SYLLOGISTIC REASONING IN FUZZY LOGIC AND ITS APPLICATION TO REASONING WITH DISPOSITIONS

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ABSTRACT

A fuzzy syllogism in fuzzy logic is defined in this paper to be an inference schema in which the major premise, the minor premise and the conclusion are propositions containing fuzzy quantifiers. A basic fuzzy syllogism in fuzzy logic is the *intersection/product syllogism*

$$\begin{array}{l} Q_1 \; A's \; are \; B's \\ Q_2 \; (A \text{ and } B)'s \; are \; C's \\ \hline (Q_1 \; dprod \; Q_2) \; A's \; are \; (B \text{ and } C)'s \quad , \end{array}$$

in which *A*, *B* and *C* are fuzzy predicates (e.g., *young men*, *blonde women*, etc.); Q_1 and Q_2 are fuzzy quantifiers (e.g., *most, many, almost all*, etc.) which are interpreted as fuzzy numbers; and Q_1 *dprod* Q_2 is the product of Q_1 and Q_2 in fuzzy arithmetic.

We develop several other basic syllogisms which may be employed as rules of combination of evidence in expert systems. Among these is the *consequent conjunction syllogism* which may be expressed as the inference schema

$$\begin{array}{l} Q_1 \; A's \; are \; B's \\ Q_2 \; A's \; are \; C's \\ \hline Q \; A's \; are \; (B \text{ and } C)'s \quad , \end{array}$$

in which Q is a fuzzy number bounded from above by Q_1 *circle* Q_2 and from below by *Oorsign* (Q_1 *dsum* Q_2 *circle* 1), where *dsum* , *circle* and *circle* are the extensions of the arithmetic operators +, − and *andsign* , respectively, to fuzzy operands. Furthermore, we show that syllogistic reasoning in fuzzy logic provides a basis for reasoning with dispositions, that is, with propositions which are preponderantly, but not necessarily always, true.

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1. Introduction

Fuzzy logic may be viewed as a generalization of multivalued logic in that it provides a wider range of tools for dealing with uncertainty and imprecision in knowledge representation, inference and decision analysis. In particular, fuzzy logic allows (a) the use of fuzzy quantifiers exemplified by *most, several, many, few, many more,* etc; (b) the use of fuzzy truth-values exemplified by *quite true, very true, mostly false*, etc; (c) the use of fuzzy probabilities exemplified by *likely, unlikely, not very likely,* etc; (d) the use of fuzzy possibilities exemplified by *quite possible, almost impossible*, etc; and (e) the use of predicate modifiers exemplified by *very, more or less, quite, extremely,* etc.

What matters most about fuzzy logic is its ability to deal with fuzzy quantifiers as fuzzy numbers which may be manipulated through the use of fuzzy arithmetic [32]. This ability depends in an essential way on the existence -- within fuzzy logic -- of the concept of cardinality or, more generally, the concept of measure of a fuzzy set. Thus, if one accepts the classical view of Kolmogoroff that probability theory is a branch of measure theory, then, more generally, the theory of fuzzy probabilities may be subsumed within fuzzy logic. This aspect of fuzzy logic makes it particularly well-suited for the management of uncertainty in expert systems [33]. More specifically, by employing a single framework for the analysis of both probabilistic and possibilistic uncertainties, fuzzy logic provides a systematic basis for inference from premises which are imprecise, incomplete or not totally reliable. In this way, it becomes possible -- as is shown in this paper -- to derive a set of rules for combining evidence through conjunction, disjunction and chaining. In effect, such rules may be viewed as instances of syllogistic reasoning in fuzzy logic; however, unlike the rules employed in most of the existing expert systems, they are not *ad hoc* in nature.

