Essays in Public Economics

by

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Abstract

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My dissertation, "Essays in Public Economics," is comprised of three chapters. The first one, titled "Altruism, Reciprocity, and Equity: A Unified Motive for Intergenerational Transfers" is to address the following question: Why do parents divide bequests equally while transferring *inter vivos* gifts unequally? Across times and places, why have there mainly been only two extreme choices of distribution of bequests: either to give them to just one child (unigeniture) or to divide them equally (equigeniture)? How can a motive for intergenerational transfers explain both "equal division puzzle" (the former) and polarized inheritance patterns (the latter)? This chapter presents a behavioral model that coherently rationalizes these empirical realities. Namely, as head of a family, a parent altruistically cares about children but also wants them to spend effort for the family. However, effort is costly and individual level of each child is unverifiable to a third party adjudicator. Given this incomplete information, there rise only two stable equilibria: either equigeniture or unigeniture. When the productivity of effort rises, the evolution of inheritance pattern from unigeniture to equigeniture occurs. So equigeniture is eventually adopted due to a rise in the productivity throughout industrialization. Furthermore, if the parent wants to counterbalance inequality among children who exert equal effort, the greater amount of inter vivos gift is transferred to a child with lower relative income compared to his siblings, while bequests remain equally divided. This model is consistent with the aforementioned empirical realities but also lends itself to further empirical tests. First of all, with a data set of pre-industrial agrarian societies, we find that a rise in the productivity of effort causes equigeniture to be chosen over unigeniture, which is consistent with the model. Second of all, through an empirical analysis on a micro-level data on *inter vivos* transfers in contemporary families, we find supporting evidence as follows: (i) income inequality among children increases the probability that their parent gives any inter vivos gift; and (ii) the amount of the gift is negatively associated with relative income of each child compared to his siblings.

The second chapter "Retirement and Exposure of Pension to Financial Market Fluctuations" studies how exposure of pension wealth to stock market fluctuations affects
retirement behavior both theoretically and empirically. Characteristics of optimal plan for retirement are elaborated with reflecting that liquidizing pension wealth is more tied to retirement decisions than non-pension wealth as well as embodying time-sensitive restrictions on availability of pension benefits. Theoretical analysis finds that exposure of pension to financial market fluctuations does not always entail perfectly symmetric response of retirement. Exposure of pension to a positive shock actually brings responses of retirement only if the magnitude of the positive shock is large enough to compensate for foregone labor earnings and demand for resources necessary for post-retirement consumptions. In particular, whereas exposure of pension to a small negative shock leads to a decrease in retirement, exposure of pension to a positive shock with the same magnitude might not yield an increase in retirement.

Next, empirical analysis is conducted with Health and Retirement Study, micro-level biennial panel data of senior workers in U.S., to examine actual retirement responses over the recent business cycle. Little evidence is found on a discernible increase in retirement rate owing to exposure of pension to the 2004 and 2006 positive shocks. However, the 2002 and 2008 negative shocks prove to lead to a decrease in retirement rate. In the view of theoretical findings, this is not self-contradictory but still can be consistent with a positive wealth effect on retirement; rather, it points to a case where these positive shocks are not sharp and large enough to bring substantive earlier retirement.

The third chapter, titled "Optimal Income Taxation and Optimal Revenue Mobilization," analyzes characteristics of nonlinear optimal income taxation and optimal revenue mobilization when the tax enforcement of a government is not costless (and thus not presumed to be perfect). The government cannot observe and verify an individual's innate ability although that ability turns out to cause inequality amongst them. This prevents the government from avoiding efficiency loss in the taxation, since each taxpayer can take advantage of private information over their own ability by reducing working hours to pretend to be less able than he truly is. Optimal income tax schedule is designed to minimize the efficiency loss from deterring such behavior to maximize social welfare. Moreover, the desired expenditure of the government is set for enhancing minimum living standard of society. In executing the tax schedule to finance this, however, tax evasion occurs due to imperfect enforcement. Although the government can verify the true amount of taxpayer's earnings, unlike their ability, it is costly to increase the enforcement rate. The optimal rate equalizes a gain of net increase in the tax revenue with a loss of decreased utility of risk-averse taxpayers from an increment in the rate. Notably, this chapter shows that aggregate loss of tax revenue can theoretically justify non-zero tax rate on top earners.
To my God the Almighty and Loving
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Chapter 1

Altruism, Reciprocity, and Equity: A Unified Motive for Intergenerational Transfers

1.1 Introduction

Intergenerational transfer has long been debated in the field of economics. For example, Adam Smith, the founding father of economics, advocated inheritance tax even though he argued against tax in general. Only in recent years, however, have economists begun to examine its motive, and there is little consensus. One of the many reasons why the motive matters is that Barro-Ricardian equivalence no longer holds when the transfer behavior does not follow Becker’s pure altruism model (1974), as Andreoni (1989) points out. That is, economic consequences of fiscal policies can differ by the motive of intergenerational transfers. Moreover, its magnitude would not be small because intergenerational transfer is still one of the main causes of wealth inequality (Kotlikoff and Summers 1981; Gale and Scholz 1994; Villanueva 2005; Piketty 2010).

In spite of its significance, a motive that offers consistent explanation on observed behavioral patterns has not yet been provided. To begin, we mainly observe only two extreme choices of distribution of bequests: either equally dividing the property

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1 For example, optimal estate taxation depends on the motive of intergenerational transfers.

2 For instance, Kotlikoff and Summers (1981) attribute 80% of US aggregate wealth to inheritances. Narrowing down to bequests and inter vivos gifts, Gale and Scholz (1994) find at least 51% of US wealth solely from these two transfers. None of these are including transfer of human capital in their analyses.

3 In fact, these two patterns of distributing bequests comprise most of inheritance patterns practiced around the world. For instance, 91.86% of 350 agrarian societies in the data from Ethnographic Atlas (Murdock 1967) have either equigeniture or unigeniture. Furthermore, this polarization is also found in data from Encyclopedia of World Cultures (Levinson 1991) that covers a wider variety of
(equigeniture) or giving all to just one offspring (unigeniture). This polarized pattern is interesting, especially to economists, since very diverse circumstances under which parents make the decision should result in a wide spread of choices for the distributions besides these two extremes. Moreover, throughout history, once the bequest pattern in a society shifts from unigeniture to equigeniture, it does not revert. Finally, at the end of this evolutionary trend, we observe that contemporary parents make inter vivos transfers unequally with bequests still divided equally; this is called "equal division puzzle." How can a motive for intergenerational transfers consistently rationalize this observed behavior? This chapter aims to present a unified model that coherently accounts for the aforementioned empirical realities.

In the first place, bequests and inter vivos gifts alike are transfers occurring within a family — the oldest and most ubiquitous and the oldest social institution. Family serves various basic needs of human beings through interactions between family members. Thus, intergenerational transfers may well reflect the interactions in the eyes of a parent. However, since they are non-market interactions, they would be better modeled in a non-standard way, as Becker (1974) first heralded. In his seminal work, a parent, whose transfer behavior is our central interest, is depicted as 'head of a family' in that his utility internalizes the utility of family members (children). Although his work is perceived as a model of altruistic motive, as a matter of fact, he maintains that the transfer is intended to lead children to work for a larger family income ("social income" in his term). A further thought on why the parent pursues a greater joint family income sheds light on another important factor in a parent's decision on the transfers: principal function of family. For most of human history, family functioned mainly as a production unit so that individual family members could subsist together. Each family member had to labor in farmland for subsistence (e.g., Millar 1970). In this light, a large family income means fulfillment of this principal role of family. Taken together, as head of a family, a parent not only altruistically cares about his children's utility but also wants the children to spend more effort to

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4Thus, it is not a simple coincidence that inheritance patterns discussed in the studies on intergenerational transfers (Stiglitz 1969; Pryor 1973; Blinder 1974; Bernheim and Severinov 2003) are only these two.

5Empirical findings from various industrialized countries — despite their institutional differences — share this feature that parents adopt equigeniture even as they distribute inter vivos gift unequally. First, this is found in the US by various data analyses (e.g., see Menchik 1980, 1988; Cox 1987; Cox and Rank 1992; Wilhelm 1996; Dunn and Phillips 1997; McGarry and Shoani 1995; McGarry 1999; Hochguertel and Ohišson 2009, etc.). At the same time, Kohli (2004) reports that this puzzling behavior is also observed in other industrialized countries - such as France, Germany, Israel, Norway, and Sweden in his review on the studies with nationally representative survey data from these countries. The finding of this stylized fact is also confirmed with other empirical studies on different datasets of the same country as well. For instance, Arrondel and Laferrière (1992) corroborate equigeniture with French microlevel data, while Olivera (2008) finds unequal transfer of inter vivos gifts in a microlevel data of European countries such as Germany, Sweden, France, Netherlands, etc.; and, Arrondel and Masson (1991) do so with French data.
fulfill the principal function of family, even though the effort is costly; and, transfers to children can give them incentive to exert that effort. By its nature, what the effort actually refers to depends on the primary function of family. With economic survival as the main function, it means physical effort for producing crops. However, it has undergone significant conversion as the primary function of family changed. In particular, a great transition of the principal function of family is brought about by industrialization. Through the course of industrialization, economic survival became less urgent and a less formidable task to a family as more of family members found better income sources outside the family farmland; moreover, many tasks that traditional families had undertaken such as education and care-giving were taken over by external institutions or commercialized (Hareven 1976). In the end, emotional support becomes the main function of family, establishing contemporary family as a "haven in a heartless world" (Lasch 1977). Accordingly, children’s effort that a parent cares about is now transformed into psychological and emotional one like affection to parents (Bernheim et al. 1985).

Notably, a parent also acquires utility directly from children’s effort when it is spent for family — whether for economic survival or for emotional support — and uses the transfer to influence the effort from children, since making an effort for others takes pains or some form of cost to them. This point is first advanced by Bernheim et al. (1985) and is called exchange motive since it views bequest as a reciprocal reward for children’s affection. This implies that any transfer scheme to induce effort from each child is hardly self-enforceable since the preference of a parent and children are not perfectly aligned. Moreover, transfer of bequests is usually executed by a third party post mortem; hence, the effort of each child must be verifiable in order for the parent to lay out the transfer scheme contingent upon children’s efforts. However, the effort level of each individual child is not verifiable to a third party. Obviously, psychological effort is unverifiable. Arguably, when manual effort of an individual child is spent for collective production in a non-market familial interaction, it is also very hard to prove (or disprove) the effort level of a child separated from other children. Thus, the transfer scheme depends upon parent’s wealth left for the transfer, which reflects children’s effort, since it is verifiable. Significantly, this incomplete information generates a moral hazard situation where children would not spend as much effort as a parent desires. As a remedy for this, the parent chooses unigeniture since the effort of the sole inheritor is immediately derived from the verifiable outcome and thus is provided at his first-best level. However, if productivity of effort is high enough, then the parent can be better off with choosing equigeniture even though all the inheritors exert second-best level of effort. Eventually, the parent ends up with one of the two stable equilibria — either unigeniture or equigeniture — depending on the productivity. In other words, the underlying force that leads the evolution from

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6Average wage of a parent alone began to be able to provide for all the other family members (Medick 1976).
unigeniture to equigeniture boils down to the productivity of effort for fulfilling the primary function of family.

In detail, the productivity of effort refers to how much an increase in parent’s utility can be produced by one unit of child’s effort spent for output of family production (whether harvested crops or psychological comfort). To begin, in pre-industrial societies where subsistence is the main task of ordinary family, it is represented by labor productivity of food production. So a rise of labor productivity such as the advance to intensive agriculture can trigger an evolution of inheritance pattern to equigeniture as observed in feudal China or India. In fact, another rise in the productivity is brought about by the transformation of child’s effort from labor to psychological support along the change in the principal function of family. That is, the same hours of a child’s effort generates a far greater increase in a parent’s utility if it is exerted for showing affection and respect to parents rather than for food production. Contrasted with manual effort, this new kind of effort produces a valuable output like "prestige and esteem" (Becker 1981) and a deep, special bonding which has no close market substitute (Cox 1987) and thus is priceless. Presumably, this rise can also be explained by the fact that the intangible process for emotional support is much less subject to the law of diminishing marginal returns than the physical process of food production. Above all, after industrialization, the inheritance pattern of a society would eventually evolve to equigeniture from unigeniture.

In addition, as a society becomes industrialized, fortunes that an individual can make become less dependent on the social class ascribed from his family; hence, the gap between children’s earnings within a family considerably widens. As a consequence, the payoffs to children who exert equal amount of effort are more likely to be unequal at the equilibrium of equigeniture. The parent can either ignore this or take a measure that preserves his equilibrium payoff. A focal point can be made on the latter if he perceives the gap to be large enough to undermine harmonious bonding between family members so that it develops a sort of equity issue; the parent may give *inter vivos* gifts to counterbalance the inequality. In particular, the amount of the gifts is negatively related with relative income of each child among their siblings whereas bequests are still equally divided regardless of the relative income.

Notably, this model of intergenerational transfers combines three motives — altruism, reciprocity (or exchange), and equity — and is consistent with the aforementioned empirical realities. Moreover, it renders itself to further empirical tests. Firstly, we can investigate data of inheritance pattern and labor productivity in pre-industrial agrarian societies. Secondly, we can also conduct empirical analysis on (i) whether income inequality among children initiates any *inter vivos* transfers and on (ii) how the amount of the gift is related to the relative income of each child.

This chapter is organized as follows. Section 1.2 thoroughly reviews previous research. Section 1.3 presents a theoretical model of intergenerational transfer behavior and characterizes equilibria. Section 1.4 empirically tests these results with various data sets. Finally, Section 1.5 concludes with discussion of the policy implications.
1.2 Critical Review of Related Literature

At first, the motive of intergenerational transfers is disregarded: bequeathing is treated as an ‘accidental’ happening due to the death that takes place earlier than parents’ expectation (Davies 1981). However, this is discredited by the findings that parents do earmark resources for bequests (e.g., Kopczuk and Lupton 2007; Kopczuk 2007). Most of all, this view of unintended bequest is not consistent with the fact that many parents do write a will long before their death.\(^7\)

Then, in an attempt to rationalize the transfer behavior, Andreoni (1989) claims that parents gain utility from behavior of giving to their children \textit{per se}, as they do from charity. However, this paternalistic view begs the question of why fairly large transfer is set to occur \textit{post mortem} even if it engenders positive utility to living parents. Moreover, this model does not offer any systematic predictions about the distribution of bequests (Kaplaw 2010).

More specific predictions that the transfer is negatively related to child’s relative income in a family are offered by Becker’s pure altruism model (1974). In his model, parents are altruistic in that they include the utility of each child into their own utility and always transfer resources to them. Through the transfers, the parents can lead their children — however selfish, or "rotten," they are — to take actions for a larger family income, which indirectly increase parents’ utility. (This is "Rotten Kid Theorem."\(^7\)) After all, the maximization of this parent’s utility begets the testable central result of negative relationship between children’s income and the transfer. However, this is challenged by various empirical findings. A great deal of studies on bequest behavior constantly find that parents divide bequests equally, regardless of income of each child (e.g., Menchik 1980; Wilhelm 1996; Arrondel and Laferrière 1992; Dunn and Phillips 1997; Light and McGarry 2004; Behrman and Rosenzweig 2004, etc.). In addition, unigeniture is not consistent with his model either.\(^8\)

On the other hand, Bernheim, Shleifer, and Summers (1985) present a model of exchange motive, taking into account that one family members can want a certain behavior which might be costly to other member (e.g., Kotlikoff and Spivak 1981; Manser and Brown 1980). In their model, since parents get utility directly from children’s effort spent for affective attention to them, they strategically utilize a share of the bequests to induce it. That is, a bequest is a reciprocal reward to the desired behavior of the child. Although this model of exchange motive allows wider forms of the rule governing division of bequests (such as unigeniture) than Becker’s model does, equigeniture are the least predicted knife-edge case just as in Becker’s model. Also, observed predominance of unigeniture in some societies is also not clearly explained by this model, either. In the end, therefore, why either of the two is predominant

\(^7\)For instance, according to the National Committee on Planned Giving in the US, most of bequest pledge makers are generally between 45 and 54 years of age.

\(^8\)Since not transferring all the other children is not an interior solution, unigeniture fails the "Rotten Kid Theorem" in the first place.
behavior in a society is hardly explained by any of models presented above.

Chu (1991) posits that parents seek to minimize the extinction probability of their family line and to raise its social class by concentrating the wealth, which lead them to choose primogeniture. Similarly, DeLong (2003) argues that primogeniture is motivated to heighten the social class of the eldest son of a family lineage. However, this perspective does not explain why parents in some societies follow primogeniture while those in others with similar (or even stronger) rigidity of class structure actually adopt equigeniture, given that all the parents have the same desire to keep their family lineage prosperous.

For a more comprehensive explanation, Bernheim and Severinov (2003) present the conditions that give rise to equigeniture and unigeniture, respectively. In their model, the utility of an altruistic parent is a weighted sum of utilities of each child which, in turn, directly depend upon the weight; however, the weight — parent’s relative affection to each child — is not known to the child. Under this asymmetric information, a bequest works as a signal for parent’s preference. Moreover, they argue that equigeniture is more likely to be prevalent in more open and mobile society since the return from parental support plays less dominant role. Although their work is pioneering in introducing incomplete information and thereby embracing the two polarized choices into a unified model, some of historical facts are left unexplained. For instance, primogeniture was practiced in England even in the seventeenth century, but equigeniture had already been adopted long before in less open and less mobile societies such as India under its strict caste system and feudal China with a definite social hierarchy. Also, their model turns mute on the distribution of inter vivos gifts.

Even when we are giving up observations across societies for the sake of attaining a model of the motive that coherently explains intergenerational transfer behavior only within a current society, the goal does not get easier to achieve. The most prominent challenge is “equal division puzzle” which refers to a seemingly inconsistent behavior of parents who divide bequests equally to their children (e.g., Menchik 1980, 1988; Wilhelm 1996; Arrondel and Laferrière 1992; Dunn and Phillips 1997; Light and McGarry 2004; Behrman and Rosenzweig 2004, etc.) and unequally distribute inter vivos gifts to them (e.g., Cox 1987; Arrondel and Masson 1991; Cox and Rank 1992; McGarry and Schoeni 1995; Dunn and Phillips 1997; McGarry 1999; Olivera 2008; Hochguertel and Ohlsson 2009, etc.).

To account for this puzzling behavior, various researchers capitalize upon Becker’s model of altruistic motive, relying on different assumptions. Firstly, Wilhelm (1996) introduces "psychic cost" from an unequal division and argues that equal division can be adopted if the payoff of unequal distribution under Becker’s altruism model does not exceed this "psychic cost." Since he does not differentiate the two forms of intergenerational transfers, this implies that inter vivos gifts should be equally distributed whenever bequests are so. To accommodate this distinction, Altonji et

Note that unlike his previous work (Bernheim et al. 1985), the exchange motive is dropped here.
al. (1997) and McGarry (1999) maintain that since altruistic parents are concern about uncertainty in children’s income (or permanent income) and liquidity constraint which the children confront, they make inter vivos transfers negatively associated with children’s current earnings while leaving bequests weakly related with the earnings. On the other hand, by differently assuming that inter vivos transfer is not publicly known, while bequest is not, Lundholm and Ohlsson (2000) claim that, under social norms of equality, altruistic parents care about their post mortem reputation far more\footnote{In their model, the consumption of children enters parent’s utility in the form of logarithm, while the payoff from post mortem reputation does a in a quadratic form. As a consequence, the marginal utility from the post mortem reputation is far larger than that from the welfare of children. For more detail, see Lundholm and Ohlsson (2000)} than about welfare of their children. In their model, one of possible equilibria is equal division of bequests with compensatory (thus unequal) inter vivos transfers. At a glance, it seems overcoming the limitation of other competing models; however, as Bernheim and Severinov (2003) point out, this model is tautological — so the model of Wilhelm (1996) is — since it relies heavily on the assumption that equal division is better for testator parents in order to get the equilibrium of equal division.

Besides little consensus in the theoretical explanations, there are also disagreements in empirical findings on how inter vivos transfer is related with children’s income. Some — like Cox (1987) and Cox and Rank (1992) — report the positive association between the amount of inter vivos gift and children’s income, whereas others - like McGarry and Schoeni (1995); Dunn and Phillips (1997); Hochguertel and Ohlsson (2009) - claim the opposite. Especially, they perceive exchange motive (Bernheim et al. 1985) as competing against altruistic motive (Becker 1974) and evince their finding to reject one of the two, announcing that their main task is to test whether a parent gives more to a child who is less well off than other children in the family or not.

However, if we try to prudently and scrupulously take above empirical findings for making the case against (or for) one of the two motives (exchange vs. altruism), none of them provide exact fitting to this purpose. First, due to limited data availability, some of them provide estimates of correlation between the amount of inter vivos gifts and income of a child’s household (McGarry and Schoeni 1995; Dunn and Phillips 1997; Cox and Rank 1992; Hochguertel and Ohlsson 2009), instead of a child’s own income. This replacement hamstrings clear identification with serious noise from earnings of other persons in a child’s household. Even when one treats individual income of children as if equal to household income of them and finds a negative correlation of it with amounts of the inter vivos gift given, one still needs to be cautious in regarding it as evidence firm enough to stand for Becker’s model since the negative correlation is found between inter vivos transfer and absolute amount of income, not relative income in the family. A simple thought experiment can clarify this point. Suppose that there are two families which are identical except for children’s income. In one, every child earns exactly the same level of income, which are fairly
low. In the other, only one child earns the same amount of low income as any child in the former family does, but all of the one’s siblings earn considerably higher incomes than the one does. Becker’s altruism model posits that the amount of *inter vivos* transfer to the one in the latter family should be higher than to any child in the former family even though incomes of recipients are exactly the same; on the contrary, a negative correlation found in the regression of the supposed children’s income against the *inter vivos* gift tells us that the amount of the gifts given to those two with the same income from the two different families will be the same.

In a nutshell, review of previous studies on the motive for intergenerational transfer behavior reveals that there are persistently large discrepancy between theory and observations and that the various studies can be eventually penetrated by the two most widely accepted and mutually competing models: altruistic motive (Becker 1974; Bernheim and Severinov 2003) and exchange motive (Bernheim et al. 1985). In fact, these two also serve as important pillars underpinning the model in this study, which will be elaborated in the following sections.

### 1.3 Theoretical Model

#### 1.3.1 Choice Environment

Family, as a basic social unit, serves its members by fulfilling its function that in turn involves effort from them, as mentioned in the beginning. So, consider a family production function $F(a_1, \cdots, a_n) : \mathbb{R}^n_+ \rightarrow \mathbb{R}$, where $i \in \{1, \cdots, n\}$ indexes children who are eligible to inherit, and $a_i$ is effort spent by a child $i$. The output of $F$ can be either crop yield or emotional betterment, according to the principal function of family. Likewise, $a_i$, input of $F$ is defined as either manual effort (for the former) or psychological effort (for the latter), depending on the output. Suppose that $F$ is continuously differentiable, strictly increasing in each argument, symmetric with $F(0, \cdots, 0) = 0$ and follows the law of diminishing marginal returns (i.e., $F_{ii} \leq 0, \forall i$). Symmetricity means that whoever works one unit of effort is treated same since it is the same input for production. Furthermore, as a parent recognizes each child individually and values their effort impartially, let $F(a_1, \cdots, a_n) = \sum_{i=1}^n f(a_i)$. This

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11A rule that decides who is an eligible inheritor varies by society. For instance, many societies do not consider an out-of-wedlock child as an eligible inheritor. To take another example, some societies do not bequeath to women who leave home after marriage. While other societies such as some matrilineal societies, son is not eligible to receive any bequests from the father. In this model, it takes this aspect as given.

12Different inputs may enter a production function differently, whereas the same input should enter it in the same way. So, the same inputs of effort $a_i$ have to be dealt identically. Put differently, whoever works, one unit of effort is treated as the same. In this sense, symmetricity assumption is standard. Moreover, effort provider is relatively homogeneous. That is, compared to a child of a different family, a child would not be significantly different from his siblings.
implies that $f' \geq 0$ and $f'' \leq 0$. Since we are studying distribution of intergenerational transfers among children, only child case\textsuperscript{13} is not of interest here; hence, let $n > 1$.\textsuperscript{14}

As a head of a family, a parent would want his family to perform its function better; hence, he prefers a larger output of $F$, which needs more of children’s effort for family even though effort is costly to them. At the same time, he also cares altruistically about welfare of each child. Therefore, these two factors constitute utility to the parent; that is, parental utility function $U_p$ is

$$U_p = \sum_{i=1}^{n} f(a_i) - \sum_{i=1}^{n} t_i + \sum_{i=1}^{n} U_i$$

(1.1)

where $t_i$ is a material reward to child $i$’s effort $a_i$, and $U_i$ is child $i$’s utility. Notice that $a_i$ is not a choice variable to the parent while $t_i$ is. Clearly, $a_i$ is chosen by a child $i$. Since effort $a_i$ takes time and energy, child $i$’s utility is

$$U_i = t_i - c(a_i) + y_i$$

(1.2)

where $y_i$ is income that child $i$ can earn from outside of family while he (or she) is spending $a_i$ for family. Notably, $y_i$ neither is directly used for family production nor affects the choice of effort $a_i$. This parameter is just meant to capture potential heterogeneous aspects among children. Suppose that $c(\cdot)$ is continuously differentiable, strictly increasing, and convex (i.e., $c'' \geq 0$) with $c(0) = c'(0) = 0$.

After all, it is noteworthy that this model dialectically synthesizes the two views that have been perceived as competing against each other: exchange motive (Bernheim et al. 1985) and altruistic motive (Becker 1974; Bernheim and Severinov 2003). This combination manifest itself via an interaction between the parent and the children. The basic time line of the interaction is as follows: (i) the parent announces a payment scheme of $t_i$s; (ii) the children take actions, $a_i$s; and, (iii) according to the scheme, $t_i$s are transferred.

First of all, insofar as the effort is disutility to the children, the preferences of both sides are not perfectly aligned. Thus, any payment scheme of $t_i$s is hardly self-enforceable. Nonetheless, if there exists a third party that executes and resolves disputes over the payment scheme like a will, it now can be enforceable. The presence of a third party, such as a judge at a probate court, makes parent’s commitment to the will credible. As a result, each child $i$ decides their own effort, $a_i$, based on $t_i$. As such, through a payment scheme of $t_i$s, the parent can manipulate the effort levels of each child even though he cannot directly choose them. Eventually, since their efforts collectively determine the total output of family production function, say $q = F(a)$,

\textsuperscript{13}There is no difference between equigeniture and unigeniture if a parent has only one offspring.

\textsuperscript{14}Nevertheless, this would not be a big issue, since the fertility rate is not yet as low as 1; for instance, in the US, the birth per women has long been around 2 for more than three decades. (Source: World Bank, World Development Indicator)
the transfer scheme of \( t_i \)'s plays a crucial role in maximizing \( U_p \), although they look like disappearing as in \( \dot{U}_p = \sum_{i=1}^{n} f(a_i) - \sum_{i=1}^{n} t_i + \sum_{i=1}^{n} t_i - \sum_{i=1}^{n} c(a_i) + \sum_{i=1}^{n} y_i = \sum_{i=1}^{n} f(a_i) - \sum_{i=1}^{n} c(a_i) + \sum_{i=1}^{n} y_i. \) To be better off, the parent will make a reward scheme \( t_i \)'s a contingent plan, since laying out constant payments induces zero effort for any given \( t_i \). In the first place, the parent would attempt to arrange a transfers scheme to directly depend on individual effort \( a_i \) of each child. Notably, for such a payment scheme to be executed, effort of each individual child should be verifiable to a third party, because unverifiable action is not suitable to enforce and thus cannot serve as contingency (Hart 1987). However, individual level of each child’s effort \( a_i \) is not verifiable. Firstly, it is clear that psychological effort is unverifiable since it is not observable. Secondly, manual effort of each child is also unverifiable since even when the parent may be able to observe the effort as an insider, a judge, outside of family, would not. Moreover, since the manual effort is put in a collective production through a non-market interaction, it is also very hard to vindicate effort level of each child separated from any other child. In addition, any child would not be adopted as a reliable witness. Therefore, a reward scheme \( t_i \)'s cannot be contingent upon effort \( a_i \).

For the best possible alternative, total output, \( q \), can serve contingency for a transfer scheme of \( t_i \)'s since it is verifiable and reflects children’s effort. In practice, \( q \) takes form of parent’s wealth left for the transfer; hence, it is observable to a third party (verifiable). Furthermore, it depends on children’s effort as \( q = \sum_{i=1}^{n} f(a_i) \). Eventually, the reward scheme \( t_i \)'s will be contingent upon total output \( q \). Let this contingent plan\(^{15} \) be an inheritance rule \( t \), which is a vector-valued function of a final output; that is, \( t = (t_1(q), \ldots, t_n(q)) \). Put another way, an inheritance rule is a contract between an altruistic but strategic parent and selfish children who can provide effort that can raise the parent’s utility. An inheritance rule can be increasing or decreasing in \( q \). However, it is unreasonable for the parent to select a decreasing inheritance rule, because if more output leads to less reward, spending effort will be discouraged, which is entirely opposite to what the parent pursues. So, we can rule out this case; that is,

\[
t_i' \geq 0 \quad \forall i
\]

Along with this monotonicity condition, it is without loss of generality to focus on piecewise continuously differentiable inheritance rule \( t \). Moreover, not only because

\(^{15}\)Insofar as the detailed information on individual effort is not verifiable to a third party, re-writing a will does not make any difference, even when we assume that re-writing the reward scheme does not incur a cost at all. First, the set of implementable actions remains the same if we consider re-writing the scheme. Recall the point of renegotiation: It makes a selfish principal better off since it reduces the payment to an agent as reservation utility after the action is taken and before uncertain outcome is realized (Hermalin and Katz 1991). In contrast, this reduction may not make the altruistic parent better off. Also, it is hard to find the proper time for renegotiation because, in this model, at the moment when the effort is spent, the output is realized without uncertainty. So, the reward scheme \( b \) is renegotiation-proof.
the sum of transfers cannot exceed total output, but also because it is neither credible
nor efficient to leave resources unused,

$$\sum_{i=1}^{n} t_i(q) = q \text{ for } \forall q \in \mathbb{R}_+. \quad (1.4)$$

As mentioned above, the reason why a transfer scheme \( t \) plays a critical role is
that it enables the parent to induce effort that determines his utility but cannot be
directly chosen by him. The way an inheritance rule \( t \) induces a certain target level of
effort levels is just to provide children incentives to implement them. In other words,
\( t \) should be compatible with children’s incentive. In the end, therefore, the parent’s
problem boils down to finding an inheritance rule \( t \) that implements \( a \) for maximizing
\( U_p \). In particular, \( a \) is implementable (or meeting incentive compatibility constraint)
if and only if there exists an inheritance rule \( t \) such that

$$U_i(t_i(F(a)), a_i) \geq U_i(t_i(F(a_i^\prime; a_{-i}), a_i^\prime) \text{ for } \forall a_i^\prime \in \mathbb{R}_+ \text{ and } \forall i. \quad (1.5)$$

Realizing complexity of the task that the parent is seeking a function \( t \) (not values
of variables) without a clue on function’s features, one may as well think that we need
to impose further restrictions on functional forms of \( f \) and \( c \) to obtain an optimal
inheritance rule in a concrete form. However, we do not have to sacrifice generality of
this model. Rather, we can proceed this analysis in a much simpler way and with more
tractability. That is, we do not need to look further than affine function because there
always exists a linear inheritance rule that implements exactly the same outcome as
any optimal inheritance rule would do.

**Lemma 1.1.** For any implementable \( a \) with \( t \), there exists an affine inheritance
rule that leads to the same effort from each child as \( t \) does.

**Proof.** First, we need to show that \( t \) is continuous at \( a \). Suppose not, then there
exists \( \delta > 0 \) such that \( t_i(F(a) + \varepsilon) - t_i(F(a) - \varepsilon) \geq \delta \) for some \( i \) and \( \forall \varepsilon > 0 \). Pick any
\( \varepsilon \in (0, \frac{\delta}{2}) \). Due to the feasibility constraint (1.4) and monotonicity condition (1.3),
aggregation of \( t_i(F(a) + \varepsilon) - t_i(F(a) - \varepsilon) \) results in \( \sum_{i=1}^{n} t_i(F(a) + \varepsilon) - \sum_{i=1}^{n} t_i(F(a) - \varepsilon) = 2\varepsilon \geq \delta \). This is a contradiction to \( \varepsilon < \frac{\delta}{2} \), so \( t \) is continuous at \( a \).

Since \( t \) is piecewise continuously differentiable, its continuity at \( a \) implies that
there exists \( \eta > 0 \) such that we have \( t_i'(F(a))f' - c'(a_i) = 0 \) for \( \forall i \) in an open set
of \( (F(a) - \eta, F(a) + \eta) \) since \( t \) is implementable. Let \( b_i \equiv \frac{c'(a_i)}{f'} = t_i'(F(a)) \) for \( \forall i \).
Since \( f \) is strictly increasing, \( b_i \) is always defined; moreover, since \( c \) is increasing as
well, \( b_i \) is positive for \( \forall i \). In addition, let \( g_i \equiv t_i - b_iF(a) \) for \( \forall i \). Now, consider a vector-valued function \( \tilde{t} \) such that \( \tilde{t}_i = g_i + b_iF(a) = g_i + bq \) for \( \forall i \). By construction, 
\( \tilde{t}_i = t_i \) for \( \forall i \) and \( \sum_{i=1}^{n} \tilde{t}_i = q \). Thus, \( \tilde{t} \) is also an inheritance rule.
Then, we want to show that this affine-sharing rule $\hat{t}$ leads the exactly same outcome as the original $t$ would implement. This is the case\footnote{In fact, we need to check S.O.C as well, and this holds when the share $b_i$s are increasing in effort, which is detailed in the next paragraph in the light of the monotonicity condition.} since $\tilde{t}_i'(F(a))f' - c'_i(a_i) = 0$ for $\forall i$ and $\tilde{t}_i' = b_i = t'_i$ for $\forall i$. QED

As the first step for finding an optimal inheritance rule $t^*$ that induces $a$ maximizing $U_p$, the parent tries to figure out how children would behave responding to an offered inheritance rule $t$, as a way of backwards induction. Each child $i$ chooses his effort $a_i$ such that

$$a_i = \arg \max_{a_i} b_i F(a_i; a_{-i}) - c(a_i)$$

since given inheritance rule $t$ is completely defined with $n$ pairs of $(b_i, g_i)$ where $i \in \{1, \ldots, n\}$ as $t_i$ is expressed as $g_i + b_i F(a)$ for $\forall i$, according to the proof of Lemma 1.1. Moreover, since both $y_i$ and $g_i$, as constant terms, do not affect the maximization of $U_i$, they are not part of its kernel as appears in (1.6). The decision rule of child $i$ is derived from the F.O.C\footnote{In fact, this is equivalent to the incentive compatibility constraint of (1.5) for child $i$.} of this program,\footnote{Since $f$ is concave and $c$ is convex, F.O.C of this maximization is sufficient.}

$$b_i f' - c'(a_i) = 0$$

since $F_i(a_i; a_{-i}) = f'$. As a stepping stone for the decision rule, or best response function, of child $i$, let

$$b_i = \frac{c'}{f'} = \gamma(a_i).$$

This is well-defined since $f$ is strictly increasing. In addition, following the logic of the monotonicity condition (1.3), we can opt out a decreasing $\gamma$ since if more effort is not rewarded more, children would not spend costly effort. Equivalently, if less effort is rewarded more, then there will be a sabotage. Thus, we can focus on $\gamma$ that is strictly increasing (i.e., $\infty > \gamma' > 0$).\footnote{Note that this monotonicity condition makes the F.O.C of $\tilde{t}_i'(F(a))f' - c'(a_i) = 0, \forall i$ in the Lemma 1 be sufficient since $\gamma' > 0$ implies that S.O.C is satisfied given the F.O.C met.} Then, we can derive its inverse function $\gamma^{-1}$ which is nothing but the best response function of the child $i$

$$a_i(b_i) = \gamma^{-1}. \quad (1.9)$$

Notice that the best response function does not depend on $g_i$. Therefore, searching for an optimal inheritance rule $t^*$ boils down to finding a vector $b^* = (b_1^*, b_2^*, \ldots, b_n^*)$ that maximizes $U_p$. In sum, the parent actually seeks to find $b^*$ that solves

$$\max_b \sum_{i=1}^n f(a_i(b_i)) - \sum_{i=1}^n c(a_i(b_i)) \text{ s.t } b_i f' - c' = 0 \text{ for } \forall i \quad (1.10)$$
Note\(^{20}\) that the constraint above is equivalent expression of incentive constraint in (1.5). More importantly, observe that \(b\) operates as the leverage with which the parent induces effort from each child. Therefore, since the parent cares about his family as long as he is alive, children’s efforts matter to him until he dies. This means that actual time of executing \(b\) is *post mortem*; in other words, \(b_i\) stands for bequest to child \(i\). As a matter of fact, \(b_i\) is a share of a total output to which a child \(i\) contributes his effort since

\[
\sum_{i=1}^{n} t_i' = \sum_{i=1}^{n} b_i = 1 \text{ and } b_i \geq 0 \text{ for } \forall i
\]

(1.11)

based on (1.3), (1.4), and, **Lemma 1.1**. Mathematically, this means that \(b\) lies in \(n\)-dimensional simplex \(\Delta^n\). In contrast, \(g_i\) can freely take any value as long as \(\sum_{i=1}^{n} g_i = 0\) due to the feasibility constraint (1.4). Since \(g_i\) is not affecting effort of each child and thus does not play a role in the parent’s optimization (1.10), it is an auxiliary and flexible term. However, since the parent impartially cares about each child, he may as well set its default value as

\[
g_i = 0 \text{ for } \forall i
\]

(1.12)

unless no specific reason to change this is provided.

### 1.3.2 Characterization of Equilibria

For an informative characterization of optimal inheritance rules, it is useful to demonstrate the crux of the problem (1.10) that the parent solves. First of all, notice that (1.10) is basically a team problem; hence, as Holmstrom (1982) points out, Pareto optimal outcome is not achievable. To show this, hypothetically suppose that individual effort levels of each child are verifiable and thus can serve as contingency for a transfer scheme. Then, the parent can induce the first-best levels of effort \(a^{FB}\) from all the children by penalizing harshly any deviation from \(a^{FB}\). In particular, \(a^{FB} = (a_1^{FB}, \ldots, a_n^{FB})\) is defined by \(f' - c'(a_i^{FB}) = 0\) for \(\forall i\). It is obvious from (1.10) that Pareto optimal levels of effort \(a^{FB}\) is not possible to implement since there must exist \(b_i \in [0, 1)\) such that \(b_i f' - c'(a_i) = 0\) due to (1.11) and \(n > 1\).

To efficiently illustrate the economic intuition underlying this failure of Pareto optimality, find that this is similar to a problem where public goods with positive externality are under-provided. In the current context, the parent plays the role of a social planner; and, each child does that of a private provider of a public good: effort for family. In the one hand, since parent internalizes costs to children, marginal private cost of effort to every child is equal to marginal cost of it to the altruistic parent (MPC=MSC in the public goods provision context). On the other hand, however,

\(^{20}\)Also, since \(\sum_{i=1}^{n} y_i\) does not affect the maximization of \(U_p\), it is not part of its kernel.
marginal private benefit of effort to each child is smaller than marginal benefit of it to the parent (i.e., \( b_i f' \leq f' \), or \( \text{MPB} \leq \text{MSB} \) in the public goods provision case). As a result, effort is hardly provided as much as the parent desires. To get more concrete sense of why, consider a parent who has two sons; and, inheritance is the channel through which he affects effort of his sons. Now, suppose the second son spent the first-best level of effort for family, whereas the first son shirked exerting strictly less effort than his brother did. However, the parent is not able to write his will that effectively punishes this exactly, since individual effort is not verifiable to a third party adjudicator. Thus, at the end of the day, some part of bequest that should have been remunerated to the second son ends up with being given to the first. In other words, marginal contribution for family by the second son’s effort is not fully compensated (\( b_i f' \leq f' \)); as a consequence, the second son loses incentive to spend his effort to the full extent. Briefly put in terms of public goods provision, a shirker can free-ride on effort of a hard worker; the parent cannot exclude out this free-riding behavior, which dampens incentive to work hard. In a nutshell, since this non-excludability originates from asymmetric information, this is a moral hazard problem where the altruistic parent suffers under-provision of effort by selfish children.

Given these circumstances, the parent still seeks a second-best solution by solving (1.10). At the outset, we can simplify (1.10) by plugging (1.9) into the object function, since invertibility entailed from no sabotage condition \( \infty > \gamma' > 0 \) makes the set of \((a, b)\) defined by the IC constraints in (1.10) equivalent to that defined by (1.9) for \( \forall i \). To begin, the first derivative of the simplified (1.10) is

\[
(f' - c') \frac{1}{\gamma'} \geq 0 \text{ for } \forall i
\] (1.13)

due to \( f' - c' \geq b_i f' - c' = 0 \). This points to a possibility that an optimal inheritance rule would take a form of corner solution. However, this does not yet give us any definite feature about \( b^* \). So, the attempt for characterizing optimal inheritance rules leads us to eye the second derivative, \( (f'' - c'') \frac{1}{\gamma'} - (f' - c') \frac{\gamma''}{(\gamma')^2} \). Furthermore, rearranging its terms, we obtain that

\[
\text{sign}\{(f'' - c'') \frac{1}{\gamma'} - (f' - c') \frac{\gamma''}{(\gamma')^2}\} = -\text{sign}\{-\frac{f'' - c''}{f' - c'} - \frac{\gamma''}{\gamma'}\}
\] (1.14)

which actually turns out to be a determinant in shaping optimal inheritance rules.

To pave the way for characterizing equilibrium inheritance rules in a more illustrative way, it is worthwhile clarifying how \( \text{sign}\{-\frac{f'' - c''}{f' - c'} - \frac{\gamma''}{\gamma'}\} \) plays a crucial role. First of all, \( f' - c' \) is marginal gain to the parent from an infinitesimal increment in effort spent for family production, and \( \gamma' \) is marginal increase in \( b_i \) as a payment for the effort. That is, the former represents marginal benefit from effort, and the latter corresponds to marginal cost of inducing the effort. However, due to disagreement in their units, a direct comparison between both is improper. Nevertheless, their growth
rates are still comparable. Namely, \(-\frac{f'' - c''}{f' - c'}\) refers to a growth rate of marginal gain from effort for family and \(-\frac{c''}{c'}\) represents the counterpart of marginal cost of inducing the effort. Obviously, the parent faces one of the three cases that are mutually exclusive and exhaustive, depending on which is greater: (i) marginal cost increases more rapidly than marginal gain \((\frac{f'' - c''}{f' - c'} < -\frac{c''}{c'})\); (ii) marginal gain increases more rapidly than marginal cost \((\frac{f'' - c''}{f' - c'} > -\frac{c''}{c'})\); (iii) both increase at the exactly same rate \((\frac{f'' - c''}{f' - c'} = -\frac{c''}{c'})\).

Moreover, in order to illustrate why the comparison between both matters for characterizing optimal inheritance rules, let the parent do a grid search for the optimization, which is more manual than analytical solving \((1.10)\) even though both methods yield the same solution. For an initial point of the search, any arbitrary vector in \(n\)-dimensional simplex \(\triangle^n\) would work. However, for the sake of efficiency, based on the clue from \((1.13)\), one of the best candidates for the starting point is be a corner solution: \(e_i\) for some \(i \in \{1, 2, \ldots, n\}\), where \(e_i\) is a \(n\)-dimensional vector whose elements are all zero but \(i\)th element taking value one. The parent begins the search by comparing the payoff at this initial point with that from the next closest grid which is a net gain from effort newly induced by an increment in \(b_j\) for some \(j \neq i\). Apparently, if marginal cost grows faster than marginal gain \((\frac{f'' - c''}{f' - c'} < -\frac{c''}{c'})\), then the payoff from the next step is smaller than that from the starting point. This keeps applying for any other additional following steps. As a result, the initial point will survive as an equilibrium. In contrast, if the opposite is true \((\frac{f'' - c''}{f' - c'} > -\frac{c''}{c'})\), then the parent would keep searching until he finds the point where there is no further room to be better off. Lastly, though very rare, if both grows at a precisely equal rate \((\frac{f'' - c''}{f' - c'} = -\frac{c''}{c'})\), the parent is indeterminate between the two. In this manner, \(\text{sign}\{\frac{f'' - c''}{f' - c'} - (\frac{c''}{c'})\}\) plays a determining role in designing an inheritance rule. Most of all, how optimal inheritance rules look like can be fully characterized by investigating these three cases.

**Proposition 1.1.** When marginal gain from effort spent for family production grows slower than marginal cost of inducing the effort from children \((\frac{f'' - c''}{f' - c'} < -\frac{c''}{c'})\), unigeniture is optimal; that is, only one child takes all the bequests at an equilibrium. Moreover, this equilibrium distribution of bequests is unique (under the condition \((\frac{f'' - c''}{f' - c'} < -\frac{c''}{c'})\)).

**Proof.** It is enough to show that the simplified \((1.10)\), denoted by \(U_p(b)\) in short, is Schur-convex in \(b\). This is equivalent\(^{21}\) to show that \((b_j - b_k)(U_{pj} - U_{pk}) > 0\) for any \(b_j \neq b_k\), since \(U_p(b)\) is continuously differentiable due to no sabotage condition

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\(^{21}\)This condition is called Schur’s condition since Schur (1923) proved it as necessary and sufficient for being Schur-convex with the case of \(I=(0,\infty)\) although Ostrowski (1952) generalized it with arbitrary open interval cases.
$(\infty > \gamma' > 0)$ and continuous differentiability of $f$ and $c$.

Pick any arbitrary $b_j \neq b_k$ from the relevant domain of $b$ (n-dimensional simplex $\Delta^n$) and, without loss of generality, suppose $b_j > b_k$. Moreover, $U_{p_{ii}} = (f'' - c'') \frac{b_i}{b_j} - (f_i - c') \frac{\gamma''}{\gamma'}$ for $\forall i$. Therefore, due to (1.14), $-\frac{f'' - c''}{f - c} < -\frac{\gamma''}{\gamma'}$ implies that $U_{p_{ii}} > 0$, which means $U_{p_{jj}} - U_{p_{kk}} > 0$. This in fact shows that giving all to only one child (unigeniture) is an equilibrium rising under the condition of $-\frac{f'' - c''}{f - c} < -\frac{\gamma''}{\gamma'}$. Let a unigeniture equilibrium be notated by $b^* = e_i$ for some $i \in \{1, 2, \ldots, n\}$, where $e_i$ is a n-dimensional vector whose elements are all zero but $i$th element taking value of one. Consequently, there can be $n$ possible equilibria, depending on who is the only heir (or heiress), all of which have the same distribution.

Then, we need to prove that this equilibrium distribution is unique under this condition ($-\frac{f'' - c''}{f - c} < -\frac{\gamma''}{\gamma'}$). To show this, suppose there exists another equilibrium $b'$ that is distributively different from $b^* = e_i$ for any given $i \in \{1, 2, \ldots, n\}$. Then, $b'$ has, at least, two strictly positive elements, all of which are smaller than one. Also, due to strict monotonicity of $f$ and $c$, $U_p(b') \neq U_p(b^*)$. By construction, $\sum_{i=1}^{m} b'_{[i]} = 1 \geq \sum_{i=1}^{m} b_{[i]}$ for all $m < n$, where $b'_{[i]}$ and $b_{[i]}$ are order statistics which are ordered from the greatest. Due to (1.11), $\sum_{i=1}^{n} b'_{[i]} = \sum_{i=1}^{n} b_{[i]} = 1$, which means that $b^*$ majorizes $b'$. By the property of Schur-convexity, this implies that $U_p(b') < U_p(b^*)$. A contradiction to the assumption that $b'$ is an equilibrium. QED

**Proposition 1.2.** When marginal gain from effort spent for family production grows faster than marginal cost of inducing the effort from children ($-\frac{f'' - c''}{f - c} > -\frac{\gamma''}{\gamma'}$), equigeniture is optimal. Moreover, this equilibrium is unique (under the condition $-\frac{f'' - c''}{f - c} > -\frac{\gamma''}{\gamma'}$).

**Proof.** By a similar token of the first part in the proof for Proposition 1.1, it is enough to show that $U_p(b)$ is Schur-concave in $b$, which is equivalent to show that for any $b_j \neq b_k$, $(b_j - b_k)(U_{p_{jj}} - U_{p_{kk}}) < 0$.

Pick any arbitrary $b_j \neq b_k$ from the relevant domain of $b$ (n-dimensional simplex $\Delta^n$) and, without loss of generality, suppose $b_j > b_k$. Moreover, $U_{p_{ii}} = (f'' - c'') \frac{b_i}{b_j} - (f_i - c') \frac{\gamma''}{\gamma'}$ for $\forall i$. Therefore, due to (1.14), $-\frac{f'' - c''}{f - c} > -\frac{\gamma''}{\gamma'}$ implies that $U_{p_{ii}} < 0$, which means $U_{p_{jj}} - U_{p_{kk}} < 0$. This shows that equal division of bequests (equigeniture) is an equilibrium rising under the condition of $-\frac{f'' - c''}{f - c} > -\frac{\gamma''}{\gamma'}$. Denote this by $b^* = \frac{1}{n} \mathbf{l}$, where $\mathbf{l}$ is a n-dimensional vector whose elements are all one.

Then, we need to show that this equilibrium $\frac{1}{n} \mathbf{l}$ is unique. To show this, suppose there rises another equilibrium $b' \neq b^*$ which maximizes (1.10). Firstly, due to strict monotonicity of $f$ and $c$, $U_p(b') \neq U_p(b^*)$. Due to (1.11), $\sum_{i=1}^{n} b'_{i} = \sum_{i=1}^{n} b_{i} = 1$. Moreover, since every n-dimensional vector whose elements sum to 1 majorizes uniform vector $\frac{1}{n} \mathbf{l}$, $b'$ majorizes $b^*$. By the property of Schur-concavity, this means

\footnote{This is shown by a mathematical induction. In brief, this makes intuitive sense because the...}
that $U_p(b') < U_p(b^*)$. A contradiction to the assumption that $b'$ is an equilibrium. 

QED

For the third case of $\frac{f''(c')}{g''(c')} = \frac{\gamma''}{\gamma'}$, which is knife-edge,$^{23}$ characterizing equilibrium/equilibria as definite as the previous two other cases is not feasible without more restrictions. In addition, an equilibrium, if any, might not be unique. More importantly, parameters, in any specific $f$ and $c$, which represent the growth rates are meeting either $-\frac{f''(c')}{g''(c')} < -\frac{\gamma''}{\gamma'}$ or $-\frac{f''(c')}{g''(c')} > -\frac{\gamma''}{\gamma'}$ for almost all the values that they can take; in other words, it is least likely for them to meet $\frac{f''(c')}{g''(c')} = \frac{\gamma''}{\gamma'}$. This ‘sharp tie’ case is much more likely to be lost by tiny perturbations in the parameters than any one of the other two cases is. In other words, the third knife-edge case is not stable, contrasted the other two which yield stable equilibria of either equigeniture or unigeniture.

In the end, the comprehensive investigation of all the possible cases discovers that the space of optimal inheritance rules has a remarkably simple landscape having only two specific stable equilibria - either equigeniture or unigeniture.$^{24}$ This is consistent$^{25}$ with polarized behavioral pattern that there mostly have been these two opposite inheritance rule practiced across diverse times and places.

