Essays in Finance, Sovereign Debt Maturity, and Debt Ownership Structure

by

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Structure

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Abstract

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This dissertation explores the relationship between sovereign debt ownership, default probabilities, and debt returns, focusing on the increasing domestic debt ownership in developed countries since the global financial crisis in 2007. It also explains, both theoretically and empirically, how changes in sovereign debt maturity structure would affect the real economy. This dissertation helps advance the study of the linkages between sovereign debt composition, asset prices and the real economy.

In the first chapter, I study the relationship between sovereign debt default and debt ownership structure. Major developed countries have experienced a significant run-up in public debt after the onset of the global financial crisis in 2008. However, the impact on sovereign debt ownership varies across countries. Specifically, the share of debt held by domestic banks has increased in GIIPS countries but declined in non-GIIPS countries. I explain the cross-country differences in debt ownership structure using a dynamic equilibrium model with strategic and non-discriminatory defaults, in which sovereign debt can serve as collateral for expanding private investments. The key insight is that the share of debt held domestically is positively correlated with the government’s incentive to default. Consequently, the model predicts that the share of domestically-held debt is strictly increasing in total debt only in highly-indebted countries whose debt has low collateral value. My result is consistent with the notion that domestic debt is a commitment device for debt repayment. The key policy implication is that changes in debt ownership are important indicators for the optimality of public debt level. Using data from
a panel of 11 countries between 2007 and 2014, I find evidence consistent with these predictions.

In the second chapter, I study the interaction between monetary and fiscal policies, and how changes in fiscal policies, such as the level of debt and debt maturity composition, would affect inflation, the real economy and asset prices. I developed a three-period equilibrium model, in which monetary policies are modelled as open market operations. In my model, inflation and the term structure of interest rates are jointly determined by monetary and fiscal policies, and therefore Sargent and Wallace (1981)’s “game of chicken” problem is avoided. I show from the model that fiscal instruments, such as the primary surplus, and the level and maturity structure of government debt, have important implications on inflations and the term structure of interest rates. I then provide robust empirical evidence on how changes in debt-maturity structure are associated with changes in future inflation using U.S. data. One percent increase in the fraction of short-term debt issued is associated with more than 0.2 percent increase in future inflation of different horizons. Empirical evidence also shows that changes in the short-end of the maturity structure has the most explanatory power over short- and medium- horizons, whereas changes in the long-end of the maturity structure has the most explanatory power over long- horizons.
# Contents

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sovereign Debt Default and Debt Ownership: Domestic Debt as a Commitment Device for Debt Repayment</td>
<td>1</td>
</tr>
<tr>
<td>1.1</td>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>Related Literature</td>
<td>5</td>
</tr>
<tr>
<td>1.3</td>
<td>Model</td>
<td>7</td>
</tr>
<tr>
<td>1.4</td>
<td>Equilibrium</td>
<td>10</td>
</tr>
<tr>
<td>1.5</td>
<td>Empirical Evidence</td>
<td>19</td>
</tr>
<tr>
<td>1.6</td>
<td>Conclusion</td>
<td>27</td>
</tr>
<tr>
<td>1.7</td>
<td>Figures and Tables</td>
<td>29</td>
</tr>
<tr>
<td>2</td>
<td>Maturity Structure of Government Debt, Asset Markets, and the Real Economy</td>
<td>41</td>
</tr>
<tr>
<td>2.1</td>
<td>Introduction</td>
<td>41</td>
</tr>
<tr>
<td>2.2</td>
<td>Literature Review</td>
<td>45</td>
</tr>
<tr>
<td>2.3</td>
<td>Model</td>
<td>48</td>
</tr>
<tr>
<td>2.4</td>
<td>Empirical evidence</td>
<td>57</td>
</tr>
<tr>
<td>2.5</td>
<td>Conclusion</td>
<td>59</td>
</tr>
<tr>
<td>2.6</td>
<td>Figures and Tables</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Bibliography</td>
<td>72</td>
</tr>
<tr>
<td>A</td>
<td>Proofs for Chapter 1</td>
<td>78</td>
</tr>
<tr>
<td>A.1</td>
<td>Proof of proposition 1</td>
<td>78</td>
</tr>
<tr>
<td>A.2</td>
<td>Proof of Proposition 2</td>
<td>78</td>
</tr>
</tbody>
</table>
List of Figures

1.1 Shares of sovereign debt held by foreign investors and domestic banks (left axis) and Debt-to-GDP ratio (right axis) 29
1.2 Government Default Policy, $a^*$ as a function of domestically-held debts, $b_D$, for a given debt level $T$. ........................ 30
1.3 Share of Debt held by Domestic Banks $b_D/T$ as a function of $T$ for a country with low collateral value of sovereign debts, $\lambda = 0.01$. ............................... 31
1.4 Share of Debt held by Domestic Banks $b_D/T$ as a function of $T$ for a country with low collateral value of sovereign debts, $\lambda = 0.2$. ............................... 32
1.5 Share of Debt held by Domestic Banks $b_D/T$ as a function of $T$ for a country with low collateral value of sovereign debts, $\lambda = 0.5$. ............................... 33
1.6 Share of Debt held by Domestic Banks $b_D/T$ for countries with high collateral values and different domestic cost of default $c_D$. ............................... 34

2.1 Average Debt Maturity of Total Federal Debt in Years. 60
2.2 Impact of Monetary Policy ($B$) on Real Short-Term Interest Rates and Inflation .......................... 61
2.3 Impact of Debt Maturity Structure ($\alpha$) on Real Short Rate and Inflation .......................... 62
2.4 Impact Level of Debt ($L$) on Real Short Rate and Inflation 63
List of Tables

1.1 Summary Statistics for, $\lambda_R$, the Collateral Value of Sovereign Debts (2007-2014) ........................................ 35
1.2 Regressions of collateral value of sovereign debt on creditor rights .......................................................... 36
1.3 Regressions debt ownership on collateral value of sovereign Debt and total debt-to-GDP ratio ....................... 37
1.4 The Effect of Changes in Debt-to-GDP Ratio and the Collateral Value of Sovereign Debt on the Share of Domestically-held Debts. ............................................ 38
1.5 Default Risk and Changes in Externally- and Domestically-held Debt-to-GDP Ratio ......................................... 39
1.6 Default risk and changes in total debt-to-GDP Ratio ......................................................................................... 40
2.1 One-Year Ahead Forecast of Inflation using Short-term Debt ........................................................................... 65
2.2 One-Year Ahead Forecast of Inflation using Long-term Debt ........................................................................ 66
2.3 Inflation Forecast at Different Horizons using Short-term Debt ....................................................................... 67
2.4 Percentage Increase in $R^2$ when Debt-Maturity Structure is included in Inflation Forecast over Different Horizons. 68
2.5 Inflation Forecast at Different Horizons using Long-term Debt ........................................................................ 69
2.6 Inflation Forecast at Different Horizons using Long-term Debt, using One-Year ahead Expected Inflation from the Surveys of Consumers ............................................. 70
| 2.7 One-Year Ahead Forecast of Inflation using Short-term Debt, using One-Year ahead Expected Inflation from the Surveys of Consumers | 71 |
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Chapter 1

Sovereign Debt Default and Debt Ownership: Domestic Debt as a Commitment Device for Debt Repayment

1.1 Introduction

It has long been recognized that changes in sovereign debt ownership have important implications for the real economy. For example, Diamond (1965) argues that, when domestically-held debts and capital are perfect substitutes, domestic debt issuance reduces investment through crowding out capital accumulation; while externally-held debts affect the real interest rate via its effects on the share of income transferred abroad. Less clear is how sovereign debt ownership is determined, and how it should change in response to changes in government borrowing. In the wake of the recent global financial crisis, during which stabilization policies adopted by the governments in major developed countries have led to a rapid and significant increase in public debts, these questions are particularly relevant.

Specifically, shares of sovereign debt held by domestic banks steadily declined over the past decade in major developed countries. This trend changed after the onset of the global financial crisis in 2008. As illustrated by figure 1.1b, sovereign debts issued by peripheral countries in the Euro Area, such as Greece, Italy, Ireland, Portugal, and Spain (the “GIIPS”), were increas-

\footnotesize{1See, for example, Abbas et al. (2014) and Reinhart and Rogoff (2011).}
ingly held by their domestic banking sectors, but the opposite was observed for non-GIIPS countries. Recent literature has focused on the former. For instance, Becker and Ivashina (2014) provide empirical evidence that GIIPS governments pressured domestic banks to purchase their debts through financial repression. Farhi and Tirole (2014) argue that domestic banks in the GIIPS countries optimally diversify as little as possible when they can count on government bailouts. However, few have paid attention, at the same time, to the continually declining shares of domestic debt in the non-GIIPS countries. Given that both GIIPS and non-GIIPS countries have experienced a significant run-up in public debt issuance during the same period, how could one explain the cross-country differences in debt ownership in a unified framework?

The paper answers this question by using a modified dynamic equilibrium model of Bolton and Jeanne (2011) to allow for strategic and non-discriminatory defaults and endogenous debt ownership. In the model, both domestic banks and foreign investors hold sovereign debts because these debts can serve as collateral for private debts, which can then be used to expand their private investments. Returns on private investments are subject to a productivity shock. Debt ownership affects the government’s incentive to default because debt repayment to foreign investors is an \textit{ex-post} loss of resources from the perspective of the domestic country, but, at the same time, default would destroy productive collateral for domestic banks and incur a direct output loss. Therefore, the welfare maximizing debt ownership depends on the trade-off between these two opposing incentives. The paper first analyzes, for an exogenous amount of public debt, the equilibrium default policy, and debt ownership. It also derives the conditions under which the equilibrium share of public debt held by domestic banks is increasing in total debts. Then, it characterizes the optimal level of public debt and the corresponding equilibrium debt ownership. Finally, using data from a panel of 11 countries between 2007 and 2013, the paper presents empirical evidence consistent with these theoretical results.

The contributions of this paper are both theoretical and empirical. Theoretically, I develop a new argument for “financial repression”, i.e. formal or informal pressure from the government placed on the domestic banking sector.
to absorb sovereign debts, as suggested empirically by Becker and Ivashina (2014). In particular, I argue that domestic debt is a commitment device for debt repayment, so the share of debt held by domestic banks is positively correlated with the government’s incentive to default. The key variables that affect this incentive are: 1) the collateral value of government debt and 2) the level of government debt. Because collateral values vary across countries, they could, therefore, explain the cross-country differences in debt ownership, despite the fact that the governments in GIIPS and non-GIIPS countries have both significantly expanded their borrowing after the global financial crisis. This paper also has an important policy implication. Specifically, I argue that the government should pay close attention to changes in debt ownership as it conducts fiscal policy, because my welfare analysis of the model shows that changes in debt ownership are key indicators for the optimality of public debt level.

There are three main theoretical results. First, the share of domestic debt in equilibrium is strictly increasing in total debt only in highly-indebted countries whose debts have low collateral value. Because sovereign debts in these countries are less productive, defaults become more desirable as their levels of total debt increase. Therefore, if these countries want to raise their debt capacities to finance increasing deficits, a larger fraction of newly issued debts must be held by the domestic banking sector, or otherwise their incentives to default are greater than those expected by the foreign investors, who would then refuse to lend. In contrast, sovereign debts are more productive in moderately-indebted countries whose debts have high collateral value. When these countries borrow more, their incentives to default, in fact, decrease, because doing so would destroy valuable collateral for private investments. For these countries, the equilibrium share of domestic debt is strictly decreasing in total debt.

Second, the equilibrium probability of default is more sensitive to an increase in total debt if the collateral value of sovereign debt is lower. In equi-

\footnote{While I assume the collateral values of sovereign debt are exogenous, one can think of countries with similar debt levels as having different expected deficits and thus different default probabilities. A version of the model in which the collateral value is endogenized as a decreasing function of debt (with different sensitivities) yields similar qualitative results.}

\footnote{It is not possible, in equilibrium, to force the domestic banks to absorb all sovereign debts in arbitrary amount. Assume a fixed level of deficit. If all debts are held by domestic banks, they would expect the government to repay with probability one because repayment of debt expands domestic investment and involves no transfer of resources to foreign investors. This effectively pins down the price of debt and, therefore, a unique debt level. In fact, it is the lower bound of the government’s debt capacity.}
librium, the default probability is the ratio of debt-repayments to foreign investors to net-loss of investment output when there is default. For countries whose debts have lower collateral value, their loss of investment output is smaller when they default on their debts. I show that, not only their incentives to default are higher, they are also more sensitive to changes in fiscal conditions. Therefore, in response to the same increase in total debt, even though only a smaller fraction of newly issued debts are held by foreign investors, their probabilities of default still increase by relatively more.

Third, if the share of domestic debt is increasing in total debt, the level of total debt is suboptimally excessive. As government borrowing increases, newly issued public debts are held, in equilibrium, by both domestic banks and foreign investors. The optimal level of government borrowing depends on the trade-offs, in states where there is no default, between a declining expected marginal contribution of sovereign debt to private investment for domestic banks and increasing debt-repayments to foreign investors. I show that when debts are “moving back” home, as observed in the GIIPS countries, this is an indicator that the later effect dominates, and the governments should reduce borrowing.

The existing empirical literature that studies the relationship between debt supply and sovereign default risk focuses on \textit{total} debt supply.\footnote{See, for example, Aizenman et al. (2013).} My theoretical results suggest that total debt supply should be used in conjunction with data on debt ownership to assess sovereign default risk. For each country in my sample between 2007 and 2014, I construct a proxy for the collateral value of sovereign debt, which, according to the model, should be observationally equivalent to the ratio of private debt issued by domestic banks to their holdings of domestic sovereign debt. This proxy is strongly and significantly correlated with the strength of legal rights index from the World Bank’s World Development Indicators, suggesting that it could capture some important aspects regarding the quality of institutions.\footnote{The strength of legal rights index measures the degree to which collateral and bankruptcy laws protect the rights of borrowers and lenders and thus facilitate lending.} Using this proxy, I then provide the first evidence that the collateral value of sovereign debt is a key determinant of debt ownership.

There are three main empirical results. First, consistent with the model, I show that sovereign debt issued by GIIPS countries, on average, does have significantly lower collateral value than that issued by non-GIIPS countries. Second, results from instrumental variable (IV) regressions of the share of

\textit{total debt supply}.
domestically-held debt and default risk on total debt and collateral value suggest that debt ownership and default risk are, indeed, more sensitive to changes in total debt if the collateral value of debt is lower. The 5-Year CDS spread is used as a proxy for default risk. The instrumental variable approach is used to mitigate the empirical problem that the proxy for collateral value is endogenous. The strength of legal rights index is used as an instrument for my proxy for collateral value.

Third, using 5-Year CDS spreads as a proxy, I show with strong statistical significance that a country's default risk increases with externally-held debts but decreases with domestically-held debts. The externally-held component of total debt-to-GDP ratio, in fact, have more predictive power on 5-Year CDS spreads than total debt-to-GDP ratio. Again, the instrumental variable approach is used because debt ownership is endogenous in my model. The result is consistent with my theory, which argues that changes in externally-held and domestically-held debt have opposite impacts on the government’s incentive to default.

I argue that the empirical results distinguishes this paper from the “risk-shifting” explanation to changes in debt ownership in the GIIPS countries. For example, Farhi and Tirole (2014) argues that due to the convex payoff structure of equity, increasing holding of risky sovereign debts could be particularly attractive to domestic banks during a sovereign debt crisis. This explanation suggests that domestically-held debt should be positively correlated with default risk. Indeed, the regression coefficient is positive when default risk is directly regressed on domestically-held debt. According to my theory, however, this is not the correct approach because of omitted-variable bias. Both default risk and domestically-held debt are negatively correlated with the collateral value of sovereign debt. This implies that, without using instrumental variables, the regression coefficient on domestically-held debts suffers an upward bias. My result shows that the regression coefficient is negative after this omitted-variable bias is corrected.

1.2 Related Literature

This paper’s main concern is the increasing share of public debt held by domestic banks during turbulent times as, for example, Broner et al. (2014). They emphasize the role of creditor discrimination, whereas this paper focuses on the non-discriminatory default. Broner et al. (2010) also assume non-discriminatory default. Their model, however, do not allow for default in
equilibrium, whereas the default probability is endogenously dependent on realized shocks in this paper. Farhi and Tirole (2014), Uhlig (2013), Crosignani (2014), and Drechsler et al. (2013) focus on risk-shifting and moral hazard, while this paper focuses on role of domestic debt as a commitment device for debt sustainability, an argument also used by the contemporaneous work of Chari et al. (2015). Chari et al. (2015) focuses on the optimality of financial repression. They show that the government finds it optimal to force domestic bankers to hold government debt when its expenditure is unusually high. In contrast, this paper focuses on explaining the cross-country differences in debt ownership in response to exogenous changes in the level of total debt.

This paper also relates to debt sustainability and foreign borrowing. Meningus (2014) argues that, if domestically-held sovereign debts are imperfectly observed, government bailout of domestic banks does not affect its ability to borrow. In his model, default is either certain, not allowed, or sunspot. This paper, on the other hand, focuses on debt ownership when the collateral value and the total level of sovereign debt differs, and the default probability is always dependent on fundamentals. Guembel and Sussman (2009) developed a political theory that suggests repayment of externally-held debts is optimal when the interest of the median voter are aligned more with the foreign creditors than with the poor domestic tax payers. This paper, instead, emphasizes the role of sovereign debts as productive collateral as a deterrent for defaults, with positive default probability in equilibrium. Bolton and Jeanne (2011) study the relative total supplies of government debt between risky and safe countries when their economies are financially integrated. They assume an exogenous probability of default and a symmetric equilibrium where domestically- and externally-held sovereign debts are always equal. This paper employs a modified version of their model to study the relationship between sovereign default and debt ownership. Therefore, different from Bolton and Jeanne (2011), both probability of default and debt ownership are endogenous in this paper.

Additionally, this paper contributes to the growing literature about the relationship between sovereign debt supply and interest-rate spreads, for example, Krishnamurthy and Vissing-Jorgensen (2012), Fontana and Scheicher (2010), and Cecchetti et al. (2010). These literatures focus on total debt supply as an explanatory variable for interest-rate spreads that measures default risk. This paper argues and demonstrates from an empirical standpoint that it is important to distinguish domestically- and externally-held debts, because their effects on government’s incentive to default are opposite.
1.3 Model

Consider a three-period model of a small open economy. This economy is populated by a benevolent government and domestic bankers. The rest of the world consists of foreign investors. All variables without subscripts belong to domestic bankers throughout the paper. Whenever needed, the subscripts “D” (“F”) represent domestic bankers (foreign investors), which shall be described below. There is a private, homogenous, and non-storable consumption good. Time is divided into three periods, \( t = 0, 1, 2 \).

**Domestic Bankers**

There is a continuum measure one of domestic bankers, of which a fraction \( w \in [0, 1] \) are productive. Productive bankers have access to an investment technology \( f(\cdot) \) at \( t = 1 \), where \( f(\cdot) \) is strictly increasing and concave with \( f(0) = 0 \). Bankers are *ex-ante* identical at \( t = 0 \), and their types will only be revealed at \( t = 1 \).

Returns on investment are subject to a non-negative productivity shock, \( \tilde{A} \geq 0 \), which has a differentiable cumulative distribution function (CDF), \( \Pi : [0, \infty) \to [0, 1] \), and probability density function, \( \pi : [0, \infty) \to \mathbb{R} \). The productivity shock, \( \tilde{A} \), is realized at \( t = 1 \) before any investment decision is made. An investment of \( I \) at \( t = 1 \) yields a (net) return of \( \tilde{A}f(I) \geq 0 \) at \( t = 2 \), but \( I \) is constrained by the amount of private deposits \( d \) that productive bankers can raise from non-productive bankers.