Our concern in this paper is with fuzzy syllogisms of the general form

$$\begin{array}{l} p(Q_1) \\ \underline{q(Q_2)} \\ r(Q) \end{array} \tag{1.1}$$

in which the major premise, $p(Q_1)$, is a fuzzy proposition containing a fuzzy quantifier Q_1; the minor premise, $q(Q_2)$, is a fuzzy proposition containing a fuzzy quantifier Q_2; and the conclusion, $r(Q)$, is a fuzzy proposition containing a fuzzy quantifier Q. For example, the *intersection/product syllogism* [32] may be expressed as

$$\begin{array}{l} Q_1\ A's\ are\ B's \\ \underline{Q_2\ (A \text{ and } B)'s\ are\ C's} \\ Q\ A's\ are\ (B \text{ and } C)'s\ \ , \end{array} \tag{1.2}$$

where A, B and C are labels of fuzzy sets, and the fuzzy quantifier Q is given by the product of the fuzzy quantifiers Q_1 and Q_2, i.e.,

$$Q = Q_1 \; dprod \; Q_2 \quad , \tag{1.3}$$

where *dprod* denotes the product in fuzzy arithmetic [7].[1] It should be noted that (3) may be viewed as an analog of the basic probabilistic identity [15]

$$p(B, C/A) = p(B/A)p(C/A, B) \quad . \tag{1.4}$$

A concrete example of the intersection/product syllogism is the following

$$\begin{array}{l} \textit{most students are young} \\ \underline{\textit{most young students are single}} \\ \textit{most}^2 \textit{ students are young } \text{and } \textit{single} \quad , \end{array} \tag{1.5}$$

where $most^2$ denotes the product of the fuzzy quantifier *most* with itself.

An important application of syllogistic reasoning in fuzzy logic relates to what may be regarded as reasoning with *dispositions*. A disposition, as its name suggests, is a proposition which is preponderantly, but not necessarily always, true. To capture this intuitive meaning of a disposition, we define a disposition as a proposition with implicit extremal fuzzy quantifiers, e.g., *most, almost all, almost always, usually, rarely, few, small fraction,* etc. This definition, should be regarded as a *dispositional definition* in the sense that it may not be true in all cases.

Examples of commonplace statements of fact which may be viewed as dispositions are: *overeating causes obesity, snow is white, glue is sticky, icy roads are slippery*, etc. An example of what appears to be a plausible conclusion drawn from dispositional premises is the following

$$\begin{array}{l} \textit{icy roads are slippery} \\ \underline{\textit{slippery roads are dangerous}} \\ \textit{icy roads are dangerous} \quad . \end{array} \tag{1.6}$$

As will be seen in Section 3, syllogistic reasoning with dispositions provides a basis for a formalization of the type of commonsense reasoning exemplified by (1.6).

The importance of the concept of a disposition stems from the fact that what is commonly regarded as *commonsense knowledge* may be viewed as a collection of dispositions [34]. It is widely recognized that commonsense knowledge plays an essential role in human reasoning and decision-making. Viewed in this perspective, one of the objectives of the present paper is to

1. More generally, a circle around an arithmetic operator represents its extension to fuzzy operands.

suggest that syllogistic reasoning in fuzzy logic may contribute to a better understanding of commonsense reasoning and its role in decision analysis.

2. Fuzzy Quantifiers, Compositionality and Robustness

As was stated in the Introduction, the concept of a fuzzy quantifier is related in an essential way to the concept of cardinality -- or, more generally, the concept of measure -- of fuzzy sets. More specifically, a fuzzy quantifier may be viewed as a fuzzy characterization of the absolute or relative cardinality of a collection of fuzzy sets. In this sense, then, a fuzzy quantifier is a second-order fuzzy predicate.

The cardinality of a fuzzy set may be defined in a variety of ways [31]. For simplicity, we shall employ the *sigma-count* for this purpose, which is defined as follows [6], [30].

Let A be a finite fuzzy subset of the university of discourse, U, with A expressed as

$$A = \mu_1/u_1 + ... + \mu_n/u_n \quad , \tag{2.1}$$

where μ_1/u_1 $, i = 1, \ldots, n$, signifies that μ_i is the grade of membership of u_i in A and + denotes the union. Then, the sigma-count of A is defined as the real number

$$\Sigma Count(A) = \Sigma_i \mu_i \quad , \tag{2.2}$$

with the understanding that the sum may be rounded, if need be, to the nearest integer. Furthermore, one may stipulate that the terms whose grade of membership falls below a specified threshold be excluded from the summation. The purpose of such an exclusion is to avoid a situation in which a large number of terms with low grades of membership become count-equivalent to a small number of terms with high membership.