In the light of the questions cast at the very beginning, these results lead us to ask what can be a driving force of the evolution from unigeniture to equigeniture. Obviously, as an increment in effort for family produces more output and requires less disutility to children (thus less disutility to the altruistic parent), the marginal gain grows faster than the marginal cost for inducing the effort. In other words, as the effort is more productive, marginal return from it is more likely to grow faster than marginal cost of it. This corollary from Proposition 1.1 and Proposition 1.2 in a way that is formal and relatable to observable data. To this end, consider$^{26}$

$$f(a_i) = \theta a_i^\frac{1}{\theta} \text{ and } c(a_i) = \frac{1}{m} a_i^m.$$  

(smallest disperseness between elements in a n-dimensional vector is achieved when it is uniform vector. For more detail, refer to Marshall and Olkin (1979).

$^{23}$Contrasted to this, in previous studies like Becker (1974), Bernheim et al. (1985), both equigeniture and unigeniture are knife-edge cases.

$^{24}$However, this model explores distributional aspects only; hence, it does not offer an explanation about why the oldest child is chosen under primogeniture while the youngest one under ultimogeniture.

$^{25}$In addition, this model also shows consistency with other aspects of bequest behavior as well. For instance, in the logic of the model, it is reasonable for the parent to intentionally write a will since it gets a third party involved and thus engenders a binding force so that both can follow through the will although their preferences are not perfectly aligned.

$^{26}$In fact, we can get the same result with with $a_i^\frac{1}{\theta}$ and $a_i^m$, but unfolding equations for the proof will be much less neat than these form. What matters is the superscript $\theta (m)$ that governs productivity. Moreover, this choice functional form can obtain some generality since $f(a_i) = \theta a_i^\frac{1}{\theta}$ is a kind of Cobb Douglas production function with other inputs, if any, constantly given.
Due to the law of diminishing marginal returns, \( \theta \geq 1 \); and, due to convexity of cost function, \( m \geq 1 \), which implies that \( a_i \in [0, 1] \). Moreover, these functions can be with a decent generality because (i) \( \theta a_i^{\frac{\theta}{\gamma}} \) is a kind of Cobb Douglas production function with other inputs, if any, given and (ii) a convex function is generally used as a cost function. First of all, \( \theta \) governs marginal productivity\(^{27} \) of family production as \( \frac{\partial MP(\theta)}{\partial \theta} > 0 \) where \( MP(\theta) = \frac{\partial f(a_i)}{\partial a_i} = a_i^{\frac{1-\theta}{\gamma}} \). An increase in \( \theta \) unequivocally means improved marginal productivity of family production in real terms. Hence, when the effort refers to manual labor for crop yield, it is possible for us to find corresponding data that captures a rise in \( \theta \). At the same time, \( \theta \) is a parameter of the productivity of effort because an increase in \( \theta \) leads one unit of child’s effort to yield more increase in the parent’s utility. On the other hand, the interpretation of an increase in \( m \) is not exactly interpreted as a decrease in marginal \( \frac{\partial \text{Total cost}}{\partial \text{output}} \) since total cost is based on expenditure on purchase from factor market. However, a reciprocal reward to effort for family production is not priced as in a usual factor market. Nevertheless, an increase in \( m \) decreases marginal disutility of effort for family; moreover, disutility from effort for family is born by the altruistic parent even though children provide effort. This implies that an increase in \( m \) means that the same amount of effort can give larger utility to the parent. Thus, \( m \) is another parameter for the productivity of effort.

Now, in the light of Proposition 1.1 and Proposition 1.2, we examine how the productivity of effort for family, captured by an increase in \( \theta \) (or \( m \)) is related with the determinant denoted by

\[
D = \frac{-f''}{f'} - c'' - (-\frac{\gamma''}{\gamma'})
\]

(1.16)

which can take either a negative value (leading to unigeniture) or a positive one (entailing equigeniture). In other words, as \( \frac{-f'' - c''}{f' - c'} - (-\frac{\gamma''}{\gamma'}) \) increases, equigeniture is more likely to rise as an optimal inheritance rule than unigeniture is. So, with a simple comparative statics, we can show that a rise in the productivity leads to the evolution from unigeniture to equigeniture.

**Proposition 1.3.** As the productivity of effort spent for family rises, equigeniture is more likely to be chosen than unigeniture is.

**Proof.** To begin, the productivity of effort spent for family is defined by how much utility of the parent is produced by a small increase in the effort. With

\[^{27}\text{As a total productivity factor (TPF) one might argue that productivity is to be parameterized only in the form of a coefficient by taking } f(a_i) = \theta a_i. \text{ This would be simpler but not proper or robust for the current analysis. Recall that we are actually dealing with utility, this simpler way of parameterizing fails to differentiate change in productivity from that in measurement unit or that from a monotone transformation.}\]
(1.15), a rise in the productivity is lead by an increase in \( \theta \) or \( m \). Moreover, as 
\[-\frac{\partial^{m-1}}{\partial \theta^{m-1}} - (-\frac{\gamma}{\gamma'}) \] increases, the probability with which \( \text{sign}\left\{-\frac{\partial^{m-1}}{\partial \theta^{m-1}} - (-\frac{\gamma}{\gamma'})\right\} \) switches to plus from minus increases. First of all, 
\[
\frac{\partial D}{\partial \theta} = \frac{1}{a_i} - \frac{\theta - 1}{\theta} \left( \frac{1}{a_i^{\gamma}} \right) \ln a_i + \left( \frac{m - 1}{\gamma} \right) a_i^{\gamma - 1} - \left( \frac{1}{\gamma'} \right) a_i^{\gamma' - 1} \ln a_i > 0 \text{ for } \forall a_i \in [0, 1] \text{ since } \ln a_i < 0 \text{ and } \theta \geq 1.
\]
Second of all, 
\[
\frac{\partial D}{\partial m} = \frac{1}{a_i} + \left( \frac{m - 1}{\gamma} \right) a_i^{\gamma - 1} - \left( \frac{1}{\gamma'} \right) a_i^{\gamma' - 1} \left( a_i^{\gamma - 1} \ln a_i \right) + \frac{\gamma - 1}{\gamma' \gamma - a_i^{\gamma' - 1}} a_i^{\gamma - 1} \ln a_i > 0 \text{ for } \forall a_i \in [0, 1] \text{ since } \ln a_i < 0 \text{ and } m \geq 1.
\]

Based on Proposition 1.1 and Proposition 1.2, this means that the evolution from unigeniture to equigeniture can be driven by a rise in the productivity of effort for family. \( \text{QED} \)

Importantly, Proposition 1.3 can shed light on equigeniture practiced in unequal pre-industrial societies: a rise in the productivity of effort may have lead parents to choose equigeniture over unigeniture, even when they do not value equality. To take an example, equigeniture was practiced in India or China at fairly early stage of history when society is unequal by rigid class system; by contrast, primogeniture was practiced even in the 17th century in England. This contrast may be explained by the fact that agricultural labor productivity in India and China was greater than its counterpart in England.

Another rise in the productivity comes along the change in the primary function of family to emotional support. The nature of production process turns thus less subject to the law of diminishing marginal returns; moreover, the output of the production — like a deep, special bonding or "prestige and esteem" (Becker 1981) — becomes more valuable as it has no close substitutes from outside of family (Cox 1987). Therefore, we can predict that the inheritance patterns of a society would eventually evolve to equigeniture from unigeniture, either through improvement of physical productivity of labor in subsisting activity or change of the main family function itself into a psychological support. Put another way, equal division of bequests is not a puzzling exception but an expectable evolutionary outcome. (See Figure1.1)

In addition, there is another change during industrialization that has a meaningful bearing on the analysis of intergenerational transfer: enlarged variation of children’s income. As industrialization is unfolded, earnings that an individual can achieve to make get less associated with social class of family; and, schools of various levels and specialties that replaced home education enable children in the same family to make more heterogeneous incomes along the development of labor market (Brenner et al. 1991). As a result, the gap in wages for which each child in a family can get paid considerably widens. Furthermore, it becomes usual that children work for earning
substantial income from outside of family at the same time when they are exerting enough (emotional) effort for well-functioning family. As a result, it becomes more feasible\footnote{In contrast, for pre-industrial societies, since family was not only a workplace but also a school, there would not be much of meaningful variation in $y_i$. Moreover, because large amount of manual labor of children was needed for its main functioning — economic survival — it was hard and infeasible for children to keep full time job outside family once they work for family farm land. Thus, we can regard $y_i = 0$ for $\forall i$ in the utility of children in pre-industrial societies.} that within-family inequality in $y_i$ is sometimes so large that it undermines harmonious bonding of family members. Moreover, at equigeniture equilibrium, this could raise an equity issue to the parent as a head of a family since the payoff to each child may end up with unequal even after all the children expend exactly equal amount of effort.

Figure 1.1: Evolution to equigeniture along the development of a society

Facing a large inequality among children’s income, however, the parent can do nothing on this since $y_i$ is given parameter that he cannot affect in the maximization of $U_p$. Moreover, if any measure taken to offset the difference distorts incentive of children, the parent would rather ignore this. However, as long as equilibrium payoff to the parent is not compromised, he may as well try to address the inequality if it grows so large that counterbalancing it can have a merit to deter a tension within family (due to unequal payoffs to the children who spend equal effort for family) which might not be helpful for emotional betterment of family. To this end, the parent can
harness $g_i$ because it does not affect effort of each child. Moreover, this auxiliary term $g_i$ can serve for equalizing the equilibrium utilities of children at equigeniture. With this specific reason, now the parent can deviate from the default homogeneous $g_i$ in (1.12) by giving more gift $g_i$ to a less well-off child.

Once the parent tries to use $g_i$ to tackle an inequity issue, the parent has to choose the timing of transferring the gift. Since $g_i$ does not affect children’s effort, payoff to the parent may remain the same regardless of when $g_i$ is transferred. Thus, the parent is indifferent between ante mortem and post mortem. However, in practice, a focal point would be made on the former. Notably, Cox (1990), Altonji et al. (1997), and McGarry (1999) find that inter vivos gift (ante mortem transfer of gift $g_i$) is useful to ease liquidity constraints that children confront in the stages of their lives before the death of their parent. Moreover, decrease in inequality among children’s income by the gift transfer can help harmonious bonding between siblings while the parent is alive. Therefore, the gift $g_i$ would be more effective when it is transferred to children before the death of the parent rather than after the death. All in all, the way how the parent achieves equal payoffs to children who exert equal effort is shown as follows.

**Proposition 1.4.** If the parent tries to equalize utilities of children at equilibrium of equigeniture, then he can transfer inter vivos gifts unequally to children and leave bequests equally to them. Moreover, when he gives inter vivos gifts, the amount of the gift is negatively associated with relative income of each child compared to their siblings.

**Proof.** At equilibrium of equigeniture, let utility of child $i$ be notated as $U_i^* = \frac{1}{n}F(\alpha_i^{equi}) - c(a_i^{equi}) + y_i$. Since every child expends the same effort $a_i^{equi}$ at the equilibrium, $\frac{1}{n}F(\alpha_i^{equi}) - c(a_i^{equi})$ is identical across all the children. Thus, if $y_i > y_j$, then $U_i^* > U_j^*$ for $\forall i \neq j$.

When the parent tries to tackle inequality in $U_i^*$’s, if any, with his equilibrium payoff $F(\alpha_i^{equi}) - \sum_{i=1}^{n} c(a_i^{equi}) + \sum_{i=1}^{n} y_i$ preserved, he can use inter vivos transfer $g_i$ to counterbalance the differences. Let $g_i = \bar{y} - y_i$, where $\bar{y}$ is the mean of children’s earnings. Then, by giving this $g_i$, all of the children can have the same utility at the equilibrium as $g_i + \frac{1}{n}F(\alpha_i^{equi}) - c(a_i^{equi}) + y_i = \frac{1}{n}F(\alpha_i^{equi}) - c(a_i^{equi}) + \bar{y}$, for $\forall i$, while preserving the parent’s payoff since $\sum_{i=1}^{n} g_i = \sum_{i=1}^{n} \bar{y} - y_i = 0$. Also, the parent’s choice of equigeniture ($b_i = \frac{1}{n}F(\alpha_i^{equi})$, for $\forall i$) is not changed. Above all, notice that, in this inter vivos transfer scheme, if income of child $i$ is larger relative to income of other child $j$ within a family, then the amount of inter vivos transfer to child $i$ is smaller than that to child $j$ as $y_i > y_j \implies g_i < g_j$ for $\forall i \neq j$. QED

Apparently, this appears to resonate with the prediction of Becker’s pure altruism model (1974): negative relation between the transfer and income of children. However,
this model is different — especially in the following two points. First, Becker’s model offers the same prediction on both *inter vivos* gift and bequest, whereas the model in this chapter does not. Second, Becker’s model further claims that children’s income relative to that of their parent also matters, while the model concerns only about relative income among their siblings.

In addition, considering that actual time interval between the will writing and the transfer of bequests can be quite long, it would be usual that at the moment when *inter vivos* gifts are transferred to children, some of their effort $a_i^{equi}$ is *already* spent after the will is written. Thus, a partial remitting of $b_i F(a^{equi})$ can cover the already spent effort and thus would not affect the remaining effort of each child. With this in mind, when the parent’s optimal choice of $g_i$ is negative, the parent might not actually take some money from children. Instead, the parent practically lets the partial remuneration of $b_i F(a^{equi})$ cancel the negative gift out each other. In other words, parents can make positive *inter vivos* gifts to less well-off children while giving nothing to relatively rich children — instead of actually receiving money from them.

Above all, notice that this rationalizes the "equal division puzzle" — seemingly inconsistent behavior of transferring *inter vivos* gifts unequally with bequests equally divided. The altruistic parent with exchange motive chooses equigeniture since the productivity of child’s effort jumps through the change in the main function of family. At the same time, facing a rise in inequality of children’s income from outside of family, *inter vivos* gifts are intended to counterbalance differences in the payoffs to children who exert equal effort, thus being distributed unequally. Under the motive for intergenerational transfers, therefore, the differing behavior would not be puzzling or self-contradictory but simply accommodating societal changes.

So far, we characterize equilibrium transfer behavior expressed with $n$ pairs of $(b_i, g_i)$. Economic reasoning underlying optimal choice of $g_i$ is clear. In contrast, how the parent ends up with choosing one of the two equilibria could still look obscure, since they are embedded implicitly in the proofs of Proposition 1.1 and Proposition 1.2. Thus, economic intuition of this will be illustrated by unfolding the decision making process of the testator parent in the following subsection.

### 1.3.3 Intuition of the Model

Recall that the parent suffers under-provision of effort for family because he cannot verify individual effort of each child to a third-party executor of his will. As an

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29 Based on average life expectancy, 79 in the US, and roughly estimating that 50 years of age is the time when a will is written (based on the figure from National Committee on Planned Giving in the US), it is about 30 years.

30 Notice that this is not of plural form (as in the title of this paper). Altruism, reciprocity, and equity altogether constitute the single transfer motive. Thus, if they are separated, then any of the results driven in this study do not hold any more. Only when they are combined into one, the analysis is meaningful.
immediate remedy for this, the parent would choose unigeniture, say primogeniture. Because the eldest son is the only one who works, from the total output $q$, the third party can immediately find exact level of effort by the eldest son. With his own effort being verifiable under primogeniture, the first son will exert his first-best level of effort since full compensation of his effort for family is now enforceable. In this manner, the parent can fix a moral hazard problem from the first son, but this remedy is not for free; that is, the parent has to give up other sons’ work. Thus, he would ponder whether a deviation of appointing another child as an heir can make him better off or not. Suppose that he deducts a tiny part $\triangle$ from the share of the eldest son (which is one) and to induce effort from another son. An increase in output may be entailed from the newly induced effort. However, at the same time, a decrease in output will be ensued by the first son. More specifically, since the parent becomes unable to reward exactly for their individual effort, he cannot effectively keep a shirking child from free-riding on effort of other children. As a result, no son will be perfectly compensated for his own contribution (i.e., $(1 - \triangle)f' \leq f'$ and $\triangle f' \leq f'$) which dampens work incentives of each son (moral hazard). Nevertheless, given the disutility (cost) of effort, if the same unit of effort yields large output, this deviation may be profitable since effort from each child increases in the output with a given portion. Therefore, as the effort gets more productive, the dampening effect on work incentive becomes less overwhelming. Eventually, if the effort is productive enough to overcome this dampening effect of moral hazard, then it is better for the parent to add another heir. Otherwise, the parent would rather remain under primogeniture. Therefore, the productivity of effort makes a decisive difference.

To put this reasoning in a more formal way, realize that the testator parent virtually makes sequential decisions on (i) whether to appoint each of eligible child as an heir and then on (ii) how to distribute the bequests among the designated heirs. Put differently, the search for optimal $b^*$ can be decomposed into two parts: at the extensive margin (the former) and at the intensive margin (the latter).

Revisiting (1.15) with this frame, beginning at the extensive margin, the parent compares the payoff under primogeniture

$$U_{p}^{primo} = \theta - \frac{1}{m}$$

(1.17)

with payoff under a deviation of taking a tiny share $\triangle$ from the eldest son to another, which is

$$U_{p}^{\triangle} = \theta\{\frac{1}{m} + (1 - \triangle)\frac{1}{m}\} - \frac{1}{m}\{\frac{m}{\theta} + (1 - \triangle)\frac{m}{\theta}\}$$

(1.18)

In sum, the parent examines the sign of the following equation:
Clearly, if $U_p^{\Delta} - U_p^{\text{primo}}$ is negative, then the parent will remain under primogeniture which establishes itself as an optimal inheritance rule. Otherwise, the parent will deviate from primogeniture. Notably, $U_p^{\Delta} - U_p^{\text{primo}}$ is positive if the marginal gain from the new heir

$$U_{p,\Delta} = \frac{1}{(m - \frac{1}{\theta})} \{ \Delta \frac{2 - \theta m}{m^{\theta - 1}} - \Delta \frac{1}{m^{\theta - 1}} \}$$

is larger than the marginal loss from the eldest son

$$U_{p,1-\Delta} = \frac{1}{(m - \frac{1}{\theta})} \{ (1 - \Delta) \frac{2 - \theta m}{m^{\theta - 1}} - (1 - \Delta) \frac{1}{m^{\theta - 1}} \}.$$  

That is,

$$\text{sign}\{U_p^{\Delta} - U_p^{\text{primo}}\} = \text{sign}\{U_{p,\Delta} - U_{p,1-\Delta}\}.$$  

Moreover, given that $1 - \Delta > \Delta$, $\text{sign}\{U_{p,\Delta} - U_{p,1-\Delta}\}$ is determined by the sign of the second derivative of $U_p$ since it decides whether $U_{pi}$ is increasing or not. That is, due to (1.14) and (1.16),

$$\text{sign}\{U_{p,\Delta} - U_{p,1-\Delta}\} = \text{sign}\{-\frac{f''}{f' - c'} + \frac{\gamma''}{\gamma'}\} = \text{sign}\{D\}.$$  

Most importantly, the determinant $D$ is more likely to be positive as the productivity of effort increases, which is expressed by an increase in $\theta$ (or $m$) since $\frac{\partial D}{\partial \theta} > 0$ and $\frac{\partial D}{\partial m} > 0$.

Therefore, a rise in the productivity of effort enables the parent to overcome the dampening effect of moral hazard due to the presence of free riders in order to deviate from primogeniture.

In the one hand, if the productivity of effort is low so that $D < 0$, the deviation of adding an heir is not profitable (i.e., $U_p^{\Delta} - U_p^{\text{primo}} < 0$); the parent would rather not appoint another heir and chooses primogeniture (as in Proposition 1.1). At the intensive margin, the first son claims 100% of the output, spending his first-best level of effort. On the other hand, if the productivity is high such that $D > 0$ and thus $U_p^{\Delta} - U_p^{\text{primo}} > 0$, then the parent will appoint another heir. As a matter of fact, the logic above applies repeatedly, resulting in all the eligible children being included in the list of heirs. To see why, given that some sons are already appointed as heir, the parent can again deduct a tiny portion $\Delta$ from the share, say $b_i$, of one of the heirs. This reallocation makes the parent better off since marginal gain from
the newly added heir \( (U_{p,\Delta}) \) is greater than the marginal loss from the heir whose \( b_i \) is deducted \( (U_{p,b_i-\Delta}) \). It is because as long as \( b_i - \Delta > \Delta \)

\[
\text{sign}\{U_{p,\Delta} - U_{p,b_i-\Delta}\} = \text{sign}\{-\frac{f'' - c''}{f' - c'} + \frac{\gamma''}{\gamma'}\} = \text{sign}\{D\}. \quad (1.24)
\]

Repeating this, the parent reaches the same decision on each child at the extensive margin given the high productivity of effort with \( D \) positive. Furthermore, at the intensive margin, the decision of how to allocate shares among the appointed heirs is shaped by the same the driving force at the extensive margin. Namely, it is always profitable for the parent to reallocate a tiny portion from a larger-share heir to a smaller-share heir. Repeating this adjustment until there is no further room to be better off eventually leads the parent to equigeniture (as in Proposition 1.2). In the end, therefore, it is the productivity of effort that derives the parent to choose either equigeniture or unigeniture.

The economic intuition underlying this reasoning can be efficiently displayed by finding a similarity with a firm’s decision on input. For maximizing its profit, a firm simultaneously seeks a larger output and a smaller cost, both of which are conflicting in a decision of input. Likewise, the altruistic parent with exchange motive pursues two conflicting goals at the same time. He wants more effort for a greater output of family production, while he also concerns disutility of the effort that children bear. On the top of this similarity, realize that if the productivity of labor input increases, the firm will employ more labor input to gain a larger profit, although this increase raises cost as well. This intuition in firm’s decision applies to the parent’s as well; that is, growth in the productivity of effort gives rise to the expansion of heirs by deviating from primogeniture.

As a matter of fact, this analogue that views \( U_p(b) \) as a profit function helps us to gain an overall perspective on how an optimal inheritance rule evolve. To offer a more visual vintage point, for two children case \( (n = 2) \), the shape of \( U_p(b) \) under the two conditions \( (-\frac{f'' - c''}{f' - c'} < -\frac{\gamma''}{\gamma'} \) and \( -\frac{f'' - c''}{f' - c'} > -\frac{\gamma''}{\gamma'} \) are depicted in separate quadrants and artificially connected in Figure 1.2. On the one hand, when the productivity of effort is low, the yielding is not large enough to cover needed reward\(^{31}\) reciprocal to the induced effort of children since marginal gain from inducing effort is overwhelmed by rise of the marginal cost of it \( (-\frac{f'' - c''}{f' - c'} < -\frac{\gamma''}{\gamma'}) \). Thus, expanding heirs (effort-provider) by distributing a share of bequests to a smaller-share child from a larger-share child incurs loss, which is a feature of a Schur convex function, as portrayed in the quadrant III in Figure 1.2 (corresponding to the Proposition 1.1). If a firm is not productive enough, then it has to shut down in a competitive market; in contrast, however, the parent does not let his family shut down due to low productivity. As a result, the best possible gain can be secured by appointing only one heir (i.e., by bequeathing all to one child, who is the eldest one under primogeniture and the youngest one

\(^{31}\)Recall that the schedule of reciprocal rewards has to meet \( b_i = \frac{c'}{f'} \).
under ultimogeniture); and, the only heir exerts his first-best level under unigeniture. On the other hand, if the productivity is high, then the return is large enough for reciprocal rewards paid to the induced effort as marginal gain from the effort grows quicker than marginal cost of inducing it \((-\frac{f''-c''}{f-c}) > -\frac{c''}{c}\). Then, appointing more heirs by distributing a share of bequests to a smaller-share child from a larger-share child is always profitable, which is a property of a Schur concave, as in the quadrant I in Figure 1.2 (corresponding to the Proposition 1.2). By this property of Schur concavity, thus, \(U_p(b)\) can be maximized by giving equal shares to all the eligible children. In other words, in this lucrative stage, the parent now can afford side effects of a moral hazard in inducing effort from children.

![Figure 1.2](image.png)

Figure 1.2: **Overall sketch of Prop 1.1. and Prop 1.2.**

In Section 1.3, a theoretical model for intergenerational transfer behavior is presented with characterization of stable equilibria and illustration of underlying economic intuition. However, this would hardly be relevant if it is inconsistent with observed behavior. Therefore, in the following section, to prove the relevance of the model, we put the theoretical results under various tests with real data.
1.4 Empirical Evidence

1.4.1 Existing Evidence

Notice that theoretical results (Proposition 1.1, 1.2, 1.3, and 1.4) of the model are consistent with the observed behavior of intergenerational transfers mentioned at the beginning of this chapter. First, through comprehensive examination on the space of optimal inheritance rules, there survives two specific stable equilibria: either equigeniture or unigeniture (Proposition 1.1 and 1.2), which is consistent with observed dominance of these two extreme inheritance rules practiced across diverse times and places. Second, Proposition 1.3 can offer an explanation of the evolutionary trend of inheritance patterns from unigeniture to equigeniture in the light of evolution of the principal function of family — from economic survival to emotional support — which sharply raises the productivity of the same unit of children’s effort for family. Third, Proposition 1.4 rationalizes "equal division puzzle" that parents divide bequests equally but inter vivos gifts unequally. The former is already validated by numerous studies with a variety of data sets from many industrialized countries such as US, France, Germany, etc. For instance of the US, Wilhelm (1996) finds that 88% of the decedents — recorded in the tax return data — divide bequests virtually equally (i.e., only within 5% difference); Menchik (1988) shows that 84.3% of the parents divide bequest exactly equally, using probate record data; Light and McGarry (2004), utilizing the National Longitudinal Surveys, report that 92.1% of those who wrote a will divide bequests equally, while Dunn and Phillips (1997) find the corresponding figure as 90% with a different data set, the Health and Retirement Study survey. To take another example, Arrondel and Laferrière (1992) show that more than 92% of the decedents divide bequests exactly equally in France, based on the tax return data. In addition, Kohil (2004) finds that equigeniture is practiced in industrialized countries like France, German, Norway, Sweden, and Israel, in his reviewing the studies of intergenerational transfer with nationally representative survey data from these countries. In stark contrast, there are disagreements on the way how inter vivos gifts are distributed, as briefly mentioned in Section 1.2, although all the existing empirical findings support that inter vivos transfer are unequally distributed. Furthermore, none empirically examined yet on the relationship between the amount of inter vivos transfers and relative income of children. Thus, this will be examined later in this chapter to test whether the amount of inter vivos transfers is negatively associated with relative income of children as specified in Proposition 1.4.

Moreover, besides empirical realities of intergenerational transfer behavior mentioned at the very beginning, as a matter of fact, the model further allows another important empirical test on its validity to be conducted. Remindedly, for many pre-industrial societies, Proposition 1.3 also offers a prediction that a rise labor productivity can lead equigeniture more likely to be chosen over unigeniture. This is also not yet tested. For the sake of a thorough scrutiny of theoretical model, empiri-
cal analyses of these two results will be presented. Following chronological order, the investigation on the relationship between inheritance patterns and labor productivity in pre-industrial societies is first implemented in the next subsection.

1.4.2 Empirical Analysis on Pre-industrial Societies

First of all, many of pre-industrial societies that we can apply Proposition 1.3 are usually agrarian since ordinary parents are settled and can have some property to inherit (such as land); hence, we examine relationship between inheritance patterns and agricultural labor productivity in pre-industrial agrarian societies. To begin, in a prompt and verbatim translation of the model, the major form of bequest would be an accumulated stack of crops, but this is perishable and thus not easily bequeathable. Moreover, the harvested crops are immediately needed for subsistence. After all, instead of a literally direct output of the grain, a parent ends up with bequeathing land that is not only one of the most valuable property in agrarian societies but also useful for an heir to yield crops later.

If we can use econometric techniques to test whether a rise in agricultural labor productivity leads the evolution to equigeniture from unigeniture in agrarian pre-industrial societies, we will evidently obtain better evidence to validate (or invalidate) the model than few pieces of historical cases. Obviously, an appropriate use of the technique requires a large number of observations of variables on inheritance patterns, labor productivity, and other relevant variables for a better and clearer identification of the effect of labor productivity on inheritance patterns. However, securing necessary and comparable data on diverse pre-industrial societies beyond very few extensively-studied societies like England is quite a challenging request. Even if we can collect data on inheritance patterns of land in diverse pre-industrial agrarian societies, we soon confront an obstacle — lack of agricultural labor productivity data that are reliable and comparable across pre-industrial societies as many as needed for a decent econometrical analysis.

In spite of this, we can still proceed this examination with a variable that not only is easily and widely applicable to diverse pre-industrial agrarian societies but also indicates a large increase in output per the same amount of effort: agricultural intensity. In fact, most of pre-industrial agricultural technology such as plow, irrigation, and natural fertilizer were introduced very early stage of human history and spread out worldwide before medieval periods and remained unchanged until industrialization (Grigg 1974). Notably, however fully a farmer utilized all the possible technology and resources, the agricultural productivity were so limited by the natural environment that some of the arable land could not be cultivated annually due to poor soil fertility. Thus, difference in how much the same hours of human labor could yield was largely defined by soil fertility. Moreover, this determining factor is measured by agricultural intensity which is based on the frequency of cropping grain per year given land (Boserup 1965; Grigg 1974; Turner and Doolittle 1978). After all,
without chemical fertilizers and machinery, the productivity of effort in pre-industrial agrarian societies is clearly larger if the agriculture is fully intensified (i.e., cropping a field annually due to better fertility of soil) than otherwise.

In fact, thorough empirical investigations with individual data validate that ‘intensive agriculture’ (i.e., cropping a field annually since improved soil fertility makes fallow unnecessary) entails higher labor productivity than less intensified one such as shifting cultivation and lanching. Hunt (2000) critically reviews the data collected by fieldwork that closely investigates family farms in traditional agricultural villages which are neither industrialized nor commercial, demonstrating that labor productivity under ‘intensive agriculture’ is considerably larger than that under shifting cultivation with fallow. To take an example from his review, Dayak farmers in Malaysia practice fallow and produce 5.47 kg of rice per day of labor (Geddes 1954), while Bang Chan farmers in Thailand yield 32.5 kg of rice from one day labor with ‘intensive agriculture’ (Janlekha 1955). Thus, we can utilize whether ‘intensive agriculture’ is implemented or not as the best possible indicator of a rise in the agricultural productivity of labor (effort).

1.4.2.1 Three Cases from History

Before doing econometrical analysis on the causality between a rise in labor productivity, captured by an indicator variable of ‘intensive agriculture,’ and adoption of equigeniture over unigeniture, case evidence from few extensively-studied societies with a more detailed data available is provided in this subsection.

[1] English settlers in America adopted equigeniture long before primogeniture was abolished in England; and, labor productivity of agriculture in America exceeded that in England when equigeniture was practiced by the settlers in America.

The settlers who immigrated from England soon dropped English primogeniture and then shifted to equigeniture about 130 years ahead of the abolishment of primogeniture in England. This clearly shows that economic environment can affect an individual’s choice on the division of bequests. Firstly, to explore how, Alston and Shapiro (1984) draw attention to the fact that New England Colonies and Middle Colonies abandoned primogeniture earlier than Southern Colonies did; and, they attribute this to the distinct characteristic of agriculture - small family farm in the former vs. large scale plantation in the latter. That is, they argue that large scale plantation in Southern Colonies realized "economies of scale in monitoring" workers and thus caused Southern colonies stick to primogeniture, whereas other colonies deviated from primogeniture earlier because the small size family farm was predominant there and thus lacked the same gain from the scale. At a glance, their argument

\[32\] Not only geographical but also socioeconomic environments are quite similar between both people in traditional agricultural village. However, it turns out that the Dayak people adopts primogeniture whereas the Bang Chan people chooses equigeniture.
seems being able to explicate the evolution from unigeniture to equigeniture at least in America, a more careful look at the historical data necessitates a second thought. For example, North Carolina in Southern Colonies chose an equal sharing rule in 1784, but Massachusetts in New England Colonies had not by then. Moreover, it is to be noted that when New Jersey in Middle colonies had not yet adopted equigeniture, most of the Southern colonies such as Virginia (in 1792), Georgia (in 1789), North Carolina, and South Carolina (both in 1791) had already chosen equigeniture (Shammas et al. 1987). Considering that large plantations still featured the agricultural production in the Southern colonies then, these historical facts are inconsistent with the explanation proposed by Alston and Shapiro (1984).

Nevertheless, their underlying notion that individual’s choice about how to divide bequests can be accounted for "their compatibility with economic efficiency" (Alston and Shapiro 1984) is still convincing. In this line of spirit, the model presented in this chapter can provide an alternative explanation for why those who once followed primogeniture in England was able to achieve the evolution to equigeniture remarkably earlier than their contemporaries in England: owing to the labor productivity growth in agriculture.\(^{33}\) In other words, the labor productivity of agriculture in the settler’s farms improved to exceed that in England so that equigeniture became more efficient choice for them. In spite of lack of reliable productivity data about the settler’s agriculture, the surpass of the productivity is evinced by findings as follows. First of all, a farmer in colonial America could produce enough corn to feast 5 to 7 persons (Perkins 1998) while an agricultural worker in England was able to feed 2.7 non-agricultural workers (O’Brien 1985) even in 1841. Furthermore, Bairoch (1976) demonstrates that the output per farmer worker in America in 1840 was 23% more than that in England. In addition, Clark (1987) shows that output per farm worker was 308 bushels a year in the Northeastern part in 1850 which was greater than its British counterpart, 206 bushels. Presumably, in addition to continuing effort to innovate for better yielding, the factors that enabled the settlers (unlike their contemporaries in England) to accomplish this surpassing growth would be not only abundance of fertile land but also interaction with Native Americans (Schwartz 1995). At the outset, early colonist settlers were not as skillful agriculturalists as the Native Americans, or Indians, were. They did not pay enough attention to crop rotation, fertilization, or proper tillage (Hurt 2002; Perkins 1988), so they soon confronted the soil erosion problem even with the bestowed fertile land. In contrast, Native Americans already had started ‘intensive agriculture’ of corn using natural fertilizers long before the contact with European settlers (Hurt 2002). Through the interaction, Native Americans introduced to the settlers their advanced methods of cultivation that were more ecologically suitable to the environment (Perkins 1988; Schwartz 1995; Vickers 2003). At the same time, settlers’ efforts to bring new technology such as the cast-iron plow, cradle and

\(^{33}\)This is consistent with the view of DeLong (2003) that the rapidly-expanding settler economy contributed to the rise of equigeniture.
scythe into the crop field were put forth as well. As a result, the settler parents could enjoy the increase in agricultural productivity that those in England could not, which could facilitate the evolution of their inheritance pattern from primogeniture to equigeniture far ahead of England.

[2] English primogeniture was abolished after a large growth in agricultural productivity during the British Agricultural Revolution.

Likewise, the abolishment of primogeniture in England in 1926 can be understood to be, in part, attributable to the agricultural productivity improvement accomplished through the British Agricultural Revolution. Many scholars approach this event just as a political outcome of the Industrial Revolution (e.g., Justman and Gradstein 1999; Bertocchi 2006). However, it is also to be noted that the British Agricultural Revolution was instrumental to the Industrial Revolution (e.g., Jones 1968; O’Brien 1977). As a matter of fact, a fairly remarkable growth of agricultural productivity of labor was made during the British Agricultural Revolution. For example, the agricultural labor force became more productive enough to feed 2.7 non-agricultural workers in 1841, contrasted to only one in 1760 (O’Brien 1985). Moreover, according to Clark, the output per a farmer worker given a day stagnated at 66 (bushel/one work day) between 1300 and 1580 and then increased to 110 by 1860 after the British Agricultural Revolution. This improvement was achieved in the course of the revolution by various innovations such as widely spread use of water-meadow irrigation (Kerridge 1968) and crop rotation with legumes like clover (Chorley 1981) which led to "unprecedented improvements in crop yields and farm output even for the contemporary" (Overton 1996). Above all, through the angle of the model, it is the considerable improvement in the labor productivity from the revolution that could catalyze the evolution toward equigeniture.

However, a caveat is that the aforementioned two cases must be carefully interpreted because both are using records of intestacy law which might not be exactly synchronized with what was actually practiced by ordinary people. Notwithstanding, in both cases, their intestacy law was based on custom and allowed complete testamentary freedom, so the testator parent could always write a will differently from it whenever he (or she) wanted. Moreover, if it was not lined with what the majority of the testators did, it then became obsolete and thus was soon abolished. Not surprisingly, therefore, for the English settlers in America, Alston and Shapiro (1984) find that "surprisingly, many wills even followed exactly the provisions of intestacy law"; likewise, according to Spufford (1976), primogeniture was actually practiced among English peasants even in the late eighteenth century.

Rather, what deserves more caution is that the evolution toward equigeniture from primogeniture occurred when both societies underwent a radical change in their social class structure as well. Thus, in these two historical cases, it would be difficult to establish a distinguishable point for the model from the other arguments that the rise of social mobility causes the evolution from primogeniture to equigeniture (Bernheim
and Severinov 2003; Bertochhi 2006) and that the pursuit for higher status in rigid rank of social class leads primogeniture (Chu 1991; DeLong 2003). That is, given these existing hypotheses, the change in social stratification that concurrently preceded the evolution as the productivity growth did can be confounding.

[3] Primogeniture was practiced in feudal England, while equigeniture was chosen in feudal Flanders and feudal China; and, although social mobility was similar in all of the three feudal societies, agricultural productivity in the latter two societies was greater than that in feudal England.

To address the issue of the confounder that the social stratification changed as the productivity did, we can compare feudal England with other feudal societies, since, in terms of social mobility, feudal England was more akin to the other feudal societies than to industrialized England. First, consider Flanders\(^3\) in the pre-industrial periods. Interestingly, studies of folk histories find that the peasants in Flanders actually practiced equigeniture even before the sixteenth century (Ladurie 1976) while those in England followed primogeniture even in the late eighteenth century (Faith 1966; Spufford 1976; Howell 1976). On the top of this, Allen (2000) shows that from 1400 to 1700 the labor productivity of agriculture in Flanders of the current Belgium area was constantly higher than that in England. In particular, the productivity was roughly 150% of that of England for the 200 years\(^3\) between 1400 and 1600, according to Allen (2000). Given similar movability of social class, therefore, this difference in inheritance patterns could be attributable to the difference in their agricultural productivity of labor.

In addition to this, we can also consider another feudal society, the Ming dynasty in medieval China, because although the institutional details were different from European ones, the social mobility of it could be similar to that of feudal England, compared to that of industrialized England. In fact, however, ordinary peasants in the feudal China followed equigeniture (e.g., Chu 1972; Wakefield 1998), entirely contrasted to counterparts in the feudal England. Again, given the similar rigidity of the social class structure, the previous models (Chu 1991; DeLong 2003; Bernheim and Severinov 2003) would not explain this contrast, either. Thus, as an alternative, we can attempt to apply the model for understanding the reason of the difference. Although due to the shortage of good data, a reliable comparison of agricultural labor

\(^{34}\)Flanders were living in current Belgium, northern France, some part of Netherlands. Equigeniture was practiced in all of these countries, even though there were some regional variations within France and Netherlands. In addition, when it comes to France, Napoleonic Code 1804 did not give full freedom in dividing the bequests. Moreover, Flanders did not comprise one of the major groups in the latter two countries. Therefore, we focus on Flanders in Belgium since available data of labor productivity is country level.

\(^{35}\)The farmers of Flanders played a leading role in agricultural development in the Middle Ages. For instance, they invented the four-field system with crop rotation with legumes. Later, in the British Agricultural Revolution, this was imported to England and contributed the agricultural productivity growth in England.
productivity between these two medieval countries throughout the Middle Ages would not be feasible, there are some indicative observations that we can pay attention to - in relation to the difference in their inheritance patterns. First, even before 750, Chinese peasants already tilled crop fields annually without fallow — using irrigation and natural fertilizers (Crisp 1993) — and followed equigeniture, whereas peasants in England had to leave at least one third of arable land as fallow even in the fifteenth century. Furthermore, Allen (2009) compares the estimates of output per agricultural worker around the Yangtz Delta area between 1600 and 1650 in the late Ming dynasty and those in medieval European countries, according to which the productivity of peasants in the Ming dynasty was higher than that in any other European countries between 1400 and 1700. After all, agricultural productivity of labor can be an explanatory factor of the stark difference in the way bequests were divided in feudal societies. Put another way, in the light of the model in this chapter, the high productivity made equigeniture more compatible for incentives of peasants in feudal Flanders or China than for those in feudal England with low productivity.

These handful pieces of historical cases presented above can be testimonies to the model, but a more systematic and rigorous test using data sets will certainly provide a more generalized and robust evidence. Thus, the next subsection brings observations of a larger number of pre-industrial societies under econometrical investigation to test Proposition 1.3.

1.4.2.2 Econometrical Analysis on Pre-industrial Societies

1.4.2.2.1 Data Overview and Key Estimand

For the sake of statistically testing whether we can establish a causality between adoption of equigeniture and the growth of agricultural labor productivity, the database of Ethnographic Atlas (Murdock 1967) is utilized. This is compiled database of fieldworks by myriads of anthropologists. This database covers numerous societies in all the continents of the world. Moreover, it provides useful information on them, including data about how much a society relies on agriculture, animal husbandry, and hunter-gathering for subsistence; actual inheritance pattern of land; type of settlement (whether nomadic or sedentary); how social class is stratified and the like. In particular, all the observations used in this analysis are pre-industrial societies. In other words, all of the societies used in this analysis have pre-industrial means of livelihood such as hunter-gathering or animal husbandry or agriculture; furthermore, in terms of production technology, none of them relies on the machinery, chemicals, or any other industrialized methods in the production processes. Furthermore, based on

36 According to historical record, equigeniture already was practiced even in the Han dynasty (Chu 1972).
37 Almost all of the data were gathered around 1900.
the rate of dependency on agriculture for subsistence in the database, a pre-industrial society is categorized as an agrarian if this rate exceeds 50%.

At the outset, in a regression analysis on agrarian pre-industrial societies, the left-hand side variable is about inheritance patterns of land. It turns out that among 350 pre-industrial agrarian societies in the data set, 91.86% of them (325 observations) have either equigeniture or unigeniture. For the remaining 8.14% of them have not yet developed specified pattern on the distribution of land although they have a certain rule about who is eligible heir (heiress). As a matter of fact, all of these 25 societies are nonsedentary. Fundamentally, there is no clear way to tell whether these societies are in transitioning their inheritance pattern from unigeniture to equigeniture (as in the aforementioned cases of colonial America and of England) so that they can represent the knife-edge case \( \frac{f''}{f' - c'} = \frac{c''}{c'} \) or not. So, for avoidance of measurement errors, it is better to use the observations of societies where inheritance pattern on the distribution of land exists.

Along with the left-hand side variable of inheritance pattern on land division, the right-hand side variables should include a measure of agricultural labor productivity for testing Proposition 1.3. Although the database Ethnographic Atlas does not have detailed statistics of the productivity, it does provide a proxy variable of a rise in the agricultural productivity of labor (effort) - indicator of ‘intensive agriculture.’ As shown above, ‘intensive agriculture’ means higher agricultural productivity of labor than otherwise. In detail, ‘intensive agriculture’ in Murdock’s dataset is defined as "crop cultivation on the permanent fields, utilizing fertilization by compost or animal manure, crop rotation, or other techniques so that falling is unnecessary" and often engaged with irrigation, which is basically following the aforementioned definition. Consequently, non-intensive agriculture in the database refers to shifting cultivation with fallow or horticulture. Thus, given the data available, we can utilize the indicator variable of ‘intensive agriculture’ for obtaining the variation in the productivity. All in all, the key estimand of this empirical investigation is a parameter capturing the effect of a rise in agricultural labor productivity — indicated by binary variable of ‘intensive agriculture’ — on adoption of equigeniture over unigeniture.

For an overview of the data, Table 1.1 presents the summary statistics of variables. To be informative, since all the variables are categorical, except for the two — population and dependency rate on agriculture for subsistence — frequencies of their subcategories are displayed, instead of their overall mean and standard deviation.

As shown in Table 1.1, although half of the observations are from Africa, the key variables — indicator of equigeniture and that of intensive agriculture — are almost evenly distributed in the data. At the same time, it is worthwhile to note that more than 100 pre-industrial agrarian societies, which would include underdeveloped and immobile societies, have already adopted equigeniture like current US or Sweden, which is not well predicted by any of previous models. Above all, this variation of inheritance rules among a large number of pre-industrial agrarian societies presents a good opportunity to implement a balanced empirical test of Proposition 1.3 in a
systematical and rigorous way which will be further elaborated in the next subsection.

Table 1.1 | Descriptive Statistics of Pre-industrial Agrarian Societies

<table>
<thead>
<tr>
<th>Variable</th>
<th>Freq.</th>
<th>Percent(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inheritance Pattern on Land Division</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equigeniture</td>
<td>122</td>
<td>50.83</td>
</tr>
<tr>
<td>Unigeniture</td>
<td>118</td>
<td>49.17</td>
</tr>
<tr>
<td>Intensity of Agriculture</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-intensive agriculture</td>
<td>127</td>
<td>52.92</td>
</tr>
<tr>
<td>Intensive agriculture</td>
<td>113</td>
<td>47.08</td>
</tr>
<tr>
<td>No.Obs</td>
<td>240</td>
<td>Mean</td>
</tr>
<tr>
<td>Dependency on Agriculture (%)</td>
<td></td>
<td>(9.62)</td>
</tr>
<tr>
<td>Population</td>
<td>240</td>
<td>1376146 (6433607)</td>
</tr>
<tr>
<td>Class Stratification</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Absence of class stratification</td>
<td>84</td>
<td>35.00</td>
</tr>
<tr>
<td>Class is based on complex aspects</td>
<td>105</td>
<td>43.75</td>
</tr>
<tr>
<td>Class is mainly based on wealth</td>
<td>51</td>
<td>21.25</td>
</tr>
<tr>
<td>Settlement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not fully sedentary</td>
<td>8</td>
<td>3.33</td>
</tr>
<tr>
<td>Permanently settled</td>
<td>232</td>
<td>96.67</td>
</tr>
<tr>
<td>Region</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Africa</td>
<td>120</td>
<td>50</td>
</tr>
<tr>
<td>Circum Mediterranean</td>
<td>33</td>
<td>13.75</td>
</tr>
<tr>
<td>East Eurasia</td>
<td>33</td>
<td>13.75</td>
</tr>
<tr>
<td>Insular Pacific</td>
<td>38</td>
<td>15.83</td>
</tr>
<tr>
<td>America</td>
<td>16</td>
<td>6.66</td>
</tr>
<tr>
<td>No.Obs</td>
<td>240</td>
<td></td>
</tr>
</tbody>
</table>

Data source: Ethnographic Atlas (Murdock 1976)

1.4.2.2.2 Identification Strategy

To begin with, as a prompt precursory check on the relationship between a rise in agricultural labor productivity (indicated by intensive agriculture) and evolution to equigeniture from unigeniture, the relative frequency of both is displayed in Table 1.2, suggesting a positive correlation between them.
<table>
<thead>
<tr>
<th></th>
<th>Intensive Agriculture</th>
<th>Non-intensive agriculture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equigeniture</td>
<td>62.8</td>
<td>40</td>
</tr>
<tr>
<td>Unigeniture</td>
<td>37.2</td>
<td>60</td>
</tr>
<tr>
<td>total (%)</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Although Table 1.2 appears to offer a supportive evidence, we need to be careful since potential effects of other relevant factors are not yet distilled out. Thus, we can control them by running regressions using other variables at hand, to clearly identify our key estimand. Moreover, since the left-hand side variable is binary, we can utilize a binary response model whose specification can be neatly expressed as follows.

\[
\text{Equigeniture}_s = 1[\beta_0 \cdot \text{Intensive Agriculture}_s + \mathbf{x}_s \delta + \varepsilon_s > 0]
\]  

(4.2.a)

where Equigeniture$_s$ is a binary variable that takes one if equigeniture is practiced in a society $s$ and zero if unigeniture is practiced; Intensive Agriculture$_s$ is also a binary variable taking one if intensive agriculture is implemented and null otherwise; $\mathbf{x}_s$ is a vector of unity and other relevant explanatory variables such as population, whether wealth is important for perpetuating social class, and the like; $\varepsilon_s$ is an error term that follows Normal distribution; and, $1[\cdot]$ is indicator function.

Given this specification, to pick a proper estimator for this analysis, one might firstly think a linear probability model as a candidate; however, this certainly provides far poorer fits than nonlinear model does since the right-hand side variables in the dataset are chiefly categorical variables (Wooldridge 2007). Thus, as a nonlinear binary response model, Probit model, which is one of the most widely adopted, is selected for estimation. On the top of this statistical model, the central interest lies at only the sign of $\beta_0$ in the light of Proposition 1.3. Therefore, estimates matters only up to scale, and one-tail tests will be conducted for inference.

Furthermore, to avoid a bias from omitted variables, other factors that are thought to have relevance to inheritance patterns are controlled as the right-hand side variables of (4.2.a). For instance, Chu (1991), Bertochhi (2006) and Howell (1976) posit that there can be an association of population and inheritance pattern in a society, although their predictions are far from aligned; hence, population variable is included in the right-hand side. Likewise, considering the other hypothesis for bequest motive (DeLong 2003; Chu 1991) that inheritance is intended for larger family wealth to buttress the social class of the family in pre-industrial societies, the variable of how wealth is important for keeping social class status, if there is any stratified one, is also included in $\mathbf{x}_s$. Under the same concern, the variables of indicating the region that a society $s$ belongs to are added in $\mathbf{x}_s$ to control potential fixed effect.

However carefully we choose the estimator and the right-hand side variables, one could raise a concern for bias or inconsistency in the identification of $\beta_0$, our key
estimand. For instance, one could suspect that the way of chopping down land ownership over generations may somehow affect whether a society can develop ‘intensive agriculture’ or not. In order to tackle the endogeneity issue, we need to introduce exogenous variation in the suspected endogenous variable, Intensive Agriculture, in a model separated from (4.2.a). Since Intensive Agriculture is also a binary variable, we can state

$$\text{Intensive Agriculture} = 1[z_s \omega + u_s > 0] \quad (4.2.b)$$

where $u_s$ is an error term that is independent of $z_s = \{x_s, y_s\}$; and, $y_s$ is disparate with $x_s$ (for the moment, we are open to the possibility that $y_s$ is empty, although it would not eventually be of interest and of use).

Whichever source that one would think as a bias comes from, notice that the problem of endogeneity is stated as $\text{corr}(\varepsilon_s, u_s) \neq 0$. Thus, given (4.2.a) and (4.2.b), we need to find a proper method that deals well with a possible nonzero correlation between the error terms to secure an unbiased (and consistent) estimate for $\beta_0$. At the outset, we can think of three candidates that are widely used for addressing endogeneity issue: two-stage least squares (hereafter 2SLS), matching estimation, and control function method.\(^{38}\)

First of all, the conventional 2SLS seems handy and plausible; however, it does not work for this analysis. That is, 2SLS fails to give us unbiased estimates when $\text{corr}(\varepsilon_s, u_s) \neq 0$, even if we can secure ideal instrumental variables $y_s$ that perfectly meet the requirements of inclusion and exclusion restrictions. To see why, notice that the first stage (4.2.a) and the second (4.2.b) alike should be nonlinear because both the left-hand side variable in (4.2.a) and the allegedly endogenous variable Intensive Agriculture, are indicator variables in the present analysis. Moreover, we know that a nonlinear regression model does not allow expectation operator to permeate through as to make $E(z_s \omega + u_s) = 0$ in the second stage equation. As a consequence, 2SLS is not able to fix the endogeneity even with a perfect instrumental variable.\(^{39}\)

Next, between the other two remaining options — matching estimation and control function method — the latter is more robust and suitable for the current analysis. First of all, in terms of key assumptions to be met, control function approach is more robust than matching estimation. Namely, to properly treat the endogeneity of $\text{corr}(\varepsilon_s, u_s) \neq 0$, matching estimation method requires $\varepsilon_s \perp u_s \mid z_s$ (and $y_s$ can possibly be empty), whereas control function method only needs $(\varepsilon_s, u_s) \perp z_s$ with non-empty $y_s$. As Heckman and Navarro-Lozano (2004) point out, the latter is more robust in that even when $\varepsilon_s$ is related with Intensive Agriculture after conditioning on $z_s$, control function approach still can fix a bias problem, while matching method

\(^{38}\)In fact, 2SLS can be understood as one of control function approach, but it is more restricted estimator since it requires linearity of both stages. Also, one might suggest GMM. However, in fact, with moment restriction specified, the actual estimator will eventually be one of the two - 2SLS or control function method.

