{F}$ and spent three minutes on the resulting differential equations in order to decide that the acceleration is not zero and to determine its direction. At the end of this unnecessary calculation he said, "I could probably have solved it just using the physics without the equations if I had stopped and thought about it."

E2 had some awareness of his tendencies to be excessively formal. Thus, at the end of his first pass through Test A, he spontaneously remarked, "My tendency in seeing a new problem...in a slight somewhat fresher situation is to fall back on what I am most sure of...usually, Newton's second law. ...If I were better at physical insight...I think that I could have answered all these questions, or a lot of it, based on physical insight.... In other words, not setting up the laws of motion, and solving them, and getting the answer out of that, but, uh, working qualitatively."

In summary, E2's main deficiency was in his application knowledge. He invoked general concept knowledge and formal mathematical knowledge in many cases where it was inefficient or even inappropriate to do so. In addition, he exhibited deficiencies in his implementation of such general and formal knowledge, and thus reached incorrect conclusions in several problems. Finally, his confidence in his methods and answers was poorly related to their correctness; indeed, the knowledge he regarded as most "tried and true" often led him to mistakes.

11. REMEDYING CONCEPTUAL DEFICIENCIES BY INSTRUCTION

The observations discussed in Section 3 indicate that students' ability to interpret the basic physics concept of acceleration is quite poor, even after they have been studying and using this concept for a couple of months in an introductory physics course. As we have seen, many of their difficulties can be traced to the fact that their available concept knowledge is incoherent and not reliably implementable in particular cases.

The work described in this section is aimed to explore how such underlying deficiencies in student's concept knowledge could be remedied by suitably designed instruction. We summarize here only the main results of this work, since some of it has already been published elsewhere (Labudde, Reif, & Quinn, 1988; Reif, 1987b).

Instructional Design Principles

The following instructional design principles were specifically formulated to overcome the previously mentioned conceptual deficiencies commonly exhibited by students:

(a) Ensure the interpretability of an unfamiliar concept by teaching explicitly a definitional *procedure* for determining the value of the concept (e.g., the definitional procedure of Figure 6 in the case of acceleration).

(b) To promote the coherence of students' new concept knowledge, have them apply this single definitional procedure to determine the concept in diverse special cases.

(c) To ensure that students' entire knowledge is globally coherent, have students use their new concept knowledge to detect, diagnose, and remedy their own misconceptions or mistakes in other error-prone situations. (For example, present students with situations where somebody claims erroneously that a particle, moving with constant speed along a curved path, has zero acceleration; or that a particle, whose velocity is zero at some instant, has zero acceleration.)

These instructional principles alone were not intended to yield optimal instruction. For example, they emphasize general ways of interpreting a concept and don't explicitly foster the compilation of case-specific knowledge. However, it seemed valuable to explore the extent to which implementation of these instructional principles would be sufficient to remedy students' major difficulties in interpreting a scientific concept.

Experimental Methods

The preceding instructional principles were implemented to teach the concept acceleration. Their implementation by a human tutor has been previously described by Labudde, Reif, & Quinn (1988). In later work, the first two principles (omitting the third one involving diagnostic activities) were also implemented in the form of a computer tutorial designed to teach acceleration.

The subjects in all these instructional investigations were students at the same stage of an introductory physics course as those used in the preceding study of novices and experts. These students were given a pretest, then the instructional intervention, and finally one or more posttests. They were asked to talk out loud when taking these tests, as well as during the instruction. The transcribed tape-recordings were subsequently analyzed.

The problems on the pretests and posttests were similar to those in Test A. (Indeed, responses on these pretests provided additional data about novice students.) However, the posttests included also some "diagnostic problems" designed to assess students' ability to detect, diagnose, and correct errors of concept interpretation. Such a diagnostic problem differed from the ones in Test A by also providing an answer, with or without accompanying reasoning, allegedly given by some other student. (This answer was often incorrect and deliberately designed to reflect a common error or misconception.) The problem then asked the following questions: Is the given answer correct? If not, what is the likely reason for the student's incorrect answer? What is the correct answer?

The test questions could be answered by students by any method of their choice. Hence it was also possible to ascertain to what extent students would spontaneously use the definitional procedure taught to them. In the computer-implemented instruction, a delayed posttest was also administered about four weeks after the instruction.

Results of the Instruction

The main results of the preceding instructional interventions were as follows.