The *relative sigma-count*, denoted by $\Sigma Count(B/A)$, may be interpreted as the proportion of elements of B in A. More explicitly,

$$\Sigma Count(B/A) = \frac{\Sigma Count(B \cap A)}{\Sigma Count(A)} \quad , \tag{2.3}$$

where $B \cap A$, the intersection of B and A, is defined by

$$\mu_{B \cap A}(u) = \mu_B(u) \text{ } andsign \text{ } \mu_A(u) \, , \quad u \text{ } member \text{ } U \quad . \tag{2.4}$$

Thus, in terms of the membership functions of B and A, the relative sigma-count of B in A is given by

$$\Sigma Count(B/A) = \frac{\Sigma_i \mu_B(u_i) \text{ } andsign \text{ } \mu_A(u_i)}{\Sigma_i \mu_A(u_i)} \quad . \tag{2.5}$$

The concept of a relative sigma-count provides a basis for interpreting the meaning of propositions of the form $p \triangleq Q\ A's\ are\ B's$, e.g., *most young men are healthy*. More specifically, the fuzzy quantifier Q in the proposition $Q\ A's\ are\ B's$ may be regarded as a fuzzy characterization of the relative sigma-count of B in A, which entails that the proposition in question may be translated as

$$Q\ A's\ are\ B's \rightarrow \Sigma Count(B/A)\ is\ Q\ . \tag{2.6}$$

The right-hand member of (2.6) implies that Q, viewed as a fuzzy number, defines the possibility distribution of $\Sigma Count(B/A)$. This may be expressed as the *possibility assignment equation* [30]

$$\Pi_X = Q\ , \tag{2.7}$$

in which the variable X is the sigma-count in question and Π_X is its possibility distribution.

As was stated earlier, a fuzzy quantifier is a second-order fuzzy predicate. The interpretation expressed by (2.6) and (2.7) shows that the evaluation of a fuzzy quantifier may be reduced to that of a first order predicate if Q is interpreted as a fuzzy subset of the real line. Thus, let us consider again the proposition $p \triangleq Q\ A's\ are\ B's$, in which A and B are fuzzy sets in their respective universes of discourse, U and V; and Q, regarded as a second-order fuzzy predicate, is assumed to be characterized by its membership function $\mu_Q(X,Y)$, with X and Y ranging over the fuzzy subsets of U and V. Then, based on (2.6) and (2.7), we can define $\mu_Q(X,Y)$ through the equality

$$\mu_Q(X,Y) = \mu_Q(\Sigma Count(X/Y))\ , \tag{2.8}$$

in the right-hand member of which Q is a unary first-order fuzzy predicate whose denotation if a fuzzy subset of the unit interval. Consequently, in the proposition $Q\ A's\ are\ B's$, Q may be interpreted as (a) a second-order fuzzy predicate defined on $U^* \times V^*$, where U^* and V^* are the fuzzy power sets of U and V; or (b) a first-order fuzzy predicate defined on the unit interval [0,1].

It is useful to classify fuzzy quantifiers into quantifiers of the first kind, second kind, third kind, etc., depending on the arity of the second-order fuzzy predicate which the quantifier represents. Thus, Q is a fuzzy quantifier of the first kind if it provides a fuzzy characterization of the cardinality of a fuzzy set; Q is of the second kind if it provides a fuzzy characterization of the relative cardinality of two fuzzy sets; and Q is of the third kind if it serves the same role in relation to three fuzzy sets. For example, the fuzzy quantifier labeled *several* is of the first kind; *most* is of the second kind; and *many more* in *there are many more A's in B's than A's in C's* is of the third kind. It should be noted that, in terms of this classification, the certainty factors employed in such experts systems as MYCIN [23] and PROSPECTOR [8] are fuzzy quantifiers of the third kind.

The concept of a fuzzy quantifier gives rise to a number of other basic concepts relating to syllogistic reasoning among which are the concepts of *compositionality* and *robustness*.