\(^{39}\)For this reason, running a 2SLS when both stages are nonlinear is sometimes called a ‘forbidden regression.’
cannot. In addition, control function approach is more flexible since it does not impose any restrictions on the relationship between $\varepsilon_s$ and Intensive Agriculture$_s$. More importantly, matching estimate is very sensitive to the choice of $z_s$; that is, Heckman and Navarro-Lozano (2004) show that if $z_s$ is not an exact set which precisely satisfies $\varepsilon_s \perp u_s \mid z_s$ used for the conditioning, then a bias can be even worse. What is worse, there is no robust way to test whether $z_s$ exactly meets $\varepsilon_s \perp u_s \mid z_s$.

Once control function approach is chosen for addressing endogeneity concern, the correlation coefficient between $\varepsilon_s$ and $u_s$ is explicitly specified as

$$\varepsilon_s = \rho u_s + e_s$$

(4.2.c)

in the simplest form. Based on the assumption that $\varepsilon_s$ and $u_s$ follow a bivariate Normal distribution, we can control a possible bias in the estimation for $\beta_0$ in (4.2.a). However, on using the control function approach in this econometrical analysis, one caveat should be made: We cannot naively regard $\varepsilon_s = \rho \text{(Intensive Agriculture} - z_s \omega) + e_s$ and plug this into the first stage (4.2.a) (as we usually do in linear models) since both Equigeniture$_s$ and Intensive Agriculture$_s$ are binary variables, instead of continuous ones. Rather, a more complicated derivation is needed to make the control function approach correctly serve for the current analysis. For details of mathematical derivations and procedure, refer to Appendix A.1. which also details properties of $e_s$ driven from that of $\varepsilon_s$ and $u_s$. Furthermore, note that since we are not imposing strict assumption on the variance of $\varepsilon_s$ and $u_s$ (thus that of $e_s$ as well), we are able to identify estimates of coefficients only up scale. However, it does not affect the quality of test since our key estimand is the sign of $\beta_0$; that is, estimates matters only up to scale to test Proposition 1.3. In addition to this endeavor for unbiasedness, for the sake of an efficient estimation, maximum likelihood estimator is chosen.

Now, for the prudently chosen estimator to effectively tackle endogeneity issue, we need a qualified variable of $y_s$ that meets two conditions: (i) non-empty $y_s$ and (ii) $(\varepsilon_s, u_s) \perp z_s$. As a matter of fact, these two can be translated into a familiar version of required conditions for a valid instrumental variable. In the one hand, in practice, non-empty $y_s$ means more than verbatim interpretation that random variable $y_s$, which is different from $x_s$, exists. In other words, $y_s$ is to be explanatory of Intensive Agriculture$_s$ in (4.2.b) since the use of $y_s$ is meaningless or non-existential if it is not relevant to Intensive Agriculture$_s$.\footnote{That is, if $y_s$ is irrelevant of Intensive agriculture$_s$ so that its coefficient estimate is not statistically significant, then it would be virtually similar to empty $y_s$.} Observe that this corresponds to relevance condition (or what is so called inclusion restriction) of a valid instrumental variable. One the other hand, the second condition of $(\varepsilon_s, u_s) \perp z_s$ can be a version of exclusion restriction (or what is so called exogeneity condition). In other words, a qualified instrument $y_s$ should be exogenous to Equigeniture$_s$ and allow no direct effect on Equigeniture$_s$. It is only through Intensive Agriculture$_s$ that $y_s$ may have an indirect correlation with Equigeniture$_s$.\footnote{That is, if $y_s$ is irrelevant of Intensive agriculture$_s$ so that its coefficient estimate is not statistically significant, then it would be virtually similar to empty $y_s$.}
Considering how Intensive Agriculture's indicates a rise in agricultural productivity of labor, as delineated above, one of the best candidates for a qualified $y_s$ would be variables of soil quality for cultivation. In particular, among various soil quality variables, soil nutrient availability and available water capacity in subsoil are selected as the instrument $y_s$ since they are one of the most critical factors in soil fertility. First of all, as many agroecologists show, raising crop relies on nutrients contained in the soil;\textsuperscript{41} hence, these two are closely related with Intensive Agriculture. Especially, without chemical fertilizers, soil fertility plays a determinant role in pre-industrial agriculture. Second of all, it is obvious that soil quality are exogenously given to the testator parents. Therefore, other than through Intensive Agriculture these environmental factors would not directly affect their choice of how to divide the land property between their heirs after their death.

Moreover, these instrumental variables $y_s$ are procured from the 30 arc-second raster database of Harmonized World Soil Database (HWSD) that combines the soil information gathered by various research institutes (European Soil Bureau European Soil Database (ESDB), Chinese Academy of Science, ISRIC-World Soil Information and the like) into 1:5,000,000 scale grid world wide map under the coordinating initiative of FAO and IIASA (International Institute for Applied Systems Analysis). Since data in the HWSD are collected several decades later data in Ethnographic Atlas were, in avoidance measurement errors, subsoil (30-100 cm deep) data — instead of topsoil data — are utilized. For detailed information on the variables of nutrient availability and available water capacity, refer to Appendix A.2.\textsuperscript{42}

\subsection*{1.4.2.2.3 Estimation Results}

In this subsection, the outcomes of estimations following identification strategies, specified in the previous subsection, are presented.

Table 1.3 displays the estimation results of Probit regressions against Equigeniture, (indicator of adopting equigeniture over unigeniture). Robust to whether fixed effect is controlled or not — as shown in the (1) and (2) of Table 1.3 — the sign of the coefficient of the Intensive Agriculture stably remains positive with a high statistical significance. Presumably, this is suggestive evidence to Proposition 1.3. In

\begin{table}[h]
\centering
\begin{tabular}{|l|l|l|}
\hline
Variable & No.Obs & Mean(St.Dev) \\
\hline
Nutrient Availability & 211 & 9.41 (9.16) \\
(unit: cmol/kg). & & \\
Available Water Capacity & 211 & 121.7 (33.12) \\
(unit: mm/m) & & \\
\hline
\end{tabular}
\caption{Descriptive Statistics of Subsoil Quality Variables from HWSD}
\end{table}

\textsuperscript{41}For further information on the nutrient required by crop cultivation, refer to Table A.2-1 in Appendix A.2.

\textsuperscript{42}Table 1.1-2
addition to this, as an indirect check of robustness, the marginal effects calculated at the mean of other variables (which is of secondary concern since only the sign of the estimates matters) can be compared with statistics in Table 1.2, where we do not find a substantial discrepancy.

**Table 1.3** Equigeniture vs. Unigeniture of Land

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensive Agriculture</td>
<td>0.544***</td>
<td>0.214</td>
<td>0.433***</td>
<td>0.171</td>
</tr>
<tr>
<td>Dependency on Agriculture</td>
<td>0.029***</td>
<td>0.011</td>
<td>0.035***</td>
<td>0.014</td>
</tr>
<tr>
<td>Wealth for Class Status</td>
<td>0.105</td>
<td>0.042</td>
<td>0.039</td>
<td>0.015</td>
</tr>
<tr>
<td>Settlement</td>
<td>-0.051</td>
<td>-0.020</td>
<td>0.500</td>
<td>0.199</td>
</tr>
<tr>
<td>Population (in one million)</td>
<td>-0.006</td>
<td>-0.002</td>
<td>-0.016</td>
<td>-0.006</td>
</tr>
<tr>
<td>Region Indicators?</td>
<td>No</td>
<td>Yes [3]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-2.007*</td>
<td>-2.521*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>-154.7</td>
<td>-134.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. Obs.</td>
<td>240</td>
<td>240</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: [1] In parenthesis is standard error of each estimate.
* indicates being statistically meaningful at significance level of 10%.
** does at the level of 5%; *** does at the level of 1%.
[2] The marginal effect is evaluated at the means of right-hand side variables.
[3] One of the region indicators is dropped to avoid multicollinearity

However, whichever implication or conclusion we can draw from the estimation results in Table 1.3, we had better be careful and wait until the endogeneity concern is addressed with qualified instrumental variables — nutrient availability and available water capacity in subsoil — using the control function method which elaborated in Section 1.4.2.2.2. Thus, Table 1.4 demonstrates the reassessed results of the estimates (now with the endogeneity explicitly controlled). Exactly speaking, the virtual estimation procedure of control function approach following (4.2.a) ~ (4.2.c) is not separated into two stages in the same way as 2SLS is so; however, although estimating (4.2.b) is an auxiliary regression, the outcomes are displayed in such a way, for an illustrative convenience.
**Table 1.4** Equigeniture vs. Unigeniture of Land  
Control function approach instrumented with variables of Nutrient Availability and Available Water Capacity

<table>
<thead>
<tr>
<th>Panel A: 2nd stage</th>
<th>(1)</th>
<th>Marginal Effect</th>
<th>(2)</th>
<th>Marginal Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dep. Binary Variable: Equigeniture of Land</strong></td>
<td>Estimate (Std Err.)</td>
<td>Effect</td>
<td>Estimate (Std Err.)</td>
<td>Effect</td>
</tr>
<tr>
<td>Intensive Agriculture</td>
<td>1.320***</td>
<td>0.337</td>
<td>1.143***</td>
<td>0.291</td>
</tr>
<tr>
<td>(0.329)</td>
<td>(0.448)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependency on Agriculture</td>
<td>0.029***</td>
<td>0.009</td>
<td>0.031***</td>
<td>0.011</td>
</tr>
<tr>
<td>(0.010)</td>
<td>(0.011)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wealth for Class Status</td>
<td>0.156</td>
<td>0.044</td>
<td>0.062</td>
<td>0.025</td>
</tr>
<tr>
<td>(0.119)</td>
<td>(0.129)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Settlement</td>
<td>−0.068</td>
<td>0.123</td>
<td>0.559</td>
<td>0.319</td>
</tr>
<tr>
<td>(0.502)</td>
<td>(0.554)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Population (in one million)</td>
<td>−0.017</td>
<td>0.101</td>
<td>−0.024</td>
<td>0.085</td>
</tr>
<tr>
<td>(0.015)</td>
<td>(0.015)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region Indicators?</td>
<td>No</td>
<td>Yes&lt;sup&gt;[3]&lt;/sup&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>−2.445**</td>
<td>−2.689&lt;sup&gt;*&lt;/sup&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1.190)</td>
<td>(1.405)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: 1st stage</th>
<th>Dep. Binary Variable: Intensive Agriculture</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Available Water Capacity</td>
<td>0.0001</td>
<td>0.00002</td>
<td>0.005*</td>
</tr>
<tr>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nutrient Availability</td>
<td>0.112***</td>
<td>0.024</td>
<td>0.081**</td>
</tr>
<tr>
<td>(0.032)</td>
<td>(0.037)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nutrient Availability&lt;sup&gt;2&lt;/sup&gt;</td>
<td>−0.002**</td>
<td>−0.0004</td>
<td>−0.001</td>
</tr>
<tr>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other RHV in 2&lt;sup&gt;nd&lt;/sup&gt; Stage are included?</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Test Stat to check weak IV</td>
<td>23.36***</td>
<td>20.55***</td>
<td></td>
</tr>
<tr>
<td>J-Test Stat and (its p-value)</td>
<td>1.49</td>
<td>1.27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.218)</td>
<td>(0.287)</td>
<td></td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>−269.5</td>
<td>−226.0</td>
<td></td>
</tr>
<tr>
<td>No. Obs</td>
<td>211</td>
<td>211</td>
<td></td>
</tr>
</tbody>
</table>

Note:  
[1] In parenthesis is standard error of each estimate.  
* indicates being statistically meaningful at significance level of 10%.  
** does at the level of 5%; *** does at the level of 1%.  
[2] The marginal effect is evaluated at the means of right-hand side variables.  
[3] One of the region indicators is dropped to avoid multicollinearity

As mentioned in the previous subsection, Intensive Agriculture<sub>s</sub> is instrumented by the variables of Nutrient Availability<sub>s</sub> and Available Water Capacity.<sub>s</sub> Especially,
since there is nonlinear relation between the former and soil fertility (see Appendix A.2.1.), a quadratic term of Nutrient Availability
\( s \) is included in \( y_s \). Although these instrumental variables make economic sense, we still need to check whether they make statistical sense or not. First of all, the instrumental variables need to be significantly related with the endogenous regressor Intensive Agriculture
\( s \). For the sake of testing this, joint tests of whether all of the instrumental variables are significant in the nonlinear regression with the other right-hand side variables in (4.2.a) against Intensive Agriculture
\( s \) are implemented; and, the test statistics are reported in the bottom part of Table 1.4, indicating that the chosen instrumental variables are not weak. In addition, although a direct test for the assumption of exogeneity is not feasible, we would still be able to indirectly check this with an over-identification test; and, the suitable one for the current nonlinear regression is Mackinnon-Davidson J-Test (Davidson and MacKinnon 1981) which is reported as well in Table 1.4, not providing evidence against the exogeneity condition.

After having gone through these tests, one might be concerned with the differences in the magnitude of the corresponding estimates between in Table 1.3 and Table 1.4. However, direct comparison of the coefficient estimates in Table 1.3 and Table 1.4 is misleading. First of all, recall from the section 1.4.2.2. that estimates are now unique up to scale, unlike in the Probit estimation whose result is presented in Table 1.3. (For mathematical details on this, refer to Appendix A.1.) That is, the estimates in Table 1.4 actually are containing scaling factor other than \( \beta_0 \), contrast to those in Table 1.3. Moreover, all the right-hand side variables in (4.2.a) except for Intensive Agriculture
\( s \) are already controlled in the auxiliary regression (4.2.b) as a first stage. In spite of this intricacy, most importantly, the differences found in the estimation results in both tables are fine for the current empirical analysis since all we need is the sign of the coefficient of Intensive Agriculture
\( s \). In fact, the estimation results before and after tackling endogeneity (using the control function method) show some similarity. First of all, juxtaposing Table 1.3 and Table 1.4, the sign of each corresponding estimate is the same, which indirectly shows robustness of the result on the key estimand. Secondly, both share the feature that the coefficients of the two variables — Intensive Agriculture
\( s \) and Dependency on Agriculture
\( s \) — remain positive with statistical significance whereas the remaining ones are statistically insignificant.

Above all, even after the endogeneity issue is properly addressed, the coefficient estimates of Intensive Agriculture
\( s \) still remain positive with statistical significance, which renders robust evidence supporting Proposition 1.3.
1.4.3 Empirical Analysis on an Industrialized Society

1.4.3.1 Data Overview and Key Estimands

As mentioned in Section 1.4.2, given proven equal division of bequests by various empirical studies, to thoroughly complete empirical tests of Proposition 1.4, what remains is investigation on the relationship between the amount of *inter vivos* gift and relative income of each child. More specifically, among parents who choose equal division of bequests, we need to test (i) whether inequality among children’s income triggers them to give *inter vivos* gifts, and (ii) whether the amount of the gifts is negatively associated with relative income of child among his siblings.

To implement a rigorous test on this, the data requirement is demanding: we need (i) individual level information of parents’ characteristics; (ii) individual characteristics of each child of a parent; (iii) the amount of *inter vivos* gift given to each of the children; and (iv) information on the bequest behavior of individual parents. Among existing datasets, the Health and Retirement Study (hereafter HRS) would be the most suitable one since it has all of the four components, unlike any other available micro-level datasets. Moreover, it is nationally representative of the US; hence, respondents of the HRS survey are not confined to very wealthy parents, contrast to those in tax return data. Not surprisingly, this dataset is extensively analyzed by other studies — such as McGarry and Schoeni (1995); Dunn and Phillips (1997); McGarry (1999); Kopzuck (2007); Hochguertel and Ohlsson (2009).

In detail, the HRS is a nationally representative and micro-level longitudinal data which bienially interviews five cohort groups in the US that range from people who were born before 1923 to people who were born around 1950. In this chapter, in order to secure largest number of observations usable for the analysis, the 2004 wave of the HRS is analyzed instead of more recent one.

Remember that we are trying to examine *inter vivos* transfer behavior of parents who divide bequests equally. The HRS dataset uniquely allows us to advance accuracy with this regard since it also provides information on whether the bequests of a respondent parent will be exactly equally divided or not — in addition to how much of *inter vivos* gift he (or she) gives to each of his (or her) children. In particular, the HRS dataset contains the information on whether a respondent parent plans to divide bequests exactly equally if he (or she) replied that he (or she) wrote a will. In addition to this, since intestacy law in the US, like any other industrialized countries, is equigeniture, those who did not write a will virtually choose equal division of bequests. As a result, the parents who are without a will or wrote a equal division will comprise the sample of this econometrical investigation. On the top to this, reminding that we only deal with parents with more than two children from the very beginning, parents with only child is not included. Besides, to avoid potential compounders, children who are less than 18 years old are not included since transfers to children aged less than 18 may be due to legal obligation, instead of from any other motive. Similarly, children who are residing with their parents are exclude out since
unless we have data of dollar value of housing and food that co-resident children are
benefiting from, the amount of *inter vivos* transfers to them is contaminated with this
noise, causing a bias. In line with this, to prevent any noise, the respondent parents
who split up and re-coupled with others are not included, either.

After this distilling out, the econometric analysis in this section is largely cen-
tered on the two key estimands: (i) the correlation between income inequality among
children and a parent’s decision of whether to give *inter vivos* transfers to at least
one of the children (to test whether equity concern triggers the use of *inter vivos*
gifts); and (ii) the correlation between the amount of *inter vivos* gift and the relative
income of each child given that the parent gives the gifts (to test whether *inter vivos*
gifts are counterbalancing income inequality among children or not). Thus, income
of an individual child is one of the most essential ingredients not only because it is
required to calculate the mean of children’s income in each family so that we can
obtain relative income of each of them, but also because it is needed for a measure of
income inequality among children. In spite of this importance, only available variable
in the HRS dataset that is the most relevant to this is roughly bracketized income of
each child’s household — only with three categories: (i) less than $10,000; (ii) higher
$10,000 and lower than $35,000; and (iii) above $35,000. Using this indicator variable
to substitute for the continuous variable — individual income of each child — has at
least two serious problems in the estimations. First, replacing individual income of
children with household income of them will lead a unpredictable bias since it entails
a substantial noise from earnings of other persons who are not a child of a respondent
parent and are not controlled both theoretically and empirically. Second, even if we
are putting aside the first problem for the moment, we cannot properly obtain the key
variable — the *relative* income of each child — since we cannot obtain the mean of
children’s income only with three bracket indicators one of which even has no upper
bound.

As a matter of fact, in empirical studies on intergenerational transfers, missing of
income data (whether parent’s or children’s) is not unusual. So, we can attempt to
consult methods that are used by other researchers to deal with this problem. Cox
(1987) uses the President’s Commission on Pension Policy survey data that does not
contain income of parents; he surrogates this with average income of the area where
each parent lives in. On the other hand, McGarry (1999) utilizes the HRS dataset
and replaces children’s individual income with the midpoint of the two end points of
the bracket\(^{43}\) that children’s household income belongs to. Instead of current income,
Hochguertel and Ohlsson (2009) estimate permanent household income of children —
using the children’s information of age, education, marital status, household income
category, employment status — to examine the correlation between this and *inter vivos*
gift and claim that the gift is compensatory.

\(^{43}\)But, it is not clear how she deals with children whose household income is greater than the
largest end point $35,000.
Unfortunately, none of the methods above achieves both unbiased estimation and statistically valid inference. Thus, rather than relying on any of them, in this analysis, multiple imputation method will be utilized since it is theoretically proven to yield unbiased estimates and valid inference (Rubin 1996). In detail, the procedure of multiple imputation is as follows: (i) generating a set of plausible estimates of the missing data based on the existing data and other predictor variables; (ii) performing any regression analysis with each complete data that is filled\(^{44}\) by one estimate (which is called an impute) in the set; (iii) repeating the second step multiple times; (vi) combining\(^{45}\) the outcomes of the regression analysis to obtain final estimation results. In this line, the method that Hochguertel and Ohlsson (2009) use is classified as single imputation which is computationally less burdensome; however, single imputation method is criticized since it regards imputed data identical to real data (Little 1988). In contrast, multiple imputation takes into account the uncertainty that both can be different and thus adds between-imputation variability to the standard errors in the fourth step of the procedure.

Although multiple imputation is statistically disciplined method with desirable properties, it is no panacea for all the cases of missing data: Careful and appropriate application is required. First of all, the model used in imputing the missed value (the first step of the procedure) is crucial and should be good; otherwise, concern on bias might arise (Rubin 1996). Fortunately, there is a well-established model\(^{46}\) in the imputation for children’s individual earnings in this analysis: Mincer equation\(^{47}\) that is corroborated by numerous other economic research with diverse datasets. In addition, the HRS dataset contains necessary information of children for a proper use of Mincer equation such as their age, education, sex, employment status, and marital status. Moreover, realizing that the HRS dataset is randomly sampled from the US population, the 2004 wave of Current Population Survey of the US (hereafter CPS) that is drawn from the same population can be used for imputation (providing

\(^{44}\) Adding more details, before the impute (the estimate from the first step) is filled in to comprise a complete data, a random error is added on it.

\(^{45}\) The valid way of combining is proposed by Little and Rubin (1987). Whichever estimate results we get from the second step, the average of them is the final point estimate. On the other hand, the variance for the inference is more complicated since it is composed of two components: within-imputation variability and between-imputation variability. For detail, see Rubin (1987).

\(^{46}\) TAXSIM model can be a good example, although it is usually used in single imputation method.

\(^{47}\) For those who do not know, Mincer equation refers to an empirical specification of the earning function which was proposed by Mincer (1974). This impacts profoundly and widely economic studies (especially labor field) since it fits the data remarkably well in most contexts. The most widely used version is as follows: \( \log(\text{earning}) = \beta_0 \text{education} + \beta_1 \text{experience} + \beta_2 \text{experience}^2 + \beta_3 \text{other relevant variables} \). In practice, experience is calculated as age-years of schooling-6. Usually, the last term - other relevant variables - includes employment status (information on whether employed or not; having full-time job or not; and the like), marital status, and gender. So, in this study, all of these will be included for imputation of missing income. In addition to this, for accommodating that zero income is forced to be dropped out since log of zero is not defined, all the zero incomes are converted into one before applying Mincer equation.
"existing data" in the first step of the procedure) since it supplies real observation of income of an individual along with their age, education, sex, employment status, and marital status that are matching to the observations in the HRS dataset (in the second step). Put another way, income of each child in the HRS dataset is imputed multiply as a missing data of CPS since children in the HRS dataset belongs to the US population and drawn from the same population. For more procedural details, since the data in the HRS is randomly sampled, this case can belong to "missing at random" (Little and Rubin 1987). Therefore, the Markov Chain Monte Carlo method that flexibly copes with arbitrary missing pattern (Schafer 1997) is chosen to create the imputes. Also, to be efficient, the second step is repeated 50 times\(^{48}\) here, although the number of imputation is usually, at most, 5 times.

In the end, all the key variables of this empirical examination are properly secured; and, the descriptive statistics of them is presented in Table 1.5. In addition to basic demographic and socioeconomic variables, other relevant variables that might affect the transfer behavior — such as contact frequency, indicator of living within 10 miles from parents’ home — are included for the sake of clear identification. When it comes to characteristics of parents such as age, sex, education, the information about head of parents’ household is used in order to avoid using the data from one family more than one time. Furthermore, for a measure of income inequality among children, Gini coefficient is used. To calculate the Gini coefficient, a simple and straightforward formula\(^{49}\) proposed by Deaton (1997) is adopted as follows:

\[
\frac{1}{\bar{y} n (n-1)} \sum_{i>j} \sum_j |y_i - y_j| \tag{1.25}
\]

where \(y_i\) is income of child \(i\), and \(\bar{y}\) is average income of all of the \(n\) children of a family.

In Table 1.5, the first column displays descriptive statistics of all the parents who choose equigeniture of bequests. Narrowing down from this, the second column reports corresponding ones of the parents who make *inter vivos* gifts to at least one of their children. At a glance, other than dollar valued variables, there seems little difference between these two groups, not giving a prompt clue on what factor triggers a parent to give any *inter vivos* gifts. Rather, to excavate how these variables may be related and can predict *inter vivos* transfer behavior calls for more sophisticated and systematic investigation, which will be demonstrated in the following subsections.

---

\(^{48}\)According to the formula presented by Rubin (1996), the asymptotic efficiency of finite times repeated imputation relative to infinite times is \(\left[1 + \left(\frac{\text{fraction of missing data}}{\text{number of imputation}}\right)\right]^\frac{1}{2}\). Hence, even when missing rate is 100% like this case, simple calculation of this formula shows that 50 repetition is enough to be asymptotic efficient.

\(^{49}\)It does not involve convoluted (numerical) integration or setting up Lorenz curve.
### Table 1.5 | Summary Statistics of Variables in the HRS

<table>
<thead>
<tr>
<th>Child-level Variable</th>
<th>All the parent</th>
<th>Given any gift</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>41.74 (8.94)</td>
<td>39.15 (8.34)</td>
</tr>
<tr>
<td>Sex (male =1)</td>
<td>0.50 (0.50)</td>
<td>0.49 (0.50)</td>
</tr>
<tr>
<td>Currently married</td>
<td>0.63 (0.48)</td>
<td>0.58 (0.49)</td>
</tr>
<tr>
<td>Education (years of schooling)</td>
<td>13.44 (2.40)</td>
<td>13.42 (2.45)</td>
</tr>
<tr>
<td>Income ($)</td>
<td>49693.5 (32882.2)</td>
<td>51760.6 (32476.2)</td>
</tr>
<tr>
<td>Inter vivos Gift ($)</td>
<td>598.18 (5540.6)</td>
<td>2330.37 (10750.9)</td>
</tr>
<tr>
<td>Contact Parents (day per year)</td>
<td>28.4 (505.5)</td>
<td>23.7 (439.7)</td>
</tr>
<tr>
<td>Live within 10 miles</td>
<td>0.34 (0.47)</td>
<td>0.32 (0.47)</td>
</tr>
<tr>
<td>No. of Children[1]</td>
<td>1.91 (1.56)</td>
<td>1.67 (1.45)</td>
</tr>
<tr>
<td>No. Obs</td>
<td>12112</td>
<td>3109</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parent-level Variable</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>68.43 (8.85)</td>
<td>66.35 (8.13)</td>
</tr>
<tr>
<td>Sex (male =1)</td>
<td>0.51 (0.50)</td>
<td>0.59 (0.49)</td>
</tr>
<tr>
<td>Currently Married</td>
<td>0.56 (0.50)</td>
<td>0.69 (0.46)</td>
</tr>
<tr>
<td>Education (years of schooling)</td>
<td>11.57 (3.56)</td>
<td>13.02 (2.82)</td>
</tr>
<tr>
<td>Income ($)</td>
<td>36237 (49085)</td>
<td>53156 (67177)</td>
</tr>
<tr>
<td>Wealth ($)</td>
<td>280226 (1453066)</td>
<td>496063 (2649944)</td>
</tr>
<tr>
<td>No. of Children</td>
<td>4.18 (2.14)</td>
<td>3.76 (1.83)</td>
</tr>
<tr>
<td>Gini Coefficient of Children’s Income</td>
<td>0.42 (0.34)</td>
<td>0.43 (0.35)</td>
</tr>
<tr>
<td>At Least One in Poor Health</td>
<td>0.14 (0.35)</td>
<td>0.09 (0.28)</td>
</tr>
<tr>
<td>No. Obs</td>
<td>3483</td>
<td>960</td>
</tr>
</tbody>
</table>

Foot note: [1] They are grandchildren to parents.

#### 1.4.3.2 Identification Strategy

To begin, since whether a parent decides to initiate an *inter vivos* transfer to any of his (her) children or not is intrinsically dichotomous, a binary response model best serves for identifying the effect of income inequality among children on the decision. Further, the specification can be put, in a concise way, as follows:

\[
\text{Give Any Gift}_p = 1[\beta_1 \cdot \text{Inequality among Children}_p + \mathbf{x}_p \delta_1 + v_p > 0] \quad (4.3.a)
\]

where Give Any Gift\(_p\) takes one if a respondent parent \(p\) gives *inter vivos* gifts to at least one of his (or her) children and takes null otherwise; Inequality among Children\(_p\) is Gini coefficient of children’s income defined by (1.25); \(\mathbf{x}_p\) is a vector of unity and other relevant explanatory variables of the parent \(p\) such as age, income, wealth, etc.; \(v_p\) is an error term that follows standard Normal distribution; and, \(1[\cdot]\) is an indicator function. In particular, Probit — one of the most widely used binary response models
is utilized here for estimation. Notice that the observation in this estimation is not based on individual child since we are exploring whether a parent’s decision on the use of *inter vivos* gifts is affected by inequality among children (Gini coefficient for children’s income) which is an aggregated information of all the children.

After examining how a parent’s decision about whether to use *inter vivos* gifts is affected by income inequality among children, the naturally ensuing question is on how to distribute the gifts among children. Especially, the other key part in empirically testing Proposition 1.4 is investigation of how the parent’s decision on the amount of *inter vivos* gift to each child is affected by the relative income of the recipient child, once the parent makes *inter vivos* transfers to tackle income inequality, if any. Since the dollar value of *inter vivos* gift is a continuous variable, the specification for this part might take the form as

\[\text{Inter vivos Gift}_{ip} = \beta_2 \cdot \text{Relative Income of Child}_{ip} + x_{ip} \delta_2 + u_{ip}\]  

(4.3.b)

where Relative Income of Child\(_{ip}\) measures how much income of a parent \(p\)'s child \(i\) is deviated from average income of the parent \(p\)'s children (measuring \(\bar{y} - y_i\)); \(x_{ip}\) is a vector of unity and other relevant explanatory variables of characteristics of both children and parents; and \(u_{ip}\) is an error term that follows Normal distribution.

On searching a proper estimator for this investigation, a standard OLS would immediately come up as it is a workhorse model. However, a closer look at the real data makes us reluctant to pick OLS as the method of choice since a considerably substantial portion of *Inter vivos Gift\(_{ip}\)* turns out to take on one specific value. That is, zero comprises 56% of the observations, which is hardly ignorable. As well known, if a single value of a continuous left-hand side variable takes fairly large, positive probability — while all the other values do not — the OLS is no longer consistent (or unbiased) estimator. Rather, a basic Tobit model is more appropriate since it validly accommodates this particular distributional feature. In fact, the observed zero in *Inter vivos Gift\(_{ip}\)* is not only occupies considerable probability mass but also is the minimum of the observations, which can be interpreted as following. First, it can be censored minus value of the gift since \(g_i\) in Proposition 1.4 (which corresponds to *Inter vivos Gift\(_{ip}\)*) can take negative values. Alternatively, in a more feasible interpretation, it can be uncensored but just represents a real choice of parents which can also be optimal in the light of the model. In other words, as you notice from the average age of parents and of children in Table 1.4, it is highly likely that large amount of children’s effort is already spent; hence, parents can rationally choose to give nothing to relatively rich children — instead of actually receiving money from them. However, regardless of the origin of the zero — whether limitation on observability or real outcome — the nonstandard mixture of discrete and continuous distribution where considerable portion of the left-hand side variable stacks up at the minimum value can be succinctly expressed in a standard Tobit model as follows, although maximum function has nothing to do with the choice mechanism at all.

\[\text{Inter vivos Gift}_{ip} = \max(0, \beta_2 \cdot \text{Relative Income of Child}_{ip} + x_{ip} \delta_2 + u_{ip})\]  

(4.3.b')
Instead of (4.3.b), with this statistical model that incorporates the peculiar distribution, we can avoid bias or inconsistency that a conventional OLS would beget. In addition, according to the standard Tobit, the distribution of the error term is still Normal, as general as in the standard OLS.

Obviously, the ultimate goal of the econometric analysis with the HRS data in this section is to empirically test Proposition 1.4. To this end, the test result should show whether income inequality of children triggers a respondent parent to bestow *inter vivos* gifts and whether the amount of the gift is negatively affected by the relative income of a recipient child when the parent decides to transfer the gifts *in the concern of* the income inequality among children. Presumably, the Probit model (4.3.a) performs a proper test for the former; however, (4.3.b’) does not do so exactly for the latter. That is, the Tobit model above does not yet coherently reflect whether income inequality among children motivated the parent to distribute *inter vivos* gifts in the first place. This may be subtle but important, since the motive of *inter vivos* transfer should consistently explain both (i) whether to give any *inter vivos* gifts and (ii) how much is given to each of child.

To incorporate this (4.3.b’), we can capitalize on the fact that *Inter vivos Gift* is observed only if the parent *p* initiates *inter vivos* transfer and thus takes one as the value of *Give Any Gift* in (4.3.a). Realizing this structure, we can take the advantage of Heckit model so that the Probit model (4.3.a) works as a selection equation in the first stage and gives inverse Mills ratios for the second stage. By having the Gini coefficient of children’s income in the first stage selection equation, the estimate for the coefficient of this selectivity variable in the second stage can test whether Inequality among Children affects a respondent parent to be a giver of *inter vivos* gift in the same equation that investigates how the amount of the gift from the same parent depends upon the relative income of a recipient child. However, unlike traditional Heckit model, the second stage is now a Tobit model, not a simple OLS. Thus, in some sense, we can call this new type of Heckit that is extended on the Tobit as Heckitized Tobit. In short, the extension is essentially based on control function approach with correlation between the error terms in both stages which is parameterized by *ρ*. Detailed mathematical derivations and estimation procedure are presented in Appendix A.3. All in all, the underlying statistical model for testing the validity of Proposition 1.4 would be

\[
*Inter vivos Gift*_{ip} = \max(0, \beta_2 \cdot \text{Relative Income of Child}_{ip} + \rho \cdot \text{Selectivity}_{ip} + x_{ip} \delta_2 + u_{ip})
\]

where Selectivity_{ip} is inverse Mills Ratio driven from the Probit model (4.3.a); and, other variables are as described above.

In addition, since income of each child is built by multiple imputation method, any analysis that utilizes the income variable is subject to the influence of the imputation. Thus, whichever regressions we run based on the aforementioned strategies, each of the estimation was repeated 50 times and combined to produce an outcome of each
estimation so that its standard error terms additionally include between-imputation variability for the sake of valid inference.

1.4.3.3 Estimation Results

In this subsection, the results of (4.3.a) and (4.3.b”) are displayed.

<table>
<thead>
<tr>
<th>Table 1.6</th>
<th>Decision of Giving Inter vivos Gifts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binary Dep. Var.: Giving Any Gift</td>
<td>Probit Estimate (Std Err.) Marginal Effect</td>
</tr>
<tr>
<td>Children’s Income Inequality</td>
<td>0.146 (0.07)** 0.046</td>
</tr>
<tr>
<td>Age</td>
<td>−0.014 (0.003)** −0.004</td>
</tr>
<tr>
<td>Sex (male=1)</td>
<td>0.143 (0.055)** 0.04</td>
</tr>
<tr>
<td>Currently Married</td>
<td>0.191 (0.059)** 0.06</td>
</tr>
<tr>
<td>No. of Children</td>
<td>−0.044 (0.012)** −0.014</td>
</tr>
<tr>
<td>Education (yrs of schooling)</td>
<td>0.081 (0.008)** 0.03</td>
</tr>
<tr>
<td>At Least One in Poor health</td>
<td>−0.207 (796.1)** −625972</td>
</tr>
<tr>
<td>Wealth ($100k)</td>
<td>0.005 (0.004) 0.001</td>
</tr>
<tr>
<td>Income ($10k)</td>
<td>0.024 (0.006)** 0.007</td>
</tr>
<tr>
<td>Constant</td>
<td>−0.767 (0.250)**</td>
</tr>
<tr>
<td>No. of Obs.</td>
<td>3483</td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>−1852.5</td>
</tr>
</tbody>
</table>

Note: [1] In parenthesis is standard error of each estimate.
* indicates being statistically meaningful at significance level of 10%.
** does at the level of 5%; *** does at the level of 1%.
[2] The marginal effect is evaluated at the means of right-hand side variables.

To begin, estimates in the Probit regression against the indicator variable of whether a respondent parent makes an *inter vivos* transfer are presented in Table 1.6. Recall that, in Table 1.5 above, except for the wealth, income, health status of
parents, we could not detect a distinction between the two groups of parents: giver vs. non-giver of *inter vivos* gifts. Now by harnessing Probit as specified in (4.3.a), we can further explore the underlying relationship between the parent’s decision and other relevant factors, beyond descriptive statistics. As shown in Table 1.6, the estimate of the impact of income inequality among the children — measured by Gini coefficient of children’s income — over the choice of giving *inter vivos* transfers turns out positive with a high statistical significance, which is consistent with the prediction of **Proposition 1.4** in the model.

Besides, when it comes to coefficient estimates of other relevant variables, we can compare these with an empirical study of McGarry (1999) since she runs a similar parent-level regression with the same left-hand side variable although income inequality among children is not addressed. First, the results on the effect of wealth, income, and health status of parents over the decision are aligned (that is, positive for the former two and negative for the latter), echoing each other and appealing to economic common senses. On the other hand, the estimate of impact of a parent’s age on the decision of giving any *inter vivos* gift is negative with statistical significance in Table 1.6, whereas the counterpart in her regression is significantly positive. However, not only because the right-hand side variables are not identical, but also because the parents analyzed in this regression are composed of equigeniture follower while her regression use all the parent respondents, it is yet hard to know where this difference comes from. Rather, this negative estimate may point to the fact that as a parent is younger, his (her) children is younger and more likely to confront liquidity constraint in their life where *inter vivos* gifts from the parent can plays more useful role than later in their life.

The supportive finding right above (positive $\hat{\beta}_1$ with statistical significance) is about an aggregated information of children: the impact of children’s income inequality instead of income of each individual child’s. As mentioned in the Section 1.4.3.2, the above estimation is just one part of the task of testing **Proposition 1.4**, we need to further explore the effect of an individual child’s relative income on the parent’s decision of the amount of the *inter vivos* gift, simultaneously linked to the decision of initiating the transfer. This constitutes the other remaining part of the testing, and the estimation results from the statistical model (4.3.b') elaborated in the previous subsection, which accommodates both the peculiar distribution of the given data and its motive, are presented in Table 1.7.

For the purpose of exposition, estimation results following (4.3.b') are also presented although it does not address the impact of income inequality on the decision of the amount of *inter vivos* gift. As mentioned in the previous subsection, it is Heckitized Tobit that closely fits the purpose of testing **Proposition 1.4**; hence, we draw some conclusions upon the results in the third and fourth column of Table 1.7.

First of all, observe that $\hat{\rho}$ the coefficient of the selectivity variable in the second stage is statistically meaningful and that $\hat{\beta}_2$ the estimate for the Gini coefficient of children’s income is positive with statistical significance in the first stage. In words,
these outcomes, as a whole, evince that the parent’s concern on inequality among children plays a directive role in the decision on the amount of the transfer in the same way as it did in the decision of giving any *inter vivos* gift in the first place. Most of all, this finding of coherency grants an empirical support to Proposition 1.4 in the model.

Table 1.7] Decision on the Amount of *Inter vivos* Gifts

<table>
<thead>
<tr>
<th></th>
<th>Tobit</th>
<th>2nd Stage</th>
<th>Heckitized Tobit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dep.Vari.: Inter vivos Gift ($)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Children’s characteristics</strong></td>
<td></td>
<td>1st Stage</td>
<td></td>
</tr>
<tr>
<td>Relative Income of Child ($)</td>
<td>$-0.065$</td>
<td>$(0.01)$***</td>
<td>$-0.065$</td>
</tr>
<tr>
<td>Age</td>
<td>$-512.2$</td>
<td>$(64.08)$***</td>
<td>$-527.1$</td>
</tr>
<tr>
<td>Sex (male=1)</td>
<td>$-611.5$</td>
<td>$(760.4)$</td>
<td>$-494.17$</td>
</tr>
<tr>
<td>Currently Married</td>
<td>$-2363.8$</td>
<td>$(776.5)$***</td>
<td>$-2317.9$</td>
</tr>
<tr>
<td>No. of Children</td>
<td>$674.9$</td>
<td>$(280.7)$**</td>
<td>$671.4$</td>
</tr>
<tr>
<td>Education (yrs of schooling)</td>
<td>$-203.2$</td>
<td>$(153.1)$***</td>
<td>$-202.3$</td>
</tr>
<tr>
<td>Contact Parents</td>
<td>$0.565$</td>
<td>$(0.807)$</td>
<td>$0.568$</td>
</tr>
<tr>
<td>Live within 10 Miles</td>
<td>$1050$</td>
<td>$(774.5)$</td>
<td>$1077$</td>
</tr>
<tr>
<td><strong>Parent’s characteristics</strong></td>
<td></td>
<td>1st Stage</td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>$543.7$</td>
<td>$(65.55)$***</td>
<td>$728.2$</td>
</tr>
<tr>
<td>Sex (male=1)</td>
<td>$-1384.1$</td>
<td>$(845.7)$***</td>
<td>$-3320.7$</td>
</tr>
<tr>
<td>Currently Married</td>
<td>$24.7$</td>
<td>$(896.5)$</td>
<td>$-2163.7$</td>
</tr>
<tr>
<td>No. of Children</td>
<td>$-2196.6$</td>
<td>$(191.8)$***</td>
<td>$-1804.7$</td>
</tr>
<tr>
<td>Education (yrs of schooling)</td>
<td>$681.9$</td>
<td>$(140.1)$***</td>
<td>$-370.6$</td>
</tr>
<tr>
<td>At Least One in Poor Health</td>
<td>$-486.6$</td>
<td>$(1234.4)$</td>
<td>$1998.7$</td>
</tr>
<tr>
<td>Wealth ($10k)</td>
<td>$7.84$</td>
<td>$(1.53)$***</td>
<td>$7.38$</td>
</tr>
<tr>
<td>Income ($10k)</td>
<td>$6.98$</td>
<td>$(53.6)$***</td>
<td>$264.4$</td>
</tr>
<tr>
<td>Selectivity</td>
<td>NA</td>
<td></td>
<td>$-16259.5$</td>
</tr>
<tr>
<td>Constant</td>
<td>$-19165$</td>
<td>$(4448)$***</td>
<td>$2247.3$</td>
</tr>
<tr>
<td>No. of Obs. (of Uncensored)</td>
<td>$3109$</td>
<td>$(1355)$</td>
<td>$3109$</td>
</tr>
<tr>
<td><strong>Dep.Var.: given any gift (=1)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Children’s Income Inequality, 8 Vari. of parent’s character included?</td>
<td>$0.105$</td>
<td>$(0.042)$***</td>
<td>Yes</td>
</tr>
<tr>
<td>No. Obs.</td>
<td>$12112$</td>
<td></td>
<td>$12112$</td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>$-17294.5$</td>
<td></td>
<td>$-15925.7$</td>
</tr>
</tbody>
</table>

Note: [1]In parenthesis is standard error of each estimate.
* indicates being statistically meaningful at significance level of 10%.
** does at the level of 5%; *** does at the level of 1%.
In addition to this, juxtaposing the two columns in Table 1.7 each of which is relied on different statistical model would be a useful exercise. As implied from statistical significance of the selectivity variables, there is found a selection bias that the simple Tobit estimator might suffer, which manifests itself in differences in the same estimates in the two columns (the first vs. the third). Nonetheless, the estimate for the impact of relative income of a recipient child on the amount of the transfer remains stably negative keeping statistical significance, thus strengthening evidence supportive of Proposition 1.4 found under (4.3.b’).

Moreover, as another way of robustness check, remind the details of the model that at equilibrium of equigeniture, equal amount of effort for familial interaction is spent by all the children although individual *inter vivos* transfer to them can be different. Thus, if there is found statistically significant relationship between contact frequency and the amount of *inter vivos* gifts in the regressions in Table 1.7, then it works as evidence against Proposition 1.4. However, it turns out that the coefficient estimate for contact frequency variable is not statistically significant even in the simple Tobit.

Lastly, albeit beyond our current focus of testing the model of this chapter, the estimation strategy Heckitized Tobit used in Table 1.7 can also be used to test Becker’s model as well. That is, by employing relative income of a recipient child as a right-hand side variable, we can answer the question — based on Becker’s model (1974) — of whether the transfer is offsetting the income difference between children *within* a family, which has not been fully explored in the previous empirical studies on his model, as criticized in Section 1.2.

### 1.5 Concluding Remarks

This chapter presents a model that can coherently explain seemingly puzzling behavioral patterns in intergenerational transfers. As head of a family, a parent altruistically cares about the welfare of children, but he also pursues fulfilling the primary function of family which depends on effort spent by children. So the parent uses a share of bequests to induce effort from them. However, since individual effort of each child is unverifiable to a third party adjudicator, children’s effort for family is not effectively compensated fully. As a result, children do not provide effort for family as much as the parent desires. In the parent’s endeavor to overcome this moral hazard problem, two stable equilibrium inheritance rules rise: either giving all to one heir (unigeniture) or dividing bequests equally (equigeniture). This is consistent with the polarized distribution of inheritance patterns across times and places. Moreover, a rise in the productivity of effort leads the evolution from unigeniture to equigeniture. More specifically, when subsistence is the principal function of family as in pre-industrial agrarian societies, a jump in agricultural labor productivity like an advance of intensive agriculture can lead the evolution to equigeniture, regardless of social class mobility. This may shed light on the contrast between equigeniture in feu-
dal China or India vs. primogeniture in England. Moreover, as the main function of family changes into emotional support along the industrialization, the ensued transformation of the nature of effort from manual one to psychological one brings another rise in the productivity (the driving force of the evolution), resulting in equigeniture adopted in industrialized societies. At the same time, in order to address a widened income inequality among children who exert equal effort for family, the parent can choose to give counterbalancing *inter vivos* gifts even though he keeps dividing bequests equally, which in fact rationalizes "equal division puzzle."

In addition to aforementioned consistency found between the theoretical model and empirical realities, there are other results that further allow us to additionally test the validity of the model. First of all, applied to pre-industrial agrarian societies where economic subsistence is the major task of ordinary families, the model predicts that a rise in the productivity of effort (labor) leads equigeniture to be chosen over unigeniture. Not only historical case studies but also econometrical investigations on pre-industrial agrarian societies around the world are conducted and find evidence supporting this prediction of the model. Second of all, in explaining "equal division puzzle" that is found in fully industrialized societies, the model offers a further specification on *inter vivos* transfer behavior as follows: (i) income inequality among children triggers the parent to render any *inter vivos* gift; and, (ii) the amount of *inter vivos* gift is negatively associated with relative income of recipient child (compared to his siblings) while bequests remain equally distributed, regardless of children's income. This is tested by econometric analysis with micro-level data on *inter vivos* transfer behavior of contemporary parents who follow equigeniture, whose findings are consistent with the model.

In sum, by synthesizing altruism, reciprocity (exchange), and equity concern into one motive of intergenerational transfers, this chapter achieves to offer a coherent explanation on seemingly inconsistent behavior in the transfers. In the end, this study can be useful for more effective analyses on policies related to intergenerational transfers. Moreover, for an interesting future work, we can re-visit Barro-Ricardian equivalence with this model to see whether still this theorem holds or not; and, if not, we can further investigate how timing of taxation matters in the light of this model.
Chapter 2

Retirement and Exposure of Pension to Financial Market Fluctuations

2.1 Introduction

By 2030, about 20% of the US population is estimated to be over 65 years old, and most of them would be retirees.\(^1\) With this rapidly aging population, importance of pension rises since pension is the main income source for retirees.\(^2\) For instance, the assets for occupational private pension funds are greater than 70% of GDP (OECD, 2011). Similarly, public pension takes up 17% of the government expenditure. On top of this, exposure of pension wealth to financial market fluctuations has been increased along with continuing shift from defined benefit (DB) plans to defined contribution (DC) plans (shown in Figure B.1 and Figure B.2 in Appendix B) since DC plans allow the exposure while DB plans do not. Moreover, the fund for providing public pension is to run out by 2035 which makes reforms on the current pension system inevitable.\(^3\) Some propose more individualized investment of public pension that can entail more exposure of pension to financial market. After all, realization of these recent changes lead us to ask the following question: How does exposure of pension wealth to financial market fluctuations affect retirement behavior?

To address the question, this chapter theoretically delineates retirement behavior based on a life-cycle model under uncertainty. Contrasted to the previous studies, pension wealth is treated separately from non-pension wealth since pension plays a

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\(^1\)source: aging statistics presented by Administration on Aging of the Department Health and Human Services.

\(^2\)Public pension and private pension and other savings comprise income sources for retirees and called ‘three-legged stool.’

distinctive role in deciding when to leave the labor force. Firstly, pension wealth is more tied to retirement decision than non-pension wealth since an individual usually starts to receive pension benefit at the time of retiring. That is, liquidizing pension wealth coincides with the transition to retirement. Secondly, there are exogenously given age-sensitive constraints on the availability of pension benefits. Most of all, from the dynamic labor supply model, the conditions of optimal time to retire are elaborated in such a way that explicitly embeds these peculiar aspects of pension wealth and enables us to do comparative statics analysis with respect to pension. The resources for post-retirement consumption, which includes pension benefit, is accumulated from working when young; and, at the end of the career, as it gets feasible for him to consume as before without working, labor earnings turn no longer attractive than full leisure from leaving the labor force. Essentially, upon securing resources necessary for post-retirement consumption that can maintain the pre-retirement living standard, an individual retires.

Based on this, we analyze effect on retirement behavior of exposure of pension to financial market fluctuations. At the outset, dichotomy of exposure stems from basic classification of pension: DB plan vs. DC plan. Pension benefit of a DB plan is pre-determined by a preset formula. Thus, beneficiary individuals do not bear the investment risk on pension wealth from financial market fluctuations. By contrast, pension benefit of a DC plan depends on investment return on DC account at the retirement. Therefore, a DC plan basically exposes pension wealth of workers to financial market fluctuations whereas a DB plan does not. Along with this, financial market fluctuations are regarded as having two phases: either a positive shock or a negative shock. A positive shock occurs when the return on pension wealth is greater than expected. Likewise, a negative shock occurs when the return on pension wealth is lower than expected. Although optimal plan for retirement is laid out at the very beginning, a worker can change this plan due to a discrepancy between expectation and realization of pension wealth which each shock generates. In addition, focus narrows down to responses of senior workers on the verge of retirement. Since young workers, who are far from the planned time to retire, have less accumulated resources and longer remaining life time, the aforementioned conditions to retire are not yet met; that is, it is not likely for them to retire in responding to a shock on pension.

First of all, let us analyze difference in retirement caused by exposure of pension to a positive shock. In particular, we are interested in comparing retirement behavior of two groups of workers — DB plan workers vs. DC plan workers — who are identical except for exposure of pension to the shock. Firstly, the positive shock affects workers who just reach the planned time to retire. On the one hand, DB plan workers retire now as planned because, without exposure, realized pension is the same as its expected value. On the other hand, DC plan workers may have an unexpected increase in their pension wealth due to exposure to the positive shock.

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4Social Security retirement benefit program is an example of this type.
However, since time is irreversible and already supplied labor cannot be undone. As a result, the DC plan workers cannot retire any earlier for responding to the positive shock. That is, they also retire now. Thus, for this cohort, there is no retirement response caused by exposure to the positive shock. Secondly, the positive shock also can influence pension of slightly younger workers who have not yet reached their planned time to retire. Were it not for the shock, they are supposed to work now; so, DB plan workers keep working now. However, DC plan workers can either work or retire now (earlier than planned) as a response to exposure to the positive shock. For deciding whether to retire now for responding to the shock, it is rational for the DC plan workers to compare given an unexpected increase in pension wealth with opportunity cost of early retirement. If they retire now, labor earnings of this period are foregone and they have to face longer periods of retirement than planned, which increases the demand for necessary resources for post-retirement consumptions. If the gain in their pension wealth is not large enough to compensate the opportunity cost, then the DC plan workers would rather keep working as the DB plan workers do. In this case, even when taking into account the elder cohort of DC workers, exposure of pension to the positive shock makes no difference in retirement. Markedly, only if the magnitude of the positive shock is large enough, actual change in retirement behavior is realized. In sum, exposure of pension to a positive shock affects retirement, but it does not always bring discernible action of retirement response. Only when the positive shock is sharp and great enough to support retirement earlier than planned, exposure of pension to it makes a difference in behavior of leaving the labor force.

