Bedrock Erosion by Granular Flow

By

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A dissertation submitted in partial satisfaction of the requirements for the degree of Doctor of Philosophy in Earth and Planetary Science in the Graduate Division of the University of California, Berkeley

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Abstract

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Field studies suggest that bedrock incision by granular flows may be the primary process cutting valleys in steep, unglaciated landscapes. The mechanisms of granular flow incision, however, are not well quantified. Here I present a suite of laboratory experiments describing processes and rates of bedrock erosion by granular flows. In the first study, experiments in a 56 cm diameter, 15 cm wide rotating drum test the hypothesis that bedrock erosion is related to grain inertial stresses which scale with shear rate and particle size. In 67 experimental runs, the eroded depth of the bedrock sample varied with inertial stresses in the granular flow to a power less than 1.0, and inversely with the bedrock strength. The second study used a new debris flow flume facility comprised of a 4-meter diameter, 80-cm wide vertically rotating drum to measure mean and fluctuating normal forces at the base of granular flows. The mean bulk force scaled with the static weight of the flow, while the variance of the force was a function of grain diameter, flow velocity, and matrix fluid properties. I show that the square of the impulse, related to kinetic energy transferred to the bed from granular collisions, can be quantified as the variance of the force signal. My results provide the first quantitative relationships between a metric for the collisional energy at the boundary and measurable properties of field-scaled flows. In the third study, I measured erosion of synthetic bedrock samples in the 4 meter diameter drum to test three models for the relationship between bedrock erosion rate and measured basal forces: (1) erosion by impact wear resulting from forces due to bulk inertial solid stress (2) erosion by sliding wear scaled by bulk normal force (3) erosion by impact wear scaled by the square of the impulse exerted on the bed. Based on my experimental observations, I propose a debris flow erosion rule that includes components of both sliding and impact wear, whose relative importance is scaled by experimentally-tested variables. Finally, as part of my investigations of controls on boundary forces and bedrock wear, I observed grain segregation processes and fluid-sediment interactions that were previously undescribed in the literature. These included lateral oscillations of the flow front and the formation of asymmetric coarse-particle gyres. I documented the first order controls on segregation by particle size, boundary conditions, fluid content, and fluid viscosity.
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Chapter 1

Introduction: Bedrock erosion by granular flow

Mass movements such as debris flows, rock avalanches, and landslides are all examples of granular flows in nature. Each of these processes alters the landscape, and over geomorphic time scales, the sum of events plays an important role in long term landscape evolution [e.g. Stock and Dietrich, 2003]. Boulder-rich debris flows scour steep landscapes, incising channels and moving sediment to lower gradients [e.g. Stock et al., 2005]. By examining the physical mechanisms of bedrock erosion by these granular flows, we obtain a richer understanding of the variables that control incision rate [e.g. Dietrich et al., 2003; Stock and Dietrich, 2006]. A process-based rule for debris flow incision guides the development of testable hypotheses for the field, laboratory, and numerical simulations. Large scale laboratory experiments with natural field materials (water, clay, silt, sand, gravel, cobbles, boulders) are indispensable for obtaining the quantity and precision of observations needed to construct such a rule. However, large-scale experiments have been extremely rare, one notable exception being the USGS debris flow flume facility where basal forces, pore pressures, and run out patterns have been quantified for ~10 m$^3$ flows of natural debris flow materials [Iverson, 1997; Logan and Iverson, 2007; Iverson et al., 2010]. Still, systematic measurements of bed erosion and analysis of the high frequency basal force fluctuations of wide grain size distribution flows are lacking. To address this gap we developed a new debris flow flume facility with an instrumented 4 meter diameter, 80 cm wide vertically-rotating drum. The following four chapters describe a series of laboratory experiments that explore and quantify erosion, boundary forces, and segregation dynamics of granular flows, and contribute toward a mechanistic rule for granular flow incision.

In Chapter 2, I report my initial small scale experiments to test an expression for debris flow incision into bedrock. I conducted laboratory experiments to test the hypothesis that bedrock erosion is related to grain collisional stresses which scale with shear rate and particle size. Granular material was placed in a 56-cm diameter, 15 cm wide rotating drum to explore the relationship between erosion of a synthetic bedrock sample and variables such as grain size, shear rate, water content, and bed strength. Grain collisional stresses were estimated as the inertial stress using the product of the squares of particle size and vertical shear rate. The uniform granular material consisted of 1 mm sand and quartzite river gravel with means of 4, 6, or 10 mm. In 67 experimental runs, the eroded depth of the bed sample varied with inertial stresses in the granular flow to a power less than 1.0, and inversely with the bed strength. The flows tended to slip on smooth boundaries, resulting in higher erosion rates than no-slip cases. We found that lateral wall resistance generated shear across the channel, producing two cells whose widths depended on wall roughness. While the hypothesized inertial stress dependency was supported with these data, the observations suggest that erosion by basal slip and grain dynamics specifically at the flow front require further investigation.
In Chapter 3, I describe experiments in the 4 meter diameter, 80 cm wide drum to learn about the mechanisms controlling the basal normal forces. The basal force is an important measure for understanding near-bed grain dynamics, bed surface erosion, and energy dissipation, but we have few quantitative measurements and analyses of what controls the force fluctuations caused by natural geophysical granular flows. The mean force is a result of the bulk properties of the flow, and the fluctuating component is related to the grain dynamics within the flow. A 15x15 cm load plate measured the bed normal force of a range of granular flows. I analyzed the bed forces generated in flows composed of geological granular material (clay, silt, sand, gravel, cobbles, water) in both narrow and wide grain size distributions. I show that the square of the impulse, related to kinetic energy transferred to the bed from the granular collisions, can be quantified as the variance of the force signal. The mean bulk force scaled with the static weight of the flow, while the variance of the force was a function of grain diameter, flow velocity, and matrix fluid properties. The tail of the distribution of bed forces had distinct shapes for the narrow and wide grain size distributions, as fit by a generalized Pareto distribution. The force variance generated by narrow and wide grain size distributions had similar power law dependence on an inertial stress scaling term when $D_{84}$ was used as the effective diameter. These results provide the first quantitative relationships in field-scaled materials between a metric for the collisional energy at the boundary and the properties of flows that might control this metric.

In Chapter 4, I report large scale laboratory experiments that were executed to test three models for the relationship between bedrock erosion rate and measured basal forces: (1) erosion by impact wear resulting from forces due to bulk inertial solid stress (2) erosion by sliding wear scaled by bulk normal force (3) erosion by impact wear scaled by the force variance on the bed. I measured erosion of synthetic bedrock samples with varying tensile strength and the associated basal normal forces. The eroded samples showed evidence of both sliding and impact wear. Water-saturated gravels conformed most strongly to the sliding wear relationship, consistent with direct observations of sliding motion and strong evidence of sliding seen on the surface of the erosion samples. Muddy mixtures more closely followed relationships of impact wear due to inertial stress and force variance (a measure of the square of impulse on the bed). Samples worn by dry gravel flow had the least sliding evidence. Sliding was favored by smooth beds, finer particles and water as the fluid component. Impact wear was dominant for rough beds, coarser sediment, and dry conditions. My experiments suggest that small changes in boundary roughness, water content and grain size could significantly change the relative importance of sliding in the flow. I propose a debris flow erosion rule that includes components of both sliding and impact wear, whose relative importance is scaled by variables measured in our experiments. These results provide the first quantitative relationships between bedrock erosion and measured basal normal forces in field-scale experiments.

In Chapter 5, I describe size segregation in the laboratory experiments. In nature, geophysical granular flows create coarse-grained fronts, lateral levees and vertically-graded profiles. Size segregation affects internal particle dynamics, boundary forces, and basal erosion, but the mechanisms responsible for the origin and maintenance of size segregation are not well understood. Although segregation studies are numerous, there are few large scale experiments using a wide grain-size distribution of natural geologic materials in viscous matrix fluid. As part of my investigation of controls on boundary forces and bedrock wear associated with shallow granular flows, I observed grain segregation processes and fluid-sediment interactions that were previously undescribed in the literature. These included a number of unexpected flow...
phenomena arising from boundary conditions and flow asymmetry, including lateral oscillations and the formation of asymmetric coarse-particle gyres. I also studied segregation of spherical glass marbles in a 56 cm diameter, 15 cm wide drum. I tracked the trajectories of particles on the flow surface and observed dynamically evolving size segregation, which was sometimes asymmetric, and sometimes spanned the front of the flow, like the coarse-grained fronts seen in nature. In shallow flows, first order controls on segregation were particle size, boundary conditions, fluid content, and fluid viscosity. Contact with the boundaries was a major factor in the trajectory of a particle. My observations suggest the fluid and solids in a debris flow may take different paths through the flow. I propose that particle segregation processes (including the tendency for a coarse front to form) are influenced by (a) boundary roughness, (b) flow velocity, (c) the ratio of grain size of interest to representative grain size for the flow, (d) the ratio of grain size to flow depth, (e) the ratio of grain size to flow width, and (f) the viscosity of the flow relative to that of water. These observations point to some important scaling and process controls for segregation which can be used for a quantitative theory for segregation and coarse front formation.

Experiments in the new debris flow flume emphasized the importance of particle-particle, particle-fluid, and particle-boundary interactions in large scale granular flows with wide grain size distributions. The experimental observations uncovered many remaining questions about granular flow dynamics, and on-going work in the flume is exploring grain-size dependent mechanics [Johnson et al., 2008], pore fluid pressure fluctuations [Kaitna et al., 2008], and detailed internal velocity profiles [Kaitna et al., 2009]. A major next step is to test the erosion of non-homogeneous substrate samples where fracture and plucking occur. The importance of boundary interaction in my erosion measurements call for more experiments that purposely explore basal sliding and bed roughness. The force plate measurement analysis advocates collaboration with numerical modelers, which will help to interpret and predict collisional force distributions at the boundary and within the flow [Yohannes et al., 2008]. In addition, my experiments can supply target data sets for numerical simulations of water-saturated or muddy flows. In the field, more wear measurements, quantification of bed topography and jointing, and measurement of flow generated force and fluid pore pressure will help to determine the scaling relationships between experiment and nature. The experiments presented here provide the foundation on which to build additional quantitative analyses of near field-scale flows, with the ultimate goal of a mechanistic, data-validated, field-applicable rule for bedrock erosion by granular flow.

References


Johnson, J. P., Hsu, L. and Dietrich, W. (2008), Clever Cobbles, Embedded Accelerometers:


Chapter 2

Experimental study of bedrock erosion by granular flows

2.1 Introduction

As rock avalanches and debris flows surge down steep valleys, they entrain loose material [e.g. Dietrich and Dunne, 1978; Pierson, 1980; Suwa and Okuda, 1980; Hungr et al., 1984; Benda, 1990] and generate granular collisions with the bed, which can cause visible wear and plucking of the underlying bedrock [Stock and Dietrich, 2003; Papa et al., 2004; Stock et al., 2005; Stock and Dietrich, 2006]. These mass failure events fall under the general category of granular flows, gravity-driven masses of discrete solids with an interstitial fluid [Campbell, 1990]. The mass removal by granular flow erosion into bedrock is minor compared to loose debris entrainment, but could be the dominant process cutting valleys into steep landscapes [e.g. Stock and Dietrich, 2003]. During debris flow passage, video cameras have recorded huge boulders sliding on and colliding with the bed [Swartz and McArdell, 2005], force plates have documented large normal and shear stress values [McArdell et al., 2007], and ultrasonic gauges have measured significant ground vibrations [e.g. Arattano and Moia, 1999; Arattano and Franzi, 2004; Itakura et al., 2005 and references therein; Huang et al. 2007], all observations suggesting potentially significant collision-driven erosion. Concrete check dams in debris-flow-dominated channels exhibit grooves, knicks, and missing blocks [Figure A1, Auxiliary Material]. Field studies have shown that after a debris flow’s passage, its channels can be swept clean to bedrock [e.g. Dietrich and Dunne, 1978; DeGraff, 1994; Howard, 1998; May and Gresswell, 2003; Stock and Dietrich, 2003; Jakob et al., 2005], then buried by colluvium in the following months instead of worn further by fluvial action [Stock et al., 2005]. This suggests that some steep bedrock channels may be exposed to greater amounts of granular wear than fluvial wear. There is also evidence of granular wear into bedrock by pyroclastic flows, another type of granular flow [e.g. Sparks et al., 1997; Calder et al., 2000; Grunewald et al., 2000]. Thus, abundant evidence exists for erosion of bedrock channels by particle collisions in granular flows. We note that erosion and entrainment of loose colluvium by granular flows has been measured and modeled [e.g. Berti et al., 2000; Egashira et al., 2001; Itoh et al., 2003; Rickenmann et al., 2003; Wang et al., 2003; Papa et al., 2004; Hungr et al., 2005], but is not the focus of this paper.

In addition to field observations, analysis of scaling relationships between channel slope and drainage area suggests a distinct topographic signature that differentiates areas dominated by granular flows from areas dominated by fluvial flows. These channel slope-drainage area relationships from varied geographic locations exhibit a break in slope, which appears to reflect a change in erosional mechanism from granular/debris flow incision to fluvial incision into bedrock [Stock and Dietrich, 2003]. Typically there is a power-law trend in low-gradient, fluvially-dominated regimes, but at slopes steeper than ~10% there may be little or no channel steepening with decreasing drainage area [Seidl and Dietrich, 1992; Montgomery and Foufoula-
Georgiou, 1993; Lague and Davy, 2003]. Hence, these steeper channels may not follow a simple power-law relationship [Stock and Dietrich, 2003]. The location of the scaling break at or above a slope of 10% coincides with the slope at which debris flow deposits are commonly found, signifying a transition from debris flow incision to deposition [Stock and Dietrich, 2003].

Though abundant research exists on bedrock incision by fluvial flows, the above observations imply that landscape evolution models in hilly and mountainous areas require a separate theory for bedrock incision by granular flows. For bouldery debris flows, Stock and Dietrich [2006] have proposed that the coarse, fluid-poor snout should exert the greatest collisional stress on the underlying bedrock, while the relatively fluid-rich, finer grained tails may be less important to channel wear. In the coarse-grained snout, the large boulders impact the bedrock channel, causing abrasion and surface-fatigue wear. Surface fatigue causes fractures that coalesce and form removable material, leading to bedrock erosion [e.g. Momber, 2004; Stock and Dietrich, 2006 and references therein]. Hence the important variables that strongly influence erosion rates are the bouldery snout length, particle size, and particle dynamics in the snout. We hypothesize that the fracture-generation and bedrock removal by surface fatigue wear is related to localized normal stresses exerted on the bed, which in turn scale with the bulk inertial stresses generated in the flow. Other important variables are bed roughness, degree of weathering and fracturing of the bedrock and the debris flow frequency, which varies with network structure. Hence the predicted erosion rate depends not only on local physics of grain collisions with the bed, but also the network structure and associated frequency of landsliding. Stock and Dietrich [2006] summarize field and literature observations that indirectly support this hypothesis. Key untested assumptions in the model are that bedrock wear rate is proportional to grain collisional stresses, and that these stresses can be approximated by the product of the squares of both particle grain size and shear rate (inertial stresses, sensu Bagnold [1954]; Iverson [1997]). Here we will use the term inertial stress when we calculate stresses using this product.

Our goal is to explain bedrock incision rates through the mechanical processes involved in erosion by grains interacting with the boundary. Understanding the mechanisms causing erosion of the bed is important because small differences in the nature of the bed and the granular flow composition could have large effects on the amount of basal wear. For example, in some cases there is evidence of intense damage after passage of a flow, but in others the flow passes over grass and pavement with little to no effect [e.g. Perez, 2001]. In this paper we explore granular wear of bedrock and its controlling variables by first briefly reviewing the relevant theory on granular flows then reporting the results of physical modeling experiments in a rotating drum. We find a power law relation between the experimental bedrock erosion and a non-dimensional inertial stress term which includes the product of the squares of grain size and shear rate normalized by the square of the tensile strength of the bedrock. Experiments also reveal that bed roughness and water content of the flow affect the tendency for the mass to slide, which increases erosion rate when it occurs. Our observations suggest that the inertial stress model may be an inadequate approximation for collisional stresses that drive bedrock wear. We explore three simple models based on grain motion: snout-impact, zero-slip, and sliding block. Although our data can not distinguish which model best explains the erosion in the drum, the analysis points to the need for further study of the local mechanics of wear erosion.
Theoretical framework

Iverson [1997] and others, following the work of Bagnold [1954], Savage and Hutter [1989], and Campbell [1990], have proposed that the grain collisional stress in high flow rates can be estimated as

\[ \sigma_i = \nu_s \rho s D_p \left( \frac{\partial u}{\partial z} \right)^2 \]  \hspace{1cm} (1)

where \( \sigma_i \) (Pa) is referred to as the inertial stress (following Bagnold’s [1954] and Iverson’s [1997] terminology), \( \nu_s \) (-) is volumetric solids concentration, \( \rho_s \) (kg/m\(^3\)) is solid particle density, \( D_p \) (m) is particle diameter, \( u \) (m/s) is velocity and \( z \) (m) is distance from the bed. The application of equation (1) to calculate inertial stress from field or flume observations is challenging. Most commonly, it is assumed that the velocity gradient term, \( \partial u/\partial z \), can be represented by the typical flow velocity divided by the total flow depth [e.g. Iverson, 1997; Iverson and Vallance, 2001; Iverson et al., 2004]. In some cases, \( z \) is better constrained as the depth of the shear band within the flow, giving a larger shear rate value [e.g. Parsons et al., 2001]. Especially problematic is determining \( D_p \), the characteristic grain diameter. According to Bagnold [1954], grain size enters the problem in several roles: the scale for the mass of the particle of interest, the scale for the vertical spacing between shear layers, the scale for the number of particles per bed area, and the scale for the number of collisions per unit time. In heterogeneous grain mixtures, it is not obvious that one grain size would serve as the scale for all of these terms, though one could argue that there is an effective grain size, \( D_e \), whose mass and concentration capture the characteristics of the size distribution [Stock and Dietrich, 2006]. Since the \( D_p \) term is squared, the choice between using the smaller size fraction or the larger size fraction as the representative diameter can change the inertial stress estimation by several orders of magnitude. A commonly used rule is to define the solid fraction as those particles that settle out during the duration of the flow, the mean diameter of this thus-defined solid fraction is used for \( D_p \) [O’Brien and Julien, 1988; Iverson, 1997].

The collisional stress and shear in the flow is also affected by the amount of basal slip that occurs between granular particles and the bed. Although it is difficult to observe the internal structure of the debris flow, there are end-members that have been hypothesized [e.g. Parsons et al., 2001; Figure 1]. In the all-slip case, the basal grains slide along the bed at the same velocity as the entire vertical column (Figure 1a). This case is like a sliding block, and has been proposed for large rockslides and rock avalanches whose deposits have preserved structural features of the original source rock during the flow [e.g. Shreve, 1968; Van Gassen and Cruden, 1989]. At the other extreme, there is no slip at the bed and the velocity profile increases to a maximum at the surface, creating high internal shear (Figure 1b). Commonly it has been argued, however, that most of the shear is close to the bed, and the interior region is close to a plug flow (Figure 1d) [Hubert and Filipov, 1989; Savage and Hutter, 1989; Genevois et al., 2001; Parsons et al., 2001; Bi et al., 2005; Brewster et al., 2005].

These diverse styles of basal interaction generate different normal stresses on the channel bed, and it seems that the style is largely situation-specific. Theoretical studies and modeling of natural granular flow velocity profiles often have a no-slip condition at the bed [e.g. Whipple,
But measured profiles in experiments often do not have zero velocity at the bed, especially with unroughened beds and spherical particles for chutes [e.g. Azanza et al., 1999; Hanes and Walton, 2000; Louge and Keast, 2001; Ancey, 2002; Iverson et al., 2004]. However, in chute flows where the bed is sufficiently rough, or particles and velocities sufficiently small, zero velocity at the bed has been observed and modeled [e.g. Parsons et al., 2001; Silbert et al., 2001; Andreotti et al., 2002; Iverson et al., 2004].

Erosion of the bed probably depends on the mechanism of movement at the boundary. Sliding wear (all-slip) is described by Archard’s Law,

\[
e_v = \frac{kWx}{H}
\]

(2)

where \(e_v\) (length\(^3\)) is eroded volume, \(k\) (-) is a nondimensional wear coefficient dependent on the materials in contact, \(W\) (mass·length / time\(^2\)) is the applied load, \(x\) (length) is the sliding distance, and \(H\) (mass / length·time\(^2\)) is the hardness of the surface being worn away. In terms of an erosion rate, this can be written as

\[
\frac{\partial \bar{z}}{\partial t} = \frac{kp_nV}{H}
\]

(3)

where \(-\partial \bar{z}/\partial t\) (length/time) is the bed wear rate, \(p_n\) (mass / length·time\(^2\)) is the normal pressure, and \(V\) (length/time) is the sliding velocity.

For low-slip, collision-dominated wear, Stock and Dietrich [2006] proposed a rate law to express the long term erosion of successive debris flows:

\[
\frac{\partial \bar{z}}{\partial t} = \frac{K_0K_1}{T_0^2E_{eff}^2} \cdot f \cdot L \left[ \cos(\theta_0) \rho_s D_{se}^2 \left( \frac{u_s}{h} \right)^w \right]^a
\]

(4)

where \(K_0\) (-) is a constant of proportionality that relates bulk inertial normal stresses to higher excursions of inertial normal stress, \(K_1\) is a proportionality constant between rock resistance and incision rate that has dimensions that vary with \(w\) and \(n\) so that the right side of the expression has units of erosion rate, \(T_0\) (Pa) is the tensile strength of the bedrock, \(E_{eff}\) (Pa) is the elastic modulus of the bedrock, \(F\) (m) is a function of the fracture spacing of the bedrock and size of eroding boulders, \(f\) (a\(^{-1}\)) is the frequency of flows over the bedrock per annum, \(D_{se}\) (m) is the effective grain size, \(u_s\) (m/s) is the surface velocity, \(h\) (m) is the flow height, \(L\) (m) is the length of the eroding flow, and \(w\) and \(n\) are empirical exponents. \(L\) represents the length of the bouldery snout only. \(D_{se}\), the effective grain size, is defined as the particle size that characterizes the collisional normal stress that causes bedrock lowering under coarse-grained debris flow fronts [Stock and Dietrich, 2006]. Therefore, \(D_{se}\) may be significantly larger than the mean grain size of the flow. In this study, we use homogeneously-sized grain flows and therefore \(D_{se} = D_p\). The square dependency on tensile strength was demonstrated by Sklar and Dietrich [2001].
Measurements and modeling

Following Stock and Dietrich [2006], we propose that the erosion rate scales with the normal stresses caused by the bulk inertial stress of the flow. Measurement of the normal force on the bed for experimental granular chute flow has been carried out by Iverson [1997] and Ahn et al. [1991]. Iverson found high frequency fluctuations in the normal stress measurement, assumed to be related to granular impacts. In natural channels, normal stresses on the bed have been reported by Berti et al. [2000] (12-17 kPa – excursions 2kPa) and McArdell and Bartelt [2007], (5-20 kPa, excursions 2kPa). The size of the sensor plate and frequency of measurement affects the ability to resolve local grain-bed contact stresses [Iverson, 1997]. Our hypothesis for bedrock erosion relates to local instantaneous point loads that are better measured by smaller plates and greater frequency than those reported in the literature for natural flows. Hence, we further hypothesize that the high local grain collsional stresses scale with the bulk inertial stress as defined in equation (1).

Authors have employed different approaches for modeling stresses and erosion at the base of a granular flow, with no general agreement so far. One method is to scale bedrock erosion with a bed shear stress value, similar to the assumption used in many bedrock incision models for fluvial flows [e.g. Howard and Kerby, 1983; Whipple and Tucker, 1999]. For example, Howard [1998] assumed that bedrock erosion by debris flows and rock avalanches is proportional to shear stress exerted on the bed. The shear stress is calculated from the Coulomb model, with a linear relationship between maximum shearing strength, \( \tau_f \), and normal stress, \( \sigma_n \), on the failure plane:

\[
\tau_f = c + \sigma_n \tan \Phi
\]  

where \( c \) (Pa) is cohesion and \( \Phi \) (degrees) is the friction angle. Howard [1998] reasoned that weathering decreases the bed cohesion value and when the surface shear exceeds a threshold value for erosion, mass loss occurs. Pitman et al. [2003] also employed a friction law at the basal contact surface to calculate the shear stress caused by the overlying flow. They proposed that if the shear stress exceeds a threshold value, then the erosion rate scales with an empirical factor fitted to experimental results. Many models assume that changes in z-direction momentum are small compared to the static weight of the mass, such that vertical stresses other than the static normal stress are neglected [e.g. Savage and Hutter, 1989]. In terrain-fitted coordinates, centripetal acceleration accounts for much, but not all, of the vertical acceleration [Iverson and Denlinger, 2001; Pudasaini et al., 2005]. In Cartesian coordinates, an estimation of vertical stresses over an irregular bed has been accomplished by modeling stresses on the bed from basal sliding as the sum of a normal component from weight and an additional term associated with vertical acceleration due to the topography of the channel [Denlinger and Iverson, 2004].

Many different flume designs have been used to examine granular flow phenomena, including straight chutes [e.g. Savage, 1984; Ahn et al., 1991; Iverson, 1997; Azanza et al., 1999; Parsons et al., 2001], conveyor belt flumes [e.g. Davies, 1990; Hubl and Steinwendtnner, 2000; Tognacca and Minor, 2000; Perng et al., 2006], “race track” flumes [e.g. Hampton, 1975] and vertically rotating (drum) flumes [e.g. Huizinga, 1996; Jan and Chen, 1997; Hotta and Ohta, 2000; Tai et al., 2000; Gray, 2001; Longo and Lamberti, 2002; Kaitna and Rickenmann, 2005]. For bedrock erosion experiments, the most useful characteristic of a flume is the ability to sustain
long periods of flow over an erodible sample – long enough to accomplish measurable wear. We chose the vertically-rotating drum design specifically for this reason that it provides long-term flow. Recirculating chute flumes can also provide long periods of flow, but do not generate a distinct flow with a snout and tail, which may have different eroding properties. In a drum, there is also the ability to adjust the velocity easily. Major [1997] noted that assumptions about the velocity profile of the flow and slip at the drum bed cause uncertainty in interpreting data from vertically rotating drums. Here we use the common approximation that the velocity gradient can be estimated as the velocity difference between the surface and the bed divided by the flow thickness [e.g. Iverson, 1997]. We note when there is observable slip at the bed and focus on estimating the average stress on the boundary from the mean flow properties.

Because of the industrial applications of rotating drums, numerous descriptions of granular media in drums exist in the engineering and physics literature [e.g. Ristow, 1996; Buchholtz and Poschel, 1997; Boateng, 1998; Ding et al., 2001]. Of relevance to our problem, there is description of the wear of liners in industrial ball mills [Radziszewski et al., 1993a; 1993b; Radziszewski, 1997]. In these studies, the rate of metal liner wear is modeled as a function of the total normal force, which has gravitational, centrifugal, and compression (slip) components. These studies typically use thick fills in which internal grain dynamics (rather than boundary effects) prevail. Nonetheless, we can use these studies to guide our analysis of velocity profiles and solid-fluid interactions.

Scaling the experiments

In order to relate experimental studies to field conditions, as we propose to do, the relative importance of inertial, viscous, and frictional forces in the experimental flows must match that found in nature [Iverson, 1997; Denlinger and Iverson, 2001; Iverson et al., 2004]. Inertial forces arise from short-term collisions between the grains, viscous forces dominate when the fluid viscosity is high, and frictional forces occur when there are sustained contacts between grains. The ratios of these forces are described by the Bagnold and Savage non-dimensional numbers, which we evaluated in our experiments and compared with the values for natural granular flows.

The Bagnold number is a measure of the ratio of inertial to viscous stresses in a granularflow. As represented by Iverson [1997], it is

$$N_{Bag} = \frac{\nu_s \rho_s D_p^2 \frac{\partial u}{\partial z}}{\nu_f \mu} \approx \frac{\nu_s \rho_s D_p^2 \frac{u_i}{h}}{\nu_f \mu} \quad (6)$$

where $\nu_f (-)$ is the volume fraction of fluid, $\mu$ (Pa·s) is the fluid viscosity, and other terms are as previously defined.

To quantify the relative importance of inertial stresses, Savage and Hutter [1989], proposed the ratio of inertial to total normal stresses:
\[ N_{str} = \frac{\rho D_p^2 \left( \frac{U}{h} \right)^2}{p_T} \]  

(7)

Where \( U \) (m/s) is the velocity difference across the shear layer and \( p_T \) (N) is the total normal stress.