After the instruction, all students could successfully implement the procedural definition of acceleration, and often invoked this procedure spontaneously.

Most students used the procedural definition judiciously in cognitively efficient ways. They used it preferentially in unfamiliar or complex cases, but otherwise resorted to compiled knowledge about familiar cases and then used the procedure only as an occasional check.

However, there were significant individual differences in students' use of the procedure after the computer-implemented instruction. For example, one student used the procedure in every problem, with resultant excellent performance. But one student never used it, relying instead on previous knowledge, and performed worse on the posttest than on the pretest. As indicated in Figure 8, students who frequently used the definitional procedure improved significantly on the posttest, while those who used it rarely deteriorated in their overall performance.

In the human-implemented instruction of Labudde et al., the effects of the instruction were more pronounced and scores on these tests improved from about 40% to 95%.

Invocation and implementation of the procedural definition of acceleration did not change significantly after a period of four to six weeks.

Insert Figure 8 about here

Some knowledge was spontaneously acquired, although it was *not* explicitly taught. For example, two students going through the computer-implemented instruction generated some knowledge about the components of acceleration and used it successfully with high reliability. For example, one of these students said, "Actually I should have seen it before: you just add a centripetal acceleration vector and a plain old vector just along the tangent. It's kind of stupid actually; I should have seen it earlier, instead of humpty-dumptying with velocity vectors."

Learning the procedural definition of acceleration seemed to produce some deeper understanding of the properties of the acceleration. For example, one student, asked whether an answer obtained by application of the procedural definition seemed to make sense, answered: "Um, yeah.... It does have a linear component because it's changing in speed; at the same time it has a centripetal acceleration due to the rotation. So I guess, if it was constant velocity, it would just be inward. But because there is a clockwise change to the positive in speed, then it should be pointed more in that direction. Boy, it's truly a good method." Similarly, another student said: "It's better to use the procedure for this problem, since with components you don't really know where the different accelerations come from, but starting with velocity vectors, you can be more sure."

Finally, students were quite effective in diagnosing and correcting their own previous errors of concept interpretation or others' errors presented to them. In the human-implemented instruction of Labudde et al., as well as in the computer-implemented instruction, students correctly detected, diagnosed, and corrected more than 80% of errors presented in the diagnostic problems.

12. INSTRUCTIONAL IMPLICATIONS

Although the work described in the preceding pages has dealt in detail with the concept acceleration, it is more broadly relevant to other scientific or mathematical concepts. Hence it seems appropriate to summarize briefly the main conclusions yielded by this work and to point out its wider implications for the teaching of scientific concepts.

Summary of Findings

The main findings emerging from our work are the following:

(1) A cognitive analysis helps to identify important features of the underlying knowledge and thought processes required to interpret a scientific concept accurately and efficiently. This underlying knowledge is quite complex, even in the case of an elementary concept such as acceleration, and deficiencies in this knowledge lead to many observed concept misinterpretations.

(2) Expert scientists usually interpret scientific concepts effectively and efficiently, and appear to have the requisite kinds of underlying knowledge. However, even experienced scientists may exhibit marked deficiencies in interpreting a basic scientific concept such as acceleration. This observation further illustrates the complexity of the interpretation task.

(3) Novice students, even after learning a scientific concept and using it for some months, often interpret it incorrectly in many situations (similarly, poor performance was observed by Trowbridge & McDermott, 1981, in their work on students' interpretation of acceleration in the simpler case of motion in one dimension). Students' poor performance can be traced to underlying concept knowledge which is highly fragmented, and which is often not implementable even if it can be correctly stated.

(4) It seems possible to design instructional interventions which remedy some of these deficiencies of students' concept knowledge—and thereby greatly improve their ability to interpret scientific concepts effectively.

Deficiencies of Prevailing Instruction

The preceding findings can be used as a basis for examining the characteristics of prevailing methods of teaching scientific concepts and for identifying some of their deficiencies.