Second of all, facing a negative shock, for workers who reach the planned time to retire, losses from the exposure of pension to the negative shock directly leads to failure in meeting the conditions to retire, falling short of necessary resources for post-retirement consumptions. As a result, DC plan workers delay retirement whereas DB plan workers retire as planned. In addition, for workers who are younger than these, they will keep working regardless of exposure to the negative, because none of them would succeed in now achieving to secure necessary resources for retirement. Eventually, a negative shock always brings a decrease in retirement. Moreover, this negative response of retirement is realized by exposure to negative shock without specific conditions on its magnitude, contrast to a positive shock. All in all, exposure of pension to financial market fluctuations leads to response of retirement in asymmetric way, due to the lack of change in retirement behavior facing a small positive shock. This new theoretical finding that existence of actual retirement responses is not perfectly symmetric is also visualized in a simple simulation exercise.

Next, this theoretical analysis is applied to biennial panel data of senior workers in the US, Health and Retirement Study (HRS), covering recent business cycle. In detail, the 2004 and 2006 expansions are regarded as positive shocks, whereas the 2002 and 2008 recessions are regarded as negative shocks. More specifically, the exposure to these shocks is captured by a binary variable that indicates whether a worker holds a DC plan that is invested into stock market in each sampling year. Following
the nature of data and keeping a link to the theoretical model, proportional hazard model is chosen for empirical analysis. Furthermore, to address bias concerns from non-random assignment of DC plans to workers, difference-in-difference estimator is also adopted. Regressions with this data show that exposure of pension to 2002 and 2008 negative shocks lead to a decrease in retirement. By contrast, statistically significant evidence is not found on a discernible increase in retirement owing to exposure of pension to either 2004 or 2006 positive shock. Noticeably, these findings from observations over the last decade are not self-contradictory. Rather, in light of theoretical results, this implies that little response of retirement may be attributed to the magnitude of those positive shocks. Put another way, if there were a sharper and greater positive shock, we could have observed a substantive increase in retirement.

In contrast to previous studies on retirement behavior, this chapter can make a contribution with the theoretical approach that articulates a channel through which pension enters into retirement decision embracing peculiar aspects of pension wealth distinct from non-pension wealth. More importantly, it is the first study showing how a positive shock on pension wealth could end up with no discernible behavioral response of retirement. Furthermore, with respect to applicability of theory to data, it enables us to properly conduct reduced form estimations by requiring information of only few periods around which retirement occurs.\(^5\)

Finally, this chapter is organized as follows. Section 2.2 reviews previous research relevant to the present study. Section 2.3 provides a behavioral model that theoretically analyzes retirement decision for characterizing optimal conditions to retire. Applying the given theoretical framework, empirical analysis is conducted in Section 2.4. Section 2.5 concludes with policy implications.

### 2.2 Review of Related Literature

To begin, there have been continuing efforts made for understanding retirement behavior. By its nature, retirement decision is complicated since it involves multi-period concerns and is related to various intricate factors such as Social Security retirement benefit program. According to various studies like Gustman and Steinmeier (1986), Rust (1989), Stock and Wise (1990), Gruber and Coile (2007), pension wealth proves to be one of the most important factors. In their dynamic labor supply model, they assume that pension benefit is preset by formula as DB plans.

In last decades, however, there has been a steady shift from traditional DB plans to DC plans, not only in terms of participants but also in terms of total asset volume (See Figure B.1 and Figure B.2 in Appendix B). For instance, Poterba, Venti, and Wise (2010) point out that defined contribution (DC) plans came to comprise 80%...
of private pensions. In fact, these plans usually are invested in stock market; and the uncertainty on the return is borne by individual workers. The rise in volatility of wealth due to its exposure to investment risk can be substantial, which is presented by Feldstein and Ranguelova (2001) and Burtless (2003) in public pension context and by Poterba et al. (2007) in private pension context. Nevertheless, built upon simulation exercises, retirement is not main focus of these analyses.

Despite of the increasing exposure of pension to financial market fluctuations, how this can affect retirement is not yet well studied theoretically and empirically. Focusing on the stock holdings of non-pension wealth, Sevak (2002) and Coronado and Perozek (2003) regard the run-ups in the late 1990’s stock market as a windfall gain and empirically study its impact on retirement behavior. In particular, in their regression analysis where the stock ownership is central variable, Coronado and Perozek (2003) report that DB plan holders retired earlier than DC plan holder in the expansion period, which is opposite to usual conjecture.

Later, this line of study is extended with releases of data covering early 2000 recession triggered by the burst of what is so called the dot com bubble. First of all, Coile and Levine (2006) investigated both boom and bust periods for separate groups differentiated by their exposure to stock market. According to their result, DC plan holders are less likely to retire in recession periods whereas there is no statistically meaningful response from these individuals found in late 90’s expansion periods. On the other hand, Gustman et al. (2010) revisit the same periods that Coile and Levine (2006) cover. They argued that both fall and rise of stock returns entails responses in retirement behavior, unlike Coile and Levine (2006); however, their result is based on simulations, contrasted to the aforementioned studies on the same topic. In addition, after recent turbulence of the Great Recession in 2008, labor force participation of old workers are discussed by Coile and Levine (2009, 2011) and Gustman et al. (2011) without investigating wealth effect on retirement behavior in this period.

Since wealth is related to earnings from labor supply, identification of its effect on retirement is subject to concern of endogeneity bias. To properly address this issue, utilizing exogenous positive shocks like lottery and inheritance on non-pension wealth, Holtz-Eakin et al. (1993), Imbens et al. (2001) and Brown et al. (2010) found negative responses in labor supply, although their estimates are different from each other by large margin. Notably, even though these studies might provide better identifications on the impact of wealth exploiting such exogenous wealth shocks, they might not be directly applicable for labor responses by exposure of pension wealth to stock market. First of all, as non-pension wealth, liquidizing this wealth is not tied up to retirement decision. Moreover, they are not subject to specific restrictions of availability that any pension — whether public or private — imposes. Nevertheless, it would be infeasible to intentionally run an experiment in pension wealth to address bias issue. Utilizing a sudden rise in Social Security benefit schedules, regarded

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6This is estimated in 2000. Moreover, this has been continuing to rise so far.
as quasi-experiment, evidence that earlier retirement is not induced by a large and unexpected increase in public pension wealth both is found by Krueger and Pischke (1992).\footnote{In simulation — not directly using observation data — Burtless (1986) also tried to examine its effect on retirement and found little evidence.}

All in all, previous studies suggest that simple textbook prediction of wealth effect on retirement might not carry over to how exposure of pension to upturns or downturns of financial market fluctuations can affect retirement.

\section{2.3 Theoretical Analysis}

\subsection{2.3.1 The Behavioral Model}

To analyze retirement behavior in the face of economic fluctuations, we can capitalize upon a life-cycle model with uncertainty. Since the time of retirement is contemplated and planned ahead, instead of being chosen on instant base, dynamic model is more suitable. At the outset, preference of an individual is represented by additively separable life time utility function. Moreover, the temporal utility function is $U_t = U(C_t, L - h_t)$ where $C_t$ is consumption and $L - h_t$ is leisure in period $t$ with $h_t$ being labor supplied for the period $t$. Since no individual is immortal, no one lives beyond period $T$. In other words, $t \in \{0, 1, \cdots, T\}$. After all, given an initial endowment $A_0$, an individual solves the following program for his life time

\begin{equation}
\max_{\{C_t, h_t\}_{t=0}^{T}} E_0 \left[ \sum_{t=0}^{T} \frac{1}{(1 + \rho)^t} U(C_t, L - h_t) \mid \Omega_0 \right] \text{ s.t. } A_t + h_t W_t \geq C_t + \frac{1}{R_t} A_{t+1} \text{ for all } t \tag{2.1}
\end{equation}

where $A_t$ is an asset held in period $t$ and yields a gross return $R_t$; $W_t$ is a wage rate in period $t$. In addition, $\rho$ is a constant time-preference parameter; and, $\Omega_t$ refers to all the available information by period $t$. For the terminal period $T$, no Ponzi game\footnote{If one wants to incorporate bequest concern, this condition is easily modified into $A_T \geq b$ ($b$: bequest) preserving main results from no bequest cases.} is allowed, so $A_T \geq 0$. Furthermore, $U$ is continuous and concave; and, marginal utility from consumption is not negatively affected by an increase in leisure; that is, $U_{sl} = \frac{\partial U}{\partial c \partial l} \geq 0$ where $l$ refers to leisure for short.\footnote{This can be an innocuous assumption. To take a justifying example, however nice meal you may eat, your marginal utility from consuming the meal will be larger if you can eat it leisurely than if you have to hastily gulf it up while working.} At the very beginning, the individual lays out an optimal path of consumption and labor supply $\{C_t, h_t\}_{t=0}^{T}$ which includes his plan on the retirement. Apparently, his optimal path for the remaining life is always subject to change in any period over the course of time as he updates newly arrived information, since he does not know future for certain but just forms a rational expectation.
This is workhorse model used in the majority of analyses on retirement behavior. In spite of its popularity, a simple characterization of optimal path of labor supply is not easily obtainable since this dynamic programming rarely has a closed form solution. However, there are numerous equivalent ways available for solving the model. For instance, to solve the same life-cycle labor supply model like above, Gustman and Steinmeier (1986) took Hamiltonian approach whereas Rust and Phelan (1997) relied on Bellman equation utilizing a value function. Not surprisingly, both methods yield equally good fits to observed retirement behavior. Nevertheless, both are complicated and require information (or assumptions when information is not available) on numerous periods before and after retirement as well. For example, to calculate a value function just one period before retirement alone, in principal, needs all the utility values till death after retirement. More fundamentally, these approaches are not effectively informative about a mechanism underpinning the retirement decision. In particular, they do not explicitly embrace the fact that usual start of receiving pension benefit is concurrent with the transition to retirement. This suggests us that we might have to look for another approach to analyze how exposure of pension to stock market fluctuations affect timing the retirement.

There is another equivalent way to solve the dynamic optimization (2.1): finding an optimal policy function. As Bellman equation approach involves finding a value function that maps into level of utility from state variables, optimal policy function approach requires us to obtain a function that maps into optimal choice variables. Instead of level of utility at retirement, we are interested in when optimal choice of labor supply evolves to zero, the optimal policy function appears to be more useful for this study. First of all, the conditions that the optimal policy function should meet for every \( t \) are as follows.

\[
U_c(C_t, L - h_t) = \lambda_t \tag{2.2}
\]

\[
U_t(C_t, L - h_t) \geq \lambda_t W_t \tag{2.3}
\]

\[
\frac{R_t}{1 + \rho} E_t[\lambda_{t+1} \mid \Omega_t] = \lambda_t \tag{2.4}
\]

where \( \lambda_t \) is a Lagrange multiplier on the intertemporal budget constraint in (2.1). Notably, once having \( \lambda_t \), information outside of the period \( t \) is not necessary; hence, we do not need to impose any assumption on retrospective and prospective consumption, wealth, wage path. As Macurdy (1981) pointed out, \( \lambda_t \) reduces required information for making decisions of consumption and labor supply for the period \( t \) in an intertemporal setting of life-cycle model. Put in our context, even though individuals do take into account of a series of consumptions, wealth, wages in the past and the future for deciding whether to retire today or not, the seemingly overwhelming demand for information for the decision is conveniently summarized and reduced into one parameter, \( \lambda_t \). Furthermore, we can take one more step that replaces \( \lambda_t \) with (2.2), which reveals the meaning of (2.4) as an intertemporal Euler equation that supports
consumption smoothing throughout life time. On top of this, since retirement is depicted as zero labor supply, note that corner solution case — when (2.3) holds with inequality rather than equality — is pertinent to our analysis.

Having this new approach in mind, realize that even though an optimal path of consumption and labor supply satisfies the above conditions for all periods over the entire life, only few of them around the transition to retirement need to be looked at for the present analysis. This implies that we do not have to actually obtain entire optimal policy function itself for the purpose of exposition on mechanism underlying the retirement decision. Rather, the optimal function’s property covering retirement periods alone would be informative enough to shed a light on how pension benefit can enter into decision of time to retire. Let’s say that an optimal time to retire lies at period \( r^* \), according to the optimal path derived from (2.1). By the nature of retirement, optimal choice of labor supply is to be zero after the period \( r^* \) during the remaining life time. In terms of the above conditions, this implies that for \( \forall t \in \{r^*, r^* + 1, \ldots, T\} \)

\[
\frac{U_t(C_t, L - h_t)}{U_c(C_t, L - h_t)} \bigg|_{h_t=0} = \frac{U_t(C_t, L)}{U_c(C_t, L)} > W_{t^*},
\]

and

\[
\frac{R_{t-1}}{1 + \rho} E_{t-1}[U_c(C_t, L) | \Omega_{t-1}] = U_c(C_{t-1}, L).
\]

The latter means that post-retirement consumptions \( \{C_t\}_{t=r^*}^T \) are smoothed out as similar level of pre-retirement consumptions.\(^{10}\) However, consumption Euler equation (2.6) also applies for all the periods before retirement. Rather, distinction from pre-retirement rises at the former (2.5). Thus, more critical is to find when the equality between left and right hand sides of (2.3) converts into the inequality of (2.5). Although we now get more concrete description of optimal time to retire, it is not yet expressed in terms of pension benefit, which is part of the post-retirement consumptions. For paving the way to show the channel through which pension benefit affects the time of retirement, notice that intertemporal budget constraint is binding for every period, so pre-retirement consumption is \( A_t - \frac{1}{R_t} A_{t+1} + h_t W_t \). Likewise, post-retirement consumption can be restated as \( A_t - \frac{1}{R_t} A_{t+1} + B_t I_B(t) \) for \( \forall t \in \{r^*, r^* + 1, \ldots, T\} \) where \( B_t \) refers to periodic pension benefit received in a period \( t \)\(^{11}\) and \( I_B(t) \) is an indicator function defined as for a period \( t \)

\[
I_B(t) = \begin{cases} 
0 & \text{if one is not eligible to receive } B_t \\
1 & \text{if one is eligible}
\end{cases}
\]

\(^{10}\)At a glance, (2.5) looks the same as labor supply decision in static model. But this is clearly distinct because this is to be met for the remaining periods and this involves intertemporal consumption Euler equation.

\(^{11}\)Rigorously speaking, \( C_T = A_T + B_T \) for the terminal period.
which is monotonously increasing, so once its value switches into one and stays so for good. First of all, although it is subtle, difference between post- and pre-retirement consumption shows that pension wealth is more tightly and directly related with labor supply than non-pension wealth. Second of all, in order to bring more relevancy to the given behavioral model, the indicator function $I_B(\cdot)$ is introduced to reflect another differentiated feature of pension wealth, contrasted to non-pension wealth: time-sensitive restrictions imposed on the availability of pension benefits. In practice, an individual is not able to receive pension benefits until specific age. This is beyond an individual’s control. To take an example, even after paying all the Social Security taxes, one cannot claim Social Security retirement benefit until a legally stipulated age (which is 62, early retirement age; ERA, in short). Furthermore, it is not legally allowed to borrow money against Social Security wealth. On the other hand, private pension plans often impose restrictions on availability of pension benefit like when a worker is fully vested. However, it is usual that these restrictions are lifted after a beneficiary worker has worked five years. As a consequence, it is ERA that plays an actual binding role in pension benefit availability. Markedly, these time-sensitive restrictions are exogenously given and out of individual’s disposal, implying that an individual has less control over time to liquidize pension wealth than non-pension wealth.

Next, let us move on how post-retirement consumption can be related with the conditions for optimal time to retire. In particular, the relationship between post-retirement consumption and the switch of (2.5) to inequality can be first clarified by the following lemma.

**Lemma 2.1.** $U_L(C_t;L)/U_C(C_t;L)$ is increasing in consumption $C_t$.

*Proof.* See Appendix B.

Notice that (2.5) is more likely to hold, as available resources for post-retirement consumption increases. It is intuitive in that wage becomes less attractive as one can consume well without working, for retirement to be chosen. That is, available resources for post-retirement consumption is the key factor for retiring. Taken these

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12Vesting refers to the amount of time one must work before earning a non-forfeitable right to his pension benefit. When he is fully vested, he can claim and receive the pension benefit even if he has left the work. Usually, if a worker leaves a firm prior to the vesting date, contributions that have been made by the employer would be lost.

13In addition to the restrictions on the starting point after which a worker becomes eligible to claim pension benefits, there are similar, if not the same, restrictions on the back end. For instance, for a worker who holds a defined contribution pension plan like 401k must start receiving pension benefits (i) in the year he retires and (ii) no later than his 70 and $\frac{1}{2}$ even if he still works then (which is called as Required Minimum Distributions, RMD). In line with this, there is a similar kind of upper-bound restriction: increment in Social Security retirement benefit due to postponement of the benefit claim is no longer available when taking up the public pension benefits is delayed after 70.
altogether, optimal time to retire (i.e., the period $r^*$) is the time when the following conditions — which are restatement of (2.5) and (2.6) respectively — start to be met: for $\forall t \in \{r^*, r^* + 1, \ldots, T\}$

$$U_t(A_t - \frac{1}{R_t} A_{t+1} + B_t I_B(t), L) \quad U_c(A_t - \frac{1}{R_t} A_{t+1} + B_t I_B(t), L) > W_{r^*}$$

(2.8)

and

$$\frac{R_{t-1}}{1 + \rho} E_{t-1}[U_c(A_t - \frac{1}{R_t} A_{t+1} + B_t I_B(t), L) \mid \Omega_{t-1}] = U_c(A_{t-1} - \frac{1}{R_{t-1}} A_t + h_{t-1} W_{t-1}, L - h_t).$$

(2.9)

It may be useful to note a bigger picture of the life-cycle model that implicitly underlies the conditions for retirement (2.8) and (2.9). First of all, before retirement, part of earnings is accumulated as the asset for post-retirement consumption. At the same time, labor supplies when young also accrue the level of pension benefit through various channels such as Social Security taxes and contribution to private pension account. Thus, at the end of career, the benefit of labor (wage) is no longer greater than utility which one can get without working, which is stated in (2.8). On the other hand, the other condition (2.9) suggests that resources for post-retirement consumptions throughout the remaining life should be large enough for an individual to maintain living standard of pre-retirement. The property of the period $r^*$, optimal time to retire, is that upon securing resources necessary for post-retirement consumption which can maintain the level of pre-retirement living, one retires after decades of working and saving.

Most importantly, as a corollary of Lemma 2.1, notice that, when we are holding other factors fixed, as available pension benefit increases (decreases), (2.8) gets more (less) likely to hold, facilitating (delaying) transition of retirement. In sum, through (2.8), we now are able to explicitly examine how pension benefits can affect retirement behavior.

Moreover, it is worthwhile to mention the implications of Lemma 2.1, (2.8), and, (2.9) made on locating the period $r^*$. First of all, according to indicator function $I_B(\cdot)$, available pension benefit discontinuously jumps at the time when a worker gets eligible to receive pension benefits. Furthermore, since the likelihood with which (2.8) holds is increasing in available pension benefit, available pension benefit discontinuously jumps at the time when the eligibility switches from $I_B(r^* - 1) = 0$ to $I_B(r^*) = 1$. As a matter of fact, the introduction of the indicator function may provide an explicit explanation on a spike of retirement rate around ERAs in various countries (Gruber and Wise, 1999). If availability of pension benefits were at one’s own proposal, many of heterogeneous workers could have retired earlier in the absence of eligible

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14In reality, timing retirement is much more complicated involving a confluence of factors like asset accumulation, health issues, and the like. For instance, even if one has already reached ERAs of all the pension plans that he holds, he might not have accumulated enough assets. Or, he might
age restrictions. In other words, the model can generate a prediction that some focal points of retirement are made around ERA even when individual workers are allowed to be heterogeneous.

### 2.3.2 Retirement and Exposure of Pension to Financial Market Fluctuations

Having discussed the optimal conditions to retire and its relationship with pension, let us move on to analyzing how exposure of pension to economic fluctuations can make a difference in retirement behavior.

At the outset, the way how pension is exposed to financial market fluctuations prerequisites to know basic classification of pension: defined benefit (DB) plan vs. defined contribution (DC) plan. DB plans provide pension benefit which is defined by a preset formula (usually depending on years of service and wage history). As a result, even when resources for funding DB plan beneficiaries are invested in financial market, pension benefit from DB plan is paid as predefined, regardless of losses or gains of the investment.\textsuperscript{15} Essentially, the risk from economic fluctuations is borne by other entity than beneficiary individuals. Social Security retirement benefit program is one example of a DB plan; and shrinking number of employers also provide private pensions of this type. By contrast, DC plans place the risk on beneficiary without any pre-determined formula. Examples of DC plan include 401k, 403b, IRA, Keogh account, and the like. Especially, a DC plan holder can expose his pension wealth to financial market fluctuation by investing his DC account in stock market. Whether losses or gains from the investment, he bears the financial risk on his pension benefit. That is, pension benefit from his DC plan depends on realized value of pension wealth at the time of retirement.

The distinction between the two types of pension suggests us a way to embody difference in exposure of pension to financial market fluctuations. Since only DC plan holders can expose their pension to financial market, let $DC_t$ denote periodic pension benefit for an individual who holds a DC plan \textit{and} invests his DC account in stock market.\textsuperscript{16} Similarly, $DB_t$ stands for periodic pension benefit from a private DB plan.

In terms of the present model, the way how losses or gains from financial market fluctuations be better off with delaying retirement after ERAs due to benefits from health insurance provided by his employer (French and Jones, 2010; Lumsdaine et al. 2001). Nevertheless, despite of this complexity, this behavioral model suggests a lens that shows how availability of pension benefits, which is exogenously given, can steer the time of retiring.

\textsuperscript{15}The payout of these benefits is guaranteed by an independent agency of Pension Benefit Guaranty Corporation (PBCG) insurance.

\textsuperscript{16}Although not all of DC plan holders invest their DC account into stock market, vast majority of them do.
fluctuations affect pension benefit is depicted as follows

\[
\frac{\partial DB_t}{\partial R_t} = 0 \text{ but } \frac{\partial DC_t}{\partial R_t} > 0
\]

(2.10)

for period \( t \). In this line, since Social Security retirement benefit program is a DB plan, we can also say that

\[
\frac{\partial SS_t}{\partial R_t} = 0
\]

(2.11)

for period \( t \). Thus, pension benefit \( B_t \) is either \( B_t = SS_t + DB_t \) or \( B_t = SS_t + DC_t \) with the latter exposed to ups and downs in stock market, contrasted to the former. Moreover, for a simple and reasonable comparison between different pension plans, periodic pension benefit is calculated with dividing pension wealth by number of periods for which the benefit is paid out. As a consequence, the amount of periodic benefit payment calculated at the period when an individual retires remains constant throughout the remaining life time, which simplifies our analysis. In other words, when an individual retires at period \( r^* \), his pension benefit is settled as \( B_t = SS_{r^*} + DB_{r^*} \) or \( B_t = SS_{r^*} + DC_{r^*} \) for \( \forall t \in \{r^*, r^* + 1, \cdots , T\} \).

Eventually, difference in retirement behavior by contrast between \( DB_{r^*} \) vs. \( DC_{r^*} \) stems from the exposure of pension to financial market fluctuations, which we will investigate here. To this end, we need to distill out any other factors as confounders with ceteris paribus assumptions such as same expected values of \( DB_{r^*} \) and \( DC_{r^*} \). In other words, we will investigate retirement behavior of two individuals (a DB plan worker and a DC plan worker) who are identical except for exposure of pension to financial market fluctuations. As a consequence, both have the same retirement plan.

Although individuals laid out optimal retirement plan as the period \( r^* \) at the very beginning, they can later change their time to retire, if there is a large discrepancy between realized value of pension wealth and ex ante expectation on it. Therefore, if there is a shock from fluctuations in the market, one might need to deviate from the plan by re-optimization. A shock triggering the change in retirement behavior would be either positive or negative, based on a gap between expected vs. realized investment return. In this study, we regard a positive shock occurring when returns in financial market yield greater than expectation on them. Likewise, when pension wealth invested in financial market yields smaller than expected, we treat

\[17\] If we allow variation in periodic benefit payments, we can regard this constant payment as average of them.

\[18\] To distill out other channel through which exposure of pension wealth to a shock can affect retirement behavior, we assume that both as same realization of wage whose fluctuations, if any, over the cycle is not big enough to switch the sign of (2.10).

\[19\] This also means that some possible changes in wage rate of the current period is not of major concern on retirement decision. Instead of static decision of labor supply, the intertemporal labor supply decision to govern the transition to retirement period which involves long history of wage profile of a worker, an increase (a decrease) wage rate of one period might not make a primary difference in retirement behavior.
this as a negative shock. Certainly, over the course of time toward the period \( r^* \), uncertainty on the retirement is sequentially resolved since \( \Omega_s \subseteq \Omega_{s+1} \cdots \subseteq \Omega_{r^*} \) \((0 \leq s < r^*)\). However, even just before the period \( r^* \), there still remains uncertainty on the resources for post-retirement consumptions, which is the key factor for the retirement decision as shown in (2.8). Hence, the change in retirement behavior by exposure to stock market can occur before reaching the period \( r^* \) as well as at the period \( r^* \). Nevertheless, periods far from \( r^* \) have little relevance; since young workers have less accumulated assets and longer remaining life than senior workers do, the conditions to retire (2.8) and (2.9) are rarely likely to be met. Thus, we will focus on the verge of retirement. Put another way, it would not be loss of generality to examine how difference in retirement behavior is made among workers who enter the period \( r^* - 1 \) and the period \( r^* \), respectively.

Let us start with how exposure of pension to a positive shock makes a difference between a DC plan worker and a DB plan worker in their exiting labor force. First of all, the positive shock can affect workers who just entered the period \( r^* \), their planned time to retire. Obviously, as pension benefit is not changed by the shock, the DB plan worker will retire at the period \( r^* \). By contrast, there can be an unexpected increase in pension benefit of the DC plan worker. The question is whether the DC plan worker responds to this through an adjustment on retirement side. Time is irreversible, and he cannot undo his already supplied labor. Therefore, the DC plan worker cannot reduce labor supply by retiring earlier than planned. As a consequence, he retires as he planned, which is the same retirement behavior of the counterpart DB plan worker. Probably, the DC plan worker may enjoy more affluent post-retirement consumptions than the counterpart DB plan worker would. After all, the DC plan worker retires at the period \( r^* \); in other words, all the workers of this cohort (who enter their period \( r^* \)) equally follow through their retirement plan, regardless of exposure of pension to the positive shock.

Second of all, the positive shock can also affect those who are younger by one period than the above workers. Entering their period \( r^* - 1 \), they are supposed to work instead of to retire, if there were not the positive shock. However, in the face of the positive shock, they either keep working or retire now (earlier than planned). On the one hand, without change in pension benefit, the DB plan worker would not change his plan and thus will work following his original plan. On the other hand, exposed to the positive shock, the DC plan worker may mull over retiring now earlier than planned, responding to an unexpected increase in pension benefit. Importantly, the DC plan worker does not always retire early facing the positive shock. It is rational for the DC plan worker to retire earlier than planned, only when he can be better off with this deviation. If he retires early, his labor earnings are foregone and he faces an increase in necessary resources for post-retirement consumption due to extended periods of retirement. As a consequence, only if the gain from exposing pension wealth to the positive shock is greater than this opportunity cost, the DC plan worker would retire early. On the one hand, if the magnitude of the positive shock
is large enough to compensate not only the earnings foregone from earlier retirement but also to meet increased necessary resources for post-retirement consumptions, the DC plan worker will respond with earlier retirement than planned. On the other hand, by contrast, when the magnitude of the positive shock is not large enough to make early retirement profitable, there would be no difference in retirement behavior lead by the exposure of pension to the positive shock among these. Especially for this latter case, after taking into account of the older cohort that the positive shock does not make a difference in retirement behavior, no change in retirement behavior of the younger cohort ends up with lack of overall retirement responses to the positive shock.

Let us move on how exposure of pension to a negative shock makes a difference in retirement behavior between a DC plan worker and a DB plan worker who are otherwise identical and thus have the same retirement plan. First of all, the negative shock can affect workers who just enter their planned time to retire, the period $r^*$. Clearly, the DB plan worker will retire at the period $r^*$ without changes in pension benefit from the shock. By contrast, because of exposure to the negative shock, the DC plan worker suffers an unexpected decrease in pension benefit. This leads to failure in meeting (2.8) and (2.9) the conditions for retiring. In other words, a sudden loss in pension wealth makes the DC plan worker fall short of the resources necessary for post-retirement consumption; hence, retiring now as planned is no longer optimal. As a consequence, the DC plan worker will delay his retirement by continuing to work. Thus, exposure of pension to the negative shock yields a negative response of retirement among this cohort. Second of all, the negative shock can also affect workers who are younger by one period. Originally, both a DB plan worker and a DC plan worker of this cohort planned to work in their period $r - 1$. Firstly, without exposure to the shock the DB plan worker does not need to re-optimize his labor and will be stick to the plan; hence the DB plan worker will work. With exposure to the negative shock, the DC plan worker will not end up with securing necessary resources for retiring; therefore, he will also keep working. So, in terms of whether a worker of this cohort retires now or not, there is no difference made. All in all, exposure of pension to the negative shock brings an increase in labor force participation due to delays in retirement.

**Proposition 2.1** When a positive shock comes, an increase in retirement can be ensued by exposure of pension to the positive shock only if magnitude of a positive shock is large enough to compensate not only the earnings foregone from earlier retirement but also to meet increased necessary resources for post-retirement consumptions.

*Proof.* See Appendix B.

**Proposition 2.2** When a negative shock comes, a decrease in retirement can be led by exposure of pension to the negative shock.
Proof. See Appendix B.

In a nutshell, unlike a positive shock, without requiring specific conditions on the size of a shock, exposure of pension to the negative shock ensues a response of less retirement than in the absence of it. In fact, so far is presented sketch of the proofs of the following propositions, which is also central result of this theoretical analysis. More technical and brief proofs are put in the Appendix B.

As a corollary of Proposition 2.1 and 2.2, it is feasible that exposure of pension to a negative shock brings responses of retirement whereas exposure of pension to a positive shock with the same magnitude does not lead to a change in retirement.

In balance, exposure of pension wealth to financial market fluctuations brings retirement responses in not-perfectly-symmetric way. First, confronting economic downturns, losses in pension benefit of a DC plan worker raises his labor supply more than a DB plan worker, due to failure in meeting the conditions for retirement. On the contrary, when economic climate turns out better than expected, a DC plan worker does not necessarily reduce their labor supply with earlier retirement than a DB plan worker. To bring retirement response, the magnitude of a positive shock needs to large enough to compensate foregone labor earnings and to finance increased post-retirement consumption. Thus, there can be case where exposure of pension to upturns of stock market brings little or no response from senior workers’ retirement.

Last but not the least, putting retirement decision in the context of dynamic allocation of labor supplies so that retirement is not on instant base, and embracing the fact that decision of liquidizing pension wealth is more tied to the retirement than that of non-pension wealth, this theoretical analysis shows that pension wealth does not play out in the exactly same way that non-pension wealth would do. In particular, by taking a new approach on investigating retirement behavior with respect to the exposure to financial market fluctuations, this study finds a case where a positive shock might lead to non-response in retirement behavior. In this light, previous studies like Krueger and Pischke (1992) showing that unexpected increases in Social Security benefit do not entail response of retirement behavior may be explained. On the other hand, note that this is not inconsistent with the previous study on non-pension wealth effect over retirement such as Imbens et al. (2001). That is, by articulating difference between two types of wealth, this does not rule out other studies that a positive shock non-pension wealth have led earlier retirement.

2.3.3 Simulation

To illustrate theoretical findings above in a more tangible way, let us do a simplified simulation exercise. Let $U(C_t, L - h_t) = \log C_t + \log(L - h_t)$. Without specific contract designs on private plans at hand, replacement rate (the ratio of periodic pension benefit over labor earnings before retirement) is assumed to be 50% since it is the rate for average workers in the US after excluding income tax, according to OECD
(2011). Apparently, we are to compare retirement behavior between two workers who are identical except for exposure of pension to financial market fluctuation. In addition, there is only one shock on pension wealth and the shock lasts one period. Since without a shock, both retire at the same period, the period \( r^* \), the impact of exposure is measured by ratio of retirement rate of a DC plan worker (exposed) over a DB plan worker (non-exposed) by the end of the period when a shock occurs. By construction, when the ratio is equal to one, it means that exposure does not bring retirement response. Moreover, if the ratio is greater than one, it means that exposure to a shock causes an increase in retirement; otherwise, delays in retirement are lead by the exposure. Lastly, to distill out any other potential confounders, let us simplify that they have no uncertainty on wages and non-pension wealth. Furthermore, exposure of pension to financial market fluctuations is dichotomous; that is, all or none of the pension wealth is invested in stock market.

The theoretical findings are not about defying wealth effect on retirement arguing that shock on pension wealth does not affect behavior of leaving the labor force at all. Rather, a point to be highlighted would be how difference in response of retirement is caused by exposure of pension to a positive shock and a negative shock, respectively. In particular, how the ratio varies by size of shocks is simulated. In detail, the magnitude of a shock (%) is calculated as difference between realized value of pension and its expected value in terms of total pension wealth held before the shock.

![Figure 2.1: Retirement and Exposure of Pension to Positive Shocks (Simulation)](image)

As displayed in Figure 2.1 and Figure 2.2, exposure of pension to shocks does
make a difference in retirement behavior. By juxtaposing the plots, we can find that the response is not perfectly symmetric, as delineated in the theoretical investigation above. In other words, there is a case where exposure to a positive shock do not end up with realizing any change in retirement behavior. In particular, this occurs when the magnitude of a positive shock is small and disappears when it increases over a certain threshold. This is contrasted to the effect of negative shocks portrayed in the Figure 2.2 which does not have such a threshold.

![Figure 2.2: Retirement and Exposure of Pension to Negative Shocks](Simulations)

By structure of discrete time model, the decision of retirement is made per period, say one year, at the begging point of a period. Facing a negative shock, whether delay by one month or three years, any alteration of retirement plan leads to change in retirement behavior. On the other hand, facing a positive shock, as mentioned above, no difference in actual action of retirement is caused by exposure to the shock among workers who just reached their planned time to retire, say 62. However, younger workers are able to make a difference. For instance, being exposed to the positive shock, those who currently are of age 58 and planned to retire at their 62 would adjust their retirement plan earlier than 62. It can be changed to 60 or 59, but the actual response of retirement is realized only when it is altered down to 58. So, when a negative shock causes a 3-month delay in retirement plan, this is observed/realized as a decrease in retirement; by contrast, when a positive shock with the same magnitude, it might not end up with any actual increase in retirement.
In addition, although it is not our main interest, one may well ask about how consumption side is affected by exposure. To address this, similar exercises were conducted and their results are put in the Appendix. The impact of exposure on post-retirement consumption is measured by ratio of periodic post-retirement consumption of a DC plan worker (exposed) over a DB plan worker (non-exposed). As mentioned above, there is more room for senior workers to adjust post-retirement consumption. As a result, effect of exposure to financial market fluctuations turns out symmetric (See Figure B.3 and Figure B.4 in the Appendix B).

By its nature, having other factors under a researcher’s control, simulation exercise can be useful to demonstrate theoretical explorations. However, whether and how this applies to actual retirement is yet to be examined and probably more interesting and relevant to policy making. Thus, the next section will empirically study how exposure of pension to both kinds of shocks can affect retirement behavior with actual observational data of individual senior workers.

2.4 Empirical Analysis

2.4.1 Data Overview

To empirically investigate how exposure of pension wealth to stock market fluctuations can affect retirement behavior, panel data of Health and Retirement Study (HRS) is adopted. This is a nationally representative biannual micro-level panel data on senior workers. In particular, four waves of the HRS (2002, 2004, 2006, and 2008) are used for the analysis.

![GDP Growth Rates (unit: %) 2000-2009](image)

Figure 2.3: Trend of GDP Growth
Figure 2.4: Rate of Returns to DC Accounts

At the outset, for the sake of linking to theoretical analysis, we need to identify which phase of economic fluctuations that each wave of the data corresponds. To this end, both stock market performance and GDP growth are based. In particular, annual data of rate of return on DC accounts is used as a measure of the investment performance (See Figure 2.4). Moreover, it is also very similar to the trend of growth rate of GDP (See Figure 2.3). Specifically, for the current analysis, 2002 and 2008 are regarded as recession periods when a negative shock comes, while 2004 and 2006 are treated as expansion periods when a positive shock comes. Under this classification, it is worthwhile to note that rate of annual return from stock market is negative for each recession period; by contrast, it is positive for each expansion period.

Even though both 2002 and 2008 are classified as a recession, the detailed feature is different. Specifically, since the Great Recession was a more surprising shock than that of the other 2002 recession which was triggered by the burst of dot com bubble in 2000, estimation with the 2008 wave has a greater potential for clear identification than with the 2002 wave. However, by including two recessions and two expansions in our sample, we can provide another opportunity of consistency check as well. In other words, we can indirectly test our theoretical model by checking whether we do not find opposite (inconsistent) responses of retirement behavior from analyzing data of two separate recession or expansion periods.

Among many, one merit of the HRS data for the purpose of this study is that comprehensive information on public and private pension — including the amount of Social Security benefit, IRA account, private pensions sponsored by the current and

\textsuperscript{20}In relation to reference dates offered by NBER’s Business Cycle Dating Committee, there exists a slight difference. That is, for the early 2000 recession, only 2001 (from March to December) is classified as a period of contraction by the Committee although growth rate of US GDP in 2002 still remained as low as 2001.
all the past employers and their type (whether DB or DC) — is available.\textsuperscript{21} Regarding our key variable — exposure of pension wealth to stock market fluctuations — an indicator of whether a worker holds a DC account could be misleading since a DC account holder does not necessarily invest the account in the stock market. However, the HRS data enables us to avoid possible noise or errors of this proxy indicator. That is, there are HRS survey questions that ask whether any of a respondent’s DC account (and/or IRA account), if any, is invested in stock market and whether part of a DC account is in the form of the company’s stock share.\textsuperscript{22} As a consequence, those who put all of their DC pension wealth in accounts which earn only interest as a usual savings account were not regarded as being exposed to stock market losses (gains) in recession (expansion) periods. After all, the key indicator variable ‘DC account in stock market’ takes value one if a respondent holds a DC account and invests any of it in stock market in a sampling year; and it takes value zero otherwise.

Another key variable is to describe the decision to retire. First of all, whether a respondent is retired or not is based on actual labor force status, instead of self-report about whether they perceive themselves as retired or not. Thus, in addition to full time workers, those who are partially retired or working part time are classified as non-retirees.\textsuperscript{23} In this account, the transition of retiring occurs if a respondent was a non-retiree in the previous wave and get retired in the current wave. Since we are examining effect that exposure of pension wealth to stock market gains or losses can exert on retirement transition, those respondents who were not non-retirees in the first place are not included in the sample used for this analysis. In other words, we are not using the data of those who remain out of labor force throughout the four waves of HRS data. In addition to this, the HRS data also contains information on the total number of years for which a respondent has worked so far. This enables us to conduct time-to-event analysis that is well suited for our empirical investigation, which will be discussed later.

On top of this, in relation to theoretical analysis, the availability of pension benefit can also play a role in retirement responding to a positive shock, as shown above. In the light of section 2.3.2, the publicly known restriction on public pension benefit availability is used as proxy to control potential effects of pension availability on retirement. In other words, a binary variable to indicate whether a respondent is over 62, ERA of Social Security benefit, is used. The HRS does not contain exact data on how respondent’s private pension stipulates on restrictions pension availability.

\textsuperscript{21}In the survey of HRS data, there are sections for pensions and assets. In particular, the sections that contain these information are as follows: J, K, L, and, Q sections. The reason why 2000 and earlier waves of HRS is not used lies at lack of coherency due to the massive change in corresponding sections like G,GG, GH, and J that contain pension information.

\textsuperscript{22}In detail, utilized items in the HRS questionnaire are as follows: (i) how a DC account is invested (ii) whether a DC account is invested in company’s stock, and (iii) whether IRA account is invested in stock market.

\textsuperscript{23}Moreover, note that those who are willing to work but do not secure a job are classified as partially retired. The observed retirement is chosen rather than forced to do.
However, it would not be concerning, since it is against law to delay the entitlement of a DC plan more than five years without having the plan vested.\textsuperscript{24} Not exposed to financial market fluctuations, deviation from optimal retirement plan would not be caused by availability constraint on DB plan benefits.

Finally, data on the amount of accurate periodical pension benefit of non-retirees is not available, specially for a DC plan; hence, pension wealth accumulated so far is calculated, instead of benefit. Firstly, the amount of pension wealth of a DC plan is straightforward, by construction; however, that of a DB plan is not. To make both comparable, present value of a DB plan pension wealth is calculated based on the amount of (expected) periodic pension benefit from a DB plan and life expectancy. Likewise, Social Security retirement benefit is also converted into Social Security wealth at present value in the same manner. In calculating these present values, real interest rate is assumed to be 3\% and the number of years of remaining life time over which the benefit is to be paid is estimated based on life expectancy data provided by Centers for Disease Control and Prevention (CDC), which is in fact used for actuarial calculation of Social Security benefit. Moreover, as the theoretical model shows, in timing retirement, pension wealth may play a role different from non-pension wealth; hence, the (constructed) pension wealth is not included in the variable of wealth.

In addition, since labor earnings can be an important factor as entered in the left hand side of (2.8), the variable of annual salary is also included in regression.

Since the sample covers six years which may change the value of one dollar, all the monetary values are converted into 2002 dollars; and, the factors for the conversion are in the Table B1. With key variables at our hand, how we can rigorously verify the effect of exposure of pension wealth to stock market fluctuations on retirement behavior will be discussed in the next subsection.

\section{Estimation Strategy}

\subsection{Proportional Hazard Model}

To begin, understanding basic features of retirement helps us select a well-suited statistical tool to analyze underlying data generating process. First of all, decades of labor supply precede the retirement, followed by consumption \textit{without} labor supply in a row. In this light, this event of exiting labor force — which is of central interest — is a change of state: from a worker into a retiree. Second of all, even if a researcher observes that an individual does not retire within sampling periods, it does not mean that the individual will not do so. In a sense that workers eventually retire in the

\textsuperscript{24}In fact, in HRS, there is a survey question that asks when is the youngest age at which a respondent can leave the current job (or business) and start receiving pension benefit. However, it is questionable whether it is legally binding age-restriction that takes effect. Since its average is 62.3 or so, and its distribution of DC plans appear to concentrate on very few of ages, two of which coincide with ERA and NRA of Social Security retirement benefit, even when respondents worked at the current job more than five years. To avoid this inconsistency, this variable is not utilized.
end, retirement behavior of the individual is just not observed by the researcher. This kind of censoring problem can cause a bias in estimation. Taking these into account, time-to-event analysis, or survival analysis, can provide appropriate statistical basis for analyzing this type of data since it properly deals with the above issues. Not surprisingly, there are exemplary studies on retirement behavior which also use time-to-event analysis such as Card and Ashenfelter (2002) and Hausman and Wise (1985).

Since we are analyzing how retirement behavior is affected by exposure of pension wealth to stock market fluctuations, occurrence of retiring is the outcome that we will investigate. As in any statistical model, the event occurrence translates into probability of whether an individual retires or not. However, it inherently means that the individual has not retired yet. Therefore, precisely speaking, the key variable would rather be the conditional probability with which a worker gets retired given the worker has stayed as a non-retiree. Put in terms of survival analysis, this is called as ‘hazard’ rate of retirement. In detail, hazard rate at specific time basically is probability to retire at the period / probability of surviving as non-retiree up to the period. In fact, the HRS data provides us the key ingredient of the rate and enables us to conduct survival analysis: the information on how many years for which a respondent has been in labor force as a non-retiree. On the other hand, instead of survival analysis, one may as well propose a more familiar alternative: probit (or logit). Probably, it seems to be easier for analyzing event occurrence; however, it is less suited for this study since it does not properly address the censoring problem. Moreover, it is less flexible in that probability with which the worker has survived in labor force is not allowed to vary by number of years for which he has spent in the labor force, although, in reality, the survival probability is more likely to decrease as one gets older as shown in Figure 2.6.  

Furthermore, if we view exposure to each shock as treatment, we are investigating difference in retirement hazard rates of the following two groups: (as treatment group) the one with a DC account invested in stock market in a sampling year vs. (as control group) the other without it in the same year. In comparing these two groups, relative difference in retirement rates informs us effect of the exposure. With these in mind, Cox proportional hazard model can be best-suited for this study. In fact, the shape of hazard rates are quite similar between the two, as shown in Figure 2.5, meeting proportionality assumption of the model.

25In addition, Cox proportional hazard model is semi-parametric while probit is parametric. Thus, hazard model impose less assumption on distribution, which also is another merit.
Figure 2.5: **Retirement Hazard Rates by Exposure of Pension to Expansion and Recession**

Note: (i) For the left panel of ‘Retirement Rates in Expansion Periods,’ retirement hazard rate is weighted based on data of workers who retire in the expansion years of 2004 and 2006 from Health and Retirement Study. (ii) For the right panel of ‘Retirement Rates in Recession Periods,’ retirement hazard rate is weighted based on data of workers who retire in the recession years of 2002 and 2008 from the same dataset HRS. (iii) Red line depicts retirement behavior of workers who own DC account and invested it in the stock market while blue line describes those whose pension wealth is not invested in the stock market. (iii) Dashed lines around solid lines of each color are borders of the 95% confidence interval of estimated retirement hazard rate at each age.

On the top of this, notice that recession and expansion are mutually disparate shocks to pension wealth; and, theoretical analysis predicts different responses of retirement ensued by each shock. Therefore, empirical analysis is to be composed of two separate parts: examination of the effect of a positive shock and that of a negative shock. Accordingly, the key interest of estimations fundamentally lies at two parameters: one that captures difference in retirement rate caused by exposure of pension to stock market gains from a positive shock (2004 and 2006) and the other that does so by exposure of pension to stock market losses from a negative shock (2002 and 2008).
Figure 2.6: **Survival in Labor Force by Exposure of Pension to Expansion and Recession**

Note: (i) For the left panel, survival probability based on Kaplan-Meier method use data of workers in the expansion years of 2004 and 2006 from Health and Retirement Study. (ii) For the right panel, data of workers in the recession years of 2002 and 2008 from the same dataset HRS. (iii) Red line depicts retirement behavior of workers who own DC account and invested it in the stock market while blue line describes those whose pension wealth is not invested in the stock market.

At the outset, before running regressions, we can quickly form a guess by drawing plots. First, Figure 2.5 shows the evolution of hazard rates over ages by the exposure to each treatment. Second, Figure 2.6 displays estimated survival probability, built based on Kaplan-Meier method, which is non-parametric. Putting this into the context of this study, the survival probability can correspond to labor force participation rate. Basically, hazard rate is anti-integral of survival function (or probability); and, these two types of plots do not seem inconsistent. Overall, these two kinds of plotting lead us to conjecture that negative shock appears to make a difference in retirement behavior while impact of positive shock is not clearly discernible. However, pension wealth is not the sole determinant of retirement. In order to find more convincing and clearer evidence by exploiting the data at hand as much as possible, we need to conduct regression analysis for distilling out influences of other factors (potential confounders).
2.4.2.2 Key Parameters

Firstly, let us start with the first part that explores how exposure of pension to a positive shock effects retirement behavior, the estimation equation is put as follows:

$$\log HR_i = \alpha_0 + \alpha_1 I(\text{pension exposed to positive shock})_i + X_i \alpha + \varepsilon_i$$ \hspace{1cm} (2.12)

where $HR_i$ is retirement hazard rate of a worker $i$; and, $\alpha_0$ is logarithm of baseline hazard rate. Since a positive shock refers to stock market gains in an expansion period, this regression will be run twice: both 2004 and 2006 wave. The sample for either year is comprised with workers (non-retirees). More importantly, $I(\text{pension exposed to positive shock})_i$ is the key indicator variable that takes value one if a respondent holds a DC account and invests any of it in stock market of an expansion period and zero otherwise (i.e., ‘DC account in stock market’ defined above in an expansion period). $X_i$ is a vector of other pertinent factors including variables of wealth, socioeconomic characteristics, a qualitative measure of risk aversion, annual labor earnings, indicators of whether a worker $i$ has health problem that limits capacity to do work, whether incentive for early retirement (early out window) is offered to a worker $i$, which industry that a worker $i$ is in.\textsuperscript{26}

Likewise, the second part that investigates impacts of exposure of pension to a negative shock over retirement can run a regression whose specification is as follows:

$$\log HR_i = \beta_0 + \beta_1 I(\text{pension exposed to negative shock})_i + X_i \beta + \varepsilon_i$$ \hspace{1cm} (2.13)

where $\beta_0$ is logarithm of baseline hazard rate; and, $X_i$ is the same vector of other pertinent factors. Since a negative shock means stock market losses for a recession period, this regression will be run twice: both 2002 and 2008 wave. Similarly, $I(\text{pension exposed to negative shock})_i$ is the key indicator variable that takes value one if a respondent holds a DC account and invests any of it in stock market and zero otherwise in a recession period (i.e., ‘DC account in stock market’ defined above in a recession period).

With specifications given above, the key parameters identifying impacts of each shock are $\alpha_1$ and $\beta_1$ which correspond to Proposition 2.1 and 2.2, respectively. Controlling other relevant variables, regression analyses following (2.12) and (2.13) would give better inference on these parameters than simple plots. Furthermore, since stock market fluctuations are very hard for an individual to predict, negative or positive returns on the pension wealth investment would be a random shock to an individual worker, which is out of one’s control. Put in the frame picked above, assignment of which treatment is exogenously given.

However, one may raise a concern that the exposure itself, or participation of treatment, is not randomly assigned. In detail, if unobserved individual heterogeneity

\textsuperscript{26}13 categories of industry is available, and in avoidance of multicollinearity, only 12 binary variables are included in $X_i$.\n
is systematically associated with this exposure and related to decision of retirement at the same time, then estimation of $\alpha_1$ and $\beta_1$ could suffer a bias. Some studies like Madrian, Brigitte and Dennis Shea (2001) find that many workers passively follow default option of DC plans, which usually lets DC account exposed to stock market through investing in mutual funds. Nonetheless, as an attempt to address case where workers actively make a decision on pension investment and factors that govern the decision are also related to timing retirement at the same time, the variables of risk aversion and industry that the individual works for (both of which are in $X_i$) are controlled.

