Intertemporal pricing, supply chain design, and consumer behavior

by

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Abstract

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Professor Ying-Ju Chen, Co-Chair

My dissertation explores the interaction between consumer behaviors and the design, pricing and management of products and services. The dissertation is comprised of four chapters. The first chapter studies how a seller’s pricing strategy can be affected by behaviors of non-fully rational consumers. These consumers are dynamically inconsistent and exhibit probabilistic decision making behaviors, which have been documented in experimental studies in economics and marketing literature. I show that consumers’ dynamic inconsistency can explain why flexible pricing plans are offered by service providers. Moreover, when fully rational consumers and non-fully rational consumers co-exist, a single pricing scheme is optimal. Such a result complements existing literature in mechanism design, as classic models suggest the seller should use a menu of pricing plans to differentiate the consumers. Numerical results are provided to demonstrate that the same result hold when both types of consumers non-fully rational and under mild conditions.

The second chapter examines how a seller should design the prices and qualities of products sold through his direct and indirect channels. I show that under the revenue sharing scheme, the seller’s optimal design depends on consumers’ sensitivities to price and quality. If the consumers are sufficiently sensitive, the seller should provide the product exclusively in the direct channel. If the consumers are sufficiently insensitive, the seller is better off providing a high quality product at a premium price in the direct channel while offering a low quality product in the indirect channel. Such quality differentiation can be eliminated in a profit sharing scheme. I also demonstrate that even when consumers are heterogeneous with privately observed sensitivities, offering a menu to induce self-selection may not be optimal for the seller’s profit.
In the third chapter, I use a two-period model to show that demand uncertainty can be the sole driver for the common practice of intertemporal pricing in the travel industry. Moreover, both increasing and decreasing pricing patterns can emerge as optimal strategies. I also identify the intrinsic incentive for service providers to deliberately create capacity shortage to induce early purchases. In the extended model, new arrivals are permitted in the second period enhance the competition. Contrary to intuition, the service provider’s expected profit is hurt since the additional arrival exacerbates his price commitment issue and results consumers strategically delay their purchases.

The last chapter investigates the effect of consumers’ limited knowledge of products on their purchasing behavior. Though online retailers put intense effort in improving web functionalities over the years, some product attributes (product quality, user friendliness, fit to consumers’ taste) cannot be communicated using the internet and must be examined physically by the consumers. Thus, their product valuations are not fully revealed until after they make the purchase. I show that when consumers are subject to both valuation uncertainty and future price uncertainty, their purchasing decisions are largely influenced by the return policies. A generous refund policy induces high-valued consumers to purchase early. However, it also invites some consumers to wait for the returns. This suggests that capacity rationing can be dampened. On the other hand, since neither the seller nor consumers can predict how many products will be returned, allowing consumer returns strengthens the seller’s credibility in not committing to pre-announced prices. This implies that the additional source of valuation uncertainty can be desirable for the seller when dealing with forward-looking consumers. A rationale for retailers do not actively engage in recertifying or remanufacturing returned products is also provided: when returns are perceived as low-quality products, the retailers can facilitate market segmentation without creating new product lines.
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Chapter 1

Intertemporal pricing with boundedly rational consumers

Flexible pricing plans are commonly observed in service industries. In this paper, we argue that the presence of flexible pricing plans can be attributed to consumers being boundedly rational – these consumers do not always select the best available option; rather, they select better options more often. In our model, the seller faces consumers who are heterogeneous in their degrees of intertemporal inconsistency - their ultimate actions can be different from their intended actions. We show that, in response to these boundedly rational consumers the seller may be able to extract more profit by setting different prices in different periods and allowing the consumers to self-select which period to pay. Moreover, a single pricing plan may emerge as an optimal pricing scheme even when the consumers are heterogeneous in their degrees of rationality and the seller is not fully aware of the consumers’ types. We further show that the pricing patterns depend primarily on the relative discounting factor between the seller and the consumers.

1.1 Introduction

It is hard to overemphasize pricing in modern service industries as it is key to firms’ profitability. As pricing becomes increasingly complex, some sellers use advanced softwares to forecast price-sensitive demand and to achieve price optimization. In addition to that, many sellers implement the “buy-now discount” strategy to induce consumers to purchase early. A sales person informs the customers that he can offer a 10% on a TV only if they make the purchase immediately; a banker promotes credit card origination
by rewarding the customers $100 for opening a credit card today and spending $799 or more for the next three months. Others may choose to offer flexible pricing plans that give consumers the options of when to pay for the services. As an example, the registration fee for the annual meeting of Allied Social Science Association (ASSA) has an early registration deadline, December 1st. Participants who pay before the date pays $75 while those who pay after the date pays $125. Moreover, American Economic Association (AEA) members enjoy an additional $15 discount. Thus, AEA members pay $60 before the deadline while they pay $125 after the deadline. The annual membership fee for AEA varies with annual income and ranges from $64 to $90. For the services that last for an extended period of time and require multiple periods of payment, many sellers also offer the consumers the flexibility of terminating the service early. A homeowner can get a fifteen-year fixed rate mortgage, which allows her to pay back the loan in monthly installments. The homeowner can pay off the mortgage prior to the expiration date with a pre-specified penalty.

At first glance, the flexible pricing plans may seem strange and detrimental from the sellers’ perspective. For example, the early termination option may disrupt the bank’s planned cash flows assuming that the mortgage is paid off in full. If the exact time for the homeowner to repay the mortgage were known, there would be no added value to the consumers and thus no point of offering a flexible contract. The homeowner and the bank can simply sign a binding contract that specifies the exact repayment plan. Likewise, conference organizers would have liked to specify a time slot for transactions to facilitate better logistics arrangement and capacity allocation. In this paper, we argue that the presence of flexible pricing plans may be attributed to the probabilistic behaviors of consumers in the payment periods. Consumers are subjective to “noise” in selecting their options, and thus they are unable to select the most favorable option due to future uncertainties. A conference participant may forget to register until they have missed the early registration deadline. Similarly, believing that the chance she will sell the house and exercise the early payoff option is small, a homeowner may still select a fifteen-year fixed rate mortgage despite the penalty associated with the early payoff option. Thus, it is conceivable that while signing a bank loan, consumers may not know for sure when they are going to pay back the principal balance and terminate the contractual relationship.

1Flexible pricing is commonly adopted by conference organizers. As an another example, the 2009 INFORMS conference on O.R. practice has an early registration deadline, April 10th. Participants who register before the deadline pays $970 while participants who register after it pays $1,070. Moreover, INFORMS members enjoy a $100 discount, that is they pay $870 to register before the deadline and $970 to register after it.
The probabilistic behavior described above is perceived as a consequence of consumers’ bounded rationality in the economics literature. In this paper, we intend to provide a unified framework that combines contract design, multiple-period pricing, probabilistic behaviors, and time discounting. Through this framework, we hope to uncover the intrinsic relationship between flexible pricing plans and consumers’ probabilistic behaviors. We also attempt to answer the following research questions. How should a seller design offer(s) facing boundedly rational consumers? Is it possible for the seller to take advantage of such boundedly rational behaviors? Does the seller prefer to deal with a rational consumer whose behavior is more predictable, or a boundedly rational consumer whose mistakes can be capitalized on?

To address these questions, we consider a stylized two-period model in which a seller intends to provide a service to the consumers who exhibit probabilistic behaviors. Both the seller and the consumers are risk neutral and value the present and future differently. The consumers are classified into two groups with heterogeneous degrees of bounded rationality. A more rational consumer puts a higher probability on selecting the option that gives her a higher net (expected) payoff. We adopt the quantal response framework introduced by McKelvey and Palfrey (1995) to model this probabilistic behavior. This framework captures the central idea of consumers “are prone to errors and biases” and “better options are chosen more often” while allows us to quantitatively characterize the degree of bounded rationality. Some researchers (Anderson et al. (1998), Lim and Ho (2007) and Su (2008)) use this framework to model consumers or retailers’ behaviors.

In our model setting, the seller offers pricing plans to consumers who are heterogeneous in their rationality at the contracting stage. Each pricing plan consists of pre-determined prices over all periods. Once a consumer accepted a pricing plan she is committed to make a purchase, though the period at which she will pay is a future choice of hers. The seller’s goal is to maximize his discounted expected profit. Note that even though consumers may be heterogeneous in rationality, sellers in real practices do not always offer multiple pricing plans. As the case of the fifteen-year fixed rate mortgage plan, all consumers are offered the same pricing plan (assuming these consumers are at the same credit risk level). Thus, one of our primary goals is to identify the operating regimes in which a single pricing plan (pooling contract) emerges as an optimal contract in this context.

Initially, we look at the “public quantal response” scenario, in which the seller is able to observe a consumer’s type and can offer the pricing plan specifically designed for her type. We first show that the seller can take advantage of the consumer’s probabilistic behaviors only if she is sufficiently irrational, i.e., with a low degree of rationality. As a sufficiently rational consumer is highly sensitive to price differences, she is less likely to
pay at the period with the less favorable price. Consequently, the benefit from setting the price of one period high and the other low is not an effective pricing strategy. On the other hand, if the consumer is relatively irrational, the seller can capitalize on her probabilistic behaviors by offering different prices at different periods.

In addition to identifying the conditions under which the seller can extract more profit, we also characterize the optimal pricing pattern. We find that the intertemporal pricing pattern depends solely on the relative time discounting factors between the seller and the consumers. In the case of a less patient seller, it is optimal to offer a more appealing second-period price as it can induce these consumers to probabilistically postpone their purchases. Counterintuitive as it sounds, this result is actually quite natural. As the seller values the future less than the consumers do, it is less costly for the seller to offer a favorable price in the second period. The seller can then increase the first-period price as long as the overall pricing plan prompts the consumers to participate. Even though a consumer purchases in the first period less often, she pays much more whenever this occurs. The optimal pricing plan thus balances the speculation over the intertemporal prices and the postponement of the consumers’ purchasing behaviors. In the case of a more patient seller, offering an appealing first-period price while setting a much higher second-period price is optimal. Lastly, we show that when both the seller and consumers value the future the same, simply setting the optimal price to be the consumers’ common valuation achieves the optimal expected profit.

Next, we turn to the “private quantal response” case, in which the degree of a consumer’s bounded rationality becomes privately observed by the consumer. We first start with the situation in which some consumers are fully rational (with deterministic purchasing decisions) and show that the seller is always better off offering a single pricing plan. The reason is as follows. A fully rational consumer always chooses to buy at the period with the more favorable price, and thus always picks the plan with the lowest price (regardless of the period) if the seller was to offer a menu of distinct pricing plans. This is less profitable from the seller’s viewpoint and thus a single pricing plan is offered at optimality. We further show that when there are many fully rational consumers, the seller’s best strategy is to abandon the option for the consumers to purchase in one of the periods. If this were not the case, at least one of the prices in the two periods must be lower than the consumer’s valuation in order to induce the consumers to participate. This implies that the seller must leave some information rent for the fully rational consumers, and the loss from giving them information rent outweighs the gain extracted from the boundedly rational ones as the proportion of the fully rational consumers become large. Consequently, the seller is better off abandoning the option of purchasing at both periods.
Finally, we investigate the scenario in the absence of fully rational consumers and find that the seller prefers to offer a single pricing plan to the consumers when either 1) there are plenty of relatively more rational consumers, 2) some consumers are sufficiently rational, or 3) the degrees of bounded rationality of the consumers are relatively homogeneous. The intuitive interpretations of these results are provided in Section 1.5. Notably, we are unaware of any other two-type screening models that gives the possibility of a single (pooling) pricing plan in the presence of information asymmetry. This demonstrates the unique insight one can extract from the formulation of bounded rationality.

We also consider two model extensions. First, we investigate the impact of competition on the seller’s pricing decisions and his expected profit. We incorporate competition by assuming that consumers have outside options that yield positive utilities. When the outside option is very appealing for a specific type of consumers, the seller is better off not to serve them. On the other hand, when the seller can afford to compete with outside options, the pricing structure is similar to that under no competition. Specifically, the seller can offer the pricing plan designed for the consumers with a reduced valuation for the service. Second, we study the case the consumers have heterogeneous valuations. We observe that if the seller wants to serve the entire market, his optimal strategy is to offer a single pricing plan to all consumers.

The rest of this paper is organized as follows. In the next section, we survey related literature. Section 1.3 describes the model. In Section 1.4 we characterize the optimal pricing plan (from the seller’s perspective) when the degree of bounded rationality is publicly known. In Section 1.5, we investigate the scenario in which the seller cannot observe the degrees of bounded rationality of the consumers. We extend our model to investigate the impact of competition on pricing decisions in Section 1.6. Section 1.7 presents another extension of the basic model, allowing the consumers to have heterogeneous valuations. We provides some concluding remarks and possible extensions in Section 1.8. All the proofs are included in the Appendix.

1.2 Literature Review

Our paper relates to a rising body of literature on modeling the behavioral or psychological effect in the mechanism design framework. In the classical literature, the agents (consumers in our context) are assumed to be fully rational; that is, when offered a set of actions, an agent always chooses the one that gives her the highest payoff. This assumption is commonly adopted in numerous papers, including supply chain contract-
ing (Burnetas et al. (2007), Corbett and de Groote (2000), Deshpande et al. (2010), Ha (2001), and Taylor and Xiao (2010)), dynamic pricing (Bansal and Maglaras (2009) and Zhang and Zenios (2008)), procurement and production planning (Iyer et al. (2005), Chaturvedi and Martínez-de-Albéniz (2009), Cachon and Zhang (2006), and Yang et al. (2009)), and auctions (Kostamis et al. (2009), Wan and Beil (2009), and Zhou et al. (2009)). This paper adds a distinguishable contribution to the extant literature on the mechanism design problem by incorporating boundedly rational behaviors to the mechanism design framework. This conceptual framework could potentially be useful in analyzing other forms of (dynamic) inconsistencies encountered in other pricing plan design problems.

The sources of bounded rationality have been uncovered by many researchers. Rubinstein (1993) introduce a form of bounded rationality that arises from the varying intelligence and the ability to process information. Very recently, a few papers have incorporated mis-perception of the likelihood of preference shift (Eliaz and Spiegler (2007)) and self-control ability (DellaVigna and Malmendier (2004)) in optimal contract design. Other types of bounded rationality, though not closely related to consumers’ purchasing behaviors, are also worth noting. O’Donoghue and Rabin (1999, 2008) argue that the propensity of procrastination reflects a time inconsistent preference. They model people’s tendency to pursue their immediate reward and delay costs until later by using a hyperbolic discounting model. Ozdenoren et al. (2006) suggest that willpower, the ability to self-regulate, depletes overtime and results intertemporal preference reversals. Orhun (2009) investigates the impact of consumers who have choice set dependent preferences on product line design. Please see Rabin (1998) for a more detailed account of how humans differ from traditional economic assumptions and DellaVigna (2009) for field evidences.

Our paper adopts the quantal response framework to study the intertemporal purchasing behavior. This framework is first proposed by McKelvey and Palfrey (1995) and elaborated by many papers. Anderson et al. (1998) apply the quantal response model to evaluate the bidding strategies while players engage in a rent-seeking behavior. The same framework is adopted by Han et al. (2001) to model the randomized consumer brand choice. Lim and Ho (2007) use the quantal response model to explain some practical pricing plans (fixed fee contract and nonlinear pricing contract) in certain business environments that could not be explained by the existing marketing literature. They argue that quantal response may arise from the consumers’ inability to perform complex computation. Su (2008) studies the impact of the retailer’s probabilistic behavior on quantity decisions in a classical newsvendor model. Chen and Hogg (2009) conduct experiments with human subjects making decisions involving risk and show that quantal
response model gives an explanation to the human subjects’ inconsistent behaviors with
the standard economic theory.

We also take the standard “multi-selves” approach to incorporate bounded rational-
ity in game-theoretical settings. Specifically, while multiple stages of decision making
are required and the economic agents are subject to limited memory, present bias, self-
control problems, and incorrect beliefs (over-/under-confidence), we can conveniently
model the multi-stage decision making as a sequential game played by multiple “selves”
with potentially different preferences and decision rules. This “time-inconsistent” frame-
work has been applied extensively in the literature of behavioral economics, see, e.g.,
Gul and Pesendorfer (2001), Ozdenoren et al. (2006), O’Donoghue and Rabin (2008),
Koszegi and Rabin (2006), and Eliaz and Spiegler (2007). Our results complement theirs
by introducing a different source for consumers’ non-standard behaviors. In addition, we
also investigate how rational players (the seller in our context) can capitalize on others’
bounded rationality by designing appropriate contracts accordingly.

1.3 The Model

We consider a stylized two-period model in which a seller intends to offer an item
or a service to consumers whose future actions are probabilistic. The consumers’ actual
actions taken in the future are sometimes inconsistent from the intended actions when
accepting the pricing plan. From the previous experience and historical data, the seller
is able to classify the consumers into two types: the future action of a type-2 consumer
is more likely to deviate from the planned action while that of a type-1 consumer is
less likely to deviate from the planned action. \( \rho_i \) represents the proportion of type-\( i \)
consumer in the market, and \( \rho_1 + \rho_2 = 1 \). All consumers have the same valuation \( \mu \)
and it is publicly known to the seller and the consumers. The seller incurs a fixed cost
\( c (<< \mu) \) for providing the service. In the basic model, we assume that consumers’
outside options give them null (zero) utilities, irrespective of their types. In Section
1.6, we extend our model to incorporate competitive outside offers that yield positive
utilities. To capture the fact that both the seller and the consumers value the current
and the future differently, we let \( \delta_s \) and \( \delta_c \) denote the discount factors of the seller and the
consumers respectively. In addition, we assume that both the seller and the consumers
are risk neutral and therefore aim at maximizing their expected utilities.

Let us now elaborate on the consumers’ decisions over periods. Suppose that, at
the contracting stage, a type-\( i \) consumer is offered a pricing plan which specifies a pair
of prices \( \{p_1, p_2\} \). Upon accepting this plan, the consumer has agreed to purchase the
service for sure and the seller has also committed to deliver at the end of the selling horizon. The consumer is, however, free to pick which period she will actually pay. She pays $p_1$ if she chooses to make the purchase in the first period while she pays $p_2$ in the second period if she has not already purchased in the first period. It is worth emphasizing that consumers cannot walk away without purchasing once they accepted the offer as our model assumes an infinite penalty amount incurs if either party fails to fulfill the contract. This kind of commitment is commonly observed in business practices. Using the mortgage loan example mentioned earlier as an example, if the homeowner does not payoff the mortgage early she must payoff at the expiration as specified in the contract; otherwise, she will be considered as default and the penalty associated is very large.

Given this pricing plan, a consumer's utility from purchasing in period 1 is $\mu - p_1$ and that in period 2 is $\mu - \delta_c p_2$. In the case that $\mu - p_1 > \mu - \delta_c p_2$, every consumer obtains a higher payoff from purchasing in period 1. A fully rational consumer simply chooses to purchase in period 1; however, a boundedly rational consumer is subject to dynamic inconsistency and may end up making the purchase in period 2. This dynamically inconsistent consumer behavior of our model reconciles with some of the empirical evidences documented by Rabin (1998) and DellaVigna (2009). We use a field experiment as an example: a credit card issuer mailed randomized credit card offers to two groups, one of which received an offer with a low introductory rate (4.9% for the first 6 months and followed by 16%) while the other received an offer with a low post rate (6.9% for the first 6 months and followed by 14%). The average balance borrowed in the first 6 months is about $2,000 and that in the next 15 months is about $1,000. Looking at the two offers ex-post, the second offer is better for the consumers as it allows them to save more on interest payment; however, the acceptance rate for the first offer is 2.5 times larger. One interpretation of this phenomenon is that the consumers who took the second offer naively believed that they will not borrow much on the credit card after the introductory period.

To account for the fact that consumers sometimes deviate from their planned action, we adopt the Quantal Response Equilibrium framework. Specifically, we apply the logit model on consumers' probabilistic behaviors. For a type-$i$ consumer, she makes the purchase in period 1 with probability

$$q_{i1} = \frac{e^{\gamma_i (\mu - p_1)}}{e^{\gamma_i (\mu - p_1)} + e^{\gamma_i (\mu - \delta_c p_2)}},$$

and in period 2 with

$$q_{i2} = 1 - q_{i1} = \frac{e^{\gamma_i (\mu - \delta_c p_2)}}{e^{\gamma_i (\mu - p_1)} + e^{\gamma_i (\mu - \delta_c p_2)}}.$$
The exponential probabilities capture the central idea of “better options are chosen more often” and its behavioral foundation is provided in McKelvey and Palfrey (1995).

Note that $\gamma_i$ is a parameter that characterizes how likely a type-$i$ consumer’s future action deviates from the optimal action. A smaller value of $\gamma_i$ indicates that type-$i$ consumer’s future action is more likely to be inconsistent with the planned action. In the mortgage example, if we assume that the homeowner is considering to pay off the mortgage at the end of the fifth year, the price $p_1$ denotes the sum of future payments (i.e., the discounted value of monthly payment from year 6 to year 15) and the early payoff penalty, pre-specified at the contracting stage. The price $p_2$ denotes the outstanding principal balance. Thus, $q_{i1}$ represents the probability that the homeowner decides to keep the mortgage and pay as the amortization schedule while $q_{i2}$ represents probability that the homeowner pays off the mortgage then.

From (1.1), we can easily observe that a type-1 consumer is more likely to purchase in period 1 than a type-2 consumer as $\gamma_1$ is larger; that is, the probability of choosing the favorable option increases in $\gamma$. This implies that the parameter $\gamma$ serves as a measure/proxy of how rational a consumer is: the higher the value is, the more rational the consumer is. As a special case, when $\gamma = \infty$, the consumer is fully rational and always chooses the action and its corresponding pricing plan that gives her the highest (expected) payoff.

The seller’s goal is to maximize his expected (discounted) payoff by offering the consumers pricing plans at the contracting stage. Initially, we assume that the seller is able to observe the consumers’ types, and therefore can offer the pricing plan specifically designed for them. We label this case the “public quantal response” scenario. Later, we turn to the case in which the seller is unable to identify the type of a specific consumer, which is referred to as the “private quantal response” scenario. In the next two sections, we characterize the seller’s optimal pricing plans for these two scenarios.

1.4 Public Quantal Response

In this section, we investigate the scenario in which the seller is able to identify the consumer’s type. Our goal is to characterize the optimal menu of pricing plans that maximizes the seller’s expected profit. This will help us understand the effect of consumers’ probabilistic behaviors alone on the seller’s expected profit.

To facilitate our analysis, let us introduce some more notation. At the contracting stage, since the consumer’s type $i$ is publicly known, the seller can offer a pricing plan $\{p_{i1}, p_{i2}\}$ to the consumer. Given this pricing plan, the consumer obtains a net expected
payoff \( u_{i1} = \mu - p_{i1} \) from purchasing in period 1 and \( u_{i2} = \mu - \delta_c p_{i2} \) in period 2. The probabilistic behavior of a consumer is then represented by the probability of purchasing in period 1:

\[
q_{i1} = \frac{e^{\gamma_i u_{i1}}}{e^{\gamma_i u_{i1}} + e^{\gamma_i u_{i2}}} \equiv \frac{1}{1 + e^{\gamma_i v_i}},
\]

where

\[
v_i \equiv u_{i2} - u_{i1} = p_{i1} - \delta_c p_{i2}
\]

represents the difference between the (discounted) utilities from obtaining the product in different periods. The absolute value of \( v_i \) can be regarded as a measure of the “(adjusted) price dispersion” between periods. Accordingly, the consumer purchases in period 2 with probability

\[
q_{i2} = \frac{e^{\gamma_i u_{i2}}}{e^{\gamma_i u_{i1}} + e^{\gamma_i u_{i2}}} = \frac{e^{\gamma_i v_i}}{1 + e^{\gamma_i v_i}}.
\]

We define \( \delta = \delta_s / \delta_c \) as the relative time discounting factor of the seller with respect to that of the buyer. We refer to the case of consumers and the seller having different time discounting factors as “heterogenous time discounting.”

In order to induce the consumers to accept the pricing plan, the seller needs to ensure that the consumer’s expected utility is at least her reservation utility. For simplicity of the analysis, we normalize her reservation utility to zero. Mathematically, this participation constraint for type-\( i \) consumer (PC-\( i \)) gives rise to the following condition:

\[
u_{i1} \cdot q_{i1} + u_{i2} \cdot q_{i2} = u_{i1} \cdot \frac{1}{1 + e^{\gamma_i v_i}} + u_{i2} \cdot \frac{e^{\gamma_i v_i}}{1 + e^{\gamma_i v_i}} \geq 0, \quad (\text{PC-}i)
\]

which, after replacing \( u_{i2} \) by \( v_i + u_{i1} \), can be simplified as follows: \( u_{i1} + v_i \cdot \frac{e^{\gamma_i v_i}}{1 + e^{\gamma_i v_i}} \geq 0 \).

Note that from (PC-\( i \)), the consumer is fully aware of her own probabilistic behavior while deciding whether to accept the pricing plan. In the literature on behavioral economics, there have been vast discussions on whether a boundedly rational agent is aware of her own bounded rationality. For example, in the self control literature, researchers investigate the behaviors of sophisticated agents who are aware of their temptations (DellaVigna and Malmendier (2004) and Gul and Pesendorfer (2001)). Likewise, a sophisticated agent in the hyperbolic discounting literature is fully aware of her present bias and is fully aware of this time inconsistency problem (O’Donoghue and Rabin (1999, 2008)). Even in the literature that incorporates unawareness, agents are assumed to be aware that they are unaware of something (Filiz-Ozbay (2008), Ozbay (2006)).
Heifetz et al. (2009), and von Thadden and Zhao (2009)). In the cognitive thinking literature, agents know that there are some foreseeable events but cannot foresee these events without making this cognitive effort (Tirole (2009)). Similarly, the literature on willpower assumes that agents can blindly impose certain wrong beliefs while making decisions even though they fully understand that this belief is completely wrong and not justifiable (Benabou and Tirole (2009)). All of the aforementioned research streams allow for some sophistication despite the accommodation of behavioral issues and bounded rationality.

Now let us consider the seller’s problem while facing a specific type-$i$ consumer, which is finding a pricing plan that solves the following problem:

$$
\Pi_i^F = \max_{p_{i1}, p_{i2}} \{p_{i1} \cdot q_{i1} + \delta_s p_{i2} \cdot q_{i2} - c\}
$$

s.t. \(u_{i1} \cdot q_{i1} + u_{i2} \cdot q_{i2} \geq 0\).

Recall that \(q_{i1} = 1/(1+e^{\gamma_i v_i})\) is the probability that type-$i$ consumer purchases in period 1 and \(p_{i1}\) is the first-period price. The first term \((p_{i1} \cdot q_{i1})\) in the objective function is thus the expected profit the seller collects from first-period purchasing and likewise the second term \((\delta_s p_{i2} \cdot q_{i2})\) is the discounted expected profit from second-period purchasing. The last term is simply the cost of providing the product. Let \(\{p_{i1}^F, p_{i2}^F\}\) represents the optimal pricing plan under the public quantal scenario. The optimal solution to this problem is summarized below.

**Proposition 1.** In the case of public quantal response and heterogeneous time discounting, when the seller is relatively less patient \((\delta < 1)\), there exists a cutoff threshold \(\gamma_{lp}\) such that:

1. For \(\gamma_i \geq \gamma_{lp}\), the seller’s optimal pricing plan is \(\{p_{i1}^F, p_{i2}^F\} = \{\mu, \infty\}\), irrespective of the consumer’s type. The seller’s optimal expected profit is \(\mu - c\).

2. For \(\gamma_i < \gamma_{lp}\), the seller’s optimal pricing plan \(\{p_{i1}^F, p_{i2}^F\}\) is type-specific. The optimal (adjusted) pricing plan exhibits a decreasing pattern, i.e., \(p_{i1}^F > \mu > \delta_s p_{i2}^F\). The seller’s expected profit is higher than \(\mu - c\). Moreover, his expected profit decreases as \(\gamma_i\) increases.

The first part of Proposition 1 shows that when the seller is relatively impatient \((\delta < 1)\) and the consumer is sufficiently rational \((\gamma_i \geq \gamma_{lp})\), the seller’s best strategy is to simply offer one option – purchasing in period 1. In real practice, sellers may need to sacrifice a bit of profit to increase customer satisfaction. Instead of abandoning the
option of purchasing in the second period, they may simply set a really high price. Given this pricing plan, all consumers must purchase immediately, and the seller collects the expected profit in the first period. The intuition of this result is as follows. In order to induce a consumer to participate, the seller must leave the consumer at least her reservation payoff. When the seller intends to increase the price in one period, he must decrease the price in the other period. This, however, makes the option of purchasing in the period with the lower price more favorable. A sufficiently rational consumer is highly sensitive to the price difference (which is reflected on the probabilities of choosing to pay in different periods), and thus an incremental price increase in one period triggers a significant change of the consumer’s purchasing behavior. Consequently, the seller is worse off attempting to increase the price. We can easily validate our result with a special case – a fully rational consumer who only purchases in the period that gives her a higher (expected) utility – the seller can extract no more than $\mu - c$ when $\delta < 1$.

The second part of Proposition 1 shows that when the seller is relatively impatient ($\delta < 1$) and the consumer is sufficiently irrational ($\gamma_i < \gamma_{lp}$), the seller can take advantage of her probabilistic behavior by offering different prices at different periods. Our intuition would suggest that the seller has an incentive to induce the consumer to purchase early when the seller is less patient. However, our results show that the optimal strategy is exactly the opposite: the seller now sets a more appealing second-period price to induce the consumers to postpone their purchases (probabilistically). Recall that the seller only needs to offer her the null expected payoff to induce the consumer’s participation when the consumer’s type is publicly known. It is less costly for the seller to offer a favorable price in the second period since the seller values the future payment less than the consumers do. In turn, the seller can increase the first-period price as long as the overall pricing plan still induces the consumers to participate. Although the consumer purchases in the first period less often, she pays much more whenever she purchases in the first period and consequently the seller can obtain a higher expected profit. This pricing plan can be easily applied to our mortgage example: the bank can set a high early termination penalty and obtain much more money from the homeowner if she decides to pay back the mortgage before the expiration date.

In summary, the complete information of a consumer’s degree of rationality enables the seller to extract more expected profit from the consumer only if the consumer is sufficiently irrational. The exact analytical expression of the cutoff threshold $\gamma_{lp}$ is provided in the appendix. Next, we look at the case where the seller is more patient.

**Proposition 2.** In the case of public quantal response and heterogeneous time discounting, when the seller is relatively more patient ($\delta > 1$) there exist a cutoff threshold $\gamma_{mp}$
such that:

1. For $\gamma_i \geq \gamma_{mp}$, the seller’s optimal pricing plan is $\{p^F_{i1}, p^F_{i2}\} = \{\infty, \mu/\delta_c\}$, irrespective of the consumer’s type. In this case, the seller’s optimal expected profit is $\delta \mu - c$.

2. For $\gamma_i < \gamma_{mp}$, the seller’s optimal pricing plan $\{p^F_{i1}, p^F_{i2}\}$ is type-specific and the optimal (adjusted) pricing plan exhibits an increasing pattern, i.e., $p^F_{i1} < \mu < \delta_c p^F_{i2}$. The seller always obtains an expected profit higher than $\delta \mu - c$. Moreover, his expected profit decreases as $\gamma_i$ increases.

Proposition 2 shows that when the seller is relatively patient ($\delta > 1$), the seller’s best strategy is to only allow the consumer to purchase the service in period 2 when the consumer is sufficiently rational ($\gamma_i \geq \gamma_{mp}$). This result corresponds to the case where a flexible term mortgage is not offered: once signed up for a fifteen-year fixed-rate mortgage, the homeowner is obligated to pay exactly as specified. When the consumers are sufficiently irrational ($\gamma_i < \gamma_{mp}$), offering a favorable first-period price is an optimal strategy for the seller. The reason is similar to that of Proposition 1. The seller prefers to induce the consumers to purchase earlier by setting an appealing first-period price, and when the consumer ends up purchasing in the second period the seller can obtain a much higher expected profit. Finally, we look at the case where the seller and the consumers are equally patient.

Proposition 3. In the case of public quantal response and homogeneous time discounting ($\delta = 1$) the seller’s optimal expected profit is $\mu - c$, one of the optimal pricing can be characterized as $\{p^F_{i1}, p^F_{i2}\} = \{\mu, \mu/\delta_c\}$.

Proposition 3 shows that when the seller and the consumers are equally patient, simply setting the time discounted price as the consumers’ valuation is the best pricing strategy. The optimal expected profit the seller can obtain is $\mu - c$ regardless of the consumer’s type. The intuition for this result is simple: since both the seller and the consumers value the future the same, the seller’s optimal strategy is to set the (adjusted) prices such that the consumers are indifferent between purchasing in period 1 or period 2.

In summary, even when the seller is fully aware of the consumer’s type, he cannot take advantage of the consumer’ probabilistic purchasing behavior and obtain a higher expected profit when the consumer is sufficiently rational. The best strategy is to allow the consumers to purchase the service in only one of the periods. The seller can obtain higher expected profit when the consumer is sufficiently irrational by setting a type
specific pricing plan. The optimal expected profit increases as the degree of rationality decreases; moreover, the (adjusted) pricing pattern depends primarily on the relative time discounting factor between the seller and the consumer. An immediate follow-up question is whether these results hold when the seller has no access to how rational the consumers are. This is investigated in the next section.

1.5 Private Quantal Response

In this section, we consider the scenario in which the seller knows the distribution of the consumers but is not able to identify a particular consumer’s type. Information asymmetry thus exists between the seller and the consumers, and the seller must induce the consumers to reveal their types by offering the appropriate incentives. According to mechanism design theory, the best strategy for the seller is to offer a menu of pricing plans (contracts) for the consumers to self-select. Potentially, the set of pricing plans could be incredibly huge; however, the Revelation Principle allows us to focus on the menu of contracts for which the consumer simply reports her type and the seller then picks a pricing plan on the consumer’s behalf (Laffont and Martimort (2002)). Specifically, let us denote \( \{p_{i1}, p_{i2}\} \) as the pricing plan intended for type-\( i \) consumer, where \( i = 1, 2 \). In the conference registration example, the participants who choose to be members of the professional societies are considered as, say, type-1 consumers. The ones who choose not to be members of the professional societies can be regarded as type-2 consumers. The pricing plans consist an early registration fee and a late registration fee. Our goals are two-fold. First, we intend to investigate whether a menu of pricing plans is necessary to maximize the seller’s expected profit. Second, if a menu is used, we will find the optimal menu. Without loss of generality, we assume that type-1 consumers are more rational than type-2 consumers, i.e. \( \gamma_1 > \gamma_2 \).

Let us now turn to the case in which the seller and the consumers have different time discounting factors. First, let us state the optimal pricing strategy for the cases that both types of consumers are sufficiently rational.

**Proposition 4.** In the case of private quantal response and heterogeneous time discounting, offer a single pricing plan is optimal. Specifically,

1. when \( \delta < 1 \) and both \( \gamma_1 > \gamma_2 \geq \gamma_{lp} \), the seller should offer pricing plan \( \{\mu, \infty\} \) to all consumers and obtain an expected profit of \( \mu - c \).

2. when \( \delta > 1 \) and both \( \gamma_1 > \gamma_2 \geq \gamma_{mp} \), the seller should offer pricing plan \( \{\infty, \mu/\delta_c\} \) to all consumers and obtain an expected profit of \( \delta \mu - c \).
when $\delta = 1$, the seller should offer pricing plan $\{\mu, \mu/\delta, c\}$ to all consumers and obtain an expected profit of $\mu - c$.

Proposition 4 is an immediate extension of the results in the previous section. Information asymmetry affects neither the seller’s expected profit nor the consumers’ information rent when they are sufficiently rational. Having characterized the optimal solution when both types of consumers are sufficiently rational, we now focus on the cases where at least one type of the consumers is sufficiently irrational ($\gamma_2 < \gamma_p$ when $\delta < 1$ and $\gamma_2 < \gamma_{mp}$ when $\delta > 1$) in the following sections.

1.5.1 A Mix of Fully Rational and Boundedly Rational Consumers

We start with the case in which some consumers are fully rational (i.e., they always choose the pricing plan that gives them the highest profit). Recall that $(p_{1i}, p_{2i})$ denotes the pricing plan intended for type-$i$ consumer, where $i = 1, 2$. $u_{i1} = \mu - p_{i1}$, $u_{i2} = \mu - \delta c p_{i2}$, and $v_i = u_{i2} - u_{i1}$ (defined in Section 1.4). Since a consumer can reject the contract if it gives a negative expected payoff, the following participation constraints must hold:

$$\max\{u_{11}, u_{12}\} \geq 0,$$  \hfill (PC-1)

$$u_{21} \cdot q_{21} + u_{22} \cdot q_{22} \geq 0,$$  \hfill (PC-2)

where (PC-1) ensures that the fully rational consumer is willing to participate (the left-hand side is the maximum payoff she can obtain from the pricing plan designed for her), and (PC-2) guarantees the participation of the boundedly rational consumer. In the presence of information asymmetry, the seller also needs to ensure that the consumer will pick the pricing plan designed for her rather than misrepresent herself. Mathematically, this implies that

$$\max\{u_{11}, u_{12}\} \geq \max\{u_{21}, u_{22}\},$$  \hfill (IC-1,2)

$$u_{21} \cdot q_{21} + u_{22} \cdot q_{22} \geq \frac{u_{11}}{1 + e^{\gamma_2 v_1}} + \frac{u_{12} \cdot e^{\gamma_2 v_1}}{1 + e^{\gamma_2 v_1}},$$  \hfill (IC-2,1)

where (IC-i,j) guarantees that type-$i$ consumer prefers the pricing plan designed for her over that for type-$j$: the left-hand side corresponds to the expected payoff a consumer chooses the pricing plan intended for her type, and the right-hand side refers to the expected payoff upon misrepresentation. Note that when a fully rational consumer misrepresents herself by choosing $(p_{21}, p_{22})$, her purchasing behavior remains rational.
and deterministic (as seen in the right-hand side of (IC-2,1). On the other hand, when the boundedly rational consumer chooses the pricing plan intended for the fully rational type, she exhibits a probabilistic purchasing behavior.

From this representation, we have adopted the standard “multi-selves” approach introduced in the economics literature that incorporates bounded rationality to game-theoretical settings. Specifically, while multiple stages of decision making are required and the economic agents are subject to limited memory, present bias, self-control problems, and incorrect beliefs (over-/under-confidence), we can conveniently model the multi-stage decision making as a sequential game played by multiple “selves” with potentially different preferences and decision rules.

The seller’s expected profit is therefore the discounted expected profit extracted from these two types of consumers. Collectively, the seller’s optimization problem can be written as follows:

\[
\Pi = \max_{p_1, p_2} \left\{ \rho_1 (p_{11} \cdot 1_{u_{11} \geq u_{12}} + \delta p_{12} \cdot 1_{u_{11} < u_{12}}) + \rho_2 (p_{21} q_{21} + \delta p_{22} q_{22}) - c \right\}
\]

s.t. (PC-1), (PC-2), (IC-1,2), and (IC-2,1),

(1.3)

where \(1_{u_{11} \geq u_{12}}\) and \(1_{u_{11} < u_{12}}\) are the indicators for the purchasing behavior of the fully rational consumer.

