A New Approach in Supply Chain Design: studies in reverse logistics and nonprofit settings

By

Gemma Berenguer-Falguera

A dissertation submitted in partial satisfaction of the requirements for the degree of Doctor of Philosophy in Engineering - Industrial Engineering and Operations Research in the Graduate Division of the University of California, Berkeley

Committee in charge:

Professor Zuo-Jun Shen, Chair
Professor George Shanthikumar
Professor Xuanming Su

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Abstract

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This dissertation contributes to the supply chain design literature by providing a novel solution approach and presenting new theoretical and applied models. Using the proposed solution approach, we are able to formulate models that consider joint facility location and inventory management problems in a stochastic environment. Due to these uncertainties, it is important to design the supply chain that better adapts to unexpected changes. The distribution network studied is a three-tiered supply chain that consists of a central plant that supplies to a set of distribution centers (DCs), which then ship to the final retailer.

In Chapter 2, we study several joint location-inventory problems. In particular, we consider cases that already exist in the literature (the uncapacitated and capacitated facility cases) and other novel ones (with correlated retailer demand, stochastic lead times, and multi-commodities). We show how to formulate these problems as conic quadratic mixed-integer problems. We compare with the existing modeling and solution methods to state that the new conic integer programming approach not only provides a more general modeling framework but also leads to fast solution times in general. Valid inequalities, including extended polymatroid and extended cover cuts, are added to strengthen the formulations and improve the computational results.

In Chapter 3, we employ the technique presented in the previous chapter to explore the supply chain design problem that incorporates reverse logistics decisions. Supply chains with returned products are receiving increasing attention in the operations management community and this chapter studies a capacitated facility location model with bidirectional flows and marginal value of time for returned products. While at the retailers’ site, products can be shipped back to the supplier for reprocessing. Each DC is capacitated and handles stocks of new and/or returned product. The model is a nonlinear mixed-integer program that
optimizes DC location and allocation between retailers and DCs. We show that it can be converted to a conic quadratic program which can then be efficiently solved. Some valid inequalities are added to the program to improve computational efficiency. We conclude by reporting numerical experiments that reveal some interesting properties of the model.

In Chapter 4, we study another applied area that is getting increasing attention in the operations management community: the nonprofit sector. Under some circumstances nonprofit practices can significantly differ from for-profit ones and this might require a different design of the supply chain. This chapter starts with a definition of the nonprofit supply chain identifying its major managerial challenges. Then, we provide a summarized comprehensive overview of current operations management literature that address each major challenge. The second part of the chapter studies a particular nonprofit supply chain design problem in the context of a humanitarian organization distributing food in a less secure area. We provide a model framework and support it with numerical experiments.
Als meus pares, Dolors i Eudald.
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Chapter 1

Introduction

1.1 Supply chain design

A supply chain is “a set of organizations directly linked by one or more of the upstream and downstream flows of products, services, finances, and information from a source to a customer” (Mentzer et al. (2001)). During the past two decades, the raise of new technologies has increased the opportunities in supply chain communication by allowing visibility through the whole chain. Therefore, the study of the entire supply chain has become possible and nowadays it is one of the major topics of study by Operations Research/Management Science (OR/MS) scholars. There are numerous definitions of supply chain management and the definition presented in this dissertation is a version adapted from Snyder and Shen (2011). Hence, supply chain management is the set of practices required to perform the functions of a supply chain to achieve a certain objective. This objective depends on the particular supply chain context, that can focus on achieving more efficiency, reducing costs, or increasing profitability, to give some examples.

These studies are challenging due to the complexity of the supply chain structure and its multiple stakeholders. The design of the supply chain is one of the most important subdisciplines and encompasses a long list of decisions. These decisions include facility location, production and distribution schedules, inventory levels, number of stages or echelons, assignments between plant and distribution center (DC) and DC and customer, supplier selection, product selection, market selection, and the type of technology to use.
Chapter 1. Introduction

Most of these decision variables are strategic in nature and this is important since the decisions made will impact the firm’s operations in the long run (for the next years or decades). Managers should select a subset of these decision variables to reduce the complexity of the problem by identifying the critical challenge(s) that the supply chain faces (or might potentially face). To illustrate, there is a categorization of supply chain design challenges in costs, time, and uncertainty (Tomlin (1999)). Some supply chains are focused on cost per product unit or shipping unit, others need to prioritize final product lead times, while the third category might need to address uncertainty in the form of demand, process, or supply uncertainty (Lee and Billington (1993)). While all categories are important, this dissertation studies the design of supply chains with a strong focus on costs and demand uncertainty.

1.2 The integrated supply chain design problem

The idea of accounting for inventory costs while making facility location decisions of a network has been developed during the past decade by different scholars. The so-called “integrated supply chain design problem” embraces this idea simultaneously considering location, shipment, and inventory decisions in the same model. Usually, these problems’ goal is to minimize costs when demand is stochastic. In order to deal with demand uncertainty some organizations aggregate stocks in a common location to better respond to demand picks in what is known as “risk pooling strategy”. This practice is mathematically modeled with an economies of scale (concave) term that makes the problem nonlinear and, thus, harder to solve. Eppen (1979) already studied the cost benefits of grouping uncertain demand of retailers in a single order to save in holding costs. Borrowing Eppen’s idea, Shen et al. (2003) and Daskin et al. (2002) were the first to introduce location-inventory models with risk pooling.

Models of the integrated supply chain family share the same structure as shown in Figure 1.1. It is a three-tiered supply chain with a central plant, a set of distribution centers (DCs) that store and distribute supplies, and, as downstream components, the retailers.

Different versions of the problem have been studied (for a review refer to Shen (2007b)). Some of the most representative ones account for uncertain supply (Qi and Shen (2007) and Cui et al. (2010)), multiple commodities (Shen (2005)), vehicle routing (Shen and Qi (2007)), customer service (Shen and Daskin (2005)), and multi-echelon distribution (Romeijn et al. (2007) and Mak and Shen (2009)). In this dissertation we study the seminal model with uncapacitated DCs (Shen
et al. (2003) and Daskin et al. (2002)) and the model that assumes that DCs are capacitated (Özsen et al. (2008)). Furthermore, we provide generalized versions of the capacitated model incorporating correlated retailer demands, stochastic lead times, and multi-commodities with correlation between commodity demand. Two other novel models based on two different applied areas are presented: a closed-loop supply chain model and a nonprofit/humanitarian delivery model.

1.3 Solution approaches to the integrated supply chain design problem

The most common modeling approaches in supply chain design are: deterministic analytical models, stochastic analytical models, economic models, and simulation models (Beamon (1998)). Depending on the modeling approach a different set of solution methodologies will need to be employed. This dissertation is based on stochastic analytical models in which at least one of the variables is unknown and is assumed to follow a probability distribution. Often, stochastic supply chain design problems result in integer or mixed-integer nonlinear optimization problems, which are hard-to-solve problems. In the past decade, La-
Chapter 1. Introduction

grangian relaxations and decomposition methods have been widely used to solve these problems. While learning these techniques and realizing that each problem requires specific algorithms to solve the relaxed subproblems\(^1\), the following question became the center of the first part of this dissertation – *Is there a common technique that allows us to solve different models from the same family of supply chain design problems with similar or better computational efficiency than the techniques employed up to now but with less intricately designed algorithms?* Conic programming seemed to be the best candidate.

Conic programming has been extensively employed to solve problems related to different areas of applied research (e.g., portfolio selection, energy planning, antenna array weight design) but, despite its apparent potential and to the best of our knowledge, no one had used this technique for supply chain design problems prior to the present work. Chapter 2 shows how different location-inventory models with stochastic demand can be solved to optimality by using *conic quadratic mixed-integer programming* (CQMIP). This approach provides flexibility to the modeler who can solve existent problems and novel versions that were not solvable before. Furthermore, the numerical experiments show comparable computational efficiency to that of other resolution methods for the same version of the model and the same numerical instances.

The cutting plane method has been extensively studied by scholars and it explores the polyhedral convex hull of the feasible solutions and constructs valid inequalities to combine with the branch and bound or branch and cut algorithm. Normally, the addition of these inequalities can dramatically improve the computational efficiency of these algorithms. The cutting plane method for solving uncapacitated facility location problems has been studied since Cornuejols et al. (1977). At the end of 1980s, it was extended to solve the capacitated facility location problem (Leung and Magnanti (1989)). Chapter 2 studies specific cutting planes that, added to the conic formulation, improve the CPU time and reduces the number of branch and bound nodes in our experiments. In particular, we suggest polymatroid valid inequalities, that are originally studied for mean-risk minimization problems with discrete decision variables, and extended cover inequalities, studied in the subfamily of nonlinear knapsack sets.

The advantages of the suggested resolution methodology go beyond the supply chains management and logistics problems. In fact, the methodology can be employed to any optimization problem that can be equivalently substituted to a second-order conic mixed-integer program. We have observed that one set of candidates are those problems that have one or multiple submodular terms in

\(^1\)for a lot of cases, a low-order polynomial algorithm is required to solve a nonlinear (concave) integer subproblem.
the objective or in the constraints. In the supply chain literature, this term represents the risk pooling effect. However in other areas, this term can have other interpretations.

Conic programming is the solution method of choice for the problems presented in this dissertation. By the application of this methodology under different supply chain design models we aim to show the potential applicability of CQMIP to a big family of supply chain design models and beyond.

1.4 Different applied areas

The study of the supply chain design problem is typically applied to the commercial setting. By commercial supply chain we refer to that supply chain that has a profit maximizing overall objective and employes the regular chain nodes such as supplier, manufacturer, retailer, and end customer. Indeed, some supply chain terminology implicitly assumes that the supply chain refers to a commercial setting with a profit maximizing objective (for example, note the use of “customer” in the definition of supply chain in Section 1.1). Nonetheless, not all organizations are commercial for-profit entities and this is reflected in the existence of a variety of supply chains beyond the commercial ones. Some examples currently studied by OR/MS scholars are humanitarian supply chains, service supply chains, health care supply chains, or green supply chains. This literature has been the inspiration to dedicate part of this dissertation to the study of some unconventional aspects of the supply chain. Thus, in Chapter 3, we study how returned products should be integrated in a (regular) forward-flow supply chain and, in Chapter 4, we study the particularities of those supply chains driven by an overall nonprofit objective.

1.4.1 Closed-loop supply chains

Profitability, demand penetration, and waste-reduction objectives are some of the major motivations of organizations to go green. Following this trend, there is a growing area of study in green and environmentally-friendly supply chains. Srivastava (2007) defines green supply chain management (GrSCM) as “integrating environmental thinking into supply chain management, including product design, material sourcing and selection, manufacturing processes, delivery of the final product to the consumers as well as end-of-life management of the product after
its useful life”. For reviews in this topic refer to Beamon (1999) and Srivastava (2007).

Closed-loop supply chains are considered a subfield in green supply chain management. These are “supply chains where, in addition to the typical “forward” flow of materials from suppliers all the way to end customers, there are flows of products back (post-consumer touch or use) to manufacturers” (Ferguson (2009)). Our work in Chapter 3 corresponds to the analysis of this type of supply chain since we integrate reverse flows with the regular forward flows in a supply chain design problem. From Ferguson’s work there are three strategic design issues related to reverse logistics networks: (1) whether the OEM or retailer should handle the collection of the used products, (2) whether products should be collected at the point of use or at the drop-off point, and (3) to find the right balance between managing a reverse logistics network for efficiency versus managing it to maximize revenue from the remanufacturing process. Our work is centered to address the third issue since besides accounting for supply chain design costs we also track the time value of money of the returned products.

1.4.2 Nonprofit supply chains

Part of my doctoral work reflects my personal interest in the nonprofit world. Humanitarian relief logistics, health care, or community services are all streams of literature in OR/MS related to nonprofit initiatives, but each one has been studied independently. My work in Chapter 4 examines the nonprofit setting as a whole motivated by the following question – Given the numerous and successful contributions made by OR/MS scholars to improve supply chain practices in the private sector, how can we achieve a similar level of rigor and success for supply chains in the nonprofit context?

As mentioned before, the definition of supply chain is typically restricted to the private sector, where terms such as “business”, “customer”, and “profits” are widely employed. However, in the past years there has been an increasing interest by the OR/MS community to nonprofit/humanitarian/social issues. From this perspective, the last chapter of this dissertation starts by providing a definition of the “nonprofit supply chain”. It refers to all those parties involved in fulfilling a beneficiary need with the objective of maximizing an overall nonprofit goal, where the difference between the revenue generated from donors and beneficiaries and the overall cost across the supply chain is not necessarily aimed to be maximized and what is maximized, instead, is its specific nonprofit goal.