New concepts are sometimes introduced by appeal to familiar experiences or analogies, and are then used without ever being adequately defined. As a result, students are left with the confusing task of using vaguely specified scientific concepts and discriminating them from equally vaguely specified concepts in everyday life. For example, the concept "force" is used in physics as a particular way of specifying how the interaction between objects depends on their properties. Yet, some physics texbooks (e.g., Sears, Zemansky, & Young, 1987; Serway, 1983) introduce this concept as one obvious from everyday experience or define it merely as a "push or pull." Hence they leave students with misleading anthropomorphic notions that obscure the significance of the force concept in physics and make it even difficult to understand why an inanimate table exerts a force. Some other books (e.g., Ohanian, 1985; Resnick & Halliday, 1977) define force in terms of acceleration in such a way as to obscure the fundamental difference between these two concepts and to make it seem that Newton's principle $m\boldsymbol{a} = \boldsymbol{F}$ is merely a tautology. Hence it is not surprising that some students, like N5 quoted in Section 5, have difficulties distinguishing between acceleration and force. Other concepts (such as time, mass, charge, etc.) are often equally poorly or ambiguously defined.

Even when concepts are unambiguously defined, the procedural knowledge needed to interpret such definitions is rarely taught. For example, most physics textbooks define acceleration as "the rate of change of velocity with time" and state this definition formally as $\boldsymbol{a} = d\boldsymbol{v}/dt$. But almost none spell out explicitly the complex procedure, illustrated in Figure 6, needed to interpret this definition; nor do they give students practice in implementing such a procedure to apply the formal definition in specific cases. Hence, as shown by our data, the formal definition remains for most students an empty statement that can be quoted, but not used.

Although concept knowledge may be coherent in the mind of an instructor or coherently represented in a textbook, it often remains fragmented in the minds of students because they themselves have not made the inferences interrelating the elements of this concept knowledge. For example, most physics textbooks apply the definition of acceleration to the circular motion of a particle with constant speed, and thus derive the result that the acceleration is then directed toward the center and has a magnitude v^2/r. But to students, who have not made this inference themselves, this result becomes merely another disconnected fact to be memorized, along with the definition of acceleration. Thus it is not too surprising that students are often led to inconsistencies and paradoxes of the kind described in Section 7.

Finally, prevailing approaches to instruction often pay inadequate attention to students' pre-existing knowledge acquired in everyday life or prior schooling. Hence they may fail to connect new scientific concept knowledge with students' prior notions, and to discriminate it from such notions, with the result that students' entire knowledge remains incoherent. For example, the concept "acceleration" in physics, just like the one denoted by the same word in everyday life, describes a change of velocity. But in physics it describes any change of velocity, whether due to a change in its magnitude or its direction; by contrast, in everyday life it describes more vaguely a change merely in the magnitude of the velocity, or sometimes just an increase in this magnitude. Persistent confusions by students can thus be expected unless one points out the distinctions and provides students with practice in making the appropriate discriminations.

Instructional Suggestions

As mentioned at the beginning of this paper, recent years have seen numerous investigations of students' naive scientific conceptions and misconceptions. In agreement with the comments made in the last paragraph, these investigations point out forcefully why science instruction must pay adequate attention to students' pre-existing knowledge. Sometimes this pre-existing knowledge may fruitfully be exploited to facilitate the acquisition of more refined scientific knowledge; it may also lead to many confusions or scientific misconceptions. But attention to students' pre-existing knowledge, although necessary, is far from sufficient to teach scientific concepts successfully. Indeed, the work described in the preceding pages indicates that the accurate and efficient interpretation of a scientific concept requires the complex kinds of knowledge indicated in Figure 3. Hence instruction must ensure that students learn explicitly all these kinds of knowledge.

Prevailing teaching methods are often inadequate because of insufficiently explicit teaching of concept knowledge. Instructors and textbooks commonly fail to recognize the complexity of the knowledge required to interpret and use a previously unfamiliar scientific concept. Hence they often fail to spend enough time and attention teaching basic concept interpretation before going on to more complex problem-solving tasks. As a result, students' concept knowledge remains inadequate and error-prone, providing a shaky basis for solving problems or for further learning. As we have seen, after two months of learning and using the concept acceleration in a conventional college physics course, students could answer correctly less than 25% of simple interpretation questions about this concept. Students had many difficulties, only few of which were due to students' confusions with everyday knowledge.

Our work suggests the following ways in which more explicit and effective teaching of scientific concepts could be implemented.

(1) After some introductory qualitative and motivating comments, give a clear definition of the concept and accompany it by an explicit procedure indicating how this concept can be interpreted in any specific case.

(2) Let students practice applying this definitional procedure to interpret the concept in a variety of special cases. Try thereby to ensure that the definition of the concept is usable by students, and that students' case-specific knowledge about the concept is coherent because they can infer it from one basic definition.