**Proposition 5.** In the case where there is a mix of fully rational and boundedly rational consumers but the seller cannot distinguish between them, it is always optimal for the seller to offer a single pricing plan. Moreover, there exists a cutoff threshold \(\rho^*_1\) such that:

1. when \(\rho_1 > \rho^*_1\) and \(\delta < 1\), the optimal pricing plan is \(\{p_{1}^S, p_{2}^S\} = \{\mu, \infty\}\), and the seller’s expected profit is \(\mu - c\). When \(\rho_1 \leq \rho^*_1\) and \(\delta < 1\), the optimal pricing plan exhibits a decreasing pricing pattern and the seller can obtain an expected profit that is higher than \(\mu - c\).

2. when \(\rho_1 > \rho^*_1\) and \(\delta > 1\), the optimal pricing plan is \(\{p_{1}^S, p_{2}^S\} = \{\infty, \mu/\delta\}\), and the seller’s expected profit is \(\delta \mu - c\). When \(\rho_1 \leq \rho^*_1\) and \(\delta > 1\), the optimal pricing plan exhibits an increasing pricing pattern and the seller can obtain an expected profit that is higher than \(\delta \mu - c\).

According to Proposition 5, the seller is always better off offering a single pricing plan in the presence of fully rational consumers. The rationale for this result is the following. Suppose the seller designs two pricing plans, A for fully rational consumers
and B for boundedly rational consumers. In order to induce the boundedly rational (type-2) consumers to participate, the seller must give the bounded rational consumers their null payoff. Thus, at least one of the prices in scheme B is lower than $\mu$. Recall that a fully rational consumer always chooses the more favorable option with certainty. Thus, a fully rational consumer always gets a strictly positive payoff from picking scheme B. If the seller wants to offer a pricing plan specific to the fully rational consumer, he needs to set one of the prices in scheme A to be the lower than the lower price in scheme B. This is not optimal since the seller can simply offer scheme B to the fully rational consumers to prevent extra loss.

Proposition 5 further shows that when there are many fully rational consumers, all the seller can do is to simply abandon the option for the consumers to purchase in one of the periods. Let us consider an alternative in which he offers a single pricing plan (a menu is never desirable from the seller’s viewpoint as discussed above). In order to induce the boundedly rational (type-2) consumer to participate, the seller must give the bounded rational consumer her null payoff. Thus, at least one of the prices in the two periods must be lower than $\mu$, and the fully rational consumer will purchase in the period with the lower price. This implies that the seller must leave some information rent for the fully rational consumers. As there are plenty of fully rational consumers, the loss from giving away information rent to the fully rational consumers outweighs the gain extracted from the boundedly rational ones. Consequently, the seller is better off by abandoning the option for the consumers to purchase in one of the periods.

It is worth mentioning that if one were naively following the conventional wisdom from the theory of incentives, such a pooling result can never be predicted, regardless of how close the two types are and what the population mix is. Our results thus put forth the unique role of bounded rationality in our mechanism design problem. Moreover, in all the aforementioned scenarios, we also observe that the pricing pattern is again determined entirely by the relative time discounting, thereby leading to a belief that this close connection is a norm rather than an exception in the context with bounded rationality.

Given the above discussions, it is then straightforward to observe that the pricing pattern depends only on the relative time discounting between the seller and the consumers. Provided that a single pricing plan is offered, offering a more favorable price in the second period is less costly for the seller if he is less patient than the consumers are while a more favorable price in the first period is less costly for the seller if he is more patient. This point has been stated in the case of public quantal response, and our result here re-confirms it when the information asymmetry is present.
1.5.2 A Mix of Boundedly Rational Consumers

Finally, let us consider the case in which all consumers are boundedly rational (i.e., \( \gamma_2 < \gamma_1 < \infty \)). In this case, all consumers exhibit the probabilistic behaviors for any given pricing plan. The seller’s maximization problem can be written as:

\[
\Pi^S = \max_{\{p_1, p_2\}} \sum_i \rho_i [p_{i1} \cdot q_{i1} + \delta s p_{i2} \cdot q_{i2}] - c
\]

s.t. (PC-i):

\[
u_{i1} q_{i1} + \nu_{i2} q_{i2} \geq 0, \forall i = 1, 2,
\]

(IC-i,j):

\[
u_{i1} q_{i1} + \nu_{i2} q_{i2} \geq \frac{\nu_{j1}}{1 + e^{\gamma_{i} v_{i}}} + \frac{\nu_{j2} \cdot e^{\gamma_{i} v_{i}}}{1 + e^{\gamma_{i} v_{i}}}, \forall i, j = 1, 2, j \neq i,
\]

where (PC-i) and (IC-i,j) are the corresponding participation and incentive compatibility constraints for type-\(i\) consumers.

With this general formulation, we observe that the seller may or may not give a menu of contracts to the consumers. The rationale of providing a menu of contracts has been made clear in the classical mechanism design literature: in the presence of information asymmetry, the principal (the seller in our context) intends to induce the agents (the consumers) to reveal their true preferences via their pricing plan selections. Since this can only be done if the contracts are differentiated, and the principal has no other way to distinguish among the agents, offering a menu of distinct contracts turns out to be the only optimal option for the principal (see Laffont and Martimort (2002) for an extensive survey).

The incentive for the seller to differentiate the consumers remains valid in our context. Nevertheless, due to the consumers’ probabilistic behavior or bounded rationality, we are able to further identify situations in which the seller may prefer offering a single pricing plan as we will describe in the next proposition. Let us first consider the case in which one type of consumers is sufficiently rational (\( \gamma_1 \geq \gamma_{lp} \)).

**Proposition 6.** In the case where both types of consumers are boundedly rational, offer a single pricing plan is optimal if type-1 consumers are sufficiently rational and the proportion of type-1 is sufficiently high:

1. when \( \delta < 1 \), \( \gamma_1 \geq \gamma_{lp} \) and \( \rho_1 \) is sufficiently large, the seller should offer a single pricing plan \( \{p_{1S}, p_{2S}\} = \{\mu, \infty\} \) to both types of consumers and obtain an expected utility of \( \mu - c \).

2. when \( \delta > 1 \), \( \gamma_1 \geq \gamma_{mp} \) and \( \rho_1 \) is sufficiently large, the seller should offer a single pricing plan \( \{p_{1S}, p_{2S}\} = \{\infty, \mu/\delta c\} \) to both types of consumers and obtain an expected utility of \( \delta \mu - c \).
In the proof of Proposition 6, we observe that the seller must always give some information rent to the relatively more rational (type-1) consumers. For any given pricing plan, consumers with a higher degree of bounded rationality make the right choice more often (than consumers with lower degree of bounded rationality do) and thus obtain a higher expected payoff. If there are many more rational consumers, the seller gives too much information rent to those relatively more rational consumers, compared to the benefit extracted from the less rational consumers. This entices the seller to simply offer one pricing plan.

To further explore the impact of consumers’ bounded rationality on the seller’s pricing plan design problem, we conduct some numerical analysis.

Numerical examples of mixed boundedly rational consumers

Let us first investigate how the seller’s expected profit changes as the population of the consumers changes. To this end, we fix a pair of \((\gamma_1, \gamma_2)\), and vary the value of \(\rho_1\) (proportion of type-1 consumers). Recall that when \(\delta < 1\) and \(\gamma_{lp} < \gamma_2 < \gamma_1\), it is optimal for the seller to abandon the option of selling in the second period and collect \(\mu - c\) as his expected profit. We thus restrict our attention to the case in which \(\gamma_2 < \gamma_{lp}\). We present generic graphs of the seller’s expected profit with different combinations of \((\gamma_1, \gamma_2)\) in the following two figures.

![Figure 1.1: \(\gamma_1 \geq \gamma_{lp}\) and \(\delta < 1\).](image1)

![Figure 1.2: \(\gamma_1 < \gamma_{lp}\) and \(\delta < 1\).](image2)

Figure 1.1 represents the case in which type-1 consumers are sufficiently rational while Figure 1.2 represents the case in which type-1 consumers are sufficiently irrational. In Figure 1.1, we observe that when \(\gamma_1 > \gamma_{lp}\), the seller’s expected profit decreases as there
are more relatively rational consumers. In this case, the seller always gets less than the expected valuation ($\mu$) from type-1 consumers. Moreover, since the maximum revenue extracted from type-2 consumers is limited, the seller pays more information rent for type-1 consumers and results in a lower expected profit as $\rho_1$ becomes higher. Eventually, the seller is better off adopting a single pricing plan to avoid paying too much information rent for type-1 consumers. This gives rise to the specific shape in Figure 1.1. When $\gamma_1 < \gamma_{lp}$, we observe from Figure 1.2 that the seller’s expected profit decreases in $\rho_1$ initially, but increases after $\rho_1$ exceeds certain threshold. Recall that if the seller knew $\gamma_1$ and $\gamma_2$ perfectly, he could obtain an expected revenue higher than $\mu$ from each type. Thus, it is conceivable that in the two extreme cases ($\rho_1 = 0$ and $\rho_1 = 1$), information asymmetry goes away and the seller can simply implement the first-best pricing plan for that type. In the intermediate case, however, such a pricing plan may lead to a lower expected profit since the seller may extract less from either type or both types of consumers; consequently, the seller may be forced to either distort the (menu of) pricing plans or simply abandon the option of purchasing in the second period. This explains the non-monotonicity in Figure 1.2.

We now investigate how the consumers’ population mix and their relative bounded rationality affect the optimal pricing plans. We again focus our discussion on the case of $\delta < 1$ and $\gamma_{lp} < \gamma_2 < \gamma_1$. Figure 1.3 shows that the seller intends to offer a single pricing plan when 1) there are many type-1 consumers and they are relatively rational, or 2) there are only a few type-1 consumers and their degree of rationality is close to that of type-2 consumers. The rationale for 1) has been stated earlier: as the consumers become more rational, it is more difficult for the seller to take advantage of the consumers’

![Figure 1.3: population mix](image1.png)

![Figure 1.4: heterogeneous rationality](image2.png)
probabilistic behaviors, and thus offering a single pricing plan is optimal. The reason for 2) may be explained as follows. When the consumers are sufficiently homogeneous in rationality, differentiating them via offering distinct contracts generates less expected profit for the seller. Thus, it is conceivable that the seller may then intend to offer a single pricing plan while facing sufficiently homogeneous consumers (with regard to the bounded rationality).

In Figure 1.4, we demonstrate that the seller prefers to offer a single pricing plan when type-1 consumers are sufficiently rational \((\gamma_1 > \gamma_{lp})\). This is in line with our results stated in Proposition 5. When some consumers are fully rational, it can be regarded as a special case \((\gamma_1 = \infty)\). More interestingly, we observe that the seller’s incentive of offering a single pricing plan is also strong if type-2 consumers are either extremely irrational or sufficiently rational; nonetheless, for intermediate cases a menu of contracts could emerge as an optimal solution. We also observe from our numerical studies that when there are many type-1 consumers, the region in which a menu of pricing plans is offered degenerates.

Recall that the seller would like to offer specific pricing plans for different types of consumers if \(\gamma_2 < \gamma_1 < \gamma_{lp}\) with public quantal response. However, based on our numerical examples, we identify certain situations in which the seller intends to offer a single pricing plan when such information becomes unobservable. In this sense, the presence of information asymmetry may simplify the pricing plan design problem.

1.6 Competition

A natural extension of our model is to investigate the impact of competition on pricing decisions. More specifically, we would like to know whether flexible pricing plans can still emerge as optimal pricing strategies. In many business practices, a seller without monopoly power may face multiple competitors. Although these competitors may use different pricing strategies (flexible pricing plans or fixed prices) and/or may offer different prices for different periods, consumers’ self-selection behaviors result in type-dependent utilities. Follow the same contract selection process described in the basic model, a consumer who receives multiple offers evaluates each of them individually by computing the expected utility and accepts the best one. Thus, the seller needs to offer a pricing plan that gives her the highest expected utility. Let \(\mu_{i0}\) be type-\(i\) consumers’ highest utility from their best outside offers. The following participation constraint captures type-\(i\) consumer’s evaluation process and only accepts the offer when

\[\mu_{i0}\]
the utility from pricing plan \( \{p_{i1}, p_{i2}\} \) generates a utility that is weakly greater than \( \mu_{i0} \):

\[
u_{i1} \cdot q_{i1} + u_{i2} \cdot q_{i2} \geq \mu_{i0}.
\]

In the public quantal case, the type of the consumer is public information. Thus, the seller’s profit maximization problem can be then written as

\[
\Pi_i^C = \max_{\{p_{i1}, p_{i2}\}} p_{i1} \cdot q_{i1} + \delta_s p_{i2} \cdot q_{i2} - c
\]

\[
\text{s.t. } u_{i1} \cdot q_{i1} + u_{i2} \cdot q_{i2} \geq \mu_{i0}.
\]

We define the competition as *intense* if one of the following conditions holds:

1. If the seller is relatively less patient (\( \delta < 1 \)) and the utility (\( \mu_{i0} \)) from the best outside option of type-\( i \) consumers is greater than \( \mu - c \).

2. If the seller is relatively more patient (\( \delta > 1 \)) and the utility (\( \mu_{i0} \)) from the best outside option of type-\( i \) consumers is greater than \( \mu - c/\delta \).

If non of the above conditions holds, we consider the competition as *moderate*. The result of the above optimization problem can be then summarized in the following proposition.

**Proposition 7.** When the competition for type-\( i \) consumers is moderate, the seller’s optimal strategy is the pricing plan designed for consumers with valuation \( \mu - \mu_{i0} \) under the no competition case. However, when the competition for type-\( i \) consumers is intense, the seller should not serve these consumers.

Proposition 7 reveals several effects of competition on seller’s pricing plan and his expected profit. For one, competition drives down the seller’s expected profit. When the competition is intense, it can be too costly for the seller to offer some types of consumers pricing plans that give them the highest utility and thus he is better off

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3We now give an example to show that our model is suitable for competitors who offer different pricing strategies as well as services that consumers value differently. Suppose there are three competitors (A, B and C) whose services are valued by consumers as \( \mu_a, \mu_b \) and \( \mu_c \) respectively. Competitor A sells the service only in the first period at price \( p_{A1} \) while Competitor B sells the service only in the second period at price \( p_{B2} \). Competitor C, on the other hand, offers a pricing plan \( \{p_{C1}, p_{C2}\} \). For a type-\( i \) consumer, her utility from accepting Competitor A’s offer is \( \mu_a - p_{A1} \) and that from Competitor B’s offer is \( \mu_b - \delta_s p_{B2} \). The consumer’s expected utility from accepting Competitor C’s offer is \( (\mu_c - p_{C1}) \cdot q_1 + (\mu_c - \delta_s p_{C2}) \cdot q_2 \), where \( q_1 = \frac{e^{\gamma_i(\mu_c - p_{C1})}}{e^{\gamma_i(\mu_c - p_{C1})} + e^{\gamma_1(\mu_c - \delta_s p_{C2})}} \) and \( q_2 = 1 - q_1 \). Thus, the best outside option (\( \mu_{i0} \)) is type-dependent and in this example equals to the maximum of the three (expected) utilities.
not to serve them. This result is not surprising as we often observe this phenomenon in business practices: sellers profile their consumers and only target the ones that are more profitable. When the competition is moderate, however, the seller can still serve the consumers and offer similar pricing plans. Specifically, he can offer the consumers the optimal pricing plan designed under no competition but with a reduced consumer valuation, \( \mu - \mu_0 \). As an example, if the seller is relatively less patient (\( \delta \leq 1 \)), he should offer \( \{p_{11}^C, p_{12}^C\} = \{\mu - \mu_0, \infty\} \) to all consumers who are sufficiently rational (\( \gamma_i \geq \gamma_{lp} \)) and obtain an expected profit is \( \mu - \mu_0 - c \). On the other hand, he should offer type-specific pricing plans \( \{p_{21}^C, p_{22}^C\} \) to all types of consumers who are sufficiently irrational (\( \gamma_i < \gamma_{lp} \)) and obtain expected profits higher than \( \mu - \mu_0 - c \).

**Corollary 1.** When the competition for type-i consumers is moderate, the higher utility the best outside option gives, the more likely the seller is to offer flexible pricing plan. The price dispersion decreases with respect to the utility of the best outside option.

The corollary comes directly from the proof of Proposition 7 shown in the appendix. The intuition of this result is the following. When there is no competition or the best outside option gives the consumers zero utility, the seller can fully take advantage of the consumers’ “mistakes” by setting a high price for the unintended action. In the mortgage example, the lender can include a high penalty for the early payoff option. This penalty, however, has to be reduced as the consumers’ outside options become more appealing. Thus, the seller has to decrease the price dispersion of the two actions. For the similar reason, the seller needs to offer flexible pricing plans to a wider range of consumers who are boundedly rational in order to to induce them to accept the offers.

### 1.7 Heterogeneous Valuations

Another extension for us to consider is that the consumers have different valuations. In this case, we again adopt our two-type framework and let \( \mu_1 \) represent the high-valuation consumers’ valuation and \( \mu_2 \) represent the low-valuation consumers’ valuation. Without loss of generality, we assume \( \mu_1 \geq \mu_2 >> c \). To isolate the affect of heterogeneous consumers’ valuations have on optimal pricing plans, we consider all consumers to have the same degree of bounded rationality, that is, \( \gamma_i = \gamma \) for both \( i = 1, 2 \). In this case, the type corresponds to the consumer’s private valuation. Given these privately observed valuations, the seller may offer a menu of pricing plans to elicit this information. Let \( \{p_{i1}, p_{i2}\} \) be the pricing for the type-i consumer, where \( p_{i1} \) is the first-period price and \( p_{i2} \) is the second-period price. Let \( u_{i1} = \mu_i - p_{i1} \) and \( u_{i2} = \mu_i - \delta_i p_{i2} \) be
the utility for a type-\(i\) consumer choosing the pricing plan designed for her. The seller’s goal is to find the optimal pricing plans, and we can write his optimization problem as:

\[
\Pi^H = \max_{\{p_1, p_2\}} \sum_i \rho_i \left[ p_{i1} \cdot q_{i1} + \delta_s p_{i2} \cdot q_{i2} \right] - c \\
\text{s.t. (PC-}\!i\!\): \quad u_{i1}q_{i1} + u_{i2}q_{i2} \geq 0, \forall i = 1, 2, \\
(\text{IC-}\!i\!, j): \quad u_{i1}q_{i1} + u_{i2}q_{i2} \geq (\mu_i - p_{j1}) \frac{1}{1 + e^{\gamma(p_{j1}-\delta_{c}p_{j2})}} + (\mu_i - \delta_{c}p_{j2}) \frac{e^{\gamma(p_{j1}-\delta_{c}p_{j2})}}{1 + e^{\gamma(p_{j1}-\delta_{c}p_{j2})}}, \forall \ i, j = 1, 2, j \neq i.
\]

The terms \(\mu_i - p_{j1}\) and \(\mu_i - \delta_{c}p_{j1}\) are the utilities of type-\(i\) consumer selecting the pricing plan designed for type-\(j\). Equations (PC-\(i\)) and (IC-\(i, j\)) are the corresponding participation and incentive compatibility constraints for type-\(i\) consumers. The result of the above optimization problem is summarized in the following proposition.

**Proposition 8.** In the case of heterogeneous valuations, the seller’s optimal strategy is to offer a single pricing plan to both types of consumers if he wants to serve the entire market.

Proposition 8 shows that the seller’s optimal strategy is to offer a single pricing plan. Recalling the probability of a consumer selecting a less favorable action is a direct result of the consumer’s degree of rationality. In our model, both types of consumers have the same degree of rationality. They make the same probabilistic purchasing choice when giving a pricing plan. Moreover, the expected utility of the high-valuation consumers is always higher than that of the low-valuation consumers by a fixed amount, namely the difference between the valuations \((\mu_1 - \mu_2)\). Thus, any pricing plan designed to induce the low-valuation consumers gives the high-valuation consumers an additional information rent of \((\mu_1 - \mu_2)\). The seller thus cannot extract more expected profit from the high-valuation consumers, and consequently he is better off providing a single pricing plan. The optimal single pricing plan is the same as the optimal pricing plan for the low-valuation consumers under the public quantal scenario. Alternatively, the seller can choose to only serve the high-valuation consumers. In this case, the private information of consumers’ valuation degenerates as well.

### 1.8 Conclusions

In this paper, we investigate the seller’s optimal design of intertemporal pricing plans while facing consumers with heterogeneous bounded rationality that arises from
consumers’ probabilistic behaviors. We show that, despite the heterogeneity among
the consumers, the seller may find it optimal to offer a single pricing plan even when
information asymmetry exists between the seller and the consumers. Moreover, the in-
tertemporal pricing pattern depends crucially on the relative time discounting. When
the seller is more (less) patient than the consumers are, he intends to induce the con-
sumers to purchase early (late) because offering favorable prices for early purchases is
less (more) costly for the seller. We further find that the seller always extracts more
expected profit from the more irrational consumers. Perhaps surprisingly, there are sit-
uations in which the seller offers specific pricing plans to different types of consumers
when the degree of bounded rationality is publicly known, but a single pricing plan is
offered instead when such information becomes unavailable. This leads to the conclusion
that the pricing plan design problem may get simplified in the presence of information
asymmetry.

Our analysis has several empirical implications. First, boundedly rational consumer
behaviors can be used to explain the phenomenon of flexible pricing plans. Propositions
1-3 predict that when a seller faces heterogenous degrees of bounded rationality and
is able to differentiate among them, he should offer sufficiently irrational consumers
flexible pricing plans. In business practices where the seller is unable to distinguish
the different types of consumers, our analysis shows that the seller need not design a
menu of pricing plans as a single contract can be optimal. Next, we show that the
optimal pricing patterns depend primarily on the relative discounting factors between
the seller and the consumers. Third, in Propositions 4-6 and the numerical studies,
we identify conditions in which the seller can offer a single pricing plan to achieve
profit maximization when he cannot differentiate consumers. These results are testable
in empirical experiments. Our extended models also show that the main insights are
robust with respect to competition and heterogenous consumer valuations. The seller
can offer similar pricing plans to consumers in a moderate competitive environment.
While he is more likely to offer pricing plans to consumers who are boundedly rational,
he needs to reduce the price dispersion to induce consumers to participate. In the
heterogeneous consumer valuation case, we show that offering a single pricing plan is
always optimal. We hope that in future research, empirical studies can be conducted to
test the relationship among sellers’ pricing decisions, consumers’ purchasing choices and
the associated profits.

Aside from empirical research, another possible extension is to apply our intertem-
poral pricing framework to selling physical products in multiple periods. In this setting,
we can give the consumers the option of not purchasing at all and this option should be
considered in each of the three periods. Thus, consumers are allowed to not signing for
the contract, exiting in the first period or exiting in the second period. As the consumers may experience different reservation utilities from exiting earlier than later, we will also need to artificially impose the probability of not purchasing. Such a relaxation of the consumers’ commitment power should be mirrored on the seller’s side as well. That is, the seller need not guarantee a delivery of the service. In the mechanism design literature, the lack of commitment power is notoriously challenging as renegotiation may take place at each period and there is no guarantee that the agents are willing to disclose their types. For more detailed discussions, please see Laffont and Tirole (1990) and Chapter 9 in Laffont and Martimort (2002). Such an analysis is thus best suitable in a dynamic framework with the seller only provides a limited capacity.

Appendix

Proofs of Propositions 1, 2 and 3.
Recall that the participation constraint can be written as

$$u_{i1} \cdot \frac{1}{1 + e^{\gamma v_i}} + u_{i2} \cdot \frac{e^{\gamma v_i}}{1 + e^{\gamma v_i}} \geq 0,$$

which, after replacing $u_{i2}$ by $v_i + u_{i1}$, can be simplified as follows: $u_{i1} + v_i \cdot \frac{e^{\gamma v_i}}{1 + e^{\gamma v_i}} \geq 0$. The seller’s problem is therefore to find a pricing plan that solves the following problem:

$$\Pi^F_i = \max_{p_{i1}, p_{i2}} \left\{ p_{i1} \cdot \frac{1}{1 + e^{\gamma v_i}} + \delta c p_{i2} \cdot \frac{e^{\gamma v_i}}{1 + e^{\gamma v_i}} - c \right\}$$

s.t. $u_{i1} + v_i \cdot \frac{e^{\gamma v_i}}{1 + e^{\gamma v_i}} \geq 0$.

Since $u_{i2} = v_i + u_{i1}$ and $\delta c p_{i2} = \delta (p_{i1} - v_i)$, for a fixed value of $v_i$, $u_{i2}$ decreases as $u_{i1}$ decreases and $p_{i2}$ increases in $p_{i1}$. Suppose that the participation constraint does not bind, the seller can decrease $u_{i1}$ while keeping $v_i$ unchanged. $u_{i2}$ decreases as a result. Since $p_{i1} = \mu - u_{i1}$ and $p_{i2} = (\mu - u_{i2})/\delta c$, both $p_{i1}$ and $p_{i2}$ increase and so does the seller’s expected profit. Thus (PC-i) binds: $u_{i1} + v_i \cdot \frac{e^{\gamma v_i}}{1 + e^{\gamma v_i}} = 0$. We can therefore represent utilities and prices in terms of $v_i$:

$$u_{i1} = -v_i \cdot \frac{e^{\gamma v_i}}{1 + e^{\gamma v_i}}, u_{i2} = u_{i1} + v_i = \frac{v_i}{1 + e^{\gamma v_i}},$$

$$p_{i1} = \mu - u_{i1} = \mu + v_i \cdot \frac{e^{\gamma v_i}}{1 + e^{\gamma v_i}},$$

$$\delta c p_{i2} = \mu - u_{i2} = \mu - \frac{v_i}{1 + e^{\gamma v_i}}.$$ (1.4)
Substituting these terms in the seller’s objective, the seller’s optimization problem becomes an unconstrained one:

$$
\Pi_i^F = \max_{v_i} \left\{ \mu \cdot \frac{1 + \delta e^{\gamma_i v_i}}{1 + e^{\gamma_i v_i}} + (1 - \delta) \frac{v_i e^{\gamma_i v_i}}{(1 + e^{\gamma_i v_i})^2} - c \right\},
$$

where the third equation follows from $\delta c_{pi2} = \mu - \frac{v_i}{1 + e^{\gamma_i v_i}}$. The first-order condition on $v_i$ leads to

$$
\frac{\partial \Pi_i^F}{\partial v_i} = (\delta - 1) \frac{e^{\gamma_i v_i}}{(1 + e^{\gamma_i v_i})^2} \left[ \gamma_i \mu - \left( 1 + \frac{\gamma_i v_i (1 - e^{\gamma_i v_i})}{1 + e^{\gamma_i v_i}} \right) \right] = 0. \quad (1.5)
$$

Let $D(v_i) = \gamma_i v_i(1 - e^{\gamma_i v_i})/(1 + e^{\gamma_i v_i})$ and observe that $D(0) = 0$ and $D(v_i)$ is negative for all other values of $v_i$. Therefore, $1 + \gamma_i v_i(1 - e^{\gamma_i v_i})/(1 + e^{\gamma_i v_i}) \leq 1$ and thus:

- When $\delta = 1$, (1.5) is satisfied for any $v_i$, thus the seller’s expected profit is $\mu - c$ regardless of the first and second period prices.

- When $\delta < 1$ and $\gamma_i > \frac{1}{\mu}$, first order derivative with respect to $v_i$ is positive for all values of $v_i$. Thus the optimal value of $v_i$ is $-\infty$, the optimal pricing plan is $\{p_{i1}^F, p_{i2}^F\} = \{\mu, \infty\}$, and the associated seller’s expected profit is $\mu - c$.

- When $\delta > 1$ and $\gamma_i > \frac{1}{\mu}$, first order derivative with respect to $v_i$ is negative for all values of $v_i$. Thus the optimal value of $v_i$ is $\infty$, the optimal pricing plan is $\{p_{i1}^F, p_{i2}^F\} = \{\infty, \delta \mu\}$, and the associated seller’s expected profit is $\delta \mu - c$.

Notice that $D(v_i)$ is symmetric with respect to the origin:

$$
D(-v_i) = \gamma_i(-v_i)(1 - e^{\gamma_i (-v_i)})/(1 + e^{\gamma_i (-v_i)}) \cdot e^{\gamma_i v_i}/e^{\gamma_i v_i} = \gamma_i v_i(1 - e^{\gamma_i v_i})/(1 + e^{\gamma_i v_i}) = D(v_i).
$$

Its derivative, $\frac{\partial D(v_i)}{\partial v_i} = \gamma_i - (2 \gamma_i v_i + e^{\gamma_i v_i})e^{\gamma_i v_i} \frac{1}{(1 + e^{\gamma_i v_i})^2}$, is negative for positive values of $v_i$ and positive for negative values of $v_i$. Thus, for all $\gamma_i < 1/\mu$, there are exactly two solutions to (1.5). Let $v_i^P > 0$ be the positive solution and $v_i^N = -v_i^P$ be the negative solution to (1.5). Now we examine the second-order conditions on $v_i$:

$$
\frac{\partial^2 \Pi_i^F}{\partial v_i^2} \bigg|_{v_i = v_i^P} = (1 - \delta) \frac{e^{\gamma_i v_i}}{(1 + e^{\gamma_i v_i})^2} \frac{\partial D(v_i)}{\partial v_i}, \quad (1.6)
$$

$$
\frac{\partial^2 \Pi_i^F}{\partial v_i^2} \bigg|_{v_i = v_i^N} = (1 - \delta) \frac{e^{\gamma_i v_i}}{(1 + e^{\gamma_i v_i})^2} \frac{\partial D(v_i)}{\partial v_i}. \quad (1.7)
$$
We see that (1.6) is negative when $\delta < 1$ and positive when $\delta > 1$ while (1.7) is negative when $\delta > 1$ and positive when $\delta < 1$. Thus $v_i^P$ is a local maximizer when $\delta < 1$ while $v_i^N$ is a local maximizer when $\delta > 1$. We now show that $\Pi_i^F$ monotonically decreases in $\gamma_i$:

$$\frac{\partial \Pi_i^F}{\partial \gamma_i} = (\delta - 1)\frac{v_i}{\gamma_i} \frac{e^{\gamma_i v_i}}{(1 + e^{\gamma_i v_i})^2} \leq 0 \text{ if } v_i > 0 \text{ and } \delta < 1,$$

$$\frac{\partial \Pi_i^F}{\partial \gamma_i} = \frac{v_i}{\gamma_i} \frac{e^{\gamma_i v_i}}{(1 + e^{\gamma_i v_i})^2} \leq 0 \text{ if } v_i < 0 \text{ and } \delta > 1.$$

Thus, there exists a cutoff type, $\gamma_{lp}$, such that the corresponding $v_{lp}(> 0)$ satisfy $\Pi_i^F(v_{lp}) = \mu - c$. When $\delta < 1$, $v_i^P$ is the global maximizer for all $\gamma_i < \gamma_{lp}$. We now look for the condition for $\gamma_{lp}$ and $v_{lp}$:

$$\Pi_i^F - (\mu - c) = (\delta - 1)\frac{e^{\gamma_i v_i}}{1 + e^{\gamma_i v_i}}(\mu - \frac{v_i}{1 + e^{\gamma_i v_i}}) = 0.$$

Thus, $\gamma_{lp}$ and $v_{lp}$ satisfy the following equations simultaneously:

$$\gamma_{lp}\mu = \frac{1 + e^{\gamma_{lp} v_{lp}}}{1 + e^{\gamma_{lp} v_{lp}}}, \mu = \frac{v_{lp}}{1 + e^{\gamma_{lp} v_{lp}}}.$$

(1.8)

Similarly, there exists a cutoff type, $\gamma_{mp}$, such that the corresponding $v_{mp}(< 0)$ satisfy $\Pi_i^F(v_{mp}) = \delta\mu - c$. When $\delta > 1$, $\gamma_{mp}$ is the global maximizer for all $\gamma_i < \gamma_{mp}$.

$$\Pi_i^F - (\delta\mu - c) = (1 - \delta)\frac{1 + e^{\gamma_i v_i}}{1 + e^{\gamma_i v_i}}(1 + \frac{\gamma_i v_i}{1 + e^{\gamma_i v_i}}) = 0.$$

Thus, $\gamma_{mp}$ and $v_{mp}$ satisfy the following equations simultaneously:

$$\gamma_{mp}\mu = \frac{1 + e^{\gamma_{mp} v_{mp}}}{1 + e^{\gamma_{mp} v_{mp}}}, \gamma_{mp}\mu = -\frac{v_{mp}}{1 + e^{\gamma_{mp} v_{mp}}}.$$

(1.9)

**Proof of Proposition 5.** First, we re-write the optimization problem as

$$\Pi^M = \max_{\{p_{i1}, p_{i2}\}} \left\{ \rho_1 \left( (\mu - u_{11}) \cdot 1_{\{u_{11} \geq u_{12}\}} + \delta (\mu - u_{12}) \cdot 1_{\{u_{11} < u_{12}\}} \right) \right. \left. + \rho_2 \left( (\mu - u_{21}) \cdot \frac{1}{1 + e^{\gamma_2 v_{12}}} + \delta (\mu - u_{22}) \cdot \frac{e^{\gamma_2 v_{12}}}{1 + e^{\gamma_2 v_{12}}} \right) - c \right\} \quad (1.10)$$

s.t. (PC-1), (PC-2), (IC-1,2), and (IC-2,1).

Notice that when $\delta < 1$, the $\{p_{i1}, p_{i2}\} = \{\mu, \infty\}, \forall \ i = 1, 2$, is a feasible solution to (1.10). The seller’s expected profit associated with this candidate solution is $\mu - c$. We thus focus on searching for a solution that yields an expected profit that is higher than $\mu - c$. Similarly, $\{p_{i1}, p_{i2}\} = \{\infty, \delta \mu\}, \forall \ i = 1, 2$, is a feasible solution to (1.10) when
\( \delta > 1 \). The seller’s expected profit associated with this candidate solution is \( \delta \mu - c \), and we search for a solution that yields a higher than \( \delta \mu - c \) expected profit. We now show that both (PC-2) and (IC-1,2) bind at optimality by contradiction. Suppose (PC-2):

\[
u_{21} + \frac{v_2 \cdot e^{\gamma v_2}}{1 + e^{\gamma v_2}} \geq 0 \]

does not bind. In this case, the seller can increase his expected profit by decreasing \( u_{21} \) and \( u_{22} \) while keeping \( v_2 \) unchanged (\( v_2 = u_{22} - u_{21} \)). This will increase the objective value without violating any constraint. Thus (PC-2) binds, i.e.,

\[
u_{21} + \frac{v_2 \cdot e^{\gamma v_2}}{1 + e^{\gamma v_2}} = 0.
\]

Given this, we can write type-2 consumers’ utilities and payments in terms of a single variable \( v_2 \):

\[
u_{21} = -v_2 e^{\gamma v_2} \left( \frac{1}{1 + e^{\gamma v_2}} \right), \quad p_{21} = \mu - u_{21} = \mu + \frac{v_2 \cdot e^{\gamma v_2}}{1 + e^{\gamma v_2}}, \quad \text{and}
\]

\[\delta_c p_{22} = \mu - u_{22} = \mu - \frac{v_2}{1 + e^{\gamma v_2}} \quad \text{and} \quad \delta_s p_{22} = \delta (\delta_c p_{22}) = \delta \mu - \frac{\delta v_2}{1 + e^{\gamma v_2}}.\]

We then consider (IC-1,2):

\[
\max\{u_{11}, u_{12}\} \geq \max\{u_{21}, u_{22}\}.
\]

This constraint must bind at optimality as well; otherwise, the seller can increase his expected profit by decreasing \( u_{11} \) and \( u_{12} \) while keeping \( v_2 \) unchanged. When \( v_2 > 0 \),

\[
\frac{v_2}{1 + e^{\gamma v_2}} > 0 > -\frac{v_2 e^{\gamma v_2}}{1 + e^{\gamma v_2}}
\]

and thus \( \max\{u_{21}, u_{22}\} = \frac{v_2}{1 + e^{\gamma v_2}} \). When \( v_2 < 0 \),

\[
0 > \frac{v_2}{1 + e^{\gamma v_2}} \quad \text{and} \quad \max\{u_{21}, u_{22}\} = \frac{-v_2 e^{\gamma v_2}}{1 + e^{\gamma v_2}}.
\]

(IC-1,2) can then be written as the following equality constraint:

\[
\max\{u_{11}, u_{12}\} = \begin{cases} \frac{v_2}{1 + e^{\gamma v_2}}, & \text{when} \ v_2 > 0, \\ \frac{-v_2 e^{\gamma v_2}}{1 + e^{\gamma v_2}}, & \text{when} \ v_2 < 0. \end{cases}
\]

In the remaining proof, we divide our analysis into two cases: 1) \( v_2 \) is positive and 2) \( v_2 \) is negative. There are two subcases in each case: \( \max\{u_{11}, u_{12}\} = u_{11} \) and \( \max\{u_{11}, u_{12}\} = u_{12} \) in each case. We now look at each of them separately:

**Case 1**: \( v_2 \) is positive.

**Subcase 1.1**: \( \max\{u_{11}, u_{12}\} = u_{11} = \frac{v_2}{1 + e^{\gamma v_2}} \). In this case, \( p_{11} = \mu - u_{11} = \mu - \frac{v_2}{1 + e^{\gamma v_2}} \). A fully rational consumer purchases in the period when the price is lower, and thus the first-period price has to be non-negative in the optimal solution; otherwise,
the seller can set the second-period price to be negative instead \( p_{12} = \mu - \frac{v_2}{1 + e^{\gamma v_2}} \)
to obtain a smaller loss. Thus, \( p_{11} \geq 0 \), i.e., \( \mu - \frac{v_2}{1 + e^{\gamma v_2}} \geq 0 \). Notice that if we set \( v_1 = v_2 \), we obtain \( u_{11} = u_{12} - v_1 = u_{12} - v_2 = u_{22} - v_2 = u_{21} \). In this case,

\[
\frac{u_{21}}{1 + e^{\gamma v_2}} + \frac{u_{22}e^{\gamma v_2}}{1 + e^{\gamma v_2}} = \frac{u_{11}}{1 + e^{\gamma v_1}} + \frac{u_{12}e^{\gamma v_1}}{1 + e^{\gamma v_1}},
\]

that is, (IC-2,1) is satisfied. The seller’s expected profit function can be written as an
unconstrained optimization function with a single variable \( (v_2) \):

\[
\Pi^S = \max_{v_2} \left\{ \rho_1 \left( \mu - \frac{v_2}{1 + e^{\gamma v_2}} \right) + \rho_2 \frac{1 + \delta e^{\gamma v_2}}{1 + e^{\gamma v_2}} \mu + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma v_2}}{(1 + e^{\gamma v_2})^2} - c \right\}. \tag{1.11}
\]

First, let us compare the seller’s expected profit with \( \mu - c \) - obtained from a candidate solution \( \{p_{i1}, p_{i2}\} = \{\mu, \infty\} \) \( \forall i = 1, 2 \) when \( \delta < 1 \):

\[
\Pi^S - (\mu - c) = \mu - \rho_1 \frac{v_2}{1 + e^{\gamma v_2}} - \rho_2 (1 - \delta) \frac{e^{\gamma v_2}}{1 + e^{\gamma v_2}} \mu + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma v_2}}{(1 + e^{\gamma v_2})^2} - \mu
\]

\[
= -\rho_1 \frac{v_2}{1 + e^{\gamma v_2}} + \rho_2 (1 - \delta) \frac{e^{\gamma v_2}}{1 + e^{\gamma v_2}} \left( \frac{v_2}{1 + e^{\gamma v_2}} - \mu \right)
\]

\[
= -\rho_1 \frac{v_2}{1 + e^{\gamma v_2}} - \rho_2 (1 - \delta) \frac{e^{\gamma v_2}}{1 + e^{\gamma v_2}} p_{11}
\]

\[
\leq -\rho_1 \frac{v_2}{1 + e^{\gamma v_2}},
\]

where the inequality follows from the fact that \( p_{11} \geq 0 \) and \( \delta < 1 \). Since \( -\rho_1 \frac{v_2}{1 + e^{\gamma v_2}} < 0 \), setting the first-period price to \( \mu - \frac{v_2}{1 + e^{\gamma v_2}} \) is never optimal when \( \delta < 1 \). Next, we compare the seller’s expected profit with \( \delta \mu - c \) - obtained from a candidate solution \( \{p_{i1}, p_{i2}\} = \{\infty, \delta \mu\} \) when \( \delta > 1 \):

\[
\Pi^S - (\delta \mu - c) \leq (1 - \delta) \frac{v_2}{1 + e^{\gamma v_2}} - \rho_2 \frac{v_2}{1 + e^{\gamma v_2}} < 0.4 \tag{1.12}
\]

\[^{4}\text{The derivation of the equation is shown below:}\]

\[
\Pi^S - (\delta \mu - c) = \rho_1 (1 - \delta) \mu + \rho_2 (1 - \delta) \mu \frac{1}{1 + e^{\gamma v_2}} - \rho_2 \frac{v_2}{1 + e^{\gamma v_2}} + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma v_2}}{(1 + e^{\gamma v_2})^2}
\]

\[
\leq \rho_1 (1 - \delta) \frac{v_2}{1 + e^{\gamma v_2}} + \rho_2 (1 - \delta) \frac{v_2}{1 + e^{\gamma v_2}} - \rho_2 \frac{v_2}{1 + e^{\gamma v_2}} + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma v_2}}{(1 + e^{\gamma v_2})^2}
\]

\[
= (1 - \delta) \frac{v_2}{1 + e^{\gamma v_2}} - \rho_2 \frac{v_2}{1 + e^{\gamma v_2}} < 0.
\]
Thus, it is optimal to set $v_2$ to infinity when the first-period price to $\mu - \frac{v_2}{1 + e^{\gamma_2 v_2}}$.