From this definition, we study the major challenges faced by nonprofit supply chains by describing how OR/MS scholars address these challenges via examples
Chapter 1. Introduction

from the literature. The challenges identified are: lack of a single performance measure, limited and uncertain funds, supply, and resources, the allocation problem when demand exceeds supply, weak demand forecasts, high value of loss and stock-out costs, and lack of intra- and inter- collaboration.
Chapter 2

A novel solution approach: conic programming

2.1 Introduction

In order to achieve significant cost savings across the supply chain, the major cost components that can impact the performance of the supply chain should be considered jointly, rather than in isolation. This is not only true for decisions at the same hierarchical level (for instance, it is well known that the inventory management scheme and the transportation strategy should be integrated), but also at different levels.

Recently, we have seen a proliferation of research on integrated facility location and inventory management models. These models simultaneously consider decisions both at the strategic (location decisions) level and tactical (inventory decisions) level. Daskin et al. (2002) and Shen et al. (2003) were the first to propose joint location-inventory models with nonlinear safety stock costs and integer location decisions. The nonlinearity arises from the risk pooling strategy used to buffer random demand at the retailers. Specifically, they consider the design of a supply chain system in which a supplier ships products to a set of retailers, each with uncertain demand. The decision problem is to determine how many distribution centers to locate, where to locate them, which retailers to assign to each distribution center (DC), how often to reorder at the distribution center, and
what level of safety stock to maintain to minimize total location, shipment, and inventory costs, while ensuring a specified level of service.

The complexity of integrated models with integer decision variables and nonlinear costs and constraints suggested the development of special-purpose heuristic algorithms for various special cases. Shen et al. (2003) outline a column generation approach while Daskin et al. (2002) propose a Lagrangian relaxation approach for this problem. Both of the approaches utilize a low-order polynomial algorithm for solving a nonlinear (concave) integer subproblem. Özen et al. (2008) study a capacitated version of the joint location-inventory problem, and they design an efficient algorithm to handle fractional terms in the objective function and nonlinear capacity constraints.

In this chapter we propose a new flexible and general approach based on recent developments in conic integer programming. In particular, we reformulate the joint location-inventory models with different types of nonlinearities as conic quadratic mixed-integer programs, which can then be solved directly using standard optimization software packages without the need for designing specialized algorithms. This approach has several advantages over the Lagrangian relaxation and column generation approaches. For the later approaches to work well, one needs to design special-purpose algorithms for solving the nonlinear sub-problems and, for their exact solutions, implement a specialized branch-and-bound algorithm that either makes use of the Lagrangian relaxation bounds or allows convenient generation of columns in the search tree. In many cases, this requires an extensive programming effort which often gives way to simpler heuristics approaches as alternative. Moreover, these special-purpose algorithms often work under simplifying assumptions on the problems and are not easily extendable to more general settings. On the other hand, as we will see in the later discussions, our proposed conic quadratic programming based approach is direct, efficient, and flexible enough to handle more general problems that have been considered before in the literature, including correlated retailer demand, stochastic lead times, and multi-commodity cases.

The main contributions in this chapter can be summarized as follows:

1. We propose a new approach to modeling and solving integrated supply chain problems with stochastic demand. The conic integer programming based approach is general, flexible, and quite efficient.

2. We show how to reformulate different types of nonlinearities arising in joint location-inventory problems as conic quadratic integer programs.
3. We address for the first time the joint location-inventory problems with distinct variance-to-mean ratio for each retailer, correlated retailer demand, stochastic lead times, and correlated multi-commodity demand.

4. We strengthen the conic quadratic integer formulations with cutting planes for their efficient solution.

5. We perform computational studies comparing the new approach with earlier ones in the literature that deal with special cases of our general model and investigate the impact of correlated demand on the supply chain design.

The rest of the chapter is organized as follows. In Section 2.2 We review the relevant literature on integrated location and inventory optimization and recent developments in conic programming. In Section 2.3 we formally define a conic quadratic mixed-integer program and review the notation and parameters used in the chapter. In Section 2.5 we address the basic uncapacitated model and give a conic quadratic mixed-integer formulation for it. In Section 2.6 we study the capacitated model and its respective equivalent conic mixed-integer reformulation. In these two sections, we also show how to utilize relevant polymatroid and cover inequalities for strengthening the conic quadratic formulations. In Section 2.7 we generalize the models to incorporate correlated retailer demand, stochastic lead times, and multi-commodities. Each model is accompanied by its equivalent conic quadratic formulation. In Section 2.8 we present our computational results with the conic quadratic MIP approach, provide comparisons with earlier studies, and investigate the impact of correlations and stochastic lead times. Finally, in Section 2.9 we conclude with a few final remarks.

2.2 Literature review

In this section we review the literature on integrated supply chain design models, especially the papers that model fixed location costs and nonlinear inventory costs. We mention some work related to multi-commodity in supply chain design and retailers’ and products’ demand correlation. Recent developments on conic integer programming are also reviewed.

Daskin et al. (2002) and Shen et al. (2003) propose the first location-inventory model with nonlinear inventory costs. They propose column generation and Lagrangian relaxation methods for its solution, respectively. Both methods employ the same sub-problem, which is solved in $O(n \log n)$ for two special cases: when the variance of the demand is proportional to the mean (as in the Poisson demand
Chapter 2. A novel solution approach: conic programming

case), or when the demand is deterministic. In these cases the objective function simplifies to one with a single nonlinear (concave) term for each retailer, which underlies the efficient solution approach. Shu et al. (2005) and Shen and Qi (2007) study more general models in which these assumptions on demand are relaxed. As a result, multiple nonlinear terms appear in the objective functions. Specifically, Shu et al. (2005) study a subproblem with two concave terms and Shen and Qi (2007) added a third term to accommodate routing costs. More general problems are studied by Qi and Shen (2007); Shen (2005); Shen and Daskin (2005); Snyder et al. (2007).

Özsen et al. (2008) consider the capacitated version of the models in Shen et al. (2003) and Daskin et al. (2002). They propose a Lagrangian relaxation based solution algorithm to solve the problem, where the Lagrangian subproblems are nonlinear integer program which include concave and fractional terms. For more detailed review on integrated location-inventory models, we refer the reader to Shen (2007b).

Multi-commodity problems have been studied in the location literature and are of our interest for the present chapter. Geoffrion and Graves (1974) utilize a Bender’s decomposition to solve multi-commodity problems with capacitated plants and DCs. Dasci and Verter (2001) consider economies of scale by introducing concave technology selection cost into the objective function of a multi-commodity location model. To handle concavity of the objective function, they solve the problem with a series of piecewise linear underestimations. In the integrated supply chain design literature, Shen (2005) presents a multi-commodity model that includes economies of scale cost terms in the objective function. The author proposes a Lagrangian-relaxation solution algorithm with a low-order polynomial algorithm to solve the Lagrangian relaxation subproblems.

Correlated demand has received much attention in the inventory management literature and it can be studied across time, sites, and products. Johnson and Thompson (1975) is among the first to study correlated demand in a single item and a single location setting. Erkip et al. (1990) consider a multi-echelon inventory system where demand is a first-order autoregressive process and is correlated across sites and time. These authors solve for the optimal safety stock level and show the impact of demand correlation over time. Charnes et al. (1995) assume that the sequence of demand is a covariance-stationary Gaussian stochastic process. The literature on supply chain problems with correlation between different products is scarce. Inderfurth (1991) studies the effects of correlation between different items on the optimal safety stock in stochastic multi-stage production/distribution systems. Fine and Freund (1990) and Goyal and Netessine
(2011) study the correlation between products in the context of product and volume flexibility.

### 2.3 Solution approach and valid inequalities

Recently, there has been a number of advances in the theory of conic integer programming. Atamtürk and Narayanan (2010) give conic mixed-integer rounding inequalities for conic quadratic mixed-integer programs and Çezik and Iyengar (2005) give convex quadratic cuts for mixed 0-1 conic programs. Atamtürk and Narayanan (2011) propose lifting methods for conic mixed integer programming. Atamtürk and Narayanan (2009) propose cover-type inequalities for submodular knapsack sets and Atamtürk and Narayanan (2008) introduce polymatroid inequalities that can help with solving special structured conic quadratic programs efficiently. We have utilized these recently introduced valid inequalities for the efficient solution of our joint location-inventory models.

A **conic quadratic mixed-integer program** (CQMIP) is an optimization problem of the form:

$$\begin{align*}
\min & \quad c'x \\
\text{s.t.} & \quad \|A_i x + b_i\|_2 \leq d_i' x + e_i, \quad i = 1, \ldots, p,
\end{align*}$$

where $x \in \mathbb{Z}^n \times \mathbb{R}^m$, $\|\cdot\|_2$ is the Euclidean norm, and all parameters are rational. Observe that a linear constraint can be written as a special case of a conic quadratic constraint by letting $A_i = b_i = 0$. Similarly, a convex quadratic constraint can be written as a special case by letting $d_i' = 0$. For an introduction to (convex) conic quadratic programming we refer the reader to Ben-Tal and Nemirovski (2001) and Alizadeh and Goldfarb (2003). In recent years there have been significant developments on the computation of conic quadratic mixed-integer programs. Due to the rise in demand for solution of CQMIP, commercial optimization software vendors such as CPLEX and Mosek have added to their offerings branch-and-bound based solvers for CQMIP.

During the last decade, conic quadratic programs have been employed to solve problems in different areas such as portfolio optimization, scheduling, and energy planning. Indeed, basic uncapacitated facility location problems have been formulated as a conic quadratic program (e.g., Kuo and Mittelmann (2004)). In this chapter, we show how to model nonlinear mixed 0-1 optimization models arising in complex supply chain design problems as conic quadratic mixed 0-1 programs and utilize the recent advances in cutting planes for their scalable solution.
Valid inequalities have been used in the literature to improve the efficiency and quality of the models’ solutions. Some of the most used valid inequalities are: clique inequalities (Leung and Magnanti (1989)), odd cycle inequalities (Leung and Magnanti (1989); Klose (2000)), submodular inequalities (Aardal et al. (1995); Klose (2000)), $(k, I, S, I)$ inequalities (Aardal et al. (1995)), flow cover inequalities (Aardal et al. (1995); Aardal (1998); Klose (2000)), knapsack cover inequalities, effective capacity inequalities, combinatorial inequalities (Aardal et al. (1995); Aardal (1998)), single depot inequalities (Aardal (1998)), lifted cover inequalities (Klose (2000)), and extended polymatroid inequalities (Atamtürk and Narayanan (2008)). Some of them are facets for the capacitated facility location problems. Some other valid inequalities are incorporated into Lagrangian-relaxation based methods to tighten the feasible region for capacitated facility location problems (Klose (2000); Miranda and Garrido (2008)).

In the context of the models presented in this dissertation, we have successfully employed polymatroid inequalities (introduced in Section 2.5.3) and extended cover inequalities (introduced in Section 2.6.3).

### 2.4 Notation

The following parameters and notation are used throughout all the dissertation:

#### Parameters and Notation

**Demand**

- $\mu_i$ : mean of daily demand at retailer $i$,
- $\sigma_i$ : standard deviation of daily demand at retailer $i$,
- $V$ : variance-covariance matrix of daily demand at retailers.

**Costs**

- $d_{ij}$ : unit cost of shipment between retailers $i$ and $j$,
- $f_j$ : annualized fixed cost of locating a DC at retailer site $j$,
- $F_j$ : fixed cost of placing an order at DC $j$,
- $a_j$ : unit cost of shipment from the central plant to DC $j$,
- $g_j$ : fixed cost per shipment from the central plant to DC $j$,
- $h$ : unit inventory holding cost per year.

**Weights**
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\[
\begin{align*}
\beta &: \text{ weight associated with the transportation costs}, \\
\theta &: \text{ weight associated with the inventory costs.} \\
\end{align*}
\]

Other parameters

\[
\begin{align*}
\chi &: \text{ days worked per year}, \\
\alpha &: \text{ service level} \\
z_{\alpha} &: \text{ standard normal deviation associated with service level } \alpha, \\
L_j &: \text{ lead time in days at DC } j. \\
\end{align*}
\]

Decision variables

\[
\begin{align*}
x_j &= \begin{cases} 
1, & \text{if a distribution center (DC) is located at retailer site } j, \\
0, & \text{otherwise}; 
\end{cases} \\
y_{ij} &= \begin{cases} 
1, & \text{if retailer } i \text{ is assigned to DC located at retailer site } j, \\
0, & \text{otherwise}. 
\end{cases}
\end{align*}
\]

2.5 Model with uncapacitated facilities

We start with the basic uncapacitated location-inventory model, which was originally studied by Daskin et al. (2002) and Shen et al. (2003). Their model assume the following:

- Shipments are direct from DCs to retailers,
- Demand at each retailer is independent and Gaussian,
- Each retailer is supplied from exactly one DC.