(3) Guide students to summarize and organize their accumulating case-specific knowledge. Try thereby to build up students' repertoire of useful compiled knowledge.

(4) Encourage students to use their new explicit knowledge about the scientific concept to confront situations which are error-prone, either because of intrinsic difficulties or potential confusions with prior knowledge. In each case, ask them to detect their mistakes of concept interpretation, to diagnose the likely reasons for them, and to correct them. Try thereby to ensure that the students' entire knowledge, newly acquired as well as pre-existing, becomes consistent and coherent.

(5) Finally, encourage students to be efficient by using their compiled case-specific knowledge in familiar situations, and relying on more general knowledge only when this is necessary for checking or for coping with more complex tasks.

As discussed in Section 11, some of the preceding instructional guidelines have been implemented by us to teach the concept acceleration, and were found to be quite successful in improving students' ability to interpret this concept effectively. We are presently trying to test experimentally the merits of some of the other guidelines, and plan also to apply these guidelines to the teaching of other scientific concepts.

ACKNOWLEDGMENTS

The work reported in this article was partially supported by grant No. MDR-8550332 from the National Science Foundation. Most of it was carried out while the senior author was at the University of California. Lisa Quinn helped substantially with the analysis of the data and with the computer-implemented instruction. Peter Labudde and David Schuster participated in some of the data analysis and contributed helpful comments. Jill H. Larkin made some very useful suggestions. Finally, Martin Kent and Mark Nakamura did the programming for the computer-implemented instruction.

REFERENCES

Caramazza, A., McCloskey, M., & Green B. (1981). Naive beliefs in "sophisticated" subjects: Misconceptions about trajectories of objects. *Cognition*, **9**, 117-123.

Clement, J. (1982). Students' preconceptions in elementary mechanics. *American Journal of Physics*, **50**, 66-71.

Cohen, R., Eylon, B., & Ganiel, U. (1983). Potential difference and current in simple circuits: A study of students' concepts. *American Journal of Physics*, **51**, 407-412.

diSessa, A. (1988). Knowledge in pieces. In G. Forman & P. B. Pufall (Eds.), *Constructivism in the computer age* (pp. 49-70). Hillsdale, NJ: Lawrence Erlbaum.

Driver, R., Guesne, E., & Tiberghien, A. (Eds.), (1985). *Children's ideas about the physical world*. Great Britain: Open University Press (Taylor & Francis).

Gunstone, R. F. (1987). Student understanding in mechanics: A large population survey. *American Journal of Physics*, **55**, 691-696.

Halloun, I. A., & Hestenes, D. (1985a). The initial knowledge state of college students. *American Journal of Physics,* **53**, 1043-1055.

Halloun, I. A., & Hestenes, D. (1985b). Common sense concepts about motion. *American Journal of Physics*, **53**, 1056-1065.

Helm, H., & Novak, J. D. (Eds.), (1983). *Proceedings of the International Seminar on Misconceptions in Science and Mathematics.* Ithaca, NY: Cornell University.

Labudde, P., Reif, F., & Quinn, L. (1988). Facilitation of scientific concept learning by interpretation procedures and diagnosis. *International Journal of Science Education*, **10**, 81-98.

McCloskey, M., Caramazza, A., & Green, B. (1980). Curvilinear motion in the absence of external forces: Naive beliefs about the motion of objects. *Science*, **210**, 1139-1141.

McDermott, L. C. (1984, July). Research on conceptual understanding in mechanics. *Physics Today*, **37**, 24-32.

Ohanian, H.C. (1985). *Physics.* New York: Norton & Co.

Reif, F. (1987a). Interpretation of scientific or mathematical concepts: Cognitive issues and instructional implications. *Cognitive Science,* **11**, 395-416.

Reif, F. (1987b). Instructional design, cognition, and technology: Applications to the teaching of scientific concepts. *Journal of Research in Science Teaching*, **24**, 309-324.

Resnick, R., & Halliday, D. (1977). *Physics* (3rd ed.). New York: John Wiley & Sons.

Rosch, E. (1978). Principles of categorization. In E. Rosch & B. B. Lloyd (Eds.), *Cognition and categorization* (pp. 27-48). Hillsdale, NJ: Lawrence Erlbaum Associates.

Sears, F. W., Zemansky, M. W., & Young, H. D. (1987). *University physics* (7th ed.). Reading, MA: Addison-Wesley.