To further tackle this concern for the sake of clear identification, we can exploit the fact that the HRS is panel data. If unobservable individual heterogeneity that affects both retirement and exposure of pension to stock market is not changing before and after a shock, then difference-in-difference estimator can resolve the issue, following the seminal work of Ashenfelter and Card (1985). Although we still need parallel assumption\footnote{Both $I(\text{after expansion})_{it}$ and $I(\text{pension exposed to stock mkt gains})_{it}$ is not correlated with $\varepsilon_{it}$.} for this estimator to work, this assumption is not much stronger than what we need for above estimation to be unbiased, which presumes that omitted variable of individual heterogeneity about decision of the exposure to stock market is related to neither retirement nor $\varepsilon_i$. Moreover, there are possible benefits: addressing the issue of unobserved individual heterogeneity and distilling out influence from entire labor market of each period by de-trending. In concrete, revisiting the estimation equation (2.17) for the first key parameter with difference-in-difference method yields

$$\log HR_{it} = \alpha_0 + \alpha_1 I(\text{after positive shock})_{it} \times I(\text{pension exposed to positive shock})_{it} + X_{it} \alpha + \varepsilon_{it} \tag{2.14}$$

where $I(\text{after positive shock})_{it}$ takes zero in 2002 and one in 2004 when expanding economy benefits stock holders. Moreover, the other indicator variable of $I(\text{pension exposed to positive shock})_{it}$ takes value one if a respondent $i$ holds a DC account invested in a booming stock market in 2004 and zero otherwise. By construction, $t$ covers two periods: either 2002 and 2004. As a consequence, $X_{it}$ can contain time-variant factors as well as time-invariant ones. In addition to variables used in (2.12), for notational simplicity, other two terms in difference-in-difference estimator, $I(\text{pension exposed to positive shock})_{it}$ and $I(\text{after positive shock})_{it}$ are included in $X_{it}$.

Notice that we do not repeat difference-in-difference estimation using 2004 and 2006 even though 2006 is also a period of expansion. Because both are subject to positive shocks, only with these two waves, we actually lack of the necessary period that is free of treatment. One may as well argue that 2002 can serve as pre-treatment period for the positive shock of 2006. However, it would not enhance the quality of identification compared to employing adjacent periods (2002 and 2004) since it will introduce more noises which arise in the middle but is not well addressed.
Next, to extend (2.13) identifying the impact of exposure of pension to a negative shock, we will conduct difference-in-difference estimation for 2008 with 2006 as pre-treatment period. By the same token as delineated right above, 2002 is not well-suited for difference-in-difference since 2002, pre-treatment period of 2002 recession is also recessional. After all, the specification of this estimation is as follows.

$$\log HR_{it} = \beta_0 + \beta_1 I(\text{after negative shock})_{it} \times I(\text{pension exposed to negative shock})_{it} + X_{it} \alpha + \varepsilon_{it}$$ \hspace{1cm} (2.15)$$

where $I(\text{after negative shock})_{it}$ takes zero in 2006 and one in 2008 when recession hits stock market harshly. Moreover, the other indicator variable of $I(\text{pension exposed to negative shock})_{it}$ values as one if a respondent $i$ holds a DC account invested in stock market clash in 2008 and as zero otherwise. Accordingly, $t$ now is either 2006 or 2008. Likewise, $X_{it}$ is a vector of the other explanatory variables in (2.13) with two indicator variables of $I(\text{after negative shock})_{it}$ and $I(\text{pension exposed to negative shock})_{it}$ added.

Ultimately, resulting estimates of $\alpha_1$ and $\beta_1$ will tell us the effect of exposure in terms of retirement hazard rate, which is virtually unit free. However, Cox proportional hazard model does not go beyond for allowing us to assess the effect in terms of time unit. Certainly, policy makers will want to know not only the retirement hazard rate but also the amount of labor supply responding to stock market fluctuations. Keeping in mind the aforementioned features of retirement observations, especially the censoring problem, a simple OLS regression with duration time in labor market as left-hand side variable would not work properly for this purpose. Furthermore, since each respondent has different starting point and ending point of their carrier, Tobit model is not perfectly appropriate due to varying upper bound by individuals — instead of being constant across them. As an alternative, structural estimations, as French (2005) or Klaauw and Wolpin (2008) does, seem unnecessarily complicated to implement relying on numerous assumptions. Rather, we can find a simple alternative within the framework of survival analysis: accelerated failure time model. Although accelerated failure time model is parametric, imposing more distributional assumption than semi-parametric Cox proportional hazard model does, it helps us to get a sense on how much more (or less) labor is supplied, if any, facing a negative or positive shock on their pension wealth. For underlying distribution, log-normal distribution is adopted. Estimation equations (2.14) and (2.15) under Cox proportional hazard model carry over, with modification of dependent variable as follows.

$$\log T_{it} = \alpha'_0 + \alpha'_1 I(\text{after positive shock})_{it} \times I(\text{pension exposed to positive shock})_{it} + X_{it} \alpha' + \varepsilon'_{it}$$ \hspace{1cm} (2.16)$$

for measuring labor response to a positive shock, where $T_{it}$ is number of years for which a worker $i$ has been in labor force as a non-retiree. Likewise, for estimating labor supply adjustment by senior workers facing a negative shock on their pension
is put as

\[ \log T_{it} = \beta'_0 + \beta'_1 I(\text{after negative shock})_{it} \times I(\text{pension exposed to negative shock})_{it} + X_{it}\beta' + \varepsilon_{it}. \] (2.17)

Having selected statistical model, descriptive statistics of all the variables to control other relevant variables for identifying the key parameters in regression analyses displayed in Table 2.1.\(^{28}\)

Last but not the least, caveat needs to be made on the interpretation of the key parameters. Drawing on the estimates of the parameters, it is tempting to interpret them in terms of elasticity (or calculate elasticity based on them). However, precisely speaking, this is misleading. Notice that we do not have information on exact amount of losses or gains of individual DC plan holders from their pension investment in stock market. We only know whether they invest (expose) any of their DC account to stock market.\(^{29}\) As a consequence, it is not feasible to properly calculate elasticity. However, as shown in Table 2.1, DC plan holders who make stock market investment of their pension account enjoy, on average, gains (losses) in expansion (recession) period. At most, what we can estimate is overall (or average) effect of the exposure of pension to each shock in financial market over retirement behavior. Therefore, relevant evidence we can take with rigor is whether exposure of pension to a positive (negative) shock in financial market leads to a decrease (an increase) in the retirement rate of workers, not how much of the decrease (increase). In other words, the sign of $\alpha_1$ and $\beta_1$ will be of main interest.

Bearing this in mind, estimation outcomes and their interpretation will be presented below.

2.4.3 Estimation Results

2.4.3.1 Main Estimation Results

Let us start with conducting regression analysis on expansion periods (2004 and 2006) with simple Cox proportional hazard model. The resulting estimates following (2.12) are presented in Table 2.2.

At the outset, since the left-hand side variable is logarithm of retirement hazard rate and logarithm is monotonously increasing function, right-hand side variables whose coefficients are estimated as taking positive (negative) values are interpreted as raising (lowering) the retirement hazard rate. In this line, we can say that wealth, early out window offered by the employer, and having health problem facilitate retirement, while not being eligible for Social Security and high earnings from the current job contribute to delaying of the exit from the labor force.

\(^{28}\)For the sake of succinct presentation, the summary statistics of variables indicating industry are separately displayed in Table B2.

\(^{29}\)For this reason, this study intentionally uses the term of 'exposure.'
Table 2.1] Summary Statistics of Variables on Retirement Behavior  
source: HRS (Health and Retirement Study) 

<table>
<thead>
<tr>
<th>Wave (year)</th>
<th>2002</th>
<th>2004</th>
<th>2006</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables</td>
<td>Mean (Std Dv)</td>
<td>Mean (Std Dv)</td>
<td>Mean (Std Dv)</td>
<td>Mean (Std Dv)</td>
</tr>
<tr>
<td>Pension exposed to stock mkt (=1)</td>
<td>0.51 (0.49)</td>
<td>0.52 (0.49)</td>
<td>0.48 (0.49)</td>
<td>0.54 (0.49)</td>
</tr>
<tr>
<td>ERA constraint (below 62) (=1)</td>
<td>0.50 (0.50)</td>
<td>0.41 (0.49)</td>
<td>0.54 (0.50)</td>
<td>0.50 (0.50)</td>
</tr>
<tr>
<td>DC plan value</td>
<td>78.73 (486.6)</td>
<td>82.73 (263.7)</td>
<td>87.99 (534.2)</td>
<td>72.20 (232.3)</td>
</tr>
<tr>
<td>DB plan value</td>
<td>75.04 (549.8)</td>
<td>78.84 (312.4)</td>
<td>82.35 (502.4)</td>
<td>81.04 (411.9)</td>
</tr>
<tr>
<td>SS wealth value</td>
<td>98.42 (77.34)</td>
<td>107.06 (139.2)</td>
<td>102.44 (104.7)</td>
<td>113.02 (199.2)</td>
</tr>
<tr>
<td>Wealth</td>
<td>416.85 (1164.4)</td>
<td>475.9 (1696)</td>
<td>530.7 (2790)</td>
<td>482.89 (768.7)</td>
</tr>
<tr>
<td>Financial wealth in stock mkt</td>
<td>51.60 (224.6)</td>
<td>78.60 (454.6)</td>
<td>81.59 (132.4)</td>
<td>68.59 (385.3)</td>
</tr>
<tr>
<td>Earnings</td>
<td>28.45 (44.83)</td>
<td>28.53 (38.40)</td>
<td>33.38 (53.05)</td>
<td>32.98 (940.2)</td>
</tr>
<tr>
<td>Early out window offered (=1)</td>
<td>0.02 (0.14)</td>
<td>0.02 (0.14)</td>
<td>0.01 (0.12)</td>
<td>0.02 (0.12)</td>
</tr>
<tr>
<td>Health problem (=1)</td>
<td>0.11 (0.31)</td>
<td>0.10 (0.30)</td>
<td>0.08 (0.27)</td>
<td>0.10 (0.31)</td>
</tr>
<tr>
<td>Risk averseness</td>
<td>3.26 (0.88)</td>
<td>3.26 (0.88)</td>
<td>3.25 (0.88)</td>
<td>3.26 (0.87)</td>
</tr>
<tr>
<td>Age</td>
<td>59.6 (6.85)</td>
<td>60.4 (6.94)</td>
<td>59.1 (7.85)</td>
<td>60.5 (7.83)</td>
</tr>
<tr>
<td>Male (=1)</td>
<td>0.45 (0.49)</td>
<td>0.45 (0.49)</td>
<td>0.46 (0.49)</td>
<td>0.45 (0.49)</td>
</tr>
<tr>
<td>Married (=1)</td>
<td>0.77 (0.42)</td>
<td>0.76 (0.43)</td>
<td>0.76 (0.43)</td>
<td>0.75 (0.43)</td>
</tr>
<tr>
<td>Education (yrs)</td>
<td>13.06 (2.89)</td>
<td>13.10 (2.85)</td>
<td>13.35 (2.84)</td>
<td>13.44 (2.76)</td>
</tr>
<tr>
<td>Years in labor force</td>
<td>37.86 (11.18)</td>
<td>39.14 (11.28)</td>
<td>37.23 (11.51)</td>
<td>38.65 (11.45)</td>
</tr>
<tr>
<td>No. of Obs</td>
<td>6930</td>
<td>6238</td>
<td>5988</td>
<td>5382</td>
</tr>
</tbody>
</table>

Note: (i) Each indicator variable takes value one if description of the variable with (=1) holds; otherwise zero. (ii) Variables of (both pension and non-pension) wealth and (annual) earnings are in 1,000 $. (iii) ‘Risk averseness’ is a categorical variable ranging from one to four (the most risk-averse); and, this is based on a question on whether a respondent would take a new job whose income is uncertain between doubling the current one and some cuts.
Nevertheless, how to communicate estimation results of hazard models might not be as familiar or straight as simple OLS. Since the left-hand side is not retirement hazard rate, taking nonlinear function (logarithm), marginal effect of a right-hand side variable is not immediate. For the purpose of more convenient and prompt interpretation, hazard rate ratio, corresponding to each estimate, is also reported in the table. The reason why ratio of hazard rates, instead of rate itself, is calculated from estimates as follows. Firstly, each estimate stands for difference in the left-hand side variable made by one unit increase in a right-hand side variable. On the top of this, the resulting difference in logarithm of hazard rates is equal to a logarithm of ratio of the two hazard rates in the difference. Ultimately, this implies that estimates in a Cox proportional hazard model measure impact of right-hand side variables over retirement in relative sense by informing us hazard rate ratio. Put in another way, the effect of a right-hand side variable appears as a proportional increase in retirement hazard rate, instead of an increase in (absolute) level of the rate.

Furthermore, when ratio of the two rates is one, both rates are equal; hence, subtracting one from the ratio reveals the effective difference made by the right-hand side variable. In this light, 1.090 in the first row and the first column of the Table 2.2 refers to ratio of retirement hazard rate of the following two groups: the one who exposes pension to 2004 positive shock vs. the other who does not in the same year. Since this is greater than one by 0.090, retirement rate of the former is higher than the latter. Specifically, 0.090 or 0.9 % of the retirement hazard rate of workers in the sample, which can be calculated as baseline hazard rate after estimation, is attributable to the the exposure to the positive shock if it is statistically significant.30 However, it is not statistically meaningful, as the pair of coefficient estimate and its standard error in the adjacent two cells show. Similarly, positive but statistically insignificant response is also found for 2006 positive shock, as shown in Table 2.2.

On the other hand, since 0.267 in the second row and the first column of the table is smaller than one by 0.733, 73.3 % of the retirement hazard rate of workers in the sample can be the marginal decrease in retirement rate due to not-being eligible for Social Security retirement benefit. That is, after controlling for age, non-availability of Social Security pension benefit is negatively associated with retirement, now having statistical significance, unlike the exposure of pension to the positive shock.

In a nutshell, estimation results, as displayed Table 2.2, show that both 2004 and 2006 positive shock bring little response of earlier retirement by exposing pension to each shock, although ERA constraint is found to play a role of deferring retirement. This result remains so when the regression is repeated in accelerated failure time model, whose results in detail are displayed in Table B3 in Appendix B. Notably, non-response to the positive shock is not evidence against wealth effect in light of Proposition 2.1.

30In fact, baseline hazard rate is not identifiable within Cox proportional hazard model. However, after pinning down coefficient estimates, it can be built utilizing the basic relationship between hazard rate (function) and survival function (which is a pdf).
Next, let us move on the estimation outcomes from simple Cox proportional hazard models on recession periods (2002 and 2008), following (2.13). The results are presented in Table 2.3.

On the one hand, similar to 2004 and 2006 expansion periods, wealth, early out window offer, and health issue expedite retirement, while non-availability of public pension benefit and earnings play as a hold-up factor for retiring in both 2002 and 2008 recession periods. In addition, in both cases, age affects retirement rate in quadratic form. On the other hand, however, exposure to a negative shock takes a substantive effect on retirement, which is different from 2004 and 2006 positive shock. That is, the key variable indicating the exposure of pension to each negative shock — both 2002 and 2008 alike — turns out statistically significant. In detail, as shown in Table 2.3, ratio of the retirement hazard rates is consistently less than one, implying that the exposure of pension to each negative shock leads workers to delay retirement with continuing work. These results carry over in accelerated failure time model, as displayed in Table B4 in Appendix B. Firstly, this response is in line with Proposition 2.2. Secondly, consistency of the results between the different recession periods is reinforcing.

In sum, 2002 or 2008 negative shock brings delay of retirement whereas 2004 or 2006 positive shock does not end up with little response of earlier retirement. As a matter of fact, this asymmetry in realized response to the two different kinds of shocks is also found in Coile and Levine (2006). At a glance, it is self-contradictory and seems to serve evidence that exposure of pension to financial market fluctuation does not affect retirement behavior. However, in light of Proposition 2.1 and 2.2, this can be consistent. In particular, the lack of realized change in retirement caused by exposure of pension to these specific positive shocks may be attributable to insufficient magnitude of those two shocks.

With respect to magnitude of a positive shock, we do not have information about individual gains or losses of pension investment in stock market, as mentioned above. Therefore, accurate assessment on this front would be difficult given data set. However, we still could attempt to do so by employing average rate of return on DC accounts as a proxy for the magnitude of the shock. As shown in Figure 2.4, the average rate of return on DC plans is 10.3 % in 2004 and 12.4 % in 2006 for positive shocks; on the other hand, it is $-10.6\%$ in 2002 and $-23.9\%$ in 2008 for negative shocks. Notably, 2002 negative shock is of similar size to two positive shocks. Therefore, with this proxy measure, in spite of its limitation, we could infer that retirement behavior would have been discernibly changed if the shock is greater than 2004 or 2006 shock without the availability constraints. Besides, as shown in Table 2.2, the proxy of availability constraint turns out to lower retirement rate in statistically meaningful way, indicating that this alone can deter these positive shock to convey realized change of retirement behavior.
Table 2.2: Retirement Rate and Exposure of Pension to Stock Market Gains in Expansion Periods

Cox Proportional Hazard Model

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Retirement hazard rate in 2004</th>
<th>Retirement hazard rate in 2006</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HR ratio</td>
<td>Coeff</td>
</tr>
<tr>
<td>Pension exposed to positive shock (=1)</td>
<td>1.090</td>
<td>0.091</td>
</tr>
<tr>
<td>ERA constraint (below 62) (=1)</td>
<td>0.267</td>
<td>-0.511</td>
</tr>
<tr>
<td>DC plan value</td>
<td>1.000</td>
<td>-0.008</td>
</tr>
<tr>
<td>DB plan value</td>
<td>1.000</td>
<td>0.022</td>
</tr>
<tr>
<td>SS wealth value</td>
<td>1.000</td>
<td>0.007</td>
</tr>
<tr>
<td>Wealth</td>
<td>1.000</td>
<td>0.030</td>
</tr>
<tr>
<td>Financial wealth in stock mkt</td>
<td>1.000</td>
<td>0.055</td>
</tr>
<tr>
<td>Earnings</td>
<td>1.000</td>
<td>-0.642</td>
</tr>
<tr>
<td>Early out window offered (=1)</td>
<td>1.670</td>
<td>0.513</td>
</tr>
<tr>
<td>Health problem (=1)</td>
<td>1.736</td>
<td>0.551</td>
</tr>
<tr>
<td>Risk averseness</td>
<td>0.968</td>
<td>-0.033</td>
</tr>
<tr>
<td>Age</td>
<td>1.231</td>
<td>0.207</td>
</tr>
<tr>
<td>Age$^2$</td>
<td>0.998</td>
<td>-0.002</td>
</tr>
<tr>
<td>Male (=1)</td>
<td>0.461</td>
<td>-0.774</td>
</tr>
<tr>
<td>Married (=1)</td>
<td>1.080</td>
<td>0.077</td>
</tr>
<tr>
<td>Education</td>
<td>0.953</td>
<td>-0.048</td>
</tr>
<tr>
<td>12 Industry indicators included?</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>No. of Obs</td>
<td>6238</td>
<td></td>
</tr>
<tr>
<td>Log likelihood ($\chi^2$)</td>
<td>$-7589.1$ (540.78)***</td>
<td></td>
</tr>
</tbody>
</table>

Note: (i) ‘Pension exposed to positive shock’ takes one if a worker exposes his pension to stock market in an expansion period (either 2004 or 2006). (ii) *** (**;and*) indicates that the estimate is statistically significant at 1% level (5% level; and 10% level) (iii) Each indicator variable takes value one if description of the variable with (=1) holds; otherwise zero. (iv) Variables of wealth and earnings are in 1,000,000 $. (v) ‘Risk averseness’ is a categorical variable ranging from one to four (the most risk-averse); and, this is based on a question on whether a respondent would take a new job whose income is uncertain between doubling the current one and some cuts.
Table 2.3] Retirement Rate and Exposure of Pension to Stock Market Losses in Recession Periods

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Cox Proportional Hazard Model</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Retirement hazard rate in 2002</td>
<td>Retirement hazard rate in 2008</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>HR ratio</td>
<td>Coef</td>
<td>(Std Err)</td>
<td>HR ratio</td>
</tr>
<tr>
<td>Pension exposed to negative shock (=1)</td>
<td>0.780</td>
<td>-0.248</td>
<td>(0.067)***</td>
<td>0.669</td>
</tr>
<tr>
<td>ERA constraint (below 62) (=1)</td>
<td>0.908</td>
<td>-0.096</td>
<td>(0.051)**</td>
<td>0.873</td>
</tr>
<tr>
<td>DC plan value</td>
<td>1.000</td>
<td>0.011</td>
<td>(0.007)</td>
<td>1.000</td>
</tr>
<tr>
<td>DB plan value</td>
<td>1.000</td>
<td>0.005</td>
<td>(0.005)</td>
<td>1.000</td>
</tr>
<tr>
<td>SS wealth</td>
<td>1.000</td>
<td>0.006</td>
<td>(0.034)</td>
<td>1.000</td>
</tr>
<tr>
<td>Wealth</td>
<td>1.000</td>
<td>0.001</td>
<td>(0.0005)**</td>
<td>1.000</td>
</tr>
<tr>
<td>Financial wealth in stock mkt</td>
<td>1.000</td>
<td>-0.007</td>
<td>(0.010)</td>
<td>1.000</td>
</tr>
<tr>
<td>Earnings</td>
<td>1.000</td>
<td>-0.189</td>
<td>(0.096)**</td>
<td>1.000</td>
</tr>
<tr>
<td>Early out window offered (=1)</td>
<td>2.032</td>
<td>0.709</td>
<td>(0.157)***</td>
<td>1.000</td>
</tr>
<tr>
<td>Health problem (=1)</td>
<td>1.460</td>
<td>0.378</td>
<td>(0.076)***</td>
<td>1.883</td>
</tr>
<tr>
<td>Risk averseness</td>
<td>0.977</td>
<td>-0.023</td>
<td>(0.030)</td>
<td>1.033</td>
</tr>
<tr>
<td>Age</td>
<td>1.199</td>
<td>0.182</td>
<td>(0.064)***</td>
<td>1.049</td>
</tr>
<tr>
<td>Age²</td>
<td>0.997</td>
<td>-0.002</td>
<td>(0.001)**</td>
<td>0.999</td>
</tr>
<tr>
<td>Male (=1)</td>
<td>0.461</td>
<td>-0.772</td>
<td>(0.067)***</td>
<td>0.518</td>
</tr>
<tr>
<td>Married (=1)</td>
<td>1.145</td>
<td>0.115</td>
<td>(0.069)</td>
<td>0.937</td>
</tr>
<tr>
<td>Education</td>
<td>0.962</td>
<td>-0.038</td>
<td>(0.010)***</td>
<td>0.967</td>
</tr>
<tr>
<td>12 Industry indicators included?</td>
<td>Yes</td>
<td></td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>No. of Obs</td>
<td>6930</td>
<td></td>
<td>5382</td>
<td></td>
</tr>
<tr>
<td>Log likelihood (χ²)</td>
<td>-9440.5 (536.17)***</td>
<td></td>
<td>-7209.9 (503.63)***</td>
<td></td>
</tr>
</tbody>
</table>

Note: (i) ‘Pension exposed to negative shock’ takes one if a worker exposes his pension to stock market in a recession period (either 2002 or 2008). (ii) *** (**;and*) indicates that the estimate is statistically significant at 1% level (5% level; and 10% level) (iii) Each indicator variable takes value one if description of the variable with (=1) holds; otherwise zero. (iv) Variables of wealth and earnings are in 1,000,000 $. (v) ‘Risk averseness’ is a categorical variable ranging from one to four (the most risk-averse); and, this is based on a question on whether a respondent would take a new job whose income is uncertain between doubling the current one and some cuts.
2.4.3.2 Robustness Checks

However, before drawing any conclusion from these estimation results, we can take another approach for the sake of clear identification. That is, to tackle the concern of unobserved individual heterogeneity, difference-in-difference method is utilized.

Table 2.4 | Retirement & Exposure of Pension to Stock Market Gains in Expansion

Diff-in-diff in Cox Proportional Hazard Model

Treat: On expansion, DC account is invested in stock market

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>HR ratio</th>
<th>Coeff</th>
<th>(Std Err)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treat × After</td>
<td>1.306</td>
<td>0.237</td>
<td>(0.393)</td>
</tr>
<tr>
<td>Pension exposed to 2004 positive shock (Treat=1)</td>
<td>0.821</td>
<td>-0.197</td>
<td>(0.169)</td>
</tr>
<tr>
<td>After (=1)</td>
<td>0.134</td>
<td>-0.208</td>
<td>(0.249)</td>
</tr>
<tr>
<td>ERA constraint (below 62) (=1)</td>
<td>0.210</td>
<td>-0.304</td>
<td>(0.048)***</td>
</tr>
<tr>
<td>DC plan value</td>
<td>1.000</td>
<td>0.005</td>
<td>(0.006)</td>
</tr>
<tr>
<td>DB plan value</td>
<td>1.000</td>
<td>0.018</td>
<td>(0.072)</td>
</tr>
<tr>
<td>SS wealth value</td>
<td>1.000</td>
<td>0.003</td>
<td>(0.023)</td>
</tr>
<tr>
<td>Wealth</td>
<td>1.000</td>
<td>0.014</td>
<td>(0.006)**</td>
</tr>
<tr>
<td>Financial wealth in stock mkt</td>
<td>1.000</td>
<td>0.010</td>
<td>(0.030)</td>
</tr>
<tr>
<td>Earnings</td>
<td>1.000</td>
<td>-0.190</td>
<td>(0.024)***</td>
</tr>
<tr>
<td>Early out window offered (=1)</td>
<td>1.788</td>
<td>0.267</td>
<td>(0.111)**</td>
</tr>
<tr>
<td>Health problem (=1)</td>
<td>1.635</td>
<td>0.492</td>
<td>(0.088)**</td>
</tr>
<tr>
<td>Risk averseness</td>
<td>0.966</td>
<td>-0.034</td>
<td>(0.036)</td>
</tr>
<tr>
<td>Age</td>
<td>1.188</td>
<td>0.172</td>
<td>(0.079)**</td>
</tr>
<tr>
<td>Age$^2$</td>
<td>0.997</td>
<td>-0.002</td>
<td>(0.001)**</td>
</tr>
<tr>
<td>Male (=1)</td>
<td>0.494</td>
<td>-0.704</td>
<td>(0.077)**</td>
</tr>
<tr>
<td>Married (=1)</td>
<td>1.070</td>
<td>0.068</td>
<td>(0.080)</td>
</tr>
<tr>
<td>Education</td>
<td>0.988</td>
<td>-0.048</td>
<td>(0.032)</td>
</tr>
<tr>
<td>12 Industry indicators included?</td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of Obs</td>
<td>13168</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log likelihood ($\chi^2$)</td>
<td>-6801.9</td>
<td>(707.35)***</td>
<td></td>
</tr>
</tbody>
</table>

Note: (i) ‘Pension exposed to 2004 positive shock (Treat)’ takes one if a worker exposes his pension to stock market in 2004; and ‘After’ takes one if an observation is from 2004 wave. (ii) *** (**;and*) indicates that the estimate is statistically significant at 1% level (5% level; and 10% level) (iii) Each indicator variable takes value one if description of the variable with (=1) holds; otherwise zero. (iv) Variables of wealth and earnings are in 1,000,000 $. (v) ‘Risk averseness’ is a categorical variable ranging from one to four (the most risk-averse); and, this is based on a question on whether a respondent would take a new job whose income is uncertain between doubling the current one and some cuts.
First of all, checking the impact of 2004 positive shock with difference-in-difference in Cox proportional hazard models, resulting estimates following (2.14) are displayed in Table 2.4.

Notably, this does not fundamentally change outcome from regression analysis following (2.12) without difference-in-difference. That is, facing a positive shock, non-availability of public pension lowers retirement rate in statistically meaningful way, whereas exposure of pension to the shock raises the rate without statistical significance. The meaning of this result within the theoretical framework, which is presented right above, can be maintained with this confirmation from difference-in-difference method. Put another way, in light of Proposition 2.1, 2004 positive shock was not large enough to make early retirement profitable without pension benefit available right now at hand. In particular, marginal effect of the eligibility constraint is 79% of retirement hazard rate in the sample since $0.210 - 1 = -0.79$. In addition, if one wants this relative (proportional) term to take a form of more specific level, a natural base rate would be baseline hazard rate by the nature of original specification in Cox proportional model.\footnote{Basically, if we rearrange a logarithm of baseline hazard rate, which enters into right-hand side of specifications under Cox proportional hazard model, to the left hand side, one can easily realize that hazard rate of retirement in fact are in unit of baseline hazard rate, although it is not uniquely identifiable within Cox proportional model.} In this sense, the availability restriction could lower retirement rate by 11.6% since the baseline hazard rate is estimated as 14.7%.

The overall result found above does not change even in accelerated time failure model, following (2.16), as shown in Table 2.5.

Although accelerated time failure model is also one of time-to-event analyses, the dependent variable is now a logarithm of number of years surviving as a non-retiree. As a consequence, the sign of each corresponding estimates is opposite to that from Cox proportional hazard model which takes a logarithm of retirement hazard rate as the dependent variable. The longer one stays in the labor force, the less likely he retires. That is, variables whose coefficients are estimated as taking positive (negative) values are contributing a worker to delay retirement more (less) in the accelerated time failure model. Moreover, the interpretation is more direct. Firstly, facing a positive shock, increase in a worker’s labor supply is positively associated with pension availability. Specifically, after controlling for other relevant factors, the marginal effect of non-eligibility of public pension benefit is 2.5 years since $\exp(0.917)$ is 2.502 from the fourth row and the first column in Table 2.5.

Second of all, let us revisit 2008 negative shock, regression yields the following result in Table 2.6.

Similar to extensional regression on the impact of 2004 positive shock, comparison of Table 2.3 and Table 2.6 reveals that qualitative results are not changed after introducing difference-in-difference method. In other words, all the explanatory variables point to the same direction in terms of sign of the estimates having statistical significance, even though taking different quantitative values. Moreover, this
conformation is kept in accelerated time failure model following (2.17), whose result is displayed below in Table 2.7.

**Table 2.5** Retirement & Exposure of Pension to Stock Market Gains in Expansion

Diff-in-diff in Accelerated Failure Time Model (Log-logistic distribution)

Treat: On expansion, DC account is invested in stock market

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>log(Years in labor force) in 2002 (Before) and 2004 (After)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treat × After</td>
<td>Coeff (Std Err)</td>
</tr>
<tr>
<td>Pension exposed to 2004 positive shock (Treat=1)</td>
<td>-0.144 (0.293)</td>
</tr>
<tr>
<td>After (=1)</td>
<td>0.079 (0.553)</td>
</tr>
<tr>
<td>ERA constraint (below 62) (=1)</td>
<td>0.917 (0.151)***</td>
</tr>
<tr>
<td>DC plan value</td>
<td>-0.0002 (0.002)</td>
</tr>
<tr>
<td>DB plan value</td>
<td>-0.004 (0.003)</td>
</tr>
<tr>
<td>SS wealth value</td>
<td>-0.003 (0.007)</td>
</tr>
<tr>
<td>Wealth</td>
<td>-0.003 (0.001)***</td>
</tr>
<tr>
<td>Financial wealth in stock mkt</td>
<td>-0.002 (0.008)</td>
</tr>
<tr>
<td>Earnings</td>
<td>0.110 (0.039)***</td>
</tr>
<tr>
<td>Early out window offered (=1)</td>
<td>-0.100 (0.048)**</td>
</tr>
<tr>
<td>Health problem (=1)</td>
<td>-0.138 (0.031)**</td>
</tr>
<tr>
<td>Risk averseness</td>
<td>0.011 (0.011)</td>
</tr>
<tr>
<td>Age</td>
<td>-0.035 (0.017)**</td>
</tr>
<tr>
<td>Age²</td>
<td>0.0002 (0.0001)**</td>
</tr>
<tr>
<td>Male (=1)</td>
<td>0.205 (0.024)*****</td>
</tr>
<tr>
<td>Married (=1)</td>
<td>-0.008 (0.025)</td>
</tr>
<tr>
<td>Education</td>
<td>0.013 (0.004)*****</td>
</tr>
<tr>
<td>12 Industry indicators included?</td>
<td>Yes</td>
</tr>
<tr>
<td>No. of Obs</td>
<td>13168</td>
</tr>
<tr>
<td>Log likelihood (χ²)</td>
<td>-1890.1 (360.40)***</td>
</tr>
</tbody>
</table>

Note: (i) ‘Pension exposed to 2004 positive shock (Treat)’ takes one if a worker exposes his pension to stock market in 2004; and ‘After’ takes one if an observation is from 2004 wave. (ii) *** (**; and *) indicates that the estimate is statistically significant at 1% level (5% level; and 10% level) (iii) Each indicator variable takes value one if description of the variable with (=1) holds; otherwise zero. (iv) Variables of wealth and earnings are in 1,000,000 $. (v) ‘Risk averseness’ is a categorical variable ranging from one to four (the most risk-averse); and, this is based on a question on whether a respondent would take a new job whose income is uncertain between doubling the current one and some cuts.
Table 2.6] Retirement & Exposure of Pension to Stock Market Losses in Recession
Diff-in-diff in Cox Proportional Hazard Model
Treat: On recession, DC account is invested in stock market

<table>
<thead>
<tr>
<th>Explanatory Variable</th>
<th>Dependent Variable</th>
<th>Retirement hazard rate in 2006 (Before) and 2008 (After)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HR ratio</td>
<td>Coeff</td>
</tr>
<tr>
<td>Treat × After</td>
<td>0.589</td>
<td>-1.238</td>
</tr>
<tr>
<td>Pension exposed to 2008 negative shock (Treat=1)</td>
<td>0.805</td>
<td>-0.247</td>
</tr>
<tr>
<td>After (=1)</td>
<td>1.501</td>
<td>1.031</td>
</tr>
<tr>
<td>ERA constraint (below 62) (=1)</td>
<td>0.815</td>
<td>-0.164</td>
</tr>
<tr>
<td>DC plan value</td>
<td>1.000</td>
<td>0.009</td>
</tr>
<tr>
<td>DB plan value</td>
<td>1.000</td>
<td>0.002</td>
</tr>
<tr>
<td>SS wealth value</td>
<td>1.000</td>
<td>0.012</td>
</tr>
<tr>
<td>Wealth</td>
<td>1.000</td>
<td>0.002</td>
</tr>
<tr>
<td>Financial wealth in stock mkt</td>
<td>1.000</td>
<td>-0.001</td>
</tr>
<tr>
<td>Earnings</td>
<td>1.000</td>
<td>-0.108</td>
</tr>
<tr>
<td>Early out window offered (=1)</td>
<td>1.182</td>
<td>0.167</td>
</tr>
<tr>
<td>Health problem (=1)</td>
<td>1.584</td>
<td>0.459</td>
</tr>
<tr>
<td>Risk averseness</td>
<td>1.079</td>
<td>0.076</td>
</tr>
<tr>
<td>Age</td>
<td>1.045</td>
<td>0.156</td>
</tr>
<tr>
<td>Age²</td>
<td>0.999</td>
<td>-0.0008</td>
</tr>
<tr>
<td>Male (=1)</td>
<td>0.543</td>
<td>-0.609</td>
</tr>
<tr>
<td>Married (=1)</td>
<td>1.023</td>
<td>0.023</td>
</tr>
<tr>
<td>Education</td>
<td>0.981</td>
<td>-0.018</td>
</tr>
<tr>
<td>12 Industry indicators included?</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No. of Obs</td>
<td>11370</td>
</tr>
<tr>
<td></td>
<td>Log likelihood ($\chi^2$)</td>
<td>-7121.6 (601.95)**</td>
</tr>
</tbody>
</table>

Note: (i) ‘Pension exposed to 2008 negative shock (Treat)’ takes one if a worker exposes his pension to stock market in 2008; and ‘After’ takes one if an observation is from 2008 wave. (ii) *** (**,and*) indicates that the estimate is statistically significant at 1% level (5% level; and 10% level). (iii) Each indicator variable takes value one if description of the variable with (=1) holds; otherwise zero. (iv) Variables of wealth and earnings are in 1,000,000 $. (v) ‘Risk averseness’ is a categorical variable ranging from one to four (the most risk-averse); and, this is based on a question on whether a respondent would take a new job whose income is uncertain between doubling the current one and some cuts.
<table>
<thead>
<tr>
<th>Table 2.7] Retirement &amp; Exposure of Pension to Stock Market Losses in Recession</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diff-in-diff in Accelerated Failure Time Model (Log-logistic distribution)</td>
</tr>
<tr>
<td>Treat: On recession, DC account is invested in stock market</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>log(Years in labor force) in 2006 (Before) and 2008 (After)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanatory Variables</td>
<td>Coeff</td>
</tr>
<tr>
<td>Treat × After</td>
<td>0.366</td>
</tr>
<tr>
<td>Pension exposed to 2008 negative shock (Treat=1)</td>
<td>-0.033</td>
</tr>
<tr>
<td>After (=1)</td>
<td>-0.160</td>
</tr>
<tr>
<td>ERA constraint (below 62) (=1)</td>
<td>0.214</td>
</tr>
<tr>
<td>DC plan value</td>
<td>-0.001</td>
</tr>
<tr>
<td>DB plan value</td>
<td>-0.0006</td>
</tr>
<tr>
<td>SS wealth value</td>
<td>-0.003</td>
</tr>
<tr>
<td>Wealth</td>
<td>-0.004</td>
</tr>
<tr>
<td>Financial wealth in stock mkt</td>
<td>0.002</td>
</tr>
<tr>
<td>Earnings</td>
<td>0.029</td>
</tr>
<tr>
<td>Early out window offered (=1)</td>
<td>-0.050</td>
</tr>
<tr>
<td>Health problem (=1)</td>
<td>-0.108</td>
</tr>
<tr>
<td>Risk averseness</td>
<td>-0.013</td>
</tr>
<tr>
<td>Age</td>
<td>-0.052</td>
</tr>
<tr>
<td>Age²</td>
<td>0.0003</td>
</tr>
<tr>
<td>Male (=1)</td>
<td>0.130</td>
</tr>
<tr>
<td>Married (=1)</td>
<td>-0.012</td>
</tr>
<tr>
<td>Education</td>
<td>-0.007</td>
</tr>
<tr>
<td>12 Industry indicators included?</td>
<td>Yes</td>
</tr>
<tr>
<td>No. of Obs</td>
<td>11370</td>
</tr>
<tr>
<td>Log likelihood ($\chi^2$)</td>
<td>-1890.1 (360.40)***</td>
</tr>
</tbody>
</table>

Note: (i) ‘Pension exposed to 2008 negative shock (Treat)’ takes one if a worker exposes his pension to stock market in 2008; and ‘After’ takes one if an observation is from 2008 wave. (ii) *** (**; and*) indicates that the estimate is statistically significant at 1% level (5% level; and 10% level). (iii) Each indicator variable takes value one if description of the variable with (=1) holds; otherwise zero. (iv) Variables of wealth and earnings are in 1,000,000 $. (v) ‘Risk averseness’ is a categorical variable ranging from one to four (the most risk-averse); and, this is based on a question on whether a respondent would take a new job whose income is uncertain between doubling the current one and some cuts.

In addition, after controlling for the concern of unobserved heterogeneity, non-availability of pension benefit still negatively affects retirement rate. It can hinder a positive shock from actually taking effect on implementing retirement plan; however, it turns out not for a negative shock. Moreover, according to theoretical framework
presented in the previous section, it does not make a difference in following through retirement plan responding to a negative shock.

In detail, exposure to the 2008 negative shock raises the ratio of retirement hazard rate by 41.1%. Furthermore, if this can be put in terms of baseline hazard rate, which is estimated as 11.2%, then the level of retirement rate is increased by 4.59%, due to the exposure of pension to the negative shock. This is modest because it lies between 9.8% change in labor force participation rate from Eaken Holtz-Eakin et al. (1993) and 1.32% change in marginal propensity to consume leisure estimated by Imbens et al. (1999). Alternatively, when translated into the amount of labor supply, negative shock on pension from stock market brings about 1.44 years more of labor supply by senior workers based on Table 2.7. Eventually, however, it should be reminded that although the discussion on estimation results are stretched to specific numbers, only the signs of the estimates on key parameters are to be picked in relation to theoretical analysis, as mentioned above.

In sum, data analyses show that 2002 and 2008 negative shocks lead to delay in retirement whereas 2004 and 2006 positive shocks do not cause a discernible change in the behavior of leaving labor force. Additionally, non-responsiveness to a positive shock on pension is also found in other studies such as Coile and Levine (2006) and Krueger and Pischke (1992). Notably, this is not self-conflicting since Proposition 2.1 and 2.2 show that exposure of pension to a positive shock links to change in retirement behavior under more restrictive condition than exposure to a negative shock does. Thus, regardless of statistical significance, the sign of $\alpha_1$ and $\beta_1$ turns out consistent with Proposition 2.1 and 2.2.

Although empirical investigation is repeated for different periods and with difference-in-difference method, there is another way of checking robustness of the results. That is, we can harness probit. First of all, the left-hand side variable is now simple indicator of whether a respondent retires or not, instead of retirement hazard rate that can vary over duration years in the labor force; hence, it is better for us to add the variable of years in the labor force onto the original specifications (2.19) and (2.20). Still, as mentioned above, probit is not best-suited for this analysis since it does not properly deal with censoring problem. Obviously, there is no established way to transform probit model into hazard model. Nevertheless, if the sign of the estimate is flipped over under probit model and if the change is not explained by reasonable conjecture on the direction of bias, it can suggest that the original statistical model might suffer a problem. Since probit deals with potential retirees as non-retirees, it is likely to under-estimate the impact of right-hand side variables over retirement behavior. Having this in mind, estimation outcomes from the modified regression under probit model are presented in Table B7 and Table B8. By comparing these with Table 2.4 and Table 2.6, inconsistency is not found, which fortifies main estimation results above relying on hazard models. Similar exercise is conducted for (2.17) and (2.18) whose results are displayed in Table B5 and Table B6, showing consistency with outcomes under hazard model as well.
Table 2.8 | Wage and Pension Type (DB vs. DC)  
Mincerian Equation (OLS)

<table>
<thead>
<tr>
<th>Wave (year)</th>
<th>2002</th>
<th>2004</th>
<th>2006</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable</td>
<td>log(hourly wage)</td>
<td>log(hourly wage)</td>
<td>log(hourly wage)</td>
<td>log(hourly wage)</td>
</tr>
<tr>
<td>Explatory Variable</td>
<td>Coeff.</td>
<td>(Std Err)</td>
<td>Coeff.</td>
<td>(Std Err)</td>
</tr>
<tr>
<td>DC plan holder (=1)</td>
<td>-0.033 (0.059)</td>
<td>0.062 (0.050)</td>
<td>0.090 (0.079)</td>
<td>-0.005 (0.050)</td>
</tr>
<tr>
<td>Education</td>
<td>0.093 (0.003)***</td>
<td>0.097 (0.003)***</td>
<td>0.098 (0.003)***</td>
<td>0.095 (0.003)***</td>
</tr>
<tr>
<td>Job experience</td>
<td>0.004 (0.001)***</td>
<td>0.002 (0.001)**</td>
<td>0.003 (0.002)*</td>
<td>0.003 (0.002)*</td>
</tr>
<tr>
<td>Job experience^2</td>
<td>-0.006 (0.002)***</td>
<td>-0.004 (0.002)**</td>
<td>-0.005 (0.001)***</td>
<td>-0.005 (0.002)***</td>
</tr>
<tr>
<td>Risk averseness</td>
<td>-0.005 (0.011)</td>
<td>-0.013 (0.010)</td>
<td>-0.055 (0.081)</td>
<td>-0.022 (0.054)</td>
</tr>
<tr>
<td>Age</td>
<td>-0.011 (0.001)***</td>
<td>-0.006 (0.001)***</td>
<td>-0.009 (0.001)***</td>
<td>-0.008 (0.001)***</td>
</tr>
<tr>
<td>Male (=1)</td>
<td>0.304 (0.023)***</td>
<td>0.224 (0.021)***</td>
<td>0.229 (0.021)***</td>
<td>0.224 (0.023)***</td>
</tr>
<tr>
<td>Married (=1)</td>
<td>0.049 (0.024)**</td>
<td>0.060 (0.022)***</td>
<td>0.050 (0.022)**</td>
<td>0.114 (0.025)***</td>
</tr>
<tr>
<td>12 Industry indicators included?</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>No. of Obs</td>
<td>6930</td>
<td>6238</td>
<td>5988</td>
<td>5382</td>
</tr>
<tr>
<td>Adjusted R^2</td>
<td>0.2493</td>
<td>0.2260</td>
<td>0.2325</td>
<td>0.2287</td>
</tr>
</tbody>
</table>

Note: (i) In avoidance of infinite negative value, zero hourly wage is replaced with one before taking logarithm. (ii) ‘DC plan holder’ takes one if a worker is offered (sponsored) with a DC account by his current employer. (iii)*** (**; and*) indicates that the estimate is statistically significant at 1% level (5% level; and 10% level). (iv) Each indicator variable takes value one if description of the variable with (=1) holds; otherwise zero. (v) ‘Risk averseness’ is a categorical variable ranging from one to four (the most risk-averse); and, this is based on a question on whether a respondent would take a new job whose income is uncertain between doubling the current one and some cuts.
Lastly, whether hourly wage is systematically related to pension type (DB or DC) is investigated since if they are related, then the link between theoretical model and data analysis is weakened. First of all, as Brown and Weisbenner (2009) pointed out, it is not usual that an employer offers more than one type of pension to employees. Furthermore, pension type probably would not be a primary factor of choosing a firm whom an individual works for. Nevertheless, whether wage and pension type is related or not has not been proven by other researchers with data so far. Therefore, although it is not a direct check of estimation results in the previous subsection, how wage rate is related with pension type is examined, adopting the mostly widely accepted Mincerian wage equation.

As shown in Table 2.8, it turns out that pension type is not associated with wage throughout sampling periods in a statistically meaningful way.

2.5 Concluding Remarks

This chapter has delineated conditions of optimal time to retire which allow for dealing pension wealth distinctly from non-pension wealth. Based on the conditions, we showed that exposure of pension to financial market fluctuations brings behavior response of retirement in asymmetric way. On the one hand, exposure of pension to a positive shock leads to a discernible increase in retirement only if the magnitude of the shock is large enough to compensate foregone earnings and increased demand for post-retirement consumption due to earlier retirement than planned. One the other hand, facing a negative shock, workers respond with delay in retirement. Overall, this implies that even when a negative shock causes a decrease in retirement, a positive shock with the same magnitude might not bring an actual increase in retirement.

This theoretical framework is applied to recent boom and bust periods. In detail, the empirical analysis finds that exposure of pension to 2002 and 2008 negative shocks led a decrease in retirement; however, for positive shocks of 2004 and 2006 expansions, little evidence on an ensued increase in retirement was found. Notably, this contrast is not a case against effect of exposure of pension to market fluctuations over retirement. Not being inconsistent with theoretical model, these estimation results rather provide an example where exposure of pension to gains from stock market does not end up with implementing earlier retirement. In the light of theoretical analysis, this finding can imply that lack of realized response may be attributable to the magnitude of 2004 and 2006 positive shocks. If there were a sudden upturn with greater magnitude in financial market, we could have observed a discernible increase in retirement.

Finally, this study can have various policy implications. Most of all, it can suggest a need to broaden our consideration in assessing economic consequences of public pension reforms that put more investment risk on individual beneficiaries than before, since most of studies on impacts of pension reforms ignored labor supply response. If its response is perfectly symmetric, then this simplifying assumption would be
fine since increase and decrease of labor supply cancels each other out in the face of positive shocks and negative shocks. If not, as shown in this chapter, it can generate misleading analysis on economic consequences of such reforms. On the top of this, considering historical fact that positive shocks have been of less magnitude than negative shocks. We can form a guess that such reforms might entail decrease in retirement in the long run. In addition to this, theoretical and empirical findings on asymmetric response of retirement to recent financial market fluctuations might provide a clue for understanding the very recent rise in labor force participation of senior workers, reversing continued decrease.
Chapter 3

Optimal Income Taxation and Optimal Revenue Mobilization

3.1 Introduction

Income tax would be more direct and thus more effective measure to tackle inequality than any other tax instrument available. This equity gain is to be carefully balanced with its efficiency loss in designing optimal income tax for the sake of maximizing social welfare. To this important end, Mirrlees (1971) presented theoretical formulae for optimal nonlinear income tax rates which enable numerical simulations (Mirrlees 1971; Tuomala 1984; Kanbur and Tuomala 1994) some of which are largely similar to actually observed tax rates. However, policy implications of optimal income taxation literature is still fairly limited, even after substantial efforts to improve its applicability (e.g., Diamond 1998; Saez 2001) have been made following his seminal work.

Importantly, these studies presume a perfect tax enforcement so that a government is able to raise an *exogenously* given amount of income tax revenue, no matter how large it is, without being concerned about tax evasion. As a matter of fact, however, the tax enforcement of governments is imperfect; hence, the tax revenue mobilization is far from trivial especially for individual income tax,\(^1\) since it is intrinsically more succumb to evasion than any other tax. First of all, commodity tax is actually collected by firms who remarkably outnumbers tax officials and can always charge the exact amount of the tax liability; therefore, there is hardly any room for individuals to evade excise taxes. Second of all, although corporate income tax might have better chances of dodging than excise tax, it is still less conducive to evasion than personal income tax, due to following reasons. Firstly, the number of taxpayers that the tax authority has to handle and monitor is far smaller for corporate income tax than for

\(^1\)For a comprehensive survey of the research on this, refer to Andreoni et al. (1998) and Slemrod (2002).
personal income tax. Secondly, a firm needs to take higher risk of being penalized for the tax evasion than an individual taxpayer does, because it potentially has much greater number of highly effective whistle blowers of cheating.

As a result, this inherently higher vulnerability to evasion causes personal income tax to suffer a larger volume of revenue leakage, leaving the income tax mobilization more sensitive to the government’s enforcement capacity than any other tax. To take an example of the US, according to Slemrod (2007, p.28), in year 2001, the estimate for evaded individual income taxes ($197 billion) is more than six times as large as that for corporate income taxes ($30 billion). Furthermore, across countries, personal income tax receipts (measured in terms of share of GDP) have shown remarkably larger variance than any other tax receipts by a substantial margin (IMF 2011). Notably, the gap in personal income tax revenue between OECD countries and low income countries, over the period from 1980 to 2009, has long exceeded 8% of GDP, whose magnitude would have a greater impact than any economic recession since World War II. In stark contrast, the corresponding one in corporate income tax revenue between the two groups of countries is only 0.9% of GDP (IMF 2011).

These observations clearly suggest that to treat the amount of tax revenue as exogenously given might not be the best approach taken. Rather, optimal income taxation can be better studied by allowing an imperfect tax enforcement and by further specifying revenue mobilization. This chapter aims to investigate characteristics of nonlinear optimal income taxation and optimal revenue mobilization when the tax enforcement of a government is not perfect and improving the enforcement rate requires some cost, under a general equilibrium framework.

Capitalizing upon the widely accepted model for nonlinear optimal income tax proposed by Mirrlees (1971), individuals of a society are different only in their earning ability. In weighting these individuals to form a social welfare function, the government will prefer to tackle the inequality if a higher weight is not given to one with higher earning ability. In this case, improvement of the social welfare can be made possible by taxation. In setting the level of desired tax expenditure financed by the taxation, not only the utility of the worst off of the society but also the extensive margin of labor supplies by low earning ability individuals is affected. Even though the innate ability is the only source of inequality of the society and thus should play a steering role in the taxation, the government is not able to levy taxes directly based on it because the government cannot observe and verify it. This causes a loss of efficiency because individuals distort their decision of working hours to hide their true ability facing an income tax schedule that tries to impose greater tax liability on higher ability individuals.

On the other hand, it is possible for the government to observe and verify an individual’s true income. However, the government does not always succeed in detecting the amount of true tax earnings; and to increase the overall rate of tax enforcement is costly. At the stand of individual taxpayers, this implies that there may be a lucrative gain from tax evasion. Essentially similar to the choice of purchasing risky
assets, taxpayers will choose the evasion level that equalizes the expected marginal utility under being penalized (after a successful detection by the government) with that under not being detected. Aligned with this, their labour supply decision is also made to equalize the marginal disutility from working with the expected utility from income.