2.5.1 Model 1

Under the assumptions listed above, the joint location-inventory model is formulated as follows:

\[
\begin{align*}
\min & \sum_{j \in J} \left( f_j x_j + \sum_{i \in I} \hat{d}_{ij} y_{ij} + K_j \sqrt{\sum_{i \in I} \mu_i y_{ij}} + q_j \sqrt{\sum_{i \in I} \sigma_i^2 y_{ij}} \right), \\
\text{s.t.} & \sum_{j \in J} y_{ij} = 1, & i \in I, \\
y_{ij} \leq x_j, & i \in I, j \in J, \\
x_j, y_{ij} \in \{0, 1\}, & i \in I, j \in J.
\end{align*}
\]

(P1)
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where $d_{ij} = \beta \chi (d_{ij} + a_j) \mu_i$, $K_j = \sqrt{2\theta h (F_j + \beta g_j)} \chi$, and $q_j = z_a \theta \sqrt{L_j h}$.

The objective of (P1) is to minimize total expected cost of location, shipment and inventory management. The first objective term is the fixed cost of locating DC $j$, $f_j x_j$. The second term is the cost of shipping from DC $j$ to the retailers and from the central plant to DC $j$, $\beta \chi \sum_{i \in I} (d_{ij} + a_j) \mu_i y_{ij}$. The third term captures the working inventory effects due to the fixed costs of placing orders and the fixed costs of shipping from the central plant to DC $j$, $\sqrt{2\theta h (F_j + \beta g_j)} \chi \sqrt{\sum_{i \in I} \mu_i y_{ij}}$. The fourth term is the expected safety stock cost at DC $j$, $z_a \theta \sqrt{L_j h} \sqrt{\sum_{i \in I} \sigma_i^2 y_{ij}}$.

Following Shen et al. (2003), we describe how the expected working inventory cost at DC $j$ in (P1) is derived. For simplicity, we drop the subscript $j$ from the formulation. The working inventory cost includes the total fixed cost of placing $n$ orders per year $F n$, the shipment cost per year $\nu (D n)$, and the average working inventory cost $h D^2 n$. There are $n$ orders per year and the annual expected demand is $D = \sum_{i \in I} \mu_i y_{ij}$. Consider the expression $F n + \beta \nu (D n) + \theta \frac{h D^2}{2 n}$. We take the derivative of this expression with respect to $n$ and we assume that $\nu(\cdot)$ is linear ($\nu(x) = ax + g$.) We obtain $F + \beta g + \beta a D^2 - \beta a D - \theta \frac{h D^2}{2 n^2} = F + \beta g - \theta \frac{h D^2}{2 n^2} = 0$. Solve for $n$, $n = \sqrt{\theta h D / 2 (F + \beta g)}$, and substitute $n$ into the above equation to get $\sqrt{2\theta h D (F + \beta g) + \beta a D} = \sqrt{2\theta h (F + \beta g)} \sqrt{\sum_{i \in I} \mu_i y_{ij}} + \beta a \sum_{i \in I} \mu_i y_{ij}$. This expression is part of the objective function in (P1).

Constraints (2.1) ensure that each retailer is assigned to exactly one DC. Constraints (2.2) guarantee that retailers are only assigned to open DCs. Constraints (2.3) define the domain of the decision variables.

As mentioned in the literature review, in order to handle the nonlinearity of the objective, Shen et al. (2003) solve (P1) by transforming it into a set-covering model and solve it using column generation approach, where the columns are generated by solving an unconstrained nonlinear subproblem on binary variables. Daskin et al. (2002) solve the same problem by designing a Lagrangian relaxation algorithm. In both of these papers the ratio of the demand variance to the mean demand is assumed to be constant for all retailers ($\sigma_i^2 / \mu_i = \gamma \forall i$). Under this assumption, (P1) would have only one square root term instead of two for each retailer, which makes the Lagrangian and column generation subproblems easier to solve. Our approach does not require this assumption.

2.5.2 A conic quadratic MIP formulation

In this section we show how to reformulate (P1) as a conic quadratic mixed-integer program (CQMIP). The advantage of the CQMIP formulation is that it can
be solved directly using standard optimization software packages such as CPLEX or Mosek.

By introducing auxiliary variables $t_{1j}, t_{2j} \geq 0$ to represent the nonlinear terms in the objective and using the fact that $y_{ij} = y_{ij}^2$, we reformulate (P1) as

$$
\begin{align*}
\min & \sum_{j \in J} \left( f_j x_j + \sum_{i \in I} \hat{d}_{ij} y_{ij} + K_j t_{1j} + q_j t_{2j} \right) \\
\text{s.t.} & \sum_{i \in I} \mu_i y_{ij}^2 \leq t_{1j}^2, \quad j \in J, \\
& \sum_{i \in I} \sigma^2_i y_{ij}^2 \leq t_{2j}^2, \quad j \in J, \\
& t_{1j}, t_{2j} \geq 0, \quad j \in J,
\end{align*}
$$

(CQMIP1)

Note that the objective of (CQMIP1) is linear and the constraints are either conic quadratic or linear, which fits into the general conic quadratic mixed integer programming model described in Section 2.3.

### 2.5.3 Polymatroid inequalities

Commercial software packages utilize a branch-and-bound algorithm for solving conic quadratic MIPs and their performance can be significantly improved by strengthening the formulations with structural cutting planes. In this section, utilizing submodularity, we will reformulate constraints (2.4) and (2.5) with polymatroid inequalities of Atamtürk and Narayanan (2008) to strengthen the convex relaxation of CQMIP1.

**Definition 1.** A function $g : 2^I \to \mathbb{R}$ is submodular if $g(S \cup i) - g(S) \geq g(T \cup i) - g(T)$ for all $S \subseteq T \subseteq I$ and $i \in I \setminus T$.

**Proposition 1.** A set function $g : 2^I \to \mathbb{R}$ defined by $g(S) = f(a(S))$, where $f(\cdot)$ is concave and $a(S)$ is the sum of the components of $a \in R_+^{|I|}$ on $S \subseteq I$, is submodular.

**Proof.** See Nemhauser and Wolsey (1999) and Shen et al. (2003).

**Definition 2.** (Schrijver (2003)) The polyhedron associated with the submodular function $g$ on $I$:

$$EP_g := \{ \pi \in \mathbb{R}^{|I|} \mid \pi(S) \leq g(S) \text{ for each } S \subseteq I \}$$
is called the extended polymatroid associated with \( g \) if \( g(\phi) = 0 \) where \( \pi(S) = \sum_{i \in S} \pi_i \).

Now, for an extended polymatroid \( EP_g \), Atamtürk and Narayanan (2008) show that the linear inequality

\[
\pi y \leq t \text{ with } \pi \in EP_g
\]

is valid for the lower convex envelope of \( g \):

\[
Q_g := \text{conv}\{(y, t) \in \{0, 1\}^{|I|} \times \mathbb{R} : g(y) \leq t \}.
\]

Hence, we can present the following definition:

**Definition 3.** (Atamtürk and Narayanan (2008)) The inequalities associated with the extended polymatroid of \( g \), \( \pi y \leq t \), \( \pi \in EP_g \) are called extended polymatroid inequalities. When the inequalities are defined by the extreme points of the extended polymatroid \( EP_g \), they are called extremal extended polymatroid inequalities of \( Q_g \).

Because \( t_j \geq 0, \forall j \in J \) and \( y_{ij}^2 = y_{ij}, \forall i \in I, j \in J \), inequalities \( \sum_{i \in I} \mu_i y_{ij}^2 \leq t_j^2 \) and \( \sqrt{\sum_{i \in I} \mu_i y_{ij}} \leq t_j \) are equivalent. The latter inequalities have a submodular form due to the concavity of the square root function and the nonnegativity of the arguments in the square root function. More precisely, the set function

\[
g(S) := \sqrt{\sum_{i \in S} \mu_i}, \quad \forall S \subseteq I
\]

is submodular.

Although there are exponentially many extremal (corresponding to extreme points \( \pi \) of \( EP_g \)) extended polymatroid inequalities, only a small subset of them is needed in the branch-and-bound search tree. It turns out that, given a solution, finding a violated polymatroid cut can be done easily. Formally, the separation problem for the extended polymatroid inequalities is defined as follows:

**Definition 4.** The separation problem associated with a combinatorial optimization problem is the problem: Given \( x^* \in \mathbb{R}^n \), is \( x^* \in \text{conv}(X) \)? If not, find an inequality \( \pi x \leq \pi_0 \) satisfied by all points in \( X \), but violated by the point \( x^* \).

Following our problem’s notation, given \((y^*, t^*) \in \{0, 1\}^{|I|} \times \mathbb{R}_+\), let

\[
\zeta = \max \left\{ \pi y^* : \pi \in EP_g \right\}.
\]

If \( \zeta > t^* \), then the extended polymatroid inequality \( \pi^* x \leq t \) for an optimal \( \pi^* \) cuts off \((y^*, t^*)\); otherwise, there exists no violated extended polymatroid inequality.
Thus, the separation problem is an optimization over an extended polymatroid, which is solved by the greedy algorithm of Edmonds (1971).

The following is the implementation of Edmond’s greedy algorithm for our separation problem. For each $j \in J$ do:

1. Given $y^*_j \in [0, 1]|I|$ and $t^*_j$, sort $y^*_{i,j}$ in non-increasing order

   \[ y^*_{(1),j} \geq y^*_{(2),j} \geq \ldots \]

2. For $i = 1, \ldots, |I|$, let $S_i = \{(1), (2), \ldots, (i)\}$ and $\pi(i) = \sqrt{\sum_{k \in S_i} \sigma^2(k)} - \sqrt{\sum_{k \in S_{i-1}} \sigma^2(k)}$

3. If $\zeta_j = \pi y^*_j > t^*_j$ we add the extended polymatroid cut $\pi y_j \leq t_j$ to the formulation.

### 2.6 Model with capacitated facilities

In this section we consider the generalization of (P1) with facility capacities and show how to reformulate it as a conic quadratic MIP. Özsên et al. (2008) present a generalization of the integrated inventory-location model (P1) by introducing inventory capacity constraints for the DCs. These constraints are defined for a $(Q, r)$ inventory control policy with type-I service level. Compared with model (P1), their model contains additional nonlinear terms:

- Nonlinear (concave) capacity constraints for each DC,
- Nonlinear (fractional) terms in the objective function.

As in the uncapacitated counterparts of Daskin et al. (2002) and Shen et al. (2003), in order to simplify the problem, Özsên et al. (2008) also assume the variance of each retailer’s demand to be proportional to the mean demand. In particular, $\sigma^2_i = \mu_i \forall i$. We do not make this assumption here.

#### 2.6.1 Model 2

Let $C_j$ be the maximum inventory capacity of DC $j$ and $Q_j$ be the reorder quantity for DC $j$. Then, the integrated inventory-location model with capacitated
facilities is formulated as the following nonlinear mixed 0-1 optimization problem:

$$\min \sum_{j \in J} \left( f_j x_j + \sum_{i \in I} \hat{d}_{ij} y_{ij} + \hat{F}_j \sum_{i \in I} \frac{\mu_i y_{ij}}{Q_j} + q_j \sqrt{\sum_{i \in I} \sigma_i^2 y_{ij} + \theta h Q_j} \right)$$

(P2) \quad \text{s.t.} \quad Q_j + z_a \sqrt{L_j} \sqrt{\sum_{i \in I} \sigma_i^2 y_{ij} + L_j \sum_{i \in I} \mu_i y_{ij}} \leq C_j, \quad j \in J, \quad (2.7)

$$Q_j \geq 0, \quad j \in J, \quad (2.8)$$

$$(2.1) - (2.3),$$

where $\hat{F}_j = (F_j + \beta g_j) \chi$.

The third term in the objective is the expected fixed cost of placing an order at DC $j$ and the expected fixed cost per shipment from the central plant to DC $j$. The fifth term is new; it is the average inventory holding cost at DC $j$.

Constraints (2.7) define the capacity of each DC to be the sum of the order quantity $Q_j$ and the reorder point. Note that in defining the DC capacity, we consider the worst-case scenario, i.e., no demand is observed during lead time.

The reorder point is the sum of the safety stock, $z_a \sqrt{L_j} \sqrt{\sum_{i \in I} \sigma_i^2 y_{ij}}$, and the expected demand during lead time, $L_j \sum_{i \in I} \mu_i y_{ij}$.