Iverson and Denlinger [2001] defined the Savage number as the ratio of inertial to gravitational (or frictional) stresses:

\[ N_{Sav} = \left( \frac{\rho D_p^2 \left( \frac{\partial u}{\partial z} \right)^2}{(\rho_s - \rho_f) gh} \right) \]  

(8)

Natural debris flows tend to be inertially dominated if above a threshold value of 0.1. Most natural flows may not be truly inertial but are frictional or transitional to inertial if fines poor (e.g. the snout), and frictional to viscous if fines-rich (e.g. the body) [Campbell, 2002; 2005]. Though these non-dimensional numbers are defined strictly for steady, uniform flows, a comparison of their relative values are useful in scaling experiments and comparing natural and experimental flows.

2.2 Experimental approach

Our experiments tested the hypothesis that erosion rate by granular flows scales with inertial stress in the flow, as represented in equation (1). We first performed experiments with single grain sizes, bimodal grain mixtures, and coarse mixtures with clay-rich fluid matrix to explore the qualitative behavior of flows in the drum. Then we focused our quantitative assessment using data with single grain sizes and simple dry or water-saturated conditions. As discussed in Section 1.1, this minimizes uncertainties associated with determining the effective grain size (because \( D_e = D_p \)) and solid fraction \( \nu_s \). Flows with more natural grain size distributions, including clay and silt, will be conducted after the relation for homogenous flows are established.

Experiments using a drum

Our drum is made from a section of PVC pipe with an inner diameter of 56 cm. Both sidewalls are composed of plexiglass, bounding a 15 cm wide channel. In order to measure erosion caused by the granular flows, an erodible sample is inserted into the drum bed, flush with the surface (Figure 2). As the drum rotates, the sample passes under the granular flow repeatedly. The flat sample area exposed to the flow is 2.5 cm (downstream) by 10 cm (cross-stream). The downstream dimension was chosen to minimize bed curvature while exceeding the length of the largest flow particle (1 cm). The magnitude of erosion was determined by weighing the erodible sample block before and after the experiment and obtaining the mass loss.
There are several consequences of using a drum for debris flow erosion experiments. First, at sufficiently high rotation rates, centrifugal force may become significant. Consider a single grain in contact with the bed of the rotating drum that is not slipping relative to the bed. In the steady state case of uniform rotational motion, the grain experiences a centripetal acceleration as a result of moving in a circle with a constant speed. This centripetal acceleration is an artifact of the drum geometry and we have attempted to minimize this effect by keeping the drum velocity, and thus the magnitude of the centripetal acceleration and force, low.

In our experiments, the radius of the drum $r_{\text{drum}}$ is 28 cm and the maximum tangential velocity of the drum bed $u_{\text{drum}}$ is 0.8 m/s, leading to a maximum centripetal acceleration of

$$a_C = \frac{u_{\text{drum}}^2}{r_{\text{drum}}} = 2.3 \text{ m/s}^2. \quad (9)$$

Thus, the magnitude of the centripetal acceleration felt by the grain is, at most, 23% of that of the gravitational acceleration (9.81 m/s$^2$) felt by the grain.

Second, the drum bed is continuously curved, influencing the bed and surface slope of the granular flow. The basal slope is used to correct the normal component to the bed for calculations of stress. Stock and Dietrich, [2006] include a bed slope effect on the inertial stress term to account for the presumed lower impact magnitude and frequency on steeper slopes. In natural steep channels the bed slope is far from constant, having knickpoints, but is not as strongly and consistently concave as a drum bed. In a drum, the basal contact below the flow is on a continuously concave slope, and in our small rotating drum, covers a range up to 90 arc degrees (Figure 3). Different authors have dealt with the bed curvature in varying ways. To calculate the total normal stress exerted on a drum bed by the granular flow, Holmes et al. [1993] divided the flow into small vertical sections and summed the stress on each section using their respective basal angles. To arrive at a single “representative” slope, Kaitna and Rickenmann [2005] took the tangential slope to the drum beneath the center of mass of the flow. The surface slope is used in calculations of shear stress exerted on the bed by the flow. Iverson and Vallance [2001] used the angle of surface inclination in their expression for intergranular normal stress on planes at depth, because the surface slope determines the pressure gradient driving the flow. To address irregular topography, Denlinger and Iverson [2004] included the influence of changes in z-momentum due to topographic variations. They defined total vertical acceleration, $g'$, to include both gravitational acceleration and down-slope and centripetal accelerations arising from irregularly shaped channels. In the drum geometry, the normal stress on the bed would be larger than on a straight chute ($g' > g$) because the flow is decelerating vertically due to impingement against the concave–up bed. Therefore, we might expect more erosion to occur in the drum than in a chute-geometry experiment.

In our experiments, we measured both the average surface slope, $\theta_s$, and the basal slope at the snout, $\theta_b$ (Figure 4). Consider a flow of constant depth in the drum, the normal stress under the flow is greatest at the flow front because $\cos \theta_b$ is maximum. Because the flow depth actually tapers at the front, this effect is diminished. In the no-slip case (Figure 1b, d), the collisional stresses occur only at the snout. There are collisional stresses farther back within the flow due to grain to grain collision, but we hypothesize (following others) that these normal stresses are
minor compared to those which occur at the snout, where particles may tumble down the snout and collide with the bed, or may remain bouncing directly on the bed for a considerable travel distance. Hence, as Stock and Dietrich [2006] proposed, we anticipate most erosion occurs as the snout sweeps across the bedrock. If no sliding occurs behind the flow front, and the only erosion occurs in impacts at the front, then the angle between the surface and basal slope at the snout is the fundamentally important slope. The basal slope at the snout is between 0° and 10° for all of our experiments, which would modify the stress values by a maximum of \(\cos 10^\circ = 0.98\). Because this value is very close to 1, we do not include it in our formulation of normal stresses. Basal slopes would need to exceed 25° to modify the normal stress by 10%. The surface slope was approximately linear with the exception of when sand was used.

A third consequence of a drum is that material cannot deposit behind the flow or laterally in levees. The grains are continually recirculated in the flow, and hence we can not examine depositional effects on debris flow processes. Nonetheless, purely erosive flows do occur in steep, canyon rivers.

In summary, drum curvature effects on bedrock wear rate by granular flows appear to be minor. None of our calculations depend explicitly on the surface slope, which we can measure. Neither do any of our calculations vary significantly with the basal slope, which we can also document. However, we might expect more erosion to occur in the drum than in a chute-geometry experiment because of the concave-up geometry of the bed.

**Testing the hypothesis of erosion by inertial stresses in a drum**

In a drum, the total length of the flow \(L_{tot}\) that passes over the sample block during the total period of the experiment \(t_{exp}\) is the product of the instantaneous length \(L_0\) and the number of times the drum is rotated. To compare experiments with differing total lengths, we non-dimensionalize erosion by dividing the average eroded depth by the total flow length:

\[
e' = \frac{e_d}{L_{tot}} = \left(\frac{e_m}{A_{block} \rho_{block}}\right) \left(\frac{L_0 t_{exp} u_{drum}}{2 \pi r_{drum}}\right)
\]

where \(e'\) is dimensionless erosion, \(e_d\) (m) is the average eroded depth of the sample block after an experiment and \(L_{tot}\) (m) is the total length of the flow over the erodible sample during the experiment. The average eroded depth, \(e_d\), is calculated from the net eroded mass, \(e_m\) (kg) divided by the area of the sample block, \(A_{block}\) (m\(^2\)) and the density of the block, \(\rho_{block}\) (kg/m\(^3\)). The total length, \(L_{tot}\), is calculated as the length of the individual flow, \(L_0\) (m) multiplied by the total period of the experiment, \(t_{exp}\) (sec), divided by the time per rotation, \(2 \pi r_{drum}/u_{drum}\) (where \(r_{drum}\) (m) is the circumference of the drum, and \(u_{drum}\) (m/s) is the speed of rotation of the drum).

To compare erosion rate with estimated inertial stress and measured bedrock strength we simplify the Stock erosion hypothesis [equation (4)] to:
where \( u_b \) (m/s) is the basal velocity of the granular flow. \( \partial z / \partial t \) is the lowering rate for one experiment. We neglect the fracture term, \( F \), because we use an erodible sample material that is not susceptible to macro-fracturing. We also leave out the frequency term, \( f \), because our experiments represent continuous flows of varying lengths, not discrete flows with a recurrence interval. We use \( E_{\text{eff}} = 2 \times 10^9 \text{ Pa} \) [Stock and Dietrich, 2006]. Because our flows do not have grain size-distinct snouts, \( L \) is equivalent to \( L_{\text{tot}} \) in the drum for the duration of one experiment. The exponent \( w \) in equation (4) is assumed to be 2 following the inertial-stress definition in equation (1), the hypothesis of Stock and Dietrich [2006], and experimental results of Hanes and Inman [1985].

Using \( [e_d \text{ per } t_{\text{exp}} = \partial z / \partial t] \) and \( [L_{\text{tot}} = L] \), we can arrange equations (10) and (11) to write:

\[
\frac{e_m}{A_{\text{block}} \rho_{\text{block}}} = K_0 K_1 \frac{E_{\text{eff}}}{T_0^2} \left[ v_s \rho_s D_e^2 \left( \frac{u_s - u_b}{h} \right)^2 \right]^{\eta}
\]

where the left-hand side is the non-dimensional erosion rate, \( e' \), and the right-hand side is a reduced form of the non-dimensional number \( N_{\text{erosion}} \), proposed by Stock and Dietrich [2006]. To apply equation (12) to our drum experiments, we made the assumption that the mean velocity and the maximum depth (\( h_{\text{max}} \)) would scale the inertial stresses in the flow. For no-slip cases, the mean velocity equals surface minus basal speed, \( u_s - u_b \). We also use the particle diameter \( D_p \), for the effective diameter, \( D_e \), because we have homogeneous grain size distributions. We thus hypothesize that erosion is proportional to the quantity \( N_{\text{SNIS}} \), (Strength-Normalized Inertial Stress):

\[
N_{\text{SNIS}} = v_s \rho_s D_e^2 \left( \frac{\partial u}{\partial z} \right)^2 \frac{E_{\text{eff}}}{T_0^2} \approx v_s \rho_s D_e^2 \left( \frac{u_s - u_b}{h} \right)^2 \frac{E_{\text{eff}}}{T_0^2}
\]

Experimental variables

We conducted erosion experiments in which we varied grain size (\( D_p \)), water content (which affects \( u_s \)), drum speed (\( u_{\text{drum}} \)), bedrock strength (\( T_0 \)), and bed roughness (which affects \( u_b \)). Because the small size of the flume presented both space and scaling issues, we did not attempt to instrument the bed with normal load sensors for directly measuring the bed stresses. Hence, our test relies on the assumptions in equation (12) regarding approximations used to estimate inertial stress.

The homogeneous granular flows were composed of gravel with mean diameter of 4, 6, or 10 mm or sand with a mean diameter of 1 mm. The quartzite gravel had high strength and experienced little breakdown during the experiments. We used only two states of water content –
dry or completely water-saturated. For water-saturated (also referred to as “wet”) flows we added water until the pore spaces between the grains were full or the water ran out the front of the snout during drum rotation.

Rotation speed of the drum varied from 0.1 to 0.8 m/s (5-27 RPM). The upper value was limited to keep the centrifugal force significantly less than the gravitational force (see Appendix). Maximum flow depth, $h_{\text{max}}$, measured at the flow center, varied from 4.5 to 10 cm with the low value chosen to keep the flow deeper than several grain diameters and the high value chosen to keep width to depth ratios comparable to natural flows (i.e. 2:1 to 8:1; e.g. as reported in Stock and Dietrich, [2006]).

Following the technique developed by Sklar and Dietrich [2001] we used a synthetic bedrock composed of silica sand and cement. This weaker sample bedrock eroded more readily than natural rocks, shortening the experimental duration needed to produce measurable wear. The erodible samples were cast into blocks 20x30x10 cm and later cut into flume-ready samples. The ratio of sand to cement varied from 10:1 to 20:1 which provided a factor of three in the tensile strength range (268-814 kPa) as measured using the Brazilian Tensile Splitting test (Table 1).

The non-dimensional Bagnold and Savage numbers for our experiments were calculated with a solid volume fraction of $\nu_s = 0.55$, which was the measured static solid volume fraction for all homogeneous size distributions of our sediment. The volume fractions were determined by mass measurement, using $\rho_s$, density of the solid particles (quartzite) = 2650 kg/m$^3$ and $\rho_f$, density of the fluid (water) = 1000 kg/m$^3$. For the non-dimensional number calculations we used $\mu$, viscosity = 0.0002 Pa·s for air and 0.001 Pa·s for water, and $\theta_b$, basal slope (at the snout) = 10º.

When the drum bed was left as the original smooth PVC surface, the granular mass typically experienced full-slip motion. In order to induce internal shearing and collisional stresses, the bed was roughened with sandpaper (80-100 grit) or wire mesh with 6-mm grid-spacing.

**Experimental procedure**

To test the hypothesis that erosion varied with estimated inertial stress and measured rock strength as described by equation (12), we conducted 67 experimental runs, 35 dry and 32 water-saturated (Table 2). The procedure for each experiment was as follows: An erodible block was weighed and placed in the flume, its top surface level with the flume bed. The total mass of grains and fluid in the flow was weighed and placed in the drum. Rotation was initiated at a prescribed drum rotation speed. During the run, we documented the position and length of the flow in the drum. Longitudinal profiles of the flow surface were made by taking digital photographs and videos of the flow through the plexiglass sidewall, then tracing the surface of the flow. The flow depth varied along the length of the flow, so for a repeatable characteristic depth measurement, we measured the maximum depth, $h_{\text{max}}$, which occurred mid-way between the snout and tail. Although particle motion at the sidewall may be different from that in the center of the flow [e.g. Davies, 1990; Hanes and Walton, 2000], we tracked the sidewall velocities for comparison with similar measurements in other studies. We noted if the flow experienced any slip at the bed. Following equation (12), we measured the surface and bed velocities to estimate the velocity gradient. Surface velocities were obtained by timing a colored particle placed in the plan-view centerline of the flow and bed velocities were determined by
tracking bed-particles in side-view videos of the flow. The duration of the experiment was chosen to be long enough to obtain significant wear, but not so long that the erodible block degraded significantly lower than the flume bed. At the end of each run, the block was removed, examined for wear marks, and weighed. Wet blocks used in saturated flows were also weighed on subsequent days until they equilibrated to their dry weight. Typically only one run was obtained from the same block before it was replaced due to extensive wear, but when possible, we obtained multiple runs from the same block to test repeatability.

2.3 Results

Flow dynamics

We observed side-view flow dynamics through the clear plexiglass sidewall and plan-view dynamics from above the flow. Initially, sediment was placed at the lowest point in the drum. As the drum began to spin, the sediment followed the bed in the direction of the spin until the angle of internal friction was exceeded, then avalanched to an equilibrium position. Figures 3 and 4 show flow outlines derived from photographs of experiments. The tail of the flow mass reaches highest in the drum for dry sand and dry gravel, and progressively less for wet gravel, muddy gravel and muddy sand (Figure 3). Hence, the position is related to the friction between the grains and the bed and the pore pressure in the flow. Shifts in the equilibrium position of the flow occurred due to small irregularities in the bottom roughness, even at the millimeter scale. Initially, some surging occurred as the result of overlapping sandpaper liner, and subsequently we eliminated overlapping to minimize bed irregularities. The surging was noticed as a noise with the same periodicity as the irregularities in the bed. Sand flows and smaller grain sizes reacted more strongly to the bed irregularities than gravel flows, presumably because the scale of the irregularity was larger with respect to sand-sized particles than to gravel particles.

The longitudinal profile tapered at the snout and tail and was relatively straight in the middle, but the tapering varied with wetness and mud content (Figures 3 and 4). Kinks in the surface slope occurred in some of the dry sand flows. The sand flows had the greatest number of grains over the flow depth and therefore followed more appreciably the curvature of the bed. In all cases, the flow front gradually tapered to one or two grain diameters deep.

For large gravel, a constant drum velocity of 0.5 m/s (18 RPM), and a constant volume of 1500 cm³, the average surface slope angles for dry, wet, and muddy-matrix (kaolinite clay and water) were 43°, 33°, and 23°, respectively (Figure 5). At the same time, the arc length of the bed covered by the flow diminished from 31, to 28, to 22 cm. For muddy sand, the surface slope decreased to about 10° and covered an arc length of 26 cm. Changing the volume of the flow did not significantly change the average surface angle. The observed variation in slope is roughly consistent with reduction in effective normal stress with increasing pore fluid pressure for a simple Coulomb material as suggested by Iverson and Vallance [2001].

At the bed, there was generally a no-slip condition. In a few cases, some slip occurred at the bed when the sandpaper was worn by many experiments. The consequence of a no-slip condition is that a strong conveyor-belt-like circulation forms with respect to the center of mass of the stationary flow (Figure 4). Particles at the bed moved towards the tail at the drum rotation speed, while particles near the surface moved downslope relative to the approximately stationary position of the flow, as reported in drum experiments with similar fill-fractions by Tai et al.,
[2000]. In the shear zone (Figure 4), there is a change in direction of particle motion – surface particles move towards the bed in the front-half of the flow and near bed particles move towards the surface in the back-half of the flow. Hence, vertical motion is not restricted to just the snout and the tail. The maximum down-slope velocity occurred at the surface of the flow, as described in numerous other natural and experimental observations [e.g. Johnson and Rodine, 1984; Campbell, 1990; Boateng, 1998; Hotta and Ohta, 2000; Longo and Lamberti, 2002].

Surface velocity \( u_s \), differs from the mean flow velocity in the drum frame, \( u \), which equals \( u_s - u_b \). Surface velocity \( u_s \) and basal velocity \( u_b \) are measured relative to the fixed laboratory reference frame, with positive values in the down-slope direction. For no-slip cases, \( u_b = u_{drum} \), but if there is slip, then \( |u_b| < |u_{drum}| \). The surface velocity was only faintly sensitive to drum speed or grain size, but varied strongly with presence of water or mud in the interstices of the solid grains. This was likely due to the surface tension between grains from the water [Tognacca and Minor, 2000]. Dry sediment surface velocities averaged 0.4 m/s, while water-saturated sediment surface velocities averaged 0.2 m/s. This implies that the drum rotated the mass to a position where the horizontal pressure gradient induced flow, and the speed of grain motion was set by internal grain dynamics. For the same drum speed \( u_{drum} \) of 0.5 m/s (18 RPM), the ratio of the surface velocity to the drum velocity \( u_s/u_{drum} \) is \(-0.8 \pm 0.04 \) one standard deviation for dry flows, and \(-0.3 \pm 0.09 \) for saturated flows. The negative sign indicates that the drum and surface velocities are in opposite directions. Increasing the drum rotation speed increased the shear rate but not dramatically the surface speed of the particles. This implies that the thicknesses of the down-slope and up-slope moving layers of the flow adjusted to satisfy mass conservation.

If the flow was deep enough, a static zone developed in the upslope-headed near-bed particles. Here, the particles had the same rotational velocity as the drum i.e. zero-velocity in the drum reference frame. This condition is referred to in drum studies as the passive region, in contrast to the active shearing levels [Boateng, 1998; Longo and Lamberti, 2002]. The thicker the flow depth in the drum, the greater the passive zone [Longo and Lamberti, 2002]. Thick passive regions in the bottom half of the depth seem unlikely in natural granular flows, and occur in the drum during unrealistic width-depth ratios. Therefore we kept our flows thin, and the passive zone was essentially only one-grain thick for the gravel flows.

The surface velocity field was strongly affected by the sidewalls. This influence has been reported in nature [e.g. Berti, 2000; Swartz and McArdell, 2005] and laboratory experiments [e.g. Hanes and Walton, 2000; Parsons et al., 2001; du Pont et al., 2003; Bi et al., 2005]. Conversely, several studies propose that wall effects are negligible in their experiments [e.g. Forterre and Pouliquen, 2001; Jain et al., 2002; Tegzes et al., 2002]. The wall resistance resulted in lateral flow circulation, a maximum down-slope velocity midway between the walls, and a distinct topographic trough along the centerline associated with the flow divergence there for muddy flows (Figure 6). In contrast to the roughened bed, particles slid against the walls i.e. there was finite slip at the sidewalls. We did not roughen the walls, as we did the bed surface, because we wanted to make observations through the clear plexiglass. For most of our experiments, both walls were plexiglass to provide symmetric wall effects. But in our first experiments, the back wall was wood and the front wall was plexiglass. This asymmetry in wall friction caused a wider lateral shear zone on the rougher wooden wall (Figure 6).

For heterogeneous flows of sand and gravel, the larger particles tended to stay near the front of the flow, where they bounced, rolled, and tended to move to the side-walls, allowing the
sand to extend furthest to the snout at the centerline. This was a result of wall friction, the upslope-moving walls tended to drag larger grains back towards the tail of the flow, where they re-entered the central flow and traveled quickly to the front.

Scaling and Erosion

Bagnold numbers for our experiments ranged from 430 to 330,000 with a dry flow mean of 110,000 and a saturated flow mean of 10,000 (Table 2). The Savage number, $N_{Sav}$ (equation (8)) ranged from 0.003 to 0.17, with a dry mean of 0.041 and a saturated mean of 0.036. Although these values do not place the flows into the inertial regime by the measure of the Savage number, they are very similar to those for other experimental and natural granular flows (Table 3, Stock and Dietrich [2006]).

During the approximately 25 minute duration of each run, individual flows (32 to 52 cm in arc length) would pass over the erodible blocks 250 to 675 times. Net erosion varied from virtually no wear to up to 2.9 mm (14.5 g). The dry flows had higher surface velocities and tended to wear a dimpled pattern into the block, suggesting a dominance of collisional wear over sliding wear. After some of the wet flows, the blocks exhibited grooves oriented in the flow-direction, suggesting sliding wear.

For dry flows, eroded depth per revolution was nearly linearly proportional to the estimated inertial stress defined in equation (1) and increased with decreasing bedrock strength (Figure 7a). The exponents change only slightly when the regression is recalculated for the dimensionless erosion, $e'$ (Figure 7b, Table 4).

Assuming that $n = 1$ in equation (12), we test the hypothesis that $e'$ varies linearly with $D_p^2$ and $(\partial u / \partial z)^2$. We evaluate the trends in $D_p^2$ (Figure 8a) and $(\partial u / \partial z)^2$ (Figure 9a) holding all other variables constant, including sample tensile strength. Given our modest number of data points, and in the case of shear rate, our inexact knowledge of the x-variable, the regression statistics are used to illustrate that we cannot reject the hypothesis that $e'$ varies linearly with $D_p^2$ and $(\partial u / \partial z)^2$. The null hypothesis, that there is no dependence of $e'$ on $D_p^2$ or $(\partial u / \partial z)^2$, is unlikely to be true because of low p-values (less than 0.07 for all regressions except for the 15C runs, where it was 0.37). For comparison, we also plotted $e'$ against $D_p$ and $(\partial u / \partial z)$, (Figures 8b and 9b). There is no significant difference between the fits for $D_p$ and $D_p^2$, or $(\partial u / \partial z)$ and $(\partial u / \partial z)^2$. The bearing of these findings is discussed in relation to different granular flow regimes in the Discussion section.

The bed roughness has a direct effect on the bed slip and shear rate, so for our regressions we use only the sandpaper-lined runs, leaving out runs with significant sliding or wire mesh. Figure 10 shows that $e'$ decreases approximately with the square of the tensile strength. The shear rate is approximately constant for the different runs and the trend is seen for all grain diameters.

For the dry runs, the proposed non-dimensional inertial stress, $N_{SNIS}$ [equation (13)], does collapse the data to a single relationship driving erosion (Figure 11). Experiments with saturated sediment quickly revealed a tendency for basal sliding. Consequently, either coarse wire mesh or new sandpaper was used in one-third of the experimental runs to roughen the bed and prevent sliding (Table 2). Nonetheless, strong sliding was noted in three cases (runs 64, 65, and 96) before we adopted this procedure and sliding was probably a component of the total motion in several other cases. When observed, sliding affects the calculation of the shear velocity by
decreasing the basal velocity, \( (|u_b| < |u_{drum}|) \) and introduces an otherwise absent sliding force on the bed. Partial sliding is an additive effect to erosion because grain collision will still occur at the flow front, then grains in the body of the flow will slide across the sample and cause further erosion, unlike the no-slip case.

To explore sliding effects, we kept track of the bed condition and sliding behavior and classified runs accordingly on a plot of dimensionless erosion, \( e' \), versus \( N_{SNIS} \) (Figure 12). Using runs with a similar sample tensile strength (sand:cement of 15:1), for a constant \( N_{SNIS} \), a rougher boundary (e.g. dry sandpaper, filled circles) inhibits sliding and also reduces the amount of erosion. The three cases with the most observed sliding (+ symbols) caused unusually high erosion for the imposed \( N_{SNIS} \). Some points that plotted initially distinctly low in erosion were later identified to be runs with new sandpaper, and thus had the least sliding.

The dry and wet experiments in which only sandpaper-lined experiments without significant sliding (29 dry and 18 wet experiments) reveal a well defined power-law dependency of erosion on \( N_{SNIS} \). The regression yields:

\[
e' \text{ (dry)} = (1 \cdot 10^{-5}) N_{SNIS}^{0.8} \tag{14}
\]
\[
e' \text{ (water-saturated)} = (3 \cdot 10^{-5}) N_{SNIS}^{0.6} \tag{15}
\]

with \( R^2 \) of 0.80 (dry) and 0.73 (saturated). The regression suggests that for the case of no sliding in our dry flows the exponent, \( n \), is close to 1. Unexpectedly, Figure 13 shows that for the same dimensionless stress, the erosion by saturated flows was higher than erosion by the dry flows. Some of this difference may be due to the wear properties of our sample blocks under wet and dry conditions. Wetting of concrete decreases the compressive strength, though often considered negligible [e.g. Popovics, 1998], but there are no data to suggest that wetting affects tensile strength (which influences erodibility). Despite eliminating cases where sliding was evident or very likely, we propose that the higher erosion rates in the wet case are due to full or partial sliding of bed particles. If this interpretation is correct, in addition to the fact that many water-saturated runs had low Savage number scaling, then the dry case is the most direct test of the hypothesis that erosion scales with inertial stress.

2.4 Discussion

Our experiments show that the bedrock incision rate increases with the particle diameter and is moderately dependent on the shear rate. Although these experiments represent considerable simplifications of natural granular flows, their similarities with natural flow scaling support the relevance of our findings to the field. The characteristics of our experimental flows are shown with data from natural and laboratory flows in Table 3. The scaling of the particle size and channel width:depth are similar to those found in nature [Stock and Dietrich, 2006]. The nondimensional numbers that characterize the inertial, viscous, and frictional forces for our experiments are also similar to those for natural dry flows and close to those for natural water-saturated flows.

Some salient differences between our experiments and nature are the uniform grain-size, absence of clays, and fluid properties. As mentioned previously, we purposely simplified our flows so we could specify the effective grain size, \( D_e = D_p \). Nonetheless, our goal was to test a
mechanistic hypothesis for the control of bedrock erosion, and make observations of the grain dynamics and their interaction with the bed.

Muddy flows in the drum developed lower surface slopes, had significantly lower shear rates (due to lower surface velocities), and correspondingly, generated lower collisional stresses for the same drum rotation speed than dry or water saturated granular flows. Because of concerns about scaling relationships, especially with regard to dynamic pore pressure development [Iverson, 1997], erosional measurements are not reported for our muddy runs. A recently completed 4-m diameter flume will allow us, however, to use 1:1 scale materials to explore the dynamics and resulting erosion of these complex flows. The larger flume will also help to decrease possible edge effects on the erodible sample, which will be several times longer than the length of the largest particle diameter.

Based on field observations, Stock and Dietrich [2006] proposed that the excursions from the static load (impacts from large boulders in the coarse snout of the debris flow) perform the erosion. In contrast, the static effective solid normal stress would be

\[ \sigma_s = (v_s \rho - v_f \rho_f) gh \cos(\theta_b) - p \]  

(16)

where \( p \) (Pa) is the nonequilibrium component of intergranular fluid pressure. The maximum static normal stress for the dry flows in the drum ranges from 0.56-1.3 kPa, and for the wet flows ranges from 0.58 to 1.0 kPa. Fluid pore pressure, \( p \), may modify the effective stress on the bed, but because we do not have a measure of this, our value is an approximation for the saturated flows. Peak solid static normal stress is estimated to be 23-93 kPa for selected flows measured in nature [Stock and Dietrich, 2006]. Solid inertial stress calculated by equation (1) ranges from 0.0008-0.60 kPa in the drum and 0.3 to 4 kPa in selected flows in nature. Hence, as shown by many others, the inertial stresses are a small fraction the total normal stress. In the absence of sliding, however, the static normal stress is a quasi-steady load, whereas the inertial stresses – especially arising from the tumbling of particles at the snout – create dynamic point-loads that can lead to surface fatigue and rock fracturing [Stock and Dietrich, 2006].