An investigation on hypothetical benchmark cases, where the government has a clairvoyant power over each individual’s innate ability and can levy lump sum taxes, reveals that the efficiency loss of income taxation stems from the lack of verifiability of a taxpayer’s innate ability. In other words, each income tax rate in the schedule fails to take effect on the intended group of taxpayers. Thus, to minimize any distortion from income taxation boils down to inducing taxpayers to voluntarily reveal their own ability. By imposing the constraints for this revelation, the optimal design of tax schedule is derived. Eventually, optimal marginal income tax rate to taxpayers of any given level of ability are set to allow efficiency loss only for compensating to taxpayers of all the higher abilities with the minimum of the surplus that those taxpayers could have enjoyed if they pretend to be of the given level of ability. Therefore, this implies that zero marginal tax rate (which incurs no efficiency loss) is levied on the highest ability taxpayers since there is no one else who is more able and thus tempted to mimic these taxpayers.

Under an optimal income tax schedule, even though the marginal tax rate on the highest ability taxpayers (who are also top earners) is zero, the expected tax payment these individuals make is the largest. Moreover, when we extend the model with a modification that the zero marginal tax rate on the top earners per se triggers a sharp hike in the cost for the tax enforcement, amendment to levy a strictly positive rate on them can be better for the social welfare. In other words, by incorporating tax revenue mobilization concerns into the derivation of optimal tax rates, we can conceive a possibility to make an argument against zero marginal tax rate on the highest income.

Along with the tax rate, the government also has to find an optimal tax enforcement rate for controlling tax evasion. On the one hand, raising the rate requires some cost, which means that an increment in the tax enforcement rate ensues not only an increase in the expected tax revenue but also the cost paid for the improvement. On the other hand, in the shoes of taxpayers, the increment in the tax enforcement rate means more risk involved with tax evasion, which leads to a decrease in their utility because they are risk averse. All in all, an optimal tax enforcement rate is set to equalize the social value on the marginal loss to taxpayers from the raised risk of evading tax with the marginal social benefit on a net increase in the expected revenue from the increment.

Additionally, by explicitly embedding tax evasion concerns into the model of optimal income taxation, this study resolves two ambiguities that have long remained puzzling in the literature on the tax evasion. That is, this chapter achieves to theoretically prove the following two statements. First, marginal tax rate is clearly positively
related to tax evasion. Second, enhanced tax enforcement rate certainly leads to decreased tax noncompliance.

Moreover, with an introduction of a unique parameter that indicates the effectiveness and efficiency of the government’s tax administration, the present study provides a theoretical proof of a positive relationship of the tax revenue and the effectiveness of the government’s operation of tax system. On the top of this, an empirical verification of this theoretical result is also presented with cross-sectional data of various countries around the world.

This study is organized as follows. Section 3.2 reviews relevant previous research. Section 3.3 displays a theoretical analysis of the best feasible design of income taxation and its revenue mobilization. Lastly, Section 3.4 concludes.

3.2 Review of Related Literature

The essential feature of income tax that it distorts individual’s labor supply incentive is rigorously formulated by Mirrlees (1971). He portrays a society as consisting of individuals endowed with unequal earning ability, which thus serves as a standard model to subsequent studies on income taxation. Nonetheless, presuming a perfect and costless enforcement of a chosen tax rates, his paper as well as its following work ignores tax evasion issue. A formal but simplistic analysis on the income tax evasion is first heralded by Allingham and Sandmo (1972). However, it disregards the labor supply choice of income taxpayers with allowing them to choose only the amount of income to declare among their own income exogenously given. Through comparative static analyses, Allingham and Sandmo (1972) find that income tax evasion is negatively associated with tax enforcement parameters — the tax audit rate and the penalty — while its relationship with income tax rate is not clear.

Obviously, in analyzing individuals’ responses to any income tax policy, a decision of working hours is more primary than that of evading some tax liabilities. So, a series of improvements upon Allingham and Sandmo model (1972) are made by integrating the labor supply choice into the original model (e.g., Weiss 1976; Pencavel 1979; Sandmo 1981). However, this progress turns out to bring puzzling ambiguities rather than clarifications. Baldry (1979), Pencavel (1979), and Horowitz and Horowitz (2000) prove that the effect of the tax audit rate over income tax evasion is ambiguous, clearly implying that improved tax enforcement can rather raise tax evasion. Taking a more general approach than these, Sandmo (1981) derives optimal linear income tax in the presence of tax evasion, instead of treating tax rate simply as one of parameters. Nevertheless, this paper fails to refute the perplexing result. While these policy relevant queries remain befuddling, a theoretically interesting result to rationalize random tax rates is found by Weiss (1976), Cowell (1981), and Stigliz (1982). That is to say, they show that, exposed to the uncertainty of being penalized, individuals can increase their labor supply compared to in the absence of tax evasion.
Along with this, elaborations on the simple random tax auditing of Allingham and Sandmo (1972) are presented by Sánchez and Sobel (1993), Reinganum and Wilde (1985), and Mookherjee and Png (1989). They derive optimal audit schemes from the maximization of a net income tax revenue — instead of social welfare — with constant cost per audit. Both Sánchez and Sobel (1993) and Reinganum and Wilde (1985) assume that taxpayers are risk neutral and argue that the audit should be conducted all the tax filing below a certain cutoff level of income. On the other hand, Mookherjee and Png (1989) allow risk averse taxpayers. They maintain that every individual, except for one with the highest tax liability, is to be randomly audited, which is virtually almost back to Allingham and Sandmo (1972); however, they add a somewhat unrealistic condition that an optimal audit scheme should reimburse a positive reward to honest taxpayers. Putting aside actual implementability of these audit policies, Slemrod and Yitzhaki (2002) and Andreioni et al. (1998) point out that the real tax audit rates and the penalties in the US are so low that the current level of tax compliance becomes inexplicably high in view of these models. Moreover, Cook (1979) criticized the validity of the audit rate in describing individual’s behavior of tax evasion.

Thus, evolving from a verbatim interpretation of the probability of tax audit, Slemrod and Yitzhaki (1987) provide a new interpretation of the tax audit rate in a theoretical model on the tax evasion — "degree of enforcement of the tax law" (p.184, Slemrod and Yitzhaki, 1987). Specifically, in Slemrod and Yitzhaki (1987), the parameter that has been regarded as the audit probability now refers to a size of the tax authority which entails some cost to enlarge. Furthermore, improving upon the previous approach, they purport to maximize the social welfare function, not the net income tax revenue, and find that the optimal size of a tax enforcement agency is set to equate the marginal excess burden with the marginal cost spent for an increase in the tax enforcement. In line with this, Sandmo (1981) actually has the parameter stand for the probability of executing the tax code and claims that, at the optimum, marginal administrative cost is larger than marginal tax revenue.

The first attempt to explicitly integrate tax evasion concern into the design of an optimal income tax rate is made by Sandmo (1981) allowing tax enforcement to be imperfect and costly. However, his model has some limitations. First of all, Sandmo (1981) arbitrarily divides individuals into two groups — either evader or non-evader — on which social weights are assigned accordingly; and, he postulates that an evader chooses only labor supplies for his (or her) two jobs that are paying the same wage rate: (i) one for irregular market where tax evasion always occurs; and (ii) the other for regular market where tax evasion is impossible. Put another way, he forges the model such that the choice of evasion is inseparable from the decision of labor supply (and job). This might have possibly caused its inability to answer

\footnote{First of all, in analyzing the decision over the uncertain outcomes of tax evasion, risk neutrality assumption is quite restrictive. Moreover, contrary to their result, the actual audit rate has long been higher for higher incomers (United States General Accounting Office, 2001).}
whether an increase in the income tax rate raises tax evasion or not. Second of all, allowing only two different social weights (one on the evader group and the other on the non-evader group) that are not directly related to inequality, the progressivity (and the equity gain) is not be well defined in his optimum income tax rate, which takes single value for all the individuals. More importantly, Sandmo (1981) did not succeed in elucidating the effect of income tax rate on the tax evasion.

Contrast to the theoretical front where efforts to clarify the effect of marginal tax rates on tax evasion were not fruitful, substantial achievements have been made on the empirical front. To begin, using the US Taxpayer Compliance Measurement Program (TCMP) data, Clotfelter (1983) find a significantly positive effect of marginal tax rate over tax evasion. This is in line with the finding of Dubin et al. (1990) from a time series data of Annual Report of the Commissioner of Internal Revenue. However, this finding is opposed to the result of Feinstein (1991) with the TCMP data but econometrical specifications different from Clotfelter (1983). Moreover, to overcome the measurement problem and endogeneity issues of observational data in these studies, lab experiments were conducted by Friedland et al. (1978), Baldry (1987), and Alm et al. (1992), all of which find a significantly positive effect of marginal tax rate over tax evasion. This positive relationship is also confirmed by the finding of Kleven et al. (2011) in a large scale field experiment.

Most of all, research done on individual income tax revenue (contrast to those on total tax revenue) is almost none, both empirically and theoretically. On the one hand, beyond simple descriptive statistics (e.g., IMF 2011), there is only one econometric analysis on personal income tax revenue that appears in Peter et al. (2010). They explore how personal income tax revenue (measured in terms of GDP) is associated with income group of a country, tax rates, corruption index, using cross country panel data spanning about 20 years. Although they treat their multi-period panel data as if it is a single period cross-section data, they find that higher tax rates tend to bring greater income tax revenues collected. On the other hand, none of the studies on optimal income taxation, whether nonlinear or linear, elaborates on the revenue consequences of the optimum tax schedule that maximizes the social welfare. First, Sandmo (1981) takes a partial equilibrium approach and assumes the required revenue exogenously given. Nevertheless, one of his results (Equation (48), p.278, Sandmo 1981) allows one to infer a positive relationship between the collected revenue and the tax enforcement. Second, Mirrlees (1971) derives an optimum income tax schedule from a general equilibrium framework so that he needs not assume an exogenous amount of the revenue to be collected. However, rather focusing on the optimal tax rates, he is silent on the properties of the resulting income tax revenue at the optimum.\footnote{This is also true for other studies on the optimal nonlinear income tax rates (e.g., Diamond 1998; Saez 2001).}
3.3 Theoretical Analysis

3.3.1 The Model

Consider a society populated by a continuum of individuals. These individuals’ preference admits a utility representation that satisfies the Von Neumann-Morgenstern axioms. In particular, their preference is represented by a function $u(x, l)$, where $x \in \mathbb{R}_+$ is a composite consumption good that represents all the commodities, and $l \in [0, 1)$ is amount of time spent for working. As usual, $u(x, l)$ is strictly concave, continuously differentiable ($C^1$ function), strictly increasing in $x$ and strictly decreasing in $l$. Since zero consumption of all the goods including one that is necessary for sustaining life can cause a death, we have $\lim_{x \searrow 0} u(x, l) = -\infty$. Likewise, since working 24 hours a day without any sleep or rest can lead to a bottomless fall of utility, let $\lim_{l \uparrow 1} u(x, l) = -\infty$. For simplicity, let none of these individuals be endowed with any wealth. Rather, they are differentiated only with their innate earning ability $n \in [0, \pi]$ which is distributed according to a continuous cumulative distribution function (hereafter, CDF) $H(n)$ with the full support and a probability density function (hereafter, PDF) $h(n)$. This identifier variable reflects one’s own innate ability in that when an individual of ability $n$ spends $l$ amount of time in working, he actually provides effective labor input of $nl \equiv L$ to a firm. With a competitive labor market wage rate given, this means that the same amount of working yields higher earnings to one with higher ability.

In addition, there are profit-maximizing firms under a perfect competition; and, their technology is characterized by a production function $F(L)$ which exhibits constant returns to scale and satisfies $F(0) = 0$. In concordance with the consumer’s demand depicted in terms of a composite commodity, assume that there are numerous firms that produce the composite good are competing each other. Furthermore, since only relative prices matter, it is without loss of generality to normalize the producer price to be one. As a result, each firm solves $\max_{L} F(L) - wL$, where $w$ is the market wage rate paid for supplied effective labor input. The first order condition (henceforth, FOC) of this optimization gives us $w = F'$. This immediately implies that each firm yields zero profit, based on the Euler’s theorem for homogeneous functions.

Above all, the government of this society respects utility of every citizen and pursues to maximize a social welfare function $W(v_0 \cdots v_n \cdots v_\pi)(henceforth, SWF)$, where $v_n$ is the indirect utility function of an individual of ability $n$. In particular, since this society is populated with a continuum of individuals, the SWF is stated as $W(v_0 \cdots v_n \cdots v_\pi) = \int_0^\pi \frac{dG}{dv_n} v_n h(n)dn$, where $\frac{dG}{dv_n}$ means a social marginal value on utility of an individual with ability $n$. Because the government respects the utility of

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4That is, if the number of all the consumption commodities is $k$, then $x = q^\prime x$, where $q \in \mathbb{R}_+^k$ is a consumer price vector and $x \in \mathbb{R}_+^k$ is a vector of the $k$ consumption goods. For more detail, refer to Hicks (1946).
each individual, $\infty > \frac{dG}{dv_n} > 0$ for $\forall n$. Furthermore, suppose that the government does not give a greater weight on individuals who enjoy higher level of utility, as modern democratic societies. This means that $-\frac{d^2}{dv_n^2} \left( \frac{dG}{dv_n} \right) \leq 0$. If the SWF is a simple sum of individuals’ utilities, then this means that the SWF is concave. In maximizing the SWF, an interpersonal lump sum transfer is not available tax instrument, as usual. Instead, the government can levy individuals income taxes on the earnings reported by them, based on a differentiable income tax schedule $T$. Most importantly, however, the government is not able to observe and verify the accurate ability of each individual at any cost, due to its nature as an inner characteristic.

The model presented so far is close to a Mirrleesian economy (Mirrlees 1971) that is widely adopted in the optimal taxation literature. However, this standard model presumes non-trivial factors of the taxation: that is, it takes both perfect tax enforcement of the government and full compliance of taxpayers for granted. These factors are of importance, since however well designed an income tax schedule is, it would not be meaningful unless it is enforced to actually collect revenue for enhancing social welfare. As a matter of fact, they are neither simple nor easy to achieve. In the first place, an established taxation system and a professional workforce who administers the system are the prerequisite of revenue mobilization per se. Moreover, even if a tax system is well operated upon carefully designed and transparent tax codes, mobilizing personal income tax is still a daunting process, since it involves gathering and handling tax files submitted by a tremendous number of individual taxpayers. For example of a country with an advanced tax system like the US, its tax collection agency Internal Revenue Service (IRS) receives 200 million tax returns each year, and the management of these files costs IRS nearly $10 billion, although this expense is only a small fraction of the total administrative cost for operating the tax system (Guyton et al. 2003). Furthermore, as the assessment of tax liability relies on individual taxpayers’ honest report of their true income, it is not difficult to learn that rational taxpayers have an incentive to understate their earnings, which necessitates an additional step of the enforcement such as tax audit. Apparently, it is possible to verify the exact amount of earnings of one taxpayer (and to penalize if a fraudulent tax evasion is detected), although it incurs some cost. However, the cost for conducting the investigations on all the taxpayers is impossible for a government to afford, even when the government performs the function of revenue mobilization very effectively. Therefore, it is feasible for a taxpayer to evade his tax liability without being fined and thus to improve his utility due to the ensuing increase in the consumption. As a result, it is far from trivial for rational taxpayers to voluntarily choose a full degree of compliance.

In order to appropriately incorporate imperfect tax enforcement into the formal analysis of income taxation, let the government face a choice about how much resource

\footnote{Also, the government can levy them excise taxes at the same time. Then, we can innocuously assume that optimal commodity taxes are chosen and already reflected on the consumer prices $q$.}
it allocates for tax administration and enforcement. Notice that this is different from most of studies on tax evasion, whether theoretical or empirical, that deal only with the tax audit rate as a choice variable through which a government controls tax compliance. This conventional but myopic approach is misleading since auditing is only small part of tax administration and enforcement. What matters more is the overall effectiveness in the tax administration and enforcement of the government. For example, reform in audit policy will be meaningless if a tax system is not well capable of processing the tax files and of collecting tax payments in the first place. The number of tax officials who properly administer taxation may be insufficient. Presumably, under a poor tax collection agency, corrupt tax officials could offer taxpayers deals of tax avoidance. Or, the complexity of tax code can deter dutiful individuals from fulfilling their tax liabilities. In whichever case, change in tax audit rate itself would not have a primary effect over individuals’ contemplation of tax evasion. By contrast, under an effective tax system, attempts to evade taxes would be difficult to make. A more resourceful and transparent tax administration hampers tax evasion even before involving tax audits.\(^6\) For instance, a third-party reporting that is well operated in many developed countries deters the tax evasion remarkably well (Alm et al. 2006, and Kleven et al. 2011), in spite of very low audit rates. Arguably, individuals’ tax evasion decision clearly is affected by the effectiveness of a third-party reporting system, not merely governed by the tax audit rate itself. Therefore, all of these factors also should be taken into account in modeling tax compliance behavior.

For the sake of an efficient and tractable analysis, however, instead of introducing several decision variables corresponding to a variety of phases that take effect together on the tax administration and enforcement, let us simplify the income tax revenue mobilization into a single stage process. Namely, at the site where individuals report their earnings for the tax purpose, the government immediately detects how much they evade their tax liabilities, with success rate \(p\) that is lower than 1. It should be noted that this rate is clearly different from (and more abstract than) the tax audit rate in the previous literature; rather, \(p\) indicates the rate of overall tax enforcement. If the government succeeds in detecting a tax evasion, it charges a fine whose amount is proportional to the unpaid tax due.\(^7\) Let this penalty rate be \(\theta > 0\). More importantly, improving the enforcement rate \(p \in (0, 1)\) takes some resources with a cost function \(\delta c(p)\), where \(c(p)\) is convex and increasing in \(p\) with \(c(0) \geq 0\). Moreover, let the cost function be invertible. This means that by choosing the resources allocated for tax administration and enforcement, the government decides the level of \(p\) which can take a value between zero and one. In this context, the parameter \(\delta\) stands for the efficiency (and effectiveness) of the government in its operation of the income tax.

\(^6\)This general approach can shed some light on the puzzle of high tax compliance with low tax audit rates and the penalties in the US, which is raised by Slemrod and Yitzhaki (2002) and Andreioni et al. (1998) in the application of these conventional models.

\(^7\)This follows Yitzhaki (1974) and is closer to reality; for instance of the US, the IRS charges a penalty of 20\% of unpaid tax liabilities for the case of a substantial understatement of income tax.
system.

On the other hand, note that the legislation of the penalty rate $\theta$ is a costless measure to deter the revenue leakage. Hence, the government will set the maximum possible level of punishment, given the legal order of the society. One might hope that the government can achieve a complete tax compliance by announcing the most severe punishment. However, in practice, it is usual that, in harmony with other penalties on heavier crimes threatening social order, the harshness of the penalty on a tax evader cannot be extreme but is fairly moderate. Therefore, even at its maximum, the punishment on a detected tax evasion is less likely to be severe enough to deter all the evading attempts.

In order to accommodate incomplete tax compliance, let us allow a taxpayer to choose $e \in [0, 1]$, the portion of his true income declared to the government. Specifically, when a taxpayer picks $e$, this means that $(1-e)$ portion of his true income is chosen to be unreported. Notably, this choice is translated into a portfolio decision of how much of income is put in the form of risky assets, since the detection rate lies between zero and one. For the risk attitude, in line with the concave utility function, individuals are risk averse. In particular, let their preference exhibit constant relative risk aversion (hereafter CRRA for short).

On the timing, it is natural that after the income tax schedules $T$ as well as the resource allocation for the tax administration and enforcement that uniquely defines $\rho$ announced by the government, individuals make decisions for their income by choosing working hours and their tax evasion rates. As mentioned above, the penalty rate $\theta$ is already set at its maximum level under the legal order of the society.

In the following subsections, the exploration on the optimal income taxation reflecting imperfect tax enforcement and compliance will be displayed in the framework described right above. After all, in the maximization of the SWF, the government shall cope mainly with (i) distortion on individuals’ labor supply incentive without observing (and verifying) their earning ability; and with (ii) costly control the lucrative temptation of tax evasions of risk averse taxpayers with limited resources.

### 3.3.2 Income Taxation in the Presence of Tax Evasion

As a way of backwards induction, the government begins the search for the optimal income taxation with analyzing individuals’ responses to a given tax policy. Exploiting imperfect tax enforcement, a rational individual will consider understatement of the income by choosing $e$ less than one. This tax evasion will turn out one of the two possible outcomes. If the tax code is enforced, then the disposable income for the composite consumption good $x$ of an individual of ability $n$ is $x^D = wnl - T(wnl) - \theta\{T(wnl) - T(ewn\ell)\}$. Otherwise, he can consume as much as $x^{ND} = wnl - T(ewn\ell)$. Thus, facing the risk of being penalized for the tax evasion,
the expected utility of the individual of ability $n$ is stated as follows.

$$pu(wnl - T(wnl) - \theta \{ T(wnl) - T(e_n, wnl_n) \}, l) + (1 - p)u(wnl - T(e_n, wnl_n), l). \quad (3.1)$$

Let this be denoted as $E[u(wnl - T(e_n, wnl_n), l)]$ (which is $E[u]$ for short). Since working hours to earn income are also to be chosen as well, given a tax schedule $T$ and $p$, this individual seeks to solve

$$\max_{e, l} E[u(wnl - T(e_n, wnl_n), l)] \quad (3.2)$$

obtaining the indirect utility function $v_n = E[u(wnl_n - T(e_n, wnl_n), l_n)]$. At the same time, this maximization automatically defines $x_n^D = wnl_n - T(wnl_n) - \theta \{ T(wnl_n) - T(e_n, wnl_n) \}$ and $x_n^{ND} = wnl_n - T(e_n, wnl_n)$. To begin with characterizing $e_n$, taking a derivative of $E[u]$ with respect to $e$ yields

$$p\theta u_1^D = (1 - p)u_1^{ND} \quad (3.3)$$

where $u_1^D$ is marginal utility from the consumption under the state of being detected and penalized; and, $u_1^{ND}$ is marginal utility from the consumption under the other state. Resonating with previous studies (e.g., Allingham and Sandmo 1972; Pencavel 1979; Sandmo 1981), the optimal evasion level is chosen to equalize the expected marginal utility from being detected with that from not being detected. However, this condition holds only for $e_n \in (0, 1)$.

For $e_n$ to be an interior solution, there should exist a set of parameters and tax policy variables such that $\frac{dE[u]}{de} |_{e=1} \leq 0$ and $\frac{dE[u]}{de} |_{e=0} \geq 0$. First, $\frac{dE[u]}{de} |_{e=1} \leq 0$ if and only if $\frac{p}{1 - p} \theta \leq 1$. With $p \in (0, 1)$ and $\theta > 0$, it is fully feasible that $\frac{p}{1 - p} \theta \leq 1$.

Second, $\frac{dE[u]}{de} |_{e=0} \geq 0$ if and only if $\frac{p}{1 - p} \theta \geq \frac{u_1(wnl_n - T(0), l_n)}{u_1(wnl_n - T(0), l_n) + \theta T(wnl_n) + \theta T(0), l_n)} = \frac{u_1^{ND}}{u_1^D}$.

In addition, if $\frac{p}{1 - p} \theta \geq \frac{u_1(wnl_n - T(wnl_n) - \theta T(wnl_n) - \theta T(wnl_n) + \theta T(0), l_n)}{u_1(wnl_n - T(0), l_n) + \theta T(wnl_n) + \theta T(0), l_n)}$, then the complete evasion of tax evasion, $e_n = 0$, is optimal. We do not know the sign of $T(0)$ at the optimum yet; however, by the nature of punishment, we know that the disposable income under being penalized cannot be larger than that under not being penalized. Because $u$ is concave, this implies that $u_1^{ND} < u_1^D$ for all $n$, meaning that $\frac{u_1(wnl_n - T(0), l_n)}{u_1(wnl_n - T(0), l_n) + \theta T(wnl_n) + \theta T(0), l_n)} < 1$ for all $n$. Therefore, it is also feasible that $\frac{p}{1 - p} \theta \geq \frac{u_1(wnl_n - T(wnl_n) - \theta T(wnl_n) + \theta T(0), l_n)}{u_1(wnl_n - T(0), l_n) + \theta T(wnl_n) + \theta T(0), l_n)}$.

Moreover, in seeking the maximum level of social welfare, the government would not choose the tax policy variables that induces the complete noncompliance (which is a suboptimal outcome); it would rather adjust them to meet $1 \geq \frac{p}{1 - p} \theta \geq \frac{u_1(wnl_n - T(wnl_n) - \theta T(wnl_n) + \theta T(0), l_n)}{u_1(wnl_n - T(0), l_n) + \theta T(wnl_n) + \theta T(0), l_n)}$. As a consequence, $

\{ (p, \theta) | \frac{dE[u]}{de} |_{e=1} \leq 0 \text{ and } \frac{dE[u]}{de} |_{e=0} \geq 0 \text{ for all } n \}$ is not empty set and equals to

$$\{ (p, \theta) | \frac{u_1(wnl_n - T(0), l_n)}{u_1(wnl_n - T(wnl_n) - \theta T(wnl_n) + \theta T(0), l_n)} \leq \frac{p}{1 - p} \theta \leq 1 \text{ for all } n \}. \quad (3.4)$$
Since either full compliance ($e_n = 1$ for $\forall n$)\(^8\) or complete noncompliance is not of our interest, we narrow the present analysis down to the cases where the condition in (3.4) is met so that we can have an inner solution $e_n \in (0, 1)$ defined by (3.3) for $\forall n$.

Even though not of the main interest, it is worthwhile to briefly illustrate the remaining cases where we have corner solutions. First, if $\frac{p}{(1-p)}\theta > 1$, then we have $e_n = 1$ for $\forall n$. Since $\frac{p}{(1-p)}$ is increasing in $p \in (0,1)$ and $\theta > 0$, this implies that as $p$ approaches 1, the perfect compliance is induced from each individual, which is basically the same as a perfect tax enforcement. Second, if $\frac{p}{(1-p)}\theta < \frac{u_1(wnl_n-T(0)l_n)}{u_3(wnl_n-T(wnl_n)+\theta T(0)l_n)}$, then the individual of ability $n$ will not pay any income tax even when his earnings are strictly positive. The complete cheating is what the government wants to avoid most. As a prompt and direct remedy, the government can raise the government can raise $p$ although this incurs some administrative costs. Moreover, it also can adjust tax rate on this individual. Interestingly, the government can prevent this undesirable situation by setting higher $-T(0)$ since

$$
\frac{d}{d-T(0)}\left(\frac{u_1(wnl_n-T(0)l_n)}{u_3(wnl_n-T(wnl_n)+\theta T(0)l_n)}\right) = \frac{u_1^{ND}u_1^D}{(u_1^T)^2} < 0.
$$

Intuitively, the greater $-T(0)$ leads to the larger variance between the disposable incomes under the two states. That is, an increase in $-T(0)$ makes the complete cheating riskier and thus less attractive to risk-averse individuals. Therefore, exploiting individuals’ risk aversion, the government can effectively deter the full tax evasion simply by augmenting $-T(0)$.

In choosing working hours, on the other hand, the marginal value of one unit of working is to be equal to the marginal disutility of it in either state. That is, if the tax code is enforced, then $u_1^D wn\{1 - (1 + \theta[1 - e_n])T'\} = -u_2$; otherwise, $u_1^{ND} wn(1 - e_n)T' = -u_2$. Taken together, the optimal condition of $l_n$ is derived as

$$
p u_1^D wn\{1 - (1 + \theta[1 - e_n])T'\} + (1 - p)u_1^{ND} wn(1 - e_n)T' + u_2 = 0 \quad (3.5)
$$

which also can be obtained from taking a derivative of $E[u]$ with respect to $l$. This in turn defines the effective labor input supply $nl_n \equiv L_n$. Notably, the marginal disutility from working is equalized with the expected marginal utility of income. In other words, this is a simple extension of standard labor supply decision which incorporates the income tax schedule as well as the uncertainty of being penalized for tax evasion, as Pencavel (1979) pointed out.

Extending the above analysis on the responses to a given tax policy of an individual to different individuals, we can prove that difference in ability is the source of inequality. That is, those with higher ability always get greater or at least the same utility as one with lower ability does.

Proposition 3.1. For an arbitrary income tax schedule $T$, $v_n$ is strictly increasing in $n$ if $l_n > 0$ and is increasing in $n$ if $l_n \geq 0$.

\(^8\)This case is already studied by Mirrlees (1971).
Proof. For an arbitrary income tax schedule $T$, pick any $n > n' \geq 0$ and suppose that $l_n > 0$. Since $u_2 < 0$, \[ E[u(wnl_n - T(e_n wnl_n), l_n)] < E[u(wnl_n - T(e_n wnl_{n'}), l_{n'})] \] This implies that $v_n > v_{n'}$ because \[ E[u(wnl_n - T(e_n wnl_n), l_n)] = v_n \] and \[ v_{n'} = E[u(wnl_{n'} - T(e_{n'} wnl_{n'}), l_{n'})] \] By the same token, if $l_n \geq 0$, then \[ E[u(wnl_{n'} - T(e_{n'} wnl_{n'}), l_{n'})] \leq E[u(wnl_n - T(e_n wnl_n), l_n)] \] This implies that $v_n \geq v_{n'}$. \hfill \blacksquare

Note that Proposition 3.1 applies even when there is virtually no taxation by setting a constant tax function that takes zero value over all the domain. However, Proposition 3.1 implies that, due to individuals’ responses, individuals who enjoyed higher utility prior to income taxation end up with higher or at least the same utility as ones who got lower utility, although the gap may be decreased by the taxation.

In addition to consumers described above, there are to be producers under a perfect competition. The input that they employ for the production is effective labor $l$, which is aggregated over the population in the society as $\int_0^\pi L_n h(n) dn$. Moreover, the behavior of each producer who maximizes their own profit is summarized as $w = F'$ (the FOC of their profit maximization). Therefore, overall production of this economy is stated as

$$F(\int_0^\pi L_n h(n) dn) = \int_0^\pi wL_n h(n) dn.$$ (3.6)

Eventually in the market, the demand for the composite consumption good cannot exceed the supply of the producers. Hence, the ex ante feasibility constraint in this economy is

$$\int_0^\pi \{px_n^D + (1-p)x_n^{ND}\} h(n) dn \leq F(\int_0^\pi L_n h(n) dn)$$ (3.7)

which has the market clearing condition met when the constraint is binding with equality. After all, the government seeks to find an income tax schedule $T(\cdot)$ and a tax enforcement rate $p$ that solves

$$\max_{T(\cdot) \& p} \int_0^\pi \frac{dG}{dv_n} v_n h(n) dn \text{ s.t. } \int_0^\pi \{px_n^D + (1-p)x_n^{ND}\} h(n) dn \leq \int_0^\pi wL_n h(n) dn$$ (3.8)

Notice that the government’s budget constraint is not added since it is redundant according to the Walras’ law.

Having a concrete and explicit statement of the problem that the government pursues to solve, let us start with the optimization with finding an optimal income tax function $T(\cdot)$ which is apparently more complicated than finding a single value of an optimal $p$. As shown in (3.5), $T$ affects individuals’ decisions of labor supply from which the total output is produced. Thus, an optimum income tax function $T$ is to
stipulate how $T$ is related with the effective labor in the first place. Regarding this critical concern, we show that an optimal income taxation schedule does not reward more of effective labor with less expected utility since that will seriously undermine the economy.

**Proposition 3.2.** If $T$ is an optimal income tax function, then $E[u(wL_n - T(e_n wL_n), \frac{L_n}{n})]$ is increasing in $L_n$ whenever $L_n > 0$.

**Proof.** By way of contradiction, suppose that there exists an optimal income taxation schedule $T$ such that $\frac{dE[u(wL_n - T(e_n wL_n), \frac{L_n}{n})]}{dL_n} < 0$ and $L_n > 0$ for some $n$. This means that $pu_1^D w\{1 - (1 + \theta - e_n \theta)T\} + (1 - p)u_1^{ND} w(1 - e_n T') + \frac{u_2}{n} < 0$. Most importantly, this implies that individuals of ability $n$ is lead to choose zero working hour $l_n = 0$ and thus $L_n = 0$ because marginal expected utility is less than marginal disutility, $pu_1^D w\{1 - (1 + \theta - e_n \theta)T\} + (1 - p)u_1^{ND} w(1 - e_n T') < -u_2$ from (3.5). A contradiction to $L_n > 0$. In addition, such a $T$ cannot be optimal: the government can always improve the social welfare upon this tax schedule by changing it to be $\frac{dE[u(wL_n - T(e_n wL_n), \frac{L_n}{n})]}{dL_n} \geq 0$ to induce any positive amount of effective labor supply from these individuals, which increases both tax revenue and total output in the economy. $lacksquare$

In addition, even if $w > 0$ and $e_n \in (0, 1)$ for $\forall n$, the domain of $T$ still contains 0 especially for the case of $n = 0$ since this yields $L_0 = 0$ with $\forall l \in (0, 1)$ any feasible working hours spent. In other words, there are individuals in the society who inevitably earn no wage income. Since they are also included as part of this society, they need to be handled as well in maximizing the welfare of the entire society; that is, the government would seek an optimal level of $T(0)$. It turns out that, at an optimum of (3.8), the government supports these individuals of no earning ability, instead of leaving them to starve. The proof for this is presented below.

**Proposition 3.3.** If the government puts some finite and positive weights to each individual in the society (i.e., $\infty > \frac{dG}{dv_n} > 0$ for $\forall n$), then $-T(0) > 0$ will be chosen at an optimum.

**Proof.** First of all, we already have that $\infty > \frac{dG}{dv_n} > 0$ for $\forall n$. Moreover, since $x \in \mathbb{R}_+$, when zero effective labor is supplied, we know that $0 - T(0) \geq 0$. By way of contradiction, suppose that there is an optimum income tax schedule such that $0 - T(0) > 0$; that is, let $T(0) = 0$. Every individual except for those of no earning ability will earn a positive income with positive hours of working to avoid $-\infty$ utility since $\lim_{x \to 0} u(x, l) = -\infty$. On the other hand, the individuals of 0 ability end up with earning no income even after spending whichever positive hours for working. Furthermore, their utility is $pu(0 - T(0) - \theta(T(0) - T(0)), l_0) + (1 - p)u(0 - T(0), l_0) =$
$u(0, l_0)$ since $0 - T(0) = 0$. This means that $u(0, l_0) = -\infty$ for any feasible $l_0$. This implies that with any possible allocations $(x, l)$ chosen by each individual under this income tax schedules laid by the government, the social welfare is $\int_0^\pi \frac{dg}{dv} v_n h(n) dn = -\infty$ as long as $T(0) = 0$ since adding any finite and positive value to $-\infty$ still ends up with $-\infty$. However, this leads to a contradiction to the assumption that we are at an optimum since the government can improve upon this income tax schedule. Obviously, the government can always strictly increase the SWF by setting $-T(0)$ slightly over 0.

**Proposition 3.3** is consistent with various social welfare programs implemented in countries around the world. First of all, $-T(0)$ can be interpreted literally such as financial aid to the disabled. However, it allows more general translations, which include welfare programs for poor families, elderly, veterans, and the like. Supporting health care for these groups can be one example, too. Furthermore, it could include the finance for public education that benefits children who do not have their own income source and are not well financed by their parents.

Nevertheless, this observation should not be misleading. In particular, one might as well be tempted to stretch a point to interpret $-T(0)$ as one measure of how much the government cares about the worst-off of the society. Probably, a greater $\frac{dg}{dv_0}$ could lead to choose a larger $-T(0)$ through $v_0 = E[u(-T(0), 0)]$, especially for a class of SWF such that $\int_0^\pi \frac{dg}{dv} dn = 1$. However, it is not that simple. First, there are other forms of SWF that does not always exhibit a positive relationship between $\frac{dg}{dv_0}$ and $-T(0)$. More fundamentally, a generous $-T(0)$ also affects the extensive margin of labor supply from individuals whose ability is near but not exactly 0 (which is proven right below as **Proposition 3.4**) and have them enjoy $E[u(-T(0), 0)]$ as well. In this light, the government could raise $-T(0)$ because of the individuals of near zero but positive ability, not because of those of no earning ability. Moreover, given $-T(0) > 0$, we have individuals whose ability is positive and near zero opt for 0 hour of working.

**Proposition 3.4.** With $-T(0) > 0$ given, $\exists n_0 > 0$ such that $L_n = 0$ for $\forall n \leq n_0$ and that $L_n > 0$ for $\forall n > n_0$.

**Proof.** First of all, consider a case where $-T(0) = 0$, we know that everybody other than zero ability individuals will work positive amount of time, as mentioned in the proof of **Proposition 3.3**. That is, if $-T(0) = 0$, $l_n > 0$ whenever $n > 0$. From **Proposition 3.1**, this implies that $v_n$ is strictly increasing in $n$ for $\forall n > 0$ when $-T(0) = 0$. Moreover, $l_n$ is continuous in $n$ since $u$ is continuously differentiable and $l_n > 0$ is defined from $pu^D wn(1 - (1 + \theta - e_n \theta)T') + (1 - p)u^{ND} wn(1 - e_n T') + u_2 = 0$. This implies that both $wnl_n - T(e_n wnl_n)$ and $v_n = E[u(wnl_n - T(e_n wnl_n), l_n)]$ are also continuous in $n$. Regarding this case of $-T(0) = 0$ as a comparator, now
$-T(0) > 0$ is introduced by the government. In comparison, due to these continuity, around 0 ability, we can always find $\pi > n' > 0$ such that $wn' l_{n'} - T(e'_n wnl_{n'}) = -T(0)$. Since $u_2 < 0$, $v_{n'} = E[u(wn' l_{n'} - T(e'_n wnl_{n'}), l_{n'})] = E[u(-T(0), l_{n'})] \leq E[u(-T(0), 0)]$. Moreover, $E[u(-T(0), 0)] = v_0 \leq v_\pi$, from Proposition 3.1. In sum, $v_{n'} < E[u(-T(0), 0)] \leq v_\pi$. Most of all, by Intermediate value theorem, there exists $n_0 \in [n', \pi]$ such that $v_{n_0} = E[u(wn_0 l_{n_0} - T(e_{n_0} wnl_{n_0}), l_{n_0})] = E[u(-T(0), 0)]$ since $v_n$ is continuous and increasing in $n$.

Next, now we need to show that any individual of ability lower than $n_0$ will choose zero hour of working. Consider any arbitrary $n''$ such that $n_0 > n'' > 0$. By way of contradiction, suppose that $l_{n''} > 0$ is optimal to an individual of ability $n''$, resulting in $L_{n''} > 0$. However, $v_{n''} = E[u(wn'' l_{n''} - T(e_{n''} wnl_{n''}), l_{n''})] < v_{n_0} = E[u(wn_0 l_{n_0} - T(e_{n_0} wnl_{n_0}), l_{n_0})] = E[u(-T(0), 0)]$, due to Proposition 3.1. This leads to a contradiction because this individual of ability $n''$ can be better off by choosing the corner solution $l_{n''} = 0$ (and thus $L_{n''} = 0$) to enjoy higher utility $E[u(-T(0), 0)]$ rather than choosing the interior solution $l_{n''} > 0$. Since $n''$ is picked arbitrarily, this holds for $\forall n \leq n_0$. That is, zero working hour will be chosen by all the individuals of ability $\forall n \leq n_0$. This implies that $L_n = 0$ for $\forall n \leq n_0$.

Lastly, we want to show that any individual of ability greater than $n_0$ will choose positive amount of labor supply. Based on Proposition 3.1, $v_n > v_{n_0}$ for $\forall n > n_0$. That is, for $\forall n > n_0$, $u(-T(0), 0) < E[u(wnnl_n - T(e_n wnl_n), l_n)]$, where $l_n > 0$ is defined by (3.5). If any individual of ability higher than $n_0$ choose zero working hour, then his utility will decrease to $u(-T(0), 0)$. In other words, this individual would not choose zero labor supply to get $-T(0)$ since he will be better off if he opts for strictly positive hours of working. ■

As a corollary to Proposition 3.4, it is immediate that all the individuals of ability less than or equal to $n_0$ obtain the same utility $E[u(-T(0), 0)]$. Interestingly, observe that

$$E[u(-T(0), 0)] = u(-T(0), 0)$$

since $E[u(-T(0), 0)] = pu(0 - T(0) - \theta\{T(0) - T(e_n 0)\}, 0) + (1 - p)u(0 - T(e_n 0), 0)$ and $e_n 0 = 0$ for any feasible value of $e_n$. It is meaningless to find $e_n$ for $\forall n \leq n_0$ since these individuals do not earn income and thus are not taxpayers. Put another way, at the extensive margin, labor supply decision dominates tax evasion decision since choosing not to work means opting for being non-taxpayer. Therefore, notice that $-T(0)$ plays a role of dividing the population into taxpayer vs. non-taxpayer since $-T(0)$ defines $n_0$ as demonstrated in the proof of Proposition 3.4. Therefore, with $-T(0) > 0$ given, finding an optimal tax function deals only with the individuals whose ability lies within $(n_0, \pi)$ rather than with the entire population.

Notice that Proposition 3.1, 3.2, 3.3, and 3.4 can be driven regardless of the observability and verifiability of each individual’s ability, since the government can verify the effective labor supply of individuals owing to the wage rate $w$ known.
However, recall that the true earning ability of each individual is private information. The government cannot observe and verify $n$ at any cost, while it can verify $e_n$ at some administrative cost. Consequently, it is a very daunting task for the government to find an optimal income tax function $T$ with being ignorant of $n$ (and $l_n$ as a result). Firstly, finding a function — instead of values of variables — cannot be easily accomplished with standard tools such as ordinary calculus. More importantly, even though an optimal tax schedule should shrewdly accommodate individuals’ responses, the government cannot identify an individual in its face due to unobservability (and unverifiability) of each individual’s ability. In this sense, the government does not know where it stands in the domain of an income tax function when designing the function. To pave the way for addressing these issues, it would be helpful to illustrate the rationale of income taxation and to describe hypothetical benchmark cases where the government has a clairvoyant power over each individual’s innate ability.

### 3.3.3 Benchmark Cases

To begin, why does the government try to levy any tax in the first place? Suppose that the government does not collect any tax by setting a constant tax function that takes zero value over all the domain. In this fiscal anarchy, there is no way to finance individuals of zero ability who are not able to earn any income for living; and, their utility (which is $-\infty$) will drive the value of SWF to a bottomless abyss. This necessitates transfer of resources to them whose legitimate form takes taxation (as shown in the proof of Proposition 3.3). As a matter of fact, however, the critical factor is the diminishing marginal utility and weakly concave SWF, not the negative infinite utility from the total deprivation of consumption. In other words, levying a tax is still socially desirable even if we modify the utility from zero consumption into taking some finite value (instead of $-\infty$) as long as the government does not have a regressive taste on inequality of the society. To demonstrate this, consider a society with $\frac{dG}{dn} = 1$ for all $n$ and $u(0, l) = -10000$ for all $l \in [0, 1)$. This means that the SWF of this society is a simple sum of every individual’s utility giving everyone equal weight. In this society, transferring one unit of consumption good from an individual of high ability to another individual of low ability still can improve the social welfare. To explain this, realize that the utility level of the high ability individual is greater than that of the low ability one, based on Proposition 3.1. This implies that the marginal utility from one unit of consumption good is higher to the low ability individual than to the high ability one, due to the concavity of utility function (i.e., diminishing marginal utility). Therefore, since the same social weight is given to both, it is clear that the marginal social gain from giving one unit of consumption good to the low ability individual is greater than the marginal social loss of taking it from the high ability individual. In fact, the marginal social net gain from this reallocation is positive as long as larger weight is not put on the higher utility (i.e., $\frac{d}{dn} \left( \frac{dG}{dn} \right) \leq 0$ which means concave SWF) which accrues to higher ability individuals (Proposition
3.1). In the end, since this allocation of redistribution may be implemented by taxes, the value of the SWF can be increased more with taxation than without it.

As well known, however, this social benefit of redistribution through taxation is usually obtained at some cost on the efficiency. To articulate this trade-off, we need a reference point where a taxation incurs no loss of efficiency. Consider a hypothetical case where the government can observe the true ability of each individual and make lump sum transfers between them with a full degree of enforcement. According to the Second fundamental welfare theorem, we know that redistribution through interpersonal lump sum transfers entails no efficiency cost. Hence, a brief description of this first-best world is worthwhile, because it can be used to effectively reveal efficiency loss that occurs in a second-best world.

First of all, with homogeneous preference and no endowment to each individual, the earning ability works as an identifier for them. Moreover, inequality of the society stems only from the difference in individual’s ability; that is, difference in earning ability engenders disparity in utilities of individuals (Proposition 3.1). Therefore, the transfer should be based on ability types. Let \( T_n \) be a lump sum transfer toward individuals of ability \( n \). Note that \( e_n = 1 \) for all \( n \) since we temporarily assume a perfect tax enforcement. Thus, an individual of ability \( n \) solves

\[
\max_l u(wn - T_n, l)
\]

whose FOC is \( u_1 wn + u_2 = 0 \). From this maximization, the indirect utility function \( v_n = u(wnl_n^{FB} - T_n, l_n^{FB}) \) is obtained. On the other hand, due to the perfect tax enforcement, there is no evasion involving uncertainty in disposable income for the consumption; hence, the resource constraint that the government faces now is

\[
\int_0^\pi x_nh(n)dn \leq \int_0^\pi wnl_n^{FB}h(n)dn.
\]

This can be restated as \( 0 \leq \int_0^\pi T_nh(n)dn \) since \( x_n = wnl_n^{FB} - T_n \). Therefore, the Lagrangian expression of the government problem of maximizing the SWF that corresponds to (3.8) above, is stated as

\[
\int_0^\pi \left\{ \frac{dG}{dv_n}v_n + \lambda T_n \right\}h(n)dn
\]

where \( \lambda \) is marginal social value of additional resources that can be used for the society. At the same time, Proposition 3.3 and 3.4 also applies here. Hence, for a given \( -T_0 > 0 \), \( n_0 \) is determined from \( v_{n_0} = u(wn_0l_{n_0} - T_{n_0}, l_{n_0}) = u(-T_0, 0) \). Most importantly, since the government observes (and verify) the ability \( n \), it can have \( T_n \) unmistakably take effect on individuals of ability \( n \) without affecting individuals of different abilities. As a result, instead of carrying the intergal over the entire population, the maximization of (3.11) can be decomposed into a simple piece of procedure to find \( T_n \) that maximizes (3.11) with treating the ability \( n \) fixed. This can be achieved by taking a derivative of (3.11) with respect to the transfer with \( n \) given. Insofar as the SWF is concave (or \( \frac{d}{dv_n}(\frac{dG}{dv_n}) \leq 0 \)), the set of such \( T_n \) obtained from this for each \( n \) constitutes an optimal transfer schedule that maximizes (3.11).
In sum, the observability of each individual’s inner ability allows us to achieve the optimization relying on a pointwise maximization. Differentiating (3.11) with respect to $T_n$ yields

$$\left\{ \frac{dG}{dv_n} \right\} + \lambda \{h(n \mid n > n_0) = 0 \}$$

for $\forall n > n_0$. Since $\frac{dv_n}{dT_n} = \frac{dw_n^{FB} - T_n}{dT_n} = -u_1$, this leads to

$$\frac{dG}{dv_n} u_1 (wn^{FB} - T_n) = \lambda.$$  \hspace{1cm} (3.13)

Put this another way, optimal lump sum transfer $T_n$ is set to equate the social value on the marginal loss from the decrease in consumption of an individual of ability $n$ due to $T_n$ (the left hand side) with the marginal social gain of the increase in tax revenue from the transfer $T_n$ (the right hand side). Furthermore, we can utilize Implicit function theorem as

$$\frac{dT_n}{dn} = \frac{dG}{dv_n} u_1 (wn^{FB} - T_n, l^{FB}) = 0.$$  \hspace{1cm} (3.14)

whose FOCs are $pu^D w_n + (1 - p)u^{ND} w_n + u_2 = 0$ for $l$ and $p\theta u^D_1 = (1 - p)u^{ND}_1$ for $e$ (which is identical to (3.3) above) entailing the indirect utility function $v_n = E[u(wn^{FB} e_n T_n, l^{FB})]$. Similarly, $-T_0 > 0$ will define $n_0$ by $v_{n_0} = E[u(wn^{FB} - e_n T_n, l_{n_0})] = u(-T_0, 0)$. Moreover, since the government still can impose interpersonal lump sum transfers with a clairvoyant power that can observe and verify ability of each individual, we can follow the same path for optimization taken right above which is pointwise maximization of the SWF. After all, optimality condition for $T_n$ is obtained as

$$\left\{ \frac{dG}{dv_n} \right\} + \lambda \{p(1 + \theta[1 - e_n]) + (1 - p)e_n\} \{h(n \mid n > n_0) = 0 \}$$

for $\forall n > n_0$. By putting $E[\lambda] = \lambda \{p(1 + \theta[1 - e_n]) + (1 - p)e_n\}$ find the similarity with (3.12) the corresponding one in a pure first-best world of a perfect tax enforcement. For one unit of $T_n$ announced, collected tax revenue may be either $(1 + \theta - \theta e_n)$ with probability $p$ or $e_n$ with probability $(1 - p)$. This is in fact no coincidence. Since the decision on tax evasion $e_n$ is essentially a choice of purchasing a risky asset, collected
tax revenue and marginal social value on it inevitably involves uncertainty. This is confirmed when we restate (3.15) as follows

$$\frac{dG}{dv_n} [p(1 + \theta[1 - e_n])u^D_1 + (1 - p)e_n u^ND_1] = E[\lambda] \tag{3.16}$$

since $$\frac{dv_n}{dT_n} = \frac{dE[u(wnTFBe_n - e_nT_n)_{\text{FBe}}]}{dT_n} = -[pu^D_1 (1 + \theta[1 - e_n]) + (1 - p)u^ND_1] e_n]$$. Optimal lump sum transfer $$T_n$$ under imperfect tax enforcement is set to equate the social value on the marginal loss from the decrease in expected consumption of an individual of ability $$n$$ due to $$T_n$$ (the left hand side) with the marginal social gain of expected increase in tax revenue from the transfer $$T_n$$ (the right hand side). Not surprisingly, the progressivity of the optimal lump sum transfers is retained since $$\frac{dT_n}{dn} = \frac{dG}{dv_n} [p(1 + \theta[1 - e_n])u^D_1 + (1 - p)e_n u^ND_1].$$

Eventually, the expected collected revenue is $$\int_{n_0}^{n} [p(1 + \theta[1 - e_n]) + (1 - p)e_n] T_n h(n \mid n > n_0) dn$$; and, this will be used to pay for the incurred enforcement cost $$\delta c(p)$$ and to finance the desired expenditure $$H(n_0)(-T_0)$$.

Most importantly, since, in income taxation, a source of efficiency loss lies at distortion on the labor supply incentive, we need to illustrate conditions of first-best choices of working hours. Recall that $$l^{FB}_n$$ is defined from

$$u_1 wn = -u_2 \tag{3.17}$$

and that $$l^{FBe}_n$$ is defined from

$$pu^D_1 wn + (1 - p)u^ND_1 wn = -u_2. \tag{3.18}$$

Notice that because these FOCs are identical to the FOCs that could have been derived under no taxation, the lump sum transfer $$T_n$$ entails no efficiency costs. Therefore, any deviation from these FOC points to occurrence of efficiency loss, which we will elaborate later.