### 2.6.2 An equivalent conic quadratic MIP model

The objective of (P2) is neither concave nor convex. Özsen et al. (2008) develop a Lagrangian relaxation based heuristic algorithm to solve this problem. In this section we show how to transform (P2) into the following equivalent conic quadratic MIP, which leads to an exact solution of the problem. Consider

$$\min \sum_{j \in J} \left( f_j x_j + \sum_{i \in I} \hat{d}_{ij} y_{ij} + q_j t_j + \frac{\theta}{2} h z_j \right)$$

(CQMIP2) \quad \text{s.t.} \quad Q_j + z_a \sqrt{L_j} t_j + L_j \sum_{i \in I} \mu_i y_{ij} \leq C_j x_j, \quad j \in J, \quad (2.9)

$$\sum_{i \in I} \sigma_i^2 y_{ij}^2 \leq t_j^2, \quad \forall j \in J, \quad (2.10)$$

$$\sum_{i \in I} H_{ij} \mu_i y_{ij}^2 + (Q_j - \frac{z_j}{2})^2 - \frac{z_j^2}{4} \leq 0, \quad j \in J, \quad (2.11)$$

$$t_j, z_j \geq 0, \quad j \in J, \quad (2.12)$$

$$(2.1) - (2.3), (2.8),$$
where \( H_j = \frac{F_j}{\theta} \).

Constraints (2.1), (2.2), (2.3), and (2.8) are still present in the transformed problem. Constraints (2.7) are linearized as (2.9). An auxiliary variable \( t_j \) is introduced for each \( j \) and defined by the constraints (2.10). Constraints (2.9) have stronger right hand sides than constraints (2.7). We linearize the objective by using \( t_j \) and the auxiliary variables for \( z_j \) for each \( j \). Variables \( z_j \) are defined by the constraints (2.11) and (2.12).

**Proposition 2.** Problem (P2) is equivalent to (CQMIP2).

**Proof.** Variables \( t_j \) and constraint (2.10) are used to substitute the terms \( \sqrt{\sum_{i \in I} \sigma_i^2 y_{ij}} \) as in (MIPCQ1). The second substitution for the third and fifth inventory terms \( \sum_{i \in I} \mu_i y_{ij} + Q_j \) follows from the following identities:

\[
\frac{\sum_{i \in I} \mu_i y_i}{Q} + Q \leq z \Leftrightarrow \sum_{i \in I} \mu_i y_i + Q^2 \leq Qz \quad \text{(as } Q > 0) \\
\Leftrightarrow \sum_{i \in I} \mu_i y_i^2 + Q^2 \leq Qz \quad \text{(as } y_i = y_i^2) \\
\Leftrightarrow \sum_{i \in I} \mu_i y_i^2 + (Q - \frac{z}{2})^2 \leq \frac{z^2}{4}.
\]

\[\square\]

### 2.6.3 Extended cover cuts

In order to strengthen formulation (CQMIP2) we add cover type inequalities derived from nonlinear knapsack relaxations of the formulation. Toward this end, consider the capacity constraints (2.9). For each \( j \), we relax the left hand side of the constraint by dropping \( Q_j \). Further, we substitute \( t_j \) with the left hand side of constraint (2.10) to arrive at the nonlinear 0-1 knapsack constraint

\[
z_\alpha \sqrt{L_j} \left( \sum_{i \in I} \sigma_i^2 y_{ij} + L_j \sum_{i \in I} \mu_i y_{ij} \right) \leq C_j. \tag{2.13}
\]

For simplicity of notation, we drop the subscript \( j \) to define the inequalities. For inequality (2.13), define the set function \( f : 2^I \to \mathbb{R} \), where

\[
f(S) = z_\alpha \sqrt{L} \sqrt{\sigma^2(S)} + L \mu(S),
\]
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\[ \sigma^2(S) := \sum_{i \in S} \sigma_i^2 \] and \[ \mu(S) := \sum_{i \in S} \mu_i. \] Using submodularity of \( f \), Atamtürk and Narayanan (2009) give cover and extended cover cuts for the submodular knapsack set,

\[ Y = \left\{ y \in \{0,1\}^{|I|} : f(y) \leq C \right\} = \left\{ y \in \{0,1\}^{|I|} : z_\alpha \sqrt{L} \sqrt{\sum_{i \in I} \sigma_i^2 y_i + L \sum_{i \in I} \mu_i y_i} \leq C \right\}. \]

They show that given a subset of indices \( S \subseteq I \) and the conic quadratic 0-1 knapsack set \( Y \), we can find valid cover inequalities that depend on the cover set.

**Definition 5.** \( S \subseteq I \) is called a cover for \( Y \) if

\[ z_\alpha \sqrt{L} \sqrt{\sum_{i \in I} \sigma_i^2(z_i) + L \mu(z_i)} > C. \]

Atamtürk and Narayanan (2009) show that for cover \( S \) the corresponding cover inequality

\[ \sum_{i \in S} y_i \leq |S| - 1 \]

is valid for \( Y \).

As with polymatroid inequalities, a separation algorithm generates the cover constraints at the root node of the branch-and-bound tree for each \( j \). Given \( y^* \in \{0,1\}^{|I|} \) a violated cover inequality can be found by solving the following nonlinear 0-1 separation problem:

\[ \zeta = \min \left\{ \bar{y}'z : z_\alpha \sqrt{L} \sqrt{\sum_{i \in I} \sigma_i^2(z_i) + L \mu(z_i)} > C, z \in \{0,1\}^{|I|} \right\}, \]

where \( \bar{y} = 1 - y^* \). If \( \zeta < 1 \), then the cover inequality corresponding to optimal \( z \) cuts off \( y^* \). We employ a heuristic algorithm based on rounding the convex relaxation of the separation as proposed in Atamtürk and Narayanan (2009).

The following is the implementation of Atamtürk and Narayanan’s cover inequality separation algorithm Atamtürk and Narayanan (2009) for our problem. Let \( \bar{y}_{ij} = 1 - y^*_{ij} \) for \( i \in I, j \in J \). For each \( j \in J \) and for each distinct pair \( i_1 \) and \( i_2 \) in \( I \) do:

1. Solve the following system of equations on variables \( \lambda \) and \( \rho \)

\[ \bar{y}_{i_1 j} = L_j \mu_i \lambda + z_\alpha^2 L_j \sigma_i^2 \rho \]
\[ \bar{y}_{i_2 j} = L_j \mu_i \lambda + z_\alpha^2 L_j \sigma_i^2 \rho \]

2. If \( (\lambda, \rho) \geq 0 \), then sort each \( i \) in non-decreasing order of \( \frac{\bar{y}_{ij}}{L_j \mu_i \lambda + z_\alpha^2 L_j \sigma_i^2 \rho} \); that is,

\[ \frac{\bar{y}_{(1)j}}{L_j \mu_{(1)} \lambda + z_\alpha^2 L_j \sigma_{(1)i}^2 \rho} \leq \frac{\bar{y}_{(2)j}}{L_j \mu_{(2)} \lambda + z_\alpha^2 L_j \sigma_{(2)i}^2 \rho} \leq \ldots \]
3. Assign $z(i) = 1$ following the established order until $z_a \sqrt{L_j} \sum_{i \in I} \sigma_i^2 z_{ij} + L_j \sum_{i \in I} \mu_i z_{ij} > C_j$.

4. If $\zeta = \bar{y} < 1$, then we add the cover cut $\sum_{i \in S} x_i \leq |S| - 1$ to the formulation, where $S$ is the ground set for $z$.

Cover inequalities can be strengthened by extending them with non-cover variables. To introduce extended cover inequalities, we first need to define the difference function and the notion of extension.

**Definition 6.** Given a set function $f$ on $I$ and $i \in I$, the difference function $\rho$ is defined as $\rho_i(S) := f(S \cup i) - f(S)$ for $S \subseteq I \setminus i$.

**Definition 7.** Let $\pi = (k(1), \ldots, k(|I| - |S|))$ be a permutation of the indices in $I \setminus S$. Define $S_\ell = S \cup \{k(1), \ldots, k(\ell)\}$ for $\ell = 1, \ldots, |I| - |S|$, where $S_0 = S$. The extension of $S$ corresponding to permutation $\pi$ is

$$E_{\pi}(S) := S \cup U_{\pi}(S), \text{ where } U_{\pi}(S) = \{k(\ell) : \rho_{k(\ell)}(S_{\ell-1}) \geq \rho_i(\emptyset) \forall i \in S\}.$$

Atamtürk and Narayanan (2009) also show that for given cover $S$ and permutation $\pi$, the corresponding extended cover inequality

$$\sum_{i \in E_{\pi}(S)} y_i \leq |S| - 1$$

is valid for $Y$. We utilize extended cover inequalities in our computations presented in Section 2.8.

### 2.7 Generalized models

In this section, we exploit the expressive power of conic programming to present more general integrated location-inventory models than considered to date. In addition to facility capacities, that have been introduced in the past, we now consider realistic aspects such as correlation between retailers’ demand, stochastic lead times, and multi-commodities.

#### 2.7.1 Model 3: Correlated demands

Let the retailer demand be a multinormal random variable with mean $\mu$ and variance-covariance matrix $V$. Generalizing the safety stock term in the previous section, in the presence of demand correlation, the safety stock at DC $j$ can be
stated as $z_a \sqrt{L_j h \sqrt{y_j'}} y_j$, where $y_{ij}$ is the assignment decision vector for the $j$th DC.

The mathematical model for the correlated demand is the same as (P2) except that the variance terms are replaced with the more general variance-covariance matrix:

$$\min \sum_{j \in J} \left( f_j x_j + \sum_{i \in I} \hat{d}_{ij} y_{ij} + \hat{F}_j \sum_{i \in I} \mu_i y_{ij} + q_j \sqrt{y_j' V y_j} + \theta h^{Q_j} \right)$$

(P3) s.t. $Q_j + z_a \sqrt{L_j} \sqrt{y_j' V y_j} + L_j \sum_{i \in I} \mu_i y_{ij} \leq C_j x_j, \quad j \in J, \quad (2.14)$

As in CQMIP2, we formulate (P3) by introducing auxiliary variables and linearizing the objective as a conic quadratic mixed 0-1 program:

$$\min \sum_{j \in J} \left( f_j x_j + \sum_{i \in I} \hat{d}_{ij} y_{ij} + q_j t_j + \frac{\theta}{2} h z_j \right)$$

(CQMIP3) s.t. $\sqrt{y_j' V y_j} \leq t_j, \quad j \in J, \quad (2.15)$

2.7.2 Model 4: Stochastic lead times

In a real life setting orders at DCs might arrive before or after the expected receiving time. Hence, in addition to correlated demand, a realistic aspect to be considered is stochastic lead times. To illustrate this situation, we define lead time between the central warehouse and each DC $j$ as a normal distribution with mean $L_j$ and standard deviation $\sigma_{L_j}$. We assume that successive lead times are independent and orders do not cross (Nahmias (1993)).

Lead time variability and correlated demands affect the amount of safety stock at the DC level. In particular, we define the safety stock as follows:

**Proposition 3.** (Nahmias (1993)) The safety stock at DC $j$ is $z_a \sqrt{L_j \sigma^2_{D_j}} + \sigma^2_{L_j} \mu_{D_j}$, where $\sigma^2_{D_j} = y_j' V y_j$ and $\mu_{D_j} = \sum_{i \in I} \mu_i y_{ij}$. 
Thus, $\mu_{D,j}^2 = y_j'y_jM y_j$ where

$$M = \begin{pmatrix} \mu_1^2 & \mu_1 \mu_2 & \ldots & \mu_1 \mu_{|I|} \\ \mu_2 \mu_1 & \mu_2^2 & \ldots & \mu_2 \mu_{|I|} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{|I|} \mu_1 & \mu_{|I|} \mu_2 & \ldots & \mu_{|I|}^2 \end{pmatrix}.$$ 

With this notation, the integrated inventory-location model with capacitated facilities, correlated demand, and stochastic lead times is then formulated as the following problem:

$$\min \sum_{j \in J} \left( f_j x_j + \sum_{i \in I} \hat{d}_{ij} y_{ij} + \hat{F}_j \frac{\sum_{i \in I} \mu_i y_{ij}}{Q_j} + \bar{q}_j \sqrt{y_j'(L_j V + \sigma_{L_j}^2 M)y_j + \theta h Q_j} \right)$$

(s.t. $Q_j + z_\alpha \sqrt{y_j'(L_j V + \sigma_{L_j}^2 M)y_j + L_j \sum_{i \in I} \mu_i y_{ij}} \leq C_j x_j$, $j \in J$)

$$2.16$$

where $\bar{q}_j = z_\alpha \theta h$.