While our procedure for calculating the inertial stress is consistent with methods used by others in field studies, it is nonetheless very simplified, especially with regard to the estimate of the velocity gradient, \( \partial u / \partial z \), and the relevance of this shear rate to the scale of stresses at the snout. Qualitative observations in the drum indicate that the velocity profile is not linear, and, as noted above, in the wet cases sliding at the base may be common. The shear rate was also higher than that reported from field studies using roughly the same calculation procedure but was similar to other smaller scale flume studies (Table 3). Perhaps the most significant approximation is the assumption that the estimated inertial stress in the body of the flow (based on maximum depth and estimated from the square of the shear rate and particle grain size) gives an adequate scale for the local collisional stresses for particles arriving at and bouncing in the snout.

The importance of sliding was not anticipated, and published field data provide little quantification of boundary sliding in grain and debris flows. In typical bedrock chutes in mountainous terrain it seems reasonable to propose that sliding occurs, but knickpoints and channel roughness may prevent prolonged sliding. Debris flows typically entrain the colluvium stored in the steep channels as they sweep downslope and scour to bedrock. It may be that the water-rich tails of many debris flows tend to drive particle sliding on the bedrock surface that was scoured clear by the snout. Video records of debris flows do show cases in which the largest
boulders in the flow appear to be sliding for durations that could produce centimeters-long grooves. Field observations on bedrock wear features after debris flows [Stock and Dietrich, 2006] suggest, however, fracture-bounded blocks are removed by glancing blows from boulders, and that perhaps erosion by debris flows is much more effective in the coarse snouts, where large boulder impacts prevail, than in the post-snout body, where sliding wear may predominate. We did see concentration of large grains at the flow front in our qualitative drum experiments with a heterogeneous grain size distribution of 4 cm diameter grains in addition to 1 cm gravel, sand, and clay, but we focused here on single grain size flows when documenting erosion. In so doing, our observations may have diminished the relative importance of collisional stresses of large grains at the snout.

The lateral circulation effects (Figure 6) that arose in the drum due to wall resistance are important to the flow dynamics and stresses on the bed, and occurs in natural flows as well [e.g. Berti et al., 2000]. We note that: (1) flows in the field also experience lateral boundary resistance, (2) the width to depth ratio in this flume (~ 2:1) does occur in small, bedrock canyon debris flows [Stock and Dietrich, 2006], and (3) inspection of videos recording grain movement suggests that boulders move more slowly near the bank, and in fact, this near bank resistance may contribute to their deposition [Arattano and Marchi, 2000; Swartz and McArdell, 2005]. The laterally directed flow circulation may contribute to wall erosion in bedrock, as evidenced by severe degradation in the side-walls of check dams in debris flow channels [Figure A2, Auxiliary Material].

While support for the collision stress driven bedrock erosion is provided here, additional improvements are possible. First, our analysis uses the entire instantaneous length of the flow, $L_0$, to scale the duration of inertial stresses. In the prevailing no-slip case, this is inaccurate, as most of the collisions of large boulders with the bed occur at the snout (Figure 14a). Even without enforcement of the no-slip condition, recirculation of large grains at the front may occur [Iverson, 2005, Figure 6.6]. Stock and Dietrich [2006] hypothesize that the appropriate length-scale is the length of the granular front. Although we do not have a coarser granular front in our uniform grain-size flows, observations in the drum suggest, roughly, that the zone of active bouncing at the snout varies with grain size, with it being about six times the grain size in the dry case and about three times the grain size in the wet case. This suggests a reformulation of $e^\prime$ (equation (10)) using $L_{act}$ (the active length of the particle bouncing at the snout) instead of $L_0$. Figure 15 shows that this estimation actually reduces the correlation between erosion and inertial stress. It is not clear whether this is because of the imprecise estimate of the active length, or because of other effects, such as the importance of sliding wear or the incompleteness of the bulk inertial stress hypothesis. In addition, because $L_{act}$ increases with larger grain diameters and faster drum speeds, it tends to covary with the inertial stress, weakening the dependency between $e^\prime$ and $N_{SNIS}$.

The low Savage numbers and the direct observations of grain motion suggest that in these experiments the assumption of an inertial stress dominance is not well supported. Inertial flows are defined as having only instantaneous bimodal collisions, but enduring contacts between particles are evident in our experiments and in natural granular flows. Also, recent literature has pointed out some shortcomings in the stress and shear rate relations found in Bagnold’s original 1954 work [Hunt et al., 2002]. An alternative to the inertial regime is the elastic regime, which is characterized as “granular materials under dense conditions where particles are in persistent contact with their neighbors and the elasticity of the material becomes an important rheological
parameter” [Campbell, 2002]. Stresses are transmitted by the compression and rotation of force chains, and friction is important. In the elastic regime, normal stress scales with the shear rate, and in our analysis, we did find a similar fit to shear rate and the shear rate squared. This may be because our experiments lie in the transitional elastic-inertial regime and therefore show variability in scaling relationships.

Alternatively, if we assume zero slip at the bed and a well-defined snout where grains tumble freely towards the bed, then the Figure 14b schematic shows that there would be bed wear only at the front-most grain, after it overrides the flow front and impacts the bed. This makes the erosion process similar to that proposed for fluvial bedload transport, where erosion is the sum of discrete impacts of single grains on the bed. Following the model of bedload incision into rock [Sklar and Dietrich, 2004], we can modify a saltation-abrasion model to write an expression for bedrock incision rate by impacting grains:

\[ e_s = V_i I_r F_e \]  \hspace{1cm} (17)

where \( e_s \) (m/s) is the bedrock incision rate, \( V_i \) (m³/impact) is the average volume of rock detached per particle impact, \( I_r \) (impacts/m² s⁻¹) is the rate of particle impacts per unit area per unit time, and \( F_e \) (-) is the fraction of the river bed made up of exposed bedrock. Following the derivation in Sklar and Dietrich [2004] and using our notation,

\[ V_i = \frac{\pi \rho D_p^3 (u_i - u_b) \cos \theta_b \theta^2}{6 k_v T_0^2} \]  \hspace{1cm} (18)

\[ I_r = \frac{n_i}{t_{\text{event}}} D_p^2 \]  \hspace{1cm} (19)

\[ F_e = 1 \]  \hspace{1cm} (20)

where \( k_v \) (-) is a dimensionless coefficient that depends on the material properties of the impacting particle, \( n_i \) is the number of impacts, \( t_{\text{event}} \) (sec/event) is the seconds per unit of time (defined as one debris-flow event), and \( D_p^2 \) (m²) is unit area (defined as the square of the length scale of the characteristic particle diameter). The average volume of rock detached per particle impact, \( V_i \), depends on the diameter and incoming velocity of the impactor and the strength of the erodible substrate. In the drum, one event is one pass of the slurry over the erodible sample, equivalent to one rotation of the drum. In nature, one event would be one debris flow or one coarse granular front. We define the fraction of the river that is exposed bedrock as 1 (fully exposed) because in the flume there is no shielding by sediment. Under these conditions, each unit area of the bed is struck by one snout particle per debris flow event (or revolution). Figure 16 shows a plot of eroded depth per event (revolution), \( e_d \) versus \( V_i I_r F_e \) as defined in equations (17)-(20). This model of bedrock erosion explains less of the variance in the data than \( N_{\text{SNIS}} \), the strength-normalized inertial stress. We note that zero-slip at the bed is unlikely in real flows, and that in our dry flows with no-slip in the body, we still observed grains bouncing at the front. This
implies that the no-slip condition illustrated by Fig. 14b is an idealized end-member situation which would underestimate the erosion of the bed.

As mentioned previously, sliding wear appears to be significant in these experiments and may contribute to bedrock erosion in the field. Our experiments were not designed to explore this process, and, in fact, considerable effort was made to avoid it. Consequently, we have few data points under pure sliding wear conditions (Figure 14c) (sliding-block model). The literature of tribology (the study of wear, friction, and lubrication) contains abundant examples of wear by pure sliding and equation (3) is a typical empirical expression relating wear to normal load, velocity and hardness. Whilst we can calculate the static normal load and relate hardness to block strength, the slip velocity is unknown except for the few cases of pure sliding. Additionally, because our experiments were not designed to test the relationship between sliding and erosion, our range in $h$ is not adequate to evaluate the trend between these two variables. These experiments, nonetheless, suggest that sliding wear may be more important than anticipated for certain combinations of flow and bed-strength conditions, and warrants further investigation. For example, changing the roughness of the sample itself may have the most influence on the amount of sliding. To discern between the different erosion models discussed here, the next step is to extend the range of flow depth, grain diameter, and bed roughness in the experiments. We plan to explore these issues in our larger scale drum study.

2.5 Summary and Conclusions

We conducted experiments in a rotating drum to explore how the erosion rate of homogeneous bedrock varies with one-dimensional, average inertial stress values for the case of dry and water-saturated single-grain-size flows. Our data indicate a strong dependence of erosion on grain diameter and a moderate dependence on the shear rate of the flow. As Sklar and Dietrich [2001] found in fluvial experiments, we found that granular flow erosion varies as the inverse of bedrock tensile strength squared. Hence, we found correlation between measured bedrock erosion and the bedrock strength-normalized inertial stress parameter (equation (13)). However, even in this simplified case, observations of lateral circulation effects and sliding suggest that the inertial stress dependency may be insufficient. Slip occurs, more commonly in water-saturated than dry flows, and surprisingly seems to cause more wear despite the reduction in the shear rate.

While approximate inertial stress estimates based on average shear rate can serve as an indicator of erosion rates, it seems likely, as previously suggested, that a significant proportion of the wear occurs just in the coarse granular snout. The flow-average inertial stress values may provide a poor indication of stresses in the snout, especially because not all experimental flows in this study or in nature lie in the inertial regime. These experiments highlight the need for more theory and observation of the dynamic loading processes at the snout. To make this analysis more mechanistic, we need direct measurements of the dynamic normal loading on the boundary and associated velocity profiles. Using a recently constructed 4-m diameter rotating drum, we intend to study flow dynamics and bedrock wear with natural-scale debris flow materials. Our small drum experiments encourage further data gathering of inertial stresses and wear, but they also point to the need to document sliding, the conditions that induce it, and the wear it causes. Another goal is to better understand the bedrock fracture properties, hence future work with an erodible material that fractures in blocks or plates would be enlightening.
2.6 Notation

\( A_{\text{block}} \) \hspace{1cm} \text{area of erodible sample, m}^2.
\( a_c \) \hspace{1cm} \text{centripetal acceleration, m/s}^2.
\( c \) \hspace{1cm} \text{cohesion of the bed, Pa}.
\( D_p \) \hspace{1cm} \text{particle diameter, m}.
\( D_e \) \hspace{1cm} \text{effective particle diameter, m}.
\( E_{\text{eff}} \) \hspace{1cm} \text{effective elastic modulus of fractured rock, Pa}.
\( e' \) \hspace{1cm} \text{dimensionless erosion}.
\( e'(L_{\text{act}}) \) \hspace{1cm} \text{dimensionless erosion calculated with } L_{\text{act}} \text{ instead of } L_0.
\( e_d \) \hspace{1cm} \text{average eroded depth of sample block in one experiment, m}.
\( e_m \) \hspace{1cm} \text{mass of eroded material from sample block in one experiment, kg}.
\( e_s \) \hspace{1cm} \text{bedrock erosion rate from a saltation-abrasion model [Sklar and Dietrich, 2004], m/event}.
\( e_v \) \hspace{1cm} \text{eroded volume, m}^3.
\( f \) \hspace{1cm} \text{frequency of debris flows over a given reach per annum, a}^{-1}.
\( F \) \hspace{1cm} \text{function of fracture spacing of bedrock channel [Stock and Dietrich, 2006], m}.
\( F_e \) \hspace{1cm} \text{fraction of bed exposed to erosion}.
\( g \) \hspace{1cm} \text{gravitational constant, m/s}^2.
\( g' \) \hspace{1cm} \text{total vertical acceleration [Denlinger and Iverson, 2004], m/s}^2.
\( H \) \hspace{1cm} \text{hardness of eroding material, Pa}.
\( h \) \hspace{1cm} \text{debris-flow surge head depth, m}.
\( h_{\text{max}} \) \hspace{1cm} \text{maximum height of flow in drum, m}.
\( I_r \) \hspace{1cm} \text{rate of particle impacts, per unit time per unit area, time}^{-1} \text{m}^{-2}.
\( K_0 \) \hspace{1cm} \text{dimensionless constant of proportionality that relates bulk inertial normal stresses to higher excursions of inertial normal stress}.
\( K_1 \) \hspace{1cm} \text{constant of proportionality between rock resistance and incision rate}.
\( k \) \hspace{1cm} \text{constant of proportionality in wear equations}.
\( k_v \) \hspace{1cm} \text{dimensionless coefficient that depends on the material properties of the impacting particle (equation (18))}.
\( L \) \hspace{1cm} \text{length of debris-flow granular front, m}.
\( L_{\text{act}} \) \hspace{1cm} \text{instantaneous active eroding length of the flow in the drum, m}.
\( L_{\text{tot}} \) \hspace{1cm} \text{total length of flow for a drum experiment, m}.
\( L_0 \) \hspace{1cm} \text{instantaneous length of flow for a drum experiment, m}.
\( N_{\text{Bag}} \) \hspace{1cm} \text{Bagnold number (ratio of solid inertial normal stress to fluid viscous stress)}.
\( N_{\text{erosion}} \) \hspace{1cm} \text{nondimensional number representing ratio of path-integrated bulk inertial solid stresses to rock resistance [Stock and Dietrich, 2006]}.
\( N_{\text{Sav}} \) \hspace{1cm} \text{Savage number (ratio of solid inertial normal stress to solid frictional stress)}.
\( N_{\text{SNIS}} \) \hspace{1cm} \text{strength-normalized inertial stress number}.
\( n \) \hspace{1cm} \text{exponent on the inertial stress term in equations (11) and (12)}.
\( n_i \) \hspace{1cm} \text{number of impacts}.
\( p \) \hspace{1cm} \text{fluid pressure, Pa}.
\( p_n \) \hspace{1cm} \text{normal pressure, Pa}.
\( p_T \) \hspace{1cm} \text{total normal stress, Pa}.
$r_{\text{drum}}$ radius of experimental drum, m.

$T_0$ rock tensile strength, Pa.

t time, s.

t$_{\text{event}}$ duration of one debris flow event, sec/event.

t$_{\text{exp}}$ duration of erosion experiment, s.

$U$ velocity difference across the shear layer, m/s.

$u$ velocity, m/s.

$u_b$ basal streamwise (along slope) velocity of debris flow, m/s.

$u_s$ surface streamwise (along slope) velocity of debris flow, m/s.

$u_{\text{drum}}$ tangential drum velocity, m/s.

$V$ sliding velocity (Archard’s law), m/s.

$V_i$ average volume of rock detached per particle impact, m$^3$/impact.

$W$ applied load, N.

$w$ exponent on the shear rate approximation (equation (4)).

$x$ sliding distance of the applied load in Archard’s Law, m.

$z$ height above bed, m.

$\partial u/\partial z$ shear strain rate of debris flow, s$^{-1}$.

$\partial z/\partial t$ bedrock surface lowering rate, m/a (meters per annum).

$\Phi$ friction angle, degrees.

$\tau_f$ shear strength of bed, Pa.

$\mu$ viscosity of phase defined as fluid in debris flow, Pa-s.

$\nu_f$ volumetric fluid concentration.

$\nu_s$ volumetric solids concentration.

$\theta_b$ basal slope angle of flow, degrees.

$\theta_s$ surface slope angle of flow, degrees.

$\theta_t$ shear layer slope, degrees.

$\rho_s$ solid particle density, kg/m$^3$.

$\rho_f$ fluid density, kg/m$^3$.

$\rho_{\text{block}}$ erodible sample density, kg/m$^3$.

$\sigma_i$ solid normal inertial normal stress, Pa.

$\sigma_n$ normal stress, Pa.

$\sigma_s$ effective static normal stress, Pa.
2.7 References


Calder, E.S., R.S.J. Sparks, and M.C. Gardeweg (2000), Erosion, transport and segregation of pumice and lithic clasts in pyroclastic flows inferred from ignimbrite at Lascar Volcano, Chile, Journal of Volcanology and Geothermal Reserach, 104, 201-235.


### Table 1. Erodible sample block tensile strength

<table>
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<th>Block</th>
<th>Strength</th>
<th>Stddev</th>
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<td>209</td>
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<tr>
<td>10B</td>
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</tr>
<tr>
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<tr>
<td>15B</td>
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<td>22</td>
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<tr>
<td>15C</td>
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<tr>
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<td>23</td>
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*Number in the block name represents the silica sand:cement ratio (e.g. 10:1, 15:1, 20:1). Measured by Brazilian Tensile Strength Test.*
Table 2. Experimental runs, measured variables, results, and non-dimensional numbers

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<th>$u_z$</th>
<th>$h_{max}$</th>
<th>$L_0$</th>
<th>$e_m$</th>
<th>$dU/dz$</th>
<th>$e'$</th>
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<th>$N_{Bag}$</th>
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ns: new sandpaper; m: wire mesh (otherwise, sandpaper); s: significant sliding observed
(a) velocities are given in the stationary laboratory reference frame. Positive flow is downstream (particle-fall direction), negative flow is upstream (drum rotation direction)
### Table 3. Nondimensional numbers for experimental and natural debris flows

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<th>Parameter</th>
<th>Drum experiment, dry (TS)</th>
<th>Drum experiment, wet (TS)</th>
<th>Large USGS flume (I&amp;V)</th>
<th>Small USGS flume (I&amp;V)</th>
<th>Conveyor belt (D)</th>
<th>Straight chute flume (P)</th>
<th>Yale Dake debris flow (I&amp;D, T)</th>
<th>Elm Rock debris flow (I&amp;D, H)</th>
<th>Acquabona debris flow (B)</th>
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Bagnold Number (Eqn 6) $N_{Ba}$: 3.24E+05 3.24E+03 2.03E+02 2.86E+03 3.14E+02 7.95E-04 4.68E-03 1.50E-08 6.36E-03
Savage Number (Eqn 8) $N_{Sa}$: 8.17E-02 3.28E-02 2.03E-01 1.33E-01 1.73E-02 2.91E-05 3.41E-02 6.38E-02 5.97E-02

TS: This Study; I&V: Iverson and Vaillance [2001]; D: Davies [1990]; P: Parsons et al. [2001]; I&D: Iverson and Denlinger [2001]; T: Takahashi [1991]; H: Hsu [1975]; B: Berri et al. [1999] and [2000]
Table 4. Regression statistics

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$^1$For a power fit, $y = bx^m$, for a linear fit, $y=mx+b$.

$^2$The specific data points are identified in the auxiliary material, Table A1.
2.9 Figures

**Figure 1.** Illustration of different velocity profiles of granular flows for different basal behavior. The vertical dotted line represents the zero-velocity line, the dashed line is the schematic velocity profile. (a) A flow with full-slip at the base moves like a sliding block. (b) A flow with zero-slip at the base experiences internal shear. (c) Partial-slip at the base is an intermediate state with positive velocity at the base. (d) Plug flow with no-slip at the base has a non-linear velocity profile with most of the shear concentrated near the bed.
Figure 2. Illustration of the vertically rotating drum used to conduct bed-erosion experiments. An erodible sample is fixed into the bed of the drum, its surface level with the bed. As the drum with radius $r_{\text{drum}}$ rotates at speed $u_{\text{drum}}$, the granular flow of height $h$ and length $L_0$ passes over the erodible surface area, $A_{\text{block}}$, exerting erosive stresses on the sample. Erosion is measured by mass differencing the sample block before and after an experiment.
Figure 3. Outlines of granular flows with different composition in the rotating drum. (a) dry gravel, (b) water-saturated gravel, (c) dry sand, (d) gravel, sand, and mud (kaolinite and water), (e) sand and mud. For a constant drum velocity of 0.55 m/s, dry and water-saturated gravel flows (a, b) occupy higher positions in the drum while flows with clayey matrix (d, e) and smaller grain size (c) tend to lower the mean basal slope of the flow.
Figure 4. (a) Slopes measured on the flow in the drum. 1: surface slope, $\theta_s$, 2: basal slope $\theta_b$, at snout, 3: shear layer slope, $\theta_t$. Here, the surface slope and shear layer slope are parallel. (b) Semi-quantitative velocity vectors for a dry gravel flow in the drum, derived from observing particles in digital videos. (c) Velocity vectors for a water-saturated flow in the drum. The solid curved line represents the schematic velocity profile and the straight dotted line represents the zero velocity position in the laboratory frame. (d) Schematic illustrating that for constant drum velocity, dry gravel flows have faster surface velocities and greater velocity gradient between the surface and base of the flow than water-saturated flows.
Figure 5. Histogram of shear zone slopes $\theta_t$ (defined in Figure 4) for wet and dry cases of various compositions. The histogram shows the systematic decrease in shear zone slope from dry gravel to wet gravel, muddy gravel, and muddy sand.
Figure 6. Plan view of the surface velocity structure of a muddy flow composed of gravel (12% by mass), sand (70%), clay (3%), and water (15%). Semi-quantitative vectors are derived from observing particles in a digital video. Wall friction at the sides of the flow lead to a pattern of maximum velocity at the axis of divergence (dotted line labeled “a”) and 3-D circulation at the sides. When grains reach the flow front or sidewall, they drop down beneath the surface flow and are recirculated to the tail of the flow. This photo illustrates the effect of different wall roughness, the top (right-bank) wall (labeled b) is wood, and the bottom (left-bank) wall (c) is plexiglass. The rougher wood produces a greater drag on the flow and greater influence than the plexiglass. The flume was later modified to have two plexiglass walls.
Figure 7 (a) Plot of eroded depth, \( e_{dr} \), per revolution versus estimated inertial stresses, \( \sigma_i \) (equation (1)) for dry conditions and three synthetic bedrock strengths (silica sand to cement ratio: strong 10:1 (circles, \( y = 8E^{-08}x^{0.67} \)), moderate 15:1 (triangles, \( y = 2E^{-07}x^{0.82} \)), weak 20:1 (squares, \( y = 3E^{-07}x^{0.90} \))). The experimental identification numbers (Table 2, column 1) for each data point in this figure and all subsequent figures are specified in Table A1. The depth eroded per revolution is calculated from the mass loss, \( e_m \), during the experiment, the number of revolutions, the density, \( \rho_{block} \), and the exposed surface area of the erodible sample, \( A_{block} \) equation (10). For the two stronger samples, erosion rate was nearly linearly dependent on the estimated inertial stress. (b) Plot of dimensionless erosion, \( e' \), versus estimated inertial stress, \( \sigma_i \) (equation (1)). Dimensionless erosion, \( e' \), is the eroded depth, \( e_{dr} \), normalized by the length of the passing flow, \( L_{tot} \). Silica sand to cement ratio: strong 10:1 (circles, \( y = 2E^{-07}x^{0.67} \)), moderate 15:1 (triangles, \( y = 5E^{-07}x^{0.86} \)), weak 20:1 (squares, \( y = 6E^{-07}x^{0.94} \)). Experiments in the drum show a trend of increasing dimensionless erosion with increasing estimated inertial stresses. The vertical scatter between different symbols reflects the change in erodible sample strength which is controlled by silica to cement ratio. See Table 4 for regression statistics.
Figure 8. (a) Plot of dimensionless erosion, $e'$, versus $D_p^2$ for experiments in which the shear rate was held constant as the grain size varied. For two of the cases the results were nearly linear. Squares: weak bedrock strength, block 20A dry, $y = 0.11x + 4E^{-06}$; Triangles: moderate, 15B wet, $y = 0.086x + 9E^{-07}$; Circles: strong, 10B dry, $y = 0.029x + 9E^{-08}$. (b) Same as (a), but, $e'$, versus $D_p$. Squares: weak bedrock strength, block 20A dry, $y = 0.0016x - 2E^{-06}$; Triangles: moderate, 15B wet, $y = 0.0013x - 3E^{-06}$; Circles: strong, 10B dry, $y = 0.0004x - 1E^{-06}$. See Table 4 for regression statistics.
Figure 9. (a) Plot of dimensionless erosion, $e'$, versus the shear rate squared, $(\partial u/\partial z)^2$. The range of shear rates achieved in the drum varies by less than an order of magnitude. If we plot points with a constant boundary roughness (no sliding or new sandpaper) the data suggests that erosion is correlated with the measured shear rate. The symbols represent different erodible sample blocks. Triangles: moderate bedrock strength, block 15B wet, $y = 1 \times 10^{-7}x + 3 \times 10^{-6}$; Diamonds: moderate, 15C wet, $y = 4 \times 10^{-8}x + 8 \times 10^{-6}$; Circles: strong, 10A dry, $y = 3 \times 10^{-9}x + 9 \times 10^{-7}$. (b) Same as (a), but, $e'$, versus $(\partial u/\partial z)$. Triangles: moderate bedrock strength, block 15B wet, $y = 2 \times 10^{-6}x - 5 \times 10^{-6}$; Diamonds: moderate, 15C wet, $y = 1 \times 10^{-6}x + 2 \times 10^{-6}$; Circles: strong, 10A dry, $y = 8 \times 10^{-8}x + 5 \times 10^{-7}$. See Table 4 for regression statistics.
Figure 10. Plot of dimensionless erosion, $e'$, versus $T_0$, the erodible sample tensile strength, for runs with a constant drum velocity. The symbols represent different grain sizes and water conditions (closed symbols: dry; open symbols: wet). The relation for all data points is $y = 11.75x - 2.44$. The tensile strength of the erodible samples was measured by Brazilian Strength Test. See Table 1 for standard deviation of the strength values. See Table 4 for regression statistics.
Figure 11. Plot of dimensionless erosion, $e'$, versus strength-normalized inertial stress, $N_{SNIS}$, for dry flow experiments, $y = 1E^{-05}x^{0.81}$. Normalizing the inertial stress term by the tensile strength squared of the erodible sample block collapses the vertical scatter of the data points compared to Figure 7b. See Table 4 for regression statistics.
Figure 12. Plot of dimensionless erosion, $e'$, versus strength-normalized inertial stress, $N_{SNIS}$, illustrating the influence of different bed boundary roughness on erosion. For a constant $N_{SNIS}$, erosion is inversely correlated to bed roughness. Roughness increases from upper-left to lower-right, from a low roughness of wet sandpaper, to wet mesh, and to high roughness of dry sandpaper. Greater bed roughness inhibits sliding and associated wear at the bed. All points plotted have moderate sample tensile strength (15:1 sand:cement ratio).
Figure 13. Plot of dimensionless erosion, $e'$ versus strength normalized inertial stress, $N_{SNIS}$, showing the difference between dry (filled circle, $y = 1E^{-05}x^{0.81}$) and water-saturated (open circle, $y = 3E^{-05}x^{0.64}$) flows. The bed roughness is constant, as only sandpaper-lined experimental results without visible sliding are plotted. Nonetheless, sliding may have caused the saturated sediment to cause more erosion than the dry. See Table 4 for regression statistics.
Figure 14. Illustration of the three different cases of basal layer dynamics for modeling erosion. Darkest gray grains indicate grains causing erosion at the instant of time represented, lighter grains indicate grains that will cause erosion at some later point in time. (a) Snout-impact model: Large grains recirculate in the snout and impact the bed, causing wear. Smaller grains behind the snout are too small and cause negligible erosion compared to the snout. $L_{\text{act}}$ is the active eroding length – the only part of the flow causing wear. (b) Zero-slip model: a surface layer of grains tumbles down the snout and strikes the bed, causing erosion, then are entrained in the recirculation towards the tail. No wear occurs in the body of the flow, because there is no-slip on the bed. This case is similar to erosion by saltating grains, the eroding length-scale is $D_p$, the particle diameter. (c) Sliding block model: all grains at the bottom of the flow slide along the bed and cause wear, the active eroding length is the entire arc length of the flow, $L_0$. 
Figure 15. Dimensionless erosion, $e'$ versus strength normalized inertial stress, $N_{SNIS}$, using the visually estimated active eroding length, $L_{act}$, instead of the entire length $L_0$, in the calculation of $e'$. Dry (filled circle, $y = 6E^{-05}x^{0.38}$) and water-saturated (open circle, $y = 0.0002x^{0.22}$) flows. Because $L_{act}$ increases with larger grain diameters and faster drum speeds, it tends to covary with the inertial stress, weakening the dependency between $e'$ and $N_{SNIS}$. See Table 4 for regression statistics.
Figure 16. Plot of eroded depth per event (rotation), $e_{ed}$, versus $VIF_e$ (defined in equations (17)-(20)) as predicted by modifying a model for bedrock wear by impacting grains. Dry (filled circle, $y = 0.0004x^{0.98}$) and water-saturated (open circle, $y = 0.0002x^{0.71}$) flows. This zero-slip model of bedrock erosion explains less of the variance in the data than $N_{SNIS}$, the strength-normalized inertial stress. See Table 4 for regression statistics.
2.10 Supplementary Materials

Figure A1. Photograph of highly degraded dams in the upper reaches of the Illgraben Torrent, Switzerland. The dams were constructed in the 1960s, and have been eroded by decades of boulder-rich debris flows. Photograph taken in May 2006.
Figure A2. Photograph of the sidewall of a check dam in the Illgraben Torrent, Switzerland. Significant forces on the sidewalls, including those from lateral accelerations at the front of boulder-rich snouts, caused erosion on the order of centimeters, exposing the steel reinforcement. The width of the dam is ~1 meter.
Table A1. The specific data points in each figure are identified by their Run ID (column 1 of Table 2 in the main text).