**Subcase 1.2:** $\max\{u_{11}, u_{12}\} = u_{12} = \frac{v_2}{1 + e^{\gamma_2 v_2}}$. In this case, $\delta_c p_{12} = \mu - u_{12} = \mu - \frac{v_2}{1 + e^{\gamma_2 v_2}}$. Moreover, (PC-1):

$$\max\{u_{11}, \delta_c u_{12}\} = \frac{v_2}{1 + e^{\gamma_2 v_2}} > 0$$

is satisfied as well. Substituting all the utilities and payments by the aforementioned functions of $v_2$, the seller’s expected profit function can be written as:

$$\Pi^S = \max_{v_2} \left\{ \rho_1 \delta \left( \mu - \frac{v_2}{1 + e^{\gamma_2 v_2}} \right) + \rho_2 \frac{1 + \delta e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \mu + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} - c \right\}. \quad (1.13)$$

Observe that when $\delta > 1$,

$$\Pi^S - (\delta \mu - c) = \rho_1 \delta \mu + \rho_2 \frac{1 + \delta e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \mu - \rho_1 \delta \frac{v_2}{1 + e^{\gamma_2 v_2}} + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} - \delta \mu$$

$$= \rho_2 (1 - \delta) \frac{1}{1 + e^{\gamma_2 v_2}} \mu - \rho_1 \delta \frac{v_2}{1 + e^{\gamma_2 v_2}} + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2}.$$

Since $\Pi^S - (\delta \mu - c) < 0$, it is optimal to set $v_2$ to $\infty$. When $\delta < 1$, (1.11) is greater than equation (1.13) and $\delta_c p_{12} = \mu - \frac{v_2}{1 + e^{\gamma_2 v_2}} \geq 0$, in which case setting the second-period price to $\mu - \frac{v_2}{1 + e^{\gamma_2 v_2}}$ is never optimal. When $\delta < 1$ and $\delta_c p_{12} = \mu - \frac{v_2}{1 + e^{\gamma_2 v_2}} < 0$, the first-order condition on $v_2$ yields

$$\frac{\partial \Pi^S}{\partial v_2} = \frac{1}{(1 + e^{\gamma_2 v_2})^2} \left[ -\rho_1 \delta (1 + e^{\gamma_2 v_2} - \gamma_2 v_2 e^{\gamma_2 v_2}) + \rho_2 (\delta - 1) \gamma_2 \mu e^{\gamma_2 v_2} \right] = 0. \quad (1.14)$$

We can rewrite (1.14) as follows:

$$1 + \frac{\gamma_2 v_2 (1 - e^{\gamma_2 v_2})}{1 + e^{\gamma_2 v_2}} = \gamma_2 \mu + \frac{\rho_1}{\rho_2} \frac{\delta}{1 - \delta} \frac{(1 + e^{\gamma_2 v_2} - \gamma_2 v_2 e^{\gamma_2 v_2})}{e^{\gamma_2 v_2}}. \quad (1.15)$$
Let \( v_2^P \) be the positive solution of (1.15), the second-order condition is as follows:

\[
\frac{\partial^2 \Pi^S}{\partial v_2^2} \bigg|_{v_2^P} = \frac{\gamma_2}{(1 + e^{\gamma_2 v_2^P})^2} \left[ \frac{\rho_1 \delta (1 + e^{\gamma_2 v_2^P}) + \rho_2 (1 - \delta) e^{\gamma_2 v_2^P} \frac{1 - e^{2\gamma_2 v_2^P} - 2\gamma_2 v_2^P e^{\gamma_2 v_2^P}}{(1 + e^{\gamma_2 v_2^P})^2}}{(1 + e^{\gamma_2 v_2^P})^2} \right],
\]

The equation above is only negative when \( \rho_1 \) is sufficiently small, that is if

\[
\rho_1 < \frac{(1 - \delta) e^{\gamma_2 v_2^P} \left( e^{2\gamma_2 v_2^P} + 2\gamma_2 v_2^P e^{\gamma_2 v_2^P} - 1 \right)}{\delta (1 + e^{\gamma_2 v_2^P})^3 + (1 - \delta) e^{\gamma_2 v_2^P} \left( e^{2\gamma_2 v_2^P} + 2\gamma_2 v_2^P e^{\gamma_2 v_2^P} - 1 \right)},
\]

we obtain that

\[
\frac{\partial \Pi^S}{\partial \gamma_2} = \frac{e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} \left[ -\rho_1 \delta v_2^P + \rho_2 \mu v_2 (\delta - 1) + \rho_2 (1 - \delta) v_2^P \frac{1 - e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right] < 0.
\]

The equation above shows that \( \Pi^S \) monotonically decreases in \( \gamma_i \).

**Case 2:** \( v_2 \) is negative.

**Subcase 2.1:** \( \max \{ u_{11}, \delta c u_{12} \} = u_{11} = -\frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \). With the same reasoning as Subcase 1.1, \( p_{11} = \mu - u_{11} = \mu + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \geq 0 \). The seller’s expected profit function can be written as

\[
\Pi^S = \max_{v_2} \left\{ \rho_1 \left( \mu + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right) + \rho_2 \mu \frac{1 + \delta e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} - c \right\}. \tag{1.16}
\]

Let us first compare the seller’s expected profit with \( \mu - c \) - obtained from a candidate

\[5\text{The derivation of the second-order derivative is as follows:}
\]

\[
\frac{\partial^2 \Pi^S}{\partial v_2^2} \bigg|_{v_2^P} = \frac{\gamma_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} \left[ \frac{\rho_1 \delta \gamma_2 v_2 - \rho_2 (1 - \delta) \gamma_2 \mu + \rho_2 (1 - \delta) \frac{\gamma_2 v_2}{(1 + e^{\gamma_2 v_2})^2} + 1 - e^{2\gamma_2 v_2} - 2\gamma_2 v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} \right] = \frac{\gamma_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} \left[ \frac{\rho_1 \delta \gamma_2 v_2^P + \rho_2 (1 - \delta) \left( 1 + \frac{\gamma_2 v_2^P (1 - e^{\gamma_2 v_2^P})}{1 + e^{\gamma_2 v_2^P}} + \frac{1 - e^{2\gamma_2 v_2^P} - 2\gamma_2 v_2^P e^{\gamma_2 v_2^P}}{(1 + e^{\gamma_2 v_2^P})^2} \right) \right] - \rho_2 (1 - \delta) \left( 1 + \frac{\gamma_2 v_2^P (1 - e^{\gamma_2 v_2^P})}{1 + e^{\gamma_2 v_2^P}} \right) \right] \]
solution \( \{p_1, p_2\} = \{\mu, \infty\} \) \( \forall i = 1, 2 \) when \( \delta < 1 \):

\[
\Pi^S - (\mu - c) = \rho_1 \left( \mu + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right) + \rho_2 \mu - \frac{1 + \delta_1 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} - \mu \\
= \rho_2 (\delta - 1) \frac{1}{1 + e^{\gamma_2 v_2}} + \rho_1 \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} < 0.
\]

Therefore, setting the second-period price \( v_2 \) to \( -\infty \) is optimal. We now compare the seller’s expected profit with \( \delta \mu - c \), which is obtained from the candidate solution \( \{p_1, p_2\} = \{\infty, \delta \mu\} \) \( \forall i = 1, 2 \) when \( \delta > 1 \):

\[
\Pi^S - (\delta \mu - c) = \rho_1 \left( \mu + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right) + \rho_2 \frac{1 + \delta_1 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} - \mu + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} - \delta \mu \\
= \rho_1 (1 - \delta) \mu + \rho_1 \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} + \rho_2 (1 - \delta) \frac{1}{1 + e^{\gamma_2 v_2}} \left[ \mu + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right] < 0.
\]

Thus, setting first-period price to \( \mu + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \) is never optimal.

**Subcase 2.2:** \( \max \{u_{11}, u_{12}\} = u_{12} = \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \). In this case, \( \delta \mu_{12} = \mu + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \). The seller’s optimization problem can be written as

\[
\Pi^S = \max_{v_2} \left\{ \rho_1 \delta \left( \mu + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right) + \rho_2 \mu - \frac{1 + \delta e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} - c \right\}. \quad (1.17)
\]

Observe that when \( \delta < 1 \),

\[
\Pi^S - (\mu - c) = \rho_1 \delta \mu + \rho_1 \delta \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} + \rho_2 \mu - \frac{1 + \delta e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} - \rho_1 \mu - \rho_2 \mu \\
= \rho_1 \mu (\delta - 1) + \rho_1 \delta \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} + \rho_2 (1 - \delta) \frac{e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} < 0.
\]

Thus, setting the second-period price \( v_2 \) to \( -\infty \) is optimal. When \( \delta > 1 \), The expression of \( \Pi^S \) in (1.16) is greater than that in (1.17) and \( \delta \mu_{12} = \mu + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \leq 0 \). Therefore, setting the second-period price to \( \mu + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \) is never optimal. When \( \delta > 1 \) and \( \delta \mu_{12} = \mu + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} > 0 \), the first-order condition on \( v_2 \) yields:

\[
\frac{\partial \Pi^S}{\partial v_2} = \frac{e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} \left[ \rho_1 \delta (1 + e^{\gamma_2 v_2} + \gamma_2 v_2) + \rho_2 \gamma_2 \mu (\delta - 1) \right] = 0. \quad (1.18)
\]
We can re-write (1.18) as
\[
1 + \frac{\gamma_2 v_2 (1 - e^{\gamma_2 v_2})}{1 + e^{\gamma_2 v_2}} = \gamma_2 \mu - \frac{\rho_1}{\rho_2} \frac{\delta}{1 - \delta} (1 + e^{\gamma_2 v_2} - \gamma_2 v_2 e^{\gamma_2 v_2}).
\] (1.19)

Let \( v_2^N \) be the negative solution of equation (1.19), the second-order condition is as follows:
\[
\left. \frac{\partial^2 \Pi_S}{\partial v_2^2} \right|_{v_2^N} = \frac{\gamma_2 e^{\gamma_2 v_2^N}}{(1 + e^{\gamma_2 v_2^N})^2} \left[ \rho_1 \delta \left(1 + e^{\gamma_2 v_2^N}\right) + \rho_2 (1 - \delta) \frac{1 - e^{2\gamma_2 v_2^N} - 2\gamma_2 v_2 e^{\gamma_2 v_2^N}}{(1 + e^{\gamma_2 v_2^N})^2} \right].
\] (1.20)
(1.20) is negative when \( \rho_1 \) is sufficiently small. Specifically,
\[
\rho_1 < \frac{\delta (\delta - 1) (1 - e^{2\gamma_2 v_2^N} - 2\gamma_2 v_2 e^{\gamma_2 v_2^N})}{(\delta + 1) (1 - e^{2\gamma_2 v_2^N} - 2\gamma_2 v_2 e^{\gamma_2 v_2^N})}.
\]

We now show that \( \Pi_S \) monotonically decreases in \( \gamma_i \):
\[
\frac{\partial \Pi_S}{\partial \gamma_2} = \frac{e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} \left[ \rho_1 \delta v_2 + \rho_2 \mu v_2 (\delta - 1) + \rho_2 (1 - \delta) v_2^2 \frac{1 - e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right] < 0,
\]
Finally, we can pin down the cutoff thresholds \( \rho_i^c \) and \( v_i^c \). They are the solutions that solve (1.21) and (1.22) for \( \delta < 1 \):
\[
\rho_1 \delta \left( \mu - \frac{v_2}{1 + e^{\gamma_2 v_2}} \right) + (1 - \rho_1) \left( \frac{1 + \delta e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \mu + (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} \right) = \mu, (1.21)
\]
\[
1 + \frac{\gamma_2 v_2 (1 - e^{\gamma_2 v_2})}{1 + e^{\gamma_2 v_2}} = \gamma_2 \mu + \frac{\rho_1}{\rho_2} \frac{\delta}{1 - \delta} \left(1 + e^{\gamma_2 v_2} - \gamma_2 v_2 e^{\gamma_2 v_2}\right), \] (1.22)
and for \( \delta > 1 \):
\[
\rho_1 \delta \left( \mu + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right) + (1 - \rho_1) \left( \frac{1 + \delta e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \mu + (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} \right) = \delta \mu,
\]
\[
1 + \frac{\gamma_2 v_2 (1 - e^{\gamma_2 v_2})}{1 + e^{\gamma_2 v_2}} = \gamma_2 \mu - \frac{\rho_1}{\rho_2} \frac{\delta}{1 - \delta} \left(1 + e^{\gamma_2 v_2} - \gamma_2 v_2 e^{\gamma_2 v_2}\right).
\]

**Proof of Proposition 6.** Since \( u_2 = u_{i1} + v_i \), (PC-i) and (IC-i,j) can be simplified as
follows:

(PC-i) : \( u_i + v_i \cdot \frac{e^{\gamma_i v_i}}{1 + e^{\gamma_i v_i}} \geq 0, \)

(IC-i,j) : \( u_i + v_i \cdot \frac{e^{\gamma_i v_i}}{1 + e^{\gamma_i v_i}} \geq u_j + v_j \cdot \frac{e^{\gamma_j v_j}}{1 + e^{\gamma_j v_j}}. \)

Now we show that (PC-2) and (IC-1,2) imply (PC-1):

\[
\begin{align*}
    u_{11} + v_1 \cdot \frac{e^{\gamma_1 v_1}}{1 + e^{\gamma_1 v_1}} & \geq u_{21} + v_2 \cdot \frac{e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \quad \text{(IC-1,2)} \\
    & \geq u_{21} + v_2 \cdot \frac{e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} + v_2 \left( \frac{e^{\gamma_1 v_2} - e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right) \\
    & \geq u_{21} + v_2 \cdot \frac{e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} + \frac{v_2 (e^{\gamma_1 v_2} - e^{\gamma_2 v_2})}{(1 + e^{\gamma_1 v_2})(1 + e^{\gamma_2 v_2})} \\
    & \geq u_{21} + v_2 \cdot \frac{e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}}. \quad \text{(PC-1)}
\end{align*}
\]

Recall that both (PC-2) and (IC-1,2) bind, otherwise the seller can raise prices to decrease consumers’ utilities until they bind while increasing his expected profit. Define \( F(v) = \frac{v(e^{\gamma_1 v} - e^{\gamma_2 v})}{(1 + e^{\gamma_1 v})(1 + e^{\gamma_2 v})} \), a positive function for all values of \( v \). From the binding (PC-2) and (IC-1,2), we obtain that

(PC-2) binds \( \Rightarrow u_{21} = -\frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}}, \quad u_{22} = \frac{v_2}{1 + e^{\gamma_2 v_2}}, \)
\[
    \Rightarrow p_{21} = \mu + \frac{v_2 \cdot e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}}, \quad \delta_s p_{22} = \mu - \frac{v_2}{1 + e^{\gamma_2 v_2}},
\]

(IC-1,2) binds \( \Rightarrow u_{11} + \frac{v_1 \cdot e^{\gamma_1 v_1}}{1 + e^{\gamma_1 v_1}} = u_{21} + \frac{v_2 \cdot e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}}, \)
\[
    \Rightarrow u_{11} = F(v_2) - \frac{v_1 \cdot e^{\gamma_1 v_1}}{1 + e^{\gamma_1 v_1}}, \quad u_{12} = F(v_2) + \frac{v_1}{1 + e^{\gamma_1 v_1}}, \)
\[
    \Rightarrow p_{11} = \mu - F(v_2) + \frac{v_1 \cdot e^{\gamma_1 v_1}}{1 + e^{\gamma_1 v_1}}, \quad \delta_s p_{12} = \mu - F(v_2) - \frac{v_1}{1 + e^{\gamma_1 v_1}}.
\]

Recall that the objective function is \( \max_{\{p_1, p_2\}} \sum_i \rho_i \left[ p_{1i} \cdot \frac{1}{1 + e^{\gamma_i v_i}} + \delta_s p_{12} \cdot \frac{e^{\gamma_i v_i}}{1 + e^{\gamma_i v_i}} \right] - c. \)

After substituting the prices and utilities by functions of \( \{v_i\} \)’s, we can express the
reduced program of the seller’s optimization problem as follows:

\[
\Pi^S = \max_{v_1, v_2} \left\{ \rho_1 \left( \frac{1 + \delta e^{v_1}}{1 + e^{v_1}} (\mu - F(v_2)) + (1 - \delta) \frac{v_1 e^{v_1}}{(1 + e^{v_1})^2} \right) + \rho_2 \left( \frac{1 + \delta e^{v_2}}{1 + e^{v_2}} \mu + (1 - \delta) \frac{v_2 e^{v_2}}{(1 + e^{v_2})^2} \right) - c \right\} \quad \text{s.t. (IC-2,1)}
\]

\[R_1 \equiv \frac{1 + \delta e^{v_1}}{1 + e^{v_1}} (\mu - F(v_2)) + (1 - \delta) \frac{v_1 e^{v_1}}{(1 + e^{v_1})^2} - c, R_2 \equiv \frac{1 + \delta e^{v_2}}{1 + e^{v_2}} \mu + (1 - \delta) \frac{v_2 e^{v_2}}{(1 + e^{v_2})^2} - c. \]

\(\Pi^S\) can then be written as follows:

\[
\Pi^S = \max_{v_1, v_2} \{ \rho_1 R_1 + \rho_2 R_2 \} \text{ s.t. (IC-2,1)}.
\]

Let \(R_1^\ast = \max_{v_1} \left\{ \frac{1 + \delta e^{v_1}}{1 + e^{v_1}} \mu + (1 - \delta) \frac{v_1 e^{v_1}}{(1 + e^{v_1})^2} - c \right\}\) represent the maximum expected profit extracted from type-1 consumer when the seller is able to identify the consumer’s type. Recall that \(\gamma_c\) is the cutoff value of \(\gamma\) above which the seller’s expected profit is bounded above by \(\mu - c\). Thus, \(R_1^\ast \leq \mu - c\) for all \(\gamma_1 \geq \gamma_c\). Since \(F(v_2) \geq 0\) for all values of \(v_2\), \(R_1 = R_1^\ast - \frac{1 + \delta e^{v_1}}{1 + e^{v_1}} F(v_2) \leq R_1^\ast\). Similarly, let

\[R_2^\ast = \max_{v_2} \left\{ \frac{1 + \delta e^{v_2}}{1 + e^{v_2}} \mu + (1 - \delta) \frac{v_2 e^{v_2}}{(1 + e^{v_2})^2} - c \right\}\]

represent the maximum revenue extracted from type-2 consumer when the consumer’s type is known. Since \(\gamma_2 < \gamma_c\), \(R_2^\ast > \mu\). From the above discussions, we obtain an upper bound of the excess expected profit the seller can obtain from offering a menu of contracts:

\[
\Pi^S - (\mu - c) = \max_{v_1, v_2} \{ \rho_1 R_1 + \rho_2 R_2 - (\mu - c) \} \text{ s.t. (IC-2,1)}
\]

\[
\leq \rho_1 (R_1^\ast - (\mu - c)) + (1 - \rho_1) (R_2^\ast - (\mu - c)).
\]

Notice that when \(\rho_1 = 1\), only type-1 consumers are present in the market. \(\Pi^S\) becomes \(\Pi^P\) and thus \(\Pi^S = \mu - c\). When \(\rho_1 = 0\), \(\Pi^S = R_2^\ast > \mu - c\).

Let \(\overline{\Pi^P}(\rho_1) = \rho_1 (R_1^\ast - (\mu - c)) + (1 - \rho_1) (R_2^\ast - (\mu - c))\) and \(\hat{\rho}_1 \in (0, 1)\) be the cutoff value such that \(\overline{\Pi^P}(\hat{\rho}_1) = 0\). For any value of \(\rho_1 \geq \hat{\rho}_1\), \(\Pi^P \leq \mu - c\). Thus, it is optimal for the seller to simply offer a single pricing plan \(\{p_{\hat{\rho}_1}, p_{\hat{\rho}_2}\} = \{\mu, \infty\}\).

\[\Box\]

**Proof of Proposition 7.** The participation constraint can be simplified as \(u_{i_1} + v_i \cdot \frac{e^{v_i}}{1 + e^{v_i}} \geq \mu_{i_0}\). This constraint must bind, thus we can re-write utilities and prices in
terms of $v_i$:

$$u_{i1} = -\frac{v_i \cdot e^{\gamma_i v_i}}{1 + e^{\gamma_i v_i}} + \mu + \mu_0; u_{i2} = u_{i1} + v_i = \frac{v_i}{1 + e^{\gamma_i v_i}} + \mu + \mu_0,$$

$$p_{i1} = \mu + \frac{v_i \cdot e^{\gamma_i v_i}}{1 + e^{\gamma_i v_i}} - \mu + \mu_0, \delta c p_{i2} = \mu - \frac{v_i}{1 + e^{\gamma_i v_i}} - \mu_0.$$

Substituting these terms in the seller’s objective, the seller’s optimization problem becomes an unconstrained one:

$$\Pi_i^F = \max_{v_i} \left\{ (\mu - \mu_0) \left(1 + \delta e^{\gamma_i v_i} \right) - \frac{v_i e^{\gamma_i v_i}}{(1 + e^{\gamma_i v_i})^2} - c \right\}.$$

The first-order condition on $v_i$ leads to

$$\frac{\partial \Pi_i^F}{\partial v_i} = (\delta - 1) \frac{e^{\gamma_i v_i}}{(1 + e^{\gamma_i v_i})^2} \left[ \gamma_i (\mu - \mu_0) - \left(1 + \frac{\gamma_i v_i (1 - e^{\gamma_i v_i})}{1 + e^{\gamma_i v_i}} \right) \right] = 0. \quad (1.23)$$

Notice that the above equation is the same as Equation 1.5 in the basic case if we replace $\mu - \mu_0$ by $\mu$. Thus, we can directly borrow results from the proofs of Propositions 1, 2 and 3:

- When $\delta = 1$, Equation 1.23 is satisfied for any $v_i$, thus the seller’s expected profit is $\mu - \mu_0 - c$ regardless of the first and second period prices.

- When $\delta < 1$ and $\gamma_i > \frac{1}{\mu - \mu_0}$, first order derivative with respect to $v_i$ is positive for all values of $v_i$. Thus the optimal value of $v_i$ is $-\infty$, the optimal pricing plan is $(p_{i1}^C, p_{i2}^C) = (\mu - \mu_0, \infty)$, and the associated seller’s expected profit is $\mu - \mu_0 - c$.

- When $\delta > 1$ and $\gamma_i > \frac{1}{\mu}$, first order derivative with respect to $v_i$ is negative for all values of $v_i$. Thus the optimal value of $v_i$ is $\infty$, the optimal pricing plan is $(p_{i1}^F, p_{i2}^F) = (\infty, \delta (\mu - \mu_0))$, and the associated seller’s expected profit is $\delta (\mu - \mu_0) - c$.

Let $v_i^{CP} > 0$ be the positive solution to Equation 1.23 and $v_i^{CN} = -v_i^{CP}$ be the negative solution. Use the same mathematical deduction as that in the proofs of Propositions 1, 2 and 3, $v_i^{CP}$ is a local maximizer when $\delta < 1$ while $v_i^{CN}$ is a local maximizer when $\delta > 1$. Thus, there exists a cutoff type $\gamma_{lp}^C$ such that its corresponding $v_{lp}^C$ satisfy
\( \Pi^C(v^C_{lp}) = \mu - \mu_{i0} - c \) and that \( v^C_{lp} \) is the global maximizer for all \( \gamma_i < \gamma^C_{lp} \). The cutoff values, \( \gamma_{lp} \) and \( v^C_{lp} \), satisfy the following equations simultaneously:

\[
\gamma^C_{lp} (\mu - \mu_{i0}) = 1 + \frac{e^{\gamma^C_{lp} v^C_{lp}} (1 - e^{\gamma^C_{lp} v^C_{lp}})}{1 + e^{\gamma^C_{lp} v^C_{lp}}}, \quad \mu - \mu_{i0} = \frac{v^C_{lp}}{1 + e^{\gamma^C_{lp} v^C_{lp}}}. \tag{1.24}
\]

It’s worth noting that if we replace the term \((\mu - \mu_{i0})\) with \(\mu\), Equation 1.24 is the same as Equation 1.8. This means that when the best outside option gives type-\( i \) consumer a utility of \( \mu_{i0} \), the pricing plan is the same as the case of no competition and the consumer’s valuation is \( \mu - \mu_{i0} \). Combining the two equations above, we obtain a relationship between \( \gamma^C_{lp} \) and \( v^C_{lp} \):

\[
1 + e^{\gamma^C_{lp} v^C_{lp}} + e^{\gamma^C_{lp} v^C_{lp}} (1 - e^{\gamma^C_{lp} v^C_{lp}}) = \gamma^C_{lp} v^C_{lp}. \tag{1.25}
\]

The equation above shows that \( \gamma^C_{lp} v^C_{lp} \) is constant regardless the value of \( \mu - \mu_{i0} \), thus \( v^C_{lp} \) decreases in \( \mu_{i0} \) while \( \gamma^C_{lp} \) increases in \( \mu_{i0} \). Similarly, there exists a cutoff type \( \gamma^C_{mp} \) such that the corresponding \( v^C_{mp} \) satisfy \( \Pi^C(v^C_{mp}) = \delta (\mu - \mu_{i0}) - c \) and that \( \gamma^C_{mp} \) is the global maximizer for all \( \gamma_i < \gamma^C_{mp} \). The cutoff values, \( \gamma^C_{mp} \) and \( v^C_{mp} \), satisfy the following equations simultaneously:

\[
\gamma^C_{mp} (\mu - \mu_{i0}) = 1 + \frac{e^{\gamma^C_{mp} v^C_{mp}} (1 - e^{\gamma^C_{mp} v^C_{mp}})}{1 + e^{\gamma^C_{mp} v^C_{mp}}}, \quad \gamma_{mp} (\mu - \mu_{i0}) = \frac{-\gamma^C_{mp} v^C_{mp}}{1 + e^{\gamma^C_{mp} v^C_{mp}}}. \tag{1.26}
\]

With the same reasoning as above, we can conclude that the absolute value of \( v^C_{mp} \) decreases in \( \mu_{i0} \) while \( \gamma^C_{mp} \) increases in \( \mu_{i0} \). Combine both cases we can conclude that the price dispersion decreases in \( \mu_{i0} \) while the cutoff valuations increase in \( \mu_{i0} \) increases. \( \square \)

**Proof of Proposition 8.** Before we solve the optimization problem, let us first compare the consumers’ expected utility for a given pricing plan \( \{p_1, p_2\} \). The expected utility for a low-valuation consumer is

\[
u_2 = (\mu_2 - p_1) \frac{1}{1 + e^{\gamma v}} + (\mu_2 - p_2) \frac{e^{\gamma v}}{1 + e^{\gamma v}} ,
\]

where \( v = p_1 - \delta p_2 \). The expected utility for a high-valuation consumer is

\[
u_1 = (\mu_1 - p_1) \frac{1}{1 + e^{\gamma v}} + (\mu_1 - p_2) \frac{e^{\gamma v}}{1 + e^{\gamma v}} .
\]
Let \( \mu_d = \mu_1 - \mu_2 \), the above equation can be written as
\[
\begin{align*}
  u_1 &= (\mu_2 + \mu_d - p_1) \frac{1}{1 + e^{\gamma v}} + (\mu_2 + \mu_d - p_2) \frac{e^{\gamma v}}{1 + e^{\gamma v}} = u_2 + \mu_d.
\end{align*}
\]

Due to the common degree of rationality, the probability of a consumer chooses to purchase in period 1 depends only on the adjusted price difference. A high-valuation consumer behaves the same as the low-valuation consumer does provided that the pricing plan induce both types of consumer to participate. Moreover, the expected utility of a high valuation consumer is higher than that of a low-valuation consumer by a fixed amount \((\mu_d)\) for a given pricing plan. With that insight in mind, we can move on to solve the problem. Let \( v_i = u_{i2} - u_{i1} = p_{i1} - \delta_i p_{i2} \). Since \( \mu_1 - p_{21} = (\mu_1 - \mu_2) + (\mu_2 - p_{21}) = \mu_d + u_{21} \) and \( \mu_1 - \delta_i p_{22} = (\mu_1 - \mu_2) + (\mu_2 - \delta_i p_{22}) = \mu_d + u_{22} \), (IC-1,2) can thus be simplified as
\[
\begin{align*}
  u_{11} + \frac{v_1 e^{\gamma v_1}}{1 + e^{\gamma v_1}} &\geq u_{21} + \frac{v_2 e^{\gamma v_2}}{1 + e^{\gamma v_2}} + \mu_d \quad \text{(IC-1,2),} \\
  u_{21} + \frac{v_2 e^{\gamma v_2}}{1 + e^{\gamma v_2}} &\geq u_{11} + \frac{v_1 e^{\gamma v_1}}{1 + e^{\gamma v_1}} - \mu_d \quad \text{(IC-2,1).}
\end{align*}
\]

It is clear that (PC-2) binds, and thus \( u_{21} = -\frac{v_2 e^{\gamma v_2}}{1 + e^{\gamma v_2}} \), \( u_{22} = u_{21} + v_2 = \frac{v_2}{1 + e^{\gamma v_2}} \). We can write \( p_{21} \) in terms of \( \mu_2 \) and \( v_2 \): \( p_{21} = \mu_2 + \frac{v_2 e^{\gamma v_2}}{1 + e^{\gamma v_2}} \). Similarly, \( \delta_i p_{22} \) can be written as \( \mu_d - \frac{v_2}{1 + e^{\gamma v_2}} \). Suppose (IC-1,2) binds, then \( u_{11} + \frac{v_1 e^{\gamma v_1}}{1 + e^{\gamma v_1}} = u_{21} + \frac{v_2 e^{\gamma v_2}}{1 + e^{\gamma v_2}} + \mu_d = \mu_d \), that is, \( u_{11} = \mu_d - \frac{v_1 e^{\gamma v_1}}{1 + e^{\gamma v_1}} \) and \( u_{12} = u_{11} + v_1 = \mu_d + \frac{v_1}{1 + e^{\gamma v_1}} \). Consequently, \( p_{11} \) equals \( \mu_2 + \frac{v_1 e^{\gamma v_1}}{1 + e^{\gamma v_1}} \), and \( \delta_i p_{12} \) equals \( \mu_2 - \frac{v_1}{1 + e^{\gamma v_1}} \). It is easy to verify that both (PC-1) and (IC-2,1) are satisfied:
\[
\begin{align*}
  \text{(PC-1)} \quad u_{11} + \frac{v_1 e^{\gamma v_1}}{1 + e^{\gamma v_1}} &= \mu_d \geq 0 \\
  \text{(IC-2,1)} \quad u_{21} + \frac{v_2 e^{\gamma v_2}}{1 + e^{\gamma v_2}} &= 0 \geq u_{11} + \frac{v_1 e^{\gamma v_1}}{1 + e^{\gamma v_1}} - \mu_d = 0.
\end{align*}
\]

The expected profit function can then be written as
\[
\Pi^H = \max_{v_1, v_2} \left\{ \rho_1 \left[ \left( \mu_2 + \frac{v_1 e^{\gamma v_1}}{1 + e^{\gamma v_1}} \right) \frac{1}{1 + e^{\gamma v_1}} + \delta \left( \mu_2 - \frac{v_1}{1 + e^{\gamma v_1}} \right) \frac{e^{\gamma v_1}}{1 + e^{\gamma v_1}} \right] + \right\}.
\]
The derivatives with respect to $v_1$ and $v_2$ are as follows:

\[
\frac{\partial \Pi^H}{\partial v_1} = (\delta - 1) \frac{e^{\gamma v_1}}{(1 + e^{\gamma v_1})^2} \left[ \gamma \mu_2 - \left( 1 + \frac{\gamma v_1(1 - e^{\gamma v_1})}{1 + e^{\gamma v_1}} \right) \right], \tag{1.29}
\]

\[
\frac{\partial \Pi^H}{\partial v_2} = (\delta - 1) \frac{e^{\gamma v_2}}{(1 + e^{\gamma v_2})^2} \left[ \gamma \mu_2 - \left( 1 + \frac{\gamma v_1(1 - e^{\gamma v_1})}{1 + e^{\gamma v_1}} \right) \right]. \tag{1.30}
\]

Let $v^p_1 > 0$ and $v^n_1 < 0$ be the two solutions for (1.29) and $v^p_2 > 0$ and $v^n_2 < 0$ for (1.30). Since (1.29) and (1.30) must be satisfied simultaneously, $v^p_1 = v^p_2 = v^p$ and $v^n_1 = v^n_2 = v^n$. For the expected profit function to achieve its maximum, we also need

\[
\frac{\partial^2 \Pi^H}{\partial v_1^2} < 0, \quad \frac{\partial^2 \Pi^H}{\partial v_2^2} < 0 \quad \text{and} \quad \frac{\partial^2 \Pi^H}{\partial v_1^2} \frac{\partial^2 \Pi^H}{\partial v_2^2} - \left( \frac{\partial^2 \Pi^H}{\partial v_1 \partial v_2} \right)^2 > 0,
\]

Thus, the local maximizers are $(v_1, v_2) = (v^p, v^p)$ when $\delta < 1$ and $(v_1, v_2) = (v^n, v^n)$ when $\delta > 1$.

**Case 1:** $\delta < 1$

\[
\Pi^H (v^p, v^p) = \frac{1 + \delta e^{\gamma v^p}}{1 + e^{\gamma v^p}} \mu_2 + (1 - \delta) \frac{v^p e^{\gamma v^p}}{(1 + e^{\gamma v^p})^2} - c.
\]

A candidate solution is the corner solution $(v_1, v_2) = (-\infty, -\infty)$, with a profit of $\mu_2 - c$. $\Pi^H (v^p, v^p)$ is a global maximum if it is greater than $\mu_2 - c$. We need to compare to the case where the seller only serve type-1 consumers. Here only (PC-1) needs to be satisfied; thus, $u_{11} = -\frac{v_1 e^{\gamma v_1}}{1 + e^{\gamma v_1}}$, $u_{12} = \frac{v_1}{1 + e^{\gamma v_1}}$. Let $\Pi_1$ be the seller’s expected profit, which can be written as

\[
\Pi_1(v_1) = \rho_1 \left[ \frac{1 + \delta e^{\gamma v_1}}{1 + e^{\gamma v_1}} \mu_1 + (1 - \delta) \frac{v_1 e^{\gamma v_1}}{(1 + e^{\gamma v_1})^2} \right] - c.
\]

where $v_1$ satisfies $\gamma \mu_1 = 1 + \frac{\gamma v_1(1 - e^{\gamma v_1})}{1 + e^{\gamma v_1}}$. Since $\Pi_1(v_1)$ decreases in $\rho_1$, there exists a cutoff value for $\rho_1$ such that $\Pi_1(v_1) = \Pi^H (v^p, v^p)$,

\[
\rho_1 \left[ \frac{1 + \delta e^{\gamma v_1}}{1 + e^{\gamma v_1}} \mu_1 + (1 - \delta) \frac{v_1 e^{\gamma v_1}}{(1 + e^{\gamma v_1})^2} \right] = \frac{1 + \delta e^{\gamma v^p}}{1 + e^{\gamma v^p}} \mu_2 + (1 - \delta) \frac{v^p e^{\gamma v^p}}{(1 + e^{\gamma v^p})^2} - c.
\]
To summarize, the seller is interested in serving high-valuation consumers when the proportion is large enough \((\rho_1 > \rho_1^c)\), and when the proportion of high-valuation consumers is low, the seller sets the price as if it only serves type 2 consumers.

**Case 2: \(\delta > 1\)** In this case, we obtain

\[
\Pi^H(v^n, v^n) = \frac{1 + \delta e^{\gamma v^n}}{1 + \gamma v^n} \mu_2 + (1 - \delta) \frac{\nu^P e^{\gamma v^n}}{(1 + e^{\gamma v^n})^2} - c.
\]

The corner solution \((v_1, v_2) = (\infty, \infty)\) gives rise to the expected payoff \(\Pi^H(\infty, \infty) = \delta \mu_2 - c\). Thus, \(\Pi^H(v^n, v^n)\) is a global maximum if it is greater than \(\delta \mu_2 - c\).
Chapter 2

Product line design, channel management, and consumer choice

During the times of economic downturns, luxury brands suffer enormously because the department stores cut the prices of their goods by as high as 70%. As the market bounces back, luxury brands fight to gain more control over their products by renting spaces in the department stores and expanding sales on their web stores. In this paper, we study how a seller should design the prices and qualities of products sold through his direct and indirect channels.