Specifically, let \( b_D \) be the amount of domestic debts (in face value) held by domestic bankers. The amount of private deposits \( d \) to be raised, after government default and the productivity shock is observed, is limited by the market value of sovereign debts held at \( t = 1 \), i.e.:

\[
d \leq \lambda p_1 b_D
\]  

(1.1)

where \( \lambda \geq 0 \) represents the *collateral* value of sovereign debt issued, and \( p_1 \) is its price at \( t = 1 \). In other words, productive bankers have to offer

---

8Non-productive Bankers plays no significant role in the model, except to supply fund to those who are productive. This assumption is reasonable to the extent that the distribution of sovereign debts within the domestic banking sector is not the focus of this paper. My model implies all bankers holds the same amount of public debt at \( t = 1 \) before private debts are issued.

9Alternatively, because the productivity shock is realized before private debts are issued, one could also assume that productive bankers can use output at \( t = 2 \) as collateral, i.e.
their existing holding of sovereign debts as collateral and receive deposits from non-productive bankers in exchange. Assume, without loss of generality, that there is no partial recovery. Sovereign debts at $t = 1$ are traded among non-productive bankers at equilibrium price $p_1$ that equals one when there is no default, and zero otherwise. Investment returns are non-negative and strictly increasing in $I$, so the above inequality is always binding in equilibrium.

The collateral value of sovereign debt, $\lambda$, is the key friction in the model because it imposes limits on the maximum amount of investments $I$. Perhaps, more importantly, it induces a downward-sloping demand curve for sovereign debts and acts as the key driver for equilibrium debt ownership. $\lambda$ can be interpreted as follows. First, along the lines of Holmström and Tirole (1993), it represents the degree to which public debt provides liquidity to expand firms’ ability to invest. Or, in Barro (1974)’s term, sovereign debts is a net-wealth to the bankers so long as it is more efficient than private debts in intertemporal transfers. Second, $\lambda$ is also positively related to the “money-like” component of sovereign debt prices that is not accounted for by standard asset pricing models, as suggested by Krishnamurthy and Vissing-Jørgensen (2012) and Greenwood et al. (2015). Based on these interpretations, it is reasonable to assume $\lambda$ is large for non-GIIPS countries such as Germany and the U.S., and $\lambda$ is small for countries at greater risk of sovereign default, such as the GIIPS countries.

Both types of domestic bankers receive an initial endowment $y_t > 0$ at both $t = 0$ and $t = 1$. They are risk-neutral and make portfolio and investment decisions so as to maximize their expected utility $U(c_0, c_1, c_2) = c_0 + c_1 + c_2$, where $c_t \geq 0$ is their consumption at date $t$. I assume they are price takers and also take as given total debt issued and the probability of default. The asset market opens only once at $t = 0$, where domestic bankers and foreign investors trade sovereign debts at $p_0$. At $t = 1$, after the productivity shock $\tilde{A}$ and their types (i.e. whether they have access to investment technology) are realized, and the government default decision is made, productive bankers

$$d \leq \theta \tilde{A}f(I) + \lambda p_1 b_0,$$

for some $\theta \in [0, 1]$. When $f(\cdot)$ is piece-wise linear, which shall be assumed, this increases the equilibrium investment by a factor of $1/(1 - \theta \tilde{A})$. But the qualitative conclusion remains unchanged. Hence, I assume $\theta = 0$ for simplicity.

The parameter $\lambda$ is exogenous and independent of endogenous variables of interest, such as total debts and the probability of default. This could seem to be at odds with empirical evidence. For example, Krishnamurthy and Vissing-Jørgensen (2012) present evidence that the “money-like” (or simply non-default) component of U.S. Treasury yields is declining in total debt supply. Hence, alternatively, one could specify $\lambda$ as a decreasing function of total debts. All that matters, however, is the existence of some exogenous differences (say, for example, quality of institutions) between countries with different $\lambda$. 

\footnote{The parameter $\lambda$ is exogenous and independent of endogenous variables of interest, such as total debts and the probability of default. This could seem to be at odds with empirical evidence. For example, Krishnamurthy and Vissing-Jørgensen (2012) present evidence that the “money-like” (or simply non-default) component of U.S. Treasury yields is declining in total debt supply. Hence, alternatively, one could specify $\lambda$ as a decreasing function of total debts. All that matters, however, is the existence of some exogenous differences (say, for example, quality of institutions) between countries with different $\lambda$.}
maximize their investments, according to inequality (1.1), by obtaining private debts from non-productive bankers backed by their holding of sovereign debts purchased at \( t = 0 \). If there is no default, the utility of a representative banker at \( t = 1 \) and \( t = 2 \) is

\[
U_{\text{No Default}} = y_1 + b_D - T + w \tilde{A} f (y_1 + \lambda b_D) \quad (1.2)
\]

where \( T \) is the amount of lump-sum tax levied by the (domestic) government to pay off its debts. If there is default, the corresponding utility is

\[
U_{\text{Default}} = (1 - c) y_1 + w \tilde{A} f ((1 - c) y_1) \quad (1.3)
\]

where \( c \in [0, 1] \) represents an exogenous domestic cost of default. This cost is introduced to make sure that default is more costly as income \( y_1 \) increases, consistent with Mendoza and Yue (2012) and Arellano and Ramarayanam (2012). In other words, I assume that \( c \) is choosen such that \( \frac{\partial}{\partial y_1} (f (y_1 + \lambda b_D) - f ((1 - c) y_1)) > 0 \) for all \( b_D \geq 0 \). At \( t = 0 \), the representative bank optimizes its portfolio by investing in domestic debts \( (b_D) \), i.e.

\[
\max_{b_D} U = \max_{b_D} y_0 - p_0 b_D \\
+ (1 - \Pi (a^*)) E [U_{\text{No Default}} | \tilde{A} > a^*] + \Pi (a^*) E [U_{\text{Default}} | \tilde{A} < a^*] \quad (1.4)
\]

where \( p_0 \) is the price of sovereign debt at \( t = 0 \), and \( a^* \) is determined by the government default policy, which shall be explained below.

---

11 If domestic bankers can trade among themselves after their types are realized at \( t = 1 \), productive bankers would purchase all sovereign debts from non-productive bankers at \( p_1 = 1 \), so their holdings increase from \( b_D \) to \( b_D / (1 - w) \). But this has no qualitative impact on the results.

12 The sources of domestic cost include: 1) capital market exclusion (e.g. Aguiar and Gopinath (2006) and Arellano and Heathcote (2010)), 2) increasing cost of borrowing (e.g. Sturzenegger and Zettelmeyer (2007)), 3) economic sanctions (e.g. Mitchener and Weidemier (2010), 4) declining international trade (e.g. Arteta and Hale (2008) and Rose (2005)), and 5) declining productivity (Mendoza and Yue (2012)).

13 See Panizza et al. (2009) for an excellent literature review. Alternatively, there are also empirical evidence about the non-linear (negative) effects of the level of debts on output growth. For example, Reinhart and Rogoff (2010) suggest that, below a certain threshold, the level of debts has little correlation with output growth. Once a certain threshold is passed, however, the level of debts and output growth are increasingly negative correlated.
Government

At $t = 0$, the government needs to finance an exogenous amount $g$ of deficits by issuing debts of amount $T$ (in face value) to both domestic bankers and foreign investors at the same market price, $p_0$. Then, after the productivity shock $\tilde{A}$ is realized at $t = 1$, the government decides whether to default on its debts. It makes the default decision to maximize the welfare of domestic bankers. If the government decides not to default, it pays off its debts by levying a lump-sum tax of $T$ on domestic bankers at $t = 2$. If the government defaults, it has to default on both domestic bankers and foreign investors. In other words, default is non-discriminatory.\footnote{The same assumption is made by Farhi and Tirole (2014) and Gennaioli et al. (2014). Brutti (2011) points out that this assumption hinges on both the anonymity and the increasing integration of secondary markets for sovereign debts, which is consistent with the large haircuts suffered by domestic financial institutions during the recent European sovereign debt crisis. Broner et al. (2010) argues that existence of functional secondary markets makes discriminatory defaults very difficult. Foreign creditors anticipate the opportunistic behaviors of the government, and therefore sell their debts in the secondary markets to domestic creditors when default is likely.}

Foreign Investors

Foreign investors are risk neutral with preferences, endowments, and investment technology identical to the representative domestic banker. Assume that their investment returns are subject to the same productivity shock, $\tilde{A}$. Foreign investors can also purchase domestic debts at $p_0$, the same market price offered to the domestic bankers. Let $b_F$ be the amount of domestic debt held by foreign investors. There is no foreign government.

1.4 Equilibrium

Government Default Policy

At $t = 1$, the government defaults if and only if $U_{\text{Default}} \geq U_{\text{No Default}}$, which leads to the following proposition.

Proposition 1. The default policy for the domestic government can be characterized by a threshold rule. Then, the government defaults if and only if

$$\tilde{A} \leq a^* \equiv \frac{T - b_D - cy_1}{w(f(y_1 + \lambda b_D) - f((1 - c)y_1))} \quad (1.5)$$
In other words, default happens if and only if the realized productivity shock, \( \tilde{A} \), is sufficiently low. This threshold \( a^* \) is monotonically decreasing in: 1) \( b_D \), the amount of debt held domestically, 2) \( \lambda \), the collateral value of domestic sovereign debts, 3) \( c \), the domestic cost of default.

**Proof.** See Appendix A.1.

This proposition characterizes the government default policy as a threshold rule. In particular, the government defaults at \( t = 1 \) if and only if the realized productivity shock \( \tilde{A} \) falls below \( a^* \).

The default threshold, \( a^* \), is given by the default incentive for domestic government per unit of default dis-incentive. In the numerator, \( T - b_D \), is the debt-repayment to foreign investors, which is an *ex-post* resource loss from the perspective of domestic bankers. The denominator represents the net-loss in investment output when there is default. Domestically-held debts, \( b_D \), can be interpreted as a commitment device for government to repay its debts. To see this, consider a case where all debts are held by foreign investors, and the domestic cost of default \( c \) is zero. This implies that the denominator of equation \( (1.5) \) equal zero, or \( a^* \) equals infinity. As shown by figure ??, the government always defaults, so no level of debt is sustainable. Therefore, some debts must be held domestically in any equilibrium.

The government has little incentive to default when: 1) these debts are valuable in terms of expanding investments, i.e. \( \lambda \) is large, 2) most debts are held by domestic bankers, i.e. \( b_D \) is large, and/or 3) the domestic cost of default \( c \) is high. As we shall see, in equilibrium, \( T - b_D \) is the amount of sovereign debts held by foreign investors. The key insight in equation \( (1.5) \) is the asymmetric impact between domestically-held debts and externally-held debts on the government’s incentive to default.

**Prediction 1.** The government’s incentive to default increases with externally-held debts but decreases with domestically-held debts.

**Portfolio Choices and Investments of Domestic Bankers**

At \( t = 0 \), domestic bankers take as given: 1) the government’s default policy \( a^* \), 2) the price of government debt, \( p_0 \), 3) total debt issued, \( T \), and maximize their utility by choosing their own debt holding, \( b_D \), according to equation \( (1.4) \). The FOC implies

\[
p_0 = (1 - \Pi(a^*)) (1 + w\lambda R E[\tilde{A} | \tilde{A} > a^*] f_{No Default}')
\]

(1.6)
where \( f'_{\text{No Default}} = f'(y_1 + \lambda b_D) \). The price of domestic (risky) debt, \( p_0 \), equals its expected payoff. If the realized productivity shock \( \hat{A} \) is above \( a^* \), the government repays its debt. This happens with probability \( 1 - \Pi(a^*) \). There is also a non-default (or liquidity) component, \( w \lambda E[A | \hat{A} > a^*]f'_{\text{No Default}} \), because domestic debts can serve as collateral to obtain private loans for expanding investments. This component is proportional to \( \lambda \), the amount of private loan allowed per unit of domestic sovereign debt holding (or collateral value).

In equilibrium, the price of government debt, \( p_0 \), must satisfy equation (1.6). Because foreign investors have the same preferences as the domestic bankers, equation (1.6) is also the pricing equation for foreign investors. Default policy, \( a^* \) must satisfy equation (1.5), so that ex-ante beliefs in default probability is consistent with the government default policy at \( t = 1 \). Debt holdings, \( b_D \) and \( b_F \) must satisfy the government budget constraints, i.e.

\[
T = b_D + b_F \quad \text{(1.7)}
\]
\[
g = p_0 T \quad \text{(1.8)}
\]

Equation (1.7) states that, when the government decides not to default, a lump-sum tax of \( T \) is levied at \( t = 2 \) on domestic bankers to repay both domestic and foreign debt holders, so \( T \) is also the total government borrowing (in face value). Equation (1.8) says that the revenue raised by total debt issuance, \( p_0 T \), must be equal to the primary deficit, \( g \).

For a given level of total debt \( T \), let \( x = (a^*, p_0, b_D, b_F) \) be the corresponding equilibrium solution that satisfies the above equations. The following assumptions are made to simplify the analysis.

**Assumption 1.** The productivity shock \( A \) is uniformly distributed in \([0,1]\).

**Assumption 2.** The investment technology \( f(\cdot) \) is piece-wise linear:

\[
f(I) = \min(k, I) + (1 - \gamma) \max(0, I - k) \quad \text{(1.9)}
\]

where \( k \) is some constants between \( y_1 \) and \( (1 - c)y_1 \), and \( \gamma \in [0, c] \).

This specification of investment technology is inspired by [Mengus (2014)] for tractability. The investment technology \( f(\cdot) \) is strictly concave. The degree of concavity is positively related by \( \gamma \), which is assumed to be bounded above by the domestic cost of default \( c \), so default is more likely when endowment \( y_1 \) is low, i.e. \( \frac{\partial a^*}{\partial y_1} < 0 \). The constant \( k \) is chosen such that the marginal productivity is higher when there is default.
Probability of Default

The equilibrium probability of default $\Pi(a^*)$ at $t = 0$ must be consistent with proposition \ref{prop:default}, the government’s default policy. It must also be consistent with equation (1.8) and (1.6), the market clearing conditions.

**Proposition 2.** Given assumptions \ref{assumption:1} and \ref{assumption:2} the equilibrium probability of default $\Pi(a^*)$ is

$$\Pi(a^*) = \frac{1}{w\lambda(1-\gamma)} \left[ \sqrt{(1 + w\lambda(1 - \gamma))^2 - w\lambda(1 - \gamma) \frac{2g}{T}} - 1 \right]$$

(1.10)

where $T$ is the total debt level, $g$ is the primary deficit, $\lambda$ is the collateral value of sovereign debt, and $\gamma \in [0, c)$ is the degree of concavity in the investment technology $f(\cdot)$. In addition, $\frac{d\Pi(a^*)}{dT} > 0$ and $\frac{d^2\Pi(a^*)}{dT d\lambda} < 0$.

**Proof.** See Appendix A.2.

In other words, $\Pi(a^*)$ can be expressed a function of total debt-to-deficit ratio, $T/g$. Clearly, equation (1.10) implies that these two variables are positively correlated. Intuitively, when debt-to-deficit ratio increases, some fraction of the newly issued debts are held by domestic bankers, so the overall impact on the government’s incentive to default is not necessarily clear. Proposition \ref{prop:default} states that the overall impact is strictly positive. Less clear is the role of the collateral value of sovereign debt on $\Pi(a^*)$. The proposition says, in equilibrium, the probability of default is more sensitive to increase in total debt if the collateral value of sovereign debt is lower. The government’s incentive to default, according to equation (1.5), is expressed as the ratio of debt-repayments to foreign investors to net-loss of investment output when there is default. For countries whose debts have lower collateral value, their costs of default, in terms of loss of investment output, are smaller. Therefore, their incentives to default are not only higher but also more sensitive to changes in fiscal conditions. Therefore, in response to the same increase in total debt, their probabilities of default increase by relatively more.

**Prediction 2.** The probability of default is more sensitive to increases in total debt if the collateral value of sovereign debt is lower.

\footnote{This result is robust to different specification of investment technology $f(\cdot)$, provided that $f(\cdot)$ is not “too concave”.}
Shares of Domestic Debt held by Domestic Bankers

One can obtain the equilibrium share of debt held by domestic banks, \( b_D/T \), in closed-form.

**Proposition 3.** The equilibrium share of sovereign debt held by domestic bankers, \( b_D/T \), given the level of total debt issued \( T \), and the collateral value of domestic debt, \( \lambda \), is

\[
b_D = \frac{1}{1 + w\lambda(1 - \gamma)a^*} - \frac{(c - \gamma)wa^*}{T(1 + w\lambda(1 - \gamma)a^*)}y_1
\]

where \( c \in (0, 1) \) is the domestic cost of default, \( \gamma \in [0, c] \) is the degree of concavity in the investment technology \( f(\cdot) \), and the government's default policy, \( a^* \), satisfies equation (1.10). Furthermore, \( b_D/T \) is strictly decreasing in \( \lambda \) and \( c \) but strictly increasing in \( \gamma \).

**Proof.** See Appendix A.3.

The equilibrium share of domestic debts, \( b_D/T \), contains two components. The first term, \( \frac{1}{1 + w\lambda(1 - \gamma)a^*} \), is the reciprocal of the minimum marginal contribution of one unit of domestic sovereign debts to output in states where there is no default, given the default policy \( a^* \). The higher the marginal contribution, the less debts are needed to be domestically held to prevent default, in equilibrium, for a given level of total debts \( T \).

To see the intuition, consider a case where the domestic cost of default is zero. If, \( \lambda \), the collateral value of the domestic sovereign debt is also zero, the government has no incentive to repays it debts at \( t = 1 \), according to equation (1.5). Why? When there is no default, the government must levy a lump-sum tax on domestic banks for debt repayments. This is a net cash outflow when the sovereign debts are not productive. Thus, the probability of default must be one, which is recognized in equilibrium, implying that all debts must be held by domestic banks. Domestic debt holding is, therefore, a commitment device for debt repayment. The higher the collateral value of sovereign debt \( \lambda \), the less incentive for the government to default, and therefore less debts are held by domestic bankers.

The second term, \( \frac{(c - \gamma)wa^*}{T(1 + w\lambda(1 - \gamma)a^*)}y_1 \), represents the impact of domestic cost of default on the equilibrium share of domestic debt. If \( c \) is high, then a

\[\text{Note that } a^* \text{ can also be interpreted as the minimum realization of productivity shock } \hat{A} \text{ such that the government does not default, so it does affect the ex-ante marginal contribution of one unit of domestic sovereign debt to output.}\]

\[\text{This implies, by assumption, } \gamma \text{ is also zero.}\]
large fraction of endowment $y_1$ is lost when the government defaults, which in turns, discourage the government from defaulting. Consequently, less debts are needed to be held by domestic bankers to sustain an equilibrium. But this effect is weaker as the amount of total debt $T$ increases because, on per-unit basis, this cost is decreasing in $T$.

How does $\gamma$, the degree of concavity in the investment technology $f(\cdot)$, affect the equilibrium debt ownership? When $\gamma$ increases, the marginal gain in outputs from not defaulting declines. Therefore, an increase in $\gamma$ have a positive impact on the both terms through the denominator because the loss of investments from government default decreases. At the same time, however, such increase also implies the effective domestic cost of default, $(1 - \gamma) - (1 - c) = c - \gamma$, declines, which negatively contributes to numerator of the second term. The overall effect is negative, implying that equilibrium share of domestic debt increases in $\gamma$.

In the empirical section, I will test the following prediction of proposition 3.

**Prediction 3.** The share of debt held by domestic bankers, $b_D/T$, is negatively related to the collateral value of sovereign debt, $\lambda$.

**Proposition 4.** Assume the domestic cost of default $c$ is strictly positive, and degree of concavity in the investment technology $f(\cdot)$, $\gamma \in [0, c)$. Given the level of total debt issued $T$, the equilibrium share of (risky) domestic debt, $b_D/T$, as defined by equation (1.11), has the following properties:

1. If a local extremum exists for $b_D/T$ (as a function of $T$), then it is also the unique global minimum.

2. For any $T > 0$, there exists $\lambda > 0$ such that for all $\lambda < \lambda$, $b_D/T$ is strictly increasing in $T$.