Figure 7  25-31, 35-37, 41-49, 52, 73-74, 78-79, 83-86, 90
Figure 8  41, 43-45, 47, 49, 57-58, 61-63, 66-67, 83-86
Figure 9  24-31, 63, 66-67, 69-70, 93-96, 105
Figure 10 36-37, 41-43, 47-49, 84-87, 108, 110-112
Figure 11 25-31, 35-37, 41-49, 52, 73-74, 78-79, 83-36, 90
Figure 12 42, 46, 48, 55-68, 73-74, 87-88, 91-96, 105, 110-113
Chapter 3

Mean and fluctuating basal forces generated by geophysical granular flows

3.1 Introduction

Debris flows, rock avalanches, and pyroclastic flows erode their boundaries as evidenced by knicks, scratches, grooves, and plucked blocks on overrun surfaces [e.g. Grunewald et al., 2000; Stock et al., 2005; Paronuzzi, 2009]. Flow-bed interactions cause wear of the boundary. To develop a mechanistic erosion law for granular flows, we must understand what causes and controls the basal forces [Dietrich et al., 2003; Gauer and Issler, 2004; Stock and Dietrich, 2006]. Knowledge of how the basal force distribution is influenced by flow properties such as grain size and bulk velocity also contributes insight into internal flow dynamics and energy dissipation at the boundaries of geophysical flows [Bartelt et al., 2006; Dufek and Bergantz, 2007].

Several examples of the force signal measured at the base of natural and experimental debris flows are shown in Figure 1. These studies have load plate areas that range from 1 cm² to 8 m², measurement frequencies that range from 1 Hz to 2000 Hz, and exhibit variations in the force signal at many different timescales [Iverson, 1997; Tecca et al., 2003; McArdell et al., 2007; this study]. In each case, the force data time series can be decomposed into the mean and fluctuations about the mean. Hsu et al. [2008], summarizing the work of others, proposed that sliding wear varies with the mean normal force (following Archard’s law) and impact wear varies with the fluctuating force component (as scaled by the inertial stress term defined by Bagnold [1954] and proposed by Stock and Dietrich [2006]). The mean normal force is equivalent to the static normal force of the mass (the product of bulk density, gravity, flow thickness, and a slope correction [e.g. Tecca et al., 2003; McArdell et al., 2007]). These forces are only strictly equivalent under steady uniform flow. Although natural flows are almost never steady and uniform, over a sufficient period, time-averaged contact forces balance the static weight of the granular flow itself [Iverson, 1997]. This mean force is linearly related to sliding wear as supported by experiments [Archard 1953; Lee and Rutter, 2004]. On the other hand, the fluctuating force component contains information about collisional dynamics, as demonstrated by small scale experiments [Longhi et al., 2002; Gardel et al., 2009]. However, in larger scale experiments with wide grain size distributions, measurements of the high frequency fluctuating force component are few. There are challenges with data interpretation as well because the fluctuation magnitude and pattern are closely linked to specific measurement system properties such as load plate size and measurement frequency [Iverson 1997; Jalali et al., 2006]. Here we report the results of a new experimental facility that allows us to explore the controls on the basal force fluctuations of large scale, wide-grain size distribution granular flows, and thus gain a greater understanding on how to predict channel erosion by impact from geophysical granular flows.
Basal normal force fluctuations due to particle impacts give a quantitative measure of kinetic energy transfer from a granular flow to the bed surface [Clark, 1991; Zenit et al., 1997a; Kleis and Hussainova, 1999; Pronk et al., 2009]. The normal component of the impact velocity is linked to wear of brittle materials by impact [Engel, 1978]. We propose that the normal force fluctuations are quantitatively related to surface impact wear. We show that the time-averaged instantaneous impulse on the load plate is the variance of the force signal, $F_V$. Results from a suite of experiments in a 4-meter diameter, 80-cm wide vertically rotating drum reveal scaling of $F_V$ with characteristic grain diameter, flow velocity, matrix viscosity, and inertial stresses. We find the distribution of force fluctuations well described by a generalized Pareto function and show how the shape and scale of the force distribution are controlled by the width and effective diameter of the grain size distribution. These relationships suggest how localized, high frequency basal forces can be estimated in field-scale granular flow.

The relationship between kinetic energy and force fluctuations

During a particle-wall collision, kinetic energy is transferred from the incoming particle to the surface. The energy lost by the particle upon collision, or the momentum transferred, is equal to the force on the boundary integrated over time, also known as the impulse [Zenit et al., 1997a; Zenit and Hunt, 1999; Pronk et al., 2009]. Depending on the magnitude of the impact and the surface properties, strain energy may be converted to broken bonds in the surface [Bayer, 2004]. In brittle materials, wear is caused by particle-surface collisions which enlarge microcracks during the passage of the impact pressure wave [Engel, 1978] (Figure 2). It follows that the rate of impact wear of bedrock by granular flows is directly related to the transfer of kinetic energy from the granular flow collisions to the bed [e.g. Clark, 1991].

The relationship between the change in kinetic energy and the force on a surface can be derived from Newton’s second law and the definitions of kinetic energy and impulse. In elastic collisions, momentum transferred to the boundary is the change in kinetic energy, $KE$, of the impactor,

$$\Delta KE = \frac{m \Delta u^2}{2}$$  \hspace{1cm} (1)

where $m$ (kg) is the mass of the impactor and $u$ (m/s) is its velocity.

Newton’s second law can be written as

$$F = m a = m \frac{\Delta u}{\Delta t}$$ \hspace{1cm} (2)

where $a$ (m/s$^2$) is acceleration and $t$ (s) is time.

The definition of impulse, $I$ (kg m/s), for a single collision is the force integrated over the duration of the collision. Then substituting from Newton’s second law,

$$I = \int F(t)dt = F\Delta t = m\Delta u$$ \hspace{1cm} (3)
For collisional impulses associated with particles within a bulk mass that imposes an average ambient load on the boundary, we use the force deviation:

\[ I_p = \int [F(t) - \bar{F}(t)] dt \tag{4} \]

Where \( I_p \) (kg m/s) is impulse due to particle collisions and \( \bar{F} \) (N) is the ambient force. For example, for a hammer hit under water, the ambient weight of the water does not contribute to the impulse from the hammer hit.

With simple algebra we relate impulse and kinetic energy

\[ I_p^2 = m^2 \Delta u^2 \tag{5} \]

\[ \frac{I_p^2}{2m} = \frac{m \Delta u^2}{2} = \Delta KE \tag{6} \]

Substituting from the relationship between impulse and force, we have an expression relating the change in kinetic energy during particle collisions and the associated force on the boundary:

\[ \Delta KE = \frac{I_p^2}{2m} = \frac{\left[ \int (F(t) - \bar{F}(t)) dt \right]^2}{2m} \tag{7} \]

Now we have a quantity that can be extracted from a force time series which is relatable to \( \Delta KE \). Note that the right hand side of Equation 7 has a similarity in form to the definition of sample variance of a finite population,

\[ \text{Var}(x) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \tag{8} \]

We later show that our data has the relevant properties such that the time average of \( I_p^2 \) can be directly quantified by the force variance, \( F_V \) (N^2). The variance of the time series of the force deviation is our metric for the kinetic energy transfer to the bed from the granular flow.

**Flow properties and force measurements**

In a granular flow, what variables influence force fluctuations and therefore kinetic energy transfer to the bed? Granular flows have been divided into regimes of momentum transfer that describe the inertial state of the flow, sometimes likened to solid, liquid, and gaseous states of matter [Jaeger et al., 1996]. The progression of regimes from static to highly inertial flows are known as jammed, quasistatic, macroviscous, elastic, rapid, and inertial flow [e.g. Bagnold,
In different flow regimes, different mechanisms transmit force to the bed, either by short-term collisions, viscous forces, or sustained contacts between grains. For example, in the inertial regime, free collisions are more likely to take place, but in the frictional regime, the particles at the bed are not colliding directly on the surface but still transmitting collision energy from elsewhere in the flow through its contact. Nondimensional numbers are useful for defining and comparing flow regimes and therefore predicting the dominant mechanisms of force transmission.

The nondimensional Bagnold number is a measure of the ratio of inertial to viscous stresses in a granular flow. As represented by Iverson [1997], it is

$$N_{Bag} = \frac{v_s \rho_s D_p^2 \frac{\partial u}{\partial z}}{\nu_f \mu} \approx \frac{v_s \rho_s D_p^2 u_s}{\nu_f \mu}$$

where $v_s$ (-) is the volume fraction of solid, $\rho_s$ (kg/m$^3$) is the solid density of solid, $D_p$ (m) is particle diameter, $u$ (m/s) is velocity, $z$ (m) is height, $\frac{du}{dz}$ (s$^{-1}$) is the shear rate, $\nu_f$ (-) is the volume fraction of fluid, $u_s$ (m/s) is surface velocity relative to the near bed velocity, $h$ (m) is total flow depth, and $\mu$ (Pa·s) is the fluid viscosity.

To quantify the relative importance of inertial stresses, Savage and Hutter [1989], proposed the ratio of inertial to total normal (gravitational or frictional) stresses.

As reported by Iverson and Denlinger [2001] it is:

$$N_{Sav} = \frac{\rho_s D_p^2 \left( \frac{\partial u}{\partial z} \right)^2}{(\rho_s - \rho_f) g h}$$

Though these nondimensional numbers are defined strictly for steady, uniform flows, a comparison of their relative values are useful in scaling experiments and comparing natural and experimental flows. Flow regime of experiments must be similar to that of natural flows in order to draw a connection between quantitative measures such as $F_v$ from laboratory to field.

Debris flows in nature are gravity-driven free-surface granular flows with a viscous matrix fluid. Most natural flows are not truly inertial, but are frictional or transitional to inertial if deficient in fines, and frictional to viscous if fines-rich [Campbell, 2002; 2005]. Natural flows are more complex than simple single-size spherical grain flows and vary greatly in grain size distribution width and maximum, water content, grain shape, matrix density and viscosity, velocity, and overrun topography. How these properties affect the mean and fluctuating basal force component is addressed in the literature from many perspectives, including granular physics [Jaeger et al., 1996; Faug et al., 2009], geophysics [Platzer et al., 2007a; 2007b], and engineering [Buffiere and Moletta, 2000; Pronk et al., 2009].

The bulk properties of large-scale granular flows are commonly quantified with basal force plates. For example, the average normal force is used to calculate the density of the flow.
[McArdell et al., 2007], or to calculate the dynamic friction angle of flows in a large drum [Kaitna et al., 2007a; 2007b]. Evolving basal normal stresses and the passage of roll waves have also been quantified in a large scale chute flow of natural geological materials [Iverson et al., 2010]. However, there are fewer studies that quantify the very high frequency force fluctuations from individual impacts on the boundary of a dense gravity driven flow [e.g. Iverson, 1997]. Some examples are studies with geophones that measure ground vibrations in natural debris flow channels [Itakura et al., 2005; Arattano and Marchi, 2008]. Huang et al. [2007] quantified the frequencies due to different parts of the flow, confirming that the vibrations were due to rocks interacting with the bed.

The force from a single collision on the boundary is directly related to the incoming particle diameter because it defines the mass for kinetic energy transfer. Particle diameters in natural granular flows are often spread over several orders of magnitude, from $10^{-6}$ to $10^0$ m [e.g. Chen et al., 2001; Perez, 2001; Hürlimann et al., 2003]. Grain size distribution may become finer as the granular mass travels from source to deposition area because impacts break down particles and create fine materials, and fines may be picked up on the path [Berti et al., 1999].

While monosized or narrow size distributions can be described adequately by a single grain diameter, $D_p$, there is not agreement on the best effective diameter ($D_{eff}$) to use in nondimensional scaling or kinetic energy calculations when the flow has a wide particle size distribution. Wide grain size flows are more complicated because the different size fractions may either damp collisions or add to momentum transfer by particle-particle collisions. It is common to partition clay and silt-sized particles with the fluid fraction of a debris flow because they would not settle out of the flow on the timescale of the event [Iverson, 1997]. Some have proposed that a higher percentile than the median, such as the 84$^{th}$ percentile of the grain size distribution ($D_{84}$) may be a better effective grain size to characterize a wide grain size distribution [Stock, 2003]. Gandhi and Borse [2004] found surface erosion to correlate well with the weighted mass particle size in low solid density flows. For a single impact, force would be expected to scale with $D_{eff}^3$, which is the same as the mass scale. In fluidized beds with vertical flow and 0.5 solid fraction, Zenit et al. [1997a] found a correlation between particle pressure (force) and diameter to the fourth power. At a dilute 0.1 solid fraction, when particles are more free to move, the scaling was diameter to the 3$^{rd}$ power. This example illustrates the importance of both particle diameter and solid fraction (which is closely related to flow regime) when predicting force fluctuations.

Like for particle size, there is an open question of what is the appropriate estimate for particle velocity when estimating kinetic energy transfer in granular flows. Zenit et al. [1997b] found a power law dependence between force and particle velocity with an exponent of 0.8 for individual particle impacts. Louge and Keast [2001] found dependence of force on the normal component of velocity with an exponent of 1.2, also for individual drops. In a fluidized bed, Pronk et al. [2009], used piezoelectric sensors to measure maximum pressure, which scaled with velocity to the 1.2 power. Jalali et al. [2006] showed the increase in the magnitude of normal force fluctuations on the wall when the rotation velocity was increased in a Couette cell geometry. One challenge is relating mean flow velocity to individual particle-bed impact velocity. The shear rate, estimated as $du/dz$, may be a better measure to connect bulk properties and fluctuation forces. In a previous drum study, the shear rate and the drum velocity for a water-saturated gravel flow were related linearly [Hsu et al., 2008; Figure S1].
The viscosity of the matrix fluid varies over several orders of magnitude in geophysical granular flows, from air (~$10^{-5}$ Pa s) to mud (~$10^1$ Pa s) [Sosio et al., 2007; Sosio and Crosta, 2009]. Joseph et al. [2001] observed that in a viscous fluid, particle velocity decreased before impacting a surface, suggesting that more viscous fluids will damp the kinetic energy transfer to the surface. The effective coefficient of restitution varied with the Stokes number, which is a measure of a particle response time to changes in fluid flow. Liao and Hsiau [2009] and Yang and Hsiau [2006] studied partially and fully saturated granular flows of beads and air, water, and glycerin mixtures and confirmed a decrease in collision force for an increase in viscous force. However, the addition of fine-grained materials can either strengthen flows or make them more mobile, depending on the rest of the grain size distribution [e.g. Iverson, 2003]. Shear-thinning and shear-thickening fluids, those that change viscosity with changing velocity or stress, add to the challenge of predicting the effect of viscosity on kinetic energy transfer. Data are sparse for force fluctuations and natural muddy particulate flows.

A possible proxy for force on the boundary is the inertial stress scaling, $\sigma_i$, which is a combination of some of the previous variables. It given by

$$\sigma_i = \nu_s \rho_f D_p^2 \left( \frac{\partial u}{\partial z} \right)^2$$

The inertial stress has been hypothesized to scale with boundary stress fluctuations and therefore with bedrock erosion beneath debris flows. This hypothesis has been supported by longitudinal channel profile modeling and laboratory erosion experiments [Stock and Dietrich, 2006; Hsu et al., 2008]. Even though the inertial stress scaling is not perfectly suited for debris flows because the regime is not completely inertial, the correlation is good. Measurements of basal force fluctuations would be valuable to confirm or reject this hypotheses.

Distribution of forces

Quantifying the distribution of basal normal force fluctuations for known flow compositions in the laboratory is a step toward predicting force distributions in the field. Even static granular flows are found to exhibit widely spatially varying contact forces on their boundary. The forces exhibit an exponential distribution which has been explained by force chains, which are linear structures of highly stressed grains [e.g. Liu et al., 1995]. The exponential distribution of boundary forces has also been observed in flowing grains [Longhi et al., 2002; Gardel et al., 2009]. The distribution of basal forces is affected by many variables, including those discussed above, and the size of the force plate in relation to the impactors [Iverson 1997; Jalali et al., 2006]. The distribution of force fluctuations in large scale granular flows with a large range of particle diameters has not been widely explored.

The high values of the force distribution are the most relevant to erosion by impacts on the boundary. For distributions with extreme values, fitting a model specifically to the tail of the force distribution instead of the entire dataset may yield a better fit to the high data points [Castillo et al., 2006]. The generalized Pareto distribution (GPD) is a general function that can be fit for a wide range of distributions with different tail shapes. The tail is defined as the data
above a chosen threshold, these values are also known as the exceedences. The probability density function is given by

\[ y = f \left( x \mid k_{\text{GPD}}, \sigma_{\text{GPD}}, \theta_{\text{GPD}} \right) = \left( \frac{1}{\sigma_{\text{GPD}}} \right) \left( 1 + k_{\text{GPD}} \frac{x - \theta_{\text{GPD}}}{\sigma_{\text{GPD}}} \right)^{-\frac{1}{k_{\text{GPD}}}} \] (12)

where \( k_{\text{GPD}} \) is the shape parameter, \( \sigma_{\text{GPD}} \) is the scale parameter, and \( \theta_{\text{GPD}} \) is the threshold parameter. There are three basic forms, if \( k_{\text{GPD}} \) is positive the tail of the distribution decreases as a polynomial, if \( k_{\text{GPD}} \) is zero the distribution is exponential, and if \( k_{\text{GPD}} \) is negative the tail is finite [Castillo et al., 2006; explanatory Figure S2]. In geophysics applications, this model has been used to model extreme rain or snow fall [e.g. Blanchet et al., 2009; Overeem et al., 2009] and wave heights [e.g. Thompson et al., 2009]. The relationship between flow properties and GPD shape and scale factors will be useful for predicting basal force distributions in granular flows where the forces cannot be measured.

Thus, existing work on basal normal forces shows the potential for extraction of much information about the granular flow from the mean and fluctuating force signal and the force distributions. However, measurements of large-scale flows with wide grain size distributions and natural materials are needed for quantitative relations that are applicable to geophysical flows in nature. We lack an extensive catalog such measurements, and so we perform experiments to fill this void and test the following hypotheses.

1. The mean force profile will match the expected static weight of the flow as calculated from the bulk density, height, and bed slope.
2. Force variance (FV), a measure of the kinetic energy transmitted to the bed, will scale with \( D_{\text{eff}}^n \) where \( n \approx 3 \). For wide grain size distributions we propose that \( D_{84} \) is a better \( D_{\text{eff}} \) than \( D_{50} \).
3. Force variance (FV) will scale with drum velocity to a power of \( \sim 1 \).
4. Force variance (FV) will decrease as the fluid matrix viscosity increases.
5. Force variance (FV) will vary with inertial stress, \( (du/dz)^2D^2 \), and D is scaled by \( D_{84} \).
6. The generalized Pareto distribution shape and scale factors of basal normal force will be quantitatively related to the grain size distribution and \( D_{84} \).

3.2 Methods

To test the relationships between basal normal force measurements and grain diameter, velocity, and matrix viscosity, we carried out experiments in a 4-meter diameter, 80-centimeter-wide vertically rotating drum (Figure 3). The drum was driven by a 20-kilowatt induction motor and controlled by a variable speed inverter drive. The maximum tangential speed was 3 m/s. 32-mm thick Plexiglass windows allowed a side view of the flow. A 6-mm thick rubber liner with channel-spanning 25-mm tall by 25-mm wide treads, spaced every 20 cm in the stream-wise direction, prevented bulk sliding of the entire mass on the flume bed. Normal force on the bed was measured with a 15x15 cm steel load plate. The large size of the flume avoided unfavorable down-scaling of sediment diameter [Denlinger and Iverson, 2001]. Each rotation of the wheel
produced a force profile running the length of the flow, obtaining multiple profiles per experimental run. Figure 3c shows the typical position of a flow during an experiment.

We report force measurements from 47 experimental runs (Table 1). We varied grain size distribution, drum velocity, and water content. Grain size distributions were assembled from a supply of sediment which ranged from clay-sized (0.0015 mm) to large cobble (150 mm diameter) (Table S1). D50 and D84 were calculated for the fraction greater than sand (>2 mm). We chose the sand-sized cutoff because we observed sand to stay in suspension in the clay-silt-water matrix fluid. The three types of grain size distributions in these experiments were: (a) narrow grain size distribution (including water-saturated gravel (WSG), dry gravel (DG), and mud + gravel (MG)), (b) wide grain size distributions with clay, silt, sand, gravel, cobbles and water, called muddy mixed flows (MM), and (c) bimodal water-saturated gravel and large cobbles (WSGB). Figure 4 shows an oblique top view of the different runs and Figure 5 shows the grain size distributions.

For some flow compositions, three different velocities were tested: 0.4, 0.8, and 1.25 m/s. This velocity range gave a bulk shear rate similar to previous experiments and natural flows (Table 2), and avoided significant centrifugal force. The shear rate is estimated as \((u_s - u_b)/h\), the difference between the velocity at the surface and the base, divided by the flow height. The matrix fluid was either air (DG), clear water (WSG), a clay-water mix (MG) or a clay-silt-sand-water mix (MM). Only a few dry experiments were completed due to excessive dust generation from particle-particle collisions.

Experimental flows had between ~450 and 1200 kilograms of material, which created a shallow flow with maximum height of ~25 cm. The different sediment size components were premixed in small batches in a cement mixer before being placed in the flume, except the silt and clay components (to avoid dust generation). Premixing was necessary because rotation in the flume was not sufficient for quickly homogenizing the coarse components. Water, silt, and clay were added to the drum immediately before the experiment, and were incorporated into the mix within a few rotations. Bulk density was measured by sample tests immediately after experiments. The density was calculated with the mass of a sample of known volume of 270 cm³ and included clasts up to ~20 mm. The bulk density range was 1.9 – 2.3 g/cm³ for the muddy mixed flows and 1.9 g/cm³ for the water-saturated gravel flows. The viscosity of the clay and water mud mixtures used as the matrix for Runs 40-47 were measured with a Bohlin Visco 88 BV, a standard coaxial cylinder viscometer. The measuring spindle diameter was 30 mm and outer cylinder diameter was 33 mm.

The height profile of the flow during the experiment was measured by an Acuity AR4000 laser scanner, which swept a laser line across the centerline of the flow with a rotating mirror. The laser was connected to a free-standing mount and slid into place in the flume (Figure 3c). Profiles were collected at 5 Hz. The load cell measurements were synchronized with laser scanner measurements through a trigger from a rotary encoder mounted in the hub of the wheel.

**Force data acquisition and processing**

The steel load plate was attached at the center to an Interface Force Model SWP10-5K-B000 Precision Force Transducer with a maximum load of 2273 kg (5000 lb). The plate and load cell assembly was attached to the flume and further stabilized by two large steel rectangular tube bars on the outside of the drum. Water-proofing was accomplished by building in a neoprene
gasket on the back side of the plate-bounding gap. The gap was filled with silicone to prevent sediment from lodging in the gap and restricting plate movement. Power and data were transferred to and from the plate through a slip ring on the hub of the wheel. Measurements were made by the analog-to-digital board at 200,000 samples/second, then averaged to 1mS, effectively sampling at 1 kHz. The force data sampling resolution was 0.4 to 1.25 mm (arc length) spacing between samples, depending on the drum speed. Because the load plate location in the drum is recorded in the metadata, it could be synchronized with the 5 Hz laser height profiles. This was necessary because the passage of the load plate introduced a temporary low-roughness, tread-less area under the flow and disturbed the height profile.

The force plate was sensitive to the ambient temperature, with drift through time as the laboratory warmed or cooled. To remedy this we zeroed the force profiles for each rotation using the data immediately before the passage of the flow over the plate. Despite the drift of the zero, we checked the calibration of the plate with a known static mass through time, and this remained constant. The resting noise of the force plate was roughly 0.2 kg (~2 N). From drop tests on the bare load plate, we found that the negative rebound associated with a peak force is about 0.25 of the initial positive value (Figure S3). We also defined the linear increase in the plate response for different masses dropped from 1 and 4 cm (Figure 6a). Impacts on the bare plate were usually represented by 1 or 2 peak values (e.g. Figure 6b, c). Thus our force data fit the common experimental finding that collision duration is usually 10s of ms long [Zenit et al., 1997a; Gardel et al., 2009; Pronk et al., 2009].

A ~15 Hz oscillation was present in all of our data (e.g. Figure 7a, d). To explore if this was a resonant response of our load plate, we tested for the plate’s resonant frequency. When subjected to a hammer-hit impulse in air and under pure water, there was a weak ringing signal at ~140 Hz and ~100 Hz, respectively (Figure 6b, c). Even with the low-pass filter decreasing the cutoff frequency from 500 Hz to 50 Hz, which should have removed any unwanted resonant response, the 15 Hz oscillation was still present. Since the oscillation was in all of our data, we attributed it to machine noise or flume vibrational response (e.g. from the chain and sprocket driving mechanism). We removed this signal with a bandstop filter which removed frequencies from 2.5 to 50 Hz. The local fluctuations around the average remain the same magnitude as before the 15 Hz signal was removed (Figure 7c, f).

The runs have different quantities of recorded data depending on their primary purpose, to test erosion or force. Data were not recorded for the entire experiment, but in one minute intervals upon manual triggering, spaced throughout the run. Runs 1-12 (Table 1), which were erosion experiments, have between 48 and 78 recorded force profiles. Runs 13-47 were completed for force measurements and have between 2-24 recorded force profiles. From the recorded force time series, we extracted both the mean and fluctuating force components for correlation with flow variables. To obtain the mean force profile $\bar{F}$, the data were binned and averaged for each one degree of arc along the drum. These bins contained 28-90 measurements depending on the drum speed. The binned profiles for the multiple rotations in a run were then averaged to create a single average force profile, similar to the method of Iverson et al. [2010].

The process of calculating the kinetic energy transfer to the bed from a force signal (Equation 7) depends on the data acquisition system, especially the number of points defining a peak. In an ideal time series of basal normal force data under a granular flow, individual impacts on the boundary would be distinguishable. This would be the case if the force was measured exactly over the contact area, and there were sufficiently frequent force measurements to fully
define the impact. Then, integration under the force curve obtains the impulse for that collision. However, if the plate area is much larger than the diameter of a typical flow particle, then the force signal produced during an experiment is the result of many collisions, and the plate is loaded with an ambient force. Also, the fluctuating force signal is related to $\Delta KE$ through the relevant mass, $m$.

Following Equation 7,

$$I_p^2 = \left[ \int (F(t) - \bar{F}(t)) \right]^2$$

We use a discretized version of this, thus

$$I_p^2 = \left[ \sum (F(t) - \bar{F}(t))(\Delta t) \right]^2$$

The time interval is our sampling interval of 0.001 seconds, which often captures the entire peak, so for the impulse associated with a collision we do not need to sum over several measurements. Hence we calculated the impulse squared for each time step:

$$I_p^2 = \left[ (F(t) - \bar{F}(t))(\Delta t) \right]^2$$

The constant $\Delta t$ can be pulled out

$$I_p^2 = \left[ F(t) - \bar{F}(t) \right]^2 (\Delta t)^2$$

$$\frac{I_p^2}{\Delta t^2} = \left[ F(t) - \bar{F}(t) \right]^2$$

To obtain the average of all the individual values for individual impulses, we sum and divide by the number of points, $n$.

$$\sum \left( \frac{I^2}{\Delta t^2} \right) = \frac{\sum [F(t) - \bar{F}(t)]^2}{n}$$

As noted earlier, observe that the right hand side of Eqn. 18 now has the form of the variance of the force

$$Var(x) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$
The variance of the force, $F_V$, has units of $N^2$. The choice of the bin size, $n$, is important for several reasons. $\bar{F}$ should not significantly vary in time within the $n$-point bin, so that $\left[F(t) - \bar{F}(t)\right]$ is an accurate calculation of the instantaneous force fluctuation from the mean. The bin size $n$ must be large enough so that dividing by $n$ and $n-1$ are similar. Peak forces from collisions should be represented by one value over the given $\Delta t$. We use $n = 140$ (5 arc degrees, 17 arc cm in the drum) to illustrate longitudinal variations and $n = 280$ (10 arc degrees, 35 arc cm) to compare $F_V$ between different runs.