Specifically, consider a fuzzy syllogism of the general form (1.1), i.e.,

$$\begin{array}{l} p(Q_1) \\ \underline{q(Q_2)} \\ r(Q) \end{array} \tag{2.9}$$

We shall say that the syllogism is *strongly compositional* if (a) Q may be expressed as a function of Q_1 and Q_1 independent of the denotations of the predicates which enter into p and q, excluding the trivial case where Q is the unit interval; and (b) if Q_1 and Q_2 are numerical quantifiers, so is Q. Furthermore, we shall say that the syllogism is *weakly compositional* if only (a) is satisfied, in which case if Q_1 and Q_2 are numerical quantifiers, Q may be interval-valued. As will be seen in the sequel, in order to achieve strict compositionality, it is necessary, in general, to make some restrictive assumptions concerning the predicates in p and q. For example, the syllogism

$$\begin{array}{l} Q_1\ A's\ are\ B's \\ \underline{Q_2\ B's\ are\ C's\qquad\qquad} \\ (Q_1\ dprod\ Q_2)\ A's\ are\ (B \text{ and } C)'s \end{array} \tag{2.10}$$

is strictly compositional if *B subset A*.

Turning to the concept of robustness, supppose that we start with a nonfuzzy syllogism of the form

$$\begin{array}{l} p(all) \\ \underline{q(all)} \\ r(all) \end{array} \tag{2.11}$$

an example of which is

$$\begin{array}{l} all\ A's\ are\ B's \\ \underline{all\ B's\ are\ C's} \\ all\ A's\ are\ C's\ . \end{array} \tag{2.12}$$

The original syllogism is *robust* if small perturbations in the quantifiers in p and q result in a small perturbation in the quantifier in r. For example, the syllogism represented by (2.12) is robust if its validity is preserved when (a) the quantifier *all* in p and q is replaced by *almost all*; and (b) the quantifier *all* in r is replaced by *almost almost all*. (In more concrete terms, this is equivalent to replacing *all* in p and q by the fuzzy number 1 *circle* ε, where ε is a small fuzzy number; and (b) replacing *all* in *r* by the fuzzy number 1 *circle* 2ε.) More generally, a syllogism is *selectively robust* if the above holds for perturbations in either the major or the minor premise, but not necessarily in both. For example, it may be shown that the syllogism expressed by (2.12) is selectively robust with respect to perturbations in the major premise but not in the minor premise. In fact, the syllogism in question is *brittle* with respect to perturbations in the minor premise in the sense that the slightest perturbation in the quantifier *all* in *q* requires the replacement of the quantifier *all* in *r* by the vacuous quantifier *none to all*.

3. Fuzzy Syllogisms and Reasoning with Dispositions

As was stated earlier, one of the basic syllogisms in fuzzy logic is the intersection/product syllogism expressed by (1.2).

In what follows, we shall employ this syllogism as a starting point for the derivation of other syllogisms which are of relevance to the important problem of combination of evidence in expert systems.

A derivative syllogism of this type is the *multiplicative chaining syllogism*

$$\begin{array}{l} Q_1\ A's\ are\ B's \\ \underline{Q_2\ B's\ are\ C's} \\ \geq(Q_1\ dprod\ Q_2)A's\ are\ C's\ \ , \end{array} \tag{3.1}$$

in which $\geq(Q_1\ dprod\ Q_2)$ should be read as *at least* $Q_1\ dprod\ Q_2$. This syllogism is a special case of the intersection/product syllogism which results when *B subset A*, i.e.,

$$\mu_B(u_i) \leq \mu_A(u_i)\ \ ,\quad u_i\ member\ U\ ,\ \ i = 1,...\ \ . \tag{3.2}$$

For, in this case $A \cap B\ equiv\ B$, and since $B \cap C$ is contained in *C*, it follows that

$$(Q_1\ dprod\ Q_2)A's\ are\ (B \text{ and } C)'s => \geq(Q_1\ dprod\ Q_2)A's\ are\ C's\ \ . \tag{3.3}$$

(It is of interest to note that if *Q* in the proposition *Q A′s are B′s* is interpreted as the degree to which *A* is contained in *B*, then the multiplicative chaining syllogism shows that, under the assumption *B subset A*, the fuzzy relation of fuzzy-set-containment is *product transitive [28], [32].)*