The equivalent conic quadratic MIP (CQMIP4) is derived by employing the same substitution technique used for (CQMIP2):

$$\min \sum_{j \in J} \left( f_j x_j + \sum_{i \in I} \hat{d}_{ij} y_{ij} + \bar{q}_j t_j + \frac{\theta}{2} h z_j \right)$$

(s.t. $Q_j + z_\alpha t_j + L_j \sum_{i \in I} \mu_i y_{ij} \leq C_j x_j$, $j \in J$)

$$2.17$$

$$\sqrt{y_j'(L_j V + \sigma_{L_j}^2 M)y_j} \leq t_j, \quad j \in J$$

$$2.18$$

2.7.3 Model 5: Multiple commodities

Since our models exhibit economies of scale terms a multi-commodity extension is of interest. Each commodity represents a specific product or product group and we employ the subindex $l \in L$ to refer to different commodities. Before introducing the model we need to define some new notation that depends on the type of commodity:

Demand
\( \mu_{il} \): mean of daily demand at retailer \( i \) for commodity \( l \),
\( \sigma_{il} \): standard deviation of daily demand at retailer \( i \) for commodity \( l \),

Costs
\( d_{ijl} \): cost per unit to ship commodity \( l \) between retailers \( i \) and \( j \),
\( F_{jl} \): fixed cost of placing an order at DC \( j \) for commodity \( l \),
\( a_{jl} \): unit cost of shipment from the central plant to DC \( j \) for commodity \( l \),
\( g_{jl} \): fixed cost per shipment from the central plant to DC \( j \) for commodity \( l \),
\( h_l \): unit inventory holding cost per unit of commodity \( l \) per year.

Other parameters
\( \alpha^l \): service level of commodity \( l \),
\( z_{\alpha^l} \): standard normal deviation associated with \( \alpha^l \),
\( L_{jl} \): lead time in days at DC \( j \) for commodity \( l \),
\( \sigma_{L_{jl}} \): standard deviation of lead time in days at DC \( j \) for commodity \( l \).

Decision variables
\[ y_{ijl} = \begin{cases} 1, & \text{if demand for commodity } l \text{ of retailer } i \text{ is assigned to DC at site } j, \\ 0, & \text{otherwise}. \end{cases} \]

\( Q_{jl} \): reorder quantity for DC \( j \) of commodity \( l \).

Under the notation defined above, the multi-commodity joint location-inventory model with capacitated facilities, stochastic lead times, and correlated retailers’ demand is formulated as follows:

\[
\begin{align*}
\min & \sum_{j \in J} \left( f_{jl} x_j + \sum_{l \in L} \left( \sum_{i \in I} \hat{d}_{ijl} y_{ijl} + \hat{F}_{jl} \sum_{i \in I} \mu_{il} y_{ijl} \right) \frac{Q_{jl}}{Q_{jl}} \right. \\
& \quad + \tilde{a}_{jl} \sqrt{y_{jl}'(L_{jl} V_l + \sigma^2_{L_{jl}} M_l) y_{jl} + \theta h_l \frac{Q^2_{jl}}{2}} \bigg) \\
\text{(P5)} \ \text{s.t.} & \quad \sum_{i \in I} \left( Q_{jl} + z_{\alpha^l} h_l \sqrt{y_{jl}'(L_{jl} V_l + \sigma^2_{L_{jl}} M_l) y_{jl}} \right. \\
& \quad \quad \left. + L_{jl} \sum_{i \in I} \mu_{il} y_{ijl} \right) \leq C_j x_j, \quad j \in J, \quad (2.19) \\
& \quad \sum_{j \in J} y_{ijl} = 1, \quad i \in I, l \in L, \quad (2.20) \\
& \quad y_{ijl} \leq x_j, \quad i \in I, j \in J, l \in L, \quad (2.21) \\
& \quad x_j, y_{ijl} \in \{0, 1\}, Q_{jl} \geq 0, \quad i \in I, j \in J, l \in L, \quad (2.22)
\end{align*}
\]
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where \( \hat{d}_{ijl} = \beta \chi (d_{ijl} + a_{jl}); \) \( \tilde{F}_{jl} = (F_{jl} + \beta g_{jl}) \chi; \) \( \tilde{q}_{jl} = z_{ol} \theta h_{il}; \) \( y_{jl} = \begin{pmatrix} y_{1jl} \\ \vdots \\ y_{Ijl} \end{pmatrix}; \) \( V_l \) is the variance-covariance matrix of retailers’ demand related to commodity \( l, \) and

\[
M_l = \begin{pmatrix}
\mu_{11}^2 & \mu_{11} \mu_{21} & \cdots & \mu_{11} \mu_{|I|1} \\
\mu_{21} \mu_{11} & \mu_{21}^2 & \cdots & \mu_{21} \mu_{|I|1} \\
\vdots & \vdots & \ddots & \vdots \\
\mu_{1|I|1} \mu_{11} & \mu_{1|I|1} \mu_{21} & \cdots & \mu_{1|I|1}^2
\end{pmatrix}.
\]

Consequently, we have the following the conic quadratic reformulation of (P5):

\[
\begin{align*}
\min \sum_{j \in J} \left( f_j x_j + \sum_{l \in L} \left( \sum_{i \in I} \hat{d}_{ijl} y_{ijl} + \tilde{q}_{jl} t_{jl} + \frac{\theta}{2} h_{il} z_{jl} \right) \right) \\
\text{CQMIP5} \quad \text{s.t.} \sum_{l \in L} \left( Q_{jl} + t_{jl} + L_{jl} \sum_{i \in I} \mu_{il} y_{ijl} \right) \leq C_j x_j, \quad j \in J, (2.23) \\
\sqrt{y_{jl}' (L_{jl} V_l + \sigma_{Ljl}^2 M_l) y_{jl}} \leq t_{jl}, \quad j \in J, l \in L, (2.24) \\
\sum_{i \in I} H_{jl} \mu_{il} y_{ijl}^2 + (Q_{jl} - \frac{z_{jl}}{2})^2 - \frac{z_{jl}^2}{4} \leq 0, \quad j \in J, l \in L, (2.25) \\
t_{jl}, z_{jl} \geq 0, \quad j \in J, l \in L, (2.26)
\end{align*}
\]

where \( H_{jl} = \frac{\tilde{F}_{jl}}{\mu_{jl}^2}. \)

2.7.4 Model 6: Multiple commodities with correlated demand

In this last model, we consider the correlation among the demand of different commodities. Under this situation and for the simplicity of notation, we assume that the correlation coefficients related to commodities’ demand are retailer-independent and they are defined as \( \rho_{t12} \forall l_1, l_2 \in L. \) Similarly, the correlation coefficients of retailers’ demand are commodity-independent and defined as \( \rho_{i12} \forall i_1, i_2 \in I. \) Further, we assume that the inventory cost and lead time parameters are the same regardless of commodity type (i.e. \( h_l = h, L_{jl} = L_j \) and \( \sigma_{Ljl}^2 = \sigma_{Lj}^2 \forall l \in L. \))
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Under the notation defined above, the multi-commodity joint location-inventory model with capacitated facilities, stochastic lead times, and correlated retailer and commodity demand is formulated as follows:

\[
\begin{align*}
\min & \sum_{j \in J} \left( f_j x_j + \sum_{l \in L} \left( \sum_{i \in I} \hat{d}_{ijl} y_{ijl} + \hat{F}_{jl} \sum_{i \in I} \mu_{ii} y_{ijl} \frac{Q_{jl}}{Q_{jl}} + \theta h \frac{Q_{jl}}{2} \right) + \tilde{q}_j \sqrt{y_{.j} (L_j U + \sigma^2_{L_j} W) y_{.j}} \right) \\
\text{s.t.} & \sum_{l \in L} \left( Q_{jl} + L_{jl} \sum_{i \in I} \mu_{il} y_{ijl} \right) + z \sqrt{y_{.j} (L_j U + \sigma^2_{L_j} W) y_{.j}} \leq C_j x_j, \quad j \in J, \quad (2.27)
\end{align*}
\]

Model 6 is the most general model we consider in this chapter and, due to the flexibility of conic quadratic MIP approach, we arrive at the following formulation using the same transformations employed in special cases presented earlier:
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\[ \min \sum_{j \in J} \left( f_j x_j + \sum_{l \in L} \left( \sum_{i \in I} \hat{d}_{ijl} y_{ijl} + \frac{\theta}{2} h_{jil} \right) + \tilde{q}_j t_j \right) \]

(CQMIP6) \quad \text{s.t.} \quad \sum_{l \in L} \left( Q_{jl} + L_j \sum_{i \in I} \mu_{il} y_{ijl} \right) + t_j \leq C_j x_j, \quad j \in J, (2.28)

\[ \sqrt{y_j^\top (L_j U + \sigma^2_{L_j} W) y_j} \leq t_j, \quad j \in J, (2.29) \]

\[ \sum_{i \in I} \tilde{H}_{jl} \mu_{i1} y_{i1j}^2 + (Q_{jl} - \frac{z_{jl}}{2})^2 - \frac{z_{jl}^2}{4} \leq 0, \quad j \in J, l \in L, (2.30) \]

\[ t_j, z_{jl} \geq 0, \quad j \in J, l \in L, (2.31) \]

(2.20) – (2.22),

where \( \tilde{H}_{jl} = \frac{\hat{F}_{jl}}{2} \).

2.7.5 Polymatroid cuts

The cuts proposed in Section 2.5 and Section 2.6 are also pertinent to the generalized models presented in this section. It is reasonable to assume that retailers’ demand are positively correlated as they are typically affected in the same direction by economic factors. Then we may employ polymatroid inequalities by reformulating the models using new binary variables for the products of binary variables. In particular, for model 3 we can replace the products \( y_{i1j} y_{i2j} \) with \( w_{i1i2j} \) by introducing constraints

\[ w_{i1i2l} \leq y_{i1j}, w_{i1i2l} \leq y_{i2j}, y_{i1j} + y_{i2j} \leq 1 + w_{i1i2j}, \quad i_1, i_2 \in I, \ j \in J. \quad (2.32) \]

These constraints ensure that \( w_{i1i2j} = 1 \) if and only if \( y_{i1j} = y_{i2j} = 1 \). Noting that \( w_{i1i2j} \) is equivalent to \( w_{i1i2j}^2 \), the safety stock at location \( j \) can now be written as

\[ z_\alpha \sqrt{L_j h} \sqrt{\sum_{(i_1i_2) \in I \times I} V_{i1i2} w_{i1i2j}^2}. \]

Thus we replace (2.15) with (2.32) and

\[ \sqrt{\sum_{(i_1i_2) \in I \times I} V_{i1i2} w_{i1i2j}^2} \leq t_j, \quad j \in J. \quad (2.33) \]
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As constraints (2.33) define the following extended polymatroid

$$EP_g = \left\{ \pi \in \mathbb{R}^{|I|\times|I|} : \sum_{(i_1 i_2) \in S} \pi_{i_1 i_2} \leq \sqrt{\sum_{(i_1 i_2) \in S} V_{i_1 i_2}}, \forall S \subseteq I \times I \right\}$$

we can now generate polymatroid cuts from $EP_g$ in the same manner as in Section 2.5.3. Benefits of these cuts for model 3 are illustrated in Section 2.8.

2.8 Computational results

In this section we present our computational results on solving the corresponding conic quadratic MIP formulations of the joint location-inventory problems discussed in the previous sections. We compare our results with the earlier approaches based on Lagrangian relaxation and column generation methods for the special cases. We also study the impact of facility capacities, stochastic lead times, and demand correlations on the solutions.

The numerical experiments in this dissertation use data from the 1990 US Census described in Daskin (1995). We employ four different data sets: a 15-node, 25-node, 88-node, and 150-node data set. The 15-node data set reports the node demand (population) of the 15 most populous US states. The 25-node data set reports the node demand of the 25 largest cities in the US. The 88-node data set reports the demand of each of the lower 48 US state capitals plus Washington DC and the 50 largest US cities (eliminating duplicates.) The 150-node data set reports the demand of the 150 most populous US cities. All data sets can be downloaded from the site: http://sitemaker.umich.edu/msdaskin/software.

We use these data files in all our experiments except for those showing the computational benefits of adding cuts (Tables 2.4 and 2.5) and for those showing scalability for our most general model (Table 2.7), in which we report the averages for ten randomly generated instances per row. Each random instance is generated by adding noise to the demand multiplying mean and standard deviation defined in the data files by $(1 + \epsilon_i) \forall i$, where $\epsilon_i$ is drawn from Uniform $[-0.1, 0.1]$. We also draw fixed cost from Uniform $[40,000, 50,000]$. See Tables A.1 and A.2 in Appendix A.1 for a summary of the parameter values used in all experiments. All computations are done on a 2.393 GHz Linux x86 computer using CPLEX 11.0.