3. Suppose $c$ is sufficiently small. There exists $\bar{\lambda}, \underline{T} > 0$ such that, for all $\lambda > \bar{\lambda}$ and $T < \underline{T}$, $b_D/T$ is strictly decreasing in $T$.

4. If $cy_1(1 + w)$ is larger (smaller) than $\frac{cw\lambda(1-\gamma)}{(1+w\lambda(1-\gamma))^2}$, $b_D/T$ is increasing (decreasing) in $T$ if $T$ is sufficiently large.

**Proof.** See Appendix A.4.

This proposition establishes the main result of this section. Changes in the equilibrium share of debt held by domestic bankers with respect to changes in
total debt $T$ depends crucially on $\lambda$, the collateral value of domestic sovereign debt, and it is determined by two opposite forces: 1) productivity of sovereign debt at non-default states and 2) effective domestic cost of default per unit of debt. Consider an increase in total debt $T$. Because the government needs to finance an exogenous deficit $g$ through debt financing. Such increase would suppress debt prices and raise the probability of default $\Pi(a^*)$ in equilibrium, according to equation (1.6). This raises the minimum payoff in states where the government does not default, which in turn, has a negative impact on the first term in equation (1.11) because less debts is needed to be held by domestic bankers to sustain an equilibrium. Clearly, this impact is positively related to the collateral value of sovereign debt $\lambda$. The impact of an increase in $T$ on the effective domestic cost of default is not as clear. On one hand, higher default probability implies that it is more likely to incur the domestic cost of default $c$, so this has a negative impact on $b_D/T$. On the other hand, such cost per unit of total debts $T$ is declining in $T$, as reflected by the denominator of the second term in equation (1.11), so this has a positive impact on $b_D/T$. The overall impact depends on $\lambda$ and the level of total debts $T$, as shall be discussed below.

Property (2) says, given strictly positive domestic cost of default $c > 0$, the second impact dominates when the collateral value of sovereign debt $\lambda$ is small, implying that $b_D/T$ is increasing in $T$. Take, for example, a GIIPS country whose debts have very small collateral value, $\lambda \approx 0$. Figure 1.3 plots its share of debt held by domestic banks, $b_D/T$ as a function of $T$. The function is strictly increasing for all level of $T$ because the impact on per-unit domestic cost of default is much stronger than that on marginal productivity of sovereign debt. Defaults are less costly as total debt $T$ increases, more debts have to be held by domestic bankers to prevent the government from defaulting. Increasing default probability does increase the marginal productivity of its debts in non-default states, but the overall impact is minimal because $\lambda$ is close to zero.

Property (3) says, if the sovereign debts of a country has high collateral value $\lambda$, and the total level of its debt $T$ is not too high, the positive impact on marginal productivity of its debt dominates the negative impact on per-unit domestic cost of default, so the share of debts held by domestic banks is decreasing in $T$. As shown by figure 1.4, for a country whose collateral value of sovereign debt is relatively high, with $\lambda = 0.2$ and the level of deficit $g = 0.5$, $b_D/T$ is strictly decreasing in $T$ when the total debt-to-deficit ratio is below 125%, during which the positive impact on marginal debt productivity dominates (i.e. figure 1.4b). Once the debt-to-deficit ratio is above 125%, however,
the negative impact on domestic cost of default dominates, so \( b_D \) becomes strictly increasing in \( T \). Figure 1.5 shows that, for countries such as Germany and the U.S., where the collateral value of sovereign debt are very high, the positive impact on marginal debt productivity always dominates for any level of debt \( T \), implying that \( b_D/T \) is strictly decreasing in \( T \). Property (1) establishes that the three examples discussed above, in fact, cover all possibilities. Finally, as \( T \rightarrow \infty \), the derivative of \( \frac{\partial b_D}{\partial T} \) is

\[
\lim_{T \to \infty} \frac{\partial}{\partial T} \left( \frac{b_D}{T} \right) = -\frac{gw(1-\gamma)\lambda}{1+w(1-\gamma)\lambda} + \frac{(c-\gamma)yt(1+w)(1+w(1-\gamma)\lambda)}{\text{Marginal Effect From Sovereign Debt Productivity}} + \frac{(1+w)(1+w(1-\gamma)\lambda)}{\text{Marginal Effect From Domestic Cost of Default}}
\]

(1.12)

Property (4) states that, for any collateral value of sovereign debt \( \lambda \), if domestic cost of default \( c \) is high relative to deficit \( g \), then \( b_D/T \) shall eventually be increasing in \( T \). Consider, again, the above example for countries whose debts has high collateral value. Figure 1.6 plots the equilibrium share of debt held by domestic bankers, when the domestic cost of debt \( c \) increases from 10% to 20% with the same \( \lambda = 0.5 \). Consistent with proposition 3, given the level of total debt \( T \), \( b_D/T \) is lower when \( c \) is high because of lower incentive to default. More importantly, \( b_D/T \) strictly decreasing in \( T \) when \( c = 10\% \). When \( c = 20\% \), however, \( b_D/T \) eventually becomes increasing in \( T \) once the debt-to-deficit reaches around 150\%. This is because the marginal decline in per-unit domestic cost of default is higher when \( c \) is high, so more sovereign debts are needed to be held by domestic bankers to sustain a equilibrium when \( T \) is high.

**Prediction 4.** The share of debt held by domestic banks, \( b_D/T \) is strictly increasing in total debts, \( T \), only in highly-indebted countries whose debts have low collateral value, \( \lambda \).

**Optimal Level of Total Debt for a given level of deficit \( g \)**

For a given level of deficit \( g \), what is the relationship between debt composition and the optimal level of debts \( T \)? Given the collateral value of sovereign debt \( \lambda \) for country R, its government chooses \( T \) to maximizes the utility of the representative banker, such that the default probability \( \Pi(a^*) \) satisfies equation (1.5) and the (equilibrium) share of domestic debts held by domestic banks, satisfies equation (1.11). In other words, suppose \( b_D^*(T) \) is the equilibrium
portfolio for domestic banks at \( t = 0 \) for given \( T \), the government’s problem at \( t = 0 \) is:

\[
\max_T U = \max_T y_0 - p_0 b_D^* \\
+ (1 - \Pi(a^*)) E[U_{\text{No Default}}|\tilde{A} > a^*] + \Pi(a^*) E[U_{\text{Default}}|\tilde{A} < a^*]
\]

subject to equilibrium conditions (1.6) - (1.11), where \( U_{\text{No Default}} \) and \( U_{\text{Default}} \) satisfies, respectively, equation (1.2) and (1.3). The following proposition characterizes the optimal level of total debts:

**Proposition 5.** The FOC to domestic government’s problem (1.13) is

\[
\left( \frac{T}{1 - \Pi(a^*)} \right) \frac{dU(T, b_D)}{dT} = \left( \frac{\lambda(1 - \gamma) E[\tilde{A} | \tilde{A} > a^*]}{b_D} \times b_D \\
- \frac{T - b_D}{T} \right)
\]

where \( T \) and \( b_D \) satisfy equation (1.11). In addition,

1. If \( \lambda \) is sufficiently small, then \( U(T, b_D) \) is strictly decreasing in \( T \).
2. Suppose \( \frac{dU(T, b_D)}{dT} \big|_{(T^*, b_D^*)} = 0 \). Then \( U(T^*, b_D^*) \) is the global maximum.
3. \( \frac{\partial}{\partial T} \left( \frac{b_D}{T} \right) \big|_{(T', b_D')} > 0 \implies \frac{dU(T, b_D)}{dT} \big|_{(T', b_D')} < 0 \).

**Proof.** See Appendix A.5.

Equation (1.14) states that the optimal level of government borrowing depends on the trade-offs, in states where there is no default, between a declining expected marginal contribution of sovereign debt to private investment for domestic banks and increasing debt-repayments to foreign investors. Issuing more sovereign debts is beneficial because domestic banks can use them as collaterals to expand investments. At the same time, however, it is also harmful because of increasing debt repayments to foreign banks, which is an *ex-post* loss of resources from the perspective of domestic bankers.

Imagine an equilibrium where all sovereign debts are held by domestic bankers. According to equation (1.5), there is no incentive for the government to default, so \( \Pi(a^*) \) is zero, which corresponds to a particular level of total
debts $T = b_D$. Suppose the government expands its borrowing. According to proposition 2, $\Pi(a^*)$ must also increase. Therefore, newly issued debts must be held, in equilibrium, by both domestic bankers and foreign investors. The debt ownership that is sustainable in equilibrium for the increased level of total debt is determined by proposition 3.

Is it optimal for the government to borrow more? Given that the increased level of debt is sustainable, when the government decides whether or not to borrow more, it cares only about the trade-offs in the no-default states. Suppose the government increases $T$ by one unit, then the lump-sum tax levied for debt-repayment obviously also increases by one unit, of which $T - b_F$ is the net transfer of resources to foreign investors. This hurts domestic bankers. Because some of the newly issued debts must be held by domestic bankers, however, increasing $T$ also beneficial because these debts help expand investments by domestic bankers. This benefit is large when the collateral value of sovereign debt $\lambda$ is high, so the government should issue more debts. On the other hand, if $\lambda$ is sufficiently small, this benefit is close to zero, so additional debt issuance is not desirable.

Property 3 characterizes the conditions under which the later is true. More importantly, it relates the optimal level of debt to debt ownership and provides an observable indicator for (suboptimally) excessive level of debts. This property states that, if it is observed that the share of debt held by domestic bankers increases as government increases its borrowing, then the level of debts is suboptimally high, and the government should reduce its borrowing.

1.5 Empirical Evidence

In this section, I provide empirical evidence for the following predictions from my model:

1. The probability of default $\Pi(a^*)$ increases with externally-held debts $b_F$ but decreases with domestically-held debts $b_D$.

2. The probability of default $\Pi(a^*)$ is more sensitive to increases in total debt $T$ if the collateral value of sovereign debt $\lambda$ is lower.

3. The share of debt held by domestic bankers, $b_D/T$, is negatively related to the collateral value of sovereign debt, $\lambda$. 
4. The share of debt held by domestic banks $b_{D}/T$ is strictly increasing in total debts $T$ only in highly-indebted countries whose debts have low collateral value $\lambda$.

**A Proxy for the Collateral Value of Sovereign Debts and its Instrument**

**Theoretical Proxy**

Because the productivity shock $\tilde{A}$ is non-negative, and the investment technology $f(\cdot)$ is strictly increasing, the marginal return from investment is strictly positive with probability one. This implies equation (1.1), the borrowing constraint, always binds in equilibrium. In other words, the collateral value of sovereign debt, $\lambda$, should be observationally equivalent to the ratio of private debt issued by domestic banks to their holdings of domestic sovereign debt. Given this, one can construct a proxy $\hat{\lambda}$ for $\lambda$:

$$\hat{\lambda} = \frac{\text{Private debt issued by domestic banks (in face Value)}}{\text{Domestic sovereign debts held by domestic Banks (in face Value)}}$$

(1.15)

where data for the denominator, domestic sovereign debts held by domestic banks, are obtained from Merler and Pisani-Ferry (2012), and data for the numerator, private debts issued by domestic banks, are obtained from Eurostat.

Proposition [4] suggests that the share of domestic debt, in equilibrium, is strictly increasing in total debt only in highly-indebted countries whose debts have low collateral value. For countries whose debts have high collateral value, the opposite is predicted by the theory as long as their total debt levels are not too high. If the theory is to be consistent with figure [1.1b], which plots the share of public debt held by domestic banks for both GIIPS and non-GIIPS countries, it is clear that $\hat{\lambda}$ must at least be smaller on average in GIIPS countries than non-GIIPS countries.

Table [1.1] provides summary statistics for $\hat{\lambda}$, with annual from 2007 to 2014. With the notable exception of Ireland and Belgium, the result is consistent with the theory. On average, $\hat{\lambda}$ for Greece, Italy, Portugal and Spain are significantly smaller than $\hat{\lambda}$ of the non-GIIPS countries (except Belgium). The sovereign debts for Greece have the smallest average collateral value with $\hat{\lambda} = 0.102$, and it is consistently the smallest throughout the sample. Average $\hat{\lambda}$ for Italy and Portugal are, respectively, 2.314 and 2.359, which is larger than that of Greece. But their values are still significantly less than their
non-GIIPS counterparts. For instance, the average \( \hat{\lambda} \) for Germany and France are, respectively, 3.385 and 5.304. For Finland, Netherlands, and the United Kingdom, their average \( \hat{\lambda} \) are even higher but so are their standard deviations.

Statistical comparison is difficult because of the size of sample is small. Additionally, to compare means of \( \hat{\lambda} \) between countries, standard t-test cannot be used because their distributions are not normal. Therefore, I use the non-parametric Wilcoxon (1946) signed rank test. The hypothesis is as follow:

\[
\begin{align*}
H_0 & : \text{Mean of } \hat{\lambda} \text{ for non-GIIPS country } i &= \text{Mean of } \hat{\lambda} \text{ for GIIPS country } j \\
H_1 & : \text{Mean of } \hat{\lambda} \text{ for non-GIIPS country } i > \text{Mean of } \hat{\lambda} \text{ for GIIPS country } j
\end{align*}
\]

for any combination of non-GIIPS countries \( i \) and GIIPS countries \( j \) in the sample. The test statistics, \( W \), equals:

\[
W = \sum_{T=2007}^{2013} \left( \text{sign}(x_{\text{non-GIIPS},T} - x_{\text{GIIPS},T}) \times \text{Rank}_{\text{non-GIIPS,GIIPS},T} \right)
\]

where \( x_{i,T} \) is the observation for country \( i \) in year \( T \), and, for country \( i \) and \( j \), \( \text{Rank}_{i,j,T} \) is the rank of the value of \( |x_{\text{non-GIIPS},T} - x_{\text{GIIPS},T}| \) in year \( T \) among the sample. For small sample, \( W \) follows a distribution with an expected value of zero and a variance \( \frac{T(T+1)(2T+1)}{6} \). Using critical values computed by Wilcoxon (1946), I conclude that again, with the exception of Belgium and Ireland, the null hypotheses are all rejected at 5% significant level.

An Instrument for \( \hat{\lambda} \)

Although \( \hat{\lambda} \) is a theoretically motivated proxy for the collateral value of sovereign debt, it is not appropriate to be used to test my empirical predictions because \( \hat{\lambda} \) is constructed, in part, by using domestically-held debts, which, by definition, cannot be independent of the share of domestically-held debt \( b_D/T \) nor total debt \( T \). Because of this joint-determination problem, an instrument for \( \hat{\lambda} \) must be used.

I use the \textit{strength of legal rights index} (creditor rights) from the World Bank’s World Development Indicators as an instrument for \( \hat{\lambda} \). The index measures the degree to which collateral and bankruptcy laws protect the rights of borrowers and lenders and thus facilitate lending. It ranges from 0 to 12, with higher scores indicating that these laws are better designed to expand access to credit. This variable is used by Gennaioli et al. (2014) to measure the
quality of financial institutions. To gauge the strength of this instrument, I run various specifications of regression of $\hat{\lambda}$ on creditor rights. Table 1.2 shows that creditor rights are positively correlated to $\hat{\lambda}$ at 1% significant level whether or not controlling for country or year fixed-effects. When I include both country and year fixed-effects and adjust for autocorrelations in the error term, as shown in column (4), one unit increase in the score for creditor rights is associated with a 19% increase in $\lambda$, and approximately 30% of the variation in $\hat{\lambda}$ is explained. The intuition for this result is clear: Better creditor rights enables the banks ability to issue private debts with less government debt as collateral. This means, according to equation (1.15), the instrument drives $\hat{\lambda}$ via the numerator.

Relationship between Default Risk, Domestic, and External Debts

Prediction 1 states that the impacts of changes in domestically-held debt ($b_D$) and externally-held debt ($b_F$) on default risk are opposite. Because both $b_D$ and $b_F$ are endogenous in my model, I instrument by the collateral value of debt ($\lambda$) and total debt ($T$), which are, by assumption, exogenous. Creditor rights and debt-to-GDP ratio are used as proxies, respectively, for $\lambda$ and $T$. To verify the strength of these instruments, table 1.3 shows the results for the regressions of $b_D$ and $b_F$ on creditor rights and debt-to-GDP ratio. Creditor rights are negatively correlated with $b_D$ at the 5% level (column (1) and (3)), and positively correlated with $b_F$ at 1% level (column (4)). A one unit increase in the score for creditor rights is associated with roughly 2% decrease in $b_D$ (column (3)) and 4% in $b_F$ (column (4)). The Debt-to-GDP ratio are positively correlated with both $b_D$ and $b_F$ at the 1% level (column (1),(2), and (4)). A one percentage point increase in the debt-to-GDP ratio is associated with a 0.174 percentage point increase in $b_D$ and a 0.865 percentage point increases in $b_F$. The R-squared are about 40% for $b_D$ and 78% for $b_F$. Overall, creditor rights and debt-to-GDP ratios are strong instruments for $b_D$ and $b_F$.

Using these instruments, I now estimate various specifications of the second-

\footnote{Gennaioli et al. (2014) argues that government defaults should lead to declines in private credit, and these decline should be larger in countries where financial institutions are more developed and banks hold more government bonds. Probability of default and externally-held debts are exogenous in their model, and their level of total debt is fixed at one. Different from their paper, I study the impact of changes in total debts and collateral values of sovereign debt on default risk and debt-ownership.}
stage pooled OLS regression:

\[ (\text{Default Risk})_{i,t} = \alpha_i + \nu_t + X'_{i,t}\theta + \beta_1(\text{Domestically-held Debt-to-GDP})_{i,t} + \beta_2(\text{Externally-held Debt-to-GDP})_{i,t} + \varepsilon_{i,t} \]  \hspace{1cm} (1.17)

In equation (1.17), the coefficient \( \alpha_i \) represents country-fixed effects, which control for all time-invariant country-specific (e.g., political or historic) factors affecting default risk. The coefficient \( \nu_t \) captures time effects, controlling for common shocks across countries (e.g., changes in world interest rates). To control for remaining possible sources of endogeneity, i.e., country-specific time-varying shocks, the vector \( X'_{i,t} \) contains variables that capture the most common predictors of an increase in default risk, which is, again, proxied by 5-year CDS Spreads (in basis points). Most of these variables are used in [Aizenman et al. (2013)]. Current account balance-to-GDP ratio and real GDP growth are included because default risk tends to be negatively correlated with output growth, as shown by [Mendoza and Yue (2012), Aizenman et al. (2013)]. It shows that inflation tends to be positively correlated with default risk. The error term, \( \varepsilon_{i,t} \), is assumed to be AR(1). The results are in table 1.5.

In the baseline specification, I restrict \( \beta_1 = \beta_2 \), i.e column (1) and (3), which equivalent to regressing 5-Year CDS spreads on total debt-to-GDP ratio. Consistent with existing literature, the parameter estimate is positive and significant, regardless of whether controls are added. According to column (3), a one percentage point increase in total debt-to-GDP ratios is associated with a 4.1 basis points increase in 5-Year CDS spreads.

Then, I decompose the total debt-to-GDP ratio into externally-held debt-to-GDP ratio and domestically-held debt-to-GDP ratio, which are instrumented by creditor rights and debt-to-GDP ratios. According to the theory, \( \beta_1 > 0 \) and \( \beta_2 < 0 \).