If, in addition to assuming that *B subset A*, we assume that Q_1 and Q_2 are *monotone increasing* [3], i.e.,

$$\begin{array}{c} \geq Q_1 = Q_1 \\ \geq Q_2 = Q_2 \end{array} \tag{3.4}$$

which is true of the fuzzy quantifier *most*, then

$$\geq (Q_1 \, dprod \, Q_2) = Q_1 \, dprod \, Q_2 \tag{3.5}$$

and the multiplicative chaining syllogism becomes

$$\begin{array}{l} Q_1 \; A's \; are \; B's \\ \underline{Q_2 \; B's \; are \; C's \qquad\qquad} \\ (Q_1 \, dprod \, Q_2) A's \; are \; C's \;\; . \end{array} \tag{3.6}$$

As an illustration, we shall consider an example in which the containment relation *B subset A* holds approximately, as in the proposition

$$p \triangleq most \; American \; cars \; are \; big \;\; . \tag{3.7}$$

Then, if

$$q \triangleq most \; big \; cars \; are \; expensive \;\; . \tag{3.8}$$

we may conclude, by employing (3.6), that

$$r \triangleq most^2 \; American \; cars \; are \; expensive \;\; ,$$

with the understanding that $most^2$ is the product of the fuzzy number *most* with itself [32].

It can readily be shown by examples that if no assumptions are made regarding *A*, *B* and *C*, then the chaining inference schema

$$\begin{array}{l} Q_1 \; A's \; are \; B's \\ \underline{Q_2 \; B's \; are \; C's} \\ Q \; A's \; are \; C's \;\; . \end{array} \tag{3.9}$$

is not weakly compositional, which is equivalent to saying that, in general, *Q* is the vacuous quantifier *none to all*. However, if we assume, as done above, that *B subset A*, then it follows from the intersection/product syllogism that (3.6) becomes weakly compositional, with

$$Q = \geq (Q_1 \, dprod \, Q_2) \;\; , \tag{3.10}$$

and, furthermore, that (3.6) becomes strongly compositional if Q_1 and Q_2 are monotone increasing.

Another important observation relates to the robustness of the multiplicative chaining syllogism. Specifically, if we assume that

$$Q_1 = 1 \; circle \; \varepsilon_1$$

$$Q_2 = 1 \; circle \; \varepsilon_2$$

where ε_1 and ε_2 are small fuzzy numbers, then it can readily be verified that, approximately,

$$Q_1 \; dprod \; Q_2 \; iso \; 1 \; circle \; \varepsilon_1 \; circle \; \varepsilon_2 \;\; , \tag{3.11}$$

which establishes that the multiplicative chaining syllogism is robust. However, in the absence of the assumption *B subset A*, the inference schema (3.9) is robust only with respect to perturbations in Q_1. To demonstrate this, assume that Q_1 = *almost all* and Q_2 = *all*. Then, from the intersection/product syllogism it follows that $Q = \geq (almost\ all)$. On the other hand, if we assume that Q_1 = *all* and Q_2 = *almost all*, then Q = *none to all*. Thus, as was stated earlier, the inference schema (3.9) is brittle with respect to perturbations in the minor premise.

The MPR chaining syllogism

In the preceding discussion, we have shown that the assumption *B subset A* leads to a weakly compositional multiplicative chaining syllogism. Another type of assumption which also leads to a weakly compositional chaining syllogism is that of *major premise reversibility* or MPR, for short. This assumption may be expressed as the semantic equivalence

$$Q_1 A's \; are \; B's \; <\!\!\longrightarrow \; Q_1 B's \; are \; A's \;\; , \tag{3.12}$$

which, in most cases, will hold approximately rather than exactly. For example,

$$most \; American \; cars \; are \; big \; <\!\!\longrightarrow \; most \; big \; cars \; are \; American \;\; .$$

It can be shown [34] that under the assumption of reversibility the following chaining syllogism holds in an approximate sense

$$\begin{array}{l} Q_1 \; A's \; are \; B's \\ \underline{Q_2 \; B's \; are \; C's \qquad\qquad} \\ \geq(0 \; circle \; ((Q_1 \; dsum \; Q_2 \; circle \; 1)) A's \; are \; C's \;\; . \end{array} \tag{3.13}$$