2.8.1 Numerical experiments on the uncapacitated case

In order to study the impact of inventory and transportation costs on the first model, we vary the values of $\beta$ and $\theta$, which are the weights of the transportation
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and inventory costs, respectively. We report computational results for different choices of \((\beta, \theta)\). Observing that when \(\theta\) is larger than \(\beta\), solution method require more time to arrive at optimality, we focus our attention on these cases. Higher values of \(\theta\) assign more weight on the nonlinear terms of the objective terms.

For the experiments reported in Table 2.1, we use the 88- and 150-node data sets. For each run (row), we report the number of nodes (retailers), the transportation and inventory weights, the number of columns generated by the algorithm of Shen et al. (2003) and the corresponding CPU time as well as the number of polymatroid cuts and CPU time of the conic integer programming approach. So that we can directly compare the results, we ran the column generation method of Shen et al. (2003) and the conic integer model (CQMIP1) on the same computer using the same version of CPLEX. No branching was necessary for either approach for this data set. Indeed, as the polymatroid cuts define the convex hull of the nonlinear subproblem of Shen et al. (2003), both approaches give to the same relaxation values. We observe in this table that the conic method clearly outperforms the column generation method for both, the 88-node and 150-node, data sets. The aggregate times showed in Table 2.2 allow us to state that the proposed conic integer programming approach is quite fast and robust.

This experiment also provides managerial insights.

**Observation 1.** When the inventory cost is relatively larger than the transportation cost, fewer DCs are opened in an optimal solution (observe DCs column in Table 2.1). Thus, under our model, a risk pooling strategy is favored when inventory costs are proportionally larger.

**Observation 2.** When transportation cost is relatively larger than inventory cost, more DCs are opened in an optimal solution (observe DCs column in Table 2.1).

2.8.2 Numerical experiments on the capacitated case

In Table 2.3, we report the results obtained by running conic integer program (CQMIP2) along with the results presented in Özsen et al. (2008). We want to caution the reader that this table is rather descriptive and we do not aim to directly compare the running time of the two approaches. The computations in Özsen et al. (2008) were done on a 1.7 GHz computer while we used a 2.393 GHz computer. In addition, we employ CPLEX software, whereas computations in Özsen et al. (2008) are based on their own code written in C++.

We report the run number, the objective value, the number of nodes explored and the CPU time for both approaches. Focusing on the results obtained by the conic integer programming approach, we observe that all our runs reach optimality.
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Table 2.1: Comparison with Shen et al. (2003).

<table>
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<th>retailers</th>
<th>$\beta$</th>
<th>$\theta$</th>
<th>DCs</th>
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<th>Conic formulation</th>
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Table 2.2: Summary statistic.

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<th>Data set</th>
<th>Aggregate time Shen et al. set covering</th>
<th>Aggregate time Conic formulation</th>
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<td>150-node</td>
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rather fast. Some instances, such as with 150 retailers, do not even require any branching. Hence, we can state that our approach performs quite well in this experiment.

We have noticed that the facility capacities in the data set from Özsen et al. (2008) were often loose. In order to see the sensitivity of our approach to the tightness of facility capacities, we performed an additional experiment. Toward this end, we first created a problem instance where the capacity for each potential DC, \( C \), was set to 19% of the total daily average demand, i.e., \( \frac{C}{\sum_{i \in I} \mu_i} \times 100 = 19\% \). Then we created additional problem instances by progressively tightening the DC capacity till reaching 16.287% (below this percentage the problem becomes infeasible). In Table 2.4 we report the CPU time in seconds and the number of nodes explored with and without adding cover and extended cover inequalities. We observe that problems generally become more difficult to solve as the capacity becomes tighter and that adding cover and extended cover inequalities reduces the solution times and the number of nodes significantly.

### 2.8.3 Numerical experiments on correlated retailer demand case

Here we investigate the impact of correlated retailer demand on the joint location-inventory problem. First, to get an insight, we illustrate the effect of retailer demand correlations on a small example from the 25-node set data using the parameter values listed in Table A.1. The dark links on Figure 2.1 show the retailer assignments in the optimal solution when there are no correlations. In this case four DC’s are opened in New York, Los Angeles, Chicago, and Houston and the expected total cost is 100,910. To see how correlations change the solution, we add correlation to the demand of the retailers served by Chicago and New York. Namely, we set the correlation between retailers Chicago, Detroit, Milwaukee, Indianapolis, and Columbus to 80% and similarly set the correlation between retailers New York, Philadelphia, Baltimore, and Washington to 80%. This naturally increases the safety stock levels that need to be kept in Chicago and New York and the cost of doing so. We see that in this case, the current solution is no longer optimal. Indeed, it is infeasible since the maximum capacity at the NY DC is smaller than the required inventory levels at this DC when accounting for correlated demands. The optimal solution (in light color) replaces the DC in Chicago with Indianapolis and uncorrelated retailers that were served from New York and Houston are assigned to Indianapolis. Thus, as expected, the optimal solution pools more of the uncorrelated demand into the same DC and
Table 2.3: Comparison with Özen et al. (2008).

<table>
<thead>
<tr>
<th>retailers</th>
<th>Özen et al. Lagrangian</th>
<th>Conic formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>objective</td>
<td>nodes</td>
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<tr>
<td>1</td>
<td>15</td>
<td>567564</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>595707</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>621764</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>630051</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>630976</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>642722</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
<td>657981*</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>661070</td>
</tr>
<tr>
<td>9</td>
<td>15</td>
<td>668430*</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>987298*</td>
</tr>
<tr>
<td>11</td>
<td>88</td>
<td>322627</td>
</tr>
<tr>
<td>12</td>
<td>88</td>
<td>327300</td>
</tr>
<tr>
<td>13</td>
<td>88</td>
<td>328702*</td>
</tr>
<tr>
<td>14</td>
<td>88</td>
<td>328808</td>
</tr>
<tr>
<td>15</td>
<td>88</td>
<td>329024</td>
</tr>
<tr>
<td>16</td>
<td>88</td>
<td>330900</td>
</tr>
<tr>
<td>17</td>
<td>88</td>
<td>333440</td>
</tr>
<tr>
<td>18</td>
<td>88</td>
<td>337911</td>
</tr>
<tr>
<td>19</td>
<td>88</td>
<td>342219*</td>
</tr>
<tr>
<td>20</td>
<td>88</td>
<td>344845*</td>
</tr>
<tr>
<td>21</td>
<td>150</td>
<td>468645</td>
</tr>
<tr>
<td>22</td>
<td>150</td>
<td>469599</td>
</tr>
<tr>
<td>23</td>
<td>150</td>
<td>469740</td>
</tr>
<tr>
<td>24</td>
<td>150</td>
<td>471320</td>
</tr>
<tr>
<td>25</td>
<td>150</td>
<td>473743</td>
</tr>
<tr>
<td>26</td>
<td>150</td>
<td>474475</td>
</tr>
<tr>
<td>27</td>
<td>150</td>
<td>474750</td>
</tr>
<tr>
<td>28</td>
<td>150</td>
<td>476508</td>
</tr>
<tr>
<td>29</td>
<td>150</td>
<td>477314</td>
</tr>
<tr>
<td>30</td>
<td>150</td>
<td>478615</td>
</tr>
</tbody>
</table>

(*) not optimal.
Chapter 2. A novel solution approach: conic programming

Table 2.4: Impact of capacities on solving (CQMIP2).

<table>
<thead>
<tr>
<th>DC capacity (% demand)</th>
<th>CPLEX time</th>
<th>nodes</th>
<th>CPLEX + Cuts time</th>
<th>nodes</th>
<th>cover cuts</th>
<th>(ext cover cuts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>67</td>
<td>4080</td>
<td>85</td>
<td>3060</td>
<td>1562</td>
<td>(0)</td>
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<tr>
<td>18.5</td>
<td>324</td>
<td>23618</td>
<td>131</td>
<td>6677</td>
<td>626</td>
<td>(0)</td>
</tr>
<tr>
<td>18</td>
<td>202</td>
<td>10219</td>
<td>169</td>
<td>9999</td>
<td>588</td>
<td>(0)</td>
</tr>
<tr>
<td>17.5</td>
<td>199</td>
<td>11941</td>
<td>129</td>
<td>7250</td>
<td>549</td>
<td>(0)</td>
</tr>
<tr>
<td>17</td>
<td>400</td>
<td>27794</td>
<td>167</td>
<td>9402</td>
<td>483</td>
<td>(0)</td>
</tr>
<tr>
<td>16.5</td>
<td>64</td>
<td>2848</td>
<td>64</td>
<td>2480</td>
<td>576</td>
<td>(103)</td>
</tr>
<tr>
<td>16.35</td>
<td>140</td>
<td>7621</td>
<td>119</td>
<td>5340</td>
<td>574</td>
<td>(104)</td>
</tr>
<tr>
<td>16.3</td>
<td>90</td>
<td>5419</td>
<td>82</td>
<td>3668</td>
<td>575</td>
<td>(104)</td>
</tr>
<tr>
<td>16.29</td>
<td>133</td>
<td>6603</td>
<td>102</td>
<td>5523</td>
<td>576</td>
<td>(104)</td>
</tr>
<tr>
<td>16.287</td>
<td>2033*</td>
<td>161154*</td>
<td>764</td>
<td>31936</td>
<td>577</td>
<td>(106)</td>
</tr>
</tbody>
</table>

(*) instance could not be solved in 2000 seconds.

reduces pooling of correlated demand to keep the inventory levels and subsequent costs low. The expected total cost is 108,948.

Continuing with this example, in order to see how correlations affect the total expected cost, this time, we introduce positive correlation between every pair of retailers. In order to investigate the impact of correlations independent from capacity considerations and high cost of facility installations, we also set the DC capacities to a very large number and reduce the annualized facility fixed cost from 10,0000 to 1,000. Figure 2.2 shows the total expected cost as well as the number of DCs opened as a function of the retailer demand correlation. The expected total cost increases monotonically from 46,095 to 50,297 as more safety stock is needed in response to increasing demand correlation. Moreover, additional DCs are opened to reduce the number of retailers supplied by the same DC. From this we can state that:

**Observation 3.** Positive demand correlation between retailers reduces the benefits of (location) risk pooling.

Finally, in Table 2.5 we report on the computational efficiency of solving (CQMIP3) with and without adding extended polymatroid cuts to the formulation as a function of demand correlation. We report the total expected cost, the CPU time in seconds, the percentage integrality gap at the root node of the search tree (rgap), and the number of branch and bound nodes explored. We observe that computational difficulty increases with higher correlation. However, the cuts are quite beneficial in reducing the computational burden.
Figure 2.1: The effect of correlated retailer demand on the supply chain.

Table 2.5: Impact of correlation coefficient on solving (CQMIP3).

<table>
<thead>
<tr>
<th>$\rho_{ij}$</th>
<th>average cost</th>
<th>CPLEX</th>
<th>CPLEX + Cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>time % rgap</td>
<td>nodes</td>
<td>time % rgap</td>
</tr>
<tr>
<td>0</td>
<td>211521</td>
<td>14</td>
<td>0.10</td>
</tr>
<tr>
<td>0.1</td>
<td>213356</td>
<td>32</td>
<td>0.32</td>
</tr>
<tr>
<td>0.3</td>
<td>215429</td>
<td>73</td>
<td>0.76</td>
</tr>
<tr>
<td>0.5</td>
<td>223102</td>
<td>302</td>
<td>2.51</td>
</tr>
<tr>
<td>0.6</td>
<td>228836</td>
<td>844</td>
<td>2.47</td>
</tr>
<tr>
<td>0.7</td>
<td>229826</td>
<td>1156</td>
<td>4.01</td>
</tr>
</tbody>
</table>
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Figure 2.2: Cost and number of DCs as a function of retailers’ demand correlation.

Table 2.6: Impact of lead time variability.