Consistent with the theory, whenever included, the coefficient on externally-held debt-to-GDP ratio is positive and statistically significant in column (2), (4) and (6). Moreover, the estimated parameters are robust across specifications. One percent increase in external debt-to-GDP ratio is associated with a roughly 11 basis points increase in 5-Year CDS spreads. The magnitude of the regression coefficient is significantly larger than that for total debt-to-GDP ratio. Perhaps, more surprisingly, the R-squared in column (3) and (4) are, respectively, 0.43 and 0.46. In other words, externally-held debt-to-GDP ratio explains more variations in the 5-Year CDS Spreads than total debt-to-GDP ratio.
This brings the focus to the role of domestically-held debt-to-GDP ratio, i.e. $\beta_2$, in column (2), (5), and (6). The coefficients vary significantly across column (5) and (6). Specifically, the coefficient is positive at the 1% significance level when externally-held debt-to-GDP is excluded. When externally-held debt-to-GDP is included, however, the coefficient becomes negative at 10% significant level. In the latter case, one percent increase in domestically-held debt-to-GDP ratio is associated with roughly 12 basis points decrease in 5-Year CDS spreads, which is consistent with prediction [1].

At the first glance, however, this result would seem to be at odd with the observation that share of domestically-held debt increases in the GIIPS countries, which are generally perceived to have higher default risks. In other words, why should domestically-held debt and default risk not be positively correlated? I argue that this is precisely what distinguishes this paper from the "risk-shifting" explanation to this observed phenomenon. For example, Farhi and Tirole (2014) argues that due to their convex payoff structure of equity, increasing holding of risky sovereign debts could be particularly attractive to domestic banks during a sovereign debt crisis. This explanation suggests that domestically-held debts should be positively correlated with default risk, $\beta_2 > 0$. Indeed, this is true when default risk is directly regressed on domestically-held debt. But my theory argues that this is not the correct approach because of omitted-variable bias. Both default risk and domestically-held debt are negatively correlated with the collateral value of sovereign debt. This implies that, without using instrumental variables, the regression coefficient on domestically-held debts suffers an upward bias. My result shows that the regression coefficient is negative after this omitted-variable bias is corrected.

**Relationship between Default Risk, Total Debt, and Collateral Value of Sovereign Debt**

Prediction [2] says that the marginal effect of changes in total debt on default risk is decreasing in the collateral value of debt. Therefore, I estimate various specifications of the pooled OLS regression:

\[
(\text{Default Risk})_{i,t} = \alpha_i + \nu_t + X'_{i,t}\theta + \beta_1(\text{Total Debt-to-GDP})_{i,t} \\
+ \beta_2(\text{Total Debt-to-GDP} \times \text{Collateral Value})_{i,t} \\
+ \beta_3(\text{Total Debt-to-GDP} \times \text{GIIPS})_{i,t} + \varepsilon_{i,t}
\]

(1.18)
where default risk is proxied by the 5-Year CDS spreads, GIIPS is a dummy variable that equals one for GIIPS countries and zero otherwise, and collateral value, measured as the ratio of private debts issued by domestic banks to their holdings of domestic sovereign debt, is instrumented by creditor rights. As in equation (1.17), the coefficient $\alpha_i$ represents country-fixed effects, the coefficient $\nu_t$ captures time-fixed effects. As controls, I also include the vector $X_{i,t}$, which contains inflation, current account balance-to-GDP ratio and real GDP growth. The error term, $\varepsilon_{i,t}$, is assumed to be AR(1).

The theory predicts that $\beta_2 < 0$. Regression results shown in table 1.6 are consistent with this prediction. When the interaction term between debt-to-GDP ratio and collateral value is included, as indicated by column (3) and (6), $\beta_2$ is negative at the 5% significance level, and $\beta_1$ is positive at 1% significance level. In other words, the marginal effect of debt-to-GDP ratio on 5-year CDS spreads depends negatively on the collateral value of sovereign debts. When the collateral value is zero, a one percentage point increase in debt-to-GDP ratio is associated with a roughly 5.4 basis points increase in 5-year CDS spreads. When the collateral value increases by one percent, however, this marginal effect drops by 1.41 basis points.

To further test the theory, I divide the countries in my sample into two groups: GIIPS and non-GIIPS. GIIPS countries include Greece, Italy, Ireland, Portugal, and Spain; non-GIIPS countries include Belgium, Finland, France, Germany, Netherlands, the United Kingdom, and the United States. Figure 1.1b shows that share of domestically-held debt increases as total debt increases in GIIPS countries, but the opposite is observed in non-GIIPS countries. Proposition 4 suggests, if the observed data is consistent with the theory, then the marginal effect of changes in total debt on default risk must be higher in GIIPS countries than non-GIIPS countries. Therefore, $\beta_3$ should be positive (when the $\beta_2$ is omitted). Results from table 1.6 is consistent with this hypothesis. When the interaction term between debt-to-GDP ratio and GIIPS dummy is included, as indicated by column (2) and (4), $\beta_3$ is positive at the 1% significance level, but $\beta_1$ is not significant. In other words, the marginal effect of debt-to-GDP ratio on 5-year CDS spreads is larger in GIIPS countries than non-GIIPS countries. For GIIPS countries, a one percentage point increase in debt-to-GDP ratio is associated with roughly 5.6 basis points increase in 5-year CDS spreads. For non-GIIPS countries, however, this marginal effect is statistically insignificant.
Relationship between Share of Domestically-held Debt, Total Debt, and Collateral Value of Sovereign Debt

Both prediction 3 and 4 concern the effect of changes in total debt and the collateral value of debt on the share of domestically-held debt. Specifically, prediction 3 states that, holding constant the level of total debt, an increase in the collateral value of debt is associated with a decline in the share of domestically-held debt. Prediction 4 says the marginal effect of changes in total debt on the share of domestically-held debt is declining in the collateral value of debt.

\[
(\text{Share of Domestically-held Debt})_{i,t} = \alpha_i + \nu_t + X'_{i,t}\theta + \beta_1(\text{Total Debt-to-GDP})_{i,t} \\
+ \beta_2(\text{Collateral Value})_{i,t} \\
+ \beta_3(\text{Total Debt-to-GDP} \times \text{Collateral Value})_{i,t} \\
+ \varepsilon_{i,t} \tag{1.19}
\]

where, again, the collateral value of sovereign debt, measured as the ratio of private debts issued by domestic banks to their holdings of domestic sovereign debt, is instrumented by creditor rights. I assume that the variation in total debt-to-GDP is exogenous, as in Aizenman et al. (2013).\(^{19}\) As in equation (1.18), the coefficient \(\alpha_i\) represents country-fixed effects, the coefficient \(\nu_t\) captures time-fixed effects. As in the previous section, I include the vector \(X'_{i,t}\) as controls, which contains inflation, current account balance-to-GDP ratio and real GDP growth. The error term, \(\varepsilon_{i,t}\), is assumed to be AR(1).

Prediction 3 predicts \(\beta_2 < 0\), and prediction 4 predicts \(\beta_3 < 0\). Regression results shown in table 1.4 are consistent with these predictions. As expected by prediction 3, the coefficient for collateral value, \(\beta_2\), is negative and significant at 10% level. When collateral value and total debt-to-GDP are both included (column(3)), a one unit increase in the collateral value is associated with a 4 percentage points decrease in the share of domestically-held debt. The coefficient for debt-to-GDP ratio, \(\beta_1\), is positive and significant at 1% level. A one percentage point increase in debt-to-GDP ratio is associated with a 0.129 percentage point increase in the share of domestically-held debt. Interestingly, when only total debt-to-GDP or collateral value is included (column (1) and (2)), the corresponding \(R^2\) is, respectively, 0.16 and 0.11. When both variables

\(^{19}\) Equation (1.19) can be viewed as estimating the first-order approximation of equation (1.11) with respect to total debt \(T\) and collateral value \(\lambda\), which are both assumed to be exogenous in the model.
are included, the $R^2$ increases by more than 50% to 0.24. This result is also consistent with the theory, which argues that both collateral value and total debt are necessary to jointly explain the variation in debt-ownership.

When the interaction term between total debt and collateral value, \((\text{Total Debt-to-GDP} \times \text{Collateral Value})_{i,t}\), is included (column (4)), both the magnitude and significance of $\beta_1$ increase. More importantly, as expected by prediction 4, the regression coefficient for the interaction term, $\beta_3$, is negative and significant at the 10% level. This implies the marginal effect of changes in total debt on share of domestically-held debt is declining as the collateral value of sovereign debt increases. When the collateral value is zero, one percent increase in debt-to-GDP ratio is associated with 0.18 percent increase in domestic debts. When the collateral value increases by one percent, however, the marginal effect reduces by 0.09 percent. In other words, if the collateral value is larger than 2, the marginal effect of changes in debt-to-GDP ratio becomes negative. The average collateral value for GIIPS (excluding Ireland) and non-GIIPS countries between 2007 and 2014 are, respectively, 1.80 and 11.35. This explains figure 1.1b where we observe that the share of domestically held-debt increases in GIIPS countries but not in non-GIIPS countries.

### 1.6 Conclusion

Given that major developed countries have experienced a significant run-up in public debts after the onset of the global financial crisis in 2008, what explains the cross-country difference in sovereign debt ownership? This paper argues that, consistent with financial repression, domestic debt is a commitment device for debt repayments. Therefore, domestically-held debts should be positively related to the government’s incentive to default. The key variables that affect this incentive are: 1) the collateral value of government debt and 2) the level of public debt. In particular, the share of debt held by domestic banks is strictly increasing in total debts only in highly-indebted countries whose debts have low collateral value. Empirical evidence is then provided for these model predictions.

As [Diamond (1965)](1965) suggests, changes in debt-ownership have important implications for the real economy. Therefore, it is crucial to understand the determinants of debt ownership. This paper relates debt ownership to the level of public debt, the collateral value of public debt, and sovereign default risk in a unified dynamic equilibrium model. An important policy implication

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20See, for example, Forslund et al. (2011) and Guscina (2008)
from the model is that the government should pay close attention to changes in debt-ownership as it conducts fiscal policy. In particular, I argue that when debts are “moving back home”, as observed in the GIIPS countries, the level of public debt is suboptimally excessive. Empirically, this paper provides evidence that changes in domestically-held debts and externally-held debts have different impacts on the default component of sovereign debt prices.

Finally, this paper is concerned mainly about the observed time-series paths of debt ownership and total public debt in major developed countries in the European Monetary Union (except the U.K. and the U.S.), so it is reasonable to simplify the model by omitting inflation and monetary policy. Nonetheless, for countries with independent currency and monetary policy, raising the inflation rate could be, and perhaps a more attractive alternative to outright default. To the best of my knowledge, no one has yet established, either theoretically or empirically, any connection between monetary policy and debt ownership. A potentially fruitful extension that allows the government to influence inflation through monetary policy is left for future research.

\[\text{21}^{\text{The empirical result of this paper is robust to exclusion of these two countries in the sample.}}\]
1.7 Figures and Tables

Figure 1.1: Shares of sovereign debt held by foreign investors and domestic banks (left axis) and Debt-to-GDP ratio (right axis)
GIIPS countries include Greece, Ireland, Italy, Portugal, and Spain. Non-GIIPS countries include Belgium, Finland, France, Germany, Netherlands, the United Kingdom, and the United States. Central bank holdings in issuing countries and debts held by domestic household, pension, and insurance companies are excluded. All series weighted by nominal GDP.
Figure 1.2: Government Default Policy, $a^*$ as a function of domestically-held debts, $b_D$, for a given debt level $T$. 
Figure 1.3: Share of Debt held by Domestic Banks $b_D/T$ as a function of $T$ for a country with low collateral value of sovereign debts, $\lambda = 0.01$. 

(a) Share of Debts held by Domestic Banks $b_D/T$ as a function of $T$

(b) Incentives that determine the Share of Debts held by Domestic Banks $b_D/T$ as a function of $T$
(a) Share of Debts held by Domestic Banks $b_D/T$ as a function of $T$

(b) Incentives that determine the Share of Debts held by Domestic Banks $b_D/T$ as a function of $T$

Figure 1.4: Share of Debt held by Domestic Banks $b_D/T$ as a function of $T$ for a country with low collateral value of sovereign debts, $\lambda = 0.2$. 
(a) Share of Debts held by Domestic Banks $b_D/T$ as a function of $T$. 

(b) Incentives that determine the Share of Debts held by Domestic Banks $b_D/T$ as a function of $T$. 

Figure 1.5: Share of Debt held by Domestic Banks $b_D/T$ as a function of $T$ for a country with low collateral value of sovereign debts, $\lambda = 0.5$. 
Figure 1.6: Share of Debt held by Domestic Banks $b_D/T$ for countries with high collateral values and different domestic cost of default $c$. 
Table 1.1: Summary Statistics for, $\hat{\lambda}_R$, the Collateral Value of Sovereign Debts (2007-2014)

The proxy for the collateral value of sovereign debts, $\hat{\lambda}_R$, is defined as the ratio of private debts issued by domestic banks to their holdings of domestic sovereign debts. Private debts issued by domestic banks are obtained from Eurostat under the financial balance sheets of financial corporations (code: nasa.10_f_bs). Holdings of domestic sovereign debts by domestic banks are obtained from Merler and Pisani-Ferry (2012). Data are all annual.

<table>
<thead>
<tr>
<th>GIIPS Countries</th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Greece</td>
<td>0.102</td>
<td>0.063</td>
<td>0.041</td>
<td>0.206</td>
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<tr>
<td>Ireland</td>
<td>15.371</td>
<td>22.975</td>
<td>0.869</td>
<td>61.18</td>
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<td>Italy</td>
<td>2.314</td>
<td>0.780</td>
<td>1.315</td>
<td>3.383</td>
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<tr>
<td>Portugal</td>
<td>2.359</td>
<td>1.255</td>
<td>1.100</td>
<td>3.740</td>
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<td>Spain</td>
<td>1.958</td>
<td>0.759</td>
<td>0.99</td>
<td>3.06</td>
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<table>
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<th></th>
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</thead>
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<td>Belgium</td>
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<td>0.161</td>
<td>0.463</td>
<td>0.882</td>
</tr>
<tr>
<td>Finland</td>
<td>46.549</td>
<td>23.594</td>
<td>19.098</td>
<td>77.915</td>
</tr>
<tr>
<td>France</td>
<td>5.304</td>
<td>0.598</td>
<td>4.53</td>
<td>6.077</td>
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<tr>
<td>Germany</td>
<td>3.385</td>
<td>0.423</td>
<td>2.661</td>
<td>3.900</td>
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<tr>
<td>Netherlands</td>
<td>15.605</td>
<td>3.866</td>
<td>10.613</td>
<td>22.389</td>
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<tr>
<td>United Kingdom</td>
<td>19.328</td>
<td>22.903</td>
<td>4.639</td>
<td>78.508</td>
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Table 1.2: Regressions of collateral value of sovereign debt on creditor rights.

This table shows regressions of the collateral value of sovereign debt $\hat{\lambda}$ on creditor rights. The collateral value of sovereign debt, $\hat{\lambda}$, is measured as the ratio of private debts issued by domestic banks to their holdings of domestic sovereign debts. Credit Rights is the strength of legal rights index from the World Bank’s World Development Indicators. It ranges from 0 to 12, with higher scores indicating that these laws are better designed to expand access to credit. Standard errors are computed as indicated in the table. ***, **, and * indicates statistical significance at the 1%, 5%, and 10% level, respectively.

<table>
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<th>(1)</th>
<th>(2)</th>
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<tbody>
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<td></td>
<td>Collateral Value</td>
<td>Collateral Value</td>
<td>Collateral Value</td>
<td>Collateral Value</td>
</tr>
<tr>
<td>Creditor Right</td>
<td>0.469***</td>
<td>0.359**</td>
<td>0.474***</td>
<td>0.193***</td>
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<tr>
<td></td>
<td>(0.0751)</td>
<td>(0.171)</td>
<td>(0.0784)</td>
<td>(0.0681)</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.513***</td>
<td>-2.173**</td>
<td>-1.275</td>
<td>-0.0664</td>
</tr>
<tr>
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<td>(0.506)</td>
<td>(0.832)</td>
<td>(0.771)</td>
<td>(0.198)</td>
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<td>Country Fixed</td>
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<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Year Fixed</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
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<td>Robust</td>
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<tr>
<td>R2</td>
<td>0.332</td>
<td>0.870</td>
<td>0.345</td>
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<td>N</td>
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<td>75</td>
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Table 1.3: **Regressions debt ownership on collateral value of sovereign Debt and total debt-to-GDP ratio**

This table shows regressions of the collateral value of sovereign debt and total debt-to-GDP ratio on debt ownership. Credit Rights is a proxy for the collateral value. It is the strength of legal rights index from the World Bank’s World Development Indicators. It ranges from 0 to 12, with higher scores indicating that these laws are better designed to expand access to credit. Standard errors are computed as indicated in the table. ***, **, and * indicates statistical significance at the 1%, 5%, and 10% level, respectively.

<table>
<thead>
<tr>
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<th>(2)</th>
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<th>(4)</th>
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<td>Externally-held Debt-to-GDP</td>
<td>Domestically-held Debt-to-GDP</td>
<td>Externally-held Debt-to-GDP</td>
</tr>
<tr>
<td>Creditor Right</td>
<td>-1.247***</td>
<td>1.319**</td>
<td>-1.970**</td>
<td>3.956***</td>
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<tr>
<td></td>
<td>(0.491)</td>
<td>(0.646)</td>
<td>(0.979)</td>
<td>(1.352)</td>
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<tr>
<td>Debt-to-GDP</td>
<td>0.174***</td>
<td>0.672***</td>
<td>0.121*</td>
<td>0.865***</td>
</tr>
<tr>
<td></td>
<td>(0.0341)</td>
<td>(0.0486)</td>
<td>(0.0719)</td>
<td>(0.0986)</td>
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<tr>
<td>Creditor Rights × Debt-To-GDP</td>
<td>0.0107</td>
<td>-0.0388**</td>
<td>0.0126</td>
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<tr>
<td>Constant</td>
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<td>(2.360)</td>
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<td>Year Fixed</td>
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<tr>
<td>R2</td>
<td>0.399</td>
<td>0.770</td>
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Table 1.4: The Effect of Changes in Debt-to-GDP Ratio and the Collateral Value of Sovereign Debt on the Share of Domestically-held Debts.

This table shows the effect changes in debt-to-GDP Ratio and the collateral value of sovereign debt on the share of domestically-held debt. The sample spans between 2007 and 2014 and covers Belgium, Finland, France, Germany, Greece, Italy, Ireland, Netherlands, Portugal, Spain, the United Kingdom, and the United States. GIIPS is a dummy variable that equals 1 for Greece, Italy, Ireland, Portugal, and Spain. Debt-to-GDP ratio is instrumented by current account balance to GDP ratio. Real GDP growth is the annual-to-annual growth rate of nominal GDP divided by inflation. Collateral Value, measured as the ratio of private debts issued by domestic banks to their holdings of domestic sovereign debts, is instrumented by the strength of legal rights index from the World Bank’s World Development Indicators. All independent variables except GIIPS and Collateral Value are expressed in percentage points. The dependent variable, 5-Yr CDS, is the 5-Year CDS spread obtained from Bloomberg, expressed in basis points. All columns include time and country fixed effects. Standard errors (in parentheses) are corrected for autocorrelation. ***, **, and * indicates statistical significance at the 1%, 5%, and 10% level, respectively.