To differentiate $F(t)$ peaks that represent collisional energy transfer versus low-level noise that would be present regardless of granular collisions above the plate, we determined a threshold, $F_{\text{thresh}}$. We used the force data immediately preceding the flow contact with the plate, $F_{\text{bare}}$ (Figure 8b), and calculate two standard deviations of that data as the threshold (dotted line).

$$F_{\text{thresh}[\text{run}]} = (2 \text{stddev of } (F_{\text{bare}[\text{run}]))$$

$F_{\text{thresh}}$ is run-specific because the drum itself will vibrate due to collisions from the particulate flow everywhere on its bed surface. The thresholds range from 1.2 to 14.4 N (Table 1). Generally $\sim 0.5$ of the data is removed after application of the threshold filter.

Figure 9 shows $F_V$ calculated for five degree bins on the flow longitudinal axis, showing that it is not constant along the length of the flow but reaches a maximum near the maximum height of the flow. To obtain a run-average characteristic $F_V$, we choose a subset of the data that was relatively constant in $F_V$ (Figure 9). We avoided the front and the back of the flow, where the flow height tapers. We used an area of relatively constant height in a 10 degree drum angle swath (35 cm arc length) starting 20 degrees behind the flow front.

Figures 7-9 illustrate steps in the process of obtaining the $F_V$ from the force plate signal. Starting with the raw force signal, we (1) subtract out the plate weight, which varies like a sine wave as the plate rotates 360° around the wheel. (2) Zero the load plate data before each rotation by finding $F_{\text{bare}}$. (3) Use a lowpass filter to check for plate resonance effects (Figures 7a, d). (4) Use a bandstop filter to filter the repeatable machine noise but retain the high frequency signal (7b, e, black line). (5) Calculate the $F_{\text{thresh}}$ (Figure 8b, dashed gray line). (6) Clip the part of the data that is under the flow by finding the front and tail position. (7) Divide the data into bins of 5 degrees (Figure 9a). (8) Calculate the $F_V$ for deviations from $\bar{F}$ which are above $F_{\text{thresh}}$ (Figure 9b) (9) Use the value for $F_V$ calculated for 20-30 degrees behind the flow front to represent the run, because it has a relatively stable mean force near the maximum height of the flow (Figure 9). Table 1 lists the representative $F_V$ for each run obtained from this procedure.

**Distribution fitting of the force fluctuations**

We fit the force exceedences, $F(t) > F_{\text{thresh}}$ to a generalized Pareto distribution function. To quantify the distribution of force measurements we use the MATLAB Statistics Toolbox function `gpfit`, which returns maximum likelihood estimates of the shape parameter and the scale parameter. The threshold we use is $F_{\text{thresh}}$. We use the data near the maximum height of the force.
profile, from 20-30 degrees behind the flow front, in order to obtain force exceedences over a relatively homogeneous section of the flow. We use all wheel rotations of the data from the experiment (multiple rotations). We fit the force data, $F$, itself instead of the metric for kinetic energy transfer, $F_\text{V}$, because it is more general and can be compared to previous quantitative studies of basal force distributions. Since $F_\text{V}$ is derived from $F$, its distribution can be derived from the distribution of $F$.

### 3.3 Results

Representative force profiles from one pass of the load plate are shown in Figures 10-12 for twelve of the experimental runs. Figure 10 shows the entire profiles, illustrating the variation in fluctuation magnitude. Figure 11 is an enlarged view of a five degree bin near the maximum height of the flow, showing details of the high frequency fluctuations. In Figure 12, the force measurements are normalized by the local mean force, emphasizing the scale of the fluctuation around the mean for the different runs. Table 2 shows the nondimensional scaling for some of our experimental end member flows and granular flows in nature. The run MM $D_{84}=110$ mm approaches the scaling values for the finer-grained body of a natural debris flow.

**Average force profiles**

The average force profile, $\bar{F}$, for each run is shown in Figure 13. In 13(a) and (b), the average force profiles are plotted against drum angle, showing the relative positions of the flow front for the gravel and muddy mix runs. Figure 13a shows that for gravel flows, as the grain diameter increases from $D = 4$ mm to $D = 21$ mm, the position of the flow front also changes, migrating upslope. In the bimodal run (dash-dot line), the addition of 150 mm large cobbles moves the front position higher upslope than for a $D = 10$ mm flow alone (thin solid line). The dry gravel has the highest front position (dotted line). These run-average profiles suggest that grain size influences the angle of internal friction and the water in the pores decreases the effective normal stress, thus changing the bulk behavior of the flow. Figure 13b shows the relative positions of the muddy mix flows. The shape of the flow is significantly influenced by a more watery matrix (dash-dot) or halving the volume of the flow (dotted line). During the muddy mix with $D_{84} = 110$ mm the flow progressively changed due to loss of mud to the walls and loss of water to evaporation. The average profile shown in Figure 13b does not reflect any single profile well.

In 13(c) and (d), the run-average profiles are normalized in two dimensions, longitudinally by the length of the flow and vertically by the maximum average force. The normalized profiles show that despite different flow front positions, the WSG flows have a characteristic shape. There are larger differences in the normalized shape of the MM flows. The largest difference in the muddy mix flows is when the water content is increased. The normalization allows straightforward comparison to flows in other geometries (e.g. chute flows). However, during the normalization, information about the position in the drum and bed slope angle is lost.

The mean load plate signal is compared to the expected normal force profile calculated from the flow height and density $\rho gh(\cos \theta_b)$, where $\rho$ (kg/m$^3$) is bulk density, $g$ (m/s$^2$) is gravity, $h$ (m), is flow height measured normal to the bed, and $\theta_b$ is the basal slope (Figure 14).
When the flow transitioned from the roughened boundary to the smooth load plate, it experienced some local slip. This disturbance caused the flow surface to shift and undulate, which is detected by the laser profile. Although both force profiles are similar, the deviation of the two lines in Figure 14 suggests that the entire flow does not have a constant density. This could be due to segregation of solids and liquids or different-sized particles. The deviation indicates that the flow is denser near the front and less dense toward the back, which is consistent with our observation that the tail of the flow in the drum is more watery than the flow front. Using a baseline density near the middle of the flow, the profiles in Figure 14 suggest up to a ~20% increase in density near the flow front. If solid and fluid density are spatially constant in the flow, this suggests that pore space is longitudinally variable in the flow.

**Experimental correlations between force fluctuations and flow properties**

The distribution of force exceedance, $|F(t) - \bar{F}(t)|$, was closely linked to flow composition. The shape factor ($k_{GPD}$) and scale factor ($\sigma_{GPD}$) for the generalized Pareto distribution for $|F(t) - \bar{F}(t)| > F_{\text{thresh}}$ are shown in Table 3 and the exceedance force distributions are shown in Figure 15. In the narrow-grain size distribution flows (Runs 1-4 and 9, water-saturated gravel and dry gravel), the shape factor is negative, indicating that the tails are finite, with a finite maximum. For wide grain size distributions (Runs 6-8, muddy mixed) the shape factor is positive for $D_{84} = 29$ mm and larger, and shape factor increases with $D_{84}$, illustrating an increasing probability of larger force excursions with larger $D_{84}$. The scale factor is related to $D_{50}$ of the water-saturated flows by a power law exponent of 1.1. For the muddy mixed flows, $D_{84}$ gives a similar scaling to the narrow grain size $D_{50}$, with power law exponents of 1.3 (Figure S4).

As grain diameter increases, we expect higher $F_V$ because of the mass and thus energy transfer per collision increases. Figure 16 shows $F_V$ vs. grain diameter, $D_{50}$ and $D_{84}$. The diamonds are narrow size-distribution water-saturated gravel and the circles are wide size-distribution muddy mixed flows. The $D_{50}$ power law exponent is much higher for muddy mixed than water-saturated gravel flows, 4.2 versus 2.1 respectively. If $D_{84}$ is used as the effective diameter the exponents are more similar, 1.7 (muddy mixed) and 2.1 (water-saturated gravel). Water-saturated gravel $D_{50} = 21$ mm may plot lower than the trend line for $F_V$ because its sediment was a different lithology and thus shape than the other gravel in Runs 1-3. The low value may have decreased the exponent on monosized water-saturated gravel. Note that both effective diameter and the width of the distribution are variables in the muddy mixed flows. However, $F_V$ varies systematically with the coarse particle scale, $D_{84}$ for both narrow and wide grain size distributions in our experiments.

As the drum velocity increases we expect a higher $F_V$ because of a higher shear rate, higher individual grain velocities, and a higher collision rate. Qualitatively, we observed that in dry gravel runs, faster drum speeds generated more dust, implying more vigorous particle-particle and particle-wall interactions. Figure 17 shows $F_V$ versus drum velocity. A power law trendline is shown for each set of experiments of constant flow composition at three velocities. Most lines have a power law exponent of 0.9 to 2.3, with water-saturated gravel flows having a lower exponent (0.9 to 1.8) than muddy-gravel flows (1.8 to 2.2). This could be because mud and gravel (MG) flows damp $F_V$ more effectively at lower velocity, increasing the overall power law
The bimodal flows have inconsistent slopes (Table 4) which is probably due to the fact that we lack sufficient data to constrain the rare high excursions, and thus they are not plotted. The matrix fluid between the gravel-sized particles in our experiments, in order of increasing viscosity, are air, water, clay-water, and clay-silt-sand-water. Figure 18a shows FV vs. the matrix viscosity. The two trendlines are fit to runs with similar D_{eff}. For the mud & gravel runs (Runs 40-47), the clay-water viscosities were measured with a viscometer, for air, water, and muddy-mix runs, the viscosity values used are representative values from the literature [Iverson, 1997; Sosio et al., 2007]. We found a weak damping effect with higher viscosity, but the effect was much lower than that of other variables such as grain diameter or drum velocity. For four orders of magnitude change in fluid viscosity, the effect on FV is only one order of magnitude. Surprisingly, the clay-water did not have more of a damping effect than clear water.

In addition to calculating the run-representative FV, we examined FV of individual rotations and looked for changes through time. Over the course of the experiment, changes in the flow could change the kinetic energy flux to the bed. Water-saturated flows tended to produce bubbles and foam, likely caused by a combination of silt-generation from colliding gravel and incorporation of air into the water by bed roughness treads sweeping into the flow. The consequence of these changes is uncertain because air bubbles would decrease the density of the matrix, while silt would increase it. As muddy mixed runs continued, the flows tended to lose water through evaporation, splash, and wall attachment. The most significant evaporation occurred in Run 10, muddy mixed half volume, most likely because it had a larger surface area to volume ratio. Decrease in D_{eff} because of grain-comminution by collision was a minor effect, as tested by sieving gravel size before and after a run.

Despite these temporal changes in the flow, the resulting changes in FV over the course of an experiment were small. We limited the duration of the experiment to minimize these effects. A slight decrease in FV was observed for Runs 2, water-saturated gravel flows (Figure 19a), implying that the silt, bubbles, and comminution of particles slightly diminished the kinetic energy flux to the bed. An increase in FV through time was observed in some muddy-mixed flows that experienced water evaporation during the course of the run (Figure 19b). Figure 19c shows that there was not any significant change in FV during Run 8, muddy mixed D_{84}=110mm. The differences in FV from run to run dominate over any change over the course of a single run.

The inertial stress scale, σ_i (Equation 11) combines several of the previous variables. The relationship between FV and the inertial stress scale is shown in Figure 20 for runs where we have analyzed the overhead video for surface velocity and therefore have a shear rate estimation (Runs 1-8). D_{50} is used in the narrow size-distribution water-saturated gravel flows while D_{84} is used for the effective diameter in the muddy mixed flows, because earlier results showed that it is a better D_{eff} for the wide-size distribution. The volume fraction we use for water-saturated gravel is 0.55 [Hsu et al., 2008] and for muddy mixed is estimated at 0.6 [e.g. Iverson, 1997], because with a wider grain size distribution, more pore spaces are filled with small solid particles than for a single size distribution. Previous studies have used the value 0.6 for natural flows [e.g. Iverson, 1997] which would shift the y-intercept but not the slope of the power law correlation. The solid density value is 2.65 g/cm³. The shear rate, du/dz, is estimated from camera surface velocity fields and drum velocity and ranges from 4.3 to 7.4 s⁻¹. The power law exponent between FV and the inertial stress scale for water-saturated gravel and muddy mixed flows are similar but offset on the y-axis. For a given bulk inertial stress, higher kinetic energy flux to the
surface occurs in the water-saturated gravel flows. The power law exponent is \( \sim 0.87 \) for the water-saturated gravel runs and 0.89 for the muddy mixed runs.

### 3.4 Discussion

The force exceedence distributions showed a distinct difference between the narrow distribution gravel flows and the wide distribution muddy flows. The gravel flows fit a function with a finite tail while the muddy flows had an exponential or polynomial decay of high values, showing how wide grain size distributions lengthen the tail of the force distribution. Other studies found “near-exponential” tails for their boundary force data for both static and flowing cases [Howell et al., 1999; Longhi et al. 2002; Jalali et al., 2006]. However, comparing basal force distributions for studies with different plate area to particle diameter ratios is still unclear [Jalali et al., 2006].

FV is a metric that can be applied to other data sets to estimate kinetic energy flux to the bed. Some things to consider when evaluating FV are the plate area to particle cross-section ratio, the choice of \( F_{\text{thresh}} \), and the machine or ambient noise. Some unique features of the drum, such as the driving mechanism of a chain and sprocket, may have influenced the vibrations and noise in the force data. For example, in Figure 10j, large fluctuations exist in the force data when the plate is not under the flow: this implies that the collisions elsewhere in the drum are reverberating through the whole system. We also found a weak dependence on force fluctuations to the drum velocity of an empty drum. To fully test the effect of this noise, we need comparisons to data from load plates that are not in a rotating drum.

Geophysical granular flows are not spatially homogeneous, and force fluctuations would be expected to vary in different regions of the flow, such as the coarse front and watery tail. In heterogeneously-sized granular flows, often the coarsest particles, which would cause the largest impacts on the bed, segregate to the flow front. The bodies and tails of debris flows are observed to have a larger fluid fraction. Kaitna et al. [2007b] measured a difference in grain size distribution, and therefore bulk density, between the front and tail of a flow of natural material (from fresh debris flow deposits) in a large rotating drum. Although there was some grain size segregation in the flows in our experiments, as observed at the flow surface and suggested by the divergence of the expected force profile calculated with constant density, segregation was not consistent. Also, we purposely analyzed a subsection of the flow (from 20-30 drum angle degrees behind the flow front position) in order to obtain force variance over a relatively homogeneous section of the flow. Instead of interpreting the drum flow as a discrete debris flow event, a more accurate view is to regard different experimental runs as different parts of a debris flow – a coarser front (Run 8, muddy mixed \( D_{84} = 110 \)) or more watery tail (Run 11, muddy mixed more fluid). The variables we explored can be measured or predicted from field observations, and therefore we can hypothesize changes in FV for different field sites. For example, effective particle diameter is related to lithology because different rock types tend to break down into different sizes. Matrix viscosity may vary due to the weathering of different lithologies which produce different amounts of clay and silt sized particles.

The geometry of a shallow granular flow in a drum is significantly different than the geometry of a shallow flow in a straight chute. The advantage of the drum geometry is that through multiple rotations it collects enough data to quantify the mean and the tail of the bed normal force distribution, which is the main objective of this study. Concerns about the relevance
of granular flow studies in drums are include: (1) the absence of deposition, and consequent recirculation of the entire flow, (2) centrifugal force and its influence on the particle dynamics in the flow [Kaitna et al., 2007b] (3) the use of the no basal slip assumption to estimate shear when slip at the bed likely occurs, and (4) a continuously changing bed slope and an unreasonably steep tail.

Debris flows commonly sweep down steep channels and entrain material down to bedrock, leaving little material behind. Hence, these flows do not deposit, and for sufficiently long run and limited slip should experience some recirculation of debris. In our drum the shear rates, as estimated by \( \frac{du}{dz} \), are similar to those estimated for natural flows (Table 2). The drum geometry is more applicable to simulating steep channels as opposed to flow in the low-gradient depositional zone. Centrifugal force is difficult to predict in a flowing media, it is not as dominant as for a solid mass attached to the bed of the drum. The expected centrifugal force, calculated for the entire flow depth and mean flow velocity, was not detected in the large drum study of Kaitna et al. [2007b]. Slip on the boundary occurs in nature, but the issue is when no slip is assumed for calculations. For our shear rate estimates we do assume that there is no slip at the bed, and we roughened the bed with large treads to reduce slip. Because there is no large roughness over the force plate, slip may be enhanced there, and our shear rate estimate may be an overestimate. But visual estimates from the side wall indicates that slip was not a dominant process in most of our flows. Although the bed slope changes continuously from the front to the back of the drum flow, most of our data were evaluated near the flow maximum height, which was usually between 0-20° bed slope angle. This local bed slope lies in the range of natural debris flow bed slopes. Twenty degrees would cause a 13% difference due to the cosine bed-normal calculation. Furthermore, the surface slope of the flows for the middle two-thirds of the flow was generally a constant value, and shifted systematically with changes in flow conditions.

The metric we proposed to scale with particle impact energy onto a surface, \( F_V \), is applicable to other high frequency measurements of basal normal forces. The analysis here can be compared to measurements in other drums [Kaitna, 2007a; b], straight chutes [Iverson 1997; Iverson et al., 2010], and nature [Kean et al., 2009]. A remaining question is the correct value of \( m \) to relate the \( I_p^2 \) and \( \Delta KE \) in Equation 7. For the collision of a single particle, \( m \) would be the mass of the particle. For the force signal from the summation of many collisions, \( m \) may be related to the mass and solid fraction above the plate area.

Several related research efforts complement this study to understand particle dynamics in granular flows. Numerical simulations of granular flows in drums and chutes provide theoretical force data for comparison with the laboratory measurements [McCoy and Tucker, 2008; Yohannes et al., 2008]. Conversely, our experiments provide target data sets for simulation of granular flows with matrix fluid. Experimental data on the accelerations of different sized rocks inside a granular flow provide more data on particle diameter dependence [Johnson et al., 2008]. Through this ongoing and future work, we will gain a better understanding of particle dynamics inside granular flows and their force signal on the boundary.

### 3.5 Summary and Conclusions

Through the use of a new, large-scale vertically rotating drum facility, we explore the mean and fluctuating normal forces at the bed for granular flows composed of materials similar to that found in the field. We performed runs with well sorted, as well as mixed grain size
material, and varied the water and mud content. The square of the collisional impulse on the boundary is a scale for the kinetic energy transferred by impactors to the bed, and this energy is responsible for wear of the bed. We show that the square of the collisional impulse on the boundary is equivalent to the variance of the basal normal force ($F_V$). Characterization of the probability density function of the forces in the maximum force zone of the flow (near the front) is used to show the effects of run conditions on the extreme force occurrences. The approximate kinetic energy imparted to the boundary as measured by $F_V$ scaled with characteristic particle diameter, drum velocity, fluid viscosity, and varied with longitudinal position and flow height. $F_V$ also scaled with an estimation of inertial stress for both narrow and wide grain size distributions as previously hypothesized for debris flows. We found that $D_{84}$ was a suitable effective diameter to relate our wide grain size distribution flows with narrow grain size distribution mean diameter. The tail of the distribution of basal forces was finite for narrow size distribution water-saturated gravel flows but exponential or polynomial for flows with a wider grain size distribution (indicating larger extreme collisional events).

These findings indicate that internal and boundary particle collisions within the flow effect both the bulk properties and the localized force fluctuations. The effect of particle dynamics is transmitted to the bed, and the high frequency basal force fluctuations can be analyzed to extract information about energy dissipation and transfer. The bed surface beneath a granular flow experiences large fluctuations in stress, which an analysis using bulk properties only will not capture. The calculation of force variance ($F_V$) as a measure of kinetic energy transfer to the surface is generalizable to other granular flow studies, and its application to other existing datasets will build a systematic database of force data. This data can be used to guide field work and numerical studies on channel erosion and energy dissipation in debris flows and rock and snow avalanches.
3.6 List of Symbols and Abbreviations

a  acceleration (m/s²)
\( \text{D}_{\text{eff}} \)  effective particle diameter (m)
\( D_p \)  particle diameter (m)
DG  dry gravel
\( F \)  processed force data (N)
\( F_{\text{bare}} \)  average force value over the bare plate immediately preceding \( F \) (the zeroed value) (N)
\( F_{\text{thresh}} \)  threshold force for calculating \( F_V \) (Eqn. 21)
\( F_i \)  force measurement, \( i^{\text{th}} \) in the time series (N)
\( F \)  mean force (N)
\( F_V \)  force variance (Eqn. 20) (N²)
g  acceleration due to gravity (m/s²)
GPD  generalized Pareto distribution
\( h \)  height (m) [in the drum, measured normal to the bed]
I  impulse (N s)
\( k_{\text{GPD}} \)  shape parameter for the generalized Pareto distribution (Eqn. 12)
KE  kinetic energy (N m)
m  mass (kg)
MG  mud and gravel
MM  muddy mixed grain size distribution
\( N_{\text{Bagnold}} \)  dimensionless Bagnold number (Eqn. 9)
\( N_{\text{Savage}} \)  dimensionless Savage number (Eqn. 10)
t  time (s)
u  velocity (m/s)
\( u_b \)  velocity at the base (m/s)
\( u_s \)  velocity at the surface (m/s)
WSG  water-saturated gravel
WSGB  water-saturated gravel, bimodal (with 150 mm cobbles)
z  bed-normal axis (m)
\( \sigma_i \)  solid inertial stress (Pa)
\( \mu \)  viscosity (Pa s)
\( \nu_f \)  volume fraction of fluid (-)
\( \nu_s \)  volume fraction of solid (-)
\( \theta_b \)  basal slope
\( \theta_{\text{GPD}} \)  threshold parameter for generalized Pareto distribution (Eqn. 12)
\( \rho_f \)  density of fluid fraction (kg/m³)
\( \rho_s \)  density of solid fraction (kg/m³)
\( \sigma_{\text{GPD}} \)  scale parameter for generalized Pareto distribution (Eqn. 12)
3.7 References


Feng, Z. and Ball, A. (1999), The erosion of four materials using seven erodents - towards an understanding, Wear 233, 674--684.


Hürlimann, M., Rickenmann, D. and Graf, C. (2003), Field and monitoring data of debris-flow


Kean, J., McCoy, S., Staley, D., Tucker, G. and Wasklewicz, T. (2009), Field observations of


3.8 Tables

Table 1 begins on the next page.
Table 1. List of runs

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WSG: water-saturated gravel
MM: muddy mixed
Table 3. F distribution Pareto fits.

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3.9 Figures

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3.10 Supplementary materials

Figure S1. Relationship between drum velocity and shear rate in a shallow drum flow.

\[
y = -16.5x + 2.25 \\
R^2 = 0.94
\]
Figure S2. Three major forms of the generalized Pareto distribution.
**Figure S3.** Load cell response to single hit peak values. Negative response is ~25% of the positive peak value.
Figure S4. Generalized Pareto Distribution shape and scale factors versus $D_{\text{eff}}$. 

\[ y = \begin{cases} 
0.0107x^{2.4744} & \text{MM D50} \\
0.0718x^{1.263} & \text{MM D84} \\
0.4566x^{1.192} & \text{WSG} 
\end{cases} \]

\[ R^2 = \begin{cases} 
0.8966 & \text{MM D50} \\
0.9424 & \text{MM D84} \\
0.9683 & \text{WSG} 
\end{cases} \]
<table>
<thead>
<tr>
<th>median mm</th>
<th><strong>Description</strong></th>
</tr>
</thead>
</table>
| 0.0015    | **Hydrite 121S**  
Manufacturer: IMERYS  
Distributor: Pacific Coast Chemical  
Kaolin clay |
| 0.0077    | **DB Float**  
Distributor: Pacific Coast Chemical  
Kaolin Clay |
| 0.022     | **Silcosil 125**  
Manufacturer: U.S. Silica  
Distributor: Pacific Coast Chemical  
Ground Silica |
| 0.4       | **Sand 60**  
RMC Pacific Sand. |
| 1         | **Sand 1c**  
RMC Pacific Sand. |
| 1.5       | **Sand 3**  
RMC Pacific Sand. |
| 2         | Coarse aquarium  
RMC Pacific Sand.  
Sieved as 2 mm median. |
| 4         | **Pami 1/4”**  
Quartzite River gravel |
| 10        | **Pami 1/2”**  
Quartzite River gravel |
| 13        | **Pami 3/4”**  
Quartzite River gravel |
| 21        | **Limestone Gravel**  
Cool Cave Quarry  
Density 2682 kg/m3 |
| 25        | **Mexican Pebbles**  
hard beach or river pebbles. 1-2 inches  
Distributor: American Stone and Soil |
| 55        | **Mexican Pebbles**  
hard beach or river pebbles. 2-3 inches  
Distributor: American Stone and Soil |
| 80        | **Mexican Pebbles** |
| Grain size distribution: | hard beach or river pebbles. 3-5 inches  
Distributor: American Stone and Soil |
|-------------------------|----------------------------------------------------------------------------------|
| 150 | **Moss rock boulders**  
basalt  
Distributor: Acapulco  
Grain size distribution: |
Chapter 4

Bedrock erosion by granular flow: large scale experiments

4.1 Introduction

The erosive power of flowing mud and rocks is observed in steep channels recently swept by debris flows. In the uplands, channels are often swept clean of vegetation and soil, leaving behind bedrock channels with a variety of wear marks [e.g. Stock et al., 2005; 2006]. Figure 1 illustrates different types of bedrock wear by debris flows in southern Arizona. The photos were all taken within one year of the erosive debris flow events, described in Youberg et al., 2008]. The scoured channels include a smoothly worn parabolic surface (Figure 1a), a broad, flat channel with decimeter-scale blocks removed from the surface (Figure 1b), and a surface with thin chips of similarly oriented rock removed. Field observations like these suggest that there are two main debris flow erosion mechanisms – sliding wear, causing smooth surfaces, and impact wear, creating and removing blocks and chips of rock. There is growing evidence from topographic analysis, laboratory experiments, and numerical simulations that bedrock incision by debris flows is fundamentally different from bedrock incision by fluvial action [e.g. Stock and Dietrich, 2003; Hsu et al., 2008; Yohannes et al., [2008]. However, the rates and mechanisms of bedrock wear by debris flows are still poorly quantified and unverified by measurements, making it difficult to apply mechanistic rules in the field or in landscape evolution models.

Debris flows have long been noted as geomorphological agents, because the sum of many erosive events over geologic time lowers bedrock channels, incising valleys and setting the pace of landscape evolution [e.g. Woodruff, 1971; Wohl and Pearthree, 1991; Eaton et al., 2003; Stock and Dietrich, 2003, 2006; Stock et al., 2005]. Recent technology such as terrestrial laser scanning can quantify the geomorphic effect of debris flow events at high resolution [e.g. Wasklewicz and Hattanji, 2009]. By relating the rate of bedrock lowering to physical erosion mechanisms in a geomorphic law that is testable and field parameterized, one can construct a rule for debris flow erosion based on measurable variables, and begin to relate process and form [Dietrich et al., 2003]. This is important because a large fraction of steep channels are debris flow dominated [Stock and Dietrich, 2003].

In this study we conduct large scale laboratory experiments to test different models of erosion mechanisms by debris flows. These models relate erosion rate to normal forces on the bed expected from bulk inertial solid stress, sliding wear, and impact wear. We focus on the effect of grain size distribution and substrate strength. We find that sliding and impact mechanisms commonly occur simultaneously, and the erosion rate is affected by flow scale, particle diameter, fluid properties, and bed roughness, in addition to the bulk flow properties such as height and mean velocity. Drawing from the experimental results, we present a debris flow erosion law which combines sliding wear and impact wear.
Surface wear by granular flow

The literature on the wear of materials, also known as the field of tribology, is extensive due to the many engineering and industrial applications. Wear of a surface by a solid-fluid mixture is at the most basic level related to energy flux to the surface [e.g. Roco and Addie, 1987; 1987b; Lyczkowski and Bouillard, 2002; Pronk et al., 2009]. Forces imparted on the boundary transfer momentum and energy flux to the surface, causing damage and removal of the erodible substrate. Due to industrial applications, there are many empirical studies of wear by slurries, which are similar to debris flows because they are mixtures of fluid and solid particles. Most empirical formulas for slurry erosion can be distilled to the form

$$e_v = ku^\beta_1 D_p^\beta_2 C^\beta_3 f(\alpha)$$

(1)

where $e_v$ is the eroded volume (m$^3$), $u$ is velocity (m/s), $D_p$ is characteristic particle diameter (m), $C$ is the solid concentration, $\alpha$ is the impact angle, and $\beta_1$, $\beta_2$, and $\beta_3$ are constants [e.g. DeSale et al., 2008]. Sometimes thresholds are included to show that a certain velocity, $u_o$, and particle diameter, $D_o$, must be exceeded before erosion commences:

$$e_v \propto (u - u_o)^\beta_1 (D - D_o)^\beta_2 C^\beta_3$$

(2)

The constants $\beta_i$ are highly dependent on the specific wear system. But the main variables of velocity, impingement angle, hardness, particle size, and concentration are repeated in many studies [e.g. Elkholy, 1983; Iwai and Nambu, 1997; Gandhi et al., 1999; Gupta et al., 1995; Tian et al., 2005; Desale et al., 2009]. An attempt to identify the relative importance of different mechanisms of slurry wear identifies three mechanisms of wear: directional impact, random collisions, and Coulombic friction [Roco and Addie, 1987]. Total wear is a sum of wear caused by particle fluctuation, fluctuation kinetic energy, and friction. Experiments have obtained coefficients for different mechanisms, and the results were employed into a stochastic model [Roco et al., 1987].