We show that under the revenue sharing scheme, when the consumers are sufficiently sensitive to price and quality, the seller should provide the product exclusively in the direct channel. When facing consumers who are sufficiently insensitive to price and quality, the seller is better off providing a high quality product at a premium price in the direct channel while offering a low quality product in the indirect channel. Such quality differentiation is eliminated in a profit sharing scheme. We also demonstrate that even when consumers are heterogeneous with privately observed sensitivities, offering a menu to induce self-selection may not be optimal for the seller’s profit.

2.1 Introduction

During the times of economic downturns, luxury brands suffer enormously because the department stores cut the prices of their goods by as high as 70%. With such deep

\footnote{Knowledge@Wharton (2009), The New High-end Consumer: ‘Please Put My Bottega Veneta Wallet in a Plain Bag’}
discounts, the luxury brands worry that the values of their goods decrease significantly. Furious with the department stores’ actions, manufacturers of luxury brands start to question the old sales model they have with the department stores. In the past decades, department stores purchase luxury goods at wholesale prices, divide and arrange each floor in accordance to the designers they carry, and set the prices as they see fit. As the luxury market bounces back, powerful luxury brands fight to gain more control over the way their products are presented and sold in the department stores. Leading brands such as Louis Vuitton and Prada have successfully implemented the concession agreements with department stores (Dodes and Passariello (2011)). With concessions, the manufacturers rent spaces within department stores (store-within-store), hire their own sales staff and set their own prices. They pay around 20–30% of sales to their landlords.

Aside from re-designing their indirect selling channels, luxury brands have also adopted the strategy of selling their products online. According to a Forrester Research report (Lewis et al. (2008)) published in early 2008, 94% out of the 178 luxury firms have a web site while only 32% of these web sites allow their visitors to make purchases. The research also shows that eight out of ten affluent consumers worldwide use the web daily for researching and buying luxury goods and services. Recognizing full-price buyers’ craving for the convenience of online shopping, the luxury brands are enriching their web stores to provide personalization services. To create the luxurious look and feel consumers experience in the stores, the brands have deployed rich media applications to present lifestyle imagery of the products and advanced merchandising tools to engage and encourage visitors to purchase. By adopting advanced software applications that smooth out international transactions and shipping logistics, they are also enjoying the expansion in their international customer base.

Similar structural changes occur in the travel industry as well. According to the 12th annual SITA/Airline Business IT Trends Survey, airlines intend to bring their direct sales up to 55% by 2013. In order to achieve this goal, Airlines have been investing heavily to implement new web functionalities such as online shopping tools, enabling changes and cancellations, and more sophisticated frequent flyer redemption programs. In addition, American Airlines attempts to bypass the Global Distribution System (GDS) by pushing the Direct Connect model. Even though the current system allows the airlines to set their own prices, it charges transaction fees every time a flight is booked in addition to the commission charged by the online travel agencies (OTA). Direct Connect allows the airline to avoid the transaction fee. In summary, both the manufacturers and the service providers (sellers hereafter) are moving away from the wholesale mechanism and adopting a dual-channel and a revenue sharing scheme. That is, products and services
(products hereafter) are offered in both the direct and indirect channels. The seller gets all the revenue from sales in the direct channel and a fixed proportion of revenue from sales in the indirect channel. However, he bears the production cost alone.

Now positioned with a greater control of price and quality of products and services, the next big question for a seller is how to design his product line. Specifically, when should a seller offer products in both his direct and indirect channels? How should he set the price and quality in these channels? Is price discrimination a profit maximization strategy?

To address these questions, we develop a stylized model that takes into account product line design, channel management and consumer choice. In this model, a seller intends to provide a product in both the direct and indirect channels. We first consider the revenue sharing scheme and identify quality inefficiency in this scheme. This result is consistent with recent finding in some industries. As an example, the group buying market in China has plummeted in 2011 as many buyers have experienced substandard service and received low quality products. We thus propose an alternative pricing scheme that can eliminate quality inefficiency. In the profit sharing scheme, the seller gets all the revenue from sales in the direct channel while bearing the production cost. However, he pays a proportion of the profit to the intermediary for the products sold in the indirect channel.

The consumers exhibit probabilistic behaviors in our model. Their purchasing decisions are divided into two stages. In the first stage, a consumer acquires product information (price, quality, etc.) in both the direct and indirect channels, and decides whether to make a purchase. Once the decision is made, the consumer picks either channel with a certain probability. Consumers are heterogeneous in their sensitivity to the price and quality. That is, while the decisions are strictly driven by the prices and qualities of the product for some people, others may have “hidden criteria” that influence their final decisions. As an example, suppose that a traveler finds a cheaper ticket offered by an OTA than the operating airline. She may choose to still purchase from the airline to accumulate loyalty points, which can be used to earn free tickets in the future. The same consumer may forego the chance of earning loyalty points and purchase from the OTA instead, when she needs a hotel in addition and when the OTA offers a very good air plus hotel package. In the luxury goods example, the direct channel refers to the brands’ online channel and the indirect refers to the store-within-store model. In the airlines example, the direct channel is the airline’s own website, while the indirect channel refers to the OTA’s websites. Other factors can also affect consumers’ purchasing decisions. One may prefer to go to the store to purchase a high-end handbag as she enjoys looking at the luxurious displays while receiving personalized...
We thus adopt the \textit{multinomial logit} (MNL) model to describe consumers’ probabilistic purchasing behavior over the direct and indirect channels. This framework captures the idea that consumers make probabilistic choice among all available alternatives. In addition, the model parameter is a proxy of a consumer’s sensitivity to price and quality differences across channels. We call a consumer \textit{sufficiently sensitive} when her purchasing decision is highly influenced by the price and quality differences across channels.\textsuperscript{4} If her utility from purchasing in one channel is higher, the probability that she makes the purchase in that channel is much higher. On the other hand, a \textit{sufficiently insensitive} consumer’s probability of purchasing from one channel does not vary much on price and quality differences across channel.

Initially, we look at the \textit{symmetric information} scenario, in which the seller is able to observe a consumer’s type and can offer a price-quality plan specifically designed for her type. Each plan comprises the price and quality for the product sold in the direct channel as well as those in the indirect channel. We first consider the \textit{revenue sharing} scheme. We show that when the consumer is \textit{sufficiently sensitive} to the price and quality differences across the channels, the seller should make the product only available in the direct channel. Moreover, the seller should provide the efficient quality level to the consumers while charging a full price for the product. A sufficiently sensitive consumer is less likely to purchase through the channel that gives her a lower net utility. Consequently, the benefit from setting differentiated prices and qualities across channels is not an effective strategy. This result matches with the business practice of some airlines, including Southwest and JetBlue. They do not sell through OTAs and thus only operate the direct channel. Their simple fare schemes enable them to sell exclusively on their own websites.

When the consumer is \textit{sufficiently insensitive} to the difference of the prices and qualities across channels, the seller can take advantage of the consumer’s probabilistic behaviors by offering a \textit{product variety} strategy. That is, he provides a higher quality product at a premium price in the direct channel and a low quality product at a discounted price in the indirect channel. This strategy seems to coincide with some recent phenomena in the travel industry. For example, TravelSiteCritic.com has documented that customers complain about airlines cancelling their Expedia reservations, even after Expedia claims that the reservations are “guaranteed”. Moreover, customers’ seat re-

\textsuperscript{4} The definition of sufficiently sensitive and sufficiently insensitive are provided in Section 2.4.
quests on Expedia are often not honored by the airlines. The above examples suggest that quality differentiation exists when purchasing from different channels. Our analysis provides a theoretical ground for this prevalent strategy and an intuitive explanation. Since the seller only gets a partial revenue from the indirect channel sales, it is less costly for the seller to set an appealing price. While the combined offer still encourages the consumers to decide to purchase, the seller enjoys a much higher profit on the sales in the direct channel.

Under our proposed profit sharing scheme, however, the product line strategy is substantially different. The quality inefficiency in the indirect channel is eliminated under the profit sharing scheme. Here the seller adopts a price discrimination strategy by setting a higher direct channel price. The practice of price discrimination are often observed in the form of coupons. Sellers post discount codes on websites such as Dealcatcher and Fatwallet. Currently, most of these sites get a referral commission or advertisement fee. Our result suggests that if the indirect channel is willing to take some fixed fee (which we normalize to zero) along with a proportion of the seller’s profit instead of his revenue, it will motivate the seller to provide a better quality in the indirect channel. This restores the production efficiency and leads to a simple product design strategy, as the seller can avoid the hassle of creating product lines and simply manipulates the selling prices through different channels.

Next, we turn to the asymmetric information case, in which the sensitivity is privately observed by the consumer. We consider the situation that there are two types of consumers: consumers who are sensitive to price and quality and consumers who are insensitive. If the utility received from purchasing in the direct channel is higher than that in the indirect channel, the sensitive consumers always make the purchase in the direct channel. The insensitive consumers, on the other hand, randomize their purchasing decisions. We show that at optimality it is not necessary to provide a menu of price-quality plans, and the seller can provide the same price and quality to all consumers. While the classic mechanism design literature suggests that a menu of plans that consist prices and qualities should be provided to consumers to self-select, such practice is rarely observed in the industries aforementioned wherein firms utilize different channels. Our analysis provides a concrete and practically relevant scenario for which a menu is not needed, even though the seller faces heterogeneous consumers and is unable to distinguish them via observable characteristics.

The underlying economic rationale for the abandonment of menu offering is as follows. A price and quality sensitive consumer always chooses to buy from the channel that provides her the higher utility. Using the result in the symmetric information case, the consumer gets a higher utility from purchasing a product in the indirect channel
as both the price and quality are lower. Thus, the direct channel only captures partial sales from the insensitive consumers. When the proportion of the sensitive consumers is large enough, the seller is better off providing the high product exclusively in the direct channel and charging a full-price for it. This strategy forces all consumers to purchase the high-end product. When the proportion of the sensitive consumers is small, the seller then offers a low quality product at a discounted price to attract both types of consumers. He can also enjoy the higher profit margin from the direct channel, which captures the sales from the insensitive consumers. While the structural properties of the optimal product line design are the same under the symmetric and asymmetric scenarios, the channel management strategy now depends on the consumer population in different segments. This suggests that the co-existence of sensitive and insensitive consumers leads to a materialistic change of the seller’s optimal strategy. Our analysis articulates the fundamental difference between heterogeneous valuations (commonly assumed in the product line design literature) and heterogeneous sensitivities. It may also provide an empirical implication on how the consumer composition can be inferred from the observable channel management strategy.

The remainder of this paper is organized as follows. In Section 2.3, we review the relevant literature. Section 2.3 introduces the basic model and Section 2.4 carries out the analysis under symmetric information and compares between the revenue sharing and the profit sharing schemes. In Section 2.5, we provide the findings under asymmetric information. Section 2.6 summarizes the results. All the proofs are relegated to the Appendix.

2.2 Literature Review

Our paper relates to a vast literature of product line design. Mussa and Rosen (1978), Maskin and Riley (1984) and Moorthy (1984) lay the foundation for the “second-degree price discrimination ” framework. They show that a monopoly seller achieves market segmentation and hence greater profitability by providing a “quality-differentiated spectrum of goods of the same generic type” to consumers with privately observed and heterogeneous valuations. The products are allocated to consumers by a process of self-selection, in which each consumer picks the price-quality pair that gives her the highest utility. Subsequent papers have adopted this framework to study other central issues in product line design, including package pricing (Gerstner and Hess (1987)), market positioning (Dobson and Kalish (1988)), competition (Balachander and Srinivasan (1994)), and communication strategies (Villas-Boas (2004)).
Villas-Boas (1998) studies how a manufacturer can design a product line to better influence the retailer to target the different products to the consumer segments intended by the manufacturer. He finds that the best strategy for the manufacturer is to increase the differences in the products being supplied. Our paper differs from the aforementioned papers as we assume that consumers are homogeneous in valuation but have different probabilistic purchasing behaviors. Moreover, in contrast with Villas-Boas (1998), the seller in our model controls the prices in both channels, and shares a fixed portion of revenue or profit with the intermediary. This fundamental difference in this model is intended to capture the current business practice.

In recent years, there are a number of papers that study channel management. Chiang et al. (2003) show that a manufacturer can increase channel efficiency of the traditional retail channel by introducing a direct channel. The double marginalization problem is alleviated as the existence of the direct channel forces the retailer to reduce its price while increasing sales volume. The retailer’s profit may even rise as a result. The analysis in Tsay and Agrawal (2004) provides similar insights to the use of direct channel. In addition, the authors point out that paying the retailer a commission for diverting all customers toward the direct channel can be more profitable for both manufacturer and the retailer. Cattani et al. (2006) consider a different scenario, in which a manufacturer commits to price on his web channel the same as the retail price. They find that if the web channel is relatively inconvenient and costly, the manufacturer is better off committing to such strategy while optimizing the wholesale price. However, they point out that when the cost of web channel decreases, the manufacturer has a strong incentive to abandon the equal-pricing strategy. Cai et al. (2011) consider the joint impact of exclusive channel and revenue sharing on the suppliers and retailers in a competitive market. The retailers provide complementary goods or services to the suppliers’ products. They show that a revenue sharing mechanism is necessary to sustain an equilibrium between the suppliers and the retailers. Our paper considers a profit sharing mechanism in addition to the revenue sharing mechanism. Moreover, the manufacturer or the service provider is the sole price maker in our model.

We adopt the multinomial logit (MNL) model, which is commonly used to describe consumers’ choices in the marketing literature. The MNL model is a discrete choice model that calculates the probability that a person chooses one alternative as a function of the attributes of all alternatives available. This model allows consumers’ utility to be decomposed into two components, decision variables that are explicit modeled and ones that are not modeled. Guadagni and Little (1983) show that calibrated MNL model of brand choice can be used to predict the effect of price, promotion and customer loyalties on the sales and market share. The MNL model has also been used to study market
shares of competing firms (Basuroy and Nguyen (1998)), the structure of optimal assortment (van Ryzin and Mahajan (1999)) and the effect of consumer search (Cachon et al. (2008)). In the assortment planning context that considers different brands and features, nested MNL model is used instead as it captures the heterogeneous group of products in a category and the substitution effect within a subgroup. Through an empirical analysis, Draganska and Jain (2006) use such a model for consumers’ preferences for lines and flavors. They find that consumers value line attribute more than flavor attributes in yogurt, thus making pricing product lines differently but all flavors within a line the same a profitable strategy. Since we consider offering one product in each channel in our paper, the MNL model is the most suitable choice.

2.3 The Model

We consider a stylized model in which a seller intends to offer products to consumers in two different channels, direct and indirect channels. The consumers’ purchasing decisions are probabilistic; that is, they purchase from the direct channel sometimes and from the indirect channel other times. Their decisions of which channel they purchase from depend on the price and the quality of the product as well as other factors that are not observable.

Let $q_d$ and $q_i$ represent the qualities of products offered in the direct channel and in the indirect channel respectively. The seller incurs a manufacturing cost of $q_d^2/2$ and $q_i^2/2$. A consumer with valuation $\theta$ receives a utility of $U_d$ from making a purchase in the direct channel:

$$U_d = \gamma \cdot u_d + \varepsilon_d,$$

where $u_d = q_d \theta - p_d$ accounts for the utility from the price and quality of the product, and $\varepsilon_d$ captures the additional utility the consumer receives from unobserved factors and it follows extreme value distribution. Note that $\gamma$ corresponds to the coefficient of the observed variables. The higher $\gamma$ is, the more sensitive the consumer is to changes in price and quality.

Similarly, $U_i$ represents the consumer’s utility from making a purchase in the indirect channel:

$$U_i = \gamma \cdot u_i + \varepsilon_i,$$

where $u_i = q_i \theta - p_i$, and $\varepsilon_i$ is also an extreme value variable. We further define the consumer’s utility dispersion as $v \equiv u_i - u_d = (p_d - p_i) - (q_d - q_r) \theta$. Applying the MNL
model, a consumer purchases from the direct channel with probability

$$\phi_d = \frac{e^{\gamma u_d}}{e^{\gamma u_d} + e^{\gamma u_i}} = \frac{1}{1 + e^{\gamma u_d}},$$  \hspace{1cm} (2.1)$$

and from the indirect channel with probability

$$\phi_i = \frac{e^{\gamma u_i}}{e^{\gamma u_d} + e^{\gamma u_i}} = \frac{e^{\gamma u_i}}{1 + e^{\gamma u_d}}. \hspace{1cm} (2.2)$$

Note that $\gamma$ characterizes how likely a consumer’s actual action deviates from the action she would have picked when only observed variables are considered. If $u_i > u_d$, $\phi_i$ is higher than $\phi_d$ as long as $\gamma$ is non-zero. This model thus captures the idea of “better options are chosen more often” and its behavioral foundation is provided in McKelvey and Palfrey (1995). Moreover, the higher the $\gamma$ is, the higher the probability of the consumer purchasing in the indirect channel is. When $\gamma$ is infinity, the consumer’s purchasing behavior is purely driven by utility from price and quality of the product. That is, she always purchases the product from the indirect channel.

To isolate the effect of consumers’ probabilistic behavior on seller’s product line design strategy, we assume all consumers have the same valuation ($\theta$) in our model. Once the consumer has decided to purchase from the direct channel, she picks the product that gives her the highest utility with certainty. In another word, she does not exhibit probabilistic behavior after selecting a channel. As an example, once a consumer makes the effort to go to the mall, she purchases the product that gives her the highest utility. Thus, there is no need to offer multiple products within a channel when all consumers have the same valuation. In other words, offering vertically differentiated products within a channel does not lead to any profit increase in our context.

**Information Structure.** The seller’s objective is to maximize his expected profit by offering the consumers price-quality plans. Initially, we assume that the seller is able to observe a consumer’s type, and therefore can offer the plan specifically designed for her. We label this case as the *symmetric information* scenario. In such a scenario, we consider two different schemes. The first one is the *revenue sharing* scheme: the seller gets all the revenue from sales in the direct channel and a fixed proportion of revenue from sales in the indirect channel. However, he bears the production cost alone. Note that the seller controls not only the product qualities but also the prices in both channels. This setup is consistent with the examples aforementioned in the introduction. As an example, airlines sell tickets both on their own websites and through OTA, who takes a commission on tickets sold. Retailers who post deals on Dealcatcher and Fatwallet also retain their
pricing powers while selling through indirect channels. For comparison purpose, we also look at the profit sharing scheme: the seller gets all the revenue from sales through the direct channel while bearing the production cost. He receives a proportion of the profit on the products sold in the indirect channel. In another word, the intermediary splits the revenue and the cost with the seller. Next, we turn to the case in which the seller is unable to identify the type of a specific consumer, which is referred to as the asymmetric information scenario. In the next two sections, we characterize the seller’s optimal price-quality plans for these two scenarios respectively.

2.4 Symmetric Information

In this section, we investigate the scenario in which the seller is able to identify a consumer’s type. Our goal is to characterize the optimal pricing that maximizes the seller’s expected profit. This will help us understand the effect of consumers’ probabilistic behaviors alone on the seller’s product line design.

At the product line design stage, the seller determines the price-quality plan \( \{p_d, q_d, p_i, q_i\} \) to a consumer with parameter \( \gamma \) and disclose the information to the consumer. The consumer learns that she receives a utility of \( u_d = q_d \theta - p_d \) when purchasing from the direct channel and a utility of \( u_i = q_i \theta - p_i \) when purchasing from the indirect channel. The probability that a consumer purchases from the direct and the indirect channels follow (2.1) and (2.2). In order to induce the consumer to enter the market, the seller needs to ensure that the consumer’s expected utility is at least her reservation utility. For simplicity of the analysis, we normalize her reservation utility to zero. Mathematically, this participation constraint (PC) gives rise to the following condition:

\[
 u_d \cdot \phi_d + u_i \cdot \phi_i = u_d \cdot \frac{1}{1 + e^{\gamma v}} + u_i \cdot \frac{e^{\gamma v}}{1 + e^{\gamma v}} \geq 0, \tag{PC}
\]

which, after replacing \( u_i \) by \( v + u_d \), can be simplified as follows: \( u_d + \frac{v e^{\gamma v}}{1 + e^{\gamma v}} \geq 0 \).

Now let us consider the seller’s profit maximization problem and derive the optimal price-quality plan under the profit sharing and the revenue sharing schemes.

2.4.1 Revenue Sharing Scheme

Under this scheme, the seller splits the revenue of the products with the intermediary and incurs the cost for products sold in the indirect channel. The seller gets all the
revenue of the products sold in the direct channel while being liable to the cost. We can then formulate the seller’s profit maximization problem as follows:

$$\Pi_{RS} = \max_{p_d, q_d, q_i, \phi_d, \phi_i} \left\{ \phi_d \cdot \left(p_d - q_d^2/2\right) + \phi_i \cdot \left(\delta p_i - q_i^2/2\right) \right\}$$

(2.3)

s.t. \(u_d \cdot \phi_d + u_i \cdot \phi_i \geq 0\).

The term \(\phi_d \cdot \left(p_d - q_d^2/2\right)\) in the objective function is the expected profit the seller gets from sales in the direct channel, with \(p_d - q_d^2/2\) represents the profit and \(\phi_d\) the probability that a consumer purchases from the direct channel. Similarly, the term \(\phi_i \cdot \left(\delta p_i - q_i^2/2\right)\) is the profit the seller obtains from sales in the indirect channel.

Note that if the consumer’s purchasing behavior is deterministic, then either \(\phi_d\) or \(\phi_i\) is one. Suppose that the consumer only purchases from the direct channel, the seller’s profit function becomes

$$\Pi_{RS} = \max_{p_d, q_d} \left\{ p_d - q_d^2/2 \right\}$$

s.t. \(u_d \geq 0\).

Since the objective function increases in \(p_d\) and \(u_d = q_d\theta - p_d\) needs to be non-negative, the seller should set the price to \(q_d\theta\) so that he leaves exactly the consumer’s reservation utility. We can rewrite it as a non-constrained one, \(\Pi_{RS} = \max_{q_d} \{q_d\theta - q_d^2/2\}\). The equation is concave with respect to \(q_d\) and thus setting its derivative to zero gives us the optimal solution for the quality. The optimal price is then \(p_d = \theta^2\) and the seller’s profit is \(\theta^2/2\). We label the quality \(\theta\) as the efficient quality and price \(\theta^2\) as the premium price.

**Definition 1.** We define the cutoff value of \(\gamma\) as \(\gamma_{RS}^{RS}\) and its associated utility dispersion as \(v^{RS}\). They satisfy the following equations simultaneously:

$$1 + e^{\gamma_{RS}^{RS} v^{RS}} (1 - \gamma_{RS}^{RS} v^{RS}) = 0,$$

$$\frac{\theta^2}{2} \left(1 + \delta\right) - \frac{v^{RS}}{1 + e^{\gamma_{RS}^{RS} v^{RS}}} = 0.$$

Further, let \(p_d^{RS}\) and \(q_d^{RS}\) be the optimal price and quality of the product sold in the direct channel, and \(p_i^{RS}\) and \(q_i^{RS}\) be those of the product sold in the indirect channel.

The seller’s product line design strategy is summarized below.

**Proposition 9.** Under the revenue sharing scheme, the seller should abandon the option of selling any product in the indirect channel for \(\gamma \geq \gamma_{RS}^{RS}\). Moreover, the seller should offer the efficient quality product at the premium price in the direct channel, i.e. \(q_d^{RS} = \theta\) and \(p_d^{RS} = \theta^2\). The seller’s optimal expected profit is \(\theta^2/2\).
Proposition 9 suggests that when the consumer is *sufficiently sensitive* \((\gamma \geq \gamma^{RS})\), the seller’s best strategy is to simply offer one option – selling an efficient quality product at a premium price. The intuition of this result is as follows. In order to induce a sufficiently sensitive consumer to participate, the seller must give the consumer at least her reservation payoff. When the seller intends to increase the price in the direct channel, he must lower the price in the indirect channel. This, however, makes the option of purchasing in the indirect channel more favorable. A sufficiently sensitive consumer is highly responsive to price differences across channels and thus an incremental price change in one channel triggers a significant change of the consumer’s purchasing behavior. Consequently, the seller is worse off attempting to increase the price in the direct channel. We can easily validate our result with a special case – a consumer with deterministic purchasing behavior. Her purchasing decision is purely driven by price and quality of the product offered, and thus she only purchases in the channel that gives her a higher utility. As shown before, the seller gets exactly \(\theta^2/2\). In real practice, a seller may be forced to provide products in both channels for the sake of customer satisfaction. Instead of abandoning the option of selling the product in the indirect channel, the seller may simply offer an efficient quality product at a very high price.

**Proposition 10.** Under the revenue sharing scheme, the seller should sell the products in both channels for \(\gamma < \gamma^{RS}\). Moreover, the seller should provide a higher quality for the product selling in the direct channel. Specifically, \(q^{RS}_d = \theta\) and \(q^{RS}_i = \delta \theta\). The optimal price \(\{p^{RS}_d, p^{RS}_i\}\) is type-specific and the price for the product in the direct channel is higher. That is, \(p^{RS}_d > \theta^2 > p^{RS}_i\). The seller’s expected profit is higher than \(\theta^2/2\) and it decreases in \(\gamma\).

Proposition 10 shows that when the consumer is *sufficiently insensitive* \((\gamma < \gamma^{RS})\), the seller can take advantage of her probabilistic behavior by offering different prices and qualities across channels. Our intuition would suggest that since the seller gets all the revenue from products sold in the direct channel, he should offer a low quality product at a lower price so that the consumer is induced to purchase in the direct channel. However, our result shows that the optimal strategy is exactly the opposite: the seller should provide a low quality product at a more appealing price in the indirect channel. Recall that in order to induce the consumer’s participation, the seller only needs to offer the consumer her reservation utility in expectation. Since the revenue of the product sold in the indirect channel is shared by the intermediary, it is less costly for the seller to offer a favorable price there. In turn, the seller can increase the price and quality of the product sold in the direct channel as long as the overall price-quality plan still attract the consumer to purchase. Although the consumer buys from the direct channel less
often, she pays much more whenever she does so. Consequently, the seller gets a higher expected profit. In addition, the more incentive the consumer is, the higher chance she ends up purchasing in the direct channel. Hence, a decrease in $\gamma$ results in a higher expected profit for the seller.

In summary, the complete information of a consumer’s sensitivity enables the seller to extract more expected profit only if the consumer is sufficiently insensitive. The exact analytical expression of the cutoff threshold $\gamma^{RS}$ is provided in the appendix. Next, we look at the profit sharing scheme.

2.4.2 Profit Sharing Scheme

Under this scheme, the seller splits both the revenue and the cost of the products sold in the indirect channel with the intermediary. Thus, we can formulate the seller’s profit maximization problem as follows:

\[
\Pi^{PS} = \max_{p_d, p_i, q_d, q_i} \left\{ \phi_d \cdot (p_d - q_d^2/2) + \phi_i \cdot \delta (p_i - q_i^2/2) \right\}
\]

\[
\text{s.t. } u_d \cdot \phi_d + u_i \cdot \phi_i \geq 0.
\]

The term $\phi_d \cdot (p_d - q_d^2/2)$ in the objective function is the expected profit the seller collects from sales in the direct channel, which is the same as that in the revenue sharing scheme. The term $\phi_i \cdot \delta (p_i - q_i^2/2)$ is the profit the seller collects from sales in the indirect channel.

**Definition 2.** We define the cutoff value of $\gamma$ as $\gamma^{PS}$ and its associated utility dispersion as $v^{PS}$. They satisfy the following equations simultaneously:

\[
1 + e^{\gamma^{PS}v^{PS}}(1 - \gamma^{PS}v^{PS}) = 0,
\]

\[
\frac{\theta^2}{2} - \frac{v^{PS}}{1 + e^{\gamma^{PS}v^{PS}}} = 0.
\]

Let $p_d^{PS}$ and $q_d^{PS}$ be the optimal price and quality of the product sold in the direct channel; and $p_i^{PS}$ and $q_i^{PS}$ be that of the product sold in the indirect channel.

The optimal solution to the above problem is summarized in the following proposition.

**Proposition 11.** Under the profit sharing scheme,

1. the seller should abandon the option of selling any product in the indirect channel for $\gamma \geq \gamma^{PS}$. Moreover, the seller should offer the efficient quality product at the premium price in the direct channel, i.e. $q_d^{PS} = \theta$ and $p_d^{PS} = \theta^2$. The seller’s optimal expected profit is thus $\theta^2/2$. 
2. the seller should sell the products in both channels for $\gamma < \gamma^{PS}$. Products with the same quality should be offered in both channels, that is, $q_{d}^{RS} = q_{i}^{RS} = \theta$. The optimal prices $\{p_{d}^{PS}, p_{i}^{PS}\}$ are type-specific and the direct channel price should be higher ($p_{d}^{PS} > \theta^{2} > p_{i}^{PS}$). The seller’s expected profit is higher than $\theta^{2}/2$ and it decreases in $\gamma$.

Proposition 11 demonstrates that when the consumer is sufficiently sensitive ($\gamma \geq \gamma^{PS}$), the seller’s best strategy is to simply offer one option – selling an efficient quality product at a premium price. The intuition of this result is the same as stated under the revenue sharing scheme. When the consumer is sufficiently insensitive ($\gamma < \gamma^{PS}$), the seller should offer the efficient quality product in both channels, but can set a higher price for the ones sold in the direct channel. The consumer buys from the direct channel less often but pays a much price whenever she does so. As a result, the seller gets a higher expected profit. In the next section, we compare the results of the two schemes.

### 2.4.3 Comparison

To facilitate our discussion, let us first classify consumers as follows:\footnote{The relationship between $\gamma^{PS}$ and $\gamma^{RS}$ is shown in the appendix.}

**Definition 3.** A consumer is **highly sensitive** if her $\gamma$ is weakly greater than $\gamma^{PS}$. She is **moderately sensitive** if her $\gamma$ is in between $\gamma^{PS}$ and $\gamma^{RS}$, that is, $\gamma^{RS} < \gamma < \gamma^{PS}$. She is **insensitive** if his coefficient $\gamma$ is less than $\gamma^{RS}$.

Let us first highlight the similarities in the two schemes. First, when consumers are highly sensitive, providing the efficient quality products at the premium price exclusively in the direct channel is the optimal strategy. The seller’s corresponding profit is $\theta^{2}/2$ under both schemes. Second, when consumers are insensitive, the optimal strategy for the seller is to use both direct and indirect channels. Furthermore, the seller can set a higher direct channel price to exploit consumers’ probabilistic purchasing behaviors. Third, the more insensitive the consumer is, the higher the seller’s expected profit gets.

Next, we summarize the differences under the two schemes below.

**Corollary 2.** Compared to the revenue sharing scheme,

1. the product inefficiency in the indirect channel is eliminated under the profit sharing scheme;
2. the optimal price in the direct channel is weakly higher under the profit sharing scheme ($p_{d}^{RS} \geq p_{d}^{PS}$);

3. the total profit is higher under the profit sharing scheme.

The most noticeable difference between the two schemes is that when it is beneficial to offer both channels to consumers, the seller does not differentiate the qualities of the product under the profit sharing scheme. Since the cost of the products in the indirect channel is solely borne by the seller under the revenue sharing scheme, he must offer a low quality product in that channel to achieve two goals: 1) lower the production cost in the indirect channel and 2) encourage consumers to purchase in the direct channel by offering a higher quality product. Thus, using a product variety strategy is optimal. When the intermediary shares the cost of the products sold in the indirect channel, however, offering products with a low quality in the indirect channel is no longer optimal. The seller can simply achieve the highest expected profit using a price discrimination strategy. Table 2.1 summarizes the optimal product line design and channel selection strategy.

Table 2.1: Summary of Optimal Strategies

<table>
<thead>
<tr>
<th>Sensitivity</th>
<th>highly sensitive</th>
<th>moderately sensitive</th>
<th>insensitive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue sharing</td>
<td>direct channel only</td>
<td>direct channel only</td>
<td>dual channel product variety</td>
</tr>
<tr>
<td>Profit sharing</td>
<td>direct channel only</td>
<td>dual channel price discrimination</td>
<td>dual channel price discrimination</td>
</tr>
</tbody>
</table>

The cutoff threshold under the profit sharing scenario is higher. This implies that when the intermediary splits the profit with the seller, it is more likely the seller will offer both channels. The intuition is as follows: Suppose the proportion of the revenue the seller gets is small. In this case, the seller must set a very low quality and price for the product sold in the indirect channel. Moreover, he may even get a negative profit. Such a concern no longer holds under the profit sharing scheme.

For the price comparison, we focus on the case where the consumer is highly insensitive. As shown in the appendix, the optimal price in the direct channel is higher and the cross channel price difference is higher under the profit sharing scheme. This suggests that when providing a low quality in the indirect channel with a lower price, the seller must also be careful not to set the price in the direct channel too high. Otherwise, he
may end up driving consumers away from making the purchase. However, when the same efficient quality product is offered in both channels, the seller can set a higher direct channel price to take advantage of the hidden criteria that result in purchases in the direct channel.

Finally, it is worth pointing out that the total channel profit is higher under the profit sharing scheme. This suggests that the profit sharing scheme not only eliminates inefficiency in product quality but also benefits the seller and the intermediary.

2.5 Asymmetric Information

In this section, we consider the scenario in which the seller is not able to identify a particular consumer’s type. However, from historical information, he knows the distribution of the consumers. Since information asymmetry exists between the seller and the consumers, the seller must induce the consumers to reveal their own types by offering the appropriate incentives. According to the mechanism design theory, the best strategy for the seller is to offer a menu of price-quality plans for the consumers to self-select. Potentially, the set of plans could be incredibly huge; however, the Revelation Principle allows us to focus on the menu for which a consumer simply reports her type and the seller then picks one on her behalf (Laffont and Martimort (2002)). Specifically, let us denote \( \{p_{kd}, q_{kd}, p_{ki}, q_{ki}\} \) as the price-quality plan intended for type-\( k \) consumer, where \( k = 1, 2 \).

Let \( \rho_k \) represent the proportion of type-\( k \) consumer in the market and \( \rho_1 + \rho_2 = 1 \).

To study the impact of the seller’s performance when facing consumers whose behaviors are significantly different, we make the following assumptions. Type-1 consumers are sensitive to price and quality of the product, and their decisions are exclusively driven by them. Their corresponding parameter, \( \gamma_1 \), is thus infinity. Type-2 consumers are insensitive to price and quality of the product. That is, \( \gamma_2 < \min\{\gamma^{PS}, \gamma^{RS}\} \). Let \( u_{kd} = q_{kd}\theta - p_{kd} \) and \( u_{ki} = q_{ki}\theta - p_{ki} \) be the utilities a type-\( k \) consumer receives from buying the product in the direct and indirect channels respectively. Then, \( v_k = u_{ki} - u_{kd} \) is the corresponding cross channel utility dispersion.

Since a type-1 consumer always buys from the channel that gives her the higher utility and her reservation utility is zero, the following participation constraint must hold:

\[
\max\{u_{1d}, u_{1i}\} \geq 0. \quad (\text{PC-1})
\]
A type-2 consumer's participation is then

\[ u_{2d} \cdot \phi_{2d} + u_{2i} \cdot \phi_{2i} \geq 0, \]

where \( \phi_{2d} = \frac{1}{1+e^{2v_2}} \) and \( \phi_{2i} = \frac{e^{2v_2}}{1+e^{2v_2}} \) represent the probabilities a type-2 consumer purchases in the direct and indirect channels respectively when she selects plan \( \{p_{2d}, q_{2d}, p_{2i}, q_{2i}\} \).

In the presence of information asymmetry, the seller also needs to ensure that the consumer will pick the price-quality plan designed for her rather than misrepresent herself. Mathematically, this implies that

\[
\max \{u_{1d}, u_{1i}\} \geq \max \{u_{2d}, u_{2i}\}. \quad (\text{IC-1,2})
\]

(IC-1,2) guarantees that a sensitive consumer prefers the pricing plan designed for her over that for an incentive consumer. The left-hand side corresponds to the payoff a consumer chooses the price-quality plan intended for her type while the right-hand side refers to the payoff upon misrepresentation as an insensitive consumer. Similarly,

\[
u_{2d} \cdot \phi_{2d} + u_{2i} \cdot \phi_{2i} \geq u_{1d} \cdot \phi_{1d} + u_{1i} \cdot \phi_{1i}. \quad (\text{IC-2,1})
\]

(IC-2,1) ensures that an insensitive consumer also prefers the price-quality plan designed for her. When she chooses the pricing plan intended for the sensitive consumers, she still exhibits a probabilistic purchasing behavior. The probabilities are captured by the right hand side of (IC-2,1). The terms \( \phi_{1d} = \frac{1}{1+e^{2v_1}} \) and \( \phi_{1i} = \frac{e^{2v_1}}{1+e^{2v_1}} \) represent the probability a type-2 consumer purchases in the direct and indirect channel respectively when she selects plan \( \{p_{1d}, q_{1d}, p_{1i}, q_{1i}\} \). We now move on to the seller’s objective function under both schemes.

**Revenue Sharing Scheme.** Recall that under this scheme, the seller splits the revenue with the intermediary but bears the cost of the product in the indirect channel. Thus, the seller’s profit optimization problem can be written as follows:

\[
\Pi^{RS} = \max_{p_{kd}, p_{ki}, q_{kd}, q_{ki}} \left\{ p_1 \left[1_{\{u_{1d}\geq u_{1i}\}} \cdot (p_{1d} - q_{1d}^2/2) + 1_{\{u_{1d}\leq u_{1i}\}} \cdot (\delta p_{1i} - q_{1i}^2/2)\right] + p_2 \left[\phi_{2d} \cdot (p_{2d} - q_{2d}^2/2) + \phi_{2i} \cdot (\delta p_{2i} - q_{2i}^2/2)\right] \right\}
\]

s.t. \( (\text{PC-1}), (\text{PC-2}), (\text{IC-1,2}), \) and \( (\text{IC-2,1}) \). (2.5)

The terms \( 1_{\{u_{1d}\geq u_{1i}\}} \) and \( 1_{\{u_{1d}\leq u_{1i}\}} \) are the indicators for the purchasing behavior of the sensitive consumers, and

\[
1_{\{u_{1d}\geq u_{1i}\}} \cdot (p_{1d} - q_{1d}^2/2) + 1_{\{u_{1d}\leq u_{1i}\}} \cdot (\delta p_{1i} - q_{1i}^2/2)
\]

is the seller’s expected profit from type-1 consumers. Similarly, \( \phi_{2d} \cdot (p_{2d} - q_{2d}^2/2) + \phi_{2i} \cdot (\delta p_{2i} - q_{2i}^2/2) \) represents his expected profit from type-2 consumers.
Proposition 12. In the case where the seller faces a mix of sensitive and insensitive consumers but cannot distinguish between them, it is always optimal for the seller to offer a single price-quality plan \( \{p_{RS}^{d}, q_{RS}^{d}, p_{RS}^{i}, q_{RS}^{i}\} \). Moreover, there exists a cutoff threshold \( \rho_{1}^{RS} \) such that:

1. For \( \rho_{1} > \rho_{1}^{RS} \), the seller should abandon the option of offering any product in the indirect channel. Moreover, the seller should offer the efficient quality product at the premium price in the direct channel, i.e. \( q_{d}^{PS} = \theta \) and \( p_{d}^{PS} = \theta^2 \). The seller’s optimal expected profit is thus \( \theta^2/2 \).