<table>
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<th>(1) Share of Domestically-held Debt</th>
<th>(2) Share of Domestically-held Debt</th>
<th>(3) Share of Domestically-held Debt</th>
<th>(4) Share of Domestically-held Debt</th>
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<tr>
<td>Debt-To-GDP</td>
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<td>0.129*** (0.0406)</td>
<td>0.179*** (0.0469)</td>
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<tr>
<td>Collateral Value</td>
<td>-4.313* (2.432)</td>
<td>-4.071* (2.292)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt-To-GDP x Collateral Value</td>
<td></td>
<td></td>
<td>-0.090* (0.0493)</td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>0.351* (0.210)</td>
<td>0.367 (0.226)</td>
<td>0.436* (0.226)</td>
<td>0.436* (0.226)</td>
</tr>
<tr>
<td>Real GDP growth</td>
<td>-0.000318 (0.0131)</td>
<td>-0.000870 (0.0141)</td>
<td>-0.00106 (0.0141)</td>
<td>0.00770 (0.0149)</td>
</tr>
<tr>
<td>Curr-To-GDP</td>
<td>0.810 (0.679)</td>
<td>1.431** (0.686)</td>
<td>0.769 (0.708)</td>
<td>-0.510 (0.986)</td>
</tr>
<tr>
<td>Constant</td>
<td>7.621*** (0.897)</td>
<td>21.04*** (0.737)</td>
<td>10.65*** (1.391)</td>
<td>10.04*** (1.395)</td>
</tr>
<tr>
<td>Country Fixed</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year Fixed</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R2</td>
<td>0.162</td>
<td>0.114</td>
<td>0.244</td>
<td>0.256</td>
</tr>
<tr>
<td>N</td>
<td>87</td>
<td>82</td>
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</table>
Table 1.5: Default Risk and Changes in Externally- and Domestically-held Debt-to-GDP Ratio

This table shows the effect of total, externally- and domestically-held debt-to-GDP ratio on default risk between 2007 and 2014. The sample covers Belgium, Finland, France, Germany, Greece, Italy, Ireland, Netherlands, Portugal, Spain, the United Kingdom, and the United States. GIIPS is a dummy variable that equals 1 for Greece, Italy, Ireland, Portugal, and Spain. Both externally-held and domestically-held debt-to-GDP ratio are instrumented jointly by the total debt-to-GDP ratio and the strength of legal rights index from the World Bank’s World Development Indicators. Real GDP growth is the annual-to-annual growth rate of nominal GDP divided by inflation. Curr-To-GDP is the current account balance to GDP ratio. All independent variables except GIIPS are expressed in percentage points. The dependent variable, 5-Yr CDS, is the 5-Year CDS spread obtained from Bloomberg, expressed in basis points. All columns include time and country fixed effects. Standard errors (in parentheses) are corrected for autocorrelation. ***, **, and * indicates statistical significance at the 1%, 5%, and 10% level, respectively.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
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<tr>
<td>5-Yr CDS</td>
<td>4.815***</td>
<td>4.709***</td>
<td>7.580***</td>
<td>10.85***</td>
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<td></td>
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<tr>
<td>(0.766)</td>
<td>(0.778)</td>
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<tr>
<td>Externally-held</td>
<td>10.90***</td>
<td>7.580***</td>
<td>10.85***</td>
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<td>(2.209)</td>
<td>(1.221)</td>
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<tr>
<td>Domestically-held</td>
<td>-11.62*</td>
<td>15.30***</td>
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<td>-12.23*</td>
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<tr>
<td>(6.923)</td>
<td>(4.402)</td>
<td>(6.592)</td>
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<tr>
<td>Inflation</td>
<td>21.55***</td>
<td>22.11***</td>
<td>21.85***</td>
<td>22.63***</td>
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<td></td>
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<tr>
<td>Real GDP growth</td>
<td>-0.282</td>
<td>-0.265</td>
<td>-0.341</td>
<td>-0.226</td>
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<tr>
<td>(0.393)</td>
<td>(0.401)</td>
<td>(0.450)</td>
<td>(0.399)</td>
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<td>Curr-To-GDP</td>
<td>1.966</td>
<td>4.031</td>
<td>15.78</td>
<td>7.917</td>
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<tr>
<td>(15.56)</td>
<td>(15.79)</td>
<td>(18.22)</td>
<td>(15.56)</td>
<td></td>
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<tr>
<td>Constant</td>
<td>-242.4***</td>
<td>-160.0***</td>
<td>-279.8***</td>
<td>-220.6***</td>
<td>-123.2***</td>
<td>-196.0***</td>
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<tr>
<td>(37.33)</td>
<td>(33.18)</td>
<td>(36.50)</td>
<td>(32.19)</td>
<td>(35.59)</td>
<td>(33.92)</td>
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<td>Yes</td>
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<tr>
<td>Year Fixed</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R2</td>
<td>0.345</td>
<td>0.401</td>
<td>0.434</td>
<td>0.464</td>
<td>0.287</td>
<td>0.493</td>
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<tr>
<td>N</td>
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<td>87</td>
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</table>
Table 1.6: **Default risk and changes in total debt-to-GDP Ratio**

This table shows the effect of total debt-to-GDP ratio on default risk between 2007 and 2014. The sample covers Belgium, Finland, France, Germany, Greece, Italy, Ireland, Netherlands, Portugal, Spain, the United Kingdom, and the United States. GIIPS is a dummy variable that equals 1 for Greece, Italy, Ireland, Portugal, and Spain. Real GDP growth is the annual growth rate of nominal GDP divided by inflation. Curr-To-GDP is the current account balance to GDP ratio. Collateral Value, measured as the ratio of private debts issued by domestic banks to their holdings of domestic sovereign debts, is instrumented by the strength of legal rights index (Creditor Rights) from the World Bank’s World Development Indicators. Therefore, Debt-to-GDP×Collateral Value is instrumented by Debt-to-GDP×Creditor Rights. All independent variables except GIIPS and Collateral Value are expressed in percentage points. The dependent variable, 5-Yr CDS, is the 5-Year CDS spread obtained from Bloomberg, expressed in basis points. All columns include time and country fixed effects. Standard errors (in parentheses) are corrected for autocorrelation. ***, **, and * indicates statistical significance at the 1%, 5%, and 10% level, respectively.

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<th>(3) 5-Yr CDS</th>
<th>(4) 5-Yr CDS</th>
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<td>Debt-To-GDP</td>
<td>4.028***</td>
<td>-0.797</td>
<td>5.729***</td>
<td>3.882***</td>
<td>-0.918</td>
<td>5.394***</td>
</tr>
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<td></td>
<td>(0.908)</td>
<td>(1.188)</td>
<td>(1.214)</td>
<td>(0.962)</td>
<td>(1.280)</td>
<td>(1.234)</td>
</tr>
<tr>
<td>Debt-To-GDP × GIIPS</td>
<td></td>
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<td></td>
<td>5.568***</td>
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<td>5.967***</td>
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<td>(1.037)</td>
<td></td>
<td>(1.205)</td>
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<td>Debt-To-GDP × Collateral Value</td>
<td>-1.430**</td>
<td>5.687***</td>
<td>-1.414*</td>
<td>(0.710)</td>
<td>(0.717)</td>
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<td>Inflation</td>
<td>0.0550</td>
<td>4.078</td>
<td>-1.609</td>
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<tr>
<td></td>
<td>(8.639)</td>
<td>(7.556)</td>
<td>(8.774)</td>
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<tr>
<td>Real GDP growth</td>
<td>-0.242</td>
<td>-0.0872</td>
<td>-0.322</td>
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<tr>
<td></td>
<td>(0.384)</td>
<td>(0.335)</td>
<td>(0.381)</td>
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<tr>
<td>Curr-To-GDP</td>
<td>15.38</td>
<td>-6.104</td>
<td>20.33*</td>
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<tr>
<td>Year Fixed</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>R2</td>
<td>0.735</td>
<td>0.808</td>
<td>0.748</td>
<td>0.745</td>
<td>0.810</td>
<td>0.763</td>
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<td>N</td>
<td>100</td>
<td>100</td>
<td>95</td>
<td>99</td>
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Chapter 2

Maturity Structure of Government Debt, Asset Markets, and the Real Economy

2.1 Introduction

Stabilization policies adopted by the Federal Reserve (Fed) and the U.S. government in response to the recent financial crisis have led to: 1) a surge of public debt, 2) increase in debt-maturity structure, and 3) exceedingly low short-term interest rates. Between 2007 and 2011, total U.S. government debt rose from 63.8% to 97.8% of gross domestic product. Perhaps more strikingly, as shown in Figure 2.1, the average maturity of outstanding debt also significantly and rapidly increased, from 52 months in October 2008 to 73 months in December 2012, as the Treasury has been methodically issuing long-term debt to reduce interest-rate risk.

There are worries that such dramatic changes will lead to expected long-term inflation (Fisher, 2011) and be “more likely to raise inflation than it is to measurably raise growth” [3]. At the same time, the effective Federal Funds rate dropped, within one-and-a-half year, from the high of 5.26% in July 2007 to 0.16% in December 2008, and it has since stayed close to zero. Conventional monetary policies are no longer effective because nominal short-term interest

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1 Data compiled from the monthly CRSP Treasury Database.
2 Minutes of the Meeting of the Treasury Borrowing Advisory Committee Of the Securities Industry and Financial Markets Association on February 5, 2013.
3 Jeffrey Lacker’s interview with reporters after his speech on “Economics After the Crisis: Reflections on a Return to Madison” on October 11, 2011.
rate is almost at the zero-lower bound. Therefore, the Fed have resorted to unconventional policies such as quantitative easing (QE) and the maturity extension program.

Nevertheless, the joint economic impacts of these policies are not clear. For example, Feldstein (2009) argues that whether a fiscal deficit is inflationary depends on monetary conditions. If the fiscal deficit is not accompanied by an increasing money supply, short-term interest rate will rise but there will not be a sustained rise in inflation. Otherwise, when large budget deficit combines with rapidly increasing money supply (from quantitative easing), as it happens in the recent financial crisis, this will necessarily lead to high inflation in the future. In a rational expectation framework, this would implies an increase in long-term nominal interest rate. Nevertheless, the 30-Year Treasury constant maturity rate remains at its all-time low at around 3%, as of December 2012. This is not the only puzzle. The maturity extension program, also known as the “Operation Twist”, was launched in October 2011 and expanded through 2012 to swap, in total, over $600 billion of short-term Treasury notes for the same amount of long-term Treasury bonds. The program increases the supply of short-term Treasury bills and reduces the supply of long-term Treasury bonds available to secondary markets, effectively countering the actions from the Treasury. If the ultimate objective of both the Fed and the Treasury is to mitigate the impact of the recent financial crisis and strengthen the pace of U.S. economic growth, it is unclear why they are conducting these seemingly contradictory policies.

This paper answers these questions by studying the interaction between monetary and fiscal policies, and how changes in fiscal policies, such as the level of debt and debt maturity composition, would affect inflation, the real economy and asset prices. To summarize, this paper provides a theoretical framework that argues for policy coordination between the monetary and the fiscal authorities, and it also argues that increasing debt maturity structure and raising government debt simultaneously would unambiguously increase future inflation. This paper also provides empirical evidence to its theoretical results. The theoretical framework is a three-period representative agent model, with rational expectation and access to a complete and frictionless financial market,

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4These two agencies have different mandates. The mandate of the Fed is stated in the 1977 Federal Reserve Act, which says the Fed should promote the goals of maximum employment, stable prices and moderate long-term interest rates. On the other hand, the Treasury focuses on the issuance of securities to the private market at the lowest cost over time, as clearly stated in a [speech] by Mary J. Miller, the Assistant Secretary for Financial Markets, on October 5, 2010.
that features both fiscal and monetary authorities in a unified framework. This model differs from traditional equilibrium models in two significant ways.

First, the monetary policies are modelled as open market operations. Monetary authority controls interest rates (mainly) by buying and selling bonds through open markets. In conventional macroeconomic models, such as [Goodfriend and King (1997), Woodford (2003), and Clarida et al. (1999)], monetary policy is represented by a rule for setting the nominal rate of interest based on inflation, output gap, and etc, also known as the Taylor (1993) rule. While the rule seems to be successfully emulating the central bank’s practice, it does not accurately describe the open market nature of the central bank’s transactions. Perhaps more importantly, such rule directly pins down the quantity of government debt needed to balance the government budget constraint, and therefore prevents us from studying the impact of QEs on asset prices and the real economy in an equilibrium setting. The model in this paper explicitly addresses this issue by modelling monetary policies as open market operations. An expansionary (contractionary) monetary policy is represented by an increase (decrease) in short-term debt holding by the monetary authority. The monetary authority influences the interest rate by manipulating the net supply of short-term debts. The model correctly replicates that an expansionary policy would reduce short-term interest rates and induce inflation. This modelling choice also allows the possibility to study the impact of QEs on asset prices and the real economy. QE is an attempt by the Fed to influence interest rates by directly manipulating the net (market) supply of various financial assets, which can be readily extended and incorporated into this model.

Second, long-term government debt is modelled to study the effect of changes in maturity composition (and therefore the impact of the maturity extension program) on both the real and nominal economy. This feature is first studied by Cochrane (2001) and Woodford (1995), which enables the fiscal authority to trade-off between current and future inflation. None of these studies, however, allows fluctuations in the term structure of real interest rates because households are assumed to be risk-neutral. Therefore, the real interest rates must be the inverse of the subjective discount rate, regardless of changes in fiscal policies. In constrast, households in my model generate utility via their holdings on the real balance of short-term debt, which links the quantity of debt and inflation to the real interest rates.

There are three main results. First, inflation and the term structure of interest rates are determined jointly by monetary and fiscal policies. Neither monetary nor fiscal policies alone is sufficient to determine the price level and debt prices. An important implication of my model is that coordination
between monetary and fiscal policies is crucial to successfully regulating the economy, as actions by monetary and fiscal authorities, if act independently with different objectives, might have unintended consequences on the overall economy. The intuition is simple. In my model, the net-supply of short-term debt, controlled by the monetary authority, determines bond prices, which in turn enters the budget constraints of the fiscal authority. Therefore, fiscal and monetary policies are inherently linked together via the interest rate channel. This avoids what Sargent and Wallace (1981) calls the “game of chicken”, where the first mover (i.e. active policies) constraints the policy options of the follower (i.e. passive policies). In my model, there is no need to distinguish between “active” and “passive” policies. Monetary authority and fiscal authority can independently set their policies. Bond prices and the price level will adjust to satisfy the intertemporal budget constraint. This feature deviates substantially from current literature, which are divided into two groups. In the first group, such as Sargent and Wallace (1981), Sims (1987), and Aiyagari and Gertler (1985), monetary policy is active and fiscal policy is passive. Fiscal disturbance do not influence equilibrium prices, interest rates, or real balances. Monetary policies are sufficient for determining the price levels. In the second group, such as Leeper (1991), Cochrane (2001), and Woodford (1995), fiscal policy is active and monetary policy is passive. Prices depend on the aggregate supply of government liability and the nominal interest rate depend on the ratio of money to debt. Money supply is forced to be adjusted so that the intertemporal budget constraints are satisfied.

Second, I show theoretically that fiscal instruments, such as surplus, the level and maturity composition of debt, do have important implications on inflation, nominal and real interest rates. As in Cochrane (2001) and Woodford (1995), a shift in the maturity structure from short-term to long-term debt slows down inflation today, yet, at the cost of raising inflation in the future. In addition, it lowers the real short rate. Additionally, an increase in the level of government debt unambiguously raises inflation today. Different from Cochrane (2001), who assumes risk-neutrality in consumption and no supply effect, such policies can also affect both real and nominal bond prices in my model. For example, an elevated level of government debt implies an increase in real (and therefore nominal) short-term interest rates, because it affects

\footnote{This seems to be supported by this remark from the U.S. Treasury, who said “… [Fed’s] decision to purchase Treasuries in the secondary market does not, and will not, impact [their] debt management strategy. As debt managers, [they] are focused on the issuance of securities to the private market at the lowest cost over time. Fed monetary policy decisions are independent of that calculus.”}
The price level and therefore the real balance of the short-term debt held by the representative household. Since the real balance of the short-term debt enters into the household's utility, the change would also affect the marginal rate of substitution, which implies both real and nominal interest rates would be affected. Therefore, an increase in maturity structure of government debt together with an increase in government debt would have an ambiguous impact on inflation today and the real short-term interest rates, but future inflation would unambiguously increase.

Third, I provide empirical evidence that changes in the maturity structure of the Treasury do have both economic and statistically significant impact on various measures of future inflation, consistent with Cochrane (2001). One percentage point increase in the fraction of short-term Treasury debt issued over total debt is associated with 0.25% increases in future inflation. An one percent increase in the fraction of long-term Treasury debt issued over total debt is associated with 0.18% decreases in future inflation. The result holds after controlling for usual predictors for future inflation, such as the Fed Funds rate, current inflation, and the total debt-to-GDP ratio. The result is robust to different measures of future inflation, such as the expected inflation measure from the Survey of Consumers conducted by the University of Michigan, and also to different horizons. Evidence shows that changes in the short-end of the maturity structure has the most explanatory power over short- and medium-horizons, whereas changes in the long-end of the maturity structure has the most explanatory power over long-horizons.

### 2.2 Literature Review

This paper is closely related to three strands of research: 1) the interaction between fiscal and monetary policies, 2) economic impact of fiscal debt and debt maturity management, and 3) effectiveness of unconventional monetary policies on the real economy.

The debate on the interaction between fiscal and monetary policies focuses on the following question: What provides the nominal anchor in a monetary economy? Traditional monetarists, such as Friedman (1970), Friedman and Schwartz (1963), McCallum and Nelson (2005), argues that monetary policies alone are sufficient to determine the price level. Meltzer (1984) even claims “in principal, monetary policy can restore price stability even if the government continues to run large deficits.” The core of their arguments lies on the equation of exchange, $M_tV_t = P_tY_t$, where $M_t$ is the amount of money, $V_t$ is
the velocity, \( P_t \) is the price level, and \( Y_t \) is the real output. By assuming a functional form on the velocity process, \( V_t \), and abstracting from changes in real interest rates, it can be shown that inflation is determined entirely by the growth rate of money. Hence, once the monetary authority sets the level of money stock, the fiscal authority is forced to adjust its borrowing and primary surplus. This is called the “monetary dominance”, in the sense of [Leeper(1991)]. Proponents of the *fiscal theory of price level*, such as [Sims (1994), Woodford (1994), and Woodford (1998)] argues, however, that the intertemporal government budget constraints are sufficient to determine the price level. The monetary policy instead has to adjust the level of monetary stock to satisfy the equation of exchange. [Sargent and Wallace (1981)] characterized the interaction between monetary and fiscal policy in terms of game theory and leadership, or in other words, who gets to go first, and the other has to comply. In reality, however, for the U.S., the Fed and the Treasury seems to act independently. My paper reflects this by arguing that the monetary authority and fiscal authority jointly determine the price level (together with the term structure of interest rates), and therefore avoids “the game of chicken”. The intuition is clear: the net-supply of short-term debts, controlled by the monetary authority, determines bond prices, which in turns enters the budget constraints of the fiscal authority. Hence the price levels and term structure are inherently linked together via the “interest rate channel.”

What are the effects of changes in government debt and debt maturity structure on the asset markets and the real economy? For debt maturity management, the focus of the literature has almost been exclusively on the relationship between maturity composition of government debt and the term structure of interest rate. Written after the first Operation Twist procedure was implemented in 1961 by the Fed, [Modigliani and Sutch (1967)] found that, even in a period in which the size of government debt is large, responsiveness of the nominal term structure to variations in age composition of government debt was at best weak. [Wallace (1967)], however, pointed out that [Modigliani and Sutch (1967)] did not allow long-term interest rates to depend on changes in maturity composition, and this would likely bias their estimates downwards. Indeed, [Okun (1963)] and [Scott (1965)] both reported that changes in the maturity of composition of the debt affect the term structure, and short-term interest rates are more sensitive to such changes than long-term rates. More recently, [Greenwood and Vayanos (2012)] found that maturity-weighted-debt-to-GDP ratio is positively related to bond yields and future returns after controlling for the short rate but, contrary to [Okun (1963) and Scott (1965)], the effect is larger on long-term interest rates. [Hamilton and Wu (2012)] used
a model of risk-averse arbitrageurs to develop measures of how the maturity structure of debt held by the public would affect the pricing of level, slope, and curvature term structure risk. [Krishnamurthy and Vissing-Jorgensen (2012)] found that changes in the supply of short (long) Treasury debts have significant impact on a variety of yield spreads that can be attributed to safety, liquidity, and default premiums.