With the two versions of first-best labor supply described in (3.17) and (3.18), it would be a natural attempt to draw a comparison between $$l^{FB}_n$$ and $$l^{FBe}_n$$. In short, it is not promptly evident whether $$l^{FB}_n$$ under a perfect tax enforcement is larger than $$l^{FBe}_n$$ under possible tax evasions. Since $$u^D_1 > u_1 > u^ND_1$$, it not immediate which first-best world leads to more working with more valued on it. If $$pu^D_1 wn + (1 - p)u^ND_1 wn > u_1 wn$$, then individuals will work harder under imperfect tax compliance, since $$-u_2 > 0$$. This resonates with the result of Weiss (1976), Cowell (1981), and Stigliz (1982) that uncertainty in tax enforcement can be desirable since individuals supplies more labor. However, they assume that the government can raise $$p$$ as much as it wants without incurred costs. In contrast, in the present study, increasing $$p$$ is costly; and, the government chooses $$p$$ to maximize the entire social welfare taking this cost into account as well.
To effectively show the optimality condition for $p$, which is a society-level variable, we maximize the SWF employing the government’s budget constraint $\int_{n_0}^{\pi} \{p(1 + \theta)(1 - e_n)\} T_n h(n \mid n > n_0) dn - \delta c(p) \geq H(n_0)(-T_0)$, instead of an ex ante feasibility constraint like (3.7), and take a derivative of the constrained maximand with respect to $p$. We get

$$-\int_{n_0}^{\pi} \frac{dG}{dp} \frac{dv_n}{dp} h(n \mid n > n_0) dn = \lambda \{ \int_{n_0}^{\pi} (1 + \theta)(1 - e_n) T_n h(n \mid n > n_0) dn - \delta c'(p) \}. \tag{3.19}$$

First of all, the right-hand side of the above optimality condition represents the marginal net social value on raising $p$, because $\int_{n_0}^{\pi} [(1 + \theta)(1 - e_n) T_n - c'(p)] h(n \mid n > n_0) dn$ is an increase in expected tax revenue ensuing from a small increase in $p$ less an increment in cost for the small increase in $p$ which is the second term in the right-hand side $\delta c'(p)$. In order to better understand the meaning of the left-hand side, (3.19) can be restated as

$$\int_{n_0}^{\pi} \frac{dG}{dv_n} (u_n^{ND} - u_n^D) h(n \mid n > n_0) dn = \lambda \{ \int_{n_0}^{\pi} (1 + \theta)(1 - e_n) T_n h(n \mid n > n_0) dn - \delta c'(p) \} \tag{3.20}$$

since $\frac{dv_n}{dp} = u(wnl_n^{FB}e - T_n - \theta \{T_n - e_n T_n\}, l_n^{FB}) - u(wnl_n^{FB}e - e_n T_n, l_n^{FB}) = u_n^D - u_n^{ND}$. Noticeably, $u_n^{ND} - u_n^D$ is a widened gap between the payoffs under being detected vs. being not-detected accruing to a taxpayer (tax evader) of ability $n$ triggered by the increment in $p$. Presumably, this reflects a loss to individuals because the increased variance of the payoffs from the two different states translates into increased risk that individuals want to avoid. Exposure to greater risk adversely affects individuals’ utility, and this enters into the government’s consideration when choosing $p$ with social weights put on the individuals. After all, optimal $p$ is set to balance the social value on the marginal loss to individuals from raised risk of being penalized with the marginal net social value on increased expected revenue. Even with this specification on the optimal $p$, however, it is still not promptly clear whether $l_n^{FB} > l_n^{Fe}$ or not without exact form of functions of $G$, $u$ and $c$. Nonetheless, the first-best working hour choice $l_n^{FB}$ from (3.18) that is free of efficiency loss can provide a benchmark for the following analysis that deals with a second best world where taxation entails efficiency loss.

### 3.3.4 Simplification of the Optimization

In order to find the solution whose outcome is closest to the benchmark case, it is worthwhile to examine the reason why the first-best outcome is not attainable. In short, what fundamentally keeps the government from achieving the first-best outcome is its inability to observe and verify intrinsic productivity of each taxpayer. Apparently, if the government can observe and verify ability of each individual, it
can have income tax mimic a pre-assessed amount of tax $T_n$, which is defined by (3.15), and be imposed accurately on an individual of ability $n$. Since $pu^D wn + (1 - p)u^D wn = -u_2$ is increasing in $n$ and $u_22 < 0$, a higher ability taxpayer (with higher $n$) works more and earns greater income $wn^{FBe}_n$. Moreover, the government can verify $e_n$, albeit costly. In this light, the government could have achieved the first-best level of social welfare by announcing the income tax schedule that levies $T_n$ on an income amounting to $wn^{I_{FBe}}_n$, preserving the first-best progressivity. Nonetheless, this is not implementable unless the tax authority can observe and verify the true ability. Facing the income tax rate that increases with one’s ability, individuals of ability $n$ will pretend to be of lower ability $n'$ (some $n'$ such that $n' < n$) by reducing their working hours to $L_{n'}$. In doing so, the individuals of ability $n$ will be clearly better off since disposable income for consumption from one hour of working increases; that is, they can take less pain of working to afford the same consumption. However, the government cannot prevent this response because it cannot tell an individual who is really of ability $n'$ from the one who reduces working hours to feign being of ability $n'$. As a consequence, this unavoidable behavioral response keeps the income tax rates pre-assessed for individuals of ability $n$ (imitating $T_n$) from taking effect on the intended group (the individuals of ability $n$). In other words, due to the asymmetric information on the inner characteristic of taxpayers, the taxation loses its base on the source of inequality — ability — and fails to obtain the first-best outcome.

Upon this realization, the government should find a way to regain the lost link between the tax rate and taxpayer’s ability. Only then, it can properly accommodate the responses of taxpayers in maximizing the SWF (and thus minimizing efficiency losses). In other words, we can narrow down the search of an optimal tax function $T$ by requiring the property\footnote{The reasoning unfolded here could have been succinctly compressed to one word ‘revelation principle’ which might be less informative to some readers. In fact, we are dealing with a nonlinear pricing problem.} that an optimum income tax rate intended for individuals of a specific ability $n$ should be correctly delivered (applied) to the individuals of the ability $n$. Equivalently, this means that an optimal income tax schedule should make every (rational) taxpayer not have a lucrative gain from falsely representing himself to be of any other ability. As mentioned above, the way individuals of ability $n$ pretend to be any other ability $n'$ is by altering their working hour. In detail, instead of $l_n$, the individuals of ability $n$ can choose $l_{n'}$ such that $n'l_{n'} = L_{n'} = n'l_n$ to feign being of ability $n'$. The government cannot detect this, whereas it can observe $wL_{n'}$ (or $wL_n$) after verifying $e_{n'}$ (or $e_n$) at some administrative cost. Therefore, an optimum income tax schedule $T$ needs to induce the individuals of ability $n$ to voluntarily choose $l_n$. Put in another way, an optimum income tax schedule $T$ should be designed to incentivize individuals to virtually reveal their true ability. This property can be met if $T$ satisfies that, for $\forall n > n_0$, $v_n = E[u(wL_n - T(e_n wL_n), \frac{L_n}{n})] \geq E[u(wL_{n'} - T(e_{n'} wL_{n'}), \frac{L_{n'}}{n'})]$, where $\frac{L_{n'}}{n} = l_{n'}$. In fact, this condition can be put in another equivalent expression
to clarify the underlying rationale that optimal income taxation is to be compatible with individuals’ incentives. That is, the utility of an individual with ability \( n \) can be restated as \( V(n' \mid n) = E[u(wL_{n'} - T(e_nwL_{n'}), \frac{L_{n'}}{n})] \). Consider \( V(n' \mid n) \) as utility accruing to an individual of ability \( n \) when he pretends to be of ability \( n' \). In this light, the necessary condition for regaining the missing link between the tax rate and its target ability can be written as \( V(n \mid n) \geq V(n' \mid n) \) and named incentive compatibility constraint (hereafter, IC constraint).

Furthermore, from Proposition 3.3, we know that the government will select some positive \(-T(0) > 0\) according to given distributional preference embedded in the SWF. As a consequence, based on Proposition 3.4, there shall be a group of individuals with ability lower than \( n_0 \) who do not work and get constant utility \( E[u(-T(0), 0)] = u(-T(0), 0) = v_{n_0} \). Notice that these individuals do not pay income tax. This implies that individuals of ability \( n > n_0 \) also have an option of no working to stop being a taxpayer. These individuals will certainly use this option if an income tax assigned to them makes their utility less than \( v_{n_0} \). Hence, to make all the individuals of ability higher than \( n_0 \) remain being taxpayers, an optimum income tax schedule should be designed such that they get utility greater than \( E[u(-T(0), 0)] \). Adopting a term from contract theory literature, this requirement can be named participation constraint or individual-rationality constraint (henceforth, IR constraint, for short). In sum, the properties that an optimum income tax schedule \( T \) must have are concisely stated as follows:

For \( \forall n > n_0 \),

\[ \begin{align*}
(\text{IC}) \quad & V(n \mid n) \geq V(n' \mid n) \text{ for } \forall n' \neq n \\
(\text{IR}) \quad & V(n \mid n) \geq V(n_0 \mid n_0) \\
\text{where } & v_{n_0} = V(n_0 \mid n_0) = E[u(-T(0), 0)].
\end{align*} \]

With these constraints introduced, the government can much simplify the task of finding an optimal income tax function. First of all, the government now can locate itself in the domain of an income tax function \( T \) as if it knows ability \( n \). Moreover, once these two sets of requirements are met, each tax rate in the schedule \( T \) takes effect correctly on the individuals of the ability whom the rate targets at, not others. Consequently, we can find each tax rate for individuals of ability \( n \) in the tax schedule \( T \), the entire set of which eventually maximizes the SWF, with treating the ability \( n \) fixed. In other words, the government can fulfill the task with simple pointwise maximizations. In addition, with ex ante budget constraint of each individual binding,\(^{10}\) once the values of \( l_n \) and \( E[x_n] \equiv px_n^D + (1 - p)x_n^{ND} \) for each \( n \) is obtained in the pointwise maximization, the tax rate for individuals of ability \( n \) is automatically determined. Because defining \( l_n \) and \( e_n \) is equivalent to defining \( l_n \) and the expected consumption \( px_n^D + (1 - p)x_n^{ND} \) since \( e_n \) is a decision on a gamble on the disposable income for the consumption. After all, therefore, optimality condition

\(^{10}\)That is, \( px_n^D + (1 - p)x_n^{ND} = p[wLn_n - T(e_nwLn_n)] + (1 - p)[wLn_n - T(wLn_n) - \theta(T(wLn_n) - T(e_nwLn_n))] \).
for such $l_n$ and $E[x_n]$ for $\forall n > n_0$ can fully characterize the optimum income tax schedule $T$.

To complete this simplification, however, there remains one issue to be addressed: For each ability, the IC and IR constraint actually involve infinite number of inequalities. This concern necessitates few more steps of reducing these constraints to more tractable ones.

**Lemma 3.1.** The IC constraints for $\forall n > n_0$ are met if and only if (i) $L_n$ is increasing in $n$; and, (ii) $V_1(n | n) = 0$ for $\forall n > n_0$ where $V_1(n | n)$ is a partial derivative with respect to the first argument of $V(n | n)$.

**Proof.** [step 1] ($\Longrightarrow$) Suppose that the IC constraints are met for $\forall n > n_0$. Namely, $V(n | n)$ is increasing in $n$, for any $n > n_0$. Put another way, this means that $n = \arg\max_t V(t | n)$ for $\forall n > n_0$. First of all, this immediately implies that $V_1(n | n) = 0$ because it is the necessary condition for $n$ to be the maximizer. Next, by way of contradiction, suppose that $L_n = L(n)$ is decreasing in $n$; that is, $L' = \frac{dL_n}{dn} < 0$. Pick any arbitrary $n > n'$ such that $L_n = L(n) < L_{n'} = L(n')$ is met if and only if (i) 

$$\frac{dL_n}{dn} \geq 0.$$ 

for $\forall t \in (n', n)$. A contradiction. This proves that $L' = \frac{dL_n}{dn} \geq 0$.

[step 2] ($\Longleftarrow$) Now assume that (i) $L_n$ is increasing in $n$ (i.e., $L' = \frac{dL_n}{dn} \geq 0$) and (ii) $V_1(n | n) = 0$, for $\forall n > n_0$. By way of contradiction, suppose that there exists $n > n' > n_0$ such that the IC constraints are not satisfied. Then, $V(n | n) \leq V(n' | n)$, which implies that $\int_{n'}^n V_1(t | n) dt < 0$. Furthermore, since $V_1(n | n) = 0$, $\int_{n'}^n V_1(t | n) dt = \int_{n'}^n V_1(t | n) dt = \int_{n'}^n u_2 L'[\frac{1}{n} - \frac{1}{n'}] dt < 0$ because $L' < 0, u_2 < 0$, and $\frac{1}{n} - \frac{1}{n'} < 0$ for $\forall t \in (n', n)$. Therefore, this proves that the IC constraints for $\forall n > n_0$ are met if (i) and (ii) are satisfied.

To briefly illustrate the intuition underlying Lemma 3.1, regard $V_1(n | n) = 0$ as a local IC constraint. However, this is not global in that $V_1(n | n) = 0$ for $\forall n > n_0$ alone does not rule out the case where the individual of ability $n$ has an incentive to choose zero supply of labor and to misrepresent himself as having zero ability. This concern can be resolved if $L_n$ is increasing in $n$. Consequently, once the ICs hold globally, any individual of ability strictly higher than $n_0$ would not have an incentive to feign to be of any other ability, including one lower than $n_0$. Therefore, Lemma 3.2 follows.
Lemma 3.2. If the IC constraints for \( \forall n > n_0 \) are met, then the IR constraints for \( \forall n > n_0 \) are satisfied.

Proof. Suppose that the IC constraints are met for \( \forall n > n_0 \). Then, from Lemma 3.1, this implies that \( V_1(n \mid n) = 0 \) for \( \forall n > n_0 \). Moreover, based on Fundamental theorem of calculus, we know that \( V(n \mid n) = V(n_0 \mid n_0) + \int_{n_0}^{n} V_1(t \mid t) + V_2(t \mid t)dt \) for \( \forall n > n_0 \) because \( V_1(t \mid t) = 0 \). Since \( V_2(t \mid t) = -\frac{t_u}{t} u_2 = -\frac{t}{t} u_2 \) is greater than zero for \( \forall t > n_0 \) due to \( u_2 < 0 \) and \( L_t > 0 \) (from Proposition 3.4), \( V(n \mid n) > V(n_0 \mid n_0) \) for \( \forall n > n_0 \).

Note that Lemma 3.2 reduces the two sets of constraints into one. This implies that what remains in this simplification is to make the two conditions specified in Lemma 3.1 more tractable for the pointwise maximization. As briefly mentioned above, the variable that we deal with is not \( L_n \) but \( l_n \). Hence, we had better transform the conditions in Lemma 3.1 into ones stated in terms of \( l_n \). Finally, utilizing the lemmas presented above, the infinite number of inequalities of the IC and IR constraints for any given \( n > n_0 \) are now trimmed into two manageable conditions.

Lemma 3.3. The the IC constraints and IR constraints for \( \forall n > n_0 \) are met if and only if (i) \( v_n = u(-T(0), 0) + \int_{n_0}^{n} \frac{1}{t} u_2 dt \) and (ii) \( \int_{n_0}^{n} \frac{1}{t} u_2 (p \frac{1}{u_1} + (1 - p) \frac{1}{u_1^{ND}}) dt \geq 0 \) for \( \forall n > n_0 \).

Proof. [step 1] \((\implies)\) Firstly, suppose that the IC constraints and IR constraints for \( \forall n > n_0 \) are met. From Lemma 3.1, this implies that \( V_1(n \mid n) = 0 \) for \( \forall n > n_0 \). Moreover, based on the Fundamental theorem of calculus, it follows that \( V(n \mid n) = V(n_0 \mid n_0) + \int_{n_0}^{n} V_1(t \mid t) + V_2(t \mid t)dt = V(n_0 \mid n_0) + \int_{n_0}^{n} \frac{1}{t} u_2 dt \) for \( \forall n > n_0 \) because \( V_1(t \mid t) = 0 \). In addition to this, by definition, \( V(n \mid n) = E[u(wL_n - T(c_n wL_n))] = v_n \) and \( V(n_0 \mid n_0) = v_{n_0} = E[u(-T(0), 0)] = u(-T(0), 0) \). Therefore, \( v_n = u(-T(0), 0) + \int_{n_0}^{n} \frac{1}{t} u_2 dt \) for \( \forall n > n_0 \). Next, since the IR constraints are met for \( \forall n > n_0 \), \( V(n \mid n) \geq V(n_0 \mid n_0) \) for \( \forall n > n_0 \). Because \( V(n \mid n) = V(n_0 \mid n) + \int_{n_0}^{n} \frac{1}{t} u_2 dt \), this means that \( \int_{n_0}^{n} \frac{1}{t} u_2 dt \geq 0 \) for \( \forall n > n_0 \). This implies \( \int_{n_0}^{n} \frac{1}{t} u_2 (p \frac{1}{u_1} + (1 - p) \frac{1}{u_1^{ND}}) dt \geq 0 \) for \( \forall n > n_0 \) because (i) both \( u_1^D \) and \( u_1^{ND} \) are always strictly positive and (ii) \( p \in (0, 1) \).

[step 2] \((\impliedby)\) Conversely, suppose that \( v_n = u(-T(0), 0) + \int_{n_0}^{n} \frac{1}{t} u_2 dt \) and that \( \int_{n_0}^{n} \frac{1}{t} u_2 (p \frac{1}{u_1} + (1 - p) \frac{1}{u_1^{ND}}) dt \geq 0 \) for \( \forall n > n_0 \). Firstly, \( v_n = u(-T(0), 0) + \int_{n_0}^{n} \frac{1}{t} u_2 dt \) for \( \forall n > n_0 \) means that \( V(n \mid n) = V(n_0 \mid n_0) + \int_{n_0}^{n} \frac{1}{t} u_2 dt \) for \( \forall n > n_0 \), because \( V(n \mid n) = v_n \) and \( V(n_0 \mid n_0) = v_{n_0} = u(-T(0), 0) \). Furthermore, since \( V_2(t \mid t) = -\frac{t_u}{t} u_2 \), this implies that \( V(n \mid n) = V(n_0 \mid n_0) + \int_{n_0}^{n} V_1(t \mid t) + V_2(t \mid t)dt \) for \( \forall n > n_0 \). However, \( V(n \mid n) = V(n_0 \mid n_0) + \int_{n_0}^{n} V_1(t \mid t) + V_2(t \mid t)dt \) for \( \forall n > n_0 \), based on
the Fundamental theorem of calculus. Therefore, this implies that \( V_1(n \mid n) = 0 \) for \( \forall n > n_0 \).

Having \( V_1(n \mid n) = 0 \) for \( \forall n > n_0 \), it is enough to show that \( L_n \) is increasing in \( n \). Because this implies that the IC constraints for \( \forall n > n_0 \) are met, based on Lemma 3.1, and this in turn implies that the IR constraints for \( \forall n > n_0 \) are satisfied, due to Lemma 3.2. By way of contradiction, suppose that \( L_n \) is decreasing in \( n \) (i.e., \( L' = \frac{dL}{dn} < 0 \)). First of all, \( \int_{n_0}^{n} \frac{-t}{L} u_2 \bigl(p \frac{1}{u_1} + (1 - p) \frac{1}{u_1} \bigr) dt \geq 0 \) for \( \forall n > n_0 \) implies that \( \int_{n_0}^{n} \frac{-t}{L} u_2 dt \geq 0 \) for \( \forall n > n_0 \) since \( u_2 < 0 \), \( (p \frac{1}{u_1} + (1 - p) \frac{1}{u_1} ) > 0 \) and \( t > 0 \) for \( \forall t > n_0 > 0 \) (based on Proposition 3.4). Recall that \( V(n_0 \mid n_0) = v_{n_0} = E[u(w_0 L_{n_0} - T(e_{n_0} w_0 L_{n_0}), l_{n_0})] = E[u(w L_{n_0} - T(e_{n_0} w L_{n_0})] = E[u(-T(0), 0)] \) and that \( V(n \mid n) = E[u(w L_{n} - T(e_{n} w L_{n})] \). Then, this implies that \( E[u(w L_{n} - T(e_{n} w L_{n})] > E[u(w L_{n_0} - T(e_{n_0} w L_{n_0})] \) for \( \forall n > n_0 \) since \( V(n \mid n) = V(n_0 \mid n_0) + \int_{n_0}^{n} \frac{-t}{L} u_2 dt \) and \( \int_{n_0}^{n} \frac{-t}{L} u_2 dt > 0 \) for \( \forall n > n_0 \). Furthermore, \( E[u(w L_{n} - T(e_{n} w L_{n})] > E[u(w L_{n_0} - T(e_{n_0} w L_{n_0})] \) for \( \forall n > n_0 \) means that \( \int_{n_0}^{n} \frac{dE[u]}{dt} \bigl( \frac{dE[u]}{dL} \bigr) dt \geq 0 \) for \( \forall n > n_0 \). However, this is a contradiction since \( \frac{dL}{dt} < 0 \) and \( \frac{dE[u]}{dt} \geq 0 \) from Proposition 3.2. Hence, this shows that \( L_n \) is increasing in \( n \). 

In fact, it is crucial to understand economic intuition underlying Lemma 3.3. To begin with the IR constraints, taxpayers have reservation utility \( u(-T(0), 0) \) from being non-taxpayer, which is the first term of the payoff \( (v_n = u(-T(0), 0) + \int_{n_0}^{n} \frac{-t}{L} u_2 dt) \) to a taxpayer of ability \( n \). Moreover, notice that the IC constraints aim to effectively generate a choice environment where each taxpayer voluntarily reveals their private information — their own intrinsic ability. To induce the revelations, however, the government needs to incentivize taxpayers to do so by compensating them with possible surplus from pretending to be of any other ability. Otherwise, they will exploit their informational advantage by the pretending. At the same time, the government would not pay more than the smallest amount of needed to avoid this. In order to find the minimum of the compensation, consider a taxpayer of ability \( n \). If this individual fakes being of a slightly lower ability \( n - \varepsilon \) (where \( \varepsilon > 0 \)), then his surplus is at least as much as \( V(n - \varepsilon \mid n) - V(n - \varepsilon \mid n - \varepsilon) \). Note the surplus from tapping informational advantage does not include \( V(n - \varepsilon \mid n - \varepsilon) \) since he gets \( v_{n-\varepsilon} = V(n - \varepsilon \mid n - \varepsilon) \) even without information asymmetry, based on Proposition 3.1. Furthermore, since ability is not discrete but continuous variable, the minimum gain that this individual acquires from pretending to be the closest lower ability one is \( \lim_{\varepsilon \to 0} \frac{V(n - \varepsilon \mid n) - V(n - \varepsilon \mid n - \varepsilon)}{\varepsilon} = V_2(n \mid n) \). Eventually, therefore, the minimum total surplus from pretending to be a taxpayer of all the other lower abilities constitutes \( \int_{n_0}^{n} V_2(t \mid t) dt \), which is the remaining last term of the payoff, since \( \int_{n_0}^{n} V_2(t \mid t) dt = \int_{n_0}^{n} \frac{-t}{L} u_2 dt \). Notably, this is paid to the individual of ability \( n \) only
because of his informational advantage on his innate ability over the government. Thus, this is an ‘informational rent’ accruing to the individual of ability \( n \). In order to explicitly state this notion, let \( \int_{n_0}^{n} -\frac{u}{t}u_2 dt = \int_{n_0}^{n} \pi_t dt \) by introducing a notation \( \pi_t = -\frac{u}{t} = V_2(t \mid t) \).

Because the informational rent is neither tax revenue nor production output, the resource needed for paying it is to be accounted separately as an additional constraint for the government’s problem (3.8). First of all, the government measures marginal value of resource based on unit of expected consumption goods — not of working; hence, after the unit conversion (i.e., dividing \( -\frac{u}{t}u_2 \) by \( p \frac{1}{u_1^2} + (1 - p) \frac{1}{u_1^2} \))

\[ \text{Informational Rent} = \int_{n_0}^{n} \pi_t(p \frac{1}{u_1^2} + (1 - p) \frac{1}{u_1^2}) dt, \]

to the government’s point of view. Since the government should satisfy the double integral in

\[ \int_{n_0}^{n} \pi_t(p \frac{1}{u_1^2} + (1 - p) \frac{1}{u_1^2}) dt \]

to the government’s point of view. Since the government should satisfy \( \int_{n_0}^{n} \pi_t(p \frac{1}{u_1^2} + (1 - p) \frac{1}{u_1^2}) dt \geq 0 \) for all the taxpayers (the condition (ii) in Lemma 3.3), after aggregating this with their population weight, we can succinctly state the newly added constraint as

\[ \int_{n_0}^{n} \pi_t(p \frac{1}{u_1^2} + (1 - p) \frac{1}{u_1^2}) dt \geq 0 \text{ for all the taxpayers.} \]

Thus, this is an informational rent accruing to the individual of ability

\[ \pi_t = -\frac{u}{t} = V_2(t \mid t). \]

Due to imperfect tax enforcement rate, the collected tax revenue in real term (measured as units of the composite goods) inevitably involves uncertainty.

\[ \pi_t \equiv -\frac{u}{t} = V_2(t \mid t). \]

Thus, this is an informational rent accruing to the individual of ability

\[ \pi_t = -\frac{u}{t} = V_2(t \mid t). \]

Due to imperfect tax enforcement rate, the collected tax revenue in real term (measured as units of the composite goods) inevitably involves uncertainty.
a maximum of $L_{PI}$) exists. Furthermore, once the maximand is concave, we know that its FOCs are sufficient. Therefore, in the next subsection, the optimum income tax schedule $T$ will be characterized by obtaining these FOCs, and optimal revenue mobilization will be coherently analyzed as well.

### 3.3.5 Characteristics of Optimal Income Taxation and Revenue Mobilization

On beginning to specify optimality conditions for income tax schedule $T$, recall that characteristics of optimal income tax rates can be fully obtained by a pointwise maximization with respect to $l_n$ and $E[x_n]$, once the IC and IR constraints are incorporated as in $L_{PI}$. Let the Lagrangian statement for point $n$ given be $L_{PI}^n$ such that

$$L_{PI}^n = \int_0^n L_{PI}^n \, dt.$$  

Then, the FOCs of $\frac{dL_{PI}^n}{dl_n} = 0$ and $\frac{dL_{PI}^n}{dE[x_n]} = 0$ will define optimum income tax rate for a taxpayer of ability $n$. Combining these two yields optimality condition of income tax rate which is stated as follows: For $\forall n > n_0$,

$$\frac{d\pi_n}{dl_n} \int_0^\pi \left[ \lambda \left( \frac{1}{u_1^D} + (1-p) \frac{1}{u_1^{ND}} \right) - \frac{d^2 G}{dv^2} \right] T(n \mid t > n_0) dt = w \ln \lambda \left[ p \left( 1 + \theta \left( 1 - e_n \right) \right) \right] + (1-p)e_n] T'(n \mid n > n_0)$$

From this key statement, it is immediate that optimum marginal income tax rate is non-negative for $\forall n > n_0$.

**Lemma 3.4.** If $T$ is an optimal income tax function, then for $\forall n > n_0, T' \geq 0$.

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13In fact, the IR and IC constraints resume the link between ability $n$ and $l_n$ and $E[x_n]$ that uniquely define the income tax rate for taxpayer of ability $n$. This enables us to invoke optimal control theory. Based on Pontryagin Maximum Principle, the necessary condition from FOC can also be sufficient.

14$\frac{dL_{PI}^n}{dl_n} = \frac{dx}{dx} \int_0^x \left[ \frac{d^2 G}{dy^2} - \lambda \left( \frac{1}{u_1^D} + (1-p) \frac{1}{u_1^{ND}} \right) \right] T(n \mid t > n_0) dt + \lambda \left[ w - \frac{d^2 x_n + (1-p)x_n^{ND}}{dE[x_n]} \right] h(n \mid n > n_0) = 0 \text{ and } \frac{dL_{PI}^n}{dE[x_n]} = \frac{dx}{dx} \frac{dE[x_n]}{dx} \int_0^x \left[ \frac{d^2 G}{dy^2} - \lambda \left( \frac{1}{u_1^D} + (1-p) \frac{1}{u_1^{ND}} \right) \right] T(n \mid t > n_0) dt + \lambda \left[ w - \frac{dE[x_n]}{dE[x_n]} \right] h(n \mid n > n_0) = 0$ since $\frac{d^2 x_n + (1-p)x_n^{ND}}{dE[x_n]} = p \frac{dx}{dx} + (1-p) \frac{dx_n^{ND}}{dl_n} = pw_1 (1 - T' - \theta T' + \theta e_n T') + (1-p) w_1 (1 - e_n T')$.

15In fact, this is mathematically isomorphic to the central result of Mirrlees (Equation (21) of Mirrlees (1971, p.181) that was discussed at length in his paper. However, the model of Mirrlees (1971) assumes that there is an individual of infinite ability $\infty$, and this study improves upon the unrealistic infinity by introducing an individual of the highest ability $\pi$ as Sadka (1976) and Seade (1977) did. At the same time, this paper allows that the government would not have an infinite power in the tax enforcement. Moreover, this paper did not impose the non-standard assumption (Assumption B, p.182 or $u_{12} = 0$) of his paper which appears to lack an appealing ground.

16This is the famous and important result that was first heralded by Mirrlees (1971) and later rigorously proven by Seade (1982). The current analysis provides a simpler proof for this without any additional non-standard assumptions that Seade (1982) imposed.
Proof. To begin with the right-hand side of the optimality condition (3.21), since the distribution of ability has a full support, \( h(n \mid n > n_0) > 0 \) for \( \forall n > n_0 \). Moreover, \( p(1 + \theta(1 - e_n)) + (1 - p)e_n > 0 \) because \( e_n \in (0, 1) \), \( \theta > 0 \) and \( p \in (0, 1) \) for \( \forall n > n_0 \). \( \lambda > 0 \) since an additional resource for public fund has a positive value to the society. Hence, \( wn\lambda[p(1 + \theta(1 - e_n)) + (1 - p)e_n]h(n \mid n > n_0) > 0 \) for \( \forall n > n_0 \). For the left-hand side, first of all, \( \frac{dw_n}{dn} = \frac{d}{dn}k(u_2) = -\frac{1}{n}u_2 - \frac{k}{n}u_{22} > 0 \) since \( u \) is decreasing in working hours \( (u_2 < 0) \) and concave \( (u_{22} < 0) \). Obviously, \( \lambda[p\frac{1}{u_1} + (1 - p)\frac{1}{u_1}] > 0 \) due to \( u_1^D > 0 \) and \( u_1^{ND} > 0 \). Furthermore, we know that \( \frac{d^2G}{dv^2} \geq 0 \) (weakly concave SWF); if \( \frac{d^2G}{dv^2} < 0 \), (3.21) is a condition for minimizing the SWF, instead of maximizing it. Therefore, \( T' \geq 0 \) for \( \forall n > n_0 \) because of the non-negative left-hand side and positive \( wn\lambda[p(1 + \theta(1 - e_n)) + (1 - p)e_n]h(n \mid n > n_0) \). In particular, the left-hand side is 0 at \( \bar{n} = n \) since \( \int_{\bar{n}}^{\pi}\lambda[p\frac{1}{u_1} + (1 - p)\frac{1}{u_1}] - \frac{d^2G}{dv^2}|h(t \mid t > n_0)dt = 0 \); hence, \( T' = 0 \) at \( \bar{n} = n \). All in all, for \( \forall n > n_0 \), \( T' \geq 0 \). \( \blacksquare \)

Most of all, it is important to understand tax rule embedded in (3.21), the mathematically notated optimality condition. The specific optimal tax rates calculated based on the optimality condition (3.21) in numerical simulations have limited applicability for reasons. The distribution of ability or any conversion of this to empirical distribution needs to rely on untestable assumptions. Likewise, the functional form of utility function, SWF can also raise controversy on the direct applications. Even if this is the case, the notion underlying the equation (3.21) of optimal income taxation rule could still be used for a practical suggestion or guidelines to policy makers.

To begin with the right-hand side of (3.21), \( wn\lambda[p(1 + \theta(1 - e_n)) + (1 - p)e_n]T'h(n \mid n > n_0) \) stands for an efficiency loss. Because it reflects how much less a taxpayer of ability \( n \) chooses his working hour in this second-best world than he would do (which is \( t_n^{F_1} \)) in the first-best world, at the margin. This observation basically emerges from drawing a comparison between (3.5) and (3.18). To lay out details by parts, firstly, if the tax code is enforced, then the real term marginal value of working \( \frac{\overline{w_n}}{u_1} \) (i.e., marginal value of working measured by marginal value of consumption) is \( wn\{1 - (1 + \theta[1 - e_n])T'\} \) in the second-best world, whereas it is \( wn \) in the first-best world. Secondly, if tax evasion is not detected, then the real term marginal value of working \( \frac{-\overline{w_n}}{u_1} \) is \( wn(1 - e_n)T' \), whereas it is \( wn \) in the first-best world. With a tax enforcement rate \( p \), the ex ante difference in these marginal values between the two worlds, one of which is free of any efficiency loss, amounts to \( wn[p(1 + \theta(1 - e_n)) + (1 - p)e_n]T' \). In other words, income taxation entails a decrease in labor supply because of a reduction in the real term marginal value of a working hour by \( wn[p(1 + \theta(1 - e_n)) + (1 - p)e_n]T' \) since this is positive based on Lemma 3.4. This points to a distortion on the price (value) of working; hence, this clearly is a loss on the efficiency side. Moreover, to the government’s point of view, this loss is weighted by the population portion of the taxpayers of ability \( n \) which is \( h(n \mid n > n_0) \) and put in terms of public resource
whose marginal social value is \( \lambda \).

Next, to effectively learn the meaning of the left-hand side of (3.21), \( \frac{dx_n}{nu_n} \int_{n}^{\bar{\pi}} [\lambda \{ p \frac{1}{u_t} + (1-p) \frac{1}{u^*_t} \} - \frac{dG}{dv} t | t > n_0 dt] \), let us run a counterfactual thought experiment. Suppose that the government suddenly exempts only the taxpayers of ability \( n \) from the income tax. Without the tax, these individuals will now provide their first-best level of working hours \( l^{Be}_n \) based on (3.18). This means that, only for this specific \( n \), the right-hand side of (3.21), which represents the efficiency loss, falls to zero. However, this efficiency gain comes with side effect. That is, this tax reform shall entice all the individuals of ability higher than \( n \) into pretending to be of ability \( n \) by reducing their working hours to be better off with the tax exemption. To prevent this fallout, the government should compensate these higher ability individuals with the possible surplus from the pretending (informational rent). Given the gain from increased working hours of ability \( n \) individuals, the government will appraise the entailed cost of paying the rent. First of all, \( \lambda \{ p \frac{1}{u_t} + (1-p) \frac{1}{u^*_t} \} = \lambda \frac{d\pi_t \{ p \frac{1}{u_t} + (1-p) \frac{1}{u^*_t} \}}{dt} \) is the (real term) marginal social loss from a small increase in the informational rent to an individual of ability \( t \); and, \( \frac{dG}{dv} = \frac{dv}{dt} \frac{d\pi_t \{ p \frac{1}{u_t} + (1-p) \frac{1}{u^*_t} \}}{dt} \) is the marginal social gain from increased utility due to the increment in the rent, since \( \frac{dn}{dt} = 1 \) from (i) in Lemma 3.3. Having the net marginal social value aggregated over all the individuals of higher abilities with their population weights, we get \( \int_{n}^{\bar{\pi}} [\lambda \{ p \frac{1}{u_t} + (1-p) \frac{1}{u^*_t} \} - \frac{dG}{dv} t | t > n_0 dt] \). In addition, since the government weighs this against the gain of a rise in working hours, unit conversion of multiplying \( \frac{dx_n}{nu_n} \) follows. In the end, \( \frac{dx_n}{nu_n} \int_{n}^{\bar{\pi}} [\lambda \{ p \frac{1}{u_t} + (1-p) \frac{1}{u^*_t} \} - \frac{dG}{dv} t | t > n_0 dt] \) represents the net marginal social loss on the informational rent compensated to all the individuals of ability greater \( n \) for preventing the drop of their working hours to mimic a taxpayer of ability \( n \). Presumably, this hypothetical reform is gainful to the government as long as the efficiency gain is greater than this loss. Therefore, when both are equal as in (3.21), the government reaches an optimum, since there is no further room to be better off.

In this line, because there is no individual of ability greater than \( \bar{\pi} \), we now know that the government does not need to consider any informational rent when the aforementioned tax reform is applied for taxpayers of ability \( \bar{\pi} \). Since there is no loss occurred in prevention of the side effect, this reform will not be compromised (like above) and just implemented. As a consequence, these individuals will provide their first-best level of working hours \( l^{Be}_n \) based on (3.18). More importantly, since no efficiency loss means zero value of the left-hand side of (3.21), the marginal income tax rate on these top ability individuals is zero \(^{17}\) at the optimum \( (T' = 0 \text{ at } \bar{\pi} = n) \), as shown in the proof of Lemma 3.4. In other words, the reason for zero marginal tax rate on the highest ability individuals is simply that \( \textit{there is no one else (with higher ability) who is tempted to mimic these individuals} \) and thus for whom the government needs to pay the informational rent — regardless of the social weights.

\(^{17}\)This is nothing short of the well known result of ‘no distortion at the top.’
\( \frac{dG}{dn} \) on them. In fact, this holds even without tax evasion concern at all.\(^{18}\) So, this also sheds a new light on the interpretation on the result of zero marginal tax rate for top earners which has been shared widely so far (e.g., Diamond 1998; Menkiew et al. 2009). Many interpret this argument as follows. Non zero marginal tax rate on the top earner is suboptimal since zero marginal tax rate "would lead the top earner to work more, raising social welfare without losing any tax revenue" (p.84, Diamond 1998). However, there are two weak points of this interpretation. First, there is no coherent link between this interpretation and the object being interpreted: the formula of optimal income tax rate, Equation (3.21) of Mirrlees (1971), which can be equivalently re-stated as

\[
\frac{dG}{dn} \int_{n}^{\bar{n}} \left( \frac{\lambda}{u_1} - \frac{dG}{dn} \right) h(t \mid t > n_0) dt = wn\lambda T'h(n \mid n > n_0) \text{ in the present context.}
\]

Second, this logic can possibly apply to other individuals of lower ability, too: A zero marginal tax rate on taxpayers of ability \( n \), ceteris paribus,\(^{19}\) could lead them to "work more, raising social welfare without losing any tax revenue."

Furthermore, viewing this finding from the reverse side discovers another economic intuition underlying the optimality condition (3.21) on the rationale of the positive left-hand side. For the sake of exposition, let us first take a simplified dichotomous angle; contrast taxpayers of ability \( \bar{n} \) vs. any other taxpayers of ability \( n < \bar{n} \). To begin, notice that compared to the first-best world, working becomes more painful to the individuals of ability \( n \) owing to the reduction of the real term marginal value of it, as described above. Why does the government, as a benevolent social planner, let them suffer? What is the purpose? To understand this, realize, from (3.5), that the marginal value of one unit of working is increasing in \( n \), which implies that the reduction is more painful to a taxpayer of ability \( \bar{n} \) than to a taxpayer of ability \( n \).\(^{20}\) Consequently, the government can achieve to deter the taxpayer of ability \( \bar{n} \) from mimicking a taxpayer of ability \( n \) effectively by the reduction, without which the taxpayer of ability \( \bar{n} \) ends up with manipulating his working hours to deceive his their true ability. As a matter of fact, this logic also applies between any two taxpayers of different abilities, since cost for such a deterrence is manifest in terms of social value on informational rent (the left-hand side), as analyzed above. Eventually, this implies that optimal income tax rule allows efficiency loss only for the purpose of inducing the taxpayers to virtually reveal their true ability. In this way, we are minimizing efficiency loss in the maximization of the social welfare, since with the least reward to the revelation the government prevents the cascading decrease in working hours of all the taxpayers seeking to pretend to be of a lower ability. As a result, at the optimum, we can achieve that more of effective labor is supplied by more able individuals (Lemma 3.1). Moreover, from this prevention, the government can

\(^{18}\)The above explanation on the meaning of the optimality condition (3.21) unfolds without relying on tax evasion concern. Rather, tax evasion is directly controlled by the tax enforcement rate \( p \).

\(^{19}\)Therefore, the marginal tax rate on individuals of higher ability still remains the same taking positive values.

\(^{20}\)In contract theory’s term, this can be called single crossing property, or Spence-Mirrlees condition.
save resource for the redistribution which otherwise could have been wasted away. Therefore, this can rationalize the positive value of the left-hand side which reflects efficiency loss.

Once knowing that the income tax is designed to raise a public fund with the least loss on efficiency, the naturally ensuing question is its distributional effect. First of all, at the level of generality of (3.21), the relationship between $T'$ and ability $n$ does not manifest specific enough to drive a definite conclusion on the progressivity of the marginal income tax rate, since this depends on given factors like distribution of ability, functional forms of $G$ and $u$ in a complicated way. In fact, in some cases, optimal marginal tax rate $T'$ might not be increasing in ability.21 Nevertheless, we still can examine whether a more able taxpayer, who turns out to be richer, pays more tax. At the outset, with an imperfect tax enforcement, a taxpayer ends up with paying either $T(w_{nl}n) + T(e_n w_{nl})$ or $T(e_n w_{nl})$; hence, let the expected value of tax payment be $E[T] = p[T(w_{nl}n) + T(e_n w_{nl})] + (1 - p)T(e_n w_{nl})$.

First of all, actual earnings ($w_L n$) are increasing in ability, since $L_n$ is increasing in $n$ (Lemma 3.1). This immediately implies that true tax liability is increasing in $n$ under the positive marginal income tax (Lemma 3.4). Consequently, as true income increases (with ability $n$), the amount of penalty, which is unpaid tax liability, rises given the same unit of evasion $1 - e$, while the gain from evasion increases as well. This means that the gamble of evading tax gets more risky to higher ability individuals. Since taxpayers have the constant relative risk aversion, this suggests that tax evasion does not get more attractive to higher ability taxpayers. After all, therefore, the ex ante value of tax payment rises as ability and thus income increase.

**Proposition 3.5.** Expected tax payment is increasing in ability; that is, $\frac{dE[T]}{dn} \geq 0$.

Proof. $\frac{dE[T]}{dn} = \frac{dp[T(w_{L}n) + T(e_n w_{L})] + (1 - p)T(e_n w_{L})}{dn} = wT'[p(1 + \theta(1 - e_n)) + (1 - p)e_n] \frac{dl_n}{dn}$. First of all, $wT'[p(1 + \theta(1 - e_n)) + (1 - p)e_n] \geq 0$ since $T' \geq 0$ due to Lemma 3.4, $e_n \in (0, 1)$ for $\forall n > n_0$, $\theta \geq 0$ and $p \in (0, 1)$. Second of all, $\frac{dl_n}{dn} \geq 0$ due to Lemma 3.1. Therefore, $\frac{dE[T]}{dn} \geq 0$. ■

As a result, the richest individual pays the largest income tax (in expectation). In fact, this progressivity in the tax payment is echoing results of the optimal income taxation without tax evasion (Mirrlees 1971). This similarity is confirmed by Freire-Serén and Panadés (2008) in their proof that, with a constant relative risk aversion preference, tax evasion does not change the actual redistributive effect of nonlinear income tax.

In addition, the government needs to find an optimal $-T(0)$ that not only completes the optimal tax schedule but also results in a demarcation between taxpayers

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21For detailed and useful discussion, refer to Tuomala (1990) and Diamond (1998).
and non-taxpayers based on Proposition 3.4. The government seeks $-T(0)$ that maximizes the SWF$^{22}$ by solving $\frac{dC_{n0}}{d-T(0)} = 0$ from which we get an optimality condition of $-T(0)$ as follows;

$$\int_0^{\pi} \frac{dG}{dv} u_1 h(n)dn = \lambda \{ H(n_0) + [-T(0)] \frac{dH(n_0)}{d-T(0)} \} \tag{3.22}$$

since $\frac{d}{d-T(0)} (\int_0^{n_0} \frac{dG}{dv} u(-T(0), 0)h(n)dn) = \int_0^{n_0} \frac{dG}{dv} u_1 h(n)dn$ and $\frac{d}{d-T(0)} (\int_0^{\pi} \frac{dG}{dv} u(-T(0), 0) + \int_0^{n_0} \pi_1 dt) h(n)dn) = \int_0^{n_0} \frac{dG}{dv} u_1 h(n)dn$ from Lemma 3.3. To begin, the left-hand side of (3.22) indicates marginal social benefit from a small increase in $-T(0)$ that turns out to have a society-wide effect. Firstly, non-taxpayers (all the individuals of ability between 0 and $n_0$) are benefited from the increase, since their consumption level is $-T(0)$ (from Proposition 3.4). Secondly, with the IR constraints met, optimal income tax schedule is set such that $u(-T(0), 0)$ is guaranteed utility (payoff) to all the taxpayers; hence, with $u_1 > 0$, the increment in $-T(0)$ eventually benefits (based on Lemma 3.3) the other remaining part of society — all the individuals of ability between $n_0$ and $\pi$. In a nutshell, therefore, an increase in $-T(0)$ lifts up the minimum standard of living which affects every one in the society. The pursuit of this social benefit generates a demand for the government to raise tax revenue, whose marginal social cost required appears at the right-hand side of (3.22). The social cost for financing the small increase in the public expenditure $-T(0)$ takes into account of not only the population of current recipients (existing non-taxpayer) $H(n_0)$ but also for an inflow into the group of non-taxpayer $[-T(0)] \frac{dH(n_0)}{d-T(0)}$ due to the increase in the $-T(0)$, since (as shown in Proposition 3.4) the generosity of $-T(0)$ affects the extensive margin of labor supply choices of individuals of ability near 0 and thus can convert these individuals into a non-taxpayer from a taxpayer. On balance, the government selects optimal $-T(0)$ that equates the marginal social benefit from an enhanced minimum living standard of the society with the marginal social cost to an increased need of tax revenue. As a consequence, the desired tax expenditure derived from this optimality condition (3.22) will be $H(n_0)[-T(0)]$.

Having fully characterized optimal income tax schedule $T$, the remaining tax policy variable the government seeks for in the maximization of the SWF is optimal rate of tax enforcement $\rho$ that entails some cost to improve. Eventually, this variable governs revenue mobilization; and, the expected tax revenue $R$ collected under imperfect

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$^{22}$Notably, without the initial value, solving the differential equation (3.21) would not pin down the resulting value of the SWF. Instead of letting initial value exogenously given, in the current analysis, the government also can virtually select it through the choice of $-T(0)$ to maximize the SWF.
tax compliance and enforcement is

\[ R = \int_{n_0}^{\bar{n}} \{ p[T(wnl_n) + \theta(T(wnl_n) - T(e_nwnl_n))] + (1-p)T(e_nwnl_n) \} h(n \mid n > n_0)dn. \]  

(3.23)

Thus, the government’s ex ante budget constraint goes as follows

\[ R - \delta c(p) \geq H(n_0)[-T(0)] \]  

(3.24)

taking account of the cost \( \delta c(p) \) for tax administration and enforcement. Since this economy is closed, at the optimum of (3.8), the constraint (3.24) will be met with the equality, following the Walras’ law. As a matter of fact, in order to explicitly obtain the optimality condition of \( p \), it is more straightforward to put the government’s budget constraint (3.24) in the place of the feasibility constraint (3.7) when solving (3.8). From this, the FOC for an optimal \( p \) turns out as follows:

\[ -\int_{n_0}^{\bar{n}} \frac{dG}{dv_n} \frac{dv_n}{dp} h(n \mid n > n_0)dn = \lambda \int_{n_0}^{\bar{n}} (1+\theta)[T(wnl_n) - T(e_nwnl_n)]h(n \mid n > n_0)dn - \delta c'(p). \]  

(3.25)

First of all, the right-hand side of (3.25) represents the marginal net social benefit from raising \( p \). Its first term \( \int_{n_0}^{\bar{n}} (1+\theta)[T(wnl_n) - T(e_nwnl_n)]h(n \mid n > n_0)dn \) is an increase in the expected tax revenue resulting from a small increase in \( p \). This is subtracted from by the second term \( \delta c'(p) \) that is an increment in the administrative and enforcement cost required for the increment in \( p \). Lastly, this net resource gain for public fund is appraised as much as \( \lambda \) per unit. Second of all, the left-hand side of the optimality condition of \( p \) indicates the marginal social loss from the small increase in \( p \). \( -\frac{du_n}{dp} = u_n^{ND} - u_n^P \) is the ensuing increase in gap between the payoffs under being detected vs. being not-detected which reduces the utility of risk-averse taxpayers. This is a loss to the government, as a benevolent social planner, which in turn is attached with a certain social weights \( dG/dv_n \) and aggregated according to population weight \( h(n \mid n > n_0) \). All in all, an optimal \( p \) is set to equalize the social value on the marginal loss to taxpayers from the raised risk of evading tax due to a small increase in \( p \) with the marginal social value on net increase in the expected revenue from the increment. In fact, note that this optimality condition (3.25) is essentially the same as (3.19) from the first-best world, except for the kind of the tax under discussion.

Notably, this result improves upon the previous studies on the optimal tax enforcement by two folds. First, the current analysis seeks \( p \) that maximize social welfare, instead of net tax revenue, without separating the tax enforcement alone from taxation (Reinganum and Wilde 1985; Mookherjee and Png 1989; Sánchez and Sobel 1993). Second, the present study derives both optimal nonlinear income tax schedule and optimal tax enforcement, instead of assuming single linear income tax rate.

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23 Apparently, \( p \) that appears in the key condition for optimal income taxation (3.21) is defined by (3.25) and nonlinear income tax rate in (3.25) is from (3.21).
exogenously given (Slemrod and Yitzhaki 1987). These improvements enable us to clarify the unresolved ambiguities on the relationship between tax evasion and policy variables, as follows.

First of all, as mentioned in Section 3.2, previous studies show that improved tax enforcement through an increase in \(p\) can raise tax evasion (Baldry 1979; Pencavel 1979; Sandmo 1981; Horowitz and Horowitz 2000). This puzzling result undermines the government’s efforts for tax enforcement. Departing from their approach that takes income tax rate as exogenously given, this study restores the intuitive result that enhancement of tax enforcement reduces tax evasion (i.e., does not raise tax evasion).

Proposition 3.6. Tax evasion is decreasing in the tax enforcement rate; that is, for any given \(n > n_0\), \(\frac{d\alpha}{dp} > 0\).

Proof. \[\frac{d\alpha}{dp} = -\frac{\theta u_{11}^D + u_{11}^{ND}}{p \theta u_{11}^D + (1-p) u_{11}^{ND} T'wml_n} \] using (3.3), based on the Implicit function theorem. This implies that \(\frac{d\alpha}{dp} > 0\) since \(u_1 > 0\), \(u_{11} < 0\) for each state, \(\theta > 0\) and \(T' \geq 0\) (from Lemma 3.4).

Another ambiguity regarding tax evasion is on whether an increase in marginal tax rate entails an increase in tax evasion. A number of empirical studies (using administrative data or laboratory experiment data) found a positive and significant correlation between marginal tax rate and tax non-compliance (Clotfelter 1983; Dubin et al. 1990; Friedland 1987; Baldry 1987; Alm et al. 1992; Kleven et al. 2011). Nonetheless, a theoretical proof has not been provided yet; the sign of correlation between the two has remained indeterminate (Pencavel 1979; Sandmo 1981). By deriving optimal income tax rate that incorporates tax evasion – as appears in (3.21) – we now can show that marginal income tax rate is negatively associated with tax compliance.

Proposition 3.7. Tax evasion is positively related with marginal income tax rate; that is, for any given \(n > n_0\), \(\frac{d\alpha}{dT} < 0\).

Proof. Based on the Implicit function theorem, \[\frac{d\alpha}{dT} = \frac{-wn\lambda[1+e_n]h(n | n > n_0)}{wn\lambda[-p\theta+(1-p)]T'h(n | n > n_0)} \] using (3.21). \(wn\lambda[-p\theta+(1-p)]T'h(n | n > n_0) \geq 0\) because (i) \(p\theta \leq (1-p)\) from (3.4) the condition of interior solution \(e_n \in (0, 1)\); (ii) \(h(n | n > n_0) > 0\) from the distribution of ability that has a full support; and (iii) \(T' \geq 0\) from Lemma 3.4. \(p\{1+\theta(1-e_a)\}+(1-p)e_n > 0\) because \(e_n \in (0, 1)\), \(\theta > 0\) and \(p \in (0, 1)\). This implies that \(\frac{d\alpha}{dT} < 0\).
the government’s operation of the tax system. Recall that, owing to the invertibility of the cost function $c$, the government chooses the overall enforcement rate $p$ actually through earmarking resources for administering and enforcing the tax code. Any change other than an increase in $p$ which raises cost for collecting income tax is captured by an increase in $\delta$. Therefore, whatever the source of an increase in $\delta$ is, an increase in $\delta$ causes a decrease in $p$ (i.e., $\frac{dp}{d\delta} < 0$); moreover, this in turn brings about a decrease in expected revenue.