2. For \( \rho_{1} \leq \rho_{1}^{RS} \), the seller should offer products with different qualities. The product in the direct channel should have a higher quality, i.e., \( q_{d}^{RS} = \theta \) and \( q_{i}^{RS} = \delta \theta \). The optimal prices \( \{p_{d}^{RS}, p_{i}^{RS}\} \) are type-specific and the direct channel price should be higher \( (p_{d}^{RS} > \theta^2 > p_{i}^{RS}) \). The seller’s expected profit is higher than \( \theta^2/2 \).

According to Proposition 12, the seller is always better off offering a single pricing plan in the presence of sensitive consumers. The rationale for this result is the following. Suppose the seller designs two price-quality plans, A for sensitive consumers and B for insensitive consumers. In order to induce the insensitive consumers to participate, the seller must give them their reservation payoff. Thus, at least one of the prices in plan B is lower than \( \theta^2 \). Recall that a sensitive consumer always chooses the more favorable option with certainty. Thus, a sensitive consumer always gets a strictly positive utility from picking plan B. If the seller wants to offer a plan specific to the sensitive consumer, he needs to set one of the prices in plan A to be lower than the lower price in plan B. This is not optimal since the seller can simply offer plan B to the sensitive consumers to prevent extra loss.

Proposition 12 further shows that when there are many sensitive consumers, all the seller can do is to only offer the efficient product at the premium price. Let us consider an alternative in which he offers a single price-quality plan as we show above that a menu is never desirable from the seller’s viewpoint. Since the indirect channel price is lower, a sensitive consumer will purchase in the indirect channel. This implies that the seller must leave some information rent for the insensitive consumers. As there are plenty of sensitive consumers, the loss from giving away information rent to them outweighs the gain extracted from the insensitive ones. Consequently, the seller is better off by abandoning the option of offering any product in the indirect channel.

Profit Sharing Scheme. Under this scheme, the seller splits both the revenue and the cost of the product with the intermediary. The seller’s profit maximization problem
resembles 2.5. The only differences are replacing term $1_{\{u_{1d} < u_{1i}\}} \cdot (\delta p_{1i} - q_{1i}^2/2)$ by term $1_{\{u_{1d} < u_{1i}\}} \cdot \delta (p_{1i} - q_{1i}^2/2)$, and replacing term $\phi_{2i} \cdot (\delta p_{2i} - q_{2i}^2/2)$ by $\phi_{2i} \cdot \delta (p_{2i} - q_{2i}^2/2)$. The two terms represent the seller’s expected profit from selling products through the indirect channel to type-1 and type-2 consumers respectively. Similar to the results presented in Section 2.4, the quality inefficiency in the indirect channel is again eliminated under the profit sharing scheme. When the intermediary shares the cost of the products sold in the indirect channel, offering a low quality product in the indirect channel is not optimal as the corresponding profit is lower. The seller achieves the highest expected profit by using a price discrimination strategy. Thus, the result is quite robust under both symmetric and asymmetric information scenarios.

2.6 Conclusions

In this paper, we investigate how a seller should design the product line distribution channel when facing consumers with probabilistic purchasing behaviors. We find that under the revenue sharing scheme, the seller is better off only providing the efficient quality product at the premium price in the direct channel when the consumers are sufficiently insensitive. On the other hand, when the consumer is sufficiently insensitive cross channel utility differences, the seller should offer the efficient quality product in the direct channel and a low quality product at a discounted price in the indirect channel. The direct channel price is also higher. Since the seller only takes a portion of the revenue for sales in the indirect channel but bears the production cost, the seller deliberately creates an inefficient quality for the indirect channel. This allows him to set an appealing price to attract consumers and provide a product variety cross channels. However, such inefficiency is eliminated when the cost is also being shared by the intermediary. Thus, under the profit sharing scheme, the best strategy is using a cross channels price discrimination strategy.

We also show that despite the heterogeneity in sensitivity among the consumers, the seller may find it optimal to offer a single price-quality plan even in the presence of information asymmetry. Our analysis shows that when the sensitive consumers are mixed with insensitive consumers, the seller’s optimal strategy depends on the proportion of each type. A sensitive consumer always purchases from the channel that provide her the higher utility. Thus, when the proportion of these sensitive consumers is large enough, the seller is better off providing the efficient product exclusively in the direct channel and charge the premium price for it. When the proportion of the sensitive consumers is small, however, the seller can then offer low quality product at a discounted price to
attract both types of consumers. He can enjoy the higher profit margin from the direct channel, which captures the sales from the insensitive consumers.

Appendix

Proof of Propositions 9 and 10. Recall that the participation constraint can be written as \( u_d \cdot \frac{1}{1 + e^{\gamma v}} + u_i \cdot \frac{e^{\gamma v}}{1 + e^{\gamma v}} \geq 0 \), which, after replacing \( u_i \) by \( v + u_d \), can be simplified as follows: \( u_d + v \cdot \frac{e^{\gamma v}}{1 + e^{\gamma v}} \geq 0 \). The seller’s problem is therefore to design qualities and prices that solve the following problem:

\[
\Pi_{RS} = \max_{p_d, p_i, q_d, q_i} \left\{ \phi_d \cdot \left( p_d - q_d^2 / 2 \right) + \phi_i \cdot \left( \delta p_i - q_i^2 / 2 \right) \right\}
\]

s.t. \( u_d + v \cdot \frac{e^{\gamma v}}{1 + e^{\gamma v}} \geq 0 \).

Since \( u_i = u_d + v \) and \( p_i = p_d - v + \left(q_i - q_d \right)\), \( u_i \) increases in \( u_d \) and \( p_i \) increases in \( p_d \) for a fixed value of \( v \). Suppose that the participation constraint does not bind, the seller can decrease both \( u_d \) and \( u_i \) while keeping \( v \) unchanged. Since \( p_d = q_d \theta - u_d \) and \( p_i = q_i \theta - u_i \), both \( p_d \) and \( p_i \) increase and so does the seller’s expected profit. Thus (PC-i) binds: \( u_d + \frac{v}{1 + e^{\gamma v}} = 0 \).

We can therefore represent utilities and prices in terms of \( v \):

\[
\begin{align*}
    u_d &= -\frac{v \cdot e^{\gamma v}}{1 + e^{\gamma v}}, \quad p_d = q_d \theta - u_d = \frac{q_d \theta - v \cdot e^{\gamma v}}{1 + e^{\gamma v}}, \\
    u_i &= \frac{v}{1 + e^{\gamma v}}, \quad p_i = q_i \theta - u_i = \frac{\delta v}{1 + e^{\gamma v}}.
\end{align*}
\]

Substituting these terms in the seller’s objective, the seller’s optimization problem becomes an unconstrained one:

\[
\Pi_{RS} = \max_{v, q_d, q_i} \left\{ \frac{1}{1 + e^{\gamma v}} \left( q_d \theta + \frac{v \cdot e^{\gamma v}}{1 + e^{\gamma v}} - q_d^2 / 2 \right) + \frac{e^{\gamma v}}{1 + e^{\gamma v}} \left( \delta q_i \theta - \frac{\delta v}{1 + e^{\gamma v}} - q_i^2 / 2 \right) \right\}.
\]

The first-order condition on \( q_d \) leads to

\[
\frac{\partial \Pi_{RS}}{\partial q_d} = \frac{1}{1 + e^{\gamma v}} \left( \theta - q_d \right) = 0 \Rightarrow q_d = \theta.
\]

Similarly, the first-order condition on \( q_i \) leads to

\[
\frac{\partial \Pi_{RS}}{\partial q_i} = \frac{e^{\gamma v}}{1 + e^{\gamma v}} \left( \delta \theta - q_i \right) = 0 \Rightarrow q_i = \delta \theta.
\]
Next, we substitute \( q_d = \theta \) and \( q_i = \delta \theta \) in the seller’s objective, the seller’s optimization problem then becomes:

\[
\Pi^{RS} = \max_v \left\{ \frac{1}{1 + e^{\gamma v}} \left( \frac{\theta^2}{2} + v \cdot e^{\gamma v} \right) + \frac{e^{\gamma v}}{1 + e^{\gamma v}} \left( \frac{\delta^2 \theta^2}{2} - \frac{\delta v}{1 + e^{\gamma v}} \right) \right\}
\]

The first-order condition on \( v \) yields

\[
\frac{\partial \Pi^{RS}}{\partial v} = (1 - \delta) \cdot e^{\gamma v} \left[ 1 + \frac{\gamma v(1 - e^{\gamma v})}{1 + e^{\gamma v}} \right] - \gamma \frac{\theta^2}{2} (1 + \delta) = 0. \tag{2.6}
\]

Let \( D(v) = \gamma v(1 - e^{\gamma v})/(1 + e^{\gamma v}) \) and observe that \( D(0) = 0 \) and \( D(v) \) is negative for all other values of \( v \). Therefore, \( 1 + \gamma v(1 - e^{\gamma v})/(1 + e^{\gamma v}) \leq 1 \). Since \( \delta < 1 \), the first order derivative with respect to \( v \) is positive for all values of \( v \) when \( \gamma > 2/(1 + \delta)/\theta^2 \). Thus the optimal value of \( v \) is \(-\infty\), and the optimal pricing plan is \( \{p^{RS}_d, p^{RS}_i\} = \{\theta^2, \infty\} \).

The associated seller’s expected profit is \( \theta^2/2 \).

Notice that \( D(v) \) is symmetric with respect to the origin:

\[
D(-v) = \gamma(-v)(1 - e^{\gamma(-v)})/(1 + e^{\gamma(-v)}) \cdot e^{\gamma v}/e^{\gamma v} = \gamma v(1 - e^{\gamma v})/(1 + e^{\gamma v}) = D(v).
\]

Its derivative, \( \frac{\partial D(v)}{\partial v} = \gamma \frac{1 - (2\gamma v + e^{\gamma v})e^{\gamma v}}{(1 + e^{\gamma v})^2} \), is negative for positive values of \( v \) and positive for negative values of \( v \). Thus, for all \( \gamma < 2/(1 + \delta)/\theta^2 \), there are exactly two solutions to (2.6). Let \( v^P \) be the positive solution and \( v^N = -v^P \) be the negative solution to (2.6). Now we examine the second-order condition on \( v \):

\[
\frac{\partial^2 \Pi^{RS}}{\partial v^2} = (1 - \delta) \frac{e^{\gamma v}}{(1 + e^{\gamma v})^2} \frac{\partial D(v)}{\partial v}. \tag{2.7}
\]

We see that (2.7) is negative when \( v = v^P \), and thus \( v^P \) is a local maximizer.

We now show that \( \Pi^{RS} \) monotonically decreases in \( \gamma \):

\[
\frac{\partial \Pi^{RS}}{\partial \gamma} = (1 - \delta) \frac{e^{\gamma v}}{(1 + e^{\gamma v})^2} \left( v^2 \frac{1 - e^{\gamma v}}{1 + e^{\gamma v}} - \frac{\theta^2}{2} (1 + \delta) \right) \leq 0 \text{ when } v > 0.
\]
Thus, there exists a cutoff type, $\gamma_{RS}$, such that the corresponding $v_{RS}(\gamma_{RS}) = \theta^2/2$. When $\delta < 1$, $v_{RS}$ is the global maximizer for all $\gamma < \gamma_{RS}$. We now look for the condition for $\gamma_{RS}$ and $v_{RS}$:

$$\Pi_{RS}^\theta = \frac{\theta^2}{2} = (1 - \delta) \frac{e^{\gamma v}}{1 + e^{\gamma v}} \left( \frac{v}{1 + e^{\gamma v}} - \frac{\theta^2}{2} (1 + \delta) \right) = 0.$$  

The cutoff values $\gamma_{RS}$ and $v_{RS}$ satisfy the following equations simultaneously:

$$\gamma_{RS} \frac{\theta^2}{2} (1 + \delta) = 1 + \frac{e^{\gamma_{RS} v_{RS}} (1 - e^{\gamma_{RS} v_{RS}})}{1 + e^{\gamma_{RS} v_{RS}}}, \quad \frac{\theta^2}{2} (1 + \delta) = \frac{v_{RS}}{1 + e^{\gamma_{RS} v_{RS}}}.$$  

(2.8)

\textbf{Proof of Propositions 11.} The seller’s problem is to design qualities and pricing that solve the following problem:

$$\Pi_{PS} = \max_{p_d, p_i, d_i, q_i} \{ \phi_d \cdot (p_d - q_d^2/2) + \phi_i \cdot (p_i - q_i^2/2) \}$$

s.t. $u_d + v \cdot \frac{e^{\gamma v}}{1 + e^{\gamma v}} \geq 0$.

Since $u_i = u_d + v$ and $p_i = p_d - v + (q_i - q_d)$, $u_i$ increases in $u_d$ and $p_i$ increases in $p_d$ for a fixed value of $v$. Suppose that the participation constraint does not bind, the seller can decrease $u_d$ while keeping $v$ unchanged. $u_i$ decreases as a result. Since $p_d = q_d \theta - u_d$ and $p_i = q_i \theta - u_i$, both $p_d$ and $p_i$ increase and so does the seller’s expected profit. Thus (PC-i) binds: $u_d + \frac{v \cdot e^{\gamma v}}{1 + e^{\gamma v}} = 0$. We can therefore represent utilities and prices in terms of $v$:

$$u_d = \frac{v \cdot e^{\gamma v}}{1 + e^{\gamma v}}, \quad p_d = q_d \theta - u_d = q_d \theta + \frac{v \cdot e^{\gamma v}}{1 + e^{\gamma v}},$$

$$u_i = \frac{v}{1 + e^{\gamma v}}, \quad p_i = q_i \theta - u_i = q_i \theta - \frac{v}{1 + e^{\gamma v}}.$$  

Substituting these terms in the seller’s objective, the seller’s optimization problem becomes an unconstrained one:

$$\Pi_{PS} = \max_{v, q_d, q_i} \left\{ \frac{1}{1 + e^{\gamma v}} (q_d \theta + \frac{v \cdot e^{\gamma v}}{1 + e^{\gamma v}} - q_d^2/2) + \frac{e^{\gamma v}}{1 + e^{\gamma v}} \delta (q_i \theta - \frac{v}{1 + e^{\gamma v}} - q_i^2/2) \right\}.$$  

The first-order condition on $q_d$ leads to

$$\frac{\partial \Pi_{PS}}{\partial q_d} = \frac{1}{1 + e^{\gamma v}} (\theta - q_d) = 0 \Rightarrow q_d = \theta.$$
Similarly, the first-order condition on \( q_i \) leads to
\[
\frac{\partial \Pi_{PS}}{\partial q_i} = \frac{e^{\gamma v}}{1 + e^{\gamma v}} \delta (\theta - q_i) = 0 \Rightarrow q_i = \theta.
\]

Next, we substitute \( q_d = q_i = \theta \) in the seller’s objective, the seller’s optimization problem then becomes:
\[
\Pi_{PS} = \max_v \left\{ \frac{1}{1 + e^{\gamma v}} \left( \frac{\theta^2}{2} + v \cdot e^{\gamma v} \right) + \frac{e^{\gamma v}}{1 + e^{\gamma v}} \delta \left( \frac{\theta^2}{2} - \frac{v}{1 + e^{\gamma v}} \right) \right\}
\]
\[
= \max_v \left\{ \frac{\theta^2}{2} \frac{1 + \delta e^{\gamma v}}{1 + e^{\gamma v}} + (1 - \delta) \frac{v \cdot e^{\gamma v}}{(1 + e^{\gamma v})^2} \right\}.
\]
The first-order condition on \( v \) yields
\[
\frac{\partial \Pi_{PS}}{\partial v} = (1 - \delta) \cdot \frac{e^{\gamma v}}{(1 + e^{\gamma v})^2} \left[ \left( 1 + \frac{\gamma v(1 - e^{\gamma v})}{1 + e^{\gamma v}} \right) - \gamma \frac{\theta^2}{2} \right] = 0. \tag{2.9}
\]
Let \( D(v) = \gamma v(1 - e^{\gamma v})/(1 + e^{\gamma v}) \) and observe that \( D(0) = 0 \) and \( D(v) \) is negative for all other values of \( v \). Therefore, \( 1 + \gamma v(1 - e^{\gamma v})/(1 + e^{\gamma v}) \leq 1 \). Since \( \delta < 1 \), the first order derivative with respect to \( v \) is positive for all values of \( v \) when \( \gamma > 2/\theta^2 \). Thus the optimal value of \( v \) is \(-\infty\), and the optimal pricing plan is \( \{ p_{d}^{PS}, p_{i}^{PS} \} = \{ \theta^2, \infty \} \). The associated seller’s expected profit is \( \theta^2/2 \).

Notice that \( D(v) \) is symmetric with respect to the origin:

\[
D(-v) = \gamma(-v)(1 - e^{\gamma(-v)})/(1 + e^{\gamma(-v)}) \cdot e^{\gamma v}/e^{\gamma v}
\]
\[
= \gamma v(1 - e^{\gamma v})/(1 + e^{\gamma v}) = D(v).
\]

Its derivative, \( \frac{\partial D(v)}{\partial v} = \gamma v(1 - e^{\gamma v})/(1 + e^{\gamma v})^2 \), is negative for positive values of \( v \) and positive for negative values of \( v \). Thus, for all \( \gamma < 2/\theta^2 \), there are exactly two solutions to (2.9). Let \( v^P \) be the positive solution and \( v^N = -v^P \) be the negative solution to (2.9).

Now we examine the second-order conditions on \( v \):
\[
\frac{\partial^2 \Pi_{PS}}{\partial v^2} = (1 - \delta) \frac{e^{\gamma v}}{(1 + e^{\gamma v})^2} \frac{\partial D(v)}{\partial v}. \tag{2.10}
\]

We see that (2.10) is negative when \( v = v^P \), and thus \( v^P \) is a local maximizer. We now show that \( \Pi_{PS}^{R} \) monotonically decreases in \( \gamma \):
\[
\frac{\partial \Pi_{PS}^{R}}{\partial \gamma} = (1 - \delta) \frac{e^{\gamma v}}{(1 + e^{\gamma v})^2} \left( v^2 \frac{1 - e^{\gamma v}}{1 + e^{\gamma v}} - \frac{\theta^2}{2} \right) \leq 0 \text{ when } v > 0.
\]
Thus, there exists a cutoff type, $\gamma^{PS}$, such that the corresponding $v^{PS}(>0)$ satisfy $\Pi^{PS}(v^{PS}) = \theta^2/2$. When $\delta < 1$, $v$ is the global maximizer for all $\gamma < \gamma^{PS}$. We now look for the condition for $\gamma^{PS}$ and $v^{PS}$:

$$\Pi^{PS} - \frac{\theta^2}{2} = (1 - \delta) \frac{e^{\gamma v}}{1 + e^{\gamma v}} \left( \frac{v}{1 + e^{\gamma v}} - \frac{\theta^2}{2}(1 + \delta) \right) = 0.$$ 

Thus, $\gamma^{PS}$ and $v^{PS}$ satisfy the following equations simultaneously:

$$\gamma^{PS} \frac{\theta^2}{2} = 1 + \frac{e^{\gamma^{PS} v^{PS}} (1 - e^{\gamma^{PS} v^{PS}})}{1 + e^{\gamma^{PS} v^{PS}}}, \quad \frac{\theta^2}{2} = \frac{v^{PS}}{1 + e^{\gamma^{PS} v^{PS}}}.$$  

(2.11)

**Proof of Definition 3.** Let us first show that $\gamma^{PS} \geq \gamma^{RS}$. Recall that the cutoff values $\gamma^{RS}$ and $v^{RS}$ satisfy the following two equations

$$1 + e^{\gamma^{RS} v^{RS}} (1 - \gamma^{RS} v^{RS}) = 0, \quad (1 + \delta) \frac{\theta^2}{2} = \frac{v^{RS}}{1 + e^{\gamma^{RS} v^{RS}}}. \quad (2.12)$$

while the cutoff values $\gamma^{PS}$ and $v^{PS}$ satisfy the following two equations

$$1 + e^{\gamma^{PS} v^{PS}} (1 - \gamma^{PS} v^{PS}) = 0, \quad \frac{\theta^2}{2} = \frac{v^{PS}}{1 + e^{\gamma^{PS} v^{PS}}}.$$  

(2.13)

Notice that the first equation in 2.12 and that in 2.13 are the same. We can thus conclude that $\gamma^{RS} v^{RS} = \gamma^{PS} v^{PS}$. Since the left hand side of the second equation in 2.12 is higher than that in 2.13, $v^{RS}$ is then higher than $v^{PS}$. As a result, $\gamma^{RS} < \gamma^{PS}$. \qed

**Proof of Corollary 2.** First, we show the direct channel price under the profit sharing scheme is weakly higher. When $\gamma > \gamma^{PS}$, the optimal direct channel prices under both schemes are $\theta^2$. When $\gamma^{PS} > \gamma > \gamma^{RS}$, the optimal direct channel price under the revenue sharing scheme is $\theta^2$, but that under the price sharing scheme is $p^{PS}_d = \theta^2 + \frac{v^{PS} e^{\gamma v}}{1 + e^{\gamma v}} > \theta^2$. When $\gamma < \gamma^{RS}$, the direct channel price under the revenue sharing scheme is $p^{RS}_d = \theta^2 + \frac{v^{RS} e^{\gamma v}}{1 + e^{\gamma v}}$. Recall that the seller’s profit function’s first-order condition on $\tilde{v}$ requires

$$1 + \frac{\gamma \tilde{v}(1 - e^{\gamma \tilde{v}})}{1 + e^{\gamma \tilde{v}}} = \gamma \frac{\theta^2}{2}(1 + \delta).$$

Similarly, the optimal direct channel price under the profit sharing scheme is $p^{PS}_d = \theta^2 + \frac{v^{PS} e^{\gamma v}}{1 + e^{\gamma v}}$. The seller’s profit function’s first-order condition on $v$ requires

$$1 + \frac{\gamma v(1 - e^{\gamma v})}{1 + e^{\gamma v}} = \gamma \frac{\theta^2}{2}. $$
Since the left side of the above functions are decreasing in $v$ and $\tilde{v}$ respectively, we can conclude that $v > \tilde{v}$ and consequently $p_d^{PS} > p_d^{RS}$.

Next, we compare the total profit under the two schemes. Let $\Pi_{RS}^t$ represent the total profit under the revenue sharing scheme.

\[
\Pi_{RS}^t = \phi_d \cdot (p_d - q_d^2/2) + \phi_i \cdot (p_i - q_i^2/2)
\]

\[
= \frac{1}{1 + e^{\gamma v}} \left( \frac{\theta^2}{2} + \frac{v \cdot e^{\gamma v}}{1 + e^{\gamma v}} \right) + \frac{e^{\gamma v}}{1 + e^{\gamma v}} \left( \delta \theta^2 - \frac{v}{1 + e^{\gamma v}} - \frac{\delta^2 \theta^2}{2} \right)
\]

\[
= \frac{\theta^2}{2} \left( \frac{1}{1 + e^{\gamma v}} + \frac{e^{\gamma v}}{1 + e^{\gamma v}} \left( \frac{\delta}{2} - \frac{\delta^2}{2} \right) \right) \leq \frac{\theta^2}{2}.
\]

Let $\Pi_{PS}^t$ represent the total profit under the profit sharing scheme.

\[
\Pi_{PS}^t = \phi_d \cdot (p_d - q_d^2/2) + \phi_i \cdot (p_i - q_i^2/2)
\]

\[
= \frac{1}{1 + e^{\gamma v}} \left( \frac{\theta^2}{2} + \frac{v \cdot e^{\gamma v}}{1 + e^{\gamma v}} \right) + \frac{e^{\gamma v}}{1 + e^{\gamma v}} \left( \frac{\theta^2}{2} - \frac{v}{1 + e^{\gamma v}} \right)
\]

\[
= \frac{\theta^2}{2}
\]

Thus, the total profit is higher under the profit sharing scheme.

**Proof of Proposition 12.** First, we re-write the optimization problem as

\[
\Pi^{RS} = \max_{p_d, p_i, q_d, q_i} \left\{ \rho_1 \left[ 1_{\{u_{d2} \geq u_{i2}\}} \cdot (p_d - q_d^2/2) + 1_{\{u_{i1} < u_{i2}\}} \cdot (\delta p_i - q_i^2/2) \right] \right. \\
\left. + \rho_2 \left[ \phi_{2d} \cdot (p_d - q_d^2/2) + \phi_{2i} \cdot (\delta p_i - q_i^2/2) \right] \right\}
\]

s.t. (PC-1), (PC-2), (IC-1,2), and (IC-2,1). (2.14)

Notice that $\{p_{kd}, q_{kd}, p_{ki}, q_{ki}\} = \{\theta^2, \theta, \infty, 0\}$, $\forall k = 1, 2$, is a feasible solution to (2.14). The seller’s expected profit associated with this candidate solution is $\theta^2/2$. We thus focus on searching for a solution that yields an expected profit that is higher than $\theta^2/2$. We now show that both (PC-2) and (IC-1,2) bind at optimality by contradiction. Suppose (PC-2) $u_{d2} + \frac{v_2 \cdot e^{\gamma v_2}}{1 + e^{\gamma v_2}} \geq 0$ does not bind. In this case, the seller can increase his expected profit by decreasing $u_{d2}$ and $u_{i2}$ while keeping $v_2$ unchanged ($v_2 = u_{i2} - u_{d2}$). This will increase the objective value without violating any constraint. Thus (PC-2) binds, i.e., $u_{d2} + \frac{v_2 \cdot e^{\gamma v_2}}{1 + e^{\gamma v_2}} = 0$. Given this, we can write type-2 consumers’ utilities and payments
in terms of a single variable $v_2$:

$$u_{2d} = \frac{-v_2 e^{7v_2}}{1 + e^{7v_2}}, \quad p_{2d} = q_{2d} - u_{2d} = q_{2d} + \frac{v_2 \cdot e^{7v_2}}{1 + e^{7v_2}},$$

$$u_{2i} = \frac{v_2}{1 + e^{7v_2}}, \quad p_{2i} = q_{2i} - u_{2i} = q_{2i} - \frac{v_2}{1 + e^{7v_2}}.$$  

We then consider (IC-1,2):

$$\max\{u_{1d}, u_{1i}\} \geq \max\{u_{2d}, u_{2i}\}.$$  

This constraint must bind at optimality as well; otherwise, the seller can increase his expected profit by decreasing $u_{1d}$ and $u_{1i}$ while keeping $v_1$ unchanged. When $v_2 > 0$, $\frac{v_2}{1 + e^{7v_2}} > 0 > -\frac{v_2 e^{7v_2}}{1 + e^{7v_2}}$ and thus $\max\{u_{2d}, u_{2i}\} = \frac{v_2}{1 + e^{7v_2}}$ when $v_2 < 0$, $-\frac{v_2 e^{7v_2}}{1 + e^{7v_2}} > 0 > \frac{v_2}{1 + e^{7v_2}}$ and thus $\max\{u_{2d}, u_{2i}\} = -\frac{v_2 e^{7v_2}}{1 + e^{7v_2}}$. (IC-1,2) can then be written as the following equality constraint:

$$\max\{u_{1d}, u_{1i}\} = \begin{cases} \frac{v_2}{1 + e^{7v_2}}, & \text{when } v_2 > 0, \\ \frac{v_2}{1 + e^{7v_2}}, & \text{when } v_2 < 0. \end{cases}$$

In the remaining proof, we divide our analysis into two cases: 1) $v_2$ is positive and 2) $v_2$ is negative. There are two subcases in each case: $\max\{u_{1d}, u_{1i}\} = u_{1i}$ and $\max\{u_{1d}, u_{1i}\} = u_{1i}$ in each case. We now look at each of them separately:

**Case 1:** $v_2$ is positive.

**Subcase 1.1:** $\max\{u_{1d}, u_{1i}\} = u_{1d} = \frac{v_2}{1 + e^{7v_2}}$. In this case, $p_{1d} = q_{1d} - u_{1d} = q_{1d} - \frac{v_2}{1 + e^{7v_2}}$. A fully rational consumer purchases in the channel that has the lower price, and thus the direct channel price must be non-negative in the optimal solution; otherwise, the seller can set the indirect channel price to be negative instead $p_{1i} = q_{1i} - \frac{v_2}{1 + e^{7v_2}}$ to obtain a smaller loss. Thus, $p_{1d} \geq 0$, i.e., $q_{1d} - \frac{v_2}{1 + e^{7v_2}} \geq 0$. Notice that if we set $v_1 = v_2$, we obtain $u_{1d} = u_{1i} - v_1 = u_{1i} - v_2 = u_{2i} = v_2$ in the channel that has the lower price. In this case,

$$\frac{u_{2d}}{1 + e^{7v_2}} + \frac{u_{2i} e^{7v_2}}{1 + e^{7v_2}} = \frac{u_{1d}}{1 + e^{7v_2}} + \frac{u_{1i} e^{7v_1}}{1 + e^{7v_1}}.$$  

That is, (IC-2,1) is satisfied. The seller’s expected profit function can be written as an unconstrained optimization function with a single variable ($v_2$):

$$\Pi_{RS} = \max_{v_2, q_{2d}, q_{2i}} \left\{ \rho_2 \left[ \frac{1}{1 + e^{7v_2}} (q_{2d} + \frac{\rho_1}{1 + e^{7v_2}} (q_{2i} - q_{2d}^2/2) + \frac{\delta q_{2i} - \delta v_2}{1 + e^{7v_2}} (q_{2i}^2/2) \right) \right\}.$$
The first-order condition on $q_d$ leads to

$$
\frac{\partial \Pi^{RS}}{\partial q_d} = \left( \rho_1 + \frac{\rho_2}{1 + e^{\gamma v}} \right) (\theta - q_d) = 0 \Rightarrow q_d = \theta.
$$

Similarly, the first order condition on $q_i$ leads to

$$
\frac{\partial \Pi^{RS}}{\partial q_i} = \rho_2 \frac{e^{\gamma v}}{1 + e^{\gamma v}} (\delta \theta - q_i) = 0 \Rightarrow q_i = \delta \theta.
$$

First, let us compare the seller’s expected profit with $\theta^2 / 2$ – obtained from a candidate solution $\{p_{kd}, p_{ki}\} = \{\theta^2, \infty\}$ $\forall$ $k = 1, 2$:

$$
\Pi^{RS} - \frac{\theta^2}{2}
= \rho_1 \frac{\theta^2}{2} - \rho_1 \frac{v_2}{1 + e^{\gamma v^2}} + \rho_2 \frac{1}{1 + e^{\gamma v^2}} \left( \frac{\theta^2}{2} + \frac{v_2}{1 + e^{\gamma v^2}} \right) + \rho_2 \frac{e^{\gamma v^2}}{1 + e^{\gamma v^2}} \left( \delta \frac{\theta^2}{2} - \frac{\delta v_2}{1 + e^{\gamma v^2}} \right) - \frac{\theta^2}{2}
= -\rho_2 \frac{e^{\gamma v^2}}{1 + e^{\gamma v^2}} (1 - \delta^2) - \rho_1 \frac{v_2}{1 + e^{\gamma v^2}} + \rho_2 \frac{v_2}{1 + e^{\gamma v^2}} \left( \frac{1}{1 + e^{\gamma v^2}} - \delta e^{\gamma v^2} \right) < 0
$$

Thus, setting the first-period price to $q_d \theta - \frac{v_2}{1 + e^{\gamma v^2}}$ is never optimal.

**Subcase 1.2**: $\max\{u_{1d}, u_{1i}\} = u_{1i} = \frac{v_2}{1 + e^{\gamma v^2}}$. In this case, $p_{1i} = q_i \theta - u_{1i} = q_i \theta - \frac{v_2}{1 + e^{\gamma v^2}}$. Moreover, (PC-1):

$$
\max\{u_{1d}, u_{1i}\} = \frac{v_2}{1 + e^{\gamma v^2}} > 0
$$

is satisfied as well. Substituting all the utilities and payments by the aforementioned functions of $v_2$, the seller’s expected profit function can be written as:

$$
\Pi^{RS} = \max_{v_2, q_{d/q_i}} \left\{ \rho_2 \left[ \frac{1}{1 + e^{\gamma v^2}} \left( q_{2d} \theta + \frac{v_2}{1 + e^{\gamma v^2}} - q_{2d}^2 / 2 \right) + \frac{e^{\gamma v^2}}{1 + e^{\gamma v^2}} \left( \delta q_{2i} \theta - \frac{\delta v_2}{1 + e^{\gamma v^2}} - q_{2i}^2 / 2 \right) \right] \right\}.
$$

The first-order condition on $q_{1i}$ leads to

$$
\frac{\partial \Pi^{RS}}{\partial q_{1i}} = \rho_1 (\delta \theta - q_{1i}) = 0 \Rightarrow q_{1i} = \delta \theta.
$$
Similarly, the first-order condition on $q_{2d}$ leads to
\[
\frac{\partial \Pi^RS}{\partial q_{2d}} = \frac{\rho_2}{1 + e^{\gamma_2 v_2}} (\theta - q_{2d}) = 0 \Rightarrow q_{2d} = \theta.
\]
and the first order condition on $q_{2i}$ leads to
\[
\frac{\partial \Pi^RS}{\partial q_{2i}} = \frac{\rho_2 \cdot e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} (\delta \theta - q_{2i}) = 0 \Rightarrow q_{2i} = \delta \theta.
\]
Let us now compare the seller’s expected profit with $\theta^2/2$ – obtained from a candidate solution $\{p_{kd}, p_{ki}\} = \{\theta^2, \infty\} \forall k = 1, 2$:
\[
\Pi^RS - \frac{\theta^2}{2} = \rho_1 \delta^2 \frac{\theta^2}{2} - \rho_1 \frac{\delta v_2}{1 + e^{\gamma_2 v_2}} + \rho_2 \frac{(1 - \delta)e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \left( \frac{v_2}{1 + e^{\gamma_2 v_2}} - \frac{\delta^2}{2} (1 + \delta) \right)
\]
\[
> 0 \text{ when } \rho_2 \text{ is large enough.}
\]

The first-order condition on $v_2$ yields
\[
\frac{\partial \Pi^RS}{\partial v_2} = \frac{\rho_1 \delta (1 + e^{\gamma_2 v_2} - \gamma_2 v_2 e^{\gamma_2 v_2}) + \rho_2 \gamma_2 e^{\gamma_2 v_2} (\delta^2 - 1) \frac{\theta^2}{2} + \rho_2 (1 - \delta) e^{\gamma_2 v_2} \left( 1 + \frac{\gamma_2 v_2 (1 - e^{\gamma_2 v_2})}{1 + e^{\gamma_2 v_2}} \right)}{(1 + e^{\gamma_2 v_2})^2} = 0.
\]

We can rewrite the first-order condition as follows:
\[
1 + \frac{\gamma_2 v_2 (1 - e^{\gamma_2 v_2})}{1 + e^{\gamma_2 v_2}} = \frac{\gamma_2}{2} \left( 1 + \frac{\delta}{\rho_2 (1 - \delta)} \frac{1 + e^{\gamma_2 v_2} - \gamma_2 v_2 e^{\gamma_2 v_2}}{e^{\gamma_2 v_2}} \right).
\]

Let $v_2^P$ be the positive solution of the above equation, the second-order condition is as follows:
\[
\frac{\partial^2 \Pi^RS}{\partial v_2^2} |_{v_2^P} = \frac{\gamma_2}{(1 + e^{\gamma_2 v_2^P})^2} \left[ \rho_1 \delta \left( 1 + e^{\gamma_2 v_2^P} \right) + \rho_2 (1 - \delta) e^{\gamma_2 v_2^P} \frac{1 - e^{\gamma_2 v_2^P} - 2 \gamma_2 v_2^P e^{\gamma_2 v_2^P}}{(1 + e^{\gamma_2 v_2^P})^2} \right].
\]

The equation above is only negative when $\rho_1$ is sufficiently small, that is if
\[
\rho_1 < \frac{(1 - \delta) e^{\gamma_2 v_2^P} \left( e^{2 \gamma_2 v_2^P} + 2 \gamma_2 v_2^P e^{\gamma_2 v_2^P} - 1 \right)}{\delta (1 + e^{\gamma_2 v_2^P})^3 + (1 - \delta) e^{\gamma_2 v_2^P} \left( e^{2 \gamma_2 v_2^P} + 2 \gamma_2 v_2^P e^{\gamma_2 v_2^P} - 1 \right)},
\]
we obtain that
\[
\frac{\partial \Pi^RS}{\partial \gamma_2} = \frac{e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} \left[ -\rho_1 \delta v_2^2 + \rho_2 \frac{\theta^2}{2} v_2 (\delta^2 - 1) + \rho_2 (1 - \delta) v_2^2 \frac{1 - e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right] < 0.
\]
The equation above shows that $\Pi^{RS}$ monotonically decreases in $\gamma_2$.

**Case 2:** $v_2$ is negative.

**Subcase 2.1:** $\max\{u_{1d}, u_{1i}\} = u_{1d} = \frac{-v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}}$. With the same reasoning as Subcase 1.1, $p_{1d} = q_d \theta - u_{1d} = q_d \theta + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \geq 0$. The seller’s expected profit function can be written as

$$\Pi^{RS} = \max_{v_2} \left\{ \rho_1 \left( \frac{\theta^2}{2} + v_2 e^{\gamma_2 v_2} \right) + \rho_2 \left( \frac{\theta^2}{2} + \frac{\delta^2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right) \right\}.$$

Let us first compare the seller’s expected profit with $\theta^2/2$ - obtained from a candidate solution $\{p_{kd}, p_{ki}\} = \{\theta^2, \infty\}$ $\forall$ $k = 1, 2$:

$$\Pi^{RS} - \frac{\theta^2}{2} = \rho_1 \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} + \rho_2 \left( \frac{\delta^2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right) \theta^2 + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} < 0.$$

Thus, setting the direct channel price to $\theta^2 + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}}$ is never optimal.

**Subcase 2.2:** $\max\{u_{1d}, u_{1i}\} = u_{1i} = \frac{-v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}}$. In this case, $p_{1i} = \theta^2 + \frac{v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}}$. The seller’s optimization problem can be written as

$$\Pi^{RS} = \max_{v_2} \left\{ \rho_1 \left( \frac{\theta^2}{2} + \frac{\delta v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right) + \rho_2 \left( \frac{\theta^2}{2} + \frac{\delta^2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right) + (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} \right\}.$$

Observe that,

$$\Pi^{RS} - \frac{\theta^2}{2} = \rho_1 \left( \frac{(\delta^2 - 1) \theta^2}{2} + \frac{\delta v_2 e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right) + \rho_2 \left( \frac{\delta^2 - 1}{1 + e^{\gamma_2 v_2}} \right) \frac{\theta^2}{2} + \rho_2 (1 - \delta) \frac{v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} < 0.$$

Thus, setting the second-period price $v_2$ to $-\infty$ is optimal.