Despite the large body of empirical work on this topic, relatively little theoretical work has been done. The empirical work of [Modigliani and Sutch (1967)], [Greenwood and Vayanos (2012)], and [Hamilton and Wu (2012)] rely on the preferred habitat theory, which assumes agents have different preferences on various assets. For example, in term structure, the theory assumes different bond investors prefer one maturity length over another, and they are only willing to purchase bonds outside of their maturity preference if a risk premium for the maturity range is available. Any change in maturity composition of government debt changes the relative bond supply and therefore the equilibrium bond premiums. These models, however, have an important drawback, i.e. there is no government budget constraint. Why is the presence of government budget constraint important? To see this, suppose the government needs to finance a fixed amount of expenditure through borrowing. The amount of (both current and future) bond supply clearly depends endogenously on (current and future) bond prices. Therefore, the supply of government debt should be exogenously specified, as in the aforementioned models. The higher the bond price, the fewer bonds are needed to finance the expenditure, and vice versa. Other models, such as, [Cochrane (2001)], incorporates the government budget constraints. But [Cochrane (2001)] is unable to shed any insight on how changes in maturity structure affect the real bond prices because his model assumes risk-neutrality, and therefore real bond prices are just trivially the inverse of the subjective discount factor, which is addressed by the model in this paper. [Krishnamurthy and Vissing-Jorgensen (2012)] developed a model that assumes agents derive utility from holding government bonds, but they do not incorporate the government budget constraints.

Lastly, this paper is also related to the effectiveness of unconventional monetary policies on the real economy. There is ample empirical evidence that policies such as QE has significant impact on long-term interest rates. [Gagnon et al. (2010)] use event study to analyze QE1 and documents large reductions in interest rates on dates associated with positive QE announcements. [Swanson et al. (2011)] presents evidence from the 1961 Operation Twist. [Krishnamurthy and Vissing-Jorgensen (2012)] uses variations in long-term Treasury supply to determine causal effects from supply to interest rates. [Vissing-Jorgensen and]
Krishnamurthy (2011) documents various channels through which unconventional monetary policies might affect interest rates. The modelling choice in my paper can possibility provide a theoretical framework for these empirical studies. In my model, an expansionary (contractionary) monetary policy is represented by an increase (decrease) in short-term debt holding by the monetary authority. The monetary authority influence the interest rate by manipulating the net supply of short-term debts. My model could be extended to study the impact of QEs on asset prices and the real economy in an equilibrium setting. So far, my model correctly replicates that an expansionary policy would reduce short-term interest rates and induce inflation.

2.3 Model

The model is based on Woodford (1998), with the addition of long-term debt and the real balance of short-term debt enters the household utility function. There are three periods: 0, 1, and 2. Assume there exists a representative household, with rational expectations and access to a complete and frictionless financial market, who receives fixed initial endowments, $y_t$, and pays lump-sum taxes, $\tau_t$, at $t = 0, 1, 2$. In each period, it can either consume or invest in short- or long-term government bonds. There are two types of bonds available at $t = 0$: a long-term bond, $B_{02}$, and a short-term bond, $B_{01}$. At $t = 1$, there is only short-term bond, $B_{12}$. Without loss of generality, all bonds are zero-coupon bonds. The household derives utilities both from consumptions and holding a real balance of short-term bonds and money. There is no uncertainty. The utility of this representative household is therefore

$$U = u(c_0) + v(B_{01}^D/P_0) + \eta(M_0/P_0) + \beta(u(c_1) + v((B_{12}^D + B_{02}^D)/P_1) + \eta(M_1/P_1)) + \beta^2 u(c_2)$$

where $c_t$ denotes real private consumption in period $t$, $u(\cdot)$ is the utility over consumption, $v(\cdot)$ is the convenience utility with $v'(\cdot) > 0$ and $v''(\cdot) < 0$, $B_{ij}^D$ is the quantity demanded for bonds (in face value) issue at $t = i$ maturing at $t = j$, $P_t$ is the price level, $M_t$ is the nominal amount of money held by the household at $t = i$, and $\beta$ is the subjective discount factor. The budget constraints for the household are then:

$$P_0c_0 + Q_{01}B_{01}^D + Q_{02}B_{02}^D + M_0 = P_0(y_0 - \tau_0) \quad (2.1)$$
$$P_1c_1 + Q_{12}B_{12}^D + M_1 = P_1(y_1 - \tau_1) + B_{01}^D + M_0 \quad (2.2)$$
$$P_2c_2 + M_2 = P_2(y_2 - \tau_2) + B_{12}^D + B_{02}^D + M_1 \quad (2.3)$$
where $P_t$ is the price level at period $t$, $\tau_t$ is the amount of lump-sum tax, and $Q_{ij}$ is the nominal price of the bond issued at $t = i$ maturing at $t = j$. The FOC conditions with respect to $B_{ij}$ and $M_i$ give the Euler equations:

\[
Q_{01} = \left[ v'(B_{01}^D/P_0) + \beta u'(c_1) \left( \frac{P_0}{P_1} \right) \right] / u'(c_0) \tag{2.4}
\]

\[
Q_{02} = \left[ \beta v' \left[ (B_{12}^D + B_{02}^D)/P_1 \right] \left( \frac{P_0}{P_1} \right) + \beta^2 u'(c_2) \left( \frac{P_0}{P_2} \right) \right] / u'(c_0) \tag{2.5}
\]

\[
Q_{12} = \left[ v' \left[ (B_{12}^D + B_{02}^D)/P_1 \right] + \beta u'(c_2) \left( \frac{P_1}{P_2} \right) \right] / u'(c_1) \tag{2.6}
\]

\[
1 = \left[ \eta'(M_0/P_0) + \beta u'(c_1) \left( \frac{P_0}{P_1} \right) \right] / u'(c_0) \tag{2.7}
\]

\[
1 = \left[ \eta'(M_1/P_1) + \beta u'(c_2) \left( \frac{P_1}{P_2} \right) \right] / u'(c_1) \tag{2.8}
\]

where $Q_{ij}$ is the price of bond $B_{ij}$.

There are also a monetary authority and a fiscal authority. The monetary authority influences both the price levels and interest rates via open market operations, i.e. buying and selling short-term bonds, at both $t = 0$ and $t = 1$. Denote the face value of short-term bonds held by the monetary authority by $B_{01}^M$ and $B_{12}^M$. The monetary authority’s budget constraints are:

\[
M_0 = Q_{01}B_{01}^M \tag{2.9}
\]

\[
M_1 - M_0 + B_{01}^M = Q_{12}B_{12}^M \tag{2.10}
\]

\[
M_2 + B_{12}^M - M_1 = 0 \tag{2.11}
\]

In other words, the monetary authority needs to issue enough money in each period to purchase its target quantities of short-term bonds.\(^6\)

On the other hand, the fiscal authority issues nominal debt (in face value), $L_t$, at both $t = 0$ and $t = 1$, to finance an exogenous amount of government expenditure, $g_t$. Let $\alpha$ be the fraction of long-term debt issued at $t = 0$. It follows that the amount of long-term debt and short-term debt issued at $t = 0$ are, respectively, $\alpha L_0$ and $(1-\alpha) L_0$, where $0 \leq \alpha \leq 1$. The fiscal authority can only issue short-term debt at $t = 1$. The fiscal authority’s budget constraints

\[^6\]One could also extend the model so the monetary authority can purchase long-term bonds, in which case the analysis would be very similar. The central point remains – both inflation and real quantities are jointly determined by both the monetary and fiscal authority.
are:

\[ P_2(\tau_2 - g_2) = L_1 + \alpha L_0 \]  
\[ P_1(\tau_1 - g_1) + Q_{12} = (1 - \alpha) L_0 \]  
\[ P_0(\tau_0 - g_0) + Q_{01}(1 - \alpha) L_0 + Q_{02} \alpha L_0 = B_{-1} \]

where \( B_{-1} \) is the initial amount of outstanding debt due at \( t = 0 \).

**Equilibrium**

An equilibrium is defined as: 1) the price-level sequence \( \{P_0, P_1, P_2\} \), 2) the consumption sequence \( \{c_0, c_1, c_2\} \), 3) the nominal bond-price sequence \( \{Q_{01}, Q_{02}, Q_{12}\} \), and 4) money supply \( \{M_0, M_1, M_2\} \), such that: a) representative household FOCs, b) fiscal authority budget constraints, c) monetary authority budget constraints, and d) bond market clearing conditions are satisfied. The fiscal policies \( \{L_0, L_1, \alpha\} \) and monetary policy \( \{B_{01}^M, B_{12}^M\} \) are taken as given by the household. The bond market clearing conditions are:

\[ B_{01}^D = (1 - \alpha) L_0 - B_{01}^M \]  
\[ B_{02}^D = \alpha L_0 \]  
\[ B_{12}^D = L_1 - B_{12}^M \]

There exists a unique equilibrium because the system is just identified. To simplify the analysis and obtain closed-form solutions, I assume the representative household is risk-neutral on real consumption and the following parameter values:

1. Constant primary surplus, i.e. \( \tau_i - g_i = s \), for \( i = 1, 2, 3 \),
2. The government rolls over the short-term debt, i.e. \( L_0 = L \), and \( L_1 = (1 - \alpha) L \).
3. Monetary policy is conducted only at \( t = 0 \), i.e. \( B_{01}^M = B \), \( B_{12}^M = 0 \),
4. The initial amount of outstanding debt, \( B_{-1} \), is assumed to be 1.
5. \( \eta \left( M_t / P_t \right) = \left( 1 - \frac{\beta}{\pi_{t+1}} \right) \frac{M_t}{P_t} \), where \( \pi_{t+1} = P_{t+1} / P_t \) is the inflation rate at \( t + 1 \).
In particular, assumption 5) and risk-neutrality on real consumption implies that equation (2.7) and (2.8) are always satisfied. Therefore, money supply have no impact on inflation and the term structure of interest rates, so one can focus on the impact of changes in government debt supply and debt maturity structure on these variables. The fiscal authority controls the level of total debt ($L$) and debt maturity structure ($\alpha$); The monetary authority controls its holding of short-term debt ($B$).

**Benchmark:** $v(\cdot) = 0$, i.e. no convenience utility

In this case, the representative household derives no utility from holding short-term bonds, which is assumed by Woodford (1998) and Cochrane (2001). The equilibrium solution is:

$$
\pi_1 = \frac{(1 - \alpha)(1 + \beta + \beta^2)L}{(1 + (1 - \alpha)\beta)}
$$

$$
\pi_2 = \beta + \frac{1}{1 - \alpha}
$$

$$
r^s_{01} = \frac{(1 - \alpha)(1 + \beta + \beta^2)L}{\beta(1 + \beta(1 - \alpha))}
$$

$$
r^s_{02} = \left[1 + \frac{1 + \beta}{\beta^2}\right]L
$$

$$
r^s_{12} = \left[1 + \frac{1}{\beta(1 - \alpha)}\right]
$$

$$
r_{01} = \frac{1}{\beta}
$$

$$
r_{02} = \frac{1}{\beta^2}
$$

$$
r_{12} = \frac{1}{\beta}
$$

(2.18)

where $r^s_{ij}$ are the (gross) nominal interest rates, and $r_{ij}$ are the (gross) real interest rates. First, neither monetary nor fiscal policies has any impact on the real economy. The real interest rates are solely determined by the subjective discount factor, $\beta$, as expected. Fiscal policies control inflation and therefore the nominal interest rates. In other words, the fiscal theory of price level,

---

\footnote{The result holds if the representative households is risk-averse on real consumption, in which case only the marginal rate of substitution matters.}
developed by Leeper (1991), Sims (1994), and Woodford (1998) with one-period debt and generalized by Cochrane (2001) to long-term debt, holds. An increase in the level of government debt \( (L) \) increases inflation \( (\pi_1) \) between \( t = 0 \) and \( t = 1 \). According to the theory, the price level in \( t = 1 \) adjusts to ensure the associated government budget constraint is satisfied. In other words, the ratio between nominal debt and price level must be equal to the present value of real primary surplus. Hence an increase in the level of nominal debt \( (L) \) would increase the price level at \( t = 1 \). Because nominal interest rates \( (r_{ij}^S) \) equals the corresponding real interest rates adjusted for the price levels, \( r_{ij} \times \pi_i / \pi_j \), nominal interest rates also increase by the same amount. As demonstrated by Cochrane (2001), in the presence of long-term debts, maturity management matters. An increase in long-term debt \( (\alpha) \) lowers tomorrow’s inflation \( (\pi_1) \) at the cost of raising future inflation \( (\pi_2) \). As such, this lowers today’s nominal short rate \( (r_{01}^S) \) while increasing tomorrow’s nominal short rate \( (r_{12}^S) \). Interestingly, the size of primary surplus, \( s \), plays no role in the equilibrium outcomes. In this frictionless economy, monetary policies plays no role in either real or nominal interest rates.

**With convenience utility:** \( v(\cdot) = \log(\cdot) \)

Next, I investigate the case where the representative household derives utility from holding short-term bonds. In contrast to the benchmark case, both monetary and fiscal policies now have significant impact on the nominal as well as the real economy.

**Real short-term interest rate,** \( r_{01} \)

One can obtain the equilibrium short-term real interest rate, \( r_{01} \), in closed-form.

**Proposition 6.** Define \( \theta = (1 + \beta(1 - \alpha))s + (1 - \alpha) \), where \( s \) is the primary surplus. The equilibrium short-term real interest rate, \( r_{01} \), is

\[
r_{01} = \frac{1}{\beta} \left( 1 - \frac{(1 - \alpha)L}{\beta \theta L^{NS} + (1 - \alpha)L} \right)
\]

(2.19)

where \( \beta \) is the subjective discount rate, \( \alpha \) is the fraction of long-term debt issued by the fiscal authority, \( L \) is the total debt (in face value), \( B \) is the amount of short-term debt held by the monetary authority, and \( L^{NS} = (1 - \alpha)L - B \) is the net supply of short-term debt. Furthermore, \( r_{01} \) increases if either \( L \) or \( \alpha \) increases or \( B \) declines.
Proof. See Appendix B.1

It is clear that the short-term real interest rate is jointly determined by both the monetary and the fiscal authority. The real short-term interest rate, \( r_{01} \), is no longer \( 1/\beta \) because the marginal rate of substitution is now affected by the representative household’s holding the real balance of short-term bond via \( v(\cdot) \). There are two channels through which policies can affect \( r_{01} \).

First, an expansionary monetary policy (i.e. increase in \( B \)) via open market operations would lead to a decrease in the net supply of short-term bonds \( L^\text{NS} \), increasing the representative household’s marginal convenience utility, and therefore decreasing the real short-term interest rates. Second, the fiscal authority can either extend the maturity of its debt (i.e. increase in \( \alpha \)) or decrease its total borrowing (i.e. decrease \( L \)). To see the intuition, recall that \( r_{01} \) depends positively on \( B^D_{01} \) and \( \pi_1 \). When \( \alpha \) increases, this implies a decline in \( B \), and from our discussion in the previous section about the fiscal theory of the price level, \( \pi_1 \) must also decline. Therefore, the aggregate impact on the real short-term interest rate (\( r_{01} \)) is negative. The logic for the impact of changes in total debt (holding maturity structure constant) on \( r_{01} \) is exactly the same.

What about the impact of changes in fiscal policies? The effect of changes in debt maturity structure (\( \alpha \)), holding the level of debt (\( L \)) constant, is shown on the left panel of figure 2.3. Assume the primary surplus (\( s \)) equals 0.2, the subjective discount rate (\( \beta \)) equals 0.96, the level of total debt (\( L \)) equals 0.5, and the amount of debt held by the monetary authority (\( B \)) is assumed to be 0.1. When the fiscal authority issues more long-term debt (\( \alpha \) increases), real short-term interest rate declines significantly because the real balance of short-term debt held by the representative household decreases.

In contrast, changes in the level of total debt \( L \) only have a relatively modest effect on the real short-term interest rate, as shown by figure 2.4. Why? Intuitively, interest rate must increase to induce the household to hold the extra debt. Equation (2.5) suggests, however, that the real interest rate also depends inversely on the price level in period 1, which increases as the level of debt increases. Consequently, these two forces cancel against each other, and therefore the real interest rate is barely affected by the debt level. This is graphically shown, in terms of demand and supply, by the right panel of figure 2.4. Assume the primary surplus (\( s \)) equals 0.2, the subjective discount rate (\( \beta \)) equals 0.96, the debt maturity structure (\( \alpha \)) equals 0.2, and the amount of debt held by the monetary authority (\( B \)) is assumed to be 0.1. An increase in debt increase the overall supply of short-term bond, which shifts the supply
curve to the right. This should decrease (increase) the real price (return) of short-term bonds. At the same time, however, demand for such bonds also increase due to an elevated level of inflation. Hence, the overall effect on the real short-rate is modest.

Inflation between \( t=0 \) and \( t=1 \), \( \pi_1 \)

One can also obtain the equilibrium inflation rate between \( t=0 \) and \( t=1 \), \( \pi_1 \), in closed-form.

**Proposition 7.** Define \( \theta = (1 + \beta(1 - \alpha))s + (1 - \alpha) \), where \( s \) is the primary surplus. The equilibrium inflation between \( t=0 \) and \( t=1 \), \( \pi_1 \), is

\[
\pi_1 = \frac{(1 - \alpha)(1 + \beta + \beta^2)L}{1 + (1 - \alpha)(\beta + \frac{1}{s})} + \frac{(1 - \alpha)L + \beta L^{NS}}{\theta L^{NS}}(1 - \alpha)L
\]

(2.20)

where \( \beta \) is the subjective discount rate, \( \alpha \) is the fraction of long-term debt issued by the fiscal authority, \( L \) is the total debt (in face value), \( B \) is the amount of short-term debt held by the monetary authority, and \( L^{NS} = (1 - \alpha)L - B \) is the net supply of short-term debt. Furthermore, \( \pi_1 \) increases if \( \alpha \) decreases or either \( B \) and \( L \) increases.

**Proof.** See Appendix B.2.

The first term of equation (2.20) is almost the same as the inflation for the benchmark case. So the effect of introducing the convenience utility \( v(\cdot) \), and therefore the impact of monetary policies on inflation for this case is represented by the second term:

\[
\frac{(1 - \alpha)L + \beta L^{NS}}{\theta L^{NS}}(1 - \alpha)L
\]

(2.21)

where \( \theta = (1 + \beta(1 - \alpha))s + (1 - \alpha) \). Differentiate equation (2.21) with respect to \( B \) shows that inflation \( \pi_1 \) increases with the amount of short-term debt held by the monetary authority \( B \), consistent with open market operations. Monetary policies affect \( \pi_1 \) via the real interest rate channel. Increasing \( B \) reduces the household’s nominal holding of short-term debt, \( B^D_{01} = (1 - \alpha)L_0 - B^M_{01} \). This also lowers the real short-term interest rates, because reducing \( B^D_{01} \) implies an increase in the marginal convenience utility. As such, inflation must increase if the present value of the primary surplus equals the present value of debt liabilities.

\[^8\text{Real bond is more attractive when inflation is high.}\]
To show this numerically, the upper panel of figure 2.2 shows the impact of changes in the amount of short-term debt held by the monetary authority on inflation and real short-term interest rates. The level of debt \( (L) \) is 0.8, of which 0.12 units are long-term debts, with a primary surplus \( (s) \) of 1.5. Consistent with the previous analysis, an expansionary monetary policy (increase in \( B \)) decreases short-term interest rate and induces inflation. The effect of changes in \( B \) on both variables appears to be quite linear. In terms of demands and supplies, an expansionary monetary policy decreases the net-supply of short-term bonds, which shifts the supply curve to the left. The demand remains unchanged, as shown by bottom panel. Consequently, there is an increase (decrease) in bond price (return). An increase in \( B \) from 0.1 units to 0.3 units would decrease the real short-term interest rate by about 7 percent. To see the impact of such expansionary policy on inflation \( (\pi_1) \), notice that, using the same set of parameters, the nominal short-rate tomorrow \( (r_{12}^n) \) and therefore the price level \( P_1 \) does not depend on \( B \). This implies the impact of changes in \( B \) on \( P_0 \) is crucial. From equation (2.14) and the assumption that the primary surplus is positive, the price level in period 0 depends positively on the net amount of debt due, \( B_{-1} - Q_{01}(1 - \alpha)L_0 - Q_{02}\alpha L_0 \). An increase in \( Q_{01} \), as it happens in an expansionary monetary policy, allows the fiscal authority to raise more revenue for selling a fixed amount of bonds. This reduces the price level in period 0 and hence inflation.