In addition to the granular flow properties, properties of the erodible material have a strong influence on the erosion rate. Hardness is a widespread metric for the eroding substrate, in particular the ratio of the particle hardness to the eroded material hardness. But this method is generally used for ductile metals. There have been attempts to relate rock hardness to engineering properties such as strength and modulus of elasticity. Linear relationships were found to be adequate [Shalabi et al., 2007]. Experiments suggest that the splitting tensile strength is a viable indicator for concrete abrasion resistance [Liu et al., 2006]. In addition, catalogs of different indices for many lithologies exist, for example for elastic modulus [Santi et al., 2000] and dynamic impact abrasion index [Al-Ameen and Waller, 1992]. Thus there are many choices for characterizing erodible substrate resistance to erosion.
Bedrock erosion by debris flow

The observation that debris flow landscapes have a different topographic signature than lower-gradient fluvially sculpted landscapes has motivated theory and field work on the geomorphic effect of debris flows and other granular flows [e.g. Stock and Dietrich, 2003; Lague and Davy, 2003]. Debris flows are mass movements with wide grain size distributions (including large boulders), muddy matrix, and high velocities [e.g. Johnson and Rodine, 1984; Iverson, 1997]. Observations in the field provide evidence that localized impacts can remove rock material. In soft rocks like the sandstones of the Oregon Coast Range, bedrock and large boulders often have knick marks indicating impact erosion. In harder rocks, knicks are rarer, but scratches may be found on smooth surfaces. These wear marks have focused studies on wear by local-impacts of boulders, especially under the coarse front of the debris flow [Stock and Dietrich, 2006; Hsu et al., 2008].

Although it is possible to create a debris flow incision law by fitting a simple power law to local channel slope and drainage area [e.g. Lague and Davy, 2003], such relationships shed no light on controls or mechanism, nor can they be used in other basins outside those used in the fit.

A process-based debris flow incision model should have some common elements:

$$\frac{dz}{dt} \propto f \cdot L \cdot G(F)$$

where $dz/dt$ is the lowering rate of a location on the landscape, $f$ (m/a) is the frequency of debris flows, $L$ (m) is the length of the erosive part of the debris flow, $G$ is some function of the force, $F$ (N), on the boundary, and $R$ is a measure of the resistance of the bedrock to erosion.

A debris flow incision model following this general model was proposed by Stock and Dietrich [2006]. For low-slip, collision-dominated wear, they proposed a rate law to express the long term erosion of successive debris flows which scales with solid inertial normal stress:

$$\frac{\partial z}{\partial t} = \frac{K_0 K_I}{T_0^2 \frac{E_{eff}}{F_{frac}}} \cdot f \cdot L \cdot \cos(\theta_b) \upsilon_s \rho_s D_{eff}^2 \left( \frac{u_s}{h} \right)^w$$

where $K_0$ (-) is a constant of proportionality that relates bulk inertial normal stresses to higher excursions of inertial normal stress, $K_I$ is a proportionality constant between rock resistance and incision rate that has dimensions that vary with $w$ and $n$ so that the right side of the expression has units of erosion rate, $T_0$ (Pa) is the tensile strength of the bedrock, $E_{eff}$ (Pa) is the elastic modulus of the bedrock, $F_{frac}$ (m) is a function of the fracture spacing of the bedrock and size of eroding boulders, $f$ (a$^{-1}$) is the frequency of flows over the bedrock per annum, $D_{eff}$ (m) is the effective grain size, $u_s$ (m/s) is the surface velocity, $h$ (m) is the flow height, $L$ (m) is the length of the eroding flow, $\theta_b$ (-) is the basal slope, $\upsilon_s$ (-) is the volume fraction of solid material, $\rho_s$ (kg/m$^3$) is the solid fraction density, and $w$ and $n$ are empirical exponents. If $L$ is defined as the coarse, bouldery, debris flow front only, then $D_{eff}$, the effective grain size, is defined as the particle size that characterizes the collisional normal stress that causes bedrock lowering under...
coarse-grained debris flow fronts [Stock and Dietrich, 2006]. Therefore, $D_{eff}$ may be significantly larger than the mean grain size of the flow.

Small-scale experiments supported the solid inertial normal stress hypothesis [Hsu et al., 2008]. Their observations, however, suggested that sliding wear was also unavoidable, and depended on the properties of the erodible surface and the fluid properties of the flow. Large-scale experiments to test this hypothesis are lacking. Previous experiments have supported an inverse squared relationship between impact abrasion and erodible substrate sample tensile strength [Sklar and Dietrich, 2001; Stock and Dietrich, 2006; Hsu et al., 2008]. The range of lithologies (and therefore rock strength) scoured by debris flows in nature is very large, and other rock properties such as joint spacing and orientation will also play a role in determining wear rates [Stock and Dietrich, 2006; Sas and Eaton, 2008].

**Models for bedrock wear by granular flows**

Based on field observations, the tribology literature, and previous debris flow erosion studies, we propose three simple models for bedrock wear by granular flows. Each is related to basal forces or stresses caused by a physical mechanism for wear. Even though multiple wear mechanisms may occur simultaneously, evaluation of the models may reveal under what conditions (e.g. grain size distribution, fluid content) a particular mechanism is dominant.

**Erosion by impact scaled by bulk inertial stresses**

The inertial stresses in a granular flow are stresses arising from collisions of grains throughout the flow (Figure 2a). As explained in a scaling argument by Bagnold [1954], normal stress is proportional to

$$\sigma_i = \nu_s \rho_s D_p \frac{\partial u}{\partial z} \left( \frac{\partial u}{\partial z} \right)^2$$  \hspace{1cm} (5)

where $\sigma_i$ (Pa) is referred to as the inertial stress (following Bagnold’s [1954] and Iverson’s [1997] terminology), $\nu_s$ (-) is volumetric solids concentration, $\rho_s$ (kg/m$^3$) is solid particle density, $D_p$ (m) is particle diameter, $u$ (m/s) is velocity and $z$ (m) is distance from the bed. Thus, one general form of this erosion model is

$$e_i = k_i \nu_s \rho_s D_p \left( \frac{\partial u}{\partial z} \right)^2$$ \hspace{1cm} (6)

Here $e_i$ is erosion rate due to inertial stresses imparted to the boundary (length/time).

Stock and Dietrich [2006] used $\sigma_i$ as the scale for force on the boundary beneath debris flows, arguing that high stress excursions on the bed are expected to scale with the bulk inertial stress. The theory is derived from uniform layers of spheres sliding over each other, which is far from a natural debris flow, and it does not account for sliding wear at the bottom. However, Hsu
et al. [in prep] found that the fluctuating component of the force measured at the base of granular flow experiments correlated well with the calculated inertial stress.

**Erosion by sliding**

Smooth channel surfaces scoured by debris flows suggest that a sliding mechanism contributes to the erosion. The sliding model approximates the debris flow as a sliding block, exerting a uniform mean normal force on the boundary (Figure 2b). Empirically, a sliding wear law was described by Archard [1953]

\[
e_{v} = \frac{kWx}{H}
\]

where \(e_v\) (length^3) is eroded volume, \(k\) (-) is a nondimensional wear coefficient dependent on the materials in contact, \(W\) (mass·length / time^2) is the applied load, \(x\) (length) is the sliding distance, and \(H\) (mass / length·time^2) is the hardness of the surface being worn away. This model would be expected to capture erosion rates in low-impact flows with predominance of sliding. When there are larger rocks that cause significant wear by impact, sliding wear may become an insignificant component of the overall erosion rate. To write this as a vertical incision rate, \(e_s\), Equation 7 becomes

\[
e_s = \frac{kWx}{HA_s}
\]

In which \(A_s\) (length^2) is the area of erosion and \(e_s\) (length/time) is the vertical erosion rate due to sliding.

**Erosion by impact scaled by impulse to the bed**

Impact wear occurs when a discrete impact on a surface removes a unit of material. At the base of a granular flow, direct collisions and indirect collisions transferred from within the flow transmit kinetic energy to the bed (Figure 2c). This may cause damage in the form of cracks and fractures. The energy lost by the particle upon collision, or the momentum transferred, is equal to the force on the boundary integrated over time, also known as the impulse [Zenit et al., 1997; Zenit and Hunt, 1999; Pronk et al., 2009]. Depending on the magnitude of the impact and the surface properties, strain energy may be converted to broken bonds in the surface [Bayer, 2004]. In brittle materials, wear is caused by particle-surface collisions which enlarge microcracks during the passage of the impact pressure wave [Engel, 1978]. It follows that the rate of impact wear of bedrock by granular flows is directly related to the transfer of kinetic energy from the granular flow collisions to the bed [e.g. Clark, 1991].

\[
e \propto \Delta KE = \frac{I_p^2}{2m} = \left[ \frac{(F_i - F_f)}{2m} \right]^2
\]
where $I_p$ is the impulse, or time-integrated force, $m$ (kg) is the mass of the impactor, $F_i$ (N) is a force measurement in a time series of force measurements, $F_{iF}$ (N) is the local average bulk normal force. See [Hsu et al., in prep] for derivation. This method weights the amount of erosion with the magnitude of the impact squared. We hypothesize that it will predict erosion well for low slip, wide grain size distribution flows. We can write a possible form of the impulse erosion law as

$$e_f = k_f \left[ \frac{\int (F_i - \overline{F}_i)^2}{2m} \right]^{1/4}$$

(10)

In which, $e_f$ is the vertical incision rate due to impulse wear.

4.2 Experimental Methods

Flume and instruments

To test the models of bedrock wear by debris flows, we measured erosion of synthetic bedrock samples in a 4-meter diameter, 80-cm wide vertically-rotating flume (Figure 3). The large size is important because natural grain-size distributions can be tested without down-scaling particle size, which can cause changes in grain-water interactions [e.g. Iverson and Denlinger, 2004]. The rotating drum design was crucial for the study of bedrock erosion because dozens of passes of the experimental flow over the erodible samples is necessary for obtaining measurable wear. Continuous rotation also allowed collection of multiple force profiles under the flow for ample data collection. The drum was driven by a 20-kilowatt (30 horsepower) induction motor and controlled by a variable speed inverter drive (maximum speed 3 m/s tangential velocity). 32-mm thick Plexiglass windows allowed a side view of the flow. There were four 60-cm wide by 60-cm long, 15-cm deep test fixture openings, one in each quadrant. Three of the openings were occupied by erodible samples and one by sensors. The erodible samples were a synthetic rock composed of a mixture of silica sand and cement, which produced different tensile strengths [Sklar and Dietrich 2001; Hsu et al., 2008]. For every run each of the three openings held a different strength material (Tables 1a and 1b). A 6-mm thick rubber liner with channel-spanning 25-mm tall by 25-mm wide treads, spaced 20 cm apart, prevented bulk sliding of the entire mass on the flume bed.

Normal force on the bed was measured by a 15x15 cm load plate with 22 kN (5000 lb) capacity. Normal load measurements were taken at 200,000 Hz frequency then averaged to 1000 Hz frequency. The size of the plate was chosen to be of the order of the largest clast size in the flume and not too large to be adversely affected by the curvature of the drum. The plate was surrounded by flexible foam and water-seal to keep debris flow material away from the gap edge. An analysis of the force time series fluctuating component and distribution are reported in Hsu et al. [in prep].

Erosion of the samples was quantified by a laser-sheet system inside the flume. A CCD camera (Canon Rebel XT) with 3456 x 2304 pixel resolution took an oblique image of a laser
line on the sample surface. The photograph recorded the deformation of the line and mapped the line to calibrated spatial topography [Rowland, 2007]. The vertical resolution was 0.52 mm/pixel. The laser line was manually stepped across the sample at 0.5 cm intervals. A map of erosion over the sample was obtained by differencing the pre- and post- experiment topographic maps (Figure 4). To minimize boundary effects of the inerodible edges, we calculated the average depth eroded only from the center 15x15 cm area of the sample – the same area as the load plate. Rock samples were replaced when they were worn a couple centimeters below the flume surface, to keep the initial erodible surface close to the arc of the flume bed.

The longitudinal height profile of the flow was measured during the experiment by an Acuity AR4000 laser. A rotating mirror swept the laser over a longitudinal profile at the center of the channel at 5 Hz, yielding a vertical resolution on our flow material of ± 2 mm. Overhead video was captured by a Canon GL2 video camera at 30 frames per second. The field of view was calibrated to make velocity field measurements on the surface of the flow.

**Erosion runs**

Here we report 22 erosion runs (Table 1). Twelve main runs have detailed quantitative erosion data to evaluate the wear models (Runs E1-12). Six runs record one total erosion depth after multiple flow velocities, but still allow an analysis of the relationship between erosion and sample strength (Runs E13-18). Four runs are included to show the patterns of surface erosion especially for flows containing cobble size clasts (Runs E19-22). In each of the 22 runs, three erodible samples of different tensile strength were eroded by the granular flow.

Runs E1-4 were water-saturated gravel (WSG) flows with narrow grain size distributions and mean diameters of 4, 10, 13, and 21 mm, respectively. Runs E5-8 were muddy mixed (MM) flows composed of water and clay, silt, gravel, and cobble sized sediment, making a muddy slurry. The wide grain size distributions had D_{84} (84th percentile diameter of the greater than 2 mm grain size distribution) of 12, 29, 38, and 110 mm. The muddy mixed flows were based on published grain size distributions of debris flow deposits [e.g. Chen et al., 2001; Perez, 2001; Hürlimann et al., 2003]. Figure 5 shows an overhead view of the different runs, showing the variation in grain size distribution. In Runs E9-12 matrix material, total volume, and surface roughness were varied. The runs were dry gravel with D=10 mm (E9), muddy mix D_{84} = 12 mm with half volume (E10), muddy mix D_{84} = 12 mm with more fluid (E11), and water-saturated gravel D=10 mm with a rougher initial erodible surface (E12). Detailed grain size plots are reported in Hsu et al., [in prep]. Runs 1-12 were all conducted at the same rotation speed, producing a bulk mean velocity of 1.25 m/s. At this velocity, centripetal acceleration was 16% of gravitational acceleration. Our primary focus was on the effect of grain size distribution, and velocity was a secondary focus with less variation.

In Runs E13-17 (consisting of water saturated gravel ranging in size from 4 to 21 mm) the bedrock sample was eroded under three different drum velocities, 0.4, 0.8, and 1.25 m/s. Runs 18-22 experiments were included to show qualitatively the wear effect of large cobble size clasts on different strengths of erodible concrete. These runs were muddy mix D_{84} = 110 mm (E19), muddy mix D_{84} = 110 mm more fluid (E20), sand, boulders, and water (E21), and dry boulders (E22). These experiments were run before the standardization of quantitative erosion measurements and cannot be used for qualitative analysis, but they are included to show qualitative wear marks.
Table 1a and b shows notes indicating additional differences between runs. A marble rock sample was embedded in one of the synthetic rock samples to test for wear marks on a natural rock sample of much higher strength (marked ‘m’ for marble) (e.g. Figure 6a). An additional 25mm x 25mm rubber tread, similar to the ones running the length of the bed, was fixed immediately upstream of one sample to test how it affected sliding and the flow field (marked ‘t’ for tread). Experiments on newly poured concrete samples may have had a weaker crust that eroded more easily, giving an erroneously high erosion value (marked ‘n’ for new).

Preparing and Running an Erosion Experiment

The erodible synthetic bedrock samples were mixed to a target strength with a specified sand:cement ratio and poured and set inside the flume. We used Type III Portland cement to expedite the setting time. Type III cement is a high-early strength Portland cement that reaches a high percentage of its maximum strength faster than normal Type I Portland cement. The sample was set into a wooden box that was bolted firmly to the base of the flume bay. The surface of the sample was screed to follow the curvature of the drum. For each sample poured, extra forms were filled with the same mixture to make samples for a Brazilian Tensile Strength test (following the procedure used by Sklar and Dietrich [2001]). These samples were tested to quantify the tensile strength at the time of erosion experiment (Table 3). The samples were used for several experiments until they were eroded significantly below the flume bed and started affecting the flow field. The strengths we tested spanned the range that was confined by the higher limit of recording measurable erosion during an experimental run and the lower limit of withstanding erosion from multiple runs before wearing significantly below the flume bed. Samples were used for multiple runs because pouring and setting new erodible samples required several weeks, making it the most time-consuming step in the experimental procedure.

To create the flow mixture for each run, the different sediment size components were premixed in small batches in a cement mixer before being placed in the flume, except the silt and clay components (to avoid dust generation). Premixing was necessary because rotation in the flume was not sufficient for quickly homogenizing the coarse components. Water, silt, and clay were added to the drum immediately before the experiment, and were incorporated into the mix within a few rotations. Experiments were run for approximately 30 minutes (180 rotations). The duration of which was limited because the flow properties changed through time, mostly due to evaporation of water or loss of fines to the side walls of the flume. (Hsu et al., in prep)]. We took normal force and longitudinal profile data throughout the experiment to track evolution of the flow as it changes occurred. The data collected by each instrument was triggered manually and each sample event lasted one minute. Typically 10-12 sample events were selected during the 30 minute run. The instruments were operated this way to control the volume of high frequency data, while still obtaining multiple repeatable flow profiles. At the end of the experiment the erodible samples were cleaned off with water, photographed, allowed to dry, and then topographically scanned with the camera-laser system.

Testing the erosion models

The application of the erosion models to our data takes into account the cyclic nature of erosion in a drum. We also evaluate the effect of erodible sample strength separately and then
incorporate it into the models. We measure directly the volume eroded, \( e_v \). For all eroded samples, we calculated a dimensionless erosion, \( e' \), by normalizing the volume of erosion with the area of erosion (\( A, \text{m}^2 \)) and the total length of the overriding flow (\( L_{tot}, \text{m} \)). This is equivalent to the eroded depth normalized by the flow length, and creates a measure of erosion that is comparable to other experimental or field measurements [Hsu et al., 2008].

\[
e' = \frac{e_v}{A \cdot L_{tot}} = \frac{e_d}{L_{tot}}
\]  

(11)

In our drum, \( L_{tot} \) is the length of the debris flow times the number of rotations of the drum for a given run.

Thus we nondimensionalize each of our three basic erosion laws by dividing by \( L_{tot} \) and write \( e'_i, e'_s, \) and \( e'_f \).

**Model for impact erosion scaled by bulk inertial stress**

The bulk inertial stress model was applied following the methods for the 56 cm diameter drum experiments of Hsu et al. [2008], where the relationship between nondimensional erosion and strength-normalized inertial stress was tested.

\[
e'_i \propto N_{SNIS} \equiv \nu_s \rho_s D_{eff}^2 \left( \frac{u_i - u_h}{h_{max}} \right)^2 \frac{E_{eff}}{T_0^2}
\]  

(12)

\[
e'_i = k_i (N_{SNIS})^n
\]  

(13)

\( \rho_s \), the density of the solid is 2650 kg/m³ and \( \nu_s \), the solid volume fraction was 0.55 for water-saturated gravel and 0.6 for muddy mixed flows [Hsu et al., in prep].

**Model for sliding erosion**

Following the sliding model in Eqn. 8, \( \bar{W} (\text{N}) \) the average normal load can be estimated by \( \rho gh (\cos \theta) A \). But because we have basal normal force measurements, we can use the measurements to calculate the bulk normal force, \( F (\text{N}) \). Hsu et al., [in prep] demonstrated that the time-averaged force approximates the bulk static force by the granular mass in the drum. Because our measure of erosion is normalized by the total length of the flow and Archard’s law is linear in force, we use the time-averaged force over the entire run, \( F_{run} \).

\[
F_{run} = \frac{\sum_{i=1}^{n} F_i}{n}
\]  

(14)
where $F_i$ (N) is the $i^{th}$ measurement in the $n$ instantaneous (1000 Hz) force measurements when the load plate is under the flow.

Instead of using $H$, hardness, as the measure of resistance to erosion, we use the square of tensile strength, which has been used in previous experiments as the bedrock resistance to erosion [Sklar and Dietrich, 2001; Hsu et al., 2008]. Using plate area, total contact length with the sample plate, $L_{tot}$, and the same normalization as for the inertial stress hypothesis, we obtain a dimensionless quantity for each experimental run, $N_{\text{slide}}$.

$$e' \propto N_{\text{slide}} \equiv F_{\text{run}} \left( \frac{E_{\text{eff}}}{A_{\text{plate}} T_0^2} \right)$$

(15)

And from Archard’s law we expect a linear relationship, i.e.

$$e'_{s} = kN_{\text{slide}}$$

(16)

**Model for impact erosion scaled by impulse**

As derived in [Hsu et al., in prep], in a force time series with a time-varying local mean, and peaks defined by one force measurement, the particle impulse squared is equal to the force variance $F_V$

$$F_V = \sum_{n} \left[ F(t) - \bar{F}(t) \right]^2$$

(17)

There is a threshold, $F_{\text{thresh}}$, established for each run from the force data to determine peaks that are above the noise level. Thus only measurements where $F_i > F_{\text{thresh}}$ are evaluated and count towards the total $n_i$. $F_V$ varies with flow height and we used a 10 drum degree segment (35 cm arc length) of the force profile where $F_V$ is relatively constant to calculate the values.

$$e' \propto N_{FV} = \sum_{n_i} \left( F - \bar{F}_i \right)^2 \left( \frac{E_{\text{eff}}}{A_{\text{plate}} T_0^2} \right)^2$$

(18)

then

$$e'_{f} = k_f (N_{FV})^m$$

(19)

In which $k_f$ is nondimensional constant. The values for each model test, $N_{\text{SNILS}}, N_{\text{slide}},$ and $N_{FV}$ are listed in Table 1.
4.3 Results

Near bed grain dynamics

The near bed grain dynamics including particle-particle collisions, particle-bed collisions, and particle-bed contact duration are closely tied to the mechanisms and rate of sample erosion. In the inaugural run of the flume, pure sliding of the mass occurred on the initial smooth steel bed. In nature, there is often a component of bouncing and collision at the flow front, so we roughened the bed by adding evenly spaced treads to the drum surface. The treads clearly reduced sliding, and observations through the Plexiglass slide wall indicated that none of the runs experienced pure sliding. However some sliding occurred on the smooth areas associated with the load plate and erodible samples as they passed under the flow. The indication of enhanced sliding was a velocity surge of the flow front over the smooth sample, since it was unimpeded by the treads.

In water-saturated gravel flows, the gravel at the front rolled, bounced, and slid ahead of the flow if the front was not completely water saturated. Behind the flow front, the gravel appeared to be in constant contact with the boundary, with lower sliding and very few bed collisions. In the muddy mixed flows with large cobbles, data from the force plate indicated that the cobbles had more collisions with the bed in the front half of the flow [Hsu et al., in prep.] These dynamics imply that most of the wear takes place near the front of the flow.

Observations of wear surfaces in the flume

After an erosion experiment, the bedrock samples exhibited different wear marks depending on the flow properties and the strength of the erodible substrate (Figure 6). The strongest evidence of sliding was grooves which resulted from enduring particle contacts with the bed (Figure 6d, f). Removal of material at localized points indicates impact wear (Figure 4 d, f). Smoothing of the surface, however, can be accomplished both by sliding and by impact of finer particles. There was evidence of both sliding and impact wear, often both in the same flow. In general, flows without large particles showed smooth surfaces (Figure 6d,f). Flows with cobble-size particles had millimeter-scale width and depth grooves and knicks (Figure 6c, e) indicating both impact and sliding wear. Harder patches in the synthetic rock often protruded and consequently protected downstream areas from wear, indicating flow-direction processes. Longitudinal grooves came at different spatial scales. Figure 6d shows longitudinal undulations of wavelength ~10 cm after Run E4 (water-saturated gravel D=21 mm). The wavelength of this undulation is much larger than the grain scale. Figure 6f shows that in close-up view, scratches on the scale of contact area of the gravel are superimposed on the larger waveforms. Appendix 1 includes photographs of all erosion samples, and erosion maps showing the change in elevation of the samples.

Erosion is surprisingly consistent in the cross-stream direction. From the center of the sample to the lateral boundaries, the depth of erosion is quite uniform. An exception is observed in the runs with the embedded marble sample (Figure 6a). Downstream of the marble sample, flow-parallel variations exist due to protection of the area immediately downstream of the high strength marble. On the marble rock sample, wear marks indicated discrete hits with no sign of sliding (Figure 6b). The marks were a sign of damage although they were so shallow that was no
measurable rock removal. No grooves were observed in the marble sample in the flume, presumably because the largest rocks were not large enough to cause enough localized force for forming grooves. However, zones of weaker minerals were removed preferentially through some process either sliding or impact.

Thus, by observing the worn samples we can see the effect of the following variables on the style and magnitude of erosion: grain size, erodible bedrock mineralogy, roughness elements upstream of the sample, and roughness of the erodible sample. We observed some outliers in our erosion values. Erosion samples that had the extra roughness tread (which tended to reduce sliding) had lower than expected erosion. A sample with an initially rough surface due to the previous experiment had higher than expected erosion. Experiments on newly poured concrete samples may have had a weaker crust that eroded more easily, giving an erroneously high erosion value. We removed these outliers before testing our erosion hypotheses, but reintroduce them into the analysis in the Discussion.

**Wear rate dependence on sample strength**

For each run, erosion rates for three erodible samples with differing strengths were measured. In some cases one of the sample results was affected due to the additional roughness tread, and were not included. These two to three erosion samples will have experienced the sample flow conditions and a plot of erosion versus strength for each run should then reveal the erosion dependency primarily on strength. Average eroded depth \( e_d (\text{mm}) = (e_v / A_{\text{plate}}) \) versus the tensile strength \( T_0 \) (kPa) of the samples is plotted in Figure 7. The average depths eroded per run ranged from 0 to 8 mm with a high outlier at 14.5 mm. In this and all following plots, the symbology is as follows: the shape indicates the type of flow: diamonds = water-saturated gravel, squares = dry gravel, circles = muddy mixed grain size distributions. The size of the symbol correlates with the \( D_{\text{eff}} \) size of the flow (\( D_{50} \) for narrow grain size distributions and \( D_{84} \) for wide grain size distributions). The gray-shading correlates with the strength of the erodible substrate, the darker the shading, the more erodible (lower tensile strength) the sample. For Figure 7 the shading is unnecessary, but the shading scheme is repeated throughout the paper.

The power law trendlines for eroded depth and tensile strength for each experimental run are plotted in Figure 7a.. Although each experimental run only had two or three data points to calculate the trend, the repeatability of the strength-erosion relationship for many different runs adds robustness to the value. Table 2 lists the power law fit parameters for each run. A subset of these without outliers is plotted. The exponent in the power law fit ranges from \(-0.26\) and \(-6.8\). This exponent can change greatly if there are only 2 or 3 points to define the line. The \(-6.8\) value line is an outlier and not plotted because one of the points is for new concrete. Low outliers include points with very little erosion, possibly below the resolution of our measurement system. Without the outliers, the mean slope is near \(-1.1\).

In Figure 7b, the eroded depth is normalized to dimensionless erosion (Equation 11). This normalization accounts for both the different run duration and instantaneous flow length of the experiments. After the normalization, for a given strength on the x-axis, larger \( D_{\text{eff}} \) runs had higher erosion (the symbols increase in size in the positive y direction). Water-saturated gravel runs generally had higher erosion than muddy mixed flows for a given sample strength. The lowest erosion was by dry flows and flows with the smallest \( D_{\text{eff}} \).
Evaluation of the different models of wear

Here we test each of the three proposed wear models. Table 3 summarizes our regression results.