We shall refer to this syllogism as the *MPR chaining syllogism*. It follows at once from (3.13) that the MPR chaining syllogism is weakly compositional and robust. A concrete instance of this syllogism is provided by the following example

$$\begin{array}{l} most \; American \; cars \; are \; big \\ \underline{most \; big \; cars \; are \; heavy \qquad\qquad} \\ 0 \; circle \; (2 \; most \; circle \; 1) \; American \; cars \; are \; heavy \;\; . \end{array} \tag{3.14}$$

The consequent conjunction syllogism

The consequent conjunction syllogism is an example of a basic syllogism which is not a derivative of the intersection/product syllogism. Its statement may be expressed as follows:

$$Q_1\ A's\ are\ B's \qquad (3.15)$$

$$\underline{Q_2\ A's\ are\ C's}$$

$$Q\ A's\ are\ (B \text{ and } C)'s\ ,$$

where

$$0\ \mathit{circle}\ (Q_1\ \mathit{dsum}\ Q_2\ \mathit{circle}\ 1) \le Q \le Q_1\ \mathit{circle}\ Q_2 \qquad (3.16)$$

>From (3.16), it follows at once that the syllogism is weakly compositional and robust.

An illustration of (3.15) is provided by the example

most students are young

most students are single
___

Q students are single and *young*

where

$$2\mathit{most}\ \mathit{circle}\ 1 \le Q \le \mathit{most} \qquad (3.17)$$

This expression for Q follows from (3.16) by noting that

$$\mathit{most}\ \mathit{circle}\ \mathit{most} = \mathit{most}$$

and

$$0\ \mathit{circle}\ (2\mathit{most}\ \mathit{circle}\ 1) = 2\mathit{most}\ \mathit{circle}\ 1\ .$$

The importance of the consequent conjunction syllogism stems from the fact that it provides a formal basis for combining rules in an expert system through a conjunctive combination of hypotheses [33]. However, unlike such rules in MYCIN [23] and PROSPECTOR [8], the consequent conjunction syllogism is weakly rather than strongly compositional. Since the combining rules in MYCIN and PROSPECTOR are *ad hoc* in nature whereas the consequent conjunction syllogism is not, the validity of strong compositionality in MYCIN and PROSPECTOR is in need of justification.

The antecedent conjunction syllogism

An issue which plays an important role in the management of uncertainty in expert systems relates to the question of how to combine rules which have the same consequent but different antecedents.

Expressed as an inference schema in fuzzy logic, the question may be stated as

$$Q_1\ A's\ are\ C's \qquad (3.18)$$

$$\frac{Q_2 \; B's \; are \; C's}{Q \; (A \text{ and } B)'s \; are \; C's} \; ,$$

in which Q is the quantifier to be determined as a function of Q_1 and Q_2.

It can readily be shown by examples that, in the absence of any assumptions about A, B, C, Q_1 and Q_2, what can be said about Q is that it is the vacuous quantifier *none to all*. Thus, to be able to say more, it is necessary to make some restrictive assumptions which are satisfied, at least approximately, in typical situations.

The commonly made assumption in the case of expert systems [8], [23] is that the items of evidence are conditionally independent given the hypothesis. Expressed in terms of the relative sigma-counts of A, B and C, this assumption may be written as

$$\Sigma Count(A \cap B/C) = \Sigma Count(A/C)\Sigma Count(B/C) \; . \tag{3.19}$$

Using this equality, it is easy to show that

$$\Sigma Count(C/A \cap B) = K\Sigma Count(C/A)\Sigma Count(C/B) \; , \tag{3.20}$$

where the factor K is given by

$$K = \frac{\Sigma Count(A)\Sigma Count(B)}{\Sigma Count(A \cap B)\Sigma Count(C)} \; . \tag{3.21}$$