What are the impacts of changes in fiscal policies on inflation and real short-term interest rates? It appears that inflation declines at an accelerated pace as the fraction of long-term debt issued \( (\alpha) \) increases, which is shown in figure 2.3. The result is consistent with the fiscal theory of price level - a shift from short-term debt to long-term debt suppress inflation today at a cost of inducing inflation in the future. Combining with the results in the previous section, it follows that expansionary monetary policy and increase in debt-maturity structure have contradicting effects on inflation, implying coordination is necessary in achieving policy goals.

For real short-term interest rates, an increase debt-maturity structure has two effects. First, it decreases the net-supply of short-term debt. Second, it induces a decline in inflation. Equation (2.5), derived from the FOC of the household problem, states that the real price of short-term bonds depends

9 The result holds so long as the sign of the primary surplus is the same as that for the net amount of debt due at \( t = 0 \), which keeps \( P_0 \) positive.

10 In this model, long-term nominal bond price, \( R_{02}^n \), decreases in response to an increase in \( B \), but its effect on price level is relatively minor because the fraction of long-term debt, \( \alpha \) is relatively small (as in the real world).
inversely on inflation $\pi_1$ because the household derives utility from holding the real balance of short-term bonds. Any decline in inflation increases the attractiveness of such bonds, and therefore demand for short-term bond increases. Both forces act in the same direction, so the short-term interest rate decreases. Hence, expansionary monetary policy and increase in debt-maturity structure have the same impact on real short-term interest rates.

To conclude, the above discussion could potentially explain the peculiar situation the U.S. economy is experiencing: interest rate remains low for an extended period of time, yet there is at most modest sign of inflation, as illustrated in the lower panel of figure 2.3. Conventional macroeconomic models are not able to produce such results.

Figure 2.4 shows the impact of changes in the level of debt $L$ on the real short-term interest rate and inflation. The fraction of long-term debt, $\alpha$, is assumed to be 0.2. In this model, inflation rises rapidly as the level of debt increases. This is consistent with Bohn (1988), which demonstrates that nominal debt increases inflation along the time-consistent path of the economy under a welfare-maximizing government. The level of debt seems to play an important role on inflation. It dominates any force exerted by the monetary authority, as shown by the lower panel of figure 2.4. When the level of debt, $L$, doubles from 0.5 to 1, inflation increases from less than zero to more than fifty percent. A comparable contractionary monetary policy barely has any effects.

Figure 2.5 shows the impact of fiscal primary surplus on the economy. The level of debt, $L$, is assumed to be 0.8 units. A decrease in deficit (increase in $s$) would decrease inflation, as predicted by the fiscal theory of price level, because more real resources are available to pay off the debts. Such a fiscal policy would raise real short-term interest rates. The right panel of figure 2.5 provides the intuition. The supply of the short-term bond is fixed as primary surplus varies. Demand, on the other hand, shifts to the left because real bond is less attractive when the price level for tomorrow (i.e. $P_1$) is low. A higher rate is needed to induce the household to hold the same amount of bond. The overall effect of running a deficit on the real short-rate is therefore positive. This suggests an expansionary monetary policy (increase in $B$) together with a decrease in primary surplus ($s$) would produce an ambiguous effect on the short-term interest rates and inflation, as shown on the lower panel of figure 2.5.
2.4 Empirical evidence

In this section, I investigate the relationship between the maturity structure of government debt and future inflation and the term-structure of interest rates, as predicted by the theory.

Data

I examine data on marketable U.S. Treasury bills, notes, and bonds. The main sample covers the period from June 1971 to December 2012. Data were collected from the CRSP historical monthly bond database on every U.S. government bond issued between 1940 and 2012. CRSP provides data on bond characteristics such as issue date, coupon rate, face value outstanding, and maturity, etc. As in Greenwood and Vayanos (2012), I break the stream of each bond’s cash flows into principal and coupon payments. Consider, for example, the 2-year bond issued in November 2003 (CRSP ID 20051130.201870) with a coupon payment of 1.875%. On the last day of January 2005, investors holding the bond were expecting two more coupon payments of $0.9375 per $100 of face value between May 2005 and November 2005, and the principal payment in November 2005 when the bond matured.

I define short-term debt to be the amount of government debt (excluding holdings by the Federal Reserve) maturing within a year, and long-term debt to be the amount of government debt (excluding holdings by the Federal Reserve) whose maturity is longer than 10 years.

Relationship between Maturity Structure and Future Inflation

First, I look at whether measures of maturity structure have significant predictive power for future inflation from the following regression:

\[
\pi_{t+h} = \beta_0 + \beta_1 \text{Maturity Structure} + \beta_2 \pi_t + X_t' \theta + \epsilon_t \tag{2.22}
\]

In equation (2.22), the coefficient \(\beta_0\) represents the level of inflation when all debt is long-term. To control the possible sources of endogeneity, the vector

---

\[11\] Treasury Inflation-Protected Securities (TIPS) are excluded because of difficulties in computing future coupon payments associated with inflation. As of February 28, 2013, it accounts for less than 8% of publicly-held marketable government debt. (U.S. Treasury 2013)
contains variables that capture the most common predictors of inflation, including the current level of inflation and the level of debt-to-GDP ratio. The Fed funds rate is also include to controlled for the effect of monetary policies. $h$ is the number of periods (in years) ahead.

Table 2.1 shows the result. In the baseline regression, I look at one-year ahead inflation forecast. Column (1) shows the regression without including any indicator about the maturity structure. Not surprisingly, all controls are statistically significantly at one percentage level. About 30 percent of all variations of one-year ahead inflation is explained by these controls alone. Column (2) shows the effects of changes in the Treasury debt maturity structure, specifically the fraction of Treasury debt due within a year, on one-year ahead inflation. First, as predicted by theory, the relationship between these two variables are positive and significant at one percent level. A one percent increase in short-term debt is associated with a 0.25 percent increase in one-year ahead inflation. Second, the maturity structure variable alone is able to explain a significant amount of variations (about 23 percent) in future inflation. To confirm this relationship is robust, column (3) regresses future inflation with both the controls and the maturity structure variable. The coefficient for debt maturity structure remains significant at the one percent level. Relative to column (1), the R-squared increases by about 10 percents when the debt-maturity structure variable is introduced in additional to the controls.

How about if one replace the fraction of short-term debt with long-term debt? The theory predicts that there should be a negative relationship between long maturity structure and future inflation. Table 2.1 shows that the result is as predicted at one percent significant level. One percent increase in the fraction of long-term debt is associated with a 0.2 percent decrease in future inflation. Also, the introduction of this debt maturity structure variable increase the R-squared by about 10 percent.

Next, I look at the ability of debt maturity structure to forecast future inflation at different horizons. Table 2.3 shows the results. When the short-end of the maturity structure is used, it is significant at one percent level for all forecasting horizons. The effect of changes in the fraction of short-term debt on future inflation is strictly increasing between $h = 12$ and $h = 48$, with a maximum coefficient of 0.24 at $h = 48$, which is four years. After four years, the effect declines monotonically. Table 2.4 shows the percentage increase in R-squared when the fraction of short-term debt is included over different

\[\text{Note that the fractions of short- and long-term debt do not sum up to one because debt with maturities between one and ten years are excluded.}\]
horizons. At $h = 12$, the increase is at 9.1%. The maximum increase happens at $h = 48$, where the percentage increase is almost 40%. Table 2.5 shows that the results are robust when the long-end of the maturity structure is used. Long-term debt, however, tends to improve forecasting power of future inflation at a longer horizon. In fact, the effect of changes in the fraction of long-term debt on future inflation is the strongest when $h = 60$ (or 5 years). One percent increase in the fraction is associated with 0.25% decline in future inflation. Table 2.7 and 2.6 shows that the results are robust to different measures of future inflation, such as the Michigan survey of expected inflation.

2.5 Conclusion

This paper studies the interaction between monetary and fiscal policies, and how changes in fiscal policies, such as the level of debt and debt maturity composition, would affect inflation, the real economy and asset prices. Using a three-period equilibrium model, in which monetary policies are modelled as open market operations, this paper avoids Sargent and Wallace (1981)'s “game of chicken” problem on policy coordination between the monetary and the fiscal authority. Inflation and real variables are jointly determined by both monetary and fiscal policies. An important implication is that coordination between the FED and Treasury is crucial to successfully regulating the economy, as actions by monetary and fiscal authorities, if act independently with different objectives, might have unintendedly consequences on the economy.

Additionally, unlike Woodford (1998) and Cochrane (2001), changes in the level of total debt and debt maturity structure in my model have important implications on real interest rates. The model is able to explain the peculiar situation U.S. economy is experiencing: interest rate remains low for an extended period of time, yet there is at most modest sign of inflation, whereas conventional macroeconomic models are not able to produce such results.

Finally, I provide robust empirical evidence on how changes in debt-maturity structure are associated with changes in future inflation using U.S. data. One percent increase in the fraction of short-term debt issued is associated with more than 0.2 percent increase in future inflation of different horizons. Empirical evidence also shows that changes in the short-end of the maturity structure has the most explanatory power over short and medium horizons, whereas changes in the long-end of the maturity structure has the most explanatory power over long horizons.
2.6 Figures and Tables

Figure 2.1: Average Debt Maturity of Total Federal Debt in Years.

The sample covers the period from 1985 to 2012. Data were collected from CRSP historical monthly bond database on every U.S. government bond. As in Greenwood and Vayanos (2012), the stream of each bond's cash flows is separated by principal and coupon payments. Debt maturities are then calculated using a weighted average of all bonds according to their face values.
Figure 2.2: Impact of Monetary Policy \( (B) \) on Real Short-Term Interest Rates and Inflation

Parameter values: The level of debt \( (L) = 0.8 \). The fraction of long-term debt \( (\alpha) = 15\% \). Primary surplus \( (s) = 1.5 \).
Figure 2.3: Impact of Debt Maturity Structure ($\alpha$) on Real Short Rate and Inflation

Parameter values: The level of debt ($L$) = 0.8. The amount of short-term debt held by the monetary authority ($B$) = 0.2. Primary surplus ($s$) = 1.5.
Figure 2.4: Impact Level of Debt ($L$) on Real Short Rate and Inflation

Parameter values: The fraction of long-term debt ($\alpha$) = 20%. The amount of short-term debt held by the monetary authority ($B$) = 0.2. Primary surplus ($s$) = 1.5.
Figure 2.5: Impact of Primary Surplus \((s)\) on Real Short Rate and Inflation

Parameter values: The level of debt \((L) = 0.8\). The fraction of long-term debt \((\alpha) = 20\%\). The amount of short-term debt held by the monetary authority \((B) = 0.2\).
Table 2.1: One-Year Ahead Forecast of Inflation using Short-term Debt

This table shows the effect of changes in short-term debt on one-year ahead inflation using U.S. data between 1971 and 2011. “Inflation\textsubscript{t}” is the year-to-year growth for the personal consumption expenditure index. “Debt-To-GDP” is the total debt to GDP ratio. Fed Funds is the federal funds rate. “% of Debt Due in Less than 1 year” is the fraction of government debt due within a year. All variables are expressed in percentage points. Robust standard errors (in parentheses) are used. ***, **, and * indicates statistical significance at the 1%, 5%, and 10% level, respectively.

<table>
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<tr>
<th></th>
<th>(1) Inflation\textsubscript{t+12}</th>
<th>(2) Inflation\textsubscript{t+12}</th>
<th>(3) Inflation\textsubscript{t+12}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation\textsubscript{t}</td>
<td>0.217*** (0.0432)</td>
<td>0.175*** (0.0431)</td>
<td></td>
</tr>
<tr>
<td>Debt-to-GDP</td>
<td>-0.0692*** (0.00857)</td>
<td>-0.00130 (0.0157)</td>
<td></td>
</tr>
<tr>
<td>Fed Funds</td>
<td>0.158*** (0.0354)</td>
<td>0.192*** (0.0353)</td>
<td></td>
</tr>
<tr>
<td>% of Debt Due in</td>
<td></td>
<td>0.255*** (0.0191)</td>
<td>0.187*** (0.0367)</td>
</tr>
<tr>
<td>Less than 1 Year</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.0473*** (0.00465)</td>
<td>-0.0536*** (0.00679)</td>
<td>-0.0466** (0.0190)</td>
</tr>
<tr>
<td>Error</td>
<td>Robust</td>
<td>Robust</td>
<td>Robust</td>
</tr>
<tr>
<td>R2</td>
<td>0.307</td>
<td>0.228</td>
<td>0.335</td>
</tr>
<tr>
<td>N</td>
<td>607</td>
<td>607</td>
<td>607</td>
</tr>
</tbody>
</table>
Table 2.2: **One-Year Ahead Forecast of Inflation using Long-term Debt**

This table shows the effect of changes in long-term debt on one-year ahead inflation using U.S. data between 1971 and 2011. “Inflation$ _t $” is the year-to-year growth for the personal consumption expenditure index. “Debt-To-GDP” is the total debt to GDP ratio. Fed Funds is the federal funds rate. “% of Debt Due in More than 10 year” is the fraction of government debt due within a year. All variables are expressed in percentage points. Robust standard errors (in parentheses) are used. ***, **, and * indicates statistical significance at the 1%, 5%, and 10% level, respectively.

<table>
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<tr>
<th></th>
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<th>(3) Inflation$_{t+12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of Debt Due in</td>
<td>-0.211***</td>
<td>-0.133***</td>
<td></td>
</tr>
<tr>
<td>More than 10 Years</td>
<td>(0.0238)</td>
<td>(0.0303)</td>
<td></td>
</tr>
<tr>
<td>Inflation$_t$</td>
<td>0.217***</td>
<td>0.173***</td>
<td>0.173***</td>
</tr>
<tr>
<td></td>
<td>(0.0432)</td>
<td>(0.0437)</td>
<td>(0.0437)</td>
</tr>
<tr>
<td>Debt-to-GDP</td>
<td>-0.0692***</td>
<td>-0.0362***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00857)</td>
<td>(0.0113)</td>
<td></td>
</tr>
<tr>
<td>Fed Funds</td>
<td>0.158***</td>
<td>0.234***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0354)</td>
<td>(0.0389)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.0715***</td>
<td>0.0473***</td>
<td>0.0534***</td>
</tr>
<tr>
<td></td>
<td>(0.00417)</td>
<td>(0.00465)</td>
<td>(0.00479)</td>
</tr>
<tr>
<td>Error</td>
<td>Robust</td>
<td>Robust</td>
<td>Robust</td>
</tr>
<tr>
<td>R2</td>
<td>0.116</td>
<td>0.307</td>
<td>0.328</td>
</tr>
<tr>
<td>N</td>
<td>607</td>
<td>607</td>
<td>607</td>
</tr>
</tbody>
</table>
Table 2.3: **Inflation Forecast at Different Horizons using Short-term Debt**

This table shows the effect of changes in short-term debt on one-year ahead inflation using U.S. data between 1971 and 2011. “Inflationₜ” is the year-to-year growth for the personal consumption expenditure index. “Debt-To-GDP” is the total debt to GDP ratio. Fed Funds is the federal funds rate. “% of Debt Due in Less than 1 year” is the fraction of government debt due within a year. All variables are expressed in percentage points. Robust standard errors (in parentheses) are used. ***, **, and * indicates statistical significance at the 1%, 5%, and 10% level, respectively.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>% of Debt Due in Less than 1 Year</strong></td>
<td>0.187***</td>
<td>0.210***</td>
<td>0.240***</td>
<td>0.367***</td>
<td>0.331***</td>
<td>0.238***</td>
</tr>
<tr>
<td></td>
<td>(0.0367)</td>
<td>(0.0383)</td>
<td>(0.0379)</td>
<td>(0.0400)</td>
<td>(0.0434)</td>
<td>(0.0456)</td>
</tr>
<tr>
<td><strong>Inflation</strong></td>
<td>0.175***</td>
<td>0.112**</td>
<td>0.177***</td>
<td>0.0455</td>
<td>-0.0227</td>
<td>-0.00378</td>
</tr>
<tr>
<td></td>
<td>(0.0431)</td>
<td>(0.0456)</td>
<td>(0.0451)</td>
<td>(0.0446)</td>
<td>(0.0496)</td>
<td>(0.0515)</td>
</tr>
<tr>
<td><strong>Debt-to-GDP</strong></td>
<td>-0.00130</td>
<td>-0.0169</td>
<td>-0.00774</td>
<td>0.0323*</td>
<td>0.0104</td>
<td>-0.0274</td>
</tr>
<tr>
<td></td>
<td>(0.0157)</td>
<td>(0.0168)</td>
<td>(0.0169)</td>
<td>(0.0178)</td>
<td>(0.0187)</td>
<td>(0.0197)</td>
</tr>
<tr>
<td><strong>Fed Funds</strong></td>
<td>0.192***</td>
<td>0.0390</td>
<td>-0.0413</td>
<td>0.0286</td>
<td>0.0645</td>
<td>0.0205</td>
</tr>
<tr>
<td></td>
<td>(0.0353)</td>
<td>(0.0376)</td>
<td>(0.0377)</td>
<td>(0.0379)</td>
<td>(0.0394)</td>
<td>(0.0409)</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>-0.0466**</td>
<td>-0.0374*</td>
<td>-0.0490**</td>
<td>-0.109***</td>
<td>-0.0865***</td>
<td>-0.0363</td>
</tr>
<tr>
<td></td>
<td>(0.0190)</td>
<td>(0.0199)</td>
<td>(0.0197)</td>
<td>(0.0206)</td>
<td>(0.0220)</td>
<td>(0.0233)</td>
</tr>
<tr>
<td><strong>Error</strong></td>
<td>Robust</td>
<td>Robust</td>
<td>Robust</td>
<td>Robust</td>
<td>Robust</td>
<td>Robust</td>
</tr>
<tr>
<td><strong>R2</strong></td>
<td>0.335</td>
<td>0.278</td>
<td>0.311</td>
<td>0.349</td>
<td>0.336</td>
<td>0.301</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>607</td>
<td>595</td>
<td>583</td>
<td>571</td>
<td>559</td>
<td>547</td>
</tr>
</tbody>
</table>
Table 2.4: Percentage Increase in $R^2$ when Debt-Maturity Structure is included in Inflation Forecast over Different Horizons.

This table shows the percentage increase in R-squared when debt-maturity structure variable is included in the following regression:

$$\pi_{t+h} = \beta_0 + \beta_1 \text{Maturity Structure} + \beta_2 \pi_t + X_t' \theta + \epsilon_t$$

The coefficient $\beta_0$ represents the level of inflation when all debt is long-term. To control the possible sources of endogeneity, the vector $X_t'$ contains variables that capture the most common predictors of inflation, including the current level of inflation and the level of debt-to-GDP ratio. The Fed funds rate is also include to controlled for the effect of monetary policies. $h$ is the number of periods (in months) ahead.