The dimensionless erosion $e'$ is plotted against the strength normalized inertial stress, $N_{SNIS}$, in Figure 8. The water-saturated gravel runs plot separately (and higher) than the muddy mixtures and therefore are treated separately in Figure 8b where regression lines are shown. The black lines show the trends for this study while the light gray lines and open symbols are for experiments in our smaller 56 cm diameter drum [Hsu et al., 2008]. The trends for the two scales do not overlap, and the power law exponent is lower in the large drum (Table 3). The largest discrepancies in erosion are at low inertial stress, where there is much higher erosion in the large drum than in the small one. In the small drum, the difference between the water-saturated and dry flows was interpreted to be due to a component of sliding wear [Hsu et al., 2008]. Visual observations support that there was an even larger component of sliding in the 4-meter drum for the water-saturated gravels. The regression $R^2$ is much higher for the muddy mixture (which perceptibly had less sliding) than for the water-saturated gravels in the big drum.

Dimensionless erosion $e'$ is plotted against the metric for sliding wear, $N_{slide}$, in Figure 9. There is a small range of $N_{slide}$ because there is a small range of flow depth. The thickness was limited to prevent the flow from becoming too long and extending well up the steep curved bed, or too thick so that static zones without shear existed in the flow. In contrast to the inertial stress case, a larger proportion of the variance in erosion values of water-saturated gravels is explained than for muddy matrix flows. This is consistent with the greater sliding apparent in the water-saturated gravels. For a given $N_{slide}$, there is a tendency for larger clasts to cause more erosion, which would be expected if the contact area for grooves is larger. More erosion would also occur if collision impact is contributing to wear. This is most clearly expressed in the systematic increase in erosion for a given grain size in the muddy matrix.

The impulse erosion hypothesis is explored by plotting the dimensionless erosion versus force variance, $F_r$ (Figure 10) and versus nondimensional force variance, $N_{FV}$. As reported in Hsu et al., [in prep.] (Chapter 3), the force variance and inertial stress terms are highly correlated, hence Figure 10 yields similar patterns to that for the large drum data in Figure 8. The force variance varies over about two orders of magnitude. As in the bulk inertial stress model, the force variance explains more of the variation in the muddy mixes flows (has a much higher $R^2$) than the water-saturated gravel flows, where sliding effects appear to be much greater.

4.4 Discussion

The contrasting regression statistics of the water-saturated gravel and the muddy matrix runs suggest sliding wear dominance and impact wear (scaled by inertial stresses or force variance) dominance, respectively. This is consistent with visual observations of the flow and of the worn sample surfaces. Our hypotheses gave explicit expectations about erosion dependencies which were not, however, matched. As Sklar and Dietrich [2001] and Stock et al. [2005] have shown, river incision into bedrock varies as the inverse square of the tensile strength. There are theoretical and empirical arguments for this to be generally true [Engle, 1978]. Individual run regressions have few data points and mostly differ from an exponent of $-2.0$ (Table 2), although the average value for the individual runs is -1.1. The relatively low range of strengths (compared
to that used by Sklar and Dietrich [2001]) limits our ability to define independently the strength dependency and it is evident in our data that grain size also matters (which prevents us from combining all the data). With these data we cannot reject the inverse square of tensile strength, hence we use this well-established relationship as part of our nondimensionalization of the proposed erosion expressions.

Archard’s sliding law is linear but our regression on the water-saturated gravels yielded a slope of 0.68. This reduced slope may be due to the additional effects of impact wear. The impact (via inertial stress or impulse) wear regressions for the muddy mixtures yielded strong correlations ($R^2 = 0.87$), collapsed the variance in grain size, but produced fairly low exponents. The force variance model, derived from theory that impact wear varies with changes in kinetic energy imparted by particle collisions, would suggest an exponent of 1.0. The low exponent of our regression may be due to additional sliding influence. We have no theoretical or empirical argument for an expected exponent for the inertial stress-based erosion model.

Results discussed below of other runs not included in this first analysis shed further light on erosion controls. We complete our discussion by proposing a multiple regression involving the linear sliding law and the inertial stress model.

**Deviations from general trends**

Some data points were omitted from the trend calculations because the flows or sample conditions differed greatly from the rest. Figures 11 a-d show the location of the outlier points when added to the plots of $e'$ versus strength, strength normalized bulk inertial stress, sliding wear, and force variance. The new points are distinguished by different symbols. The new data are dry gravel (squares), extra roughness tread before the sample (open symbols), high water content muddy mixture (circles with gray outline), and rough initial sample surface (diamonds with gray outline). The dry gravel, high water content, and extra tread resulted in less erosion than the general trends and the initial rough surface resulted in more erosion than expected from the trends predicted from the other experiments. Here we discuss further each of these outliers because they offer further insight into erosion controls.

Run E9 (dry gravel, D=10 mm) is represented by the three squares that correspond to the three samples plates each with different strengths (Figure 11 a, c, d). The dry gravel run showed the least evidence of sliding and thus should be the most representative of impact-only wear. Sliding was observed by a forward surge of motion where the flow crossed over the drum section with the erodible samples or force plate (one per quadrant), which was distinctly smoother. The dry gravel flow was less disturbed by the smooth samples than the water-saturated gravel flows, with little surging. The eroded sample surfaces had little evidence of sliding wear after the dry gravel experiment. The dry gravel data show little dependence on $N_{slide}$ strength normalized average force (i.e. the sliding model) and while following the muddy mixture trend, systematically plotted lower than the muddy cases. (Figures 11 c, d).

In response to the noticeable surging where the flow crossed the smooth plates where sample ports and pressure plates were mounted, explored the effect of boundary roughness by adding an extra tread to one sample. The extra tread, which was the same size as all of the other treads in the flume, was placed immediately upstream of an erodible sample, forming a barrier to sliding where there was previously ~15 cm of smooth bed upstream of the sample. This effectively decreased the sliding immediately before the sample. The tread was present for five
experimental runs (all with the same sample strength). The extra tread data points are plotted as open circles and diamonds on Figure 11. In Figure 11c, $e'$ versus $N_{slide}$, water-saturated gravel points with the extra tread (open diamonds) plot well below their trendline for points without the tread. This further supports the conclusion that the water saturated gravel had a significant component of sliding wear. In the force variance plot, Figure 11d, the muddy mix (open circles) plot near the trendline, supporting the reasoning that sliding wear was less important in these muddy mix runs. In contrast, the water-saturated (open diamonds) plot well below their trendline in 11d. Hence, the location of the points with the extra tread further support that sliding wear was a larger component of wear in the water-saturated gravel runs than the muddy mix runs.

A few tests showed that the initial surface topography of the erosion samples installed in the drum affected the amount of erosion. We repeated water-saturated gravel runs with D=10 mm twice (Runs E2 and E12). The first time the sample surface was rough due to the mm-scale topography left by the preceding experiment with large clasts. The second time the surface was smooth. The roughness left by the preceding experiment left protrusions that were eroded away more quickly. In Figure 11 c and d, the points representing E2 (diamonds with gray outline) which had high initial roughness plots above the trend line for water-saturated gravel. On the rough surface, the high points were likely preferentially planed off, causing a larger average depth of erosion. An implication of this observation is that in nature, if the surface is first damaged and roughened by an initial flow (e.g. by a coarse granular front of a debris flow) then the scouring by the subsequent finer flow may be more effective than if the flow front had not roughened the surface.

Some of the variance in the data used in the establishing the regressions may have been caused by changing rock sample conditions. The synthetic rock samples, when first poured and set, appear to have a weaker crust at the surface. We could not detect a systematic difference of these runs from the others, so this effect is of minor importance.

Figure 11b compares the results of the large drum and small drum experiments. Table 4 compares the nondimensional scaling of the water-saturated gravel runs in the two drums. Strength normalized inertial stress values covered nearly the same range in the two drums, but the erosion was systematically higher and the exponent of the power law relationship was much lower in the large drum. The primary difference between the two drums appears to be a much larger sliding component in the large drum. In the small drum the bed surface was covered with sandpaper, which created a continuous uniform resistance. In the drum, resistance was created by spaced treads interrupted by four smooth panels for the force measurements and erosion samples. As mentioned above we observed visible surging of the flow on the smooth panels. The convergence toward similar erosion values at higher erosion rates may reflect the increasing role of impact wear.

Although more sliding was evident in the large drum, we argue that this does not make it less relevant to debris flows in the field. In our experience, valleys cut into bedrock are often quite smooth in extended patches. Sliding could dominate debris flow wear in many cases. This suggests that a general debris flow incision law would account for both impact and sliding processes.
Combined sliding and impact erosion law

To assign a possible relative weighting to sliding and impact wear erosion in our data, we propose an erosion rule that combines both mechanisms. We use the version of the impact model scaled by bulk inertial stress because it does not require force measurements from a load plate and so can be estimated outside of the drum.

\[ e^\prime = s_1(kN_s)^{n_1} + s_2(k_N N_{SNIS})^{n_2} \] (20)

Log transformation of the variables retains the linearity of the relations and we perform a multiple linear regression on the following expression

\[ \log_{10} e^\prime = s_0 + s_1(\log_{10} N_{slide}) + s_2(\log_{10} N_{SNIS}) \] (21)

where \( s_1 \) and \( s_2 \) are a function of variables that determine the relative importance of sliding or impact wear, and \( s_0 \) is a constant. Our experiments suggest that fluid viscosity and boundary roughness are variables that would control \( s_1 \) and \( s_2 \). Excluding the outliers, our data best fits \( s_0 = -5.08, s_1 = 0.55 \) and \( s_2 = 0.16 \) for water-saturated gravel, and fits \( s_0 = -5.12, s_1 = 0.17 \) and \( s_2 = 0.19 \) for muddy mixed flows (Table 3, Figure 12).

We can approximate where natural geophysical flows plot on our nondimensional erosion figures. An estimate of dimensionless erosion, based on observations on a marble sample placed in the Illgoben torrent for two debris flow seasons, is 1 mm of erosion for 10 km of flow: \( e^\prime = e_d/L = (10^{-6} \text{ m} / 10^4 \text{ m}) = 10^{-10} \) [Hsu et al., 2006]. Or, using the stones worn at the top of the check dams built in 1960, 1 cm erosion over 50 years and an estimated 5 km per year of debris flow: \( e^\prime = e_d/L = 10^{-5} \text{ m} / 250000 \text{ m} \approx 10^{-11} \). These field-estimated \( e^\prime \) are lower than the laboratory \( e^\prime \) on the weaker synthetic rock. However, to normalize the field eroded depth, we used the length of the entire flow, which decreases the value. Neither of these field estimates include plucking, structure-controlled, or weathered layer erosion mechanisms, which could increase the depth of erosion and thus \( e^\prime \) by several orders of magnitude.

We can also approximate the value of impact and sliding parameters in the field. Estimated values in field debris flows are \( \nu_s = 0.6, \rho_s = 2650 \text{ kg/m}^3, D_{eff} = 1 \text{ m, } \text{dU/dz} = 2 \text{ m s}^{-1} / 2 \text{ m} = 1 \text{ s}^{-1}, \) [Iverson, 1997; McArdell et al., 2007], Young’s modulus = 10^9 Pa, and a tensile strength for marble or granite ~10 MPa [Sklar and Dietrich, 2001]. These values would yield an \( N_{SNIS} \) of 10^2. Using 10 kPa as a value for normal stress at the bed of a natural debris flow [McArdell et al., 2007], the bulk average normal force would be ~2000 N. The force variance model is harder to estimate in the field because it employs normal force measurements. However, because the particle diameters and velocities are generally larger in the field, we can assume that the \( F_V \) would be higher, meaning that \( e^\prime \) would also be higher.

These erosion experiments focused on grain size and size distribution as the primary variables to differentiate experimental runs. There are many other variables to test quantitatively, such as erodible substrate roughness, joint spacing, and degree of weathering. A more advanced model of the eroding substrate proposes battering and aging layers, and has been employed in fluvial bedrock erosion [Chatanantavet and Parker, in press]. In addition, the flow itself has many variables to quantify such as particle shape and fluid viscosity.
Field measurements of erosion to estimate $e'$, $N_{SNIS}$, $N_{slide}$, or $N_{FV}$ values, as well as detailed basal normal force time series are the next step. These types of measurements have been initiated at the Illgraben catchment [McArdell et al., 2007; Hsu et al., 2006], with normal and shear force measurements and wear marks observed on natural bedrock samples, indicating millimeters of wear on exposed surfaces over two years. Discrete element modeling could elucidate particle dynamics at the base of a flow to compare sliding and impact mechanisms [Yohannes et al., 2008].

4.5 Summary and Conclusions

In a large-scale vertically rotating drum flume, we measured erosion of synthetic bedrock samples by granular flows of varying grain size distributions that were either dry, water-saturated, or had a muddy matrix. For the muddy mixture, we used size distributions similar to that found in naturally occurring debris flows, including grain sizes up to 150 mm. The flows with the largest clasts had dimensionless scaling numbers similar to that of natural flows. For each run, three synthetic rock sample plates of varying tensile strength were tested for erosion. The eroded synthetic rock samples showed distinct evidence of sliding wear by scratches, grooves and flow-parallel structures behind obstacles. Particle impact marks were also observed on a marble rock sample and in flows with cobble-sized rocks. Localized contacts between particles and the bed were important for both sliding and impact wear, as seen by variations in groove width and impact mark size. Both types of wear occurred simultaneously, pointing to a need to include both mechanisms in a bedrock erosion law by debris flows.

To explore the relationship between bedrock erosion rate by granular flows and basal forces, we tested three models: impact wear scaled by bulk inertial solid stress, sliding wear, and impact wear scaled by impulse on the boundary (via force variance). The contrasting regression statistics of the water-saturated gravel runs and the muddy matrix runs suggest sliding wear dominance in water-saturated case and a impact wear dominance in the muddy matrix case. Dry granular flows in both these experiments and previous smaller drum tests support a dominance of impact wear. This is consistent with visual observations of the flow and of the worn sample surfaces. Our observations indicate that sliding and impact wear occur concurrently in debris flows, with sliding dominating in smoother boundary and finer grained flows and impact wear dominating in dry, muddy and rough boundary flows with large grain sizes. Exposed upland canyons recently swept by debris flows often reveal extensive reaches of relatively smooth bedrock, so it is likely that sliding due to local areas of smooth surfaces, as observed in the large drum, is common in natural flows.

Our results suggest that an expression to estimate bedrock erosion incision by debris flows should include the effects of sliding and impact erosion. Hence, both flow depth (scaling sliding wear) and flow shear (scaling impact wear for both inertial stresses) should be included in such an expression. Boundary roughness has a first order control on which mechanism dominates and surprisingly, for the same flow depth and shear rate, sliding wear causes more erosion than particle impact for the sample strengths tested here. Relatively small scale variations in boundary smoothness can affect both flow and wear processes. Water content and amount of fines also strongly influences the relative dominance of sliding versus wear processes. Large grains increase both the inertial stress and the force variance, thus increase impact wear. We used a multivariate linear fit to our nondimensional parameters for sliding and impact wear, and defined
expressions for granular flow erosion. Although further field parameterization is needed to refine this model, this work begins to quantify physical erosion mechanisms in steep channels that are dominated by debris flows.

4.6 Notation

\( A \) area, m\(^2\).
\( C \) solid concentration, (-).
\( D_p \) particle diameter, m.
\( D_{eff} \) effective particle diameter, m.
\( D_o \) threshold particle diameter, m.
\( E_{eff} \) effective elastic modulus of fractured rock, Pa.
\( e' \) dimensionless erosion.
\( e_d \) eroded depth, m.
\( e_i, e_s, e_f \) vertical erosion rate by inertial, sliding, and force variance models
\( e_v \) eroded volume, m\(^3\).
\( f \) frequency of debris flows over a given reach per annum, a\(^{-1}\).
\( F \) basal normal force, N.
\( F_i \) \( i^{th} \) force measurement in a force profile in the large drum, N.
\( F \) mean component of force, N.
\( F' \) fluctuating component of force, N.
\( F_V \) force variance, N\(^2\).
\( F_{frac} \) function of fracture spacing of bedrock channel, m.
\( G \) function of basal normal force
\( g \) gravitational constant, m/s\(^2\).
\( H \) hardness of eroding material, Pa.
\( h \) debris-flow depth (normal to the bed), m.
\( h_{max} \) maximum height of flow in drum (normal to the bed), m.
\( I_p \) impulse from particles, kg m s\(^{-1}\).
\( K_0 \) dimensionless constant of proportionality that relates bulk inertial normal stresses to higher excursions of inertial normal stress.
\( K_1 \) constant of proportionality between rock resistance and incision rate.
\( k, k_i, k_s, k_{fv} \) constant of proportionality in wear equations.
\( KE \) kinetic energy (J)
\( L \) length of debris-flow erosive segment, m.
\( L_{tot} \) total length of flow for a drum experiment, m.
\( m \) mass, kg
\( N_{Bag} \) Bagnold number (ratio of solid inertial normal stress to fluid viscous stress).
\( N_{Sav} \) Savage number (ratio of solid inertial normal stress to solid frictional stress).
\( N_{slide} \) dimensionless number in sliding block model quantifying sliding wear
\( N_{SNIS} \) strength-normalized inertial stress number.
\( n \) exponent on the inertial stress term in Equation 4.
\( R \) resistance to erosion
$s_1, s_2$ coefficients in Eqns 20 and 21 quantifying relative importance of sliding and impact wear, based on experimental observations.

$T_0$ rock tensile strength, Pa.

t time, s.

$u$ velocity, m/s.

$u_0$ threshold velocity, m/s.

$u_b$ basal streamwise (along slope) velocity of debris flow, m/s.

$u_s$ surface streamwise (along slope) velocity of debris flow, m/s.

$u_{drum}$ tangential drum velocity, m/s.

$V$ sliding velocity (Archard’s law), m/s.

$W$ applied load, N.

$w$ exponent on the shear rate term in Equation 4.

$x$ sliding distance of the applied load in Archard’s Law, m.

$z$ height above bed, m.

$\partial u/\partial z$ shear strain rate of debris flow, s$^{-1}$.

$\partial z/\partial t$ bedrock surface lowering rate, m/a (meters per annum).

$\alpha$ impact angle.

$\beta_1, \beta_2, \beta_3$ constants in empirical slurry erosion rule (Eqns. 1 and 2).

$\mu$ viscosity of phase defined as fluid in debris flow, Pa⋅s.

$\nu_f$ volumetric fluid concentration.

$\nu_s$ volumetric solids concentration.

$\theta_b$ basal slope angle of flow, degrees.

$\theta_s$ surface slope angle of flow, degrees.

$\theta_t$ shear layer slope, degrees.

$\rho_s$ solid particle density, kg/m$^3$.

$\rho_f$ fluid density, kg/m$^3$.

$\sigma_l$ solid normal inertial normal stress, Pa.
4.7 References


Desale, G. R., Gandhi, B. K. and Jain, S. C. (2009), Particle size effects on the slurry erosion of aluminium alloy (AA 6063), *Wear* 266(11-12), 1066--1071.


Hsu, L., Dietrich, W., Sklar, L. and Kaitna, R. (in prep.), Mean and fluctuating basal forces generated by geophysical granular flows.


Lancaster, S. T. and Casebeer, N. E. (2007), Sediment storage and evacuation in headwater valleys at the transition between debris-flow and fluvial processes, *Geology* 35(11), 1027--


4.8 Tables

Table 1 begins on the next page.
**Table 1a.** Experimental runs E1-E12.

$D_{50}$ and $D_{84}$: 50th and 84th percentile of the >2mm sediment; drum vel: tangential drum velocity; visc: fluid viscosity; numrevs: number of revolutions for the duration of the experiment; $L_0$: instantaneous flow length; $h_{max}$: maximum height of flow, measured by laser; neg depth: average depth eroded in 15x15 cm² center of erodible sample; $e^*$: dimensionless erosion; IS: inertial stress; avg F: $\bar{F}$; $F_Y$: force variance; $T_0$: tensile strength of erodible sample; $N_{SNIS}$: strength normalized inertial stress; $N_{slide}$: dimensionless sliding number; $N_{F_Y}$: dimensionless force variance number; Notes: m: marble, t: extra roughness tread, n: new concrete sample.

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Table 2. Strength trends

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WSG: water saturated gravel
MM: muddy mixed
WSGB: water saturated gravel bimodal (with 150 mm cobbles)

Data fit to $e_d = a(T_0)^b$ where $e_d$ is eroded depth and $T_0$ is tensile strength

*Starred data are not included in the plot.
Table 3. Regression results

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<td>[Hsu et al., 2008]</td>
<td>WSG</td>
<td>power</td>
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<tr>
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<td>WSG</td>
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<td>0.17</td>
<td>0.19</td>
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</table>

WSG: water-saturated gravel  
MM: muddy mix  
DG: dry gravel

Power law fit: Data fit to e’ = a(x)^b₁ where x = N_{SNIS}, N_{slide}, or N_{FV}

Multivariate linear fit: e’ = a + b₁(N_{slide}) + b₂(N_{SNIS})

npts: number of points to calculate the fit and R²
Table 4. Scaling parameters for the 56 cm diameter drum (Small drum), 4 meter diameter drum (Large Drum), and Illgraben torrent debris flow.

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<th>Field Illgraben</th>
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<td>wet</td>
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<td>0.014</td>
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<td>density fluid (kg/m³)</td>
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<td>viscosity fluid (Pa s)</td>
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<td>volume fraction solid (-)</td>
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<td>volume fraction fluid (-)</td>
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<td>basal slope (degrees)</td>
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<td>height of flow (m)</td>
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</table>

Bagnold number: $N_{bag} = \frac{v_s \rho_s D_p^2 \left( \frac{\partial u}{\partial z} \right)^2}{v_f \mu}$

Savage number: $N_{sav} = \left( \frac{\rho_s D_p^2 \left( \frac{\partial u}{\partial z} \right)^2}{(\rho_s - \rho_f)gh} \right)$ [See Chapter 3]
4.9 Figures

**Figure 1.** Three different examples of recently scoured bedrock channels in southern Arizona. Photos taken ~8 months after scour. (a) Smoothed, abraded channel, Santa Catalina Mountains. (b) Smoothed and joint-block plucked channel, Santa Rita Mountains. (c) Centimeter-to-decimeter chips of rock removed, highlighted with white arrows. Black arrow points to pencil for scale, oriented in the downstream direction (Huachuca Mountains).
Figure 2. Three models for the dominant mechanism of bedrock erosion by debris flows: (a) impact wear due to forces arising from bulk solid inertial stress, (b) sliding block wear, (c) impact wear by scaled by impulse to the bed caused by colliding grains.
Figure 3. Components of the four meter diameter vertically rotating debris flow flume. (a) a: steel frame, b: supporting wheels, c: motor for the chain and sprocket drive system, d: inverter control box, e: bed roughness tread, f: sample or instrument bays, one per quadrant. (b) Instrument system includes f: three erodible sample bays, g: load plate, h: laser profiler for measuring the dynamic flow (light grey lines approximate range), i: video camera, j: camera-topo system for measuring bed topography after erosion experiments [Hsu et al., in prep]. (c) Flow during an experiment viewed through the Plexiglass window, showing typical flow position. Laser profiler is suspended over the flow by boom secured outside the flume. The flow is composed of water-saturated gravel, $D_{50} = 13$mm, at drum velocity $1.25$ m/s.
Figure 4. Eroded samples and wear maps for two different experimental flows. (a) and (b) show top oblique view of two flows, (a) Run E5 muddy mix $D_{84} = 12$mm, (b) Run E22 dry boulders (150 mm diameter) with no matrix. (c) and (d) show the erosion sample after each experiment. (e) and (f) show the wear map (depth of erosion), which is the differenced laser scan topography before and after the experiment.
Figure 5. Top oblique views of a subset of the experimental flows. Flow is away from the camera. (a-d) Runs E1-4, Water-saturated gravel with D = 4, 10, 13, and 21 mm (e-f) Runs E5-8, muddy mix D_{84} = 12, 29, 38, 110 mm, wide grain size distributions with water.
Figure 6. Photos of eroded samples. (a) Erosion sample after Run E8, muddy mix $D_{84} = 110$ mm, 1290 kPa synthetic rock sample plus embedded marble sample. (b) Close-up of marble sample, showing white circular impact marks and weaker minerals preferentially worn away. (c) Worn sample after Run E8, muddy mix $D_{84} = 110$ mm, (sample strength = 1186 kPa) showing impact marks and grooves. (d) Worn sample after Run E4, water-saturated gravel $D = 21$ mm, 1463 kPa, showing flow-oriented wear. (e) Close-up of panel c, showing grooves, and also air bubbles inherent to the concrete sample. (f) Close up of panel d, showing faint surface scratches oriented vertically in the photograph.
**Figure 7.** Erosion versus sample strength. The symbol shape indicates experiment type: diamonds are water saturated gravel runs, circles are natural (wide) grain size distribution runs, squares are dry gravel runs. The symbol size correlates with the effective mean coarse diameter (4 mm to 110 mm). The symbol grey-shade indicates sample strength, darker colors are weaker and more erodible, lighter colors are stronger and less erodible (although this corresponds to the x-axis in this graph, the same shading is maintained for the rest of the paper). (a) The data are plotted in dimensional format to show the range of erosion depth results. The average slope on the power law fit is – 1.1 (See Table 2). (b) Strength data points now plotted against nondimensional erosion (subset of all erosion data points with the necessary information. Note that after non-dimensionalization, for a given strength, the points line up from largest effective diameter causing the most erosion and to smallest effective diameter causing the least erosion.
Figure 8. Impact wear scaled by bulk inertial solid stress model. (a) Nondimensional erosion versus inertial stress (b) Nondimensional erosion versus strength normalized inertial stress ($N_{SNIS}$) for both the current experiments and the smaller drum experiments [Hsu et al., 2008] (Equation 12). Symbol shape, size, and shading follow Figure 7, except the open symbols are the small drum experiments. WSG: water saturated gravel, MM: muddy mix, DG: dry gravel. See Table 3 for regression statistics.
Figure 9. Sliding block model. Symbol shape, size, and shading follow Figure 7. (a) Nondimensional erosion versus average force. Note that for a given average force, the points are arranged with weaker (darker grey) samples exhibiting more erosion. (b) Nondimensional erosion versus strength-normalized average force per rotation ($N_{\text{slide}}$) (Equation 15). WSG: water-saturated gravel, MM: muddy mix. See Table 3 for regression statistics.
Figure 10. Impact wear scaled by impulse model. Symbol shape, size, and shading follow Figure 7. (a) Nondimensional erosion versus force variance ($F_V$). (b) Nondimensional erosion versus $N_{FV}$, strength-normalized force variance (Equation 18). See Table 3 for regression statistics.
Figure 11. Model evaluation with outliers added to the plots. (a) Strength. The arrow points to a new (fresh) sample (‘n’ in Table 1), which had high erosion due to a weaker crust. (b) Impact wear scaled by bulk inertial solid stress (c) Sliding wear (d) Impact wear by impulse. New symbols: Squares are dry gravel (E9), open symbols samples with the extra roughness tread (‘t’ in Table 1), gray outline circles are muddy mix with more fluid (E11), gray outline diamonds are water saturated gravel D = 10 mm with initial rough surface (E12).
Figure 12. Multivariate linear fit of dimensionless erosion with both sliding and impact parameters, $N_{slide}$ and $N_{SNIS}$. (Equation 21).
4.10 Supplementary Materials

Appendix figures:
Two .pdf documents contain all of the erosion photographs and erosion maps. Contact hsu.leslie@gmail.com.
Chapter 5

Size-dependent particle trajectories and segregation in shallow granular flows

5.1 Introduction

Debris flows exhibit puzzling particle dynamics due to their wide grain size distribution and their viscous fluid matrix. Common observations in natural debris flows include size-dependent particle velocity [Prochaska et al., 2008], inverse grading [Naylor, 1980; Legros, 2002], the formation and recirculation of bouldery fronts [e.g. Massimo, 2000], and lateral trajectories near the front of the flow [Swartz and McArdell, 2005]. These field observations have formed a widely-accepted schematic of typical debris flow kinematics with separate circulation cells in the front and body, a component of lateral velocity, and secondary flow in the body (Figure 1) [e.g. Savage, 1979; Pierson, 1986; Davies, 1990; Blair and MacPherson, 1998; Parsons et al., 2001; Iverson, 2005; Gray and Chuganov, 2006; Gray and Ancey, 2009].

Size segregation of particles in debris flows causes spatial variations in boundary forces and the resulting channel erosion [Hsu et al., in prep]. The accumulation of coarse boulders at the front of a debris flow causes a frictional front that may hold back or slow the fluid body of the flow [Parsons et al., 2001]. A coarse front also exerts higher localized impact stresses on the bed, affecting bedrock erosion [Stock and Dietrich, 2006]. However, a coarse front is not present in every debris flow, and a study using channel bank seismometers suggested that a well-defined bouldery front was not formed until the flow reached the low-gradient fan [Massimo, 2000]. A better understanding of the segregation mechanisms in debris flows would help explain the evolution of flow dynamics in natural flows and the processes responsible for segregated deposits observed in the field. Several questions are: How large does a particle need to be relative to others to segregate, and what determines the critical number of particles necessary to form and maintain a coarse-grained front? What conditions in the flow and on the boundaries of the flow encourage size segregation and what conditions discourage it? When segregation occurs, how does it affect the bulk properties of the flow?