Serway, R. A. (1983). *Physics for scientists and engineers/with modern physics*. Philadelphia, PA: Saunders College Publishing.

Trowbridge, D. E., & McDermott, L. C. (1980). Investigation of student understanding of the concept of velocity in one dimension. *American Journal of Physics*, **48**, 1020-1028.

Trowbridge, D. E., & McDermott, L. C. (1981). Investigation of student understanding of the concept of acceleration in one dimension. *American Journal of Physics*, **49**, 242-253.

Viennot, L. (1979). Spontaneous reasoning in elementary mechanics. *European Journal of Science Education*, **1**, 205-221.

APPENDIX 1: ACCELERATION TEST A

The following is a summary of the 13 problems in Test A. Each problem was accompanied by a diagram of the kind indicated in Figure 4. Furthermore, each question asked subjects to indicate the confidence in their answer on a three-point scale where 0 indicated being uncertain and mostly guessing, 1 indicated being halfway certain certain, and 2 indicated being completely certain.

[A1] After being given an initial push, a sled travels with decreasing speed up along a straight hill inclined at some fixed angle with the horizontal.

(a) What is the acceleration of the sled when it passes the point A on its way up along the hill? (Zero? Not zero?)

(b) If the acceleration is not zero, indicate its direction (as precisely as possible) by an arrow drawn from the point A.

The preceding questions a and b can be indicated by the notation (a) (b) Acceleration of sled at A? This abbreviated notation is used in the subsequent problems.

[A2] A pendulum, consisting of a heavy bob attached to a rod, is released from rest at the point A indicated in the diagram. As the bob descends with increasing speed along a circular arc, it passes the point B where the rod is horizontal.

(a) (b) Acceleration of bob at B?

[A3] The horizontal racetrack illustrated in the diagram consists of two straight sections joined to two elliptical sections with centers at the points C and C'. A car travels around this racetrack with constant speed in a counter-clockwise sense.

(a) (b) Acceleration of car at A?

(c) (d) Acceleration of car at B?

[A4] A passenger train and a freight train travel initially with the same constant speed along parallel straight tracks. At a certain instant, they both start slowing down. Moving with uniformly decreasing speed, the passenger train requires 40 seconds before it comes to rest. Moving with uniformly decreasing speed, the freight train requires 30 seconds before it comes to rest.

(a) While the trains are slowing down, how large is the magnitude of the acceleration of the passenger train compared to that of the freight train? (The same? Larger? Smaller?)

[A5] The bob of a pendulum swings back and forth along a circular arc, as indicated in the diagram. Consider the bob at the instant when it is at its highest point A where its speed is zero before it descends again down along the circular arc.

(a) (b) Acceleration of bob at A?

[A6] The horizontal racetrack illustrated in the diagram consists of two straight sections joined to two elliptical sections with centers at the points C and C'. A car travels around this racetrack with decreasing speed in a counter-clockwise sense.

(a) (b) Acceleration of car at A?

(c) (d) Acceleration of car at B?

[A7] The bob of a pendulum, fastened to the ceiling by a string, swings back and forth along a circular arc. The bob moves along the circular arc from left to right, reaching its maximum speed at the point C where the bob is at its lowest point and the string is vertical.

(a) (b) Acceleration of bob at C?

[A8] After being given an initial push, a sled travels with decreasing speed up along a straight hill inclined at some fixed angle with the horizontal. The sled reaches its highest point B, where its speed is zero, and then descends back along the hill with increasing speed.

(a) (b) Acceleration of sled at B?

[A9] A car travels with constant speed along the horizontal spiral-shaped road shown in the diagram, passing first the point A, then the point B, and finally the point C which is the entrance to a parking lot.

(a) (b) Acceleration of car at A?

(c) (d) Acceleration of car at B?

(e) What is the magnitude of the acceleration of the car at the point B compared to that at the point A? (The same? Larger? Smaller?)

[A10] The bob of a pendulum swings back and forth along a circular arc. As the bob descends with increasing speed from left to right along the circular arc, it passes through the point B indicated in the diagram.

(a) (b) Acceleration of bob at B?

[A11] A car travels around a horizontal circular track with constant speed in a clockwise sense, passing the point P at some time t.

(a) What is the acceleration of the car at the point P? (Zero? Not zero?) Consider an alternative situation where the car moves around the track with a constant speed which is twice as large as in the original situation.