**Proposition 3.8.** An increase in $\delta$ (a decrease in the government’s effectiveness over operating tax system) leads to a decrease in expected tax revenue collected; that is, $\frac{dR}{d\delta} < 0$.

**Proof.** To begin, $\frac{dR}{d\delta} = \frac{dR}{dp} \frac{dp}{d\delta}$. Firstly, $\frac{dR}{dp} = \int_{n_0}^{\pi} (1 + \theta)[T(wnl_n) - T(e_nwnl_n)]h(n | n > n_0)dn > 0$. Secondly, $\frac{dp}{d\delta} = -\frac{c'(p)}{c''(p)}$ using (3.25) and the Implicit function theorem. Since $c(p)$ is convex and increasing in $p$, $c'(p) > 0$ and $c''(p) \geq 0$. Therefore, $\frac{dp}{d\delta} < 0$ since $\delta > 0$. This implies that $\frac{dR}{d\delta} < 0$. ■

Improving upon previous studies of the kind, the current analysis explicitly embodies overall cost for administering and enforcing tax codes into optimal nonlinear income taxation problem. Furthermore, a notable innovation would be the parameter of efficiency of the government’s tax collection, which can vary by society, on the cost function; and, a potentially policy relevant finding on its consequence on revenue mobilization is established theoretically (Proposition 3.8). Admittedly, despite this innovation and some progress of clarifying the lingering ambiguities, the current analysis does not draw a clear-cut conclusion on two queries. First, the effect of increased tax enforcement rate on the labor supply is not determinate in the current study, just as the previous studies on this matter, since the sign of both $\frac{dl_n}{dp}$ and $\frac{dt_n}{dp}$ remain ambiguous.\textsuperscript{24} Second, it is not promptly evident whether $l_n^{SB}$ under a perfect tax enforcement (where $e_n = 1$ for $\forall n$) is larger than $l_n^{SB,e}$ in the presence of tax evasions, as in the benchmark cases. Apparently, we know that, from (3.21), if $p\{1 + \theta(1 - e_n)\} + (1 - p)e_n < 1$, then $l_n^{SB} < l_n^{SB,e}$; otherwise, $l_n^{SB} > l_n^{SB,e}$. Nevertheless, with the present level of generality, we cannot exclude out either of the case in advance.

Lastly, the explicit account on the cost for tax administration and enforcement in this study enables an experimental extension that can provide a new perspective on one of few famous but controversial results in optimal taxation literature: Zero

\textsuperscript{24}Without further restrictions on the cross-partial derivatives of the utility function, we are not able to settle down the sign of $\frac{dt_n}{dp}$ since $\frac{dt_n}{dp} = \frac{wnw_n \{\frac{1}{1 - (1 + \theta - e_n)T}\} - w_nw_n^{D}0(1 - e_nT)}{wu_{11}[wn\{1 - (1 + \theta - e_n)T\}] + (1 - p)u_{11}^{D}w_n\{1 - (1 + \theta - e_n)T\} + u_{12}w_n\{1 - (1 + \theta - e_n)T\} + u_{12}w_n\{1 - (1 + \theta - e_n)T\} + u_{12}}$. As a result, either we are for $\frac{dt_n}{dp}$ since $L_n = nl_n$. 
marginal tax rate on top earners — individuals of ability \( \bar{n} \) — in the society (Lemma 3.4). This result was theoretically reassured by Seade (1976) and Sadka (1976), following Mirrlees (1971). On the one hand, this result was criticized as "a mere theoretical curiosity" (Menkiew 2009) having little practical relevance. On the other hand, the efforts to resolve the controversy on this were made. Numerical simulations, taking specific assumptions on functional forms and the ability distribution, were conducted (Tuomala 1984; Kanbur and Tuomala 1994) to show that this result is very local and applying only to the very top which may be singleton. Another measure of introducing individuals of unlimited ability (that is, \( \bar{n} \to \infty \)) is taken as well (Mirrlees 1971; Saez 2002), although it turns out that as ability approaches infinite, the marginal tax rates still goes to zero. Regarding this matter, the present study could provide a more systematic ground for the non-zero marginal tax rate on the top earners by exploring a possibility that zero marginal tax rate on the richest in the society can cause an increase in \( \delta \).

After reaching the optimum with \( p \) and tax function \( T \) defined by (3.21), (3.22), and (3.25), let us consider a new situation as follows. Namely, the government’s announcement that the marginal income tax rate on the richest people is zero — starkly contrasted to strictly positive tax rate on poorer people — now triggers an increase in \( \delta \). For instance, the zero rate on the top earners per se works as a tipping point for the majority of tax payers to become quite uncooperative to the government, claiming that the most privileged in the society are doing less part for the society than they do and unduly dumping the tax burden on them. This sense of unfairness could make tax collection harder than under the case of non-zero marginal tax rate on the richest. As a consequence, the same amount of money spent on operating tax system would not yield the same degree of improvement in tax enforcement rate \( p \), which translates into an increase in \( \delta \). Facing this situation, the government is now willing to levy a positive marginal tax on the top earners as long as a loss from this alteration is smaller than a gain from getting \( \delta \) back to the previous level. To make a decision, the government needs to assess the marginal social loss from the increase in \( \delta \). In this light of the current study, the marginal loss on the social welfare amounts to \(-\lambda c(p)\) (based on the Envelope theorem) by taking a derivative with respect to \( \delta \) of the Lagrangian expression of the government’s maximization of SWF with the budget constraint of (3.24) at its optimal level. To eliminate this loss, the government will consider assessing a positive marginal income tax rate, say \( T_{\pi} > 0 \), on the richest. The marginal social value of the cost of this measure amounts to \( w\bar{\pi}\lambda[p\{1 + \theta(1 - e_{\pi})\} + (1 - p)\epsilon_{\pi}]T_{\pi}h(\bar{\pi} | n > n_0) \), which was zero before, from (3.21). On balance, facing this new situation, the government would better impose \( T_{\pi} > 0 \) the marginal tax rate on the richest as long as \( \lambda c(p) \geq w\bar{\pi}\lambda[p\{1 + \theta(1 - e_{\pi})\} + (1 - p)\epsilon_{\pi}]T_{\pi}h(\bar{\pi} | n > n_0) \).
3.4 Concluding Remarks

This chapter theoretically investigates optimal nonlinear taxation when individuals can evade tax. In the first place, as long as the government does not put higher weights on richer individuals than poorer ones, the taxation per se can be better for social welfare. On the other hand, however, it can undermine the labor supply incentives. The question, therefore, is how to minimize the distortion on labor given social benefit of taxation. Fundamentally, this study captures that distortion with a type of nonlinear pricing framework by introducing a reasonable assumption that the government cannot verify innate ability of each individual. The only reason for allowing efficiency loss in optimal income taxation is for deterring these higher ability individuals from reducing their working hours to pretend to be less able than they truly are. Thus, the optimal rate equalizes a gain of net increase in the tax revenue with a loss of decreased utility of risk-averse taxpayers from an increment in the rate. As a consequence, zero marginal tax rate is levied on the highest ability taxpayer. More importantly, this theoretical result of zero marginal tax rate on the top earners may be modified when the tax enforcement cost is affected by the announcement of zero rate on the richest per se.
Bibliography


[78] IMF Fiscal Affairs Department (2011) "Revenue Mobilization in Developing Countries" International Monetary Fund.


Appendix A

Appendices for Chapter 1

A.1 Identification Strategy When Both Dependent Variable and Endogenous Variable Are Binary

For notational convenience, let the variable Equigeniture = $l_2$ and Intensive agriculture = $l_1$. Since this analysis is unfolded at the population level the observation index $s$ is omitted. As mentioned in Section 1.4.2, both are indicator variable. Thus,

(a) $l_2 = 1[\beta_0 \cdot l_1 + x\delta + \varepsilon > 0]$
(b) $l_1 = 1[z\omega + u > 0]$
(c) $\varepsilon = pu + e$

where $z = \{x, y\}$

Note that endogeneity arises if and only if $corr(\varepsilon, u) \neq 0$.

Let $(\varepsilon, u) \perp z$ and $(\varepsilon, u)$ follow bivariate normal with mean zero with $Var(\varepsilon) = \theta_1^2$, $Var(u) = \theta_2^2$, (So, $Cov(\varepsilon, u) = \rho \theta_1^2$)

As a consequence, $e \sim N(0, \theta_1^2 - \rho^2 \theta_2^2)$ and $e \perp (l_1, z)$.

We will opt for maximum likelihood estimator (herafeter MLE) and utilize control function approach in that the error term $u$ of equation, (b), enters into the main equation (a) to tackle the problem of correlation between $l_1$ and $\varepsilon$ for the sake of consistent estimates of $\beta_0$. This requires us to clarify the likelihood function and, in turn, the density $f$ on which the likelihood function is based.

$$f(l_2, l_1 | z) = f(l_2 | l_1, z) f(l_1 | z) = f(l_2 | l_1, z) f(l_1 | z, u) f(u | z)$$

and

$$P(l_2 = 1 | l_1 = 1, z) = E(l_2 | l_1 = 1, z)$$
$$= E(E(l_2 | z, u) | l_1 = 1, z) \text{ by Law of Iterated Expectation.}$$
$$= E(P(l_2 = 1 | z, u) | l_1 = 1, z)$$
So, we first need to know \( P(l_2 = 1 \mid z,u) \) as well as density of \( u \) given \( z \) and \( l_2 = 1 \).

\[
P(l_2 = 1 \mid z,u) = P(\beta_0 \cdot l_1 + x\omega + e > 0 \mid z,u)
\]

\[
= P\left( \frac{e}{(\theta_1^2 - \rho^2 \theta_2^2)^{\frac{1}{2}}} > \frac{-y\omega + \beta_0 l_1 + pu}{(\theta_1^2 - \rho^2 \theta_2^2)^{\frac{1}{2}}} \mid z,u \right) \text{ by (c)}
\]

\[
= \Phi\left( \frac{y\omega + \beta_0 l_1 + pu}{(\theta_1^2 - \rho^2 \theta_2^2)^{\frac{1}{2}}} \right)
\]

Since \( l_1 = 1 \) if and only if \( u > -z\omega \), the density is \( \frac{\phi\left(\frac{u}{\theta_2}\right)}{P(u > -z\omega)} = \frac{\phi\left(\frac{u}{\theta_2}\right)}{\Phi\left(\frac{y\omega + \beta_0 l_1 + pu}{(\theta_1^2 - \rho^2 \theta_2^2)^{\frac{1}{2}}}\right)} \).

Therefore, \( P(l_2 = 1 \mid l_1 = 1, z) \)

\[
= E\left( \Phi\left( \frac{y\omega + \beta_0 l_1 + pu}{(\theta_1^2 - \rho^2 \theta_2^2)^{\frac{1}{2}}} \right) \mid l_1 = 1, z \right)
\]

\[
= \int_{-\infty}^{\infty} \Phi\left( \frac{y\omega + \beta_0 l_1 + pu}{(\theta_1^2 - \rho^2 \theta_2^2)^{\frac{1}{2}}} \right) \phi\left( \frac{u}{\theta_2} \right) du \quad (B.1)
\]

As a corollary, \( P(l_2 = 0 \mid l_1 = 1, z) = 1 - P(l_2 = 1 \mid l_1 = 1, z) \). (B.2)

Using the same token,

\[
P(l_2 = 1 \mid l_1 = 0, z) = \int_{-\infty}^{\infty} \Phi\left( \frac{y\omega + \beta_0 l_1 + pu}{(\theta_1^2 - \rho^2 \theta_2^2)^{\frac{1}{2}}} \right) \phi\left( \frac{u}{\theta_2} \right) du \quad (B.3)
\]

and

\[
P(l_2 = 0 \mid l_1 = 0, z) = 1 - P(l_2 = 1 \mid l_1 = 0, z). \quad (B.4)
\]

Therefore, the log-likelihood function for observation \( s \), notated as \( \ell_s(\beta_0, \omega, \rho, \theta_1, \theta_2) \), which we will use for our MLE, can be written in a neat way, like following:

\[
\ell_s(\beta_0, \omega, \rho, \theta_1, \theta_2) = r_s \cdot l_{1s} \cdot \ln P(r = 1 \mid l_1 = 1, z)_s + (1 - r_s)l_{1s} \ln P(r = 0 \mid l_1 = 1, z)_s + r_s(1 - l_{1s}) \ln P(r = 1 \mid l_1 = 0, z)_s + (1 - r_s)(1 - l_{1s}) \ln P(r = 0 \mid l_1 = 0, z)_s
\]

, where \( \ln P(r = 1 \mid l_1 = 1, z)_s \) is the value evaluated at each observation \( s \), using (B.1) - (B.4). And \( r = 1 \) iff \( l_2 = 1 \).

Finally, the estimates are calculated from \( \arg \max \sum \ell_s(\beta_0, \omega, \rho, \theta_1, \theta_2) \)

Notably, since \( e \perp (l_1, z) \), we can get a consistent estimate for \( \beta_0 \). However, although estimates are consistent, they are only identifiable up to scale. Nevertheless, only the sign is of central interest in the paper, this does not matter. In addition to this, the asymptotic variance is driven from the Hessian of \( \sum \ell_s(\beta_0, \omega, \rho, \theta_1, \theta_2) \); and, by the Cramer-Rao Inequality, MLE gives the efficient estimator for inference.

### A.2 Soil Quality Data from HWSD

Definitions of soil quality variables of nutrient availability and available water capacity used for instrumental variables are as follows.

**A.2.1** Nutrient Availability in Subsoil (unit: cmol/kg): This variable is also called as total exchangeable bases and measures nutrient availability as the sum of exchangeable cations of Na (sodium), Ca (calcium), Mg (magnesium), and K (potassium). in a subsoil, often showing a linear correlation with pH level that is also important in the soil for crop cultivation. It is known that there is nonlinear relationship between the
suitability for crop cultivation since when this variable is divided by overall exchange capacity of soil, which is called base saturation, the critical values are like following:

<table>
<thead>
<tr>
<th>base saturation</th>
<th>soil condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;20%</td>
<td>similar to extreme acid pH</td>
</tr>
<tr>
<td>20-50%</td>
<td>correspond with acid conditions</td>
</tr>
<tr>
<td>50-80%</td>
<td>ideal condition for most crops</td>
</tr>
<tr>
<td>&gt;80%</td>
<td>often calcareous, sometimes saline</td>
</tr>
</tbody>
</table>

(source p.15 manual of HWSD 2009)

The ability to retain soil nutrient is important to increase frequency of gain cropping - intensification of agriculture - can be illustrated by showing how nutrient is exhausted in the cultivation.

<table>
<thead>
<tr>
<th>Table A.2-1</th>
<th>Nutrient Removed by Crop Cultivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>yield</td>
<td>Removed Nutrient (kg/ha)</td>
</tr>
<tr>
<td></td>
<td>(tons/ha)   N  P  K  Ca  Mg</td>
</tr>
<tr>
<td>Corn</td>
<td>7.0         128 20 37 14 11</td>
</tr>
<tr>
<td>Rice</td>
<td>8.0         106 32 20 4  1</td>
</tr>
<tr>
<td>Wheat</td>
<td>5.0         80  22 20 2.5 8.0</td>
</tr>
<tr>
<td>Cassava</td>
<td>16          64  21 100 41 21</td>
</tr>
<tr>
<td>Beans</td>
<td>1           31  3.5 6.6 - -</td>
</tr>
<tr>
<td>Bananas</td>
<td>10          19  2 54 23 30</td>
</tr>
<tr>
<td>Pineapple</td>
<td>12.5        9  2.3 29 3  3</td>
</tr>
</tbody>
</table>


A.2.2] Available Water Capacity (unit: mm/m): This variable measures water available for extraction by plant roots and consistently measured by the procedure developed by FAO and can be classified into 7 classes according to the capacity, from 0 mm/m to 150 mm/m. “It can affect both productivity, e.g. crop yields, and inputs, e.g. mulching measures necessary, or amounts of irrigation water required.” (FAO 1976)

A.3 Heckitized Tobit

For notational convenience, let the indicator variable Give any gift \( p_i = s \) and continuous variable Inter vivos gift \( t_{ip} \). In addition to this, let the right-hand side variables in the (4.3.a) be \( X_2 \) and those in (4.3.b") be \( X_1 \). Since this analysis is unfolded at the population level the observation indexes are omitted. Then, the statistical models in Section 4.3.2 can be abstractly written as follows.

(a) \( y = \max(0, X_1 \beta_2 + u) \)
(b) \( s = 1[X_2 \delta + v > 0] \)
(c) \( u = \rho v + e \)

Suppose that \( u \mid X \sim \text{Normal} (0, \sigma^2) \); \( y \) is observed only when \( s = 1 \); \( (u,v) \) is independent of \( X = (X_1, X_2) \) with zero mean; \( v \sim \text{Normal}(0,1) \)

Note that selection occurs iff \( \rho \neq 0 \). Also note that it follows that \( e \sim \text{Normal} (0, \sigma^2 - \rho) \).

Again, for notational convenience, let \( \sigma^2 - \rho = \theta^2 \). We want to know \( E(y \mid X, s = 1) \).

\[
E(y \mid X, s = 1) = P(s = 1 \mid X)E(y \mid X, s = 1)
\]

\[
= P(s = 1 \mid X)P(y > 0 \mid X, s = 1)E(y \mid X, y > 0, s = 1)
\]

First, when looking at the first two term,

\[
P(s = 1 \mid X) = P(v > -X_2 \delta \mid X) = P(v > -X_2 \delta)
\]

\[
P(y > 0 \mid X, s = 1) = P(u > -X_1 \beta_2 \mid X, s = 1)
\]

\[
= P(e > -X_1 \beta_2 - \rho v \mid v > -X_2 \delta)
\]

implies

\[
P(s = 1 \mid X)P(y > 0 \mid X, s = 1)
\]

\[
= P(e > -X_1 \beta_2 - \rho v \mid v > -X_2 \delta)P(v > -X_2 \delta)
\]

\[
= P(e > -X_1 \beta_2 - \rho v \& v > -X_2 \delta)
\]

Let set \( A = \{ v \mid v > -X_2 \delta \} = \{ v, X_2 \mid s = 1 \} \)

and \( B = \{ v, e \mid e > -X_1 \beta_2, v > -X_2 \delta \} = \{ v, e, X_1 \mid y > 0 \} \)

If \( y > 0 \) is not the case, we have either \( s = 1 \) or \( s \neq 1 \), i.e. \( B^c \supset A^c \). Equivalently, the fact that \( y \) is observed in \( y > 0 \) implies \( s = 1 \). Therefore, \( B \subset A \), thus \( P(e > -X_1 \beta_2 - \rho v \& v > -X_2 \delta) = P(B \cap A) = P(B) \)

Thus, \( P(e > -X_1 \beta_2 - \rho v \& v > -X_2 \delta) = P(e > -X_1 \beta_2 - \rho v) = \Phi(\frac{X_1 \beta_2 + \rho v}{\theta}) \)

Next, turning to the last term

\[
E(y \mid X, y > 0, s = 1) = X_1 \beta_2 + E(u \mid X, y > 0, s = 1)
\]

\[
= X_1 \beta_2 + E(\rho v + e \mid X, y > 0, s = 1)
\]

\[
= X_1 \beta_2 + \rho v + E[E(e \mid X, v, y > 0, s = 1)]
\]

\[
= X_1 \beta_2 + \rho v + E[E(e \mid X, v, y > 0)]
\]

the fact that \( y \) is observed in \( y > 0 \) implies \( s = 1 \).

\[
= X_1 \beta_2 + \rho v + E[e \mid X, v, e > -X_1 \beta_2 - \rho v]
\]

\[
= X_1 \beta_2 + \rho v + \theta E(\frac{e}{\theta} \mid \frac{e}{\theta} > \frac{-X_1 \beta_2 - \rho v}{\theta})
\]

\[
= X_1 \beta_2 + \rho v + \theta \phi(\frac{X_1 \beta_2 + \rho v}{\theta})
\]

All in all, \( E(y \mid X, s = 1) = \Phi(\frac{X_1 \beta_2 + \rho v}{\theta})(X_1 \beta_2 + \rho v) + \theta \phi(\frac{X_1 \beta_2 + \rho v}{\theta}) \)

and \( E(v \mid X, s = 1) = E(v \mid v > -X_2 \delta) = \frac{\phi(X_2 \delta)}{\Phi(X_2 \delta)} \)

Based on this underlying model derivation, estimation procedure of the Heckitized Tobit is as follows.

<Step1>
Obtain estimate of $\hat{v}$ from a Probit $s = 1[\mathbf{X}_2 \delta + v > 0]$ as $\hat{v} = \frac{\phi(X_2 \delta)}{\Phi(X_2 \delta)}$.

**Step 2**

With $\hat{v}$ do MLE based on (i) $f(0 \mid \mathbf{X}_j, v) = P(y_j = 0 \mid \mathbf{X}_j, v) = 1 - \Phi\left(\frac{X_j \beta_2 + \rho v}{\sigma}\right)$

and (ii) $f(y_j = \eta \mid \mathbf{X}_j, v) = f^*(y_j = \eta \mid \mathbf{X}_j, v)$ if $y > 0$, where $f^*(\eta \mid \mathbf{X}_j, v) = \frac{1}{\sigma} \phi\left(\frac{\eta - X_j \beta_2 - \rho v}{\sigma}\right)$

($\eta$ is placeholder of random variable $y$).

The log-likelihood is

$$l_j(\beta_2, \sigma, \rho, \theta) = 1[y_j = 0] \log[1 - \Phi\left(\frac{X_j \beta_2 + \rho v}{\sigma}\right)] + 1[y_j > 0] \{\log[\phi\left(\frac{y - X_j \beta_2 - \rho v}{\sigma}\right)] - \log(\theta)\}$$

**Step 3**

Do inference in the same way as in standard MLE.
Appendix B

Appendices for Chapter 2

Lemma 2.1 \( U_l(C_t, L) \) is increasing in consumption \( C_t \).

Proof. It is enough to show that \( \frac{\partial U_l(C_t, L)}{\partial C_t} > 0 \). Moreover, \( \frac{\partial U_l(C_t, L)}{\partial C_t} = \frac{U_{cl} - U_{cc}}{U_{cl}} \), which is positive since \( U \) is concave in consumption and \( U_{cl} \geq 0 \). □

Proposition 2.1 When a positive shock comes, an increase in retirement can be ensued by exposure of pension to the positive shock only if magnitude of a positive shock is large enough to compensate not only the earnings foregone from earlier retirement but also to meet increased necessary resources for post-retirement consumptions.

Proof. Firstly for those who enter period \( r^* \), a positive shock is realized as \( R_{r^*} \geq E_{r^*-1}[R_{r^*} | \Omega_{r^*-1}] \). Due to (2.10) and the ceteris paribus assumption that \( E_{r^*-1}[DB_{r^*} | \Omega_{r^*-1}] = E_{r^*-1}[DC_{r^*} | \Omega_{r^*-1}] \), this means that \( SS_{r^*}I_{SS}(r^*) + DC_{r^*}I_{DC}(r^*) \geq E_{r^*-1}[SS_{r^*} + DC_{r^*} | \Omega_{r^*-1}] \) and \( SS_{r^*}I_{SS}(r^*) + DB_{r^*}I_{DB}(r^*) = E_{r^*-1}[SS_{r^*} + DB_{r^*} | \Omega_{r^*-1}] \) since \( I_{SS}(r^*)I_{DB}(r^*) = 1 \) and \( I_{SS}(r^*)I_{DC}(r^*) = 1 \). Due to Lemma 2.1, this rise in pension benefit helps both DB plan worker and DC plan worker meeting the conditions (2.8) and (2.9) which was expected to hold.

Secondly, for those who enter period \( r^* - 1 \), the positive shock is realized as \( R_{r^*-1} \geq E_{r^*-2}[R_{r^*-1} | \Omega_{r^*-2}] \). To begin, it should be noted that before arrival of any shock, they are supposed to work instead of retiring. In other words,

\[
E_{r^* - 2}[U_l(A_{r^*-1} - \frac{1}{R_{r^*}} A_{r^*} + B_{r^*-1}I_B(r^* - 1), L)| \Omega_{r^*-2}] \leq E_{r^* - 2}[W_{r^*-1} | \Omega_{r^*-2}]
\]

(B.1)

where \( B_{r^*-1} = SS_{r^*-1} + DB_{r^*-1} \) or \( B_{r^*-1} = SS_{r^*-1} + DC_{r^*-1} \) that are mechanically calculated by dividing the current pension wealth by the remaining life time i.e., \( T + r^* + 2 \). In order to bring earlier retirement, the positive shock should be sufficient
to turn this into

\[
U_t(A_{r^*-1} - \frac{1}{R_{r^*}} A_{r^*} + B_{r^*-1} I_B(r^* - 1), L) > W_{r^*-1}
\]  \hspace{1cm} \text{(B.2)}

for \( \forall t \in \{r^*-1, r^*, r^* + 1, \cdots, T\} \). And

\[
E_{r^*-1}[U_c(A_{r^*-1} - \frac{1}{R_{r^*}} A_{r^*} + B_{r^*-1} I_B(r^* - 1), L) \mid \Omega_{r^*-1}]
\]

\[
= \frac{1 + \rho}{1 + \rho} U_c(A_{r^*-2} - \frac{1}{R_{r^*-1}} A_{r^*-1} + h_{r^*-2} W_{r^*-2}, L - h_{r^*-2})
\]  \hspace{1cm} \text{(B.3)}

If these two additional conditions are not met in addition to (2.8), and (2.9), the DC plan worker will not respond to the positive shock with earlier retirement; that is, no difference is made by the exposure of pension to the shock. Only when the magnitude of the shock is large enough to meet all the above additional conditions, a discernible change in retirement is followed. ■

**Proposition 2.2** When a negative shock comes, a decrease in retirement can be led by exposure of pension to the negative shock.

*Proof.* Firstly, for those who enter period \( r^* \), a DC plan worker is more likely to delay retirement since \( SS_{r^*} I_{SS}(r^*) + DC_{r^*} I_{DC}(r^*) < E_{r^*-1}[SS_{r^*} + DC_{r^*} \mid \Omega_{r^*-1}] = E_{r^*-1}[SS_{r^*} + DB_{r^*} \mid \Omega_{r^*-1}] \) failing (2.8) or (2.9).

Secondly, for those who enter period \( r^* - 1 \), (2.8) is not met regardless of the exposure since \( SS_{r^*-1} + DC_{r^*-1} < E_{r^*-2}[SS_{r^*-1} + DC_{r^*-1} \mid \Omega_{r^*-2}] \). Thus, they will keep working. ■
Figure B.1: Shift from DB plans to DC plans

Figure B.2: Rise in Size of DC accounts (Source: Form 5500 filings with the U. S Department of Labor)
Figure B.3: **Post-retirement Consumption and Exposure of Pension to Positive Shocks (Simulation)**

Figure B.4: **Post-retirement Consumption and Exposure of Pension to Negative Shocks (Simulation)**
Table B1 | Factors used for conversion to 2000 dollars

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor</td>
<td>1</td>
<td>1.045</td>
<td>1.097</td>
<td>1.171</td>
<td>1.251</td>
</tr>
</tbody>
</table>

source: Consumer Price Index by Bureau of Labor Statistics
Table B2: Summary Statistics on Industry Indicator Variables

source: HRS (Health and Retirement Study)

<table>
<thead>
<tr>
<th>Variables</th>
<th>2002 Mean (Std Dv)</th>
<th>2004 Mean (Std Dv)</th>
<th>2006 Mean (Std Dv)</th>
<th>2008 Mean (Std Dv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture/ Forestry/ Fishery</td>
<td>0.02 (0.16)</td>
<td>0.03 (0.17)</td>
<td>0.02 (0.16)</td>
<td>0.02 (0.13)</td>
</tr>
<tr>
<td>Mining/ Construction</td>
<td>0.05 (0.21)</td>
<td>0.04 (0.21)</td>
<td>0.05 (0.22)</td>
<td>0.04 (0.19)</td>
</tr>
<tr>
<td>Manufacturing non-durable</td>
<td>0.05 (0.22)</td>
<td>0.04 (0.20)</td>
<td>0.04 (0.20)</td>
<td>0.03 (0.18)</td>
</tr>
<tr>
<td>Manufacturing durable</td>
<td>0.08 (0.27)</td>
<td>0.07 (0.26)</td>
<td>0.07 (0.25)</td>
<td>0.06 (0.23)</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.06 (0.23)</td>
<td>0.05 (0.22)</td>
<td>0.05 (0.22)</td>
<td>0.04 (0.20)</td>
</tr>
<tr>
<td>Wholesale</td>
<td>0.03 (0.18)</td>
<td>0.03 (0.18)</td>
<td>0.04 (0.19)</td>
<td>0.03 (0.17)</td>
</tr>
<tr>
<td>Retail</td>
<td>0.10 (0.31)</td>
<td>0.11 (0.31)</td>
<td>0.10 (0.31)</td>
<td>0.08 (0.27)</td>
</tr>
<tr>
<td>Finance/ Insurance/ Real Estate</td>
<td>0.07 (0.25)</td>
<td>0.07 (0.25)</td>
<td>0.07 (0.26)</td>
<td>0.06 (0.23)</td>
</tr>
<tr>
<td>Repair Service Business</td>
<td>0.07 (0.26)</td>
<td>0.07 (0.25)</td>
<td>0.06 (0.25)</td>
<td>0.05 (0.21)</td>
</tr>
<tr>
<td>Personal Service</td>
<td>0.05 (0.21)</td>
<td>0.05 (0.22)</td>
<td>0.05 (0.21)</td>
<td>0.03 (0.17)</td>
</tr>
<tr>
<td>Entertainment / Recreation</td>
<td>0.01 (0.13)</td>
<td>0.02 (0.13)</td>
<td>0.02 (0.14)</td>
<td>0.01 (0.11)</td>
</tr>
<tr>
<td>Professional and Related Service</td>
<td>0.28 (0.45)</td>
<td>0.29 (0.45)</td>
<td>0.29 (0.46)</td>
<td>0.23 (0.42)</td>
</tr>
<tr>
<td>Others</td>
<td>0.09 (0.29)</td>
<td>0.09 (0.29)</td>
<td>0.09 (0.28)</td>
<td>0.30 (0.54)</td>
</tr>
<tr>
<td>No. of Obs</td>
<td>6930</td>
<td>6238</td>
<td>5988</td>
<td>5382</td>
</tr>
</tbody>
</table>

Note: (i) Each indicator variable takes value one if a respondent worker in the sample is working for the industry; otherwise zero.
Table B3] Retirement Rate and Exposure of Pension to Stock Market Gains in Expansion Periods

Accelerated Failure Time Model (Log-logistic distribution)

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>log(Years in labor force) in 2004</th>
<th></th>
<th>log(Years in labor force) in 2006</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef</td>
<td>(Std Err)</td>
<td>Coef</td>
<td>(Std Err)</td>
</tr>
<tr>
<td>Pension exposed to positive shock (=1)</td>
<td>-0.018</td>
<td>(0.016)</td>
<td>-0.036</td>
<td>(0.071)</td>
</tr>
<tr>
<td>ERA constraint (below 62) (=1)</td>
<td>0.165</td>
<td>(0.028)**</td>
<td>0.197</td>
<td>(0.030)**</td>
</tr>
<tr>
<td>DC plan value</td>
<td>0.002</td>
<td>(0.002)</td>
<td>0.002</td>
<td>(0.002)</td>
</tr>
<tr>
<td>DB plan value</td>
<td>-0.004</td>
<td>(0.003)</td>
<td>-0.005</td>
<td>(0.004)</td>
</tr>
<tr>
<td>SS wealth value</td>
<td>-0.003</td>
<td>(0.006)</td>
<td>-0.002</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Wealth</td>
<td>-0.007</td>
<td>(0.002)**</td>
<td>-0.004</td>
<td>(0.001)**</td>
</tr>
<tr>
<td>Financial wealth in stock mkt</td>
<td>-0.013</td>
<td>(0.005)**</td>
<td>-0.009</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Earnings</td>
<td>0.159</td>
<td>(0.036)**</td>
<td>0.176</td>
<td>(0.033)**</td>
</tr>
<tr>
<td>Early out window offered (=1)</td>
<td>-0.131</td>
<td>(0.048)**</td>
<td>-0.100</td>
<td>(0.060)*</td>
</tr>
<tr>
<td>Health problem (=1)</td>
<td>-0.144</td>
<td>(0.022)**</td>
<td>-0.122</td>
<td>(0.027)**</td>
</tr>
<tr>
<td>Risk averseness</td>
<td>0.007</td>
<td>(0.009)</td>
<td>0.011</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Age</td>
<td>-0.015</td>
<td>(0.007)**</td>
<td>-0.014</td>
<td>(0.005)**</td>
</tr>
<tr>
<td>Age²</td>
<td>0.0005</td>
<td>(0.0001)**</td>
<td>0.0002</td>
<td>(0.0001)**</td>
</tr>
<tr>
<td>Male (=1)</td>
<td>0.185</td>
<td>(0.019)**</td>
<td>0.176</td>
<td>(0.019)**</td>
</tr>
<tr>
<td>Married (=1)</td>
<td>-0.022</td>
<td>(0.019)</td>
<td>-0.013</td>
<td>(0.020)</td>
</tr>
<tr>
<td>Education</td>
<td>0.015</td>
<td>(0.003)**</td>
<td>0.023</td>
<td>(0.003)**</td>
</tr>
</tbody>
</table>

12 Industry indicators included? | Yes | Yes |

| No. of Obs | 6238 | 5988 |
| Log likelihood (χ²) | -1894.9 (397.38)** | -7209.9 (503.63)** |

Note: (i) ‘Pension exposed to positive shock’ takes one if a worker exposes his pension to stock market in an expansion period (either 2004 or 2006). (ii) *** (**; and * indicates that the estimate is statistically significant at 1% level (5% level; and 10% level) (iii) Each indicator variable takes value one if description of the variable with (=1) holds; otherwise zero. (iv) Variables of wealth and earnings are in 1,000,000 $. (v) ‘Risk averseness’ is a categorical variable ranging from one to four (the most risk-averse); and, this is based on a question on whether a respondent would take a new job whose income is uncertain between doubling the current one and some cuts.
Table B4] Retirement Rate and Exposure of Pension to Stock Market Losses in Recession Periods

Accelerated Failure Time Model (Log-logistic distribution)

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>log(Years in labor force) in 2002</th>
<th>log(Years in labor force) in 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pension exposed to negative shock (=1)</td>
<td>0.102 (0.052)**</td>
<td>0.168 (0.037)***</td>
</tr>
<tr>
<td>ERA constraint (below 62) (=1)</td>
<td>0.073 (0.018)***</td>
<td>0.018 (0.003)***</td>
</tr>
<tr>
<td>DC plan value</td>
<td>-0.001 (0.002)</td>
<td>-0.001 (0.001)</td>
</tr>
<tr>
<td>DB plan value</td>
<td>-0.001 (0.001)</td>
<td>-0.002 (0.002)</td>
</tr>
<tr>
<td>SS wealth value</td>
<td>-0.004 (0.007)</td>
<td>-0.008 (0.004)**</td>
</tr>
<tr>
<td>Wealth</td>
<td>-0.001 (0.001)</td>
<td>-0.0003 (0.0003)</td>
</tr>
<tr>
<td>Financial wealth in stock mkt</td>
<td>-0.002 (0.002)</td>
<td>-0.001 (0.003)</td>
</tr>
<tr>
<td>Earnings</td>
<td>0.029 (0.010)**</td>
<td>0.118 (0.032)***</td>
</tr>
<tr>
<td>Early out window offered (=1)</td>
<td>-0.105 (0.036)***</td>
<td>0.025 (0.007)***</td>
</tr>
<tr>
<td>Health problem (=1)</td>
<td>-0.089 (0.017)***</td>
<td>-0.170 (0.023)***</td>
</tr>
<tr>
<td>Risk averseness</td>
<td>0.006 (0.006)</td>
<td>-0.009 (0.009)</td>
</tr>
<tr>
<td>Age</td>
<td>-0.039 (0.011)***</td>
<td>-0.010 (0.003)***</td>
</tr>
<tr>
<td>Age²</td>
<td>0.0002 (0.0001)**</td>
<td>0.0002 (0.0001)**</td>
</tr>
<tr>
<td>Male (=1)</td>
<td>0.200 (0.014)***</td>
<td>0.162 (0.019)***</td>
</tr>
<tr>
<td>Married (=1)</td>
<td>-0.019 (0.015)</td>
<td>0.011 (0.020)</td>
</tr>
<tr>
<td>Education</td>
<td>0.018 (0.002)***</td>
<td>0.012 (0.003)***</td>
</tr>
<tr>
<td>12 Industry indicators included?</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>No. of Obs</td>
<td>6930</td>
<td>5382</td>
</tr>
<tr>
<td>Log likelihood (χ²)</td>
<td>-2876.2 (528.93)***</td>
<td>-1959.7 (385.60)***</td>
</tr>
</tbody>
</table>

Note: (i) ‘Pension exposed to negative shock’ takes one if a worker exposes his pension to stock market in a recession period (either 2002 or 2008). (ii) *** (**;and*) indicates that the estimate is statistically significant at 1% level (5% level; and 10% level) (iii) Each indicator variable takes value one if description of the variable with (=1) holds; otherwise zero. (iv) Variables of wealth and earnings are in 1,000,000 $. (v) ‘Risk averseness’ is a categorical variable ranging from one to four (the most risk-averse); and, this is based on a question on whether a respondent would take a new job whose income is uncertain between doubling the current one and some cuts.
## Table B5: Retirement and Exposure of Pension to Stock Market Losses in Expansion Periods

**Probit Model**

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Whether a worker retires in 2004</th>
<th>Whether a worker retires in 2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coef</td>
<td>(Std Err)</td>
<td>Coef</td>
</tr>
<tr>
<td>Marginal Effect</td>
<td></td>
<td>Marginal Effect</td>
</tr>
<tr>
<td><strong>Explanatory Variables</strong></td>
<td></td>
<td><strong>Explanatory Variables</strong></td>
</tr>
<tr>
<td>Pension exposed to positive shock (=1)</td>
<td>0.044 (0.043)</td>
<td>0.013</td>
</tr>
<tr>
<td>ERA constraint (below 62) (=1)</td>
<td>-0.359 (0.070)**</td>
<td>-0.105</td>
</tr>
<tr>
<td>DC plan value</td>
<td>-0.006 (0.005)</td>
<td>-0.001</td>
</tr>
<tr>
<td>DB plan value</td>
<td>0.023 (0.063)</td>
<td>0.005</td>
</tr>
<tr>
<td>SS wealth value</td>
<td>0.005 (0.015)</td>
<td>0.001</td>
</tr>
<tr>
<td>Wealth</td>
<td>0.015 (0.006)**</td>
<td>0.003</td>
</tr>
<tr>
<td>Financial wealth in stock mkt</td>
<td>0.017 (0.013)</td>
<td>0.004</td>
</tr>
<tr>
<td>Earnings</td>
<td>-0.297 (0.082)**</td>
<td>-0.075</td>
</tr>
<tr>
<td>Early out window offered (=1)</td>
<td>0.456 (0.128)**</td>
<td>0.138</td>
</tr>
<tr>
<td>Health problem (=1)</td>
<td>0.403 (0.061)**</td>
<td>0.117</td>
</tr>
<tr>
<td>Risk averseness</td>
<td>-0.014 (0.023)</td>
<td>-0.003</td>
</tr>
<tr>
<td>Age</td>
<td>0.103 (0.034)**</td>
<td>0.026</td>
</tr>
<tr>
<td>Age²</td>
<td>-0.002 (0.001)**</td>
<td>-0.0001</td>
</tr>
<tr>
<td>Years in labor force</td>
<td>0.014 (0.002)**</td>
<td>0.003</td>
</tr>
<tr>
<td>Male (=1)</td>
<td>-0.042 (0.051)</td>
<td>-0.010</td>
</tr>
<tr>
<td>Married (=1)</td>
<td>0.013 (0.049)</td>
<td>0.003</td>
</tr>
<tr>
<td>Education</td>
<td>-0.034 (0.008)**</td>
<td>-0.008</td>
</tr>
</tbody>
</table>

|          | Yes                        | Yes                        |
| 12 Industry indicators included? |                                  |                                  |
| No. of Obs | 6238                       | 5988                       |
| Log likelihood ($\chi^2$) | -2399.6 (494.45)** | -2856.9 (684.4)** |

Note: (i) ‘Pension exposed to positive shock’ takes one if a worker exposes his pension to stock market in an expansion period (either 2004 or 2006). (ii) **(*** and ***) indicates that the estimate is statistically significant at 1% level (5% level; and 10% level) (iii) Each indicator variable takes value one if description of the variable with (=1) holds; otherwise zero. (iv) Variables of wealth and earnings are in 1,000,000 $. (v) ‘Risk averseness’ is a categorical variable ranging from one to four (the most risk-averse); and, this is based on a question on whether a respondent would take a new job whose income is uncertain between doubling the current one and some cuts.
<table>
<thead>
<tr>
<th>Dependend Variable</th>
<th>Whether a worker retires in 2002</th>
<th>Whether a worker retires in 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coeff (Std Err)</td>
<td>Marginal Effect</td>
</tr>
<tr>
<td>Pension exposed to negative shock (=1)</td>
<td>-0.271 (0.040)**</td>
<td>-0.027</td>
</tr>
<tr>
<td>ERA constraint (below 62) (=1)</td>
<td>-0.233 (0.073)**</td>
<td>-0.063</td>
</tr>
<tr>
<td>DC plan value</td>
<td>0.016 (0.074)</td>
<td>0.004</td>
</tr>
<tr>
<td>DB plan value</td>
<td>0.002 (0.003)</td>
<td>0.0005</td>
</tr>
<tr>
<td>SS wealth value</td>
<td>0.004 (0.002)</td>
<td>0.001</td>
</tr>
<tr>
<td>Wealth</td>
<td>0.006 (0.002)**</td>
<td>0.0001</td>
</tr>
<tr>
<td>Financial wealth in stock mkt</td>
<td>0.003 (0.006)</td>
<td>0.0009</td>
</tr>
<tr>
<td>Earnings</td>
<td>-0.061 (0.027)**</td>
<td>-0.016</td>
</tr>
<tr>
<td>Early out window offered (=1)</td>
<td>0.538 (0.119)**</td>
<td>0.173</td>
</tr>
<tr>
<td>Health problem (=1)</td>
<td>0.316 (0.055)**</td>
<td>0.093</td>
</tr>
<tr>
<td>Risk averseness</td>
<td>-0.002 (0.021)</td>
<td>-0.001</td>
</tr>
<tr>
<td>Age</td>
<td>0.097 (0.031)**</td>
<td>0.025</td>
</tr>
<tr>
<td>Age^2</td>
<td>-0.0003 (0.0001)**</td>
<td>-0.0001</td>
</tr>
<tr>
<td>Years in labor force</td>
<td>0.016 (0.002)</td>
<td>0.004</td>
</tr>
<tr>
<td>Male (=1)</td>
<td>-0.043 (0.046)</td>
<td>-0.011</td>
</tr>
<tr>
<td>Married (=1)</td>
<td>0.054 (0.068)</td>
<td>0.014</td>
</tr>
<tr>
<td>Education</td>
<td>-0.020 (0.007)**</td>
<td>-0.005</td>
</tr>
<tr>
<td>12 Industry indicators included?</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

No. of Obs 6930 5382
Log likelihood (χ^2) -2887.3 (519.41)** -2382.4 (584.19)**

Note: (i) ‘Pension exposed to negative shock’ takes one if a worker exposes his pension to stock market in a recession period (either 2002 or 2008). (ii) *** (**; and*) indicates that the estimate is statistically significant at 1% level (5% level; and 10% level) (iii) Each indicator variable takes value one if description of the variable with (=1) holds; otherwise zero. (iv) Variables of wealth and earnings are in 1,000,000 $. (v) ‘Risk averseness’ is a categorical variable ranging from one to four (the most risk-averse); and, this is based on a question on whether a respondent would take a new job whose income is uncertain between doubling the current one and some cuts.
Table B7 | Retirement & Exposure of Pension to Stock Market Gains in Expansion

Diff-in-diff in Probit Model

Treat: On expansion, DC account is invested in stock market

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Coeff</th>
<th>(Std Err)</th>
<th>Marginal Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treat × After</td>
<td>0.116</td>
<td>(0.158)</td>
<td>0.039</td>
</tr>
<tr>
<td>Pension exposed to 2004 positive shock (Treat=1)</td>
<td>−0.054</td>
<td>(0.039)</td>
<td>−0.019</td>
</tr>
<tr>
<td>After (=1)</td>
<td>−0.139</td>
<td>(0.041)**</td>
<td>−0.157</td>
</tr>
<tr>
<td>ERA constraint (below 62) (=1)</td>
<td>−0.410</td>
<td>(0.046)**</td>
<td>−0.102</td>
</tr>
<tr>
<td>DC plan value</td>
<td>0.002</td>
<td>(0.003)</td>
<td>0.001</td>
</tr>
<tr>
<td>DB plan value</td>
<td>0.019</td>
<td>(0.049)</td>
<td>0.006</td>
</tr>
<tr>
<td>SS wealth value</td>
<td>0.005</td>
<td>(0.005)</td>
<td>0.002</td>
</tr>
<tr>
<td>Wealth</td>
<td>0.004</td>
<td>(0.002)**</td>
<td>0.001</td>
</tr>
<tr>
<td>Financial wealth in stock mkt</td>
<td>0.003</td>
<td>(0.003)</td>
<td>0.001</td>
</tr>
<tr>
<td>Earnings</td>
<td>−0.761</td>
<td>(0.055)**</td>
<td>−0.275</td>
</tr>
<tr>
<td>Early out window offered (=1)</td>
<td>0.176</td>
<td>(0.091)**</td>
<td>0.065</td>
</tr>
<tr>
<td>Health problem (=1)</td>
<td>0.257</td>
<td>(0.045)**</td>
<td>0.103</td>
</tr>
<tr>
<td>Risk averseness</td>
<td>−0.007</td>
<td>(0.016)</td>
<td>−0.002</td>
</tr>
<tr>
<td>Age</td>
<td>0.071</td>
<td>(0.023)**</td>
<td>0.025</td>
</tr>
<tr>
<td>Age²</td>
<td>−0.002</td>
<td>(0.001)**</td>
<td>−0.0001</td>
</tr>
<tr>
<td>Years in labor force</td>
<td>0.008</td>
<td>(0.001)**</td>
<td>0.002</td>
</tr>
<tr>
<td>Male (=1)</td>
<td>−0.007</td>
<td>(0.035)</td>
<td>−0.002</td>
</tr>
<tr>
<td>Married (=1)</td>
<td>−0.006</td>
<td>(0.034)</td>
<td>−0.002</td>
</tr>
<tr>
<td>Education</td>
<td>−0.035</td>
<td>(0.005)**</td>
<td>−0.012</td>
</tr>
<tr>
<td>12 Industry indicators included?</td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of Obs</td>
<td>13168</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log likelihood ($\chi^2$)</td>
<td>−4751.7 (658.3)**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: (i) ‘Pension exposed to 2004 positive shock (Treat)’ takes one if a worker exposes his pension to stock market in 2004; and ‘After’ takes one if an observation is from 2004 wave. (ii) *** (**; and *) indicates that the estimate is statistically significant at 1% level (5% level; and 10% level). (iii) Each indicator variable takes value one if description of the variable with (=1) holds; otherwise zero. (iv) Variables of wealth and earnings are in $1,000,000$. (v) ‘Risk averseness’ is a categorical variable ranging from one to four (the most risk-averse); and, this is based on a question on whether a respondent would take a new job whose income is uncertain between doubling the current one and some cuts.
Table B8] Retirement & Exposure of Pension to Stock Market Losses in Recession

Diff-in-diff in Probit Model

Treat: On expansion, DC account is invested in stock market

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Coeff</th>
<th>(Std Err)</th>
<th>Marginal Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treat × After</td>
<td>-0.138</td>
<td>(0.056)**</td>
<td>-0.028</td>
</tr>
<tr>
<td>Pension exposed to 2008 negative shock (Treat=1)</td>
<td>-0.022</td>
<td>(0.096)</td>
<td>-0.001</td>
</tr>
<tr>
<td>After (=1)</td>
<td>0.246</td>
<td>(0.035)**</td>
<td>0.056</td>
</tr>
<tr>
<td>ERA constraint (below 62) (=1)</td>
<td>-0.373</td>
<td>(0.153)**</td>
<td>-0.078</td>
</tr>
<tr>
<td>DC plan value</td>
<td>0.001</td>
<td>(0.003)</td>
<td>0.003</td>
</tr>
<tr>
<td>DB plan value</td>
<td>-0.003</td>
<td>(0.002)</td>
<td>0.001</td>
</tr>
<tr>
<td>SS wealth value</td>
<td>0.005</td>
<td>(0.008)</td>
<td>0.001</td>
</tr>
<tr>
<td>Wealth</td>
<td>0.002</td>
<td>(0.001)**</td>
<td>0.0003</td>
</tr>
<tr>
<td>Financial wealth in stock mkt</td>
<td>0.003</td>
<td>(0.002)</td>
<td>0.001</td>
</tr>
<tr>
<td>Earnings</td>
<td>-0.142</td>
<td>(0.057)**</td>
<td>-0.322</td>
</tr>
<tr>
<td>Early out window offered (=1)</td>
<td>0.705</td>
<td>(0.103)**</td>
<td>0.207</td>
</tr>
<tr>
<td>Health problem (=1)</td>
<td>0.849</td>
<td>(0.032)**</td>
<td>0.252</td>
</tr>
<tr>
<td>Risk averseness</td>
<td>0.006</td>
<td>(0.015)</td>
<td>0.001</td>
</tr>
<tr>
<td>Age</td>
<td>0.176</td>
<td>(0.024)**</td>
<td>0.035</td>
</tr>
<tr>
<td>Age²</td>
<td>-0.001</td>
<td>(0.0001)**</td>
<td>-0.0002</td>
</tr>
<tr>
<td>Years in labor force</td>
<td>0.018</td>
<td>(0.001)**</td>
<td>0.004</td>
</tr>
<tr>
<td>Male (=1)</td>
<td>-0.025</td>
<td>(0.030)</td>
<td>-0.023</td>
</tr>
<tr>
<td>Married (=1)</td>
<td>0.007</td>
<td>(0.005)</td>
<td>0.020</td>
</tr>
<tr>
<td>Education</td>
<td>-0.097</td>
<td>(0.032)**</td>
<td>-0.002</td>
</tr>
<tr>
<td>12 Industry indicators included?</td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

No. of Obs 11370
Log likelihood ($\chi^2$) -5415.1 (634.86)**

Note: (i) ‘Pension exposed to 2008 negative shock (Treat)’ takes one if a worker exposes his pension to stock market in 2008; and ‘After’ takes one if an observation is from 2008 wave. (ii) *** (**;and*) indicates that the estimate is statistically significant at 1% level (5% level; and 10% level). (iii) Each indicator variable takes value one if description of the variable with (=1) holds; otherwise zero. (iv) Variables of wealth and earnings are in 1,000,000 $. (v) ‘Risk averseness’ is a categorical variable ranging from one to four (the most risk-averse); and, this is based on a question on whether a respondent would take a new job whose income is uncertain between doubling the current one and some cuts.