<table>
<thead>
<tr>
<th>$\sigma_{L,i}$</th>
<th>cost</th>
<th>DCs</th>
<th>opened DCs</th>
<th>closed DCs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>101868</td>
<td>4</td>
<td>New York, L.A., Chicago, Houston</td>
<td>-</td>
</tr>
<tr>
<td>0.1</td>
<td>125231</td>
<td>4</td>
<td>Philadelphia, Indiana</td>
<td>New York, Chicago</td>
</tr>
<tr>
<td>0.2</td>
<td>134529</td>
<td>5</td>
<td>New York, San Diego, Baltimore</td>
<td>Philadelphia, L.A.</td>
</tr>
<tr>
<td>0.3</td>
<td>141152</td>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.4</td>
<td>147305</td>
<td>5</td>
<td>Philadelphia, San Antonio</td>
<td>New York, Houston</td>
</tr>
<tr>
<td>0.5</td>
<td>152090</td>
<td>6</td>
<td>Houston, San Francisco</td>
<td>San Antonio</td>
</tr>
</tbody>
</table>

2.8.4 Numerical experiments on stochastic lead time case

One of the main effects of considering stochastic lead times in our model is the increment of safety stock cost and, consequently, the increment of expected total cost on the supply chain structure. Table 2.6 shows this impact under the optimal supply chain design per each case. In particular, we report the number of DCs employed as the lead time standard deviation increases by 0.1 in the 25-node data set assuming uncorrelated demands. We also report which DCs are opened and closed with respect to the previous run. The number of active DCs increases since the system is capacitated.

Figures 2.3 and 2.4 capture the simultaneous impact of retailers’ correlated demands and lead time variability on costs and number of DCs, respectively. Hence, we present two three-dimensional graphs that are created from adding a
Figure 2.3: Cost as a function of retailers’ demand correlation and lead time variability.

third axis that accounts for lead time variability to the two-dimensional Figure 2.2. Note that Figure 2.2 shows the particular case in which the standard deviation of the lead time is 0. In particular, the retailers’ correlation factor ($\rho_{i1}$) and the lead time standard deviation ($\sigma_{L_j}$) increase 0.1 per each experiment.

Note that since we are adding positive extra parameters inside the square root of the safety stock term, we observe that:

Observation 4. *The optimal total cost of the supply chain increases when retailers’ demand correlation and/or stochastic lead time variability increase.*

We observed this in Figure 2.3 where we go from a value of 46,095 for the uncorrelated and fixed lead time case to a value of 76,129 for the perfectly correlated with 0.5 standard deviation lead time case. This represents a 65% increase in costs with respect to the uncorrelated-fixed lead time cases versus a raise of the 67% that would represent to keep the base case supply chain configuration.

Figure 2.4 describes a boost of the number of opened DCs, from 15 DCs to 18, when increments are applied in both directions. Also observe that, given the same lead time standard deviation, for the most correlated cases the number of opened DCs is higher compared with lowest correlated cases. This general preference to build new DCs as opposed to keep pooled inventory (i.e. diversification) directly depends on the specific relative parameter values.
2.8.5 Numerical experiments on the multi-commodity case

It is interesting to study the scalability of our most general model. To do so, we increase the number of commodities \( L \) and observe the CPU times and the number of nodes explored in the search tree in two different experiments. Experiment 1 assumes uncorrelated demand between different retailers and different products and a fixed known lead time of one at each DC (i.e. block matrices \( U \) and \( W \) are diagonal). Experiment 2 assumes non-zeros in all the elements of our block matrices. In particular, there is a correlation of 0.1 between different retailers, 0.1 between different products, and a 0.1 standard deviation of all DCs’ lead times. In each run we take the average of ten random instances based on some randomly generated parameters (refer to Table A.1).

As expected, we observe a better computational performance for experiment 1 compared to experiment 2, due to sparsity of the correlation matrices and fixed lead time. Overall, we observe a good scalability as a function of \(|L|\).

For the 88-node data set when \(|L| = 3\) (or more), the models become too large (18 million columns, 6 million rows, and more than 6 million binary variables) to solve. Our computer runs out of 8GB memory during model generation with IBM ILOG Concert Technology. This suggest that for solutions of very large scale models, column and/or row generation methods would be needed.

Finally, we show an example of the impact of product correlation on the 25-node supply chain with two products. To isolate the correlation effect, we assume
that lead time is fixed, retailer demand is uncorrelated, and all DCs are uncapacitated. The latter assumption causes both products to share the same DCs and assignments between DC-retailer. This way we are able to exclusively focus on the impact of product correlation. The links and circled cities in dark color on Figure 2.5 show the optimal supply chain design when demand of both products is uncorrelated. In this case, there are nine opened DCs and the expected total cost is 148,521. Next, we assume that the products have 80% correlation. As in the retailer demand correlation case, if we keep the same assignments as in the uncorrelated case, the safety stock levels increase causing this design to be no longer optimal. If we keep the same design as in the uncorrelated case, total costs would reach 159,062 which represents a 0.35% increase compared with the new optimal expected total cost of 158,503. DCs in Detroit, Seattle, and San Jose are no longer considered for the new optimal design (in light color). Retailers assigned to Detroit are reassigned to Chicago and a new DC in San Francisco supplies San Jose and Seattle.

Note that in this experiment positive correlation causes a pooling effect (DCs go from nine to seven) as opposed to the diversified effect resulted in Figures 2.2 and 2.4 when retailers’ demand correlation increased. From these two results we can conclude that the decision to build more or less DCs compared to the uncorrelated base case depends on the specific trade-off between fixed location and safety stock costs (DCs are uncapacitated). If location fixed costs are more relevant a pooling strategy will be considered and a diversified strategy when the contrary occurs.

### Table 2.7: Scalability of model 6.

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th></th>
<th>variables (binary)</th>
<th>conic constraints</th>
<th>Experiment 1</th>
<th>Experiment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>time</td>
<td>nodes</td>
<td>time</td>
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<tr>
<td>1</td>
<td>725 (650)</td>
<td>50</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<tr>
<td>2</td>
<td>1400 (1275)</td>
<td>75</td>
<td>3</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2075 (1900)</td>
<td>100</td>
<td>14</td>
<td>0</td>
<td>361</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>2750 (2500)</td>
<td>125</td>
<td>18</td>
<td>0</td>
<td>256</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>3425 (3150)</td>
<td>150</td>
<td>31</td>
<td>0</td>
<td>102</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>6800 (6275)</td>
<td>275</td>
<td>173</td>
<td>0</td>
<td>233</td>
<td>5</td>
</tr>
<tr>
<td>15</td>
<td>10175 (9400)</td>
<td>400</td>
<td>474</td>
<td>0</td>
<td>695</td>
<td>6</td>
</tr>
</tbody>
</table>
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Figure 2.5: The effect of correlated product demand on the supply chain.

2.9 Conclusions

In this chapter we describe a conic integer programming approach to stochastic facility location and inventory management models with risk pooling. This new approach not only reasonably leads to similar or better computational solution times than previous column generation and Lagrangian based methods, but more importantly allows one to model more general problems than considered up to now. The solution algorithms developed in Shen et al. [2003], Daskin et al. [2002], and Ozsen et al. [2008] assume that the mean and variance of demand are uniform across retailers. Shu et al. [2005] and Shen and Qi [2007] offer more flexible methods and allow the proportion of retailer mean and variance to be non-uniform. However, these latter references solve the uncapacitated facility problem with uncorrelated demands. In this work, we remove these restrictive assumptions and consider other novel generalized aspects and still are able to solve the problems efficiently.

We solve each of the models by recasting them as equivalent conic quadratic mixed-integer programs, which can be solved to optimality using commercial software packages. This reduces the burden of developing special purpose algorithms for each special case. However, in cases where optimality cannot be easily reached, we may employ valid cuts to improve the computational performance.
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We also make some general recommendations about how best to respond to changes in the model structure. If all demands are uncorrelated, we show that a risk pooling strategy deals effectively with increasing inventory costs. Another situation has been observed when dealing with uncapacitated DCs and incremental positive correlated demands. A diversified location strategy is appropriate when safety stock costs are more relevant than other costs such as fixed location costs. Yet, a pooling strategy can be optimal if fixed costs are relatively higher.

The conic integer programming approach introduced in this chapter is versatile and can be applied to other nonlinear supply chain models as well. In particular, our approach could be extended to study pure inventory management models or integrated production and transportation planning models, to name a few. Finally, other integrated location-inventory models with vehicle routing, service, or unreliable supply would benefit from the conic integer programming approach as well.
Chapter 3

Applied area I: Reverse Flows

3.1 Introduction

In the increasingly competitive global manufacturing environment, the success of a corporation depends on its ability to favorably manage its supply chains. A supply chain includes all the components necessary to design, fabricate, distribute, sell, support, use, and recycle (or dispose of) a product. Competitive and regulatory pressures present new challenges in supply chain management. Consequently, green supply chains, reverse supply chains, closed-loop supply chains, and sustainable supply chains are getting more attention. In particular, supply chain managers are interested in economically handling returned products by reusing them to obtain numerous financial benefits (Blackburn et al. (2004)).

Companies face time and cost trade-offs in the implementation of integrated supply chains. Time-varying prices of returned products, especially for time-sensitive and short life-cycle products, complicate the problem. For example, consumer electronics products such as PCs can lose its value at rates of 1% per week (Guide and Van Wassenhove (2009)). In response, it would be preferable to shorten the flow time of returned products in a reverse supply chain. This strategy results in more profits from the salvage value of returned products by reentering them into the market as quickly as possible. Meanwhile, batch processing of products to benefit from economies of scale is also a recommended strategy but requires a slower flow time supply chain. Thus, the time versus cost trade-off enters into planning.

We consider a three-tiered supply network: one supplier, some distribution centers (DCs) with capacity limitation, and retailers. The product can simultaneously flow in two directions. The forward direction is the flow of the retailer’s order of the product from the DC. In turn, the DCs get replenished from the
supplier based on the specified inventory policy. The reverse direction is the flow of returned products from the retailers to the corresponding DCs and then back to the supplier to be reprocessed. Note that the DCs can hold stocks of both new and returned products. Figure 3.1 illustrates the structure of the three-tiered supply network.

![Diagram of the three-tiered supply chain network](image)

**Figure 3.1:** The three-tiered supply chain network II.

The main contribution of this chapter is the in-depth study of the impact DC capacities and reverse flows have on an integrated supply chain design problem where flows of both directions and a balance between efficiency and responsive costs are considered. We also employ an efficient resolution method for the resulting complex model, which is based on the cutting plane method. The proposed method explores the convex hull of the feasible solutions and adds valid inequalities to improve computational efficiency.

The remainder of the chapter is organized as follows. In section 3.2, we present a literature review on the integrated forward/reverse network design and the capacitated facility location problem. Section 3.3 develops a nonlinear mixed-integer programming formulation of the supply chain. In section 3.4, the model is converted into a conic quadratic mixed-integer program to be solved efficiently. Subsequently, some valid inequalities are developed to improve the computational efficiency of the branch and cut algorithm and the quality of the solution. Next, in section 3.5 we explore the behavior of the supply chain under this optimization strategy through computational experiments on real data. In the last section, we conclude and discuss future research avenues.
Chapter 3. Applied area I: Reverse Flows

3.2 Literature review

Many variants of the facility location problem (FLP) have appeared in the literature. Extensions of the basic model have been addressed including LTL (less than truckload) vehicle routing, inventory management, robustness, and reliability. For a summary, see the articles by Daskin (1995), Langevin and Riopel (2005), and the papers by Shen (2007a) and Melo et al. (2009).

The integrated study on forward/reverse supply chain network design is an emerging topic. The interested reader can refer to the work of Guide and Van Wassenhove (2009) and Akçalı et al. (2009), that provide comprehensive reviews on closed-loop supply chains. Three generic uncapacitated integrated closed-loop supply chain design models were developed in Sahyouni et al. (2007) that minimize the fixed locating costs and transportation costs. Lu and Bostel (2007) presented a two-level uncapacitated location problem with three types of facilities that minimize fixed setup costs and transportation costs. Ko and Evans (2007) proposed a genetic algorithm-based heuristic to solve a multi-period, two-echelon, multi-commodity, capacitated facility location model. The integrated aspects of optimizing a forward and reverse network were considered simultaneously. Üster et al. (2007); Easwaran and Üster (2009, 2010) studied multi-product closed-loop supply chain network design problems. They located collection centers and finite-capacity manufacturing facilities while coordinating the forward and reverse flows in the network so as to minimize the processing, transportation, and fixed location costs. Pishvaae et al. (2010) developed a bi-objective mixed integer program to minimize the total costs and maximize the responsiveness of an integrated forward/reverse logistics network. Pishvaae et al. (2011) proposed a robust optimization model for closed-loop supply chain network design based on the recent extensions in robust optimization theory.