(b) At the point P, what is the magnitude of the acceleration of the car in the new situation compared to that in the original situation? (Larger? Smaller? The same?)

(c) The magnitude of the acceleration in the new situation is K times as large as in the original situation. What is the value of K? (1/4? 1/2? 1? 2? 4? None of these?)

(d) If the car's acceleration at the point P is not zero, what is its direction in the new situation compared to that in the original situation? (The same? Different?)

(e) If the direction of the acceleration is not the same, consider the angle between the car's acceleration and velocity at the point P. What is this angle in the new situation compared to that in the original situation? (Larger? Smaller?)

[A12] The bob of a pendulum oscillates back and forth along the circular arc ABCDE shown in the diagram. The pendulum bob then passes any point on the arc when it moves from left to right. On the way back, it then passes the <u>same</u> point again while moving from right to left with a velocity having the same magnitude, but the opposite direction.

(a) What is the direction of the acceleration of the bob at the point D, when the bob moves from left to right, compared to its direction at D when the bob moves from right to left? (The same? Opposite? Neither?)

[A13] A car travels around a horizontal <u>circular</u> track with <u>increasing</u> speed in a clockwise sense, passing the point P at some time t.

(a) What is the acceleration of the car at the point P? (Zero? Not zero?)
Consider an alternative situation where the car moves around the track in such a way that, in the vicinity of the point P, its speed at any instant is twice as large as its speed in the original situation.

(b) (c) (d) (e) <u>Same questions as in Problem A11.</u>

APPENDIX 2: ACCELERATION TEST B

The following is a summary of the 6 problems in Test B. Each question was preceded by the following instruction: "Whenever your answer to a question is ‘yes’, give a specific example illustrating this possibility. Whenever your answer is ‘no’, explain why this is not possible." Furthermore, each question asked subjects to indicate the confidence in their answer on the same kind of three-point scale as that used in Test A.

[B1] A particle moves with constant speed.

(a) Can the acceleration of the particle be zero? (Yes? No?)
(b) Can the acceleration of the particle be non-zero? (Yes? No?)
(c) Can the direction of the particle’s acceleration be along its velocity? (Yes? No?)
(d) Can the direction of the particle’s acceleration be opposite to its velocity? (Yes? No?)
(e) Can the direction of the particle’s acceleration be perpendicular to its velocity? (Yes? No?)
(f) Can the direction of the particle’s acceleration be at an angle neither parallel nor perpendicular to its velocity? (Yes? No?)

[B2] A particle moves along a curved path.

(a) (b) (c) (d) (e) (f) Same questions as in Problem B1.

[B3] (Comparison of two moving particles)

(a) Two particles move at some instant with the same velocity. Can the particles then have different accelerations? (Yes? No?)
(b) Two particles move at some instant with the same acceleration. Can the particles then have different velocities? (Yes? No?)

[B4] A particle moves with increasing speed.

(a) (b) (c) (d) (e) (f) Same questions as in Problem B1.

[B5] A particle moves along a straight path.

(a) (b) (c) (d) (e) (f) Same questions as in Problem B1.

[B6] At some instant, the speed of a particle is zero.

(a) At this instant, can the acceleration of the particle be zero? (Yes? No?)
(b) At this instant, can the acceleration of the particle be non-zero? (Yes? No?)

FIGURE CAPTIONS

Figure 1: Some of the motions used in Test A to assess ability to interpret acceleration.

The numbers between brackets refer to the test problems listed in Appendix 1: (a) Motion along an inclined plane. (b) Motion of a pendulum bob. (c) Motion along a horizontal racetrack. (d) Motion along a horizontal spiral-shaped road. (e) Motion of a horizontal pendulum. (f) Motion along a horizontal circle.

Figure 2: Overall performance of the expert scientists and novice students on the 13 situation-specific problems of Test A and the 6 more general problems of Test B.

Figure 3: Knowledge features facilitating interpretation of a scientific concept.

Figure 4: Kinds of knowledge used by experts and novices answering the acceleration problems of Test A.

Figure 5: (a) Tangential and perpendicular components of the acceleration. (b) Acceleration a obtained by vectorial addition of these components.

Figure 6: Major steps of the procedure implementing the definition of the concept "acceleration."

Figure 7: Relation between the correctness of subjects' concept interpretations and their confidence in these interpretations.

Figure 8: Students' performance before and after computer-implemented instruction on the definitional procedure for acceleration.