We now show that $\Pi^{RS}$ monotonically decreases in $\gamma_2$:

$$\frac{\partial \Pi^{RS}}{\partial \gamma_2} = \frac{e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} \left[ \rho_1 \delta v_2^2 + \rho_2 \mu v_2 (\delta - 1) + \rho_2 (1 - \delta) v_2^2 \frac{1 - e^{\gamma_2 v_2}}{1 + e^{\gamma_2 v_2}} \right] < 0.$$
Finally, we can pin down the cutoff thresholds $\rho_{1}^{RS}$ and $v^{RS}$. They are the solutions that solve (2.15) and (2.16):

\[
\begin{align*}
\rho_1 \left( \frac{\theta^2}{2} - \frac{v_2}{1 + e^{\gamma_2 v_2}} \right) + (1 - \rho_1) \left( \frac{1 + \delta^2 e^{\gamma_2 v_2} \theta^2}{1 + e^{\gamma_2 v_2}} \frac{\theta^2}{2} + \frac{(1 - \delta) v_2 e^{\gamma_2 v_2}}{(1 + e^{\gamma_2 v_2})^2} \right) &= \frac{\theta^2}{2}, \quad (2.15) \\
1 + \frac{\gamma_2 v_2 (1 - e^{\gamma_2 v_2})}{1 + e^{\gamma_2 v_2}} &= \gamma_2 (1 + \delta) \frac{\theta^2}{2} + \frac{\rho_1}{\rho_2} \frac{\delta}{1 - \delta} \frac{(1 + e^{\gamma_2 v_2} - \gamma_2 v_2 e^{\gamma_2 v_2})}{e^{\gamma_2 v_2}}, \quad (2.16)
\end{align*}
\]
Chapter 3

Intertemporal pricing, capacity rationing, and demand uncertainty

In the travel industry, service providers such as airlines and hotels do not pre-announce future prices and consumers cannot observe the remaining capacity; these two distinguishing features may prevent service providers in this industry from credibly committing to predetermined prices. According to Coase conjecture, this time inconsistency issue gives rise to the suboptimality of intertemporal pricing; yet, it is a commonly adopted strategy in the travel industry. In this paper, we use a two-period model to show that demand uncertainty can be the sole driver for intertemporal pricing and that both increasing and decreasing pricing patterns can emerge as optimal strategies. We also identify the intrinsic incentive for service providers to deliberately create capacity shortage to induce early purchases.

In the extended models, we demonstrate that when the service provider discounts the second-period utility more than the consumers do, he may obtain a lower expected profit. Moreover, even though the new arrivals in the second period enhance the competition, they may induce the service provider to focus more on the second-period selling. Thus, the service provider’s commitment issue is exacerbated and ultimately his expected profit is hurt.

3.1 Introduction

The travel industry has adopted intertemporal price discrimination as one of the strategies to increase profits (Talluri and van Ryzin (2005)). Consumers are encouraged
to book tickets early to obtain the services in lower rates and they often observe that the costs increase as the departure dates get closer. As an example, Travelocity offers a stay at the Kona Kai Resort in San Diego for $99 if reserved one month in advance; however, the price goes up to $169 when booked one week in advance.  

Last minute deals with deep discounts also exist. The fare of American Airlines’ weekend vacation package - including round-trip ticket between St. Louis and San Francisco and hotel stays - can be as high as $788 booking three weeks prior to the departure date. It is documented that the fare was lowered to $447 one day before the traveling day.  

A number of operations researchers have provided justifications for the business practice of dynamic pricing and capacity rationing in the seasonal goods industries. Su (2007) shows that optimal pricing policies are determined jointly by heterogeneous valuations and patience. Both mark-up and mark-down pricing policies can be optimal. Aviv and Pazgal (2008) demonstrate numerically that when the rate of valuation decline is relatively high and the heterogeneity in the consumers’ valuations is small, price segmentation is effective in revenue maximizing even with strategic consumers. Liu and Xiao (2008a) argue that rationing can be a profitable strategy when there is a large high-value customer segment and customers have high levels of risk aversion. Other papers have examined ways to sustain the practice of intertemporal price discrimination. Su (2008) demonstrate that a decentralized supply chain can be used as either a quality or price commitment device to improve the service provider’s profit. Yin et al. (2009) study the impact of the inventory display format on the retailer’s expected revenue and have concluded that the Displaying One (DO) unit at a time format is more beneficial to the retailer than Displaying All (DA) units. Lai et al. (2009) discover that a posterior price matching policy can eliminate strategic consumers’ waiting incentive and thus allows the service provider to increase price in the regular selling season.

Compared with the seasonal goods products, travel-related services have some distinguishing features. First, service providers such as airlines and hotels typically do not pre-announce prices, as their booking systems continuously review reservations and update prices to react to the future demand. Second, consumers cannot observe the remaining capacity. Unlike seasonal goods that are usually displayed in the store, travel-related services are sold online and thus the only information disclosed to the consumers is whether the service is still available. Since a service provider can set different prices over time, he has an incentive to reduce the prices in the future to attract customers with lower valuations to purchase. Strategic consumers who expect lower prices in the

\[\text{Source: http://www.travelocity.com.}\]

\[\text{Source: http://www.kiplinger.com/features/archives/2006/05/lastminute.html.}\]
future will then postpone their purchase decisions; consequently, the service provider is competing against his future selves. This intense competition prevents the service provider from creditably committing to predetermined prices. As pointed out by Coase (1972), this *time inconsistency* issue results in the suboptimality of dynamic pricing. He further argues that the optimal strategy for a monopolist who faces consumers with heterogenous valuations is to set a constant price equal to the marginal cost over all selling periods.\(^3\)

The clear discrepancy between Coase conjecture and the business practice of dynamic pricing in the travel industry has intrigued us to re-examine the profitability of intertemporal pricing when the service provider cannot creditably commit to future prices. A crucial assumption in Coase’s model is that *the service provider, as well as every consumer, perfectly knows the demand curve*. Although this assumption is appropriate when the service provider has perfect information regarding the consumers’ valuations and preferences, it is certainly a strong and over-simplified assumption in the cases of airline tickets and hotel rooms as the demand of the consumers can vary significantly from one day to the next. In this paper, we introduce “demand uncertainty” as a natural and relaxation, and show that it can be the sole driver for using dynamic pricing as a profit maximizing strategy. In addition to the primary objective, our paper also investigates the impact of demand uncertainty on the service provider’s optimal pricing patterns and his capacity choice.

As we intend to demonstrate the impact of demand uncertainty on Coase conjecture in the simplest possible setting, we revise the two-period model proposed by Bulow (1982) in which a risk-neutral service provider with a fixed capacity intends to sell a product to risk-neutral consumers with heterogeneous valuations. Consumers learn their valuations at the beginning of the first period and these valuations stay unchanged throughout both periods. Both the service provider and the consumers are fully rational and always make the decisions that optimize their utilities. The service provider is unable to pre-commit to a price pattern at the beginning of selling horizon; consequently, the time inconsistency issue expands in its full force. As the only departure from the model

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\(^3\)This prediction, now well-known as the “Coase conjecture,” has been rooted in the economics literature and has been evaluated by various researchers, including Ausubel and Deneckere (1989), Besanko and Winston (1990), Bulow (1982), Gul et al. (1986), Skreta (2006), and Stokey (1981). Economists have proposed different reasons for the rationale of intertemporal price discrimination. Bond and Samuelson (1984) show that depreciation and replacement sales reduce the service provider’s incentive to cut price while Kahn (1986) demonstrates that an increasing cost structure can also achieve the same result. Bulow (1986) shows that the service provider can reduce his time inconsistency problem by reducing the durability of its output. See Waldman (2003) for an excellent survey.
in Bulow (1982), we assume that consumers’ valuations for the services are independently and identically distributed and they are privately observed by the consumers. This thus creates the demand uncertainty from the perspectives of both the service provider and the consumers.

In our basic model, the market size is publicly known, all consumers enter the market in the first period, and consumers’ valuations follow a uniform distribution. Since consumptions occur at the end of the selling horizon (as a unique feature of travel-related services), neither consumers nor the service provider time discount their second-period utilities. We show that even in the absence of commitment power, the service provider finds it profitable to charge different prices over the two periods. To understand this result, let us revisit the classical setting (by Bulow (1982)) in which demand uncertainty is absent. Consider a supposed equilibrium in which a subset of consumers purchase in the first period and others wait for the second period. If a high-valuation consumer who is supposed to purchase immediately unilaterally deviates from this equilibrium and waits for the second period, he perfectly predicts that the service provider will realize that there is one extra unit at the beginning of the second period and therefore reduce the second-period price. This consumer then prefers to purchase at the market clearing price in the second period. This eliminates the hope of sustaining an equilibrium in which intertemporal price discrimination is adopted. In the presence of demand uncertainty, however, this argument no longer applies. As neither the service provider nor the consumers is able to perfectly predict the residual demand in the second period, regardless of whether a deviation occurs, waiting for the second period may result in capacity rationing. From the perspective of a deviating consumer, the economic trade-off between buying now and waiting remains the same. However, deviation is not a risk-free arbitrage opportunity since getting the product at a lower price is no longer a sure thing. This creates the room for the service provider to credibly implement the intertemporal price discrimination.

Demand uncertainty also yields other managerial insights. Firstly, both increasing and decreasing pricing patterns can emerge as optimal pricing plans. Since the optimal pricing polices are determined by the realizations of consumers’ valuations, the service provider can use the remaining capacity to refine his belief of the valuation distribution of the consumers present in the second period. If the remaining capacity is scarce, the service provider learns that more consumers have relatively high valuations and thus may have the incentive to set the second-period price higher than first-period price. Conversely, when the remaining capacity is abundant, the service provider needs to clear the remaining items and thus has offer a deep discount. In the existing literature, a few papers provide justifications for the observed increasing pricing patterns. Among many
other papers, Talluri and van Ryzin (2005) attribute this phenomenon to the tendency of high-valuation high-uncertainty customers purchase closer to the time of service in the travel business. Hence, demand is less price-sensitive close to the time of service. Bensaid and Lesne (1996) provide a different viewpoint: “when the monopolist lowers its first period price, it increases the number of initial consumers and consequently the quality of its future production through the network externality. In turn, this could allow a credible increase of it future prices, at least if network externalities are of sufficient magnitude.” Our paper demonstrates that demand uncertainty alone can result in increasing pricing pattern.

Secondly, we show that the service provider can cause the capacity rationing effect by setting the first-period price to be greater than the expected second-period price. Since the service provider does not commit prices in advance, the high valuation consumers have to consider the trade-off between paying a premium price to guarantee the service and waiting for a potentially lower price but subject to the risk of not obtaining the service. Lastly, it is worth noting that excluding the cost concern, there is an inherent saturation effect on the capacity expansion. This suggests that optimal capacity exists below the market size and that the service provider can deliberately create capacity scarcity to optimize his expected profit.

We further extend our model to evaluate the robustness of our results, and compare the numerically solved optimal prices and expected profits to those in the basic model. Specifically, we investigate the impact of time discounting, valuation distribution, and additional consumers in the second period on the service provider’s optimal pricing strategies. We find that when the service provider time discounts the second-period utility less than the consumers do, he may obtain a higher expected profit than the basic model without time discounting. On the other hand, when the service provider discounts the second-period utility more than the consumers do, his expected profit is strictly lower than that in the basic model. When the consumers’ valuations are concentrated at the extremes, the service provider is able to obtain a higher expected profit by enlarging the price discrepancy between periods even though the average consumer valuation remains unchanged. We also find that the new arrivals in the second period may enhance the competition; however, since they may induce the service provider to focus more on the second-period selling, it may exacerbate the service provider’s commitment issue and ultimately hurts his expected profit.

Our paper contributes to the existing literature on pricing in the travel industries in the following ways. First, our model captures several unique characteristics of the travel industry. Like many papers in the existing revenue management literature, we recognize the impact of demand uncertainty on sellers’ revenues. Thus, we explicitly
model demand uncertainty by letting the consumer valuations to be privately observed by the consumers. In each period, the seller is able to update his belief about the valuation distribution of the consumers who are still remaining in the market. Other papers (such as Petruzzi and Dada (2002) and Cope (2007)) use the bayesian updating approach to obtain more accurate estimates on the aggregated demand uncertainty. They also show that sellers who learn demand overtime and implement dynamic pricing strategies can achieve large improvements on their revenues over static pricing strategies.

In the literature that focuses on airline and hotel industry, a common assumption is that consumers arrive in different periods or time epochs (see Koenigsberg et al. (2008) and Lin (2004)). Having realized that additional consumer arrivals alone can create rationing effect in the case of limited capacity, we allow all consumers to arrive at the beginning of the selling horizon. We are then able to show that demand uncertainty alone can create rationing effect. Later in the extension, we demonstrate that additional arrival exacerbates the capacity rationing effect and increases the seller’s revenue. Many papers that study strategic consumer behaviors (Besanko and Winston (1990) and Su (2007)) have demonstrated that the time-discounting factor can be a key determinant of pricing structure. In our basic model, we took out this factor in order to isolate the impact of demand uncertainty. We also study the impact of time-discounting in our model extension and show that an impatient seller can hurt his own profit.

Next, we would like to point out that some of our findings in the paper may seem to coincide with many of the existing literature (such as Liu and Xiao (2008a) and Su (2007)). Nevertheless, an important common theme of these papers is that the seller has full commitment power and thus the multi-period problem collapses into a one-shot optimization problem that determines the prices in all periods. While the full commitment power assumption fits nicely to industries such as retail and fashion, it is less applicable to industries. Instead, we use the dynamic pricing setting to allow the seller to dynamically adjust the prices as he learns about the demand and to optimize his expected profits period by period. We are able to show that demand uncertainty can be a sole driver of all possible pricing patterns observed in the pricing practice of the travel industry.

The remainder of this paper is organized as follows. In Section 3.2, we introduce the basic model. In Section 3.3, we carry out the equilibrium analysis, identify the optimal strategies for both the consumers and the service provider, and highlight the main results. We then discuss some extensions on the basic model and their implications in Section 3.4, and conclude the paper in Section 3.5. The proofs are included in the Appendix.
3.2 The Model

We consider a stylized two-period model in which a monopoly service provider wishes to sell products to $N$ risk-neutral consumers. The service provider’s capacity, denoted by $C$, is fixed ex ante. Note that this assumption is appropriate for travel industries as the number of seats in an airplane and the number of rooms in a hotel are exogenously given by the physical constraint. With the same argument, we also assume that there is no production cost as the travel industries cannot produce additional units. For simplicity, we do not consider operational cost either, and it is a trivial exercise to incorporate this into our framework. Each consumer demands at most one unit, and they are heterogeneous with regard to their willingness to pay (valuation). This valuation is privately observed by the consumer but is unknown to the service provider as well as the other consumers. From the service provider’s viewpoint, the consumers’ valuations are independently and identically distributed. Initially, we adopt the uniform distribution between 0 and 1, as the prior distribution of these valuations. Later in Section 3.4, we incorporate other distributions to evaluate its impact on the equilibrium behavior of the service provider and the consumers. With these unobservable valuations, the service provider effectively faces an uncertain demand curve upon determining prices.

The service provider is allowed to sell the products to the consumers in at most two periods, and his goal is to maximize his expected profit.\(^4\) The service provider is unable to observe the consumers’ valuations and has a fixed capacity to sell. In compliance with the business practice of airline ticket pricing, we assume that the service provider can only set one price in each period and that he cannot commit to predetermined prices. Since there are only two periods, he first sets a price at the beginning of the first period. If after the first period is over and there are some capacity left over, he is then allowed to set another price for the remaining units. We assume that the service provider cannot commit to a specific price pattern and therefore must decide the prices period by period. Note that this commitment issue is especially pronounced when the remaining capacity is not publicly observed or equivalently, the service provider cannot credibly disclose this information to the consumers. To focus exclusively on the impact of the capacity on the service provider’s pricing strategies, we do not consider time discounting in the basic model. In Section 3.4, we briefly discuss how the equilibrium changes in response to the time discounting from either the service provider or the consumers.

\(^4\)This simplifies the analysis and allows us to present everything in analytical form. Arguably, this is the simplest way to investigate the consumers’ “buy-now-versus-wait” decisions. It is possible to extend our results to the alternative setting with more than two periods following the same procedure we derive the equilibrium.
Figure 3.1: Sequence of Events

Figure 3.1 shows the sequence of events: $N$ consumers enter the market; each privately observes a valuation ($v$), which is drawn i.i.d. from $U[0,1]$ distribution. The service provider is given (exogenously) the total capacity ($C$). He first announces the first-period price ($p_1$) to all consumers. Given this price and the privately observed valuation, each consumer determines whether to purchase immediately or postpone the purchasing decision to the second period. If a consumer determines to purchase immediately, he expresses this intention to the service provider. The service provider then collects the responses from the consumers and awards the products. If the number of consumers who intend to purchase is less than the capacity ($C$), each consumer who expresses the willingness to purchase obtains one unit and pays the first-period price ($p_1$). Otherwise, we assume that the capacity is allocated among those consumers with equal probability and the game ends. If there are remaining capacity ($k$) after the first period ends, the service provider determines the second-period price ($p_2$) based on the number of consumers still remaining in the market ($N_2$) and $k$. The consumers who remain in the market then determine whether to purchase. A consumer who intends to purchase indicates so to the service provider. The service provider awards the remaining capacity among the consumers that intend to purchase. A consumer obtains one unit for sure if the number of intended consumers does not exceed the remaining capacity. Otherwise, each consumer obtains the product with equal probability. The notations used in this paper are summarized in Table 3.1.

In the next section, we analytically derive the optimal strategies for the service

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5The micro-foundation of this assumption is provided by Cachon and Swinney (2009): conceptually, suppose that the consumers form a queue in front of the service provider and they all have equal probability to be in the front of the queue. Thus, consumers who express the willingness to purchase get to obtain one unit with equal probability.
Table 3.1: Summary of Notation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Total number of consumers at the beginning of the first period</td>
</tr>
<tr>
<td>$N_2$</td>
<td>Number of consumers remaining in the second period</td>
</tr>
<tr>
<td>$C$</td>
<td>Total capacity available at the beginning of the first period</td>
</tr>
<tr>
<td>$k$</td>
<td>Remaining capacity at the beginning of the second period</td>
</tr>
<tr>
<td>$p_1$</td>
<td>First-period price</td>
</tr>
<tr>
<td>$p_2$</td>
<td>Second-period price</td>
</tr>
<tr>
<td>$v$</td>
<td>Consumer’s valuation</td>
</tr>
<tr>
<td>$v_1(p_1)$</td>
<td>Cutoff valuation given the first-period price</td>
</tr>
<tr>
<td>$Q_1(i, \cdot)$</td>
<td>Probability of $i$ consumers willing to purchase in the first period</td>
</tr>
<tr>
<td>$Q_2(j, \cdot)$</td>
<td>Probability of $j$ consumers willing to purchase in the second period</td>
</tr>
</tbody>
</table>

provider and the consumers in equilibrium. Following this, we demonstrate numerically the equilibrium properties of the optimal pricing patterns and the optimal capacity choice.

### 3.3 Equilibrium Analysis

In this section, we characterize the equilibrium behavior of the service provider and the consumers. Given the two-period setting, the appropriate solution concept is perfect Bayesian equilibrium (Fudenberg and Tirole (1991)). In a perfect Bayesian equilibrium, players’ strategies and beliefs satisfy the following conditions:

**Definition 4.** Bayesian updating: *Players have correct initial beliefs. Moreover, after observing players’ actions at a stage, they use Bayes’ rule to update only the corresponding beliefs.*

**Definition 5.** Sequential rationality: *Given their beliefs, the players’ actions must be the best responses.*

Specifically, in our context, this implies that upon learning the number of items sold in the first period, the service provider can update his belief of consumers’ valuation distribution and set the price accordingly to maximize his second-period expected profit. We further assume that consumers’ purchasing strategies are symmetric and have a cutoff valuation structure: For any posted first-period price $p_1$, there exists an associated cutoff
valuation $v_1$ such that all consumers whose valuations are above $v_1$ make the purchase in the first period, while the ones whose valuations are below $v_1$ wait until the second period to make the decision. The symmetric equilibrium is arguably the focal point for a symmetric game (as we intend to analyze in this paper). We later verify that the cutoff valuation structure is indeed an equilibrium.

3.3.1 Analytical Results

Now we show the analytical results using backward induction. We start with the consumers’ purchasing behaviors in the second period for any given second-period price. We then formulate a strategy for the service provider to determine the optimal second-period price to maximize the expected profit of the second period. Following this, we investigate consumers’ strategy on choosing between purchasing immediately in the first period and waiting for the second period to make the decision. We characterize the equilibrium exclusively on a cutoff valuation, which is a function of the first-period price. Based on the cutoff valuation, we compute the service provider’s expected profit for both periods and obtain the service provider’s optimal pricing strategy for the first period by maximizing his expected two-period profit. The following subsections implement our plan to derive the equilibrium behaviors.

Consumers’ Optimal Purchasing Strategy in the Second Period

Let us first consider the consumers’ optimal purchasing strategy in response to the second-period price ($p_2$) set by the service provider. For the consumers whose valuations are below $p_2$, they are better off walking away from the market since obtaining the product results in a negative utility. On the other hand, for the consumers whose valuations are above $p_2$, expressing the willingness to purchase the product is a better option since it results in a non-negative utility. Note that the consumers may get a zero utility if there are more consumers who wish to buy the product than the available inventory, but in any case this does not alter their optimal strategy in the second period.

Service Provider’s Optimal Pricing Strategy in the Second Period

We now characterize the optimal second-period pricing strategy given the first-period price ($p_1$). Let $v_1(p_1)$ be the cutoff valuation such that a consumer whose valuation is above $v_1(p_1)$ purchases the product in the first period, while one whose valuation is below
$v_1(p_1)$ postpones the purchasing decision. Let us assume for a moment that this cutoff valuation is unique; later, we prove the uniqueness of $v_1$ for any given $p_1$ in Lemma 1.

Given this cutoff valuation and the proposed equilibrium behavior, the consumers’ valuations who remain in the second period are uniformly distributed between 0 and $v_1(p_1)$. Define $k \leq C$ as the available capacity at the beginning of the second period. If $k$ is zero, there is no more item to sell; for completeness, we assume that the service provider simply sets the price at the maximum valuation. If $k$ is greater than zero, $C - k$ consumers bought the products in the first period, and thus there are $N_2 = N - (C - k)$ consumers remaining in the market in the second period. Since consumers’ valuations are independently and identically distributed as $U[0, v_1]$, the probability that $j$ consumers are willing to purchase the product at $p_2$ is binomial:

$$Q_2(j, N_2, v_1, p_2) = C_{j}^{N_2} \cdot \left( \frac{p_2}{v_1} \right)^{N_2-j} \cdot \left( 1 - \frac{p_2}{v_1} \right)^j,$$

where $\frac{p_2}{v_1}$ is the probability that a consumer’s valuation is below $p_2$, $1 - \frac{p_2}{v_1}$ is the probability that the consumer’s valuation is above $p_2$ (but below $v_1$), and $C_{j}^{N_2} = \frac{N_2!}{j!(N_2-j)!}$ is the binomial coefficient. The service provider’s corresponding expected profit is

$$[\pi_2[j] = Q_2(j, N_2, v_1(p_1), p_2(p_1, k)) \cdot \min(j, k) \cdot p_2(p_1, k),$$

where the expression $\min(j, k)$ captures the fact that the service provider can only sell up to $k$ products.

As the number of consumers who are willing to pay $p_2$ can be any (integer) value between 0 and $N_2$, the service provider’s expected profit for the second period can be written as

$$\pi_2 = \sum_{j=0}^{N_2} [\pi_2[j] = \sum_{j=0}^{N_2} Q_2(j, N_2, v_1(p_1), p_2(p_1, k)) \cdot \min(j, k) \cdot p_2(p_1, k).$$

The optimal second-period price, denoted by $p_2^*(p_1, k)$, maximizes the expected value of the second-period expected profit:

$$p_2^*(p_1, k) = \arg \max_{p_2} \pi_2.$$
Given this optimal second-period price, the full expression for the probability that \( j \) consumers are willing to buy at the optimal second-period price can be written as

\[
Q_2(j, N_2, v_1(p_1), p^*_2(p_1, k)) = C^j_{N_2} \cdot \left( \frac{p^*_2(p_1, k)}{v_1(p_1)} \right)^{N_2-j} \cdot \left( 1 - \frac{p^*_2(p_1, k)}{v_1(p_1)} \right)^j.
\]

For ease of notation, we refer to this probability as \( Q_2(j, N_2, v_1, p^*_2) \).

### Consumers’ Purchasing Strategy in the First Period

We now move on the the first period. At the beginning of this period, the service provider announces the first-period price \( (p_1) \) for the product. A consumer with valuation \( x \) can either make the purchase in the first period or wait for the announcement of \( p_2 \) in the second period. As defined in the previous section, \( v_1(p_1) \) is the cutoff valuation such that a consumer only makes the purchase in the first period if his valuation is above \( v_1(p_1) \). Since consumers’ valuations are uniformly distributed between 0 and 1, with probability \( v_1(p_1) \) a consumer’s valuation is below \( v_1(p_1) \) and with probability \( 1 - v_1(p_1) \) his valuation is above \( v_1 \). Thus, the probability that \( i \) consumers out of a total of \( N \) consumers wish to purchase the product at price \( p_1 \) is

\[
Q_1(i, N, p_1) = C^i_N \cdot (v_1(p_1))^{N-i} \cdot (1 - v_1(p_1))^i.
\]

Given the symmetric strategy assumption among all consumers, the first-period utility of a consumer whose valuation is above \( v_1(p_1) \) depends on the valuations of the other consumers. Let \( n \) represent the number of consumers (other than the consumer himself) who want to make the purchase at price \( p_1 \). The corresponding probability is \( Q_1(n, N-1, p_1) \) and \( n \) can take any (integer) value between 0 and \( N-1 \). The consumer gets the product for sure if \( n < N \), but he gets the product with probability \( \frac{C_n}{i+1} \) if \( n \geq N \). The expected utility from buying in the first period, denoted by \( \pi^b_1(x, v_1(p_1)) \), can be expressed as follows:

\[
\pi^b_1(x, p_1) = (x - p_1) \left[ \sum_{n=0}^{N-1} \left( Q_1(n, N-1, p_1) \cdot \min \left( 1, \frac{C_n}{n+1} \right) \right) \right]. \tag{3.3}
\]

For a consumer whose valuation \( (x) \) is below \( v_1(p_1) \), she needs to anticipate the second-period price. If the number of consumers who express the willingness to purchase in the first period is greater than the total capacity \( (i \geq C) \), the consumer’s utility is simply zero. If \( i < C \), the number of remaining inventory \( (k) \) is \( C - i \) and the corresponding optimal second-period price is \( p^*_2(p_1, C - i) \). Two scenarios may occur: 1) \( p^*_2 > x \). The consumer then prefers to walk away and his expected utility is just zero.
2) \( p_2^* \leq x \). The consumer’s utility depends on \( j \), the number of others who also wish to purchase. A deviating consumer can get the product for sure if \( j < C - i \). Otherwise, she gets the product with probability \( \frac{C - i}{j + 1} \).

Based on the above reasoning, the expected utility of the consumer with valuation \( x \) from waiting for the second period, denoted by \( \pi^w_1(x, v_1(p_1)) \), can be written as

\[
\pi^w_1(x, p_1) = \sum_{i=0}^{C-1} \left[ \max\{x - p_2^*(p_1, C - i), 0\} \cdot Q_1(i, N - 1, p_1) \cdot \left( \sum_{j=0}^{N-1-i} Q_2(j, N - 1 - i, v_1, p_2^*) \cdot \min\{1, \frac{C - i}{j + 1}\} \right) \right].
\] (3.4)

In Equation (3.4), the expression \( Q_1(i, N - 1, p_1) \) represents the probability that \( i \) consumers out of the other \( N - 1 \) consumers make the purchase in the first period. Note that \( i \) can only range from 0 to \( C - 1 \) here; otherwise there will be no inventory left for the second period. The term \( Q_2(j, N - 1 - i, v_1, p_2^*) \) is the probability that \( j \) out of the \( N - 1 - i \) other consumers remaining in the market in the second period want to purchase at \( p_2^*(p_1, C - i) \). Observe that both \( \pi^b_1(x, p_1) \) and \( \pi^w_1(x, p_1) \) are functions of the cutoff valuation \( v_1(p_1) \). Thus, \( v_1(p_1) \) solves

\[ \pi^b_1(v_1(p_1), p_1) = \pi^w_1(v_1(p_1), p_1). \] (3.5)

In the following lemma, we show that this cutoff level is unique.

**Lemma 1.** The value of \( v_1(p_1) \) is unique when at least one of the functions \( \pi^b_1(x, p_1) \) and \( \pi^w_1(x, p_1) \) is strictly positive.

As shown in the Appendix, \( \pi^b_1(x, p_1) - \pi^w_1(x, p_1) \) monotonically increases in the customer’s valuation \( x \). This implies that if a consumer with valuation \( x \) wants to purchase the service at \( p_1 \), all consumers with valuations higher than \( x \) get an even higher (positive) surplus in utility if they purchase in the first period.

**Service Provider’s Optimal Pricing Strategy in the First Period**

Next, we characterize the service provider’s optimal first-period pricing strategy. To this end, we first derive the service provider’s two-period expected profit given \( p_1 \) and \( C \), denoted by \( \pi(p_1, C) \), as follows:

\[
\pi(p_1, C) = \sum_{i=C}^{N} Q_1(i, N, p_1) \cdot C \cdot p_1 + \sum_{i=0}^{C-1} Q_1(i, N, p_1) \cdot \left( i \cdot p_1 + \sum_{j=0}^{N-i} Q_2(j, N - i, v_1, p_2^*) \cdot \min(j, C - i) \cdot p_2^*(p_1, C - i) \right).
\]
In the above expression, $Q_1(i, N, p_1) = C_i^{N-1} \cdot v_1(p_1)^{N-i} \cdot (1 - v_1(p_1))^i$ is the probability that $i$ consumers want to make the purchase in the first period at $p_1$. If $i$ is greater than or equal to $C$, the service provider’s profit is $C \cdot p_1$ since he can only sell up to $C$ units and there is no more sales in the second period. On the other hand, if $i$ is less than $C$, there are $C - i$ items left in the second period and thus the service provider can sell up to $C - i$ units. The term $Q_2(j, N - 1 - i, p_2)$ is the probability that $j$ consumers want to make the purchase in the second period at a price of $p_2(p_1, C - i)$. Given the expression of $\pi(p_1, C)$, the service provider chooses the first-period price $p_1^*$ that maximizes her expected profit, i.e.,

$$p_1^*(C) = \arg \max_{p_1} \pi(p_1, C) \text{ and } \pi^*(C) = \max_{p_1} \pi(p_1, C).$$

We have now completed the analytical derivations of the equilibrium behaviors of consumers and obtained the optimal pricing strategies for the service provider. In the next section, we illustrate their structural properties using numerical solutions.

### 3.3.2 Numerical Studies and Observations

In this section, we answer the research questions posted in the introduction. The observations are provided in the same backward induction order as the one used in Section 3.3. First, We show the service provider’s behavior in the second period. Given the remaining capacity at the beginning of that period, what should the optimal second-period price be? Next, we move to the first period and explain the behavior of the consumers. The intuitive and analytical reasons for the dominance of cutoff valuation over the first-period price are provided. Numerical examples are then used to demonstrate that increasing pricing pattern as well as decreasing pricing pattern can emerge as the optimal pricing strategy. We also discuss the relationship between the first-period price and the average second-period price and investigate whether there exists an optimal choice for capacity.

In the first observation, we show the relationship between the remaining capacity, the cutoff valuation and the optimal second-period price. This observation is in line with literature (Talluri and van Ryzin (2005)).

**Observation 1.** The optimal second-period price decreases in the remaining inventory and increases in first-period cutoff valuation; that is, $p_2^*$ decreases in $k$ and increases in $v_1$. 


We plot the optimal second-period price as a function of remaining inventory \((k)\) for different values of the cutoff valuations \((v_1(p_1))\) in Figure 3.2.\(^7\) As illustrated, the optimal second-period price monotonically decreases in remaining inventory. Recall that the valuation of the consumers who remain in the market in the second period is uniformly distributed between 0 and \(v_1\). The number of consumers who are willing to pay at \(p_2\) is inversely proportional to it. Thus, the more remaining units there are, the deeper discounts the service provider has to offer to sell the remaining units. It is also clear that the optimal second-period price increases in the first-period cutoff valuation for each given remaining inventory level. As the cutoff valuation determines the upper bound of consumers’ valuations who remain in the second period, the service provider can always set a higher price for consumers who have higher valuations.

The next observation focuses on the relationship between the first-period price and the cutoff valuation. It is an immediate result of the symmetric and cutoff valuation structure of our model.

**Observation 2.** The first-period cutoff valuation is (weakly) greater than the first-period price; that is, \(v_1(p_1) \geq p_1\).

Intuitively, this result is easy to understand. Consumers whose valuations are less than \(p_1\) would not purchase the item as their net expected utilities in the second period would be at least zero; thus, the first-period cutoff price needs to be greater than or

\(^7\)We use the following parameters in this numerical study: \(N=20\) and \(C=10\).
equal to first period price. Moreover, $v_1(p_1)$ does not always equal to $p_1$ as the expected second-period price is lower and thus consumers may still obtain the item in the latter period despite rationing effect. We prove this result analytically in the Appendix.

Before we move on to the next observation, let us first consider the following example. Suppose that there are 5 consumers present in the first period, and the service provider provides 3 items. The optimal first-period price is set to be $p_1^* = 0.54$ and the first-period cutoff valuation is $v_1(p_1^*) = 0.88$. With a probability of 0.36, one consumer will purchase the item in the first period. In this case, there are 2 items left in the second period and 4 consumers remaining in the market; thus, the optimal second-period price is $p_2^* = 0.51$. With a probability of 0.10, two consumers will purchase the item in the first period. In this instance, there is only 1 item left and 3 consumers remaining in the market; thus, the optimal second-period price is $p_2^* = 0.55$. This leads to our next observation regarding the optimal pricing patterns.

**Observation 3.** *In the presence of demand uncertainty, intertemporal prices are optimal and both increasing and decreasing pricing patterns can emerge as optimal pricing plans.*

To understand this result, we provide graphic representation in Figure 3.3. The valuations of the consumers who remain in the second period are uniformly distributed between zero and $v_1(p_1)$. Thus, the service provider can use the remaining capacity to infer the actual realization of the consumers’ valuations. If the remaining capacity is scarce, the service provider learns that more consumers have relatively high valuations and thus has the incentive to set the second-period price higher than first-period price. Conversely, when the remaining capacity is abundant, the service provider needs to clear the remaining items and thus have to offer a deep discount. There have been papers in the existing literature that justify the increasing pricing pattern. Among many other papers,
Talluri and van Ryzin (2005) argue that high-valuation high-uncertainty customers tend to purchase closer to the time of service in the travel business. Hence, demand is less price-sensitive close to the time of service. Bensaid and Lesne (1996), on the hand, state that when the monopolist lowers its first period price, it increases the number of initial consumers and consequently the quality of its future production through the network externality. In turn, this could allow a credible increase of it future prices, at least if network externalities are of sufficient magnitude. Our paper demonstrates that demand uncertainty alone can trigger increasing pricing pattern.

Having looked at the relationship between the optimal first-period price and second-period price, we are interested in finding the relationship between the optimal first-period price and the expected second-period price. This result is shown analytically in the Appendix.

![Figure 3.4: First-Period Price, Expected Second-Period Price, Cutoff Valuation](image)

**Figure 3.4:** First-Period Price, Expected Second-Period Price, Cutoff Valuation

**Observation 4.** The optimal first-period price is greater than the expected value of the optimal second-period price, that is \( p_1^* \geq E p_2^* \).

We plot the first-period cutoff valuation and the expected second-period price against the first-period price in Figure 3.4.\(^8\) We find that the expected second-period price \( E p_2 \) monotonically increases in the cutoff valuation \( (v_1) \). This is because a higher cutoff valuation leads to more consumers that intend to wait for the second period, and consequently the competition for actually obtaining the product becomes more intense in the second period; this in turn allows the service provider to set a higher second-period

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\(^8\)We use the following parameters in this numerical study: \( N=20 \) and \( C=4 \).
price $p_2$. Moreover, recall that the cutoff valuation ($v_1$) monotonically increases in $p_1$ (i.e., the higher the first-period price is, the higher the cutoff valuation is). This implies that a higher price induces more people to wait for the second period. Collectively, we also observe that $E p_2$ is monotonically increasing in $p_1$.

Let us now look at the impact of capacity on the optimal first-period price. We illustrate the relation between the optimal first-period prices ($p_1^*$) for uniformly distributed valuations and the total capacity ($C$) in Figure 3.5.\footnote{We use the following parameter in this numerical study: $N=20$.}

**Observation 5.** The optimal first-period price decreases in total capacity.

We observe that as $C$ increases, the optimal first-period price drops rapidly at first and then levels off. This suggests that when the capacity is relatively small ($C \ll N$), the service provider sets a fairly high price in the first period and a very low price in the second period, thereby creating an intense competition only in the second period. For the high-valuation consumers, the chance of getting the item at the second period is slim; thus, they are better off making the purchase in the first period. However, when the capacity is comparable to the total number of consumers in the market, the service provider needs to clear off as many items as possible at the end of the selling period. Therefore, he has to set a low first-period price and get as many consumers to purchase in the first period as possible.

![Figure 3.5: Optimal First-Period Prices](image1)

![Figure 3.6: Optimal Two-Period Profits](image2)
the service provider’s expected profit. As demonstrated in Figure 3.6,\textsuperscript{10} the expected profit increases first as $C$ increases but then levels off when $C$ is comparable to $N$, the total number of consumers in the market.

\textbf{Observation 6.} The service provider’s expected profit increases in total capacity only when it is scarce.

The intuition of this observation is very similar to that of the previous one. When the capacity is relatively small ($C \ll N$), the service provider does not need to worry about clearing the inventory. Thus, he can set a fairly high first-period price and a very low expected second-period price. High valuation consumers anticipating the high risk of not being able to obtain the service in the second period end up buying in the first period. Consequently, the service provider can obtain a very high profit per item in the first period and clear off the remaining inventory in the second period. As the number of inventory increases, the effect of capacity rationing is reduced. The service provider’s need of clearing the inventory drives down the optimal first-period price. We thus see the diminishing marginal profit in the service provider’s expected profit function. Notice that in our model setting, we do not consider the cost for providing each unit explicitly. This suggests that if the cost is linear or convex in units, the optimal the service provider can deliberately create capacity shortage to reach a maximum expected profit.

We have demonstrated the consumers’ equilibrium behaviors and the service provider’s optimal pricing strategies in the basic model. A natural question is how sensitive these results are with respect to our model characteristics. To address this issue, we in the next section extend our model to evaluate the robustness of our results.

\section{Model Extensions}

In this section, we consider some variants of our model characteristics, and compare the numerically solved optimal prices and expected profits to the basic model.