The segregation mechanisms in natural geophysical flows are not well understood for several reasons. First, natural debris flows are infrequent and the observation period is brief. Longer observation periods are possible in the laboratory, but most experimental flows use idealized particles and simplified particle size distributions within a channel that is much smaller than natural debris flow channels, all of which can affect the segregation mechanisms. Few studies mimic the conditions of a natural channelized debris flow, including a distinct flow front, realistic scaling of large particles to flow depth and width, and a wide grain size distribution including fine sediment, and fluid matrix.

During our investigation of controls on boundary forces and bedrock wear associated with granular flows in a four-meter diameter vertically rotating drum, we observed grain
segregation processes and fluid-sediment interactions that were previously undescribed. To further explore segregation processes, we executed additional smaller scale experiments in a smaller 56 cm diameter vertically rotating drum with spherical glass marbles. We focused on the ways in which the flow front and the formation of a coarse-grained front were affected by the particle size distribution of the flow, the boundary roughness, and fluid content. In this paper, we describe these exploratory experiments and introduce some scaling terms related to size-segregation in shallow granular flows.

Size segregation in granular flows

Studies of segregation in granular flows are copious, due to many industrial and geophysical applications. Several reviews describe the main mechanisms of segregation [e.g. Ottino and Khakhar, 2000; Vallance and Savage, 2000; Aranson and Tsimring, 2006]. The basic mechanisms were identified in a binary mixture of spheres flowing down a roughened inclined chute and an analysis of particle movement between shearing layers of spheres [Savage and Lun, 1988]. In the process of kinetic sieving, smaller particles fall down a layer (random fluctuating sieve) and larger particles move up through layers (squeeze expulsion), and these movements act in concert to conserve mass.

In natural geophysical flows like debris flows, particle size segregation is also influenced by a wide grain size distribution, particle shape, the viscosity of the matrix fluid, boundary drag, and channel topography. There is a long history of observing debris flow deposits in the field, and inferring or speculating the mechanisms that would cause such forms [Naylor, 1980; Fisher 1971; Rodine and Johnson, 1976; Johnson and Rodine, 1984; Kim et al., 1995; Blair and McPherson, 1998; Sohn et al., 1999; Kim and Lowe 2004; Zanuttigh and Lamberti, 2007]. In addition there are field and large scale experiments that directly observed particle kinematics [Pierson, 1981; Major, 1997; 1998; Major and Iverson, 1999; Logan and Iverson, 2007].

The mechanisms of size segregation in debris flows have been studied in the laboratory, but the experimental conditions have been far from realistic. Yamagishi et al. [2003] described segregation in vibrating binary mixtures of spherical particles in various diameter ratio mixtures and modeled the segregation with a discrete element modeling method. Sunamura [2007] observed the upward trajectory of a single large particle as it moved downstream in a flow of smaller particles by the mechanism of rolling up and over smaller grains. Prochaska et al. [2008] and Swartz and McArdell [2005] made field observations for short durations and tracked the trajectory of large boulders in the flow. Prochaska et al. [2008] found size-dependent velocities to be dependent on a measure of the ability of particles to rearrange themselves within the flow. There is segregation-mobility feedback where the evolving particle-size distribution affects both the bulk motion and the motion of individual particles [Gray and Ancey, 2009].

One of the most complete analyses of the segregation of a coarse front employed a simple force balance framework [Suwa, 1988]. In the experiments, a bore of water was released on different sized glass particles in a chute flume to observe what mechanisms focused large particles to the front of a flow. The water entrained the mixed particles and the largest particles concentrated at the front. Large particles did not accumulate in the body or rear because of the repulsive action of other particles in the flow. The mechanism of frontal focusing was then analyzed through a force balance on a spherical boulder in the flow, at different stages in the flow such as initiation of motion and acceleration, flow down steep and gentle slopes, and
deceleration and stoppage. The effect of collisional forces between boulders in a flow was different in different parts of the flow, and was proposed to be significant at the front but minor in the main body of the flow.

The force balance on a particle includes downstream-driving forces including gravity and particle collisions, and upstream resisting forces including boundary drag and particle collisions. A particle’s size and surface area affects the balance of these forces. The bulk flow field may concentrate particles of a certain size together, and once this occurs, the segregation will persist if the particles are stable in this new condition. Some have suggested that a segregated state possesses lower entropy [Aranson and Tsimring, 2006].

Variables affecting segregation

Previous work suggests that the variables of particle size distribution, boundary roughness and slip, and fluid properties will be important for determining the likelihood for segregation. The size of a particle relative to the particles around it and relative to the flow height are the principle variables we examine. Particle size distributions for debris flows commonly have a range of diameters over several orders of magnitude from clay to boulder size. The ratios $D_i/D_m$, $D_i/h$, and $D_i/w$ define the diameter $D_i$ of a particle of interest -- which we call the intruder particle -- relative to three quantities: (a) the diameter of a characteristic matrix particle, $D_m$ (b) the height of the flow in the vicinity of the intruder particle $h$, and (c) the flow width $w$. These ratios yield information about whether the intruder particle is often in contact with the bed ($D_i \sim h$), or if the intruder particle momentum is unlikely to be affected by individual particle collisions ($D_i \gg D_m$), and the minimum number of particles that would be required to span the flow front ($N_i = w/D_i$).

The roughness of the boundary is a major factor controlling the particle dynamics of shallow granular flows [Zheng and Hill, 1998; Silbert et al., 2002; Goujon et al., 2003]. Roughness may influence segregation of particles by direct friction on the intruder particle, by affecting the granular temperature of the surrounding particles, and by affecting the velocity profile through the flow. Roughness also dictates friction and slip on the boundary, which in turn control internal shear and the segregation mechanisms that are driven by shear of particle layers. In nature, the boundary roughness can vary greatly, both at the scale of the particles and at larger scales, such as meter-scale knickpoints in the channel.

Interstitial fluid in granular flows plays an important role in the bulk flow behavior [Major, 2000]. Viscous fluid generally inhibits segregation [Vallance and Savage, 2000]. The flow field of the fluid may have a large effect on the motion of the solid particles [Humphrey, 1990]. In debris flows, the fluid is a mixture of water and fine materials that makes a dense and viscous matrix. The amount and nature of the fluid evolves during the passage of a debris flow [Pierson, 1980; Imaizumi et al., 2005]. Understanding the fluid role in segregation mechanisms will help to predict spatially-varying segregation in different parts of debris flows.

To explore how these variables affect segregation mechanisms, we first create a simple system and then add variables that are more representative of natural debris flows. In experiments with spherical glass marbles in the small drum, we focus on segregation of coarse intruder particles to the front and sides of the flow for a range of intruder particle size. In experiments in the large drum, we then examine how boundary roughness, geologic material
grain size distribution, and fluid content influence segregation and other phenomena, including lateral oscillations and the formation of asymmetric coarse-particle gyres.

5.2 Experimental methods

We used two rotating drum flumes to study particle trajectories and segregation in shallow granular flows (Figure 2). A small drum with diameter 56 cm and 15 cm width was used to observe flows of glass marbles. A large drum with diameter of 4 meters and 80 cm width was used to observe flows of geologic granular materials such as cobbles, gravel, sand, silt, clay, and water.

In the small drum, all flows reported here were dry with no fluid. We used glass marbles with particle diameters of 5, 13.8, 25, 34, 40, and 50 mm. The drum velocities were between 12-16 RPM (0.35 – 0.47 m/s). The boundary condition was a sandpaper-lined bed and Plexiglass side walls. Particle trajectories were captured with a digital camera at 30 frames per second, with side and plan views. We used ImageJ [Abramoff et al., 2004] to analyze the position and velocity of particles.

In the large drum, the experiments used a wide range of geologic granular materials, from clay sized to 15-cm diameter cobbles. All flows included fluid water, because excessive dust was generated by the collisions in dry flows. The drum velocities ranged between 2-6 RPM (0.4 – 1.25 m/sec). The channel bed was a hard rubber surface over a steel drum, with channel-spanning treads 2.5 cm high and 2.5 cm wide spaced 20 cm apart. One wall (left bank) was steel and the other (right bank) was Plexiglass. Particle trajectories were documented using an overhead digital video camera at 30 frames per second. In addition, a laser profiler obtained longitudinal height profiles at the centerline 5 times per second. The laser profiler captured the position of the flow front and the thickness of the flow during dynamic conditions. To obtain quantitative measures of the particle velocity field, we used a particle tracking method described in Rogers et al. [2007], which is efficient for many-particle systems with diffuse boundaries and irregular shape.

Experimental runs

In the small drum, the experiments addressed the question of what mechanisms influence the trajectory of larger (intruder) particle with diameter $D_i$ in a flow of smaller matrix particles with diameter $D_m$. One group of experiments examined what happened to the trajectory of one single intruder particle as the ratio of the intruder diameter to matrix diameter ($D_i/D_m$) changed. A second group of experiments examined what happened when the number of intruder particles, $N_i$ increased, and examined if a $D_i/D_m$ threshold was crossed where the intruder particles segregated into coarse-grained front, which we define as channel-spanning intruder particles at the flow front for 75% of the run duration. The matrix particles had a diameter of 13.8 mm and $D_i/D_m$ varied from 1.8 to 3.6. The maximum height of the flows was five matrix particles, or 70 mm.

In the large drum, we varied particle size distribution, drum velocity, channel boundary properties, and fluid content, while using natural geologic materials for the flow. The large size of the 4-meter diameter drum is beneficial because the sediment was not scaled down in size,
allowing better imitation of natural debris flows. We used 500-1000 kg of material that formed a shallow flow 20-30 cm in maximum height with shear throughout the depth of the flow.

Table 1 lists the experiments described here and the associated video files.

5.3 Observations

Trajectories of single large intruder particles

The experiments with one large intruder particle in a mass of smaller matrix particles showed a size-dependent particle trajectory in the flow. With the drum spinning and the 13.8 mm diameter matrix particles in a steady-state flow, the intruder particle was dropped near the tail of the flow over the flow centerline. The intruder quickly moved to the front of the flow, then to the side wall (e.g. Figure 4a). The longitudinal position was measured on a coordinate system measured in units of drum angle, where 0° is the 6 o’clock position and each degree is one arc degree of the wheel. The drum angle is the same as the tangential slope at that point (e.g. Figure 3e). The drum angle increases positively toward the tail of the flow, so 90° was at the 3 o’clock position in the small drum. The trajectory of the intruder particle followed a loop driven by wall drag (Figure 3f). As $D_i/D_m$ decreases from the largest intruder particle, the trajectory of the intruder spans a larger segment of the drum angle coordinate (Figure 3a-d, Videos A-D). When $D_i/D_m = 1$ (the intruder is the size of the matrix particles), then the intruder has the full range of the flow. Table 2 shows the mean position and skewness of the position. The size-dependent particle trajectories are the result of the forces exerted on the intruder particle by the boundaries and matrix particles. Sometimes the intruder particle stalled in a semi-stable position in the middle of its loop. This occurred if the longitudinal forces on the particle balanced in the lab frame, e.g., upstream-driving wall drag balanced downstream-driving particle collisions.

We roughened the bed with channel-spanning 1 millimeter diameter wooden sticks with 28 mm longitudinal spacing. This roughened bed generated much higher fluctuation velocities in the matrix particles, reducing the rolling and increasing particle collisions with the bed and with each other. The increased fluctuation velocity affected the lateral position of the single intruder particle. Instead of staying at the wall, the intruder particle would stay in the center of the channel for extended periods of several seconds, as it was laterally buffeted by collisions from the matrix particles (Figure 4b, Video E).

In summary, at larger values of $D_i/D_m$, the intruder particle was more likely to segregate to the front of the flow and less likely to follow the recirculating bulk velocity field of the flow. The smaller intruder particles spanned a larger longitudinal range because they were more affected by the wall drag and bulk flow field. Only the largest intruders remained at the front of the flow, insignificantly affected by the wall drag and bulk flow. The force balance on the intruder particle was significantly affected by roughening the boundary of the drum, which led to less rolling and higher velocity fluctuations and collisions of the particles.

Segregation and formation of a coarse front by multiple large intruder particles

When additional intruder particles were added, they tended to take the same trajectory as the single intruder, causing the particles of similar size to repeatedly collide. These collisional interactions would force some particles out of sorted patches and re-enter the path of the bulk
flow. Large intrusioners clustered along the sidewalls (Figure 4c). The intrusioners in the upstream-most positions were sometimes forced away from the wall by downstream particle collisions, at which point they quickly moved to the front (Figure 4d). Once at the front, they tended to drift to the sidewall, but if sufficient numbers of intrusioners were present, a threshold was crossed and the intrusioner particles spanned the flow front (Figure 4e), creating a coarse front [Huerto, 2008]. The threshold number of particles required to create a coarse front is related to $D_i/D_m$, where smaller $D_i/D_m$ require a larger number of intrusioners to maintain the coarse front (Figure 4f). If $D_i/D_m$ is too small, a coarse front is not formed, because the smaller intrusioner particles never segregate from the matrix particles (Figure 4g, h). Having enough intrusioner particles to entirely span the width of the flow, $N_i = D_i/w$ is not sufficient in itself to form a coarse front, even if a single particle of $D_i$ would tend to stay at the front (Figure 4c). Huerto [2008] developed a mass ratio threshold of matrix to intrusioner particles for persistent coarse front formation, though this may be drum-geometry specific.

**Segregation of geologic material flows**

In the large drum, we examined flows of geologic sediment with different particle size distributions, boundary roughness, and fluid content and properties, and observed how these additional variables further changed the trajectory of particles in the flow. We ran an experiment with a water-saturated bimodal particle size distribution where the $D_i/D_m$ ratio was $150 \text{ mm} /10 \text{ mm} = 15$. Following the observations in the small drum experiments, this $D_i/D_m$ should form a coarse front if there are enough particles.

Drum velocity affected the large particle trajectories. With faster drum speeds, the intrusioner particles were less likely to stay at the front. Figure 5a (Video O) shows the intrusioner particles at the flow front at a drum velocity of 0.4 m/s. When the drum velocity was increased to 1.25 m/s, the intrusioner particles moved to the sides and further upstream in the flow (Figure 5b, Video P). The higher drum velocity produced a greater circulation rate and flux of small particles traveling down the centerline, which could push the large intrusioner particles away from the middle of the flow front toward the sides, where wall drag could act on them. Although we expected to see a formation of a coarse front by the 150 mm cobbles in the gravel flow, the upstream drag from the boundary roughness treads and wall drag prevented the cobbles from staying near the flow front. Additionally, the non-spherical shape of the cobbles and gravel probably modified the $D_i/D_m$ thresholds for coarse front formation that were determined in spherical flows. Therefore, we did not see persistent coarse fronts.

The combined effects of the walls, fluid, and particle size are seen in the velocity fields measured at the surface of the flow. In the case of water-saturated 4 mm gravel (Figure 6a, Video Q) wall drag creates a circulation cell at the left bank wall. Wall drag on the particles is accentuated by the fluid flow field which also experiences drag from the wall, and the 4 mm particles are strongly affected by the fluid velocity field. In comparison, when the gravel diameter is increased to 13mm, the wall and water drag is not large enough to influence the downward momentum of the particles, and there is not a significant lateral component in the velocity field (Figure 6b, Video R). When the grain size distribution is widened to include fine sediment of clay, silt, and sand size (muddy mixed, Video S), we see a greater lateral component of velocity, spanning almost the entire width of the flow (Figures 6c and 6d). In these figures, $D_{84}$ of the gravel and larger particles is 38mm. Figures 6c and 6d show an oscillation of the
lateral component, which is described in the next section. Here the two images represent the surface velocity fields three seconds apart in the same experiment.

**Asymmetries and oscillations in segregated flows**

In wide grain size distribution flows in the large drum, we observed the unexpected phenomena of laterally oscillating flow fronts and asymmetric coarse particle gyres. These phenomena reflect a combination of the mechanisms discussed in the previous section.

A short period lateral oscillation was controlled by the smooth areas in the drum for muddy mixed flows of wide grain size distribution (Video S, Run E7). Due to the position of sample and instrument bays, in each quadrant there were smooth areas 80 cm long without the roughness treads. The flow at the sidewalls was continuously dragged upstream by the boundary. When the flow passed over a smooth section without treads, the fluid-rich center surged ahead, filling the void at one side wall caused by the drag (Figure 7a, left bank). At the next smooth section the opposite wall of the flow had a larger void and was therefore filled (Figure 7b, right bank). The arrows in Figure 7 show the direction of the flow near the centerline, pointing toward the void about to be filled. Lateral segregation probably enhanced this process because larger clasts moved to the boundaries, leaving a more fluid center flow that surged ahead over the smooth segments of the bed.

A longer period lateral oscillation occurred with a coarse-clast circulation gyre in a wide grain size distribution muddy flow with $D_{84} = 110$ mm (Video T, Run E8). The segregation of coarse particles near the front of the flow was asymmetric, occupying about half of the channel width (Figure 8a). The coarse gyre developed, circulated driven by wall drag, and after some duration dispersed, temporarily eliminating the asymmetry (Figures 8b, c). When the gyre reformed, it formed on the opposite wall (Figure 8d). This back-and-forth oscillation consistently switched walls and also increased in periodicity as water evaporated from the flow, changing the viscosity of the fluid matrix. The periodicity of the oscillation ranged from 17 to 113 seconds, and the gyre also consistently stayed at one wall longer than the other (Figure 8e). When the gyre existed, the fluid matrix had little penetration into the pore spaces of the gyre, and as a consequence the fluid and solid fractions of the flow had separate flow fields.

### 5.4 Discussion

Segregation of the coarse particles towards the front and sides in our experiment arose when basal friction and upslope drag of the near-bed flow (of fluid and smaller particles) were smaller than the downslope pull of gravity and push of fluid and smaller particles in the near-surface flow. Wall resistance led to lateral pressure gradients which directed flows towards the walls and led to gyres that collected larger stones. In sufficient numbers coarse-grain concentrations formed at the front or sides. Importantly, the effects of gravity, bed resistance, viscous forces, and particle collisions contributed to the coarse particles and the bulk flow taking different trajectories. It appeared that the coarser the particle was, the less strictly it followed the fluid matrix path. Hence, the particle circulation system and the fluid circulation system do not match. Such relations are missing in the typical schematic given in Figure 1.

We observed an unexpected relationship between fluid viscosity and this incongruence of the fluid and solid flow fields. The trajectories of the 4mm particles in water-saturated gravel...
followed the fluid circulation which deviated towards the wall as the front was approached (Figure 6a). The 13 mm particles, however, did not follow this fluid path (Figure 6b), and instead tended to head straight to the front. When the interstitial fluid was more viscous with the addition of clays, silts, and sands, it exerted a higher viscous force on the gravel particles, causing the trajectory of larger particles in our muddy mix ($D_{84} = 38$ mm) to coincide with the fluid flow field for the most part. But increased viscosity (due to fluid loss during the experiment) eventually reduced the flow into the pore spaces of a granular mass (illustrated with muddy mix $D_{84} = 110$ mm, Video T). At that point, the flow fields of the fluid and solid were completely separated, maintaining segregation of a coarse particle gyre.

The rotating drum geometry has many advantages for studying particle trajectories in granular flows, the largest being that the observation period is long and stationary, as opposed to the transient flow down a chute. The dynamics at the distinct flow front can be measured for a long period of time. However, there are major differences between a drum flow and a channelized granular flow in nature. The first is that no deposition is allowed in the drum, so all of the material recirculates, which may cause artificially high shear rates. However, the shear rates in these experiments, as estimated by $\frac{du}{dz}$, are similar to those estimated for natural flows [Hsu et al., in prep]. Second, we keep our drum velocity low to avoid significant centrifugal effects.

Field Observations

The degree of segregation in natural debris flows can vary from flow to flow even at the same location. The Illgraben torrent is a site in Switzerland with multiple debris flows per year and instrumentation that records video and measures basal forces during flow events [Hürlimann et al., 2003; McArdell et al., 2007]. A video camera is triggered when a flow is sensed upstream, capturing the flow front and variations in flow height, flow velocity, grain size distribution and maximum boulder size. Coarse fronts are not always present. Figures 9a and b show flows of different velocity, both with a coarse bouldery front. Figure 9c shows a watery flow front without notable coarse segregation, but Figure 9d shows the same event less than two minutes later, with a cluster of boulders in a matrix fluid that appears to have increased in viscosity and density considerably.

The increase in fluid viscosity and density, either because of sediment input from bank failure or bulking up from sediments in the channel, is analogous to the drying out in experiment muddy mix $D_{84} = 110$ mm in that the solid to fluid fraction ratio increased. The clustering of boulders did not happen until the thickening of the fluid matrix.

Using the video images, we estimated the boulder diameters at the flow front as they passed over the lip of the check dam for these and other flows that had adequate video footage and velocity information (Table 3). Observations here show that bouldery fronts usually have $D/h$ for the largest boulders of ~0.75. The width of the Illgraben channel is 10 m, making $D/w$ ~0.17.

5.5 Summary and Conclusions

During a laboratory investigation on the forces and resulting wear of bedrock in granular flows, we encountered a strong tendency for particle size segregation to occur. This led us to
explore what might affect coarse particle trajectories through the granular flow and to become clustered into similar sizes, especially at the flow front and sides. In small-scale, spherical, dry granular bimodal flows, particles were more likely to circulate in a limited domain near the front in experiments with larger coarse-grained particles. Bed friction and wall resistance set up opposing lateral circulation fields through the flow. These processes caused coarse and fine particles to take different trajectories through the granular flow.

The addition of water and fines (creating a muddy fluid) in granular flows with geologic material and natural grain size distributions in our large drum revealed other processes. The flow of the fluid and the particles could differ greatly. Although lateral wall resistance still gave rise to lateral pressure gradients and lateral circulation cells, our experiments suggest that if the particles are sufficiently coarse they may tend to deviate from the fluid flow paths. Coarser particles in a flow would be pushed downstream by near surface granular and fluid flow and pulled downstream by gravity but pushed upstream by near bed flow and bed frictional resistance. The relative balance of forces dictates whether particles move toward the front, sides or upslope. This force balance is affected by fluid viscosity. We suggest that the ratio of grain size of interest to representative grain size for the flow, grain size to flow depth, grain size to flow width, and viscosity of the flow relative to that of water in addition to boundary roughness and flow velocity all influence segregation processes, including the tendency for a coarse front to form.

Most surprising was the discovery of periodic instability of the flow front when the flow was composed of a natural grain size distribution with a muddy matrix. The simultaneous opposing lateral circulation cells gave way to an alternating single dominant gyre with coarse grain concentrations which developed a periodic shift from side to side. This dynamic appears to be an instability associated with lateral segregation and is likely grain size distribution dependent.

Our findings suggest fluids and solids may take different paths through a debris flows. Larger grains may resist traveling with the circulation patterns in the longitudinal and lateral directions of fluid (and finer sediment) transport. Concentration of particles in gyres to the side may alter the circulation patterns. Flows with higher fluid viscosity tend to damp the separation of solid and fluid flow paths, unless the viscosity is so high that it no longer enters into the pore spaces of coarse clusters and forms a separate fluid flow path.

Further work is needed to apply simple relationships observed in the laboratory to natural geophysical granular flows. What is the best way to quantify the relationship between $D_i$ and a wide size distribution matrix material? Are there mechanisms in the drum that do not exist in straight chutes? What mechanisms lead to the break up of the oscillating coarse gyre? Field data collection about maximum $D_i$ at the flow front, and quantification of boundary conditions in field, including large scale roughness like knickpoints, will constrain the parameter spaces that need to be tested in more detail. Discrete element modeling of particle interactions in drums and other geometries will help determine the influence of boundary conditions and quantify particle contact forces [e.g. Yohannes et al., 2008]. Quantification of segregation mechanisms in fluid-rich wide grain size distribution geologic material will provide insight on spatially-varying energy dissipation, forces on the boundary, erosive ability, and also on the segregated deposits left in the geologic record.
5.6 References


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Swartz, M. & McArdell, B. (2005), Motion of large particles in debris flows(05943)'Geophysical Research Abstracts'.


## 5.7 Tables

### Table 1. List of experiments

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<th>Location</th>
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<td>A</td>
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<td>B</td>
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<td>N</td>
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<td>Y</td>
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Run numbers refer to Chapter 4 experiment numbering scheme. For videos, contact hsu.leslie@gmail.com.
Table 2. Small drum intruder particle position statistics

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<tr>
<th>$D_i$</th>
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$D_i$: intruder particle diameter (mm), mean position, 1 standard deviation, and skewness of the probability distribution functions of the intruder particle position, measured in units of drum angle (Fig. 3e and 3f).
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H_max: maximum front height (m), V_front: front velocity (m/s), D: front boulder diameter (m), D/H_max: D/h ratio at the front.
5.8 Figures

Figure 1:
Typical debris flow schematics, inferred from laboratory and field observations. (a) Side view showing coarse recirculating front and finer recirculating body. Larger particles migrate to the top causing inverse grading. Coarsest particle diameter close to height of flow. (b) Top view: symmetrical lateral movement at flow front from center to sidewalls. (c) Internal slice in the body of flow with secondary circulation to replenish material in the center of the flow.
Figure 2:
Two vertically rotating drums used to observe segregation and particle trajectories in shallow granular flows. (a) 56 cm diameter, 15 cm wide small drum (b) 4m diameter, 80 cm side large drum.
Figure 3:
Single intruder particle experiments in the small drum with dry spherical glass marbles. (a)-(d) probability distribution functions of the position of the large intruder particle (50, 40, 34, and 25 mm respectively) in a matrix of 13.8 mm particles. (e) side view to scale (f) plan view showing an example of a loop taken by a 34 mm intruder particle. The x-axis for all panels is in units of drum angle, where 0 is at the 6 o’clock position and the angle corresponds to the tangent to the drum bed.
**Figure 4:**
Plan-view images in the small drum with dry spherical glass marbles. Flow is toward the bottom of the frame. Large intruder particle behavior in a flow of smaller 13.8 mm diameter matrix particles. All runs are at 16 RPM (0.47 m/s). (a) One 50 mm intruder particle on a smooth bed has a restricted trajectory at the side wall. (b) Same as (a) but with a rough bed, increasing the velocity fluctuations of the matrix particles. The 50 mm intruder particle moves to the center of the flow. (c) Six 50 mm intruders on a smooth bed remain stably at the sidewall, do not bridge front, and do not form coarse front. (d) Six 50 mm intruders again, but with a larger amount of matrix particles. Intruders are pushed away from the wall and sometimes bridge the flow front, but do not form a stable coarse front. (e) Seven 50 mm intruders on a smooth bed form a coarse front (the front is bridged >75% of the time). (f) Eight 40 mm intruders on a smooth bed do not form a stable coarse front. (g) Two 25 mm intruders (outlined with a dotted line) follow their own trajectory loops on either side wall and do not interact. (h) Fifty-nine 25 mm intruders interact with each other but do not segregate from the 13.8 mm matrix particles.
Figure 5
Bimodal run with natural rocks and water in the large drum. 150 mm large cobbles in a matrix of 10 mm gravel and water. (a) 0.4 m/s drum velocity maintained a coarse front. (b) Increasing the velocity to 1.25 m/s disrupted the coarse front, as larger particles were moved to the side walls and dragged by the walls away from the front.
Figure 6
Surface velocity fields of flows in the four-meter drum, flow toward top of page. (a) Water-saturated gravel, diameter = 4 mm, large asymmetric lateral component because of fluid and wall drag. (b) Water-saturated gravel, diameter = 13 mm. Larger gravel is not affected as much by fluid and wall drag. (c) - (d) Muddy mixture D_{84} = 38 mm, showing oscillating lateral component of velocity field.
Figure 7
Lateral oscillations of the flow front in a muddy wide grain size distribution. Top view of muddy mixture $D_{84} = 38$ mm, flow towards the top of the frame. Frames are approximately 2.7 seconds apart and show lateral oscillation from one side to the other.
Figure 8
Oscillation of an asymmetric coarse gyre from one wall to another. Top view of muddy mixture $D_{84} = 110$ mm, flow towards the top of the frame. Frames (a)-(d) show the oscillation, with a transition period (b, c) where the coarse particles are spanning the front or dispersed in the flow. (e) Oscillation period of the coarse gyre in the big wheel. The increasing periods of oscillation coincide with the loss of fluid by evaporation.
Figure 9
Four different flow fronts at the Illgraben torrent, Switzerland. (a) Coarse front, 2.6 m height, 7.9 m/s velocity. (b) Coarse front, 2.6 m height, 3.1 m/s velocity. (c) Absence of a coarse front at 3.5 m/s velocity. (d) Same event as (c) but a cluster of boulders later in the flow when the matrix has thickened substantially. Maximum height of the flow is 3.3 m.
Appendix

A true story about my paper (or cat)