<table>
<thead>
<tr>
<th>Horizon, $h$</th>
<th>$R^2$ with Debt-Maturity Structure</th>
<th>$R^2$ without Debt-Maturity Structure</th>
<th>Percentage Increase</th>
</tr>
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<tr>
<td>12</td>
<td>0.307</td>
<td>0.335</td>
<td>9.1%</td>
</tr>
<tr>
<td>24</td>
<td>0.241</td>
<td>0.278</td>
<td>15.4%</td>
</tr>
<tr>
<td>36</td>
<td>0.263</td>
<td>0.331</td>
<td>18.3%</td>
</tr>
<tr>
<td>48</td>
<td>0.252</td>
<td>0.349</td>
<td>38.5%</td>
</tr>
<tr>
<td>60</td>
<td>0.266</td>
<td>0.336</td>
<td>26.3%</td>
</tr>
<tr>
<td>72</td>
<td>0.266</td>
<td>0.301</td>
<td>13.2%</td>
</tr>
</tbody>
</table>
Table 2.5: Inflation Forecast at Different Horizons using Long-term Debt

This table shows the effect of changes in long-term debt on one-year ahead inflation using U.S. data between 1971 and 2011. “Inflation$_t$” is the year-to-year growth for the personal consumption expenditure index. “Debt-To-GDP” is the total debt to GDP ratio. Fed Funds is the federal funds rate. “% of Debt Due in More than 10 year” is the fraction of government debt due in more than year. All variables are expressed in percentage points. Robust standard errors (in parentheses) are used. ***, **, and * indicates statistical significance at the 1%, 5%, and 10% level, respectively.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of Debt Due in More than 10 Years</td>
<td>-0.133***</td>
<td>-0.145***</td>
<td>-0.179***</td>
<td>-0.240***</td>
<td>-0.248***</td>
<td>-0.244***</td>
</tr>
<tr>
<td></td>
<td>(0.0303)</td>
<td>(0.0345)</td>
<td>(0.0380)</td>
<td>(0.0420)</td>
<td>(0.0431)</td>
<td>(0.0438)</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.173***</td>
<td>0.123***</td>
<td>0.195***</td>
<td>0.0762</td>
<td>0.0292</td>
<td>0.0211</td>
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<td>(0.0437)</td>
<td>(0.0461)</td>
<td>(0.0456)</td>
<td>(0.0463)</td>
<td>(0.0496)</td>
<td>(0.0503)</td>
</tr>
<tr>
<td>Debt-to-GDP</td>
<td>-0.0362***</td>
<td>-0.0533***</td>
<td>-0.0420***</td>
<td>-0.0309*</td>
<td>-0.0350**</td>
<td>-0.0400**</td>
</tr>
<tr>
<td></td>
<td>(0.0113)</td>
<td>(0.0134)</td>
<td>(0.0150)</td>
<td>(0.0165)</td>
<td>(0.0166)</td>
<td>(0.0168)</td>
</tr>
<tr>
<td>Fed Funds</td>
<td>0.234***</td>
<td>0.0721*</td>
<td>-0.0114</td>
<td>0.0705*</td>
<td>0.0991**</td>
<td>0.0583</td>
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<td>(0.0389)</td>
<td>(0.0407)</td>
<td>(0.0402)</td>
<td>(0.0407)</td>
<td>(0.0414)</td>
<td>(0.0419)</td>
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<tr>
<td>Constant</td>
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<td>0.0739***</td>
<td>0.0776***</td>
<td>0.0837***</td>
<td>0.0871***</td>
<td>0.0919***</td>
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<tr>
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<td>(0.00479)</td>
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</tr>
<tr>
<td>R2</td>
<td>0.328</td>
<td>0.264</td>
<td>0.290</td>
<td>0.293</td>
<td>0.308</td>
<td>0.306</td>
</tr>
<tr>
<td>N</td>
<td>607</td>
<td>595</td>
<td>583</td>
<td>571</td>
<td>559</td>
<td>547</td>
</tr>
</tbody>
</table>
Table 2.6: Inflation Forecast at Different Horizons using Long-term Debt, using One-Year ahead Expected Inflation from the Surveys of Consumers

This table shows the effect of changes in long-term debt on one-year ahead inflation using U.S. data between 1971 and 2011. “Inflation,” is the expected inflation for one-year ahead using the Survey of Consumer conducted by the University of Michigan. “Debt-To-GDP” is the total debt to GDP ratio. Fed Funds is the federal funds rate. “% of Debt Due in More than 10 Year” is the fraction of government debt due in more than year. All variables are expressed in percentage points. Robust standard errors (in parentheses) are used. ***, **, and * indicates statistical significance at the 1%, 5%, and 10% level, respectively.

<table>
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<tr>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
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</thead>
<tbody>
<tr>
<td>% of Debt Due in</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>More than 10 Years</td>
<td>-12.64***</td>
<td>-15.50***</td>
<td>-19.01***</td>
<td>-22.53***</td>
<td>-19.48***</td>
<td>-17.07***</td>
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<td></td>
<td>(1.644)</td>
<td>(1.888)</td>
<td>(2.072)</td>
<td>(2.224)</td>
<td>(2.332)</td>
<td>(2.363)</td>
</tr>
<tr>
<td>Inflation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>22.36***</td>
<td>16.27***</td>
<td>10.94***</td>
<td>7.372***</td>
<td>6.188**</td>
<td>4.138</td>
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<td>(2.681)</td>
<td>(2.639)</td>
<td>(2.525)</td>
<td>(2.806)</td>
<td>(2.845)</td>
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<td>Debt/GDP</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td>-2.947***</td>
<td>-2.116**</td>
<td>-0.920</td>
<td>-1.633*</td>
<td>-2.213**</td>
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<td>(0.755)</td>
<td>(0.828)</td>
<td>(0.874)</td>
<td>(0.897)</td>
<td>(0.919)</td>
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<td>fedfund</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.839***</td>
<td>0.237</td>
<td>-4.670**</td>
<td>-6.292***</td>
<td>-2.601</td>
<td>-0.0446</td>
</tr>
<tr>
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<td>(2.034)</td>
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<td>(2.178)</td>
<td>(2.138)</td>
<td>(2.223)</td>
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</tr>
<tr>
<td>Constant</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.022***</td>
<td>7.348***</td>
<td>8.064***</td>
<td>8.404***</td>
<td>7.922***</td>
<td>7.610***</td>
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<tr>
<td></td>
<td>(0.349)</td>
<td>(0.355)</td>
<td>(0.337)</td>
<td>(0.334)</td>
<td>(0.361)</td>
<td>(0.350)</td>
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<td>Error</td>
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<td>Robust</td>
<td>Robust</td>
<td>Robust</td>
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<tr>
<td>R2</td>
<td>0.541</td>
<td>0.460</td>
<td>0.471</td>
<td>0.496</td>
<td>0.471</td>
<td>0.446</td>
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<tr>
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<td>420</td>
<td>420</td>
<td>420</td>
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</table>
Table 2.7: One-Year Ahead Forecast of Inflation using Short-term Debt, using One-Year ahead Expected Inflation from the Surveys of Consumers

This table shows the effect of changes in short-term debt on one-year ahead inflation using U.S. data between 1971 and 2011. “Inflation_{t+12}” is the expected inflation for one-year ahead using the Survey of Consumer conducted by the University of Michigan. “Debt-To-GDP” is the total debt to GDP ratio. Fed Funds is the federal funds rate. “% of Debt Due in Less than 1 year” is the fraction of government debt due within a year. All variables are expressed in percentage points. Robust standard errors (in parentheses) are used. ***, **, and * indicates statistical significance at the 1%, 5%, and 10% level, respectively.

<table>
<thead>
<tr>
<th>% of Debt Due in Less than 1 Year</th>
<th>(1) Inflation_{t+12}</th>
<th>(2) Inflation_{t+12}</th>
<th>(3) Inflation_{t+12}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 1 Year</td>
<td>21.96***</td>
<td>15.08***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.630)</td>
<td>(2.604)</td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>25.86***</td>
<td>25.30***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.584)</td>
<td>(2.491)</td>
<td></td>
</tr>
<tr>
<td>Debt/GDP</td>
<td>-4.948***</td>
<td>-0.0523</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.591)</td>
<td>(1.019)</td>
<td></td>
</tr>
<tr>
<td>fedfund</td>
<td>2.120</td>
<td>4.658**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.961)</td>
<td>(1.939)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-3.515***</td>
<td>5.005***</td>
<td>-2.360*</td>
</tr>
<tr>
<td></td>
<td>(0.542)</td>
<td>(0.345)</td>
<td>(1.315)</td>
</tr>
<tr>
<td>Error</td>
<td>Robust</td>
<td>Robust</td>
<td>Robust</td>
</tr>
<tr>
<td>R2</td>
<td>0.303</td>
<td>0.476</td>
<td>0.515</td>
</tr>
<tr>
<td>N</td>
<td>420</td>
<td>420</td>
<td>420</td>
</tr>
</tbody>
</table>
Bibliography


Appendix A

Proofs for Chapter 1

A.1 Proof of proposition 1

Government in country R defaults if and only if $U_{\text{Default}} \geq U_{\text{No Default}}$, i.e.

$$
(1 - c)y_1 + w_R\tilde{A}f((1 - c)y_{1R}) \geq y_{1R} + b_F + b_{RR} - T_R + w_R\tilde{A}f(y_{1R} + \lambda b_D) \equiv y_{1R} + b_D - T_R + w_R\tilde{A}f\left(y_{1R} + \lambda b_D\right)
$$

$$
\Rightarrow \quad \tilde{A} \leq \frac{(T - b_D) - cy_1}{w(f(y_1 + \lambda b_D) - f((1 - c)y_1))}
$$

Define $a^*$ such that the above inequality holds. Hence, government defaults if and only if the realized productivity shock is less than $a^*$. Moreover, it is clear that $a^*$ is strictly decreasing in $b_{RR}$, $c$, and $\lambda_R$. $a^*$ is strictly increasing in $T_R$ and, if $f(\cdot)$ is strictly concave, also in $b_F$.

A.2 Proof of Proposition 2

Recall that $T = b_F + b_D$. Note that $\Pi(a^*) = a^*$. From equation (1.8) and (1.6), the market clearing conditions,

$$
1 - a^* + \frac{w\lambda(1 - \gamma)(\mu + \sigma)^2 - (a^*)^2}{2} = \frac{g}{T}
$$

$$
\Rightarrow -\left(\frac{w\lambda(1 - \gamma)}{2}\right)(a^*)^2 - a^* + \left(\frac{1}{2\sigma} + \frac{w\lambda(1 - \gamma)^2}{2} - \frac{g}{T}\right) = 0
$$
Suppose the quadratic equation above have a solution in $[0, 1]$. Then

$$\Pi(a^*) = \left(\frac{1}{w\lambda(1-\gamma)}\right) \times \left[ \sqrt{(1 + w\lambda(1-\gamma))^2 - w\lambda(1-\gamma)\frac{2g}{T} - 1} \right]$$

where the positive root is discarded because $\Pi(a^*) > 0$. A solution exists if $T$ is sufficiently large. Specifically, $T \in \left[\frac{g}{2(1+0.5\times)}, \infty\right)$ implies $\Pi(a^*) \in [0, 1)$. In addition,

$$\frac{\partial \Pi(a^*)}{\partial T} = \frac{g}{T^2(1 + a^*w\lambda(1-\gamma))} > 0 \quad (A.1)$$

And it is also clear that $\frac{d^2 \Pi(a^*)}{dTd\lambda} < 0$.

### A.3 Proof of Proposition 3

Given the level of total debts issued by country R, $T$, the equilibrium solution $\mathbf{x} = (a^*, p_{R0}, b_D, b_F)$ must satisfy equation (1.5), (1.6), (1.7), (1.8). This gives a system of four non-linear equations, one can obtain

$$b_D = \frac{T - cy_1(1 + a^*w)}{1 + a^*w\lambda} \quad (A.2)$$

$$b_F = T - b_D \quad (A.3)$$

$$a^* = \frac{1}{w\lambda(1-\gamma)} \left[ \sqrt{(1 + w\lambda(1-\gamma))^2 - w\lambda(1-\gamma)\frac{2g}{T} - 1} \right] \quad (A.4)$$

$$p_{R0} = \frac{g}{T} \quad (A.5)$$

Also, from equation (1.10),

$$\frac{\partial a^*}{\partial T} = \frac{g}{T^2(1 + a^*w\lambda)} > 0 \quad (A.6)$$

$$\frac{\partial a^*}{\partial \lambda} = \frac{(a^* - a_L)}{\lambda(1 + a^*w\lambda)} > 0 \quad (A.7)$$

It follows that $b_D/T$ is strictly decreasing in $\lambda$ and $c$. 
A.4 Proof of Proposition 4

Assume, without loss of generality, that \( \gamma = 0 \). Note that \( \frac{b_D}{T} = \frac{1}{1 + w\lambda} > 0 \) as \( T \to \infty \). Differentiate \( b_d \) with respect to \( T \) yields

\[
\frac{\partial b_D}{\partial T} = \frac{1}{(1 + w\lambda a^*)^2} \times \left( (1 + w\lambda a^*) - (T\lambda + cy_1(1 - \lambda))wa_T^* \right)
\]

To prove the first property, compute the second derivative of \( b_D \) with respect to \( T \).

\[
(1 + w\lambda a^*)^4 \times \frac{\partial}{\partial T} \left( \frac{\partial b_D}{\partial T} \right) = - (1 + w\lambda a^*)^2 \left( T\lambda + cy_1(1 - \lambda) \right)wa_T^{*2} - 2(1 + w\lambda a^*)w\lambda a_T^* \left( 1 + w\lambda a^* \right)
\]

\[
- (T\lambda + cy_1(1 - \lambda))wa_T^*
\]

Note that \(-a_{TT}^* = \frac{2}{T}a_T + \frac{a_T^2 w\lambda}{1 + a^* w\lambda} > 0\), from which it is clear that \( \frac{\partial b_D}{\partial T} \). Hence, given \( \lambda \geq 0 \), \( b_D(T) \) is a convex function of \( T \). Define \( h(T) = b_D(T)/T \). If \( T_0 \) is a local extrema for \( h(T) \), then \( h'(T_0) = 0 \). Next, we need to show that \( h''(T_0) > 0 \). Differentiate \( h'(T) \) with respect to \( T \) yields

\[
T^2 h'(T) = \frac{\partial b_D}{\partial T}T - b_D
\]

\[
\Rightarrow T^2 h''(T) + 2Th'(T) = \frac{\partial^2 b_D}{\partial T^2} T
\]

Because \( h'(T_0) = 0 \) and \( \frac{\partial^2 b_D}{\partial T^2} > 0 \), \( h''(T_0) \) must also be strictly positive. It follows from the differentiability of \( h(\cdot) \) that there can at most be one \( T_0 \) such that \( h'(T_0) = 0 \). Note that

\[
T^2(1 + w\lambda a^*)^2 \frac{\partial h}{\partial T} = -a_T w\lambda T^2 + cy_1 a_T wT(\lambda - 1) + cy_1(1 + a^* w)(1 + a^* w\lambda)
\]

If \( \lambda \) is small, then the first two terms are close to zero, implying \( \frac{\partial h}{\partial T} \) is positive, which proves property 2). Suppose \( c \) is small. Then, if \( \lambda \) is large and \( T \) is small, the last two terms are close to zero, implying \( \frac{\partial h}{\partial T} \) is negative, which proves property 1). Take the limit as \( T \to \infty \), property 4) immediately follows.
A.5 Proof of proposition 5

Again, assume without loss of generality, that $\gamma = 0$. Applying the Envelope theorem to the government’s problem (1.13) to obtain

$$
\frac{dU(T, b_D)}{dT} = p_0 \left( \frac{b_D}{T} \right) - (1 - \Pi(a^*))
$$

(A.8)

where $p_0$ satisfies equation (1.6) and $b_D/T$ satisfies equation (1.11). Equation (1.14) immediately follows. Suppose $(c - \gamma) > 0$. Then $b_D/T$ is strictly less than one when $\lambda = 0$, implying that $\frac{dU(T, b_D)}{dT} < 0$. Because $b_D/T$ is a continuous function of $\lambda$, property 1 must be true.

Let $h = b_D/T$ and suppose $\frac{dU(T, b_D)}{dT} = 0$. Then

$$
\left( \frac{T^2}{g} \right) \frac{d^2U(T, b_D)}{dT^2} = -h + Th_T + \frac{1}{1 + w\lambda a^*}
$$

$$
= \frac{cy_1(1 + wa^*)}{T(1 + w\lambda a^*)} + Th_T
$$

(A.9)

From equation (1.11), we know that

$$
Th_T = \frac{1 - cy_1w\lambda a_T - Thw\lambda a_T - (1 + w\lambda a^*)h}{1 + w\lambda a^*}
$$

where $a_T > 0$ satisfies equation (A.6). It follows that

$$
(1 + w\lambda a^*) \times (A.9) = \frac{2cy_1(1 + wa^*)}{T} - cy_1w\lambda a_T - Thw\lambda a_T
$$

$$
= -cy_1w\lambda a_T
$$

$$
< 0
$$

The second equality is derived from the assumption that $\frac{dU(T, b_D)}{dT} = 0$ and equation (1.11). Therefore, $\frac{dU(T, b_D)}{dT}|_{(T^*, b_D^*)} = 0$ implies that $U(T^*, b_D^*)$ is a local maximum. Because $U(T^*, b_D^*)$ is continuous with respect to $T$, the intermediate value theorem implies $U(T^*, b_D^*)$ must also be the global maximum, which proves property 2.

To prove property 3, we know from equation (A.9) that $U(T^*, b_D^*) = 0$ implies $h_T < 0$. Suppose $(T', b_D')$ satisfies (1.11) but $\frac{dU(T', b_D')}{dT}|_{(T', b_D')} > 0$. Property 1 from proposition 4 implies $T' > T^*$. But then from property 2 of proposition 5, $\frac{dU(T, b_D)}{dT}|_{(T, b_D)} < 0$. 


Appendix B

Proofs for Chapter 2

B.1 Proof of Proposition 6

Differentiate the real short-term interest rate, as defined by equation (2.19), with respect to $B$, yields

$$\frac{\partial r_{01}}{\partial B} = -\frac{1}{\beta} \left( \frac{\theta \beta (1 - \alpha)}{(\beta \theta L^{NS} + (1 - \alpha)L)^2} \right) < 0$$

$$\frac{\partial r_{01}}{\partial L} = \frac{1}{\beta} \left( \frac{\beta \theta (1 - \alpha)B}{(\beta \theta L^{NS} + (1 - \alpha)L)^2} \right) > 0$$

$$\frac{\partial r_{01}}{\partial \alpha} = \frac{1}{\beta} \left( \frac{\beta \theta B}{(\beta \theta L^{NS} + (1 - \alpha)L)^2} \right) > 0$$

B.2 Proof of Proposition 7

From equation (2.20), we have

$$\pi_1 = \frac{(1 - \alpha)(1 + \beta + \beta^2)L}{1 + (1 - \alpha)\left(\beta + \frac{1}{s}\right)} + \frac{(1 - \alpha)L + \beta L^{NS}}{\theta L^{NS}}(1 - \alpha)L$$

For the first term,

$$\frac{(1 - \alpha)(1 + \beta + \beta^2)L}{1 + (1 - \alpha)\left(\beta + \frac{1}{s}\right)}$$

It is clearly decreasing in $\alpha$, increasing in $L$, and constant in $B$. For the second term, let

$$f(\alpha, L, B) = \frac{(1 - \alpha)L + \beta L^{NS}}{\theta L^{NS}}(1 - \alpha)L$$
Then, we can compute

$$\frac{\partial f(\alpha, L, B)}{\partial B} = \frac{((1-\alpha)L)^2}{\theta L^{NS}} > 0$$

$$\left(\frac{1}{(1-\alpha)L}\right) \frac{\partial f(\alpha, L, B)}{\partial L} = \frac{(1-\alpha)B}{(L^{NS})^2} > 0$$

$$\left(\frac{1}{(1-\alpha)L}\right) \frac{\partial f(\alpha, L, B)}{\partial (1-\alpha)} = \frac{BL}{(L^{NS})^2} > 0$$

Combing the results for both the first and second terms, one can conclude that $\pi_1$ increases if $\alpha$ decreases or either $B$ and $L$ increases.