\subsection{Time Discounting}

In our basic model, we abstract away the time discounting of consumers since it is not the main driving force in the framework of the classical Coase conjecture. However, it is certainly interesting to investigate the effect of time discounting on the optimal pricing and the expected profit. Booking in advance may give consumers additional benefit, as

\textsuperscript{10}We use the following parameter in this numerical study: $N=20$. 

for example, leisure travelers who purchase airline tickets in advance can better plan the rest of their trips and thus obtain additional utility. To model this aspect, we introduce $\delta_s$ and $\delta_c$ as the discounting factors for the service provider and the consumers, respectively. The equilibrium analysis essentially follows the same procedure and the results are almost identical to those in the basic model. Specifically, the expression for the optimal second-period price (equation (3.2)), the consumer’s expected utility from purchasing in the first period (equation (3.3)), and the consumer’s expected utility from waiting for the second period (equation (3.4)) all apply to this alternative setting directly. With time discounting, the cutoff value $v^0_1(p_1)$ solves the implicit function:

$$
\pi^0_1(x, p_1) = \delta_c \pi^w_1(x, p_1),
$$

and the service provider’s optimal expected profit becomes

$$
\pi^\delta(p_1, C) = \sum_{i=C}^{N} Q_1(i, N, p_1) \cdot Cp_1 + \sum_{i=0}^{C-1} P^1(i, N, p_1) \cdot 
\left(i \cdot p_1 + \sum_{j=0}^{N-i} Q_2(j, N - i, v^\delta_1(p_2)) \cdot \min(j, C - i) \cdot \delta_s \cdot p_2(p_1, C - i)\right).
$$

Figure 3.7: Optimal First-Period Prices

Figure 3.8: Optimal Two-Period Expected Profits

We consider two cases: 1) both the seller and the consumers have the same time discounting on the second-period utility ($\delta_s = \delta_c = 0.8$), and 2) consumers discount the
second-period utility while the service provider does not ($\delta_s < 1, \delta_c = 1$). Figures 3.7 and 3.8 correspond to the first case while Figures 3.9 and 3.10 correspond to the second one. The case that the service provider discounts future more than the consumers has been extensively investigated in the existing literature and therefore is omitted. This case is straightforward and can be provided upon request.

In Figure 3.7 we plot the optimal first-period prices as a function of available capacity. We observe that the prices for the case without time discounting weakly dominate the prices for the case with time discounting when the capacity is much smaller than the total number of consumers, whereas the relationship switches when the capacity is comparable to the total number of consumers. The impact of these prices on the expected profits is documented in Figure 3.8. Specifically, the two-period expected profit for the case without time discounting is slightly higher than that for the case with time discounting when the capacity is relatively small, while the two-period expected profit for the case with time discounting is much higher when the capacity is comparable to the number of consumers. As we have shown in the analysis, the service provider wants to achieve two goals when setting the first-period price. First, he would like to induce as many consumers to purchase in the first period as possible since the expected second-period price will be lower. Second, he intends to learn the realizations of the consumers’ valuations in order to set an appropriate second-period price.

Let us now discuss how these two goals affect the service provider’s pricing decision.

\[\text{Figure 3.9: Optimal First-Period Prices} \quad \text{Figure 3.10: Optimal Two-Period Expected Profits}\]

\[\text{Figure 3.9: Optimal First-Period Prices} \quad \text{Figure 3.10: Optimal Two-Period Expected Profits}\]

\[\text{Figure 3.9: Optimal First-Period Prices} \quad \text{Figure 3.10: Optimal Two-Period Expected Profits}\]

\[\text{Figure 3.9: Optimal First-Period Prices} \quad \text{Figure 3.10: Optimal Two-Period Expected Profits}\]

\[\text{Figure 3.9: Optimal First-Period Prices} \quad \text{Figure 3.10: Optimal Two-Period Expected Profits}\]

\[\text{Figure 3.9: Optimal First-Period Prices} \quad \text{Figure 3.10: Optimal Two-Period Expected Profits}\]

\[\text{Figure 3.9: Optimal First-Period Prices} \quad \text{Figure 3.10: Optimal Two-Period Expected Profits}\]

\[\text{Figure 3.9: Optimal First-Period Prices} \quad \text{Figure 3.10: Optimal Two-Period Expected Profits}\]

\[\text{Figure 3.9: Optimal First-Period Prices} \quad \text{Figure 3.10: Optimal Two-Period Expected Profits}\]
When the capacity is small, the service provider is more concerned about the first objective and therefore tries to sell as many items as possible in the first period. Even though the competition in the second period is high, consumers with valuations in the high end still incline to wait for the second period due to the time discounting. Thus, the service provider has to set the first-period price lower in the case with time discounting to induce the consumers with high valuations to purchase early. Otherwise, consumers’ time discounting on the second-period utility will drive down the second-period price and result in a lower expected profit. On the other hand, when the capacity is high, the service provider does not intend to clear all inventory and thus the second-period price does not go down by much. In such a scenario, the service provider can then take advantage of consumers’ discounting by setting a relatively higher first-period price.

In Figure 3.9, we plot the optimal first-period prices as a function of available capacity. We observe that the prices for the case without time discounting dominate that for the case with the service provider’s time discounting. This confirms our intuition: as the service provider wants to sell as many units as possible in the first period, he has to lower the first-period price to induce the consumers to make the purchase. From Figure 3.10, we observe that the expected profit is much lower, this suggests that impatient seller can hurt his own profit when facing strategic consumers. Moreover, there is a significant drop when the capacity is high because the service provider discounts the second-period expected profit and the high capacity makes competition in the second period incredible.

### 3.4.2 Beta Distributed Consumers’ Valuations

In the basic model, we assume that consumers’ valuations are uniformly distributed to obtain simple characterizations of the equilibrium behaviors. However, the downside of this is that it imposes implicitly a strong structure over the consumers’ composition. For example, it fails to capture the scenarios in which many consumers have very similar valuations for a product/service or their valuations fall into two extremes. To investigate these alternative scenarios, we adopt the Beta distribution for the consumers’ valuation distribution in this subsection. This family of distributions is sufficiently flexible to accommodate various scenarios. For example, when $\alpha = \beta = 2$, it has a U-shaped distribution on $[0,1]$; this corresponds to the case where most consumers’ valuations are concentrated in the middle of the valuation range. Similarly, the case $\alpha = \beta = 0.5$ has a bimodal shape, which corresponds to the case where consumers’ valuations are dispersed and concentrated at the extreme points of the valuation range.

Let us indicate the modifications required for this alternative distributional assump-
Define $f(x; \alpha, \beta)$ and $F(x; \alpha, \beta)$ as the probability distribution function and cumulative distribution function for Beta distribution, respectively. When the second-period price $p_2$ is announced by the service provider, consumers’ optimal purchasing strategy is to purchase only if their valuations are above $p_2$. Thus, the second-period price based on the first-period price $p_1$ and the remaining inventory level $k$ becomes:

$$p_2(p_1, k) = \arg \max_{p_2} \left( \sum_{j=0}^{N-(C-k)} Q_2(j, N-(C-k), v_1, p_2) \cdot \min(j, k) \cdot p_2 \right),$$

where

$$Q_2(j, N-(C-k), v_1, p_2) = C_j^{N-(C-k)} \cdot \left( \frac{F(p_2; \alpha, \beta)}{F(v_1(p_1); \alpha, \beta)} \right)^{N-(C-k)-j} \cdot \left( 1 - \frac{F(p_2; \alpha, \beta)}{F(v_1(p_1); \alpha, \beta)} \right)^j.$$

In this equation, $\frac{F(p_2; \alpha, \beta)}{F(v_1(p_1); \alpha, \beta)}$ is the conditional probability that a customer’s valuation is below $p_2$ given that his valuation is below the cutoff value $v_1$. For a consumer with valuation $x$, his utility from purchasing the product in the first period is

$$\pi_1^b(x, p_1) = \max \{x - p_1, 0\} \cdot \sum_{i=0}^{N-1} \left( Q_1(i, N-1, p_1) \cdot \min \left\{ 1, \frac{C}{i+1} \right\} \right).$$
where \( Q_1(i, N - 1, p_1) = C^N_j \cdot F(v_1(p_1); \alpha, \beta)^{N-1-i} \cdot (1 - F(v_1(p_1); \alpha, \beta))^i \). The expected utility for the consumer to wait until the second period to make a decision is

\[
\pi^w_1(x, p_1) = \sum_{i=0}^{C-1} \left[ \max\{x - p_2(p_1, C - i), 0\} \cdot Q_1(i, N - 1, p_1) \cdot \left( \sum_{j=0}^{N-1-i} Q_2(j, N - 1 - i, v_1, p_2) \min\{1, \frac{C-i}{j+1}\} \right) \right].
\]

The cutoff valuation \( v_1(p_1) \) solves the implicit function \( \pi^b_1(x, p_1) = \pi^w_1(x, p_1) \), and the service provider’s expected profit function remains unchanged.

In Figure 3.11,\(^{12}\) we plot the optimal first-period price as a function of total capacity \( C \) for three distributions: Beta(2, 2), Beta(0.5, 0.5), and U[0, 1]. From Figure 3.11, we notice that the curve for Beta(0.5, 0.5) supersedes that for U[0, 1], which dominates Beta(2, 2). The following explanation might be helpful in understanding this result. Recall that valuations distributed as Beta(0.5, 0.5) have either really high or really low extreme values. Thus, the service provider can set a much higher first-period price and still give the high-valuation consumers a positive (expected) utility. At the same time, the service provider can set a low second-period price to induce the low-valuation consumers to purchase then. Additionally, this creates very intensive competition in the second period and makes it less appealing for high-valuation consumers to wait for the second period. This explains why the curve for Beta(0.5, 0.5) is the highest among the three distributions.

In Figure 3.12, we plot the corresponding expected profits for the service provider. The expected profit for Beta(0.5, 0.5) is much higher than that for Beta(0.5, 0.5) and U[0, 1] distribution since the optimal price for Beta(2, 2) is much higher. Between Beta(0.5, 0.5) and U[0, 1] there is no dominance result, however. The expected profit for U[0, 1] is higher that that for Beta(2, 2) when the capacity is small, while the ranking is reversed when the capacity is large. With a small capacity, the service provider who faces consumers with U[0, 1] distributed valuations can set a higher first-period price, since there are more consumers with high valuations. With a large capacity, Beta(2, 2) distributed consumer valuations allow the service provider to sell more capacity since there are more consumers with mid to high valuations.

3.4.3 Additional Arrivals in the Second Period

In the majority of papers on Coase conjecture, the consumers are assumed to be present throughout the game. Nevertheless, in reality it is possible that some new

\(^{12}\)The other parameters used in these numerical studies are: N=20, C=10.
consumers may arrive at a later moment, and it might be intriguing to see how the service provider sets the prices in response to these new arrivals. Given our two-period setting, we can conveniently let $M$ be the additional consumers who enter the market in the second period. As in the basic model, we assume that their valuations are independently and identically distributed and follow a uniform distribution over $[0, 1]$.

In such a scenario, for a consumer with valuation $x$ and cutoff valuation $v_1(p_1)$, the utility function from buying in the first period is not affected; that is,

$$\pi^b_1(x, p_1) = \max(x - p_1, 0) \left[ \sum_{i=0}^{N-1} P^1(i, N - 1, p_1) \cdot \min(1, \frac{C}{i + 1}) \right].$$

where $Q^1(i, N - 1, p_1) = C^i_{N-1}(v_1(p_1))^{N-1-i}(1 - v_1(p_1))^i$. The expected utility function from purchasing in the second period is

$$\pi^w_1(x, p_1) = \sum_{i=0}^{C-1} \max\{x - p_2(p_1, C - i), 0\} \cdot Q_1(i, N - 1, p_1)$$

$$\left[ \sum_{m=0}^{C} \sum_{j=0}^{N-1-i} R(m, M, p_2) \cdot Q_2(j, N - 1 - i, v_1, p_2) \cdot \min(1, \frac{C - i}{m + j + 1}) \right],$$

13 This is merely one possible scenario. For example, we could allow the range for the valuations of these new arrivals to differ from $[0, 1]$. This can be easily incorporated into our framework after slight modifications.
where \( R(m, M, p_2) = C_m^M (p_2)^{M-m}(1-p_2)^m \). The service provider’s expected profit is

\[
\pi(p_1, C) = \sum_{i=C}^{N} Q_1(i, N, p_1) \cdot C p_1 + \sum_{i=0}^{C-1} Q_1(i, N, p_1) \cdot \left( i \cdot p_1 + \sum_{m=0}^{C} \sum_{j=0}^{C-i} R(m, M, p_2) \cdot Q_2(j, N-i, v_1, p_2) \cdot \min(m+j, C-i) \cdot p_2(p_1, C-i) \right).
\]

We plot the optimal pricing with new arrivals in the second period and compare it with our basic model without new arrivals in Figure 3.13.\[14\] The corresponding optimal expected profits are drawn in Figure 3.14. From these two figures, we can see that the prices for the new arrivals in the second period are higher than those without arrivals in the second period when the capacity is relatively small (\( C \ll N \)). This is because new arrivals in the second market create more intense competition, making the option of purchasing the item in the second period undesirable for the consumers with high valuations. The service provider thus can take advantage of that by setting a higher first-period price. Since the consumers with high valuations still purchase in the first period, the service provider is able to obtain a higher expected profit. When the capacity is relatively large, the consumers have a much higher chance to obtain the product despite the increased competition in the second period. In this case, the service provider can no longer take advantage of the additional arrivals; on the contrary, he even has to set a lower first-period price to induce consumers to purchase in the first period. This implies that the presence of new arrivals is a double-edged sword for the service provider from the profit perspective.

### 3.5 Conclusions

Our paper argues that demand uncertainty can be the sole driver for dynamic pricing and that both increasing and decreasing pricing patterns can emerge as optimal pricing strategies. Moreover, we identify the intrinsic incentive for the service providers to deliberately create capacity shortage to induce strategic consumers to purchase early. In the extended models, the impact of time discounting, valuation distribution and additional consumers in the second period is investigated. We show that when the service provider discounts the second-period utility more than the consumers, he may obtain a lower expected profit. We also find that even though the new arrivals enhance

\[14\] We use the following parameters in this numerical study: \( N=20, M=10, C=10 \).
the second period’s competition; they may induce the service provider to focus on the second-period selling and thus exacerbate his commitment issue and hurt his expected profit.

**Appendix.**

**Proof of Lemma 1.** First observe that for a given $p_1$, both $\pi_i^b(x, p_1)$ and $\pi_i^w(x, p_1)$ are monotonically increasing in $x$:

\[
\frac{\partial \pi_i^b(x, p_1)}{\partial x} = \sum_{i=0}^{N-1} Q_1(i, N - 1, p_1) \cdot \min(1, \frac{C}{i + 1}) > 0,
\]

\[
\frac{\partial \pi_i^w(x, p_1)}{\partial x} = \sum_{i=0}^{C-1} Q_1(i, N - 1, p_1) \cdot \left[ \sum_{j=0}^{N-1-i} Q_2(j, N - 1 - i, v_1, p_2) \cdot \min(1, \frac{C - i}{j + 1}) \right] > 0.
\]

When both functions are positive, it is sufficient to that $\pi_i^b(x, p_1) - \pi_i^w(x, p_1)$ increases in $x$, that is \( \frac{\partial \pi_i^b(x, v_1(p_1))}{\partial x} - \frac{\partial \pi_i^w(x, v_1(p_1))}{\partial x} \geq 0 \).

\[
\frac{\partial \pi_i^b(x, v_1(p_1))}{\partial x} = \sum_{i=0}^{C-1} Q_1(i, N - 1, p_1) + \sum_{i=C}^{N-1} Q_1(i, N - 1, p_1) \cdot \frac{C}{i + 1} \geq \sum_{i=0}^{C-1} P^1(i, N - 1, p_1),
\]

\[
\frac{\partial \pi_i^w(x, v_1(p_1))}{\partial x} \leq \sum_{i=0}^{C-1} Q_1(i, N - 1, p_1) \cdot \sum_{j=0}^{N-1-i} Q_2(j, N - 1 - i, v_1, p_2) \cdot 1 \leq \sum_{i=0}^{C-1} P^1(i, N - 1, p_1).
\]

Thus if $v_1(p_1)$ satisfies $\pi_i^b(v_1(p_1), v_1(p_1)) = \pi_i^w(v_1(p_1), p_1)$, $\pi_i^b(x, v_1(p_1)) \geq \pi_i^w(x, p_1)$ for all $x > v_1(p_1)$ while $\pi_i^b(x, v_1(p_1)) \leq \pi_i^w(x, p_1)$ for all $x < v_1(p_1)$. Consequently, $v_1(p_1)$ is unique when at least one of $\pi_i^b(x, p_1)$ and $\pi_i^w(x, p_1)$ functions is positive. \( \blacksquare \)

**Proof of Observation 2.** Recall Lemma 1 stated that both $\pi_i^b(x, p_1)$ and $\pi_i^w(x, p_1)$ are monotonically increasing in $x$ and that $v_1(p_1)$ is unique. This means that $\pi_i^b(x, p_1)$ and $\pi_i^w(x, p_1)$ only intersect once. Since $\pi_i^b(p_1, p_1) = 0$ and $\pi_i^w(p_1, p_1) \geq 0$, $v_1(p_1) \geq p_1$. \( \blacksquare \)

**Proof of Observation 4.** To show this result analytically, we first re-write $\pi_i^w(x, p_1)$ as:

\[
\pi_i^w(x, p_1) = E \left[ \max(x - p_2, 0) \cdot \min \left\{ 1, \frac{C - i}{j + 1} \right\} \right]
\]

\[
\leq E \left[ \max(x - p_2, 0) \right] = E \left[ (x - p_2)1_{(x > p_2)} \right]
\]

\[
\leq E \left[ (x - p_2) \right] = x - Ep_2.
\]
Suppose \( p_1^* < E \rho_2 \), for a consumer whose valuation \( x \) is less than \( v_1^*(p_1^*) \), \( \pi_1^b(x, p_1^*) = x - p_1^* > x - E \rho_2 \geq \pi_1^w(x, p_1) \). This means that the consumer is better off purchasing the item in the first period and violates the equilibrium structure of this problem. Thus, \( p_1^* \geq E \rho_2 \) must hold. \( \Box \)
Online retailers in recent years have experienced significant growth; however, they still face three major challenges. First, consumers’ valuations are uncertain. The lack of direct inspection often creates a misfit between the product features and the consumers’ preferences. Second, in accordance with this, consumers then desire to have the option of getting a refund when such a misfit occurs. Last, consumers are more sophisticated nowadays. They anticipate deep discounts in future prices and delay their purchases strategically. It is thus not only important for the retailers to find the optimal refund policy jointly with pricing but also necessary to decide what to do with the returned products.

We develop a model that takes into account both consumer valuation uncertainty and strategic consumer behavior, and derive the optimal pricing and return policy for a retailer in a dynamic pricing framework. We identify three effects of a generous return policy on consumers who have heterogeneous valuations. First of all, a return policy with a small restocking fee encourages the high-valued consumers to purchase early. Next, the combination of consumer valuation uncertainty and returns creates the inventory uncertainty that allows the retailer to credibly generate rationing risk. Further, a generous return policy lessens the capacity rationing effect in the latter period, and may induce some consumers to wait. Our paper also provides a rationale for the business practice that retailers do not actively engage in recertifying or remanufacturing returned products. This is because when returns are perceived as low-quality products, the retailers can facilitate market segmentation without creating new product lines.
4.1 Introduction

Online retailers in recent years have experienced significant growth. The e-commerce product sales in the U.S. have achieved $142.5 billion in 2010 and they are expected to reach $279 billion in 2015. Retailers who sell a product over multiple periods, however, still face three major challenges. First and foremost, consumers’ valuations are uncertain. Products, as described by Lal and Sarvary (1999), can have both digital and nondigital attributes. The digital attributes, such as brand, size and usage, can be communicated via the Internet. However, the nondigital attributes, such as the product quality, whether it is user friendly to use and whether it fits the consumers’ taste, can only be physically inspected or trial-used by the consumers. Thus, they may incur dis-utilities when such misfits occur. According to the online survey from Forrester Research, consumers rank books, hotel reservations, and airline tickets as the top three categories that they prefer to purchase online, while they rank footwear, electronics, and household products as the bottom three categories. This further confirms that the nondigital attributes are important decision factors of the consumers’ purchasing decisions.

Such an uncertainty leads to the second challenge for the retailers – consumers want to have the option of getting a refund in case they dislike the product. According to MarketSherpa’s survey, 90% of the consumers who visit an e-commerce site leave without purchasing. Among them, 49% state that return policy is a major factor that keeps them from doing more online shopping. Recognizing its importance, many online retailers have adopted different return policies. While department stores that operate dual channels\(^1\) such as Macy’s, Nordstrom, and etc. provide full refunds at local stores, online retailers such as Amazon.com and Overstock.com only allow partial refunds. They charge 20%–50% handling fees, which vary depending on the product categories. In addition to finding the optimal pricing and refund policy, retailers also need to decide what to do with the returned products. Recently, online retailers like Newegg and Buy.com have adopted recertification processes for some returned products. Though it is costly for the retailers to do so, it gives consumers the comfort of knowing that a thorough inspection process has been completed and consequently encourages them to purchase. When the products are returned due to minor problems such as missing manuals, torn packages or damaged boxes, the retailers may want to simply sell the products “as-is”.

Last, consumers have become increasingly sophisticated over the years. While advanced technology has enabled retailers to vary their prices over time, online forums and reviews have provided consumers enough information to recognize this fact. Con-

\(^1\)dual channels include both the physical stores and online stores.
sumers anticipate deep discounts in future prices. Consequently, they delay their purchases strategically in hope of getting a better deal. As pointed out by Su (2008), Liu and Xiao (2008a) and Cachon and Swinney (2009), some retailers have recognized the negative impact of strategic consumers on their profitability and have adopted markdown optimization software to set prices in response to available inventory.

In this paper, we develop a model that takes into account consumer valuation uncertainty and strategic consumer behavior, and derive the optimal pricing and return policy for a retailer in a dynamic pricing framework. Though there has been vast literature on strategic consumer behavior, to the best of our knowledge no prior work has ever incorporated all three ingredients into a unified framework. Through this framework, we intend to answer the following research questions. What is the optimal pricing strategy and return policy? Should the retailer provide full refund, partial refund or no refund? How does the capacity choice affect the retailer’s refund policy? Should the retailer choose to go through the recertification/remanufacturing process or simply sell them as products with a lower quality?

Our basic model describes a stylized retailer who wishes to sell a durable good in two periods. The total capacity is fixed prior to the selling horizon and the retailer cannot replenish his inventory. All consumers are present at the beginning of the selling horizon. The initial consumer valuations are independently and identically distributed. At the beginning of the first period, the retailer announces both the first-period price and the restocking fee. We do not put any restriction on the restocking fee; thus, the retailer is free to pick any of the following three policies: full refund, partial refund and no refund. Consumers who intend to purchase in the first period indicate their interests to the retailer, and the products are then randomly allocated among these consumers. Once a consumer obtains the product, she receives an add-on valuation. If this valuation results in a net valuation that is lower than the first-period price minus the restocking fee, the consumer chooses to return the product by the end of the first period. In our basic framework, the retailer recertifies/remanufactures the product and sells it as new in the second period. The retailer is not bound to the posted price policy; thus, he is entitled to adjust the second-period price based on the available inventory and his updated belief of consumers’ valuation distribution for revenue maximization.

We identify three effects of a generous return policy on consumers who have heterogeneous valuations. First, a return policy with a small restocking fee encourages the high-valued consumers to purchase early. Such a policy not only provides consumers an insurance effect but also helps achieve a higher allocation efficiency in the early period. Second, the combination of consumers’ initial valuation uncertainty and add-on valuation uncertainty creates demand uncertainty. As Coase (1972) points out, strate-
gic consumers who expect lower prices in the future postpone their purchase decisions. Consequently, the retailer is competing against his future selves and such competition prevents the service provider from credibly committing to non-posted prices. This time inconsistency issue results in the suboptimality of dynamic pricing. However, we show that demand uncertainty creates a source for the retailer to credibly generate rationing risk. Third, a generous return policy lessens the capacity rationing effect in the latter period, and some consumers are induced to wait.

Our model also finds the tight connection among pricing, restocking fee and the capacity choice. While the first-period price decreases as the capacity gets large, the ratio between the restocking fee and the first-period price need not be monotonic in capacity. If the inventory appears to be scarce, the retailer can afford a generous refund policy. Namely, he can charge a very high first-period price while providing a “nearly risk-free” return policy. This pricing strategy induces high-valued consumers to purchase early as their valuation risk is largely endured by the retailer. As the capacity becomes abundant, a refund policy that encourages returns is no longer desirable. On one hand, the retailer needs to lower the first-period price to encourage more consumers purchase early. On the other hand, he must increase the restocking fee to deter consumers from returning the product. Finally, when the capacity is close to the market size, the pressure to clear the inventory becomes dominant over pricing. He must decrease both the first-period price and the restocking fee to induce all consumers to purchase early.

In the extended model, we allow the retailer to sell the returned product without any recertification or remanufacturing process. The main difference in the sequence of events is that the retailer announces both the prices for the remaining new products and the returns at the beginning of the second period. Naturally, the consumers perceive the returns as low-quality products, and may prefer to purchase the new ones over the returns or vice versa depending on their valuations and the prices. We find that, the retailer can still charge a high first-period price when the capacity is scarce. However, he no longer needs to provide the “nearly risk-free” return policy. Since the high-valued

\footnote{Coase further argues that the optimal strategy for a monopolist who faces consumers with heterogeneous valuations is to set a constant price equal to the marginal cost over all selling periods. This prediction, now well-known as the “Coase conjecture,” has been rooted in the economics literature and has been evaluated by various researchers, including Ausubel and Deneckere (1989), Besanko and Winston (1990), Bulow (1982), Gul et al. (1986), Skreta (2006), and Stokey (1981). Economists have proposed different reasons for the rationale of intertemporal price discrimination. Bond and Samuelson (1984) show that depreciation and replacement sales reduce the service provider’s incentive to cut price while Kahn (1986) demonstrates that an increasing cost structure can also achieve the same result. Bulow (1986) shows that the service provider can reduce his time inconsistency problem by reducing the durability of its output. See Waldman (2003) for an excellent survey.}
consumers have a strong preference on the new products over the returns, the rationing effect of the new products in the second period is intensified and outweighs the valuation risk. As a result, the consumers are motivated to purchase early despite a much higher restocking fee. Returns can be effectively treated as lower-quality products and help the retailer to further segment consumers by intensifying the rationing effect in the latter period. Our numerical analysis shows that selling returns as used yields a profit gap of 4%–6%, which is not a significant impact on the retailer’s performance. Given that our basic model normalizes the cost of recertification/ remanufacturing to zero, we can comfortably conclude that the retailer is better off not going through the process if the cost associated is sufficiently high.

We organize the remainder of this paper as follows. Section 4.2 provides a brief literature review and Section 4.3 introduces the basic model and notation. In Section 4.4, we carry out the equilibrium analysis for the case of selling returns as new, identify the optimal strategies for both the consumers and the retailer, and highlight the main results. We then discuss the case of selling returns as used, and compare the results against the previous case in Section 4.5. Section 4.6 concludes the paper. Detailed analysis is provided in the Appendix.

4.2 Literature Review

Our paper relates to three streams of literature. The first studies various forms of return policies. Davis et al. (1995) look at the most special form of return, namely the money-back guarantees (MBG), and show that such a policy may increase a retailer’s profit. The reason behind that is intuitive, MBG removes consumers’ misfit risk and increases their willingness to pay, consequently the retailer can charge higher prices. Hess et al. (1996) then point out that full refund policy can be abused by consumers and negatively impact the retailer’s profitability. They find that retailers can eliminate inappropriate returns by imposing non-refundable charges. Davis et al. (1998) recognize the same inappropriate return issues and propose to impose hassles or restrictions on consumers who wish to return products. A subsequent paper by Chu et al. (1998) then compare the “no-questions-asked refund policy” against “no refunds policy” and “verifiable problems only policy” and find the first one is optimal as the others will hurt the retailer’s volume. Auidwijk and Ketzenberg (2010) then report that an intermediate return policy is optimal in a wide range of operating environments. Our paper puts no restricts on the form of refund, that is no refund policy, partial refund policy and full refund policy could all emerge as optimal solutions. However, our numerical analysis
shows that partial refund is optimal. This result coincides with empirical evidence presented in Chu et al. (1998), as partial refunds often come in the form of restocking fee, which can range from 15%–30% and is much higher than the actual cost.

The second body of literature focuses on the impact of consumer valuation uncertainty on retailers’ performances. As pointed out by Gu and Tayi (2011), selling apparel, shoes and accessories online is most challenging for two reasons. First, whether a consumer keeps the product depends largely on how well the product fits the consumer’s personal taste. Second, the purchase decision and her final decision of whether to keep the product are made in two different times, and thus fit-uncertainty plays a crucial role in consumer’s decision of whether to return the product. Liu and Xiao (2008b) find that when consumers are more ex ante uncertain about the value of a product, a retailer is better off providing a menu of return policies and allow consumers to self-select. Shulman et al. (2009) show that providing consumers with product fit information that will resolve their valuation uncertainty and eliminate returns is not always optimal for the retailer, even if he can provide the product information with no additional cost. Dana (1998) and Xie and Shugan (2001) also provide more discussions on the impact of consumer valuation uncertainty in their papers.

Finally, our paper relates to the growing body of literature that studies the effect of strategic consumer behaviors. Retailers increasingly become aware of forward looking consumers, who expect deep discounts in future prices and strategically delay their purchases. Such behaviors not only reduce retailers ability of price discriminate consumers over multiple periods but also hurt the retailers’ profitability significantly. Aviv and Pazgal (2008) point out that when the retailers ignore strategic consumer behaviors, they are subject to a 20% loss in revenue. Liu and Xiao (2008a) show that deliberately understocking products can create a rationing risk for high-valued consumers and induce them to purchase early. Such a strategy is optimal when the number of high-valued consumers is sufficiently large and the benefits of inducing consumers to purchase at high prices outweigh the lost sales due to capacity shortage. Swinney (2011) finds that when dealing with the presence of strategic consumers, a second procurement opportunity before the sale period is only beneficial for the retailer if prices are increasing overtime. Su (2009) suggest an alternative way in dealing with strategic consumers, that is, the seller guarantees product availability and high service level which increases consumers’ willingness to pay. Akan et al. (2009) consider both forward looking consumers and valuation uncertainty, and argue that it is optimal to induce all consumers to purchase before they learn their true valuations and allow them to pick a return policy for a partial refund. Our paper considers the effect of both strategic consumer behaviors and valuation uncertainty on retailers’ pricing and refund decisions.
Unlike the aforementioned papers, the retailer in our setting does not have commitment power and the price path depends on updated valuation distribution and capacity.

4.3 The Model

We consider a stylized two-period model in which a monopoly retailer wishes to sell products to \( N \) risk-neutral consumers, all of whom enter the market at the beginning of the selling horizon and remain until either they obtain the product or to the end. The retailer’s capacity, denoted by \( C \), is chosen prior to the selling horizon and cannot be replenished. We assume that the capacity information is public knowledge and that there is no production cost or operational cost as it is a trivial exercise to incorporate these costs into our model. At the beginning of the selling horizon, each consumer privately observes an initial valuation \((v)\). We denote the distribution and density of \( v \) by \( F \) and \( f \). The retailer announces the first-period price \( p_1 \) as well as the refund amount \( f \) to all consumers; thus, the restocking fee is \( p_1 - f \). Given the pricing and the privately observed initial valuation, each consumer determines whether to purchase immediately or postpone the decision to the next period. If a consumer wishes to purchase immediately, she expresses this intention to the retailer. The retailer then collects the responses from all consumers and awards the products using the following mechanism: if the number of consumers who intend to purchase is less than the capacity \((C)\), each consumer who expresses the willingness to purchase obtains one unit and pays the first-period price \((p_1)\); otherwise, the products are allocated among those consumers with equal probability. \(^3\)

For the consumers who obtain the product in the first period, each of them receives an additional valuation, \( \varepsilon \). Let us denote the distribution and density of \( \varepsilon \) by \( G \) and \( g \). If a consumer’s total valuation \( v + \varepsilon \) ends up being lower than the first-period price minus the restocking fee, the consumer returns the product by the end of the first period. One exception is when the refund amount is zero; in this case, there is no incentive for any consumer to return the product. In the basic model, we assume that the retailer recertifies or remanufactures the returned products and then sells them as new. That is, the consumers are not able to tell the difference between the returns and the new products. To highlight the interplay between intertemporal pricing and consumer

\(^3\)The micro-foundation of this assumption is provided by Cachon and Swinney (2009): conceptually, suppose that the consumers form a queue in front of the retailer and they all have equal probability to be in the front of the queue. Thus, consumers who express the willingness to purchase get to obtain one unit with equal probability.
returns, we abstract away the cost associated with this recertification/ remanufacturing process. In Section 4.5, we allow the retailer to sell the returned product without any recertification or remanufacturing process, and compares the differences in the retailer’s pricing strategy, return policy and profitability.

![Sequence of Events](image)

Figure 4.1: Sequence of Events

We assume that the second period sale is final and that the retailer does not accept any returns. This is a reasonable assumption as many retailers have final sales at the end of seasons and do not allow consumers to return the products. We further assume away the salvage value of the returns, so that the retailer must sell the products to consumers who are already present in the market. At the beginning of the second period, the retailer observes the remaining capacity \((C_2)\) and the number of consumers who are still in the market \((N_2)\), and discloses the information to all consumers.\(^4\) The retailer then selects the second-period price \((p_2)\) that maximizes the expected second-period profit based on \(N_2\) and \(C_2\). A consumer who intends to purchase indicates so to the retailer, who then awards the remaining capacity among the consumers who intend to purchase using the same mechanism as described above. We demonstrate the sequence of events

\(^4\)It is worth noting that even if the retailer does not disclose all information to the consumers, they can still infer the information from the announced second-period price in the rational expectation equilibrium. In Section 4.4, we demonstrate that the consumers' purchasing decisions only depend on the second-period price and their initial valuations.
in Figure 4.1 and summarize the notation used in the basic model in Table 4.1.

Table 4.1: Summary of Notation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of consumers at the beginning of the first period</td>
</tr>
<tr>
<td>$C$</td>
<td>Total capacity at the beginning of the first period</td>
</tr>
<tr>
<td>$p_1$</td>
<td>First-period price</td>
</tr>
<tr>
<td>$f$</td>
<td>Refund amount</td>
</tr>
<tr>
<td>$v$</td>
<td>Consumer’s initial valuation, realized prior to purchase</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Consumer’s add-on valuation, realized after purchase</td>
</tr>
<tr>
<td>$r$</td>
<td>Number of returns by the end of the first period</td>
</tr>
<tr>
<td>$v_1(p_1, f)$</td>
<td>Cutoff valuation given the first-period price and the refund amount</td>
</tr>
<tr>
<td>$Q_1(n_1, \cdot)$</td>
<td>The probability that $n_1$ consumers want to purchase in the first period</td>
</tr>
<tr>
<td>$Q_{1r}(m_1, \cdot)$</td>
<td>The probability that $m_1$ consumers return the purchase in the first period</td>
</tr>
<tr>
<td>$N_2$</td>
<td>Number of consumers remaining in the second period</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Remaining capacity at the beginning of the second period</td>
</tr>
<tr>
<td>$p_2$</td>
<td>Second-period price</td>
</tr>
<tr>
<td>$Q_2(n_2, \cdot)$</td>
<td>The probability that $n_2$ consumers want to purchase in the second period</td>
</tr>
</tbody>
</table>

In the next section, we analytically derive the optimal pricing scheme for the retailer and the corresponding purchasing strategy for the consumers. Following this, we numerically demonstrate the equilibrium properties of the optimal pricing patterns, purchasing strategies and the capacity choices.

4.4 Resell Returns as New

We first discuss the case in which the retailers recertify/ remanufacture the returns and sell them as new products in the second period. This model fits best with returned clothes due to misfit, duplicate purchases or simply buyer’s remorse. In all of the scenarios just mentioned, the quality of the product remains the same as the new ones, and thus the retailer’s handling costs is negligible. It also applies to consumer electronics and appliances as the cost of the recertification processes is relatively small when compared to the value of the products.
4.4.1 Analytical Results

We derive the sub-game perfect equilibrium using backward induction. Starting from the second period, we first characterize the consumers’ purchasing behaviors for any given second-period price. We then formulate a strategy for the retailer to determine the optimal second-period price to maximize the expected profit of the second period. Next, we investigate consumers’ strategy on choosing between purchasing immediately in the first period and postponing the decision to the second period. We derive the equilibrium exclusively on a cutoff valuation, which is a function of both the first-period price and the refund amount. Based on this cutoff valuation, we compute the retailer’s expected profit for both periods and obtain the retailer’s optimal pricing strategy and return policy for the first period by maximizing his expected two-period profit. The following subsections implement our plan to characterize the equilibrium behaviors. For ease of exposition, we use the uniform distribution $U[v, \overline{v}]$ for the consumer’s initial valuation because its posterior distribution is also uniform. We also indicate how the result can be modified if a general distribution is used.

Consumers’ Optimal Purchasing Strategy in the Second Period

Let us first consider the consumers’ optimal purchasing strategy in response to the second-period price ($p_2$). In a rational expectation equilibrium, the consumers’ purchasing decisions depend solely on their initial valuations and the second-period price, regardless of whether the retailer discloses the information on the remaining capacity and number of consumers. We assume that the mean of the add-on valuation ($\varepsilon$) is zero. Thus, obtaining the products yields a negative utility for consumers whose initial valuations are below $p_2$. They are better off walking away from the market. For the consumers whose valuations are above $p_2$, however, expressing their willingness to purchase the product is a better option. Note that the consumers may get a zero utility if there are more consumers who intend to buy the product than the available inventory, but in any case this does not alter their optimal strategy in the second period. In addition, the consumers’ best response is irrespective of whether the seller publicly announces his remaining inventory, nor does it depend on the consumers’ estimation of the number of other consumers in the market.

Retailer’s Optimal Pricing Strategy in the Second Period

We now characterize the optimal second-period pricing strategy given the first-period price ($p_1$) and the refund amount ($f$). Let $v_1(p_1, f)$ be the cutoff valuation such that a
consumer intends to purchase in the first period only if her observed valuation is above \( v_1 \). Let \( n_1 \) denote the number of consumers who intend to purchase in the first period, \( r \) the number of returns, and \( N_2 \) the number of consumers remaining in the second period. As the retailer can sell up to \( C \) items in the first period, we need to consider two scenarios: 1) all consumers who intend to purchase in the first period obtain the item for certain and 2) not every consumer who intends to purchase in the first period obtains the item for certain.