The presence of this factor has the effect of making the inference schema (3.18) non-compositional. One way of getting around the problem is to employ -- instead of the sigma-count -- a count defined by

$$\rho\Sigma Count(B) = \frac{\Sigma Count(B)}{\Sigma Count(\neg B)} \tag{3.22}$$

$$\rho\Sigma Count(B/A) = \frac{\Sigma Count(B/A)}{\Sigma Count(\neg B/A)} \; , \tag{3.22}$$

in which $\neg B$ denotes the negation of B (or, equivalently, the complement of B, if B is interpreted as a fuzzy set which represents the denotation of the predicate B). These counts will be referred to as $\rho sigma-counts$ (with ρ standing for *ratio*) and correspond to the odds which are employed in PROSPECTOR [8]. Thus, expressed in words, we have

$$\rho\Sigma Count(B) \triangleq Ratio \; of \; B's \; to \; non\text{-}B's \tag{3.24}$$

$$\rho\Sigma Count(B/A) \triangleq Ratio \; of \; B's \; to \; non\text{-}B's \; among \; A's \; . \tag{3.25}$$

In terms of $\rho sigma-counts$, it can readily be shown that the assumption expressed by (3.19) entails the equality

$$\rho\Sigma Count(C/A \cap B) = \rho\Sigma Count(C/A)\rho\Sigma Count(C/B)\rho\Sigma Count(\neg C) \tag{3.26}$$

This equality, then, leads to what will be referred to as the *antecedent conjunction syllogism*

$$\begin{array}{l} \textit{ratio of } C\textit{'s to non-}C\textit{'s among } A\textit{'s is } R_1 \\ \underline{\textit{ratio of } C\textit{'s to non-}C\textit{'s among } B\textit{'s is } R_2} \\ \textit{ratio of } C\textit{'s to non-}C\textit{'s among } (A \text{ and } B)\textit{'s is } R_1 \textit{ dprod } R_2 \textit{ dprod } R_3 \end{array} \tag{3.27}$$

where

$$R_3 \triangleq \textit{ratio of } C\textit{'s to non-}C\textit{'s} \ .$$

It should be noted that this syllogism may be viewed as the fuzzy logic analog of the likelihood ratio combining rule in PROSPECTOR [8].

In the foregoing discussion, we have focused our attention on some of the basic syllogisms in fuzzy logic which may be employed as rules of combination of evidence in expert systems. Another important function which these and related syllogisms may serve is that of providing a basis for reasoning with dispositions, that is, with propositions in which there are implicit fuzzy quantifiers.

The basic idea underlying this application of fuzzy syllogisms is the following. Suppose that we are given two dispositions

icy roads are slippery

slippery roads are dangerous .

Can we infer from these dispositions what appears to be a plausible conclusion, namely:

icy roads are dangerous ? (3.28)

As a first step, we have to restore the suppressed fuzzy quantifiers in the premises. For simplicity, assume that the desired restoration may be accomplished by prefixing the dispositions in question with the fuzzy quantifier *most*, i.e.,

icy roads are slippery $\rightarrow$ *most icy roads are slippery*

slippery roads are dangerous $\rightarrow$ *most slippery roads are dangerous* .

Next, if we assume that the proposition *most slippery roads are dangerous* satisfies the major premise reversibility condition, i.e.,

most icy roads are slippery $<\!\!\rightarrow$ *most slippery roads are icy* ,

then by applying the MPR chaining syllogism (3.13), we have

most icy roads are slippery (3.29)

<u>*most slippery roads are dangerous*</u>

(2*most circle* 1) *icy roads are dangerous* .

Finally, on suppressing the fuzzy quantifiers in (3.29), we are led to the chain of dispositions

$$\begin{array}{l} \textit{icy roads are slippery} \\ \underline{\textit{slippery roads are dangerous}} \\ \textit{icy roads are dangerous} \ , \end{array} \tag{3.30}$$

which answers in the affirmative the question posed in (3.29), with the understanding that the implicit fuzzy quantifier in the conclusion of (3.29) is 2*most circle* 1 rather than *most*.

Concluding remark

This paper may be viewed as an initiation of a study of syllogistic reasoning in the context of fuzzy logic. Such reasoning has a direct bearing on the rules of combination of evidence in expert systems and, in addition, provides a basis for inference from commonsense knowledge by viewing such knowledge as a collection of dispositions.

The results presented in this paper are preliminary in nature. The issue of syllogistic reasoning in fuzzy logic has many ramifications which remain to be explored.

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