In the first scenario, the retailer faces \( N_2 = N - n_1 \) consumers in the second period. Since the consumer’s initial valuation follows an uniform distribution and the period-one cutoff valuation is \( v_1 \), the consumer valuations in the second period are uniformly distributed between \( v \) and \( v_1 \). Let \( C_2 \) denote the remaining inventory in this period, \( C_2 = C - n_1 + r \). The goal of the retailer is to pick a price \( (p_2) \) to maximize his expected profit in the second period. For any given \( p_2 \), the probability that a consumer’s valuation is above \( p_2 \) is \( \lambda_2 \equiv \frac{v_1 - p_2}{v_1 - v} \) while the probability that her valuation is below \( p_2 \) is \( \bar{\lambda}_2 \equiv 1 - \lambda_2 \).5 Moreover, the probability that \( n_2 \) consumers are willing to purchase the item at \( p_2 \) is binomial:

\[
Q_2(n_2, N_2, v_1, p_2) = C_{n_2}^{N_2} \cdot (\lambda_2)^{N_2-n_2} \cdot (\bar{\lambda}_2)^{n_2},
\]

where \( C_{n_2}^{N_2} = \frac{N_2!}{n_2!(N_2-n_2)!} \) is the binomial coefficient. The retailer’s expected profit can thus be expressed as

\[
\Pi_2(p_2) = \sum_{n_2=0}^{N_2} Q_2(n_2, N_2, v_1, p_2) \cdot (n_2 \land C_2) \cdot p_2,
\]

where \( n_2 \land C_2 = \min\{n_2, C_2\} \).

In the second scenario, the retailer faces consumers whose valuations fall into two different segments. That is, there are \( n_1 - C \) high-valued consumers whose valuations are uniformly distributed between \( v_1 \) and \( \bar{v} \). The rest of the \( N - n_1 \) consumers are low-valued and their valuations are uniformly distributed between \( v \) and \( v_1 \). Since the initial inventory is cleared out in the first period, the available inventory in the second period (\( C_2 \)) is the same as the number of returns (\( r \)). While the retailer’s goal is still to maximize his expected profit in the second period, he needs to consider two strategies: one is to only serve the high-valued consumers by setting \( p_2 \) to be above \( v_1 \), and the other is to serve all consumers by setting \( p_2 \) to be lower than \( v_1 \).

5For a general distribution \( F \), \( \lambda_2 \equiv \frac{F(v_1) - F(p_2)}{F(v_1) - F(v)} \)
Lemma 2. The retailer’s expected profit can be expressed as

$$
\Pi_2 (p_2) = \begin{cases} 
\sum_{n_2=0}^{n_1-C} Q_{2h}(n_2, n_1 - C, v_1, p_2) \cdot (n_2 \land r) \cdot p_2 & \text{if } p_2 \ge v_1 \\
\sum_{n_2=0}^{N-n_1} Q_{2a}(n_2, N - n_1, v_1, p_2) \cdot [(n_1 - C + n_2) \land r] \cdot p_2 & \text{if } p_2 < v_1.
\end{cases}
$$

(4.2)

(4.2) corresponds to the retailer’s profit when he only serves the high-valued consumers. Specifically, the term $Q_{2h}(n_2, n_1 - C, v_1, p_2)$ is the probability that $n_2$ out of the $n_1 - C$ consumers decide to purchase at $p_2$, given that the first-period cutoff valuation ($v_1$). The term $(n_2 \land r)$ indicates that the retailer can only sell up to the number of returns since the initial sale cleared the inventory. The bottom equation corresponds to the retailer’s profit when he decides to serve the entire market. In this case, all consumers who intended to purchase in the first period but did not obtain the item still want to purchase. In addition to those $n_1 - C$ consumers, the probability that $n_2$ out of the $N - n_1$ low-valued consumers want to purchase at $p_2$ is $Q_{2a}(n_2, N - n_1, v_1, p_2)$. Again, the retailer can only sell up to the number of returns for the same reason as stated above.

In both scenarios, the optimal second-period price, denoted by $p^*_2$, maximizes the retailer’s second-period expected profit:

$$p^*_2 = \arg \max_{p_2} \{ \Pi_2 (p_2) \}.
$$

(4.3)

Consumers’ Purchasing Strategy in the First Period

Having characterized both the retailer’s pricing strategy and the consumers’ purchasing strategy, we now move to the first period. The retailer announces both the first-period price ($p_1$) for the service and the refund amount ($f$). A consumer with valuation $v$ can either make the purchase in the first period or wait for the announcement of $p_2$ in the second period. As defined in the previous section, $v_1(p_1, f)$ is the cutoff valuation such that a consumer only makes the purchase in the first period if her valuation is above $v_1(p_1, f)$. Since consumers’ valuations are uniformly distributed between $\underline{v}$ and $\bar{v}$, with probability $\lambda_1 \equiv \frac{\bar{v} - v_1}{\bar{v} - \underline{v}}$ a consumer’s valuation is above $v_1$ and with probability $\bar{\lambda}_1 \equiv 1 - \lambda_1$ her valuation is below $v_1$. Thus, the probability that $n_1$ consumers out of $N$ consumers who intend to purchase the product at price $p_1$ and refund amount $f$ is

$$Q_1(n_1, N, v_1) = C_{n_1}^N \cdot (\bar{\lambda}_1)^{N-n_1} \cdot (\lambda_1)^{n_1}.$$
The first-period utility of a consumer whose valuation is above \( v_1 \) depends on the val-
uations of the other consumers. Let \( m_1 \) represent the number of consumers other than the consumer herself who intend to purchase at \( p_1 \). The corresponding probability is \( Q_1(m_1, N - 1, v_1) \) and \( m_1 \) can take any (integer) value between 0 and \( N - 1 \). While the consumer gets the product for sure if \( m_1 < C \), she gets the product with probability \( C / (m_1 + 1) \) if \( m_1 \geq C \).

Recall that each consumer’s add-on valuation \( \varepsilon \) follows a distribution of \( g(\cdot) \) and it is resolved at the end of the first period. If the consumer’s realized valuation \( v + \varepsilon \) is less than the refund amount and the refund is greater than zero, she returns the purchase right away with no additional cost incurs. The retailer treats the returned item as new.

Thus, the expected probability of a return is \( \lambda = \frac{1}{\pi_{v_1}} \int_{v_1}^{f} G(f - v)dv \) and the probability of \( r \) returns given \( m_1 \) other consumers who intend to purchase in the first period is

\[
Q_{1r}(m_1, r) = C_{m_1 \wedge C} \cdot (\lambda)^r \cdot (\overline{\lambda})^{m_1 \wedge (C-r)}.
\]

For a consumer whose valuation \( v \) is above \( v_1 \), her expected utility from making the purchase, denoted by \( \pi_{1b}(v, v_1) \), can be written as follows:

\[
\pi_{1b}(v, v_1) = EU(v) \cdot \left[ \sum_{m_1=0}^{N-1} Q_1(m_1, N - 1, v_1) \cdot \left( 1 \wedge \frac{C}{m_1 + 1} \right) \right], \quad (4.4)
\]

where \( EU(v) = f - p_1 + \int_{\varepsilon \geq f - p_1} ((\varepsilon + v) - (f - p_1)) g(\varepsilon) d\varepsilon \) and represents the consumer’s expected valuation if she obtains an item in the first period. The second term in the brackets is the probability that she can obtain the item successfully. The consumer, being strategic, also anticipates the second-period price and her probability of getting the product in the second period. Thus, her expected utility from waiting for the second period, \( \pi_{1w}(v, v_1) \), can be written as

\[
\pi_{1w}(v, v_1) = \sum_{m_1=0}^{N-1} Q_1(m_1, N - 1, v_1) \cdot \left( \sum_{r=0}^{m_1 \wedge C} Q_{1r}(m_1, r) \cdot \pi_2(v, p_2^r, C_2, N_2) \right), \quad (4.5)
\]

where \( \pi_2(p_2^*, C_2, N_2) \) is the consumer’s expected utility in the second period given that there are \( N_2 = N - (m_1 \wedge C) \) consumers and \( C_2 = C - (n \wedge C) + r \) remaining capacity. Therefore,

\[
\pi_2(v, p_2^*, C_2, N_2) = (v - p_2^*)^+ \cdot \sum_{m_2=0}^{N_2-1} Q_2(m_2, N_2 - 1, v_1, p_2^*) \cdot \left( 1 \wedge \frac{C_2}{m_2 + 1} \right).
\]
Observe that both $\pi_{ib}(v, v_1)$ and $\pi_{iw}(v, v_1)$ are functions of the cutoff valuation $v_1(p_1, f)$. Thus, $v_1$ solves

$$\pi_{ib}(v_1, v_1) = \pi_{iw}(v_1, v_1).$$

(4.6)

**Retailer’s Optimal Pricing Strategy in the First Period**

Next, we characterize the retailer’s optimal first-period pricing strategy. To this end, we first derive the retailer’s two-period expected profit given $p_1$ and $f$, denoted by $\Pi(p_1, f)$, as follows:

$$\Pi(p_1, f) = \sum_{n_1=0}^{N} Q_1(n_1, N, v_1) \cdot \left( (C \land n_1) \cdot p_1 + \sum_{r=0}^{C \land n_1} Q_{1r}(C \land n_1, r) \cdot (-fr + \Pi_2(p_2)) \right),$$

where $\Pi_2$ follows (4.1) if $n_1 \leq C$ and (4.2) if $n_1 > C$. Given the expression of $\Pi(p_1, f)$, the retailer chooses the first-period price $p_1^*$ and refund $f^*$ that maximize her expected profit, i.e.,

$$(p_1^*, f^*) = \arg \max_{p_1, f} \Pi(p_1, f) \quad \text{and} \quad \Pi^* = \Pi(p_1^*, f^*).$$

Having completed the analytical derivations of the equilibrium behaviors of consumers and obtained the optimal pricing strategies for the retailer, we are ready to summarize the three effects of a generous refund policy. First, it induces higher-valued consumers to purchase early as it provides an insurance effect and reduces consumers’ valuation risks. This effect is captured by the expressions of $EU(v)$ in (4.4). Second, it exacerbates the demand uncertainty in the second period, as the number of returns $r$ is not a priori predictable. This subsequently allows the retailer to credibly commit to a dynamic pricing mechanism with capacity rationing, and it creates another driving force for the high-valued consumers to purchase early. Last, it reduces capacity rationing risk in the second period which may result some consumers to strategically delay their purchases. In the following section, we are going to demonstrate our findings via numerical analysis.

### 4.4.2 Observations

In this section, we answer the research questions posted in the introduction. First, we show the retailer’s first-period optimal pricing strategy. Given the initial capacity and the market size, what should be the price of the product? Should the retailer offer a generous or stringent refund policy? Next, we explain the consumers’ first-period
purchasing behaviors in response to the retailer’s pricing. Numerical examples\(^6\) are used to demonstrate the non-monotonic patterns. We also discuss the relationship between the pricing under the refund policy and that under no refund policy.

![Figure 4.2: Optimal Pricing Strategy](image1)

![Figure 4.3: Optimal Refund Ratio](image2)

**Observation 1.** The optimal first-period pricing strategy \(\{p_1, f^\} \) need not to be monotonic in capacity.

Figure 4.2 plots the optimal first-period price and the refund amount as a function of capacity. It is clear that the more scarce the capacity is, the higher the first-period price can be. On the other hand, the refund amount displays a non-monotonic pattern. To further demonstrate the relationship between the refund amount and the first-period price, we plot the ratio of the two with respect to capacity in Figure 4.3. When the capacity is relatively small compared to the market size, the retailer provides a “nearly risk-free” refund policy by setting a very high refund ratio. Since purchasing in the first period gives the consumers an opportunity to learn the add-on value of the product, offering a generous refund policy reduces their valuation risk in case they dislike the product. In another word, if the consumers’ add-on valuation results a low total valuation, they can return the products and only incur a small loss. Thus, such a generous refund policy encourages high-valued consumers to purchase early. Since the initial capacity is small, any returned item can be sold easily in the second period to the consumers who are still in the market. In the case that there are many high-valued consumers.

---

\(^6\)We use the following parameters for numerical analysis: Number of consumers, \(N = 20\). Consumers’ initial valuation (\(v\)) follows a uniform distribution: \(U(2, 4)\). Consumers’ add-on valuation (\(\varepsilon\)) follows a normal distribution: \(N(0, 1)\).
consumers who did not obtain the products in the first period, the second-period price may even be adjusted upwards.

It is also worthwhile to note that the generous refund policy remains optimal until the capacity becomes sufficiently large. When the capacity is abundant, the retailer needs to offer a more stringent refund policy. That is, in addition to a lower first-period price, he needs to offer a much lower refund amount to deter consumers from returning the products. In this situation, the retailer has more pressure to clear the inventory. Excess returns from the first period will push down the second-period price even further, and consumers will then wait strategically for the deep discount.

The retailer has to adjust the refund policy when the capacity is close to the market size. The retailer knows that with the option of return, his pressure of clearing inventory becomes dominant over loss caused by returns. Thus, his goal is to encourage as many consumers to purchase as possible by lowering the first-period price while increasing the refund amount.

Observation 2. Consumers’ optimal first-period cutoff valuation can also be non-monotonic in capacity.

As shown in Figure 4.4, the strategic consumers respond to the retailer’s pricing strategy \(\{p_t^*, f_t^*\}\) in the following way. When the capacity is relatively scarce, the retailer sets a higher first-period price while offering a high refund ratio. High-valued consumers are motivated to purchase early by two factors: 1) purchasing early can help them learn the true valuation while only incurring a small loss in the event that they dislike the products, and 2) there are capacity rationing risks in both periods. Though it is likely
that the inventory is cleared in the first period, the lower-valued consumers choose to wait as they anticipate that the price will likely be lower if there are returns. It is worth noting that if the demand is greater than the total capacity and there are returns available at the second period, the retailer may choose to only serve the high-valued consumers and set the price to be even higher than the first-period price. As the capacity gets larger, the retailer adjusts the first-period price downwards while maintaining a high refund ratio to encourage more consumers to purchase in the first period. The consumers, being strategic, realize that there may not be available inventory in the second period. Since they are protected by the generous refund policy, they are better off purchasing early.

When the capacity becomes abundant, being able to sell more inventory becomes dominant in the retailer’s goal of profit maximization. Therefore, he sets a stringent refund policy to discourage consumers from returning the product. As the valuation risk gets large, the consumers are hesitant to purchase in the first period and they expect the second-period price to be much lower. As a result, they adjust the cutoff valuation upward and only high-valued consumers purchase early. When the capacity is close to the market size, the retailer adjusts the first-period price to be even lower while offering a higher refund amount to encourage everyone to purchase early. Thus, we see that the cutoff valuation drops to the lowest value.

**Observation 3.** Offering a relatively generous refund policy is more profitable to the retailer than offering no refund when the handling cost is small.

Figure 4.5 shows that the retailer’s profit is higher when offering refund than when offering no refund. A generous refund policy allows the retailer to set a higher first-period price, as we can see in Figure 4.2. While the difference between the two prices are not big, the consumers’ response are drastically different. Figure 4.4 demonstrates the difference of the cutoff valuations under the two prices. This indicates that a relatively generous refund policy really helps mitigate consumers’ hesitance in purchasing early as consumers feel that the valuation risk is shared between the retailer and themselves. Even though the retailer’s marginal profit decreases in both types of pricing, the advantage of offering partial refund is more salient when the capacity is large. In the next section, we consider the case that the retailer sells the product without any recertification or remanufacturing process. Our goal is to compare the equilibrium outcomes of the retailer’s pricing strategy, return policy and profitability, and comment on the impact of positive recertification/ remanufacturing cost.
4.5 Sell Returns as Used

We now discuss the case that the retailer sells the returns as lower-quality products in the second period. To incorporate this feature, we use a discount factor (\(\beta\)) to represent the lower valuations the consumers have on the returned products. Thus, a consumer in the second period has a valuation of \(v\) for the new products and \(\beta v\) for the returns. The retailer is free to pick the second-period prices for both the new and the used products depend on their respective inventory levels. This case is suitable for most sellers in the retail industry. The consumers who remain in the market then determine whether to purchase and which one to purchase.

4.5.1 Analytical Results

Similar to the case of selling returns as new, we characterize the equilibrium using backward induction. We start with the consumers’ purchasing behaviors in the second period for any given second-period prices, both for the new items and the returned items. We then formulate a strategy for the retailer to determine the optimal second-period prices to maximize his expected profit. Afterwards, we move to the first period and investigate consumers’ strategy on choosing between purchasing immediately in the first period and waiting for the second period to make the decision. Since the process is very similar to the case of selling returns as new, we highlight the main difference in this section and defer the derivations to the Appendix.

**Consumers’ Optimal Purchasing Strategy in the Second Period.** Let us first consider the consumers’ optimal purchasing strategy in response to the second-period prices, \(p_{2n}\) for the new products and \(p_{2r}\) for the returns. Since a consumer with valuation \(v\) gets a utility of \(v - p_{2n}\) from purchasing a new item and \(\beta v - p_{2r}\) from a returned item, we assume that \(p_{2n} \geq p_{2r}/\beta\) to preserve the idea that new items are sold at a premium price. To define a meaningful cutoff valuation, we let \(v_2 \equiv \min\{\max\{(p_{2n} - p_{2r})/\beta, v, p_{2n}\}, \bar{v}\}\) to ensure that the cutoff is within the bounds. If a consumer’s valuation is higher than \(p_{2n}\), she gets a positive utility from purchasing both a new item and a returned item. However, she intends to purchase a new item only if her valuation is above \(v_2\). If her valuation is lower than \(p_{2r}/\beta\), she is better off walking away without any purchase. We use the following table to summarize consumers’ optimal purchasing strategy in this period.

Table 4.2 indicates that if the returned products are perceived to have a lower quality,
there is a natural market segmentation in the second period. This turns out to be potentially beneficial for the retailer, as he need not create a separate product line when facing consumers with heterogeneous preferences. In the following analysis, we embed this market segmentation idea to the retailer’s and consumers’ decision making processes.

**Retailer’s Optimal Pricing Strategy in the Second Period.** In order to characterize the optimal second-period pricing strategy given the first-period price \( p_1 \) and the refund amount \( f \), we again need to consider two scenarios. The first one is that all consumers who intend to purchase in the first period obtain an item for certain. The retailer faces \( N_2 = N - n_1 \) consumers in the second period. The valuations of these consumers are uniformly distributed between \( v \) and \( v_1 \). Let \( C_2 \) denote the remaining new inventory and thus \( C_2 = C - n_1 \). The goal of the retailer is to pick prices for the new items \( p_{2n} \) and for the returned items \( p_{2r} \) to maximize his expected profit in the second period.

The second one is that not every consumer who intends to purchase in the first period obtains an item for certain. The retailer faces two segments of consumers, \( n_1 - C \) of them are high-valued consumers and the rest of the \( N - n_1 \) consumers are low-valued consumers. The high-valued consumers are uniformly distributed between \( v_1 \) and \( \bar{v} \) while that of the low-valued consumers are uniformly distributed between \( v \) and \( v_1 \). Since the only available inventory is the same as number of returns \( r \), the retailer only needs to select one price \( p_{2r} \) to maximize his expected profit in the second period. He selects the strategy that generates the higher expected second-period profit from the following: 1) only serve the high-valued consumers by setting \( p_{2r} \) to be above \( \beta v_1 \) and 2) serve all consumers by setting \( p_{2r} \) to be lower than \( \beta v_1 \).

**Consumers’ Purchasing Strategy in the First Period.** We now move to the first period and characterize the consumers’ symmetric purchasing strategy. Recall that the retailer announces both the first-period price \( p_1 \) for the item and the refund amount \( f \) at the beginning of this period. A consumer with valuation \( v \) can either purchases in the first period or defers the decision until the announcement of \( p_{2n} \) and \( p_{2r} \) in the second period. As defined previously, \( v_1(p_1, f) \) is the cutoff valuation such that a consumer only makes the purchase in the first period if her valuation is above this cutoff. For the consumer with this cutoff valuation, we first write down the expected utility from

<table>
<thead>
<tr>
<th>valuation</th>
<th>strategy</th>
<th>no purchases</th>
<th>purchase returns</th>
<th>purchase new products</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &lt; p_{2r}/\beta )</td>
<td>( p_{2r}/\beta \leq x &lt; v_2 )</td>
<td>( v_2 \leq x )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
purchasing in the first period and that from waiting for the second period to make the decision. Then, we equate the two equations to find the cutoff valuation.

**Retailer’s Optimal Pricing Strategy in the First Period.** Finally, we are able to characterize the retailer’s optimal first-period pricing strategy. We first derive the retailer’s two-period expected profit given $p_1$ and $f$, denoted by $\pi(p_1, f)$, as follows:

$$\Pi(p_1, f) = \sum_{n_1=0}^{N} Q_1(n_1, N, v_1) \cdot \left[ (n_1 \wedge C) \cdot p_1 + \sum_{r=0}^{n_1 \wedge C} Q_{1r}(n_1 \wedge C, r) \cdot (-f r + \Pi_2) \right], \quad (4.7)$$

where $\Pi_2$ corresponds to the retailer’s expected second-period profit and follows (4.8) if $n_1 \leq C$ and (4.10) if $n_1 > C$ provided in the appendix. Given the expression of $\Pi(p_1, f)$, the retailer chooses the first-period price $p_1^*$ and refund $f^*$ that maximize her expected profit, i.e.,

$$(p_1^*, f^*) = \arg \max_{p_1, f} \Pi(p_1, f) \quad \text{and} \quad \Pi^* = \Pi(p_1^*, f^*).$$

We have now completed the analytical derivations of the equilibrium behaviors of consumers and obtained the optimal pricing strategies for the retailer.

### 4.5.2 Observations

In this section, we explore numerically the effect of selling returns as lower-quality ones on both the retailer’s pricing strategy and the consumers’ purchasing strategy in the first period.

**Observation 4.** When returns are perceived to have a lower quality, the retailer can offer a much lower refund amount.

We compare the optimal first-period prices under the two cases, selling returns as used vs. selling returns as new. As we can see in Figure 4.6, the first-period prices in the case of selling returns as used are consistently lower when the capacity is scarce, though the differences are quite small. However, the refund amounts in the case of selling returns as used are much lower. This shows that when the returns are perceived to have a lower quality, high-valued consumers prefer the new products. This preference leads to a higher capacity rationing for the new product in the second period. Consequently, high-valued consumers are motivated to purchase early. In response to these consumers’ purchasing strategy, the retailer no longer needs to provide the “nearly risk-free” refund policy. He can compensate the possible loss of the consumers if they dislike the product
by setting a slightly lower first-period price. When the capacity becomes abundant, the retailer needs to lower the first-period price as well as the refund amount to achieve the following: 1) encourage more consumers with higher valuations to purchase early, and 2) discourage them from returning the products. It is also clear that the first-period prices under the two cases are the same while the refund amounts are much lower in the case of quality discount. The same interpretation described above applies as the retailer can transfer the add-on valuation risk to the high-valued consumers who prefer new products. When the capacity is close to the market size, the rationing effect of the new products in the second period looses its power and the pricing strategy under the two cases are almost the same.

Figure 4.6: Optimal Pricing Strategy with a Quality Discount Factor ($\beta = 0.8$)

Figure 4.7: Optimal Pricing Strategy with a Quality Discount Factor ($C = 3$)

Figure 4.7 explores more on the effect of the quality discount factor ($\beta$). This plot is generated under the scenario that the capacity is scarce. We can see that the first-period prices stay constant as long as the consumers have lower valuation on returns, i.e. $\beta < 1$. Compare to the case where consumers treat returns as new, the first-period prices are slightly lower. The refund amount drops sharply as $\beta$ gets smaller. This finding echoes with the argument presented in the previous paragraph: The retailer can take advantage of the preference high-valued consumers have on new products and transfer the add-on valuation risk to them. The smaller the discount factor is, the stronger preference the high-valued consumers has on new products. Thus, the lower refund amount the retailer needs to provide.

**Observation 5.** When the returns are perceived to have lower quality, capacity
rationing risk for the new products outweighs the valuation risk for higher-valued consumers. Consequently, these consumers are encouraged to purchase early despite a lower refund amount.

We compare the optimal cutoff valuations under the two cases, selling returns as used vs. selling returns as new. Figure 4.8 shows that the cutoff values are higher in the later case. The difference is smaller when the capacity is scarce but becomes much larger when the capacity is abundant. This demonstrates that facing a smaller capacity, the rationing effect is intensified as higher-valued consumers prefer the new products. As more capacity becomes available, consumers anticipate that there will be inventory available in the second period and the price can be lower. They then adjust the cutoff valuation so that only really high-valued consumers purchase, the ones in the middle range choose to forego the opportunity to learn their add-on valuations due to the lower refund amount. When the capacity is close to the market, the retailer sets a really low first-period price and higher refund amount to induce everyone to purchase. The cutoff valuation thus reduces to the lowest value.

We also plot the consumers’ cutoff valuations with respect to the discount factor in Figure 4.9. Though these cutoff valuations monotonically decrease in $\beta$, the values do not vary much when capacity is scarce. This can be interpreted as only consumers with sufficiently high valuations strongly prefer the new items over the returns.

**Observation 6.** When returns are perceived to have a lower quality, selling them separately does not impact retailer’s profit significantly.
We plot the retailer’s profit with respect to capacity under the two case, selling returns as used vs. selling returns as new. As we can see in Figure 4.10, the two curves are very close to each other. Figure 4.11 provides a different perspective; it plots the ratio of the retailer’s profit when selling returns as used against that when selling returns as new with respect to the quality discount factor, $\beta$. While the retailer’s profit monotonically increases in $\beta$, the profits do not vary drastically. The retailer’s performance is affected by 4% – 6%. Thus, when the cost of recertification or remanufacturing process is sufficiently large, the retailer may choose to forgo this process and simply sell the returns separately. In business practice, we notice that retailers often go through the recertification processes for items that have high values, such as consumer electronics and appliances. The costs associated with these processes are relatively small while the consumers have high valuations for the used items. On the other hand, books and household items with minor issues are often sold without any remanufacturing processes.

4.5.3 Comparison

In our basic model, we assume that there is no cost associated with the recertification/ remanufacturing process. Suppose that there is a cost associated, the retailer then need to choose between the two options: (1) recertify or remanufacture the returned products and sell them as new and (2) sell the returns as lower-quality products. As we have illustrated in the previous section, the latter strategy does not impact the retailer’s profitability significantly, and thus may be a preferred strategy when the recer-
tification/remanufacturing cost is sufficiently high. The intuition is as follows. Selling returns as used products effectively create a differentiated product in the second period. Such a separation between the new (higher-quality) and the used (lower-quality) products help the retailer further segment the remaining consumers in the second period. Specifically, the higher-valued consumers who strictly prefer the new products while the lower-valued consumers who strictly prefer the returns. Moreover, higher-valued consumers are encouraged to purchase early as the capacity risk in the second period becomes more pronounced. As a result, the seller can lower the refund amount significantly and transfer the valuation risk to the consumers. Thus, we provide a rationale for the business practice that some retailers do not actively engage in recertifying or remanufacturing the returned products: Selling returns as lower-quality products can facilitate retailers in market segmentation without physically creating new products lines.

4.6 Conclusions

Our paper documents the profitability of an online retailer’s refund policy in the presence of both consumer valuation uncertainty and strategic consumer behavior. We demonstrate the tight connection among capacity choice, price and refund amount, and how these decisions affect consumers’ purchasing behaviors. We identify three effects of a generous return policy. First, it induces higher-valued consumers to purchase early as it provides an insurance effect and reduces consumers’ valuation risk. As a result, it increases the allocation efficiency as only consumers with truly high valuations get to keep the products. Second, it exacerbates the demand uncertainty in the second period, and allows the retailer to credibly commit to a dynamic pricing mechanism. As each consumer acts independently based on her net valuation, the remaining capacity cannot be perfectly predicted and thus is only realized at the beginning of the second period. This creates another driving force for the high-valued consumers to purchase early. Last, it reduces the second-period capacity rationing risk, and may induce mid-valued consumers to strategically delay their purchases. These consumers anticipate that returns increase the availability of the product in the second period, and consequently drive the second-period price downward.

We also offer a new rationale for the business practice that some retailers do not actively engage in recertification or remanufacturing process. Selling returns as lower-quality products effectively create a new line of products in the second period. This helps the retailer further segment consumers and set the second-period price to maximize his expected profit. Moreover, higher-valued consumers who have a strong preference on the
new products are induced to purchase early as the capacity rationing risk in the second period becomes more pronounced. As a result, the retailer can lower the refund amount significantly and transfer some of the valuation risk to the consumers.

Appendix.

In the appendix, we provide the proof of Lemma 2 and the detailed analysis for Section 4.5.

Proof of Lemma 2. When $p_2 \geq v_1$, none of the low-valued consumers intends to purchase. In addition, the probability that $n_2$ out of the $n_1 - C$ high-valued consumers are willing to purchase the item at is

$$Q_{2h}(n_2, n_1 - C, v_1, p_2) = C^{n_1-C} \cdot \left(\lambda_{2h}\right)^{n_1-C-n_2} \cdot \left(\lambda_{2h}\right)^{n_2},$$

where $\lambda_{2h} = \frac{v_1 - p_2}{v_1}$ and $\lambda_{2h} = 1 - \lambda_{2h}$. When $p_2 < v_1$, all of the high-valued consumers intend to purchase. Moreover, the probability that $n_2$ out of the $N - n_1$ low-valued consumers are willing to purchase is

$$Q_{2a}(n_2, N - n_1, v_1, p_2) = C^{N-n_1} \cdot \left(\lambda_{2a}\right)^{N-n_1-n_2} \cdot \left(\lambda_{2a}\right)^{n_2},$$

where $\lambda_{2a} = \frac{v_1 - p_2}{v_1}$ and $\lambda_{2a} = 1 - \lambda_{2a}$. □

Equilibrium Derivations for Section 4.5. Let $n_1$ denote the number of consumers who intend to purchase in the first period, $r$ the number of returns, and $N_2$ the number of consumers remaining in the second period. As the retailer can sell up to $C$ items in the first period, we consider two scenarios arise naturally.

In the first scenario, all consumers who intend to purchase in the first period obtain an item for certain ($n_1 \leq C$). For any given $p_{2n}$, the probability that a consumer’s valuation is above $v_2$ is $\lambda_{2n} \equiv \frac{v_1 - p_{2n}}{v_1}$ while the probability that her valuation is below $p_{2n}$ is $\lambda_{2n} \equiv 1 - \lambda_{2n}$. Thus, the probability that $n_{2n}$ consumers are willing to purchase new items at $p_{2n}$ is

$$Q_{2n}(n_{2n}, N_2, v_2, p_{2n}) = C^{N_2} \cdot \left(\lambda_{2n}\right)^{N_2-n_{2n}} \cdot \left(\lambda_{2n}\right)^{n_{2n}}.$$

Let $\lambda_{2r} \equiv \frac{v_2 - p_{2r}/\beta}{v_2}$ denote the probability that a consumer’s valuation is in between $p_{2r}/\beta$ and $v_2$, and let $\lambda_{2r} \equiv 1 - \lambda_{2r}$. The probability that $n_{2r}$ consumers are willing to purchase returned items at $p_{2r}$ is

$$Q_{2r}(n_{2r}, N_2, v_2, p_{2r}) = C^{N_2} \cdot \left(\lambda_{2r}\right)^{N_2-n_{2r}} \cdot \left(\lambda_{2r}\right)^{n_{2r}}.$$

Thus, the retailer’s expected profit is

$$\Pi_2(p_{2n}, p_{2r}) = \sum_{n_{2r}=0}^{N_2} \left[ Q_{2n}(n_{2n}, N_2, v_2, p_{2n}) \cdot \left( \sum_{n_{2r}=0}^{N_2-n_{2n}} Q_{2r}(n_{2r}, N_2, v_2, p_{2r}) \cdot (n_{2r} \wedge r) \cdot p_{2r} \right) \right].$$

(4.8)

The optimal second-period prices, $p_{2n}^*$ and $p_{2r}^*$, maximize the second-period expected profit:

$$\left( p_{2n}^*, p_{2r}^* \right) = \arg \max_{p_{2n}, p_{2r}} \{ \Pi_2(p_{2n}, p_{2r}) \}.$$  

(4.9)

In the second scenario, not every consumer who intends to purchase in the first period obtains an item for certain ($n_1 > C$). The retailer faces two segments of consumers, $n_1 - C$ of them are high-valued consumers and the rest of the $N - n_1$ are low-valued consumers. The former are uniformly distributed between $v_1$ and $\bar{v}$ while the latter are uniformly distributed between $v$ and $v_1$. Since the only available inventory is the same as the number of returns ($r$), the retailer only needs to select one price ($p_{2r}$) to maximize his expected profit in the second period. He considers two strategies: one is to only serve the high-valued consumers by setting $p_{2r}$ to be above $\beta v_1$, and the other is to serve all consumers by setting $p_{2r}$ to be lower than $\beta v_1$.

When $p_{2r} \geq \beta v_1$, the probability that $n_{2r}$ out of the $n_1 - C$ high-valued consumers intend to purchase the item at is

$$Q_{2rh}(n_2, n_1 - C, v_1, p_{2r}) = C_{n_2}^{m_1-C} \cdot (\bar{\lambda}_{2h})^{m_1-n_2} \cdot (\lambda_{2h})^{n_2},$$

where $\lambda_{2h} = \frac{p_{2r} - p_{2r}/v_1}{v_{1} - \bar{v}}$ and $\bar{\lambda}_{2h} = 1 - \lambda_{2h}$. When $p_{2r} < \beta v_1$, all of the high-valued consumers intend to purchase. Moreover, the probability that $n_{2r}$ out of the $N - n_1$ low-valued consumers are willing to purchase is

$$Q_{2ra}(n_2, N - n_1, v_1, p_{2r}) = C_{n_2}^{N-n_1} \cdot (\bar{\lambda}_{2a})^{N-n_1-n_2} \cdot (\lambda_{2a})^{n_2},$$

where $\lambda_{2a} = \frac{v_{1} - p_{2r}/v_1}{v_{1} - \bar{v}}$ and $\bar{\lambda}_{2a} = 1 - \lambda_{2a}$. The retailer’s expected profit can thus be expressed as

$$\Pi_2(p_{2r}) = \begin{cases} \sum_{n_{2r}=0}^{n_1-C} Q_{2rh}(n_2, n_1 - C, v_1, p_{2r}) \cdot (n_2 \wedge r) \cdot p_{2r} & \text{if } p_{2r} \geq \beta v_1 \\ \sum_{n_{2r}=0}^{N-n_1} Q_{2ra}(n_2, N - n_1, v_1, p_{2r}) \cdot (n_1 - C + n_2) \wedge r \cdot p_{2r} & \text{if } p_{2r} < \beta v_1 \end{cases}$$

(4.10)
The optimal second-period price, denoted by \( p_{2r}^* \), maximizes the retailer’s second-period expected profit:

\[
p_{2r}^* = \arg \max_{p_{2r}} \{ \Pi_2 (p_{2r}) \}.
\]

Having characterized the optimal second-period pricing, we can move to the first period by first characterize the cutoff valuation. Given the announced first-period price \( (p_1) \), the refund amount \( (f) \), and the cutoff valuation \( (v_1) \) at the beginning of this period, the probability that \( n_1 \) consumers out of \( N \) consumers intend to purchase is

\[
Q_1(n_1, N, v_1) = \binom{N}{n_1} \cdot \left( \lambda_1 \right)^{N-n_1} \cdot (\lambda_1)^{n_1},
\]

where \( \lambda_1 \equiv \frac{v_1 - p_1}{f - p_1} \) and \( \lambda_1' \equiv 1 - \lambda_1 \) represent the probabilities that a consumer’s valuation are above and below \( v_1 \) respectively. As the analysis we carry out in Section 4.4, the consumer’s expected utility from purchases in the first period is

\[
\pi_{1b}(v, v_1) = EU(v) \cdot \left[ \sum_{m_1=0}^{N-1} Q_1(m_1, N-1, v_1) \cdot \left( 1 + \frac{C}{m_1 + 1} \right) \right],
\]

where \( EU(v) = f - p_1 + \int_{\varepsilon>v} ((\varepsilon + v) - (f - p_1)) g(\varepsilon)d\varepsilon \) and represents the consumer’s expected valuation if she obtains an item in the first period. The second term in the bracket is the probability that she can obtain the item successfully. The consumer also anticipates the second-period price as well as computes her expected utility from waiting for the second period:

\[
\pi_{1w}(v, v_1) = \sum_{m_1=0}^{N-1} Q_1(m_1, N-1, v_1) \cdot \left( \sum_{r=0}^{m_1 \land C} Q_2(m_1, r) \cdot \pi_2 (v, p_{2n}^*, p_{2r}^*, C_2, r, N_2) \right),
\]

where \( \pi_2 (v, p_{2n}^*, p_{2r}^*, C_2, r, N_2) \) is the consumer’s expected utility in the second period given that there are \( N_2 = N - (m_1 \land C) \) consumers and \( C_2 = C - (m_1 \land C) \) remaining capacity. Therefore,

\[
\pi_2 (p_{2n}^*, p_{2r}^*, C_2, r, N_2) = 1_{\{C_2>0,x\geq v_2\}} \cdot EU_1 + 1_{\{C_2>0,x<v_2\}} \cdot EU_2 + 1_{\{C_2=0,p_{2r}^*\geq \beta v_1\}} \cdot EU_3 + 1_{\{C_2=0,p_{2r}^*< \beta v_1\}} \cdot EU_4.
\]

The first term in the above equation,

\[
EU_1 = (v - p_{2n}^*)^+ \cdot \sum_{m_{2n}=0}^{N_{2n}-1} Q_2n(m_{2n}, N_2-1, v_2, p_{2n}^*) \cdot \left( 1 + \frac{C_2}{m_{2n} + 1} \right)
\]
represents the consumer’s expected utility given that there are new items available in the second period and she wants a new item. Similarly,

\[ EU_2 = (\beta v - p_{2r}^*)^+ \cdot \sum_{m_{2n}=0}^{N_2-1} \sum_{m_{2r}=0}^{N_2-1-m_{2n}} Q_{2r}(m_{2r}, N_2 - 1, v_2, p_{2r}^*) \cdot \left(1 \wedge \frac{r}{m_{2r} + 1}\right) \]

represents the consumer’s expected utility given that she wants to purchase one of the returns even though there are new items available. The third term in the above equation,

\[ EU_3 = (\beta v - p_{2r}^*)^+ \cdot \sum_{m_{2r}=0}^{m_1-C} Q_{2r}(m_{2n}, m_1 - C, v_1, p_{2r}^*) \cdot \left(1 \wedge \frac{r}{m_{2n} + 1}\right) \]

thus represents the consumer’s expected utility when there are only returned items available and the price is set to serve only those who expressed interest to purchase in the first period but did not actually get the item. Finally,

\[ EU_4 = (\beta v - p_{2r}^*)^+ \cdot \sum_{m_{2n}=0}^{N-m_1} Q_{2r}(m_{2r}, N - m_1, v_1, p_{2r}^*) \cdot \left(1 \wedge \frac{r}{m_{2r} + m_1 - C + 1}\right) \]

represents the consumer’s expected utility when there are only returns available and the price is set to serve both segments of consumers.

Since both \( \pi_{1b}(v, v_1) \) and \( \pi_{1w}(v, v_1) \) are functions of the cutoff valuation \( v_1(p_1, f) \). Thus, \( v_1 \) solves

\[ \pi_{1b}(v_1, v_1) = \pi_{1w}(v_1, v_1). \] (4.11)
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