Capacity restrictions in the facility location problem are a natural extension of the original problem and play a critical role. The capacitated facility location problem (CFLP) and its variants are well studied in the literature. For a review, please refer to the book of Mirchandani and Francis (1990). We note that most solution algorithms for capacitated facility location problems are adaptations of algorithms for uncapacitated problems. Therefore, heuristics such as Lagrangian relaxation-based algorithms Holmberg et al. (1999); Daskin et al. (2002); Langevin and Riopel (2005); Sahyouni et al. (2007); Lu and Bostel (2007); Özsen et al. (2008); Liu et al. (2010), Benders decomposition-based solution approaches Üster et al. (2007); Easwaran and Üster (2009, 2010) and meta-heuristics such as genetic algorithm Ko and Evans (2007), tabu search Easwaran and Üster (2009), memetic algorithm Pishvaee et al. (2010) are used extensively.
3.3 Problem formulation

There are three types of distribution centers in the network: forward (new products), reverse (returned products), and joint DCs (both new and returned products). We determine the DC locations among potential sites and the assignment of retailers to the DCs. The objective is to minimize the fixed charges of locating the distribution centers, working inventory costs, transportation costs, and the value loss of returned products.

To benefit from the risk pooling strategy, inventories are not kept at the retailers’ sites but at the DCs. The DCs can fill retailer demand and can store returned products temporarily. An approximation to the \((Q, R)\) model with Type-I service (Özsen et al. (2008)) is used for managing the stock of new products. The inventory policy followed by the return products is an approximation of the EOQ since not always the EOQ formula will provide the optimal quantity (the system is capacitated).

To exploit economies of scale in transportation costs, returned products will be shipped back to the supplier for reprocessing after a predetermined quantity at the DCs is reached. At the same time, getting returned products back to the market quickly will bring more profit. Blackburn et al. (2004) investigated reverse supply chains for commercial returns (in particular, products returned by customers for any reason within 90 days of sale). In a real example, for $1000 worth of product returns nearly half the product value (\(> 45\%\)) is lost in the return process by waiting for the product to be reprocessed. Indeed, a returned consumer product could wait in excess of 3.5 months before it is sent to disposition in a real case. Thus, we analyze the trade-off between efficiency and responsive costs when designing a forward/reverse supply chain network.

Before proposing the model, some important assumptions are followed. First of all, customer demands are Poisson distributed. Thus, variances of daily demand and returns are identical to the means of daily demand \((\mu_i^F)\) and returns \((\mu_i^R)\), respectively, for each retailer \(i\). \(I\) corresponds to the set of retailers by \(i\) and \(J\) corresponds to the set of candidate DC sites indexed by \(j\). Further, demands at the retailers are uncorrelated over time and across retailers. Demand of returned products is independent from new product’s demand. The model also assumes that there is sufficient transportation capacity but controls capacity at each DC. In the following lines we define the parameters and variables.

**Parameters and Notation**

*Demand*

\[
\mu_i^F, \mu_i^R : \text{mean (daily) volumes of new and returned products at retailer } i,
\]
Chapter 3. Applied area I: Reverse Flows

Costs
- \( f^F_j, f^R_j \): fixed (yearly) costs of locating a forward/reverse DC at site \( j \),
- \( F^F_j, F^R_j \): fixed costs of placing an order of new/returned products at DC \( j \),
- \( g^F_j, g^R_j \): fixed shipping costs from supplier to DC \( j \) for new/returned products,
- \( a^F_j, a^R_j \): unit cost to ship from supplier to DC \( j \) for new/returned products,
- \( d_{ij} \): unit cost to ship from DC \( j \) to retailer \( i \) in forward/reverse flows,
- \( S^C_j \): fixed location cost savings at joint DC \( j \),
- \( WI^F,R_j \): the total annual cost of working inventory at forward/reverse DC \( j \),

Other parameters
- \( \beta \): weight associated with transportation cost in forward/reverse flows,
- \( \theta \): weight associated with inventory cost in forward/reverse flows,
- \( W \): weight factor associated with loss in value of returned products,
- \( \gamma \): returned products’ marginal value of time,
- \( \alpha \): desired percentage of retailers orders satisfied,
- \( z_\alpha \): standard normal deviate such that \( P(z \leq z_\alpha) = \alpha \),
- \( h \): inventory holding cost per unit of products per year for each DC,
- \( L_j \): lead time in days at a DC \( j \),
- \( \chi \): number of days in a year.

Decision Variables
- \( X^F_j \): 1, if candidate location \( j \) is selected as a forward DC, 0 otherwise,
- \( X^R_j \): 1, if candidate location \( j \) is selected as a reverse DC, 0 otherwise,
- \( X^C_j \): 1, if candidate location \( j \) is selected as a joint DC, 0 otherwise,
- \( Y^F_{ij} \): 1, if demand of new products of retailer \( i \in I \) is served by DC \( j \), 0 otherwise,
- \( Y^R_{ij} \): 1, if returned products of retailer \( i \in I \) is collected by DC \( j \), 0 otherwise,
- \( Q^F_j, Q^R_j \): shipment quantity of new and returned products at DC \( j \).

In summary, model \((P)\) is:
\[
\min_{X,Y,Z} Z = \sum_{j \in J} \left\{ f_j^F X_j^F + \sum_{i \in I} \beta \chi d_{ij} \mu_i^F Y_{ij}^F + \theta h z_a \sqrt{L_j \sum_{i \in I} \mu_i^F Y_{ij}^F} + W I_j^F (D_j^F) \right\}
\]
\[
+ \sum_{j \in J} \left\{ f_j^R X_j^R + \sum_{i \in I} \beta \chi d_{ij} \mu_i^R Y_{ij}^R + W I_j^R (D_j^R) \right\} - \sum_{j \in J} S_j^C X_j^C
\]
\[
+ W \sum_{j \in J} R(Q_j^R),
\]  
\[
\text{s.t.} \quad \sum_{j \in J} Y_{ij}^F = 1, \sum_{j \in J} Y_{ij}^R = 1, \quad \forall i \in I, (3.2)
\]
\[
Y_{ij}^F \leq X_j^F, Y_{ij}^R \leq X_j^R, \quad \forall i \in I, \forall j \in J, (3.3)
\]
\[
X_j^C \leq X_j^F, X_j^C \leq X_j^R, \quad \forall j \in J, (3.4)
\]
\[
Q_j^F + z_a \sqrt{L_j \sum_{i \in I} \mu_i^F Y_{ij}^F} + L_j \sum_{i \in I} \mu_i^F Y_{ij}^F + Q_j^R \leq C_j, \quad \forall j \in J, (3.5)
\]
\[
Q_j^F, Q_j^R \geq 0, \quad \forall j \in J, (3.6)
\]
\[
X_j^F, X_j^R, X_j^C \in \{0, 1\}, \quad \forall j \in J, (3.7)
\]
\[
Y_{ij}^F, Y_{ij}^R \in \{0, 1\}, \quad \forall i \in I, \forall j \in J. (3.8)
\]

where,
\[
WI_j^F (D_j^F) = \frac{F_j^F D_j^F}{Q_j^F} + \beta (g_j^F + a_j^F Q_j^F) \frac{D_j^F}{Q_j^F} + \frac{\theta h}{2} Q_j^F, \quad \forall j \in J, (3.9)
\]
\[
WI_j^R (D_j^R) = \frac{F_j^R D_j^R}{Q_j^R} + \beta (g_j^R + a_j^R Q_j^R) \frac{D_j^R}{Q_j^R} + \frac{\theta h}{2} Q_j^R, \quad \forall j \in J, (3.10)
\]
\[
D_j^F = \chi \sum_{i \in I} \mu_i^F Y_{ij}^F, D_j^R = \chi \sum_{i \in I} \mu_i^R Y_{ij}^R, \quad \forall j \in J.
\]

The objective function (3.1) consists of four parts: cost of forward flows, cost of reverse flows, savings from co-location of forward and reverse DCs, and the time value of returned products.

The first part sums the costs of handling new products including the fixed charge of locating forward DCs, the DC-to-retailer shipping costs, the safety stock costs to ensure customer satisfaction, and the working inventory cost. The working inventory cost of new products is formulated as equation (3.9) which is the sum...
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of the fixed costs for handling orders, the DC-to-supplier shipping costs, and the average order holding costs per year. The detailed explanation of equation (3.9) can be referred to Shen et al. (2003).

The second part of the objective contains the costs of the reverse flows. Except for the safety stock costs, it has the same cost components as the first part of the objective. The working inventory cost of returned products is represented as equation (3.10). We suppose that the return rates of used products are constant among different retailers and are integrated in the definition of $D^R_j$. Geyer et al. (2007) provide a discussion of calculating return rates in practical settings.

The third part of the objective represents the fixed cost savings created by the co-location of forward and reverse DCs at the same site. Note that normally the cost saving must be less than the minimum of the fixed charges of forward and reverse DCs. We, therefore, assume that $S^C_j \leq \min\{f^F_j, f^R_j\}$ (Sahyouni et al. (2007)).

The fourth part concerns the time value of returned products. $R(Q^R_j)$ is the average value loss of returned product per year and it is related to returned product’s marginal value of time. Derivation of the formula of $R(Q^R_j)$ is given at the end of this section.

Constraints (3.2) ensure that each retailer is served by exactly one DC. Constraints (3.3) state that a retailer can only be assigned to an open DCs. Constraints (3.4) stipulate that if a DC is assigned to serve both forward and reverse flows, i.e. a joint DC, then it acts as not only a forward DC but a reverse DC as well. Consequently, cost savings occur. Note that forward (reverse) DCs refer to stand-alone forward (reverse) DCs and forward (reverse) facilities at joint DCs throughout the rest of this chapter. Constraints (3.5) are the capacity restrictions of each DC $j$ (further described in the next paragraph.) Constraints (3.6) are nonnegative constraints. Constraints (3.7) and (3.8) are standard integrality constraints.

Özsen et al. (2008) point out that the capacity of a DC must withstand the worse-case scenario because the amount of space the warehouse needs is proportional to peak inventory. In particular, this happens when there is no demand of new products and no shipment of returned products during the replenishment lead time. Thus, the capacity constraints can be formulated as:

$$Q^F_j + z \alpha \sqrt{L_j \sum_{i \in I'} \mu^F_i} + L_j \sum_{i \in I'} \mu^F_i + Q^R_j \leq C_j, \quad \forall j \in J,$$

where $I'$ is set of retailers served by DC $j$. The first and fourth terms represent the order quantity of new products and returned products, respectively. The second term is the safety stock under the assumption of Normal demands and it covers
stock-outs that occur with a probability of $\alpha$ or less. Note that $z_\alpha$ is a standard Normal deviate such that $P(z \leq z_\alpha) = \alpha$. The third term is the average demand during lead times.

3.3.1 Derivation of the average value loss of returned product

We start the derivation by defining the average value loss of returned product per year ($R(Q^R_j)$) for a linear decay rate. Next, we study this measure for an exponential decay rate and we conclude that the linear version can be a good enough approximation.

Based on Blackburn et al. (2004), the marginal value of time (MVT), denoted by $\gamma$, can be represented by the slopes of the lines in Figure 3.2. The greater $\gamma$ is the more sensitive the price of returned products is to time. $R(Q^R_j)$ can be defined in a similar way as we calculate inventory holding cost of returned products in a lot sizing problem. Since, on average, there will be $Q^R_j/2$ returned product of inventory on hand per year, the average value loss of returned products is defined as:

$$R(Q^R_j) = \frac{V\gamma}{2}Q^R_j.$$  \hfill (3.11)

Figure 3.2: Differences in marginal value of time for returns Blackburn et al. (2004)

Moreover, we can also define $R(Q^R_j)$ to tightly approximate the well-studied case of returned products with exponential price decay function. In this case

$$Price(t) = Ve^{-\gamma t},$$
where $t$ denotes the time the unit is held at DC and $V$ denotes the initial value of returned products. This case is extensively used in the literature to investigate the loss in value of returned products (Guide et al. (2006); Blackburn and Scudder (2009)).

The salvage value of returned products (with batch size equal to $Q^R_j$) can be found by integrating over $[0, Q^R_j]$, 

$$
\int_0^{Q^R_j} \text{Price} \left( \frac{Q^R_j - q \chi Q^R_j}{Q^R_j} - \frac{Q^R_j}{D^R_j} \right) dq.
$$

Then, the loss in value per batch equals

$$
VQ^R_j - \int_0^{Q^R_j} \text{Price} \left( \frac{Q^R_j - q \chi Q^R_j}{Q^R_j} - \frac{Q^R_j}{D^R_j} \right) dq.
$$

Subsequently, the loss in value of returned products, $R(Q^R_j)$, can be determined multiplying by the number of shipments to the supplier per year, $D^R_j Q^R_j$:

$$
R(Q^R_j) = \frac{D^R_j}{Q^R_j} \left\{ VQ^R_j - \int_0^{Q^R_j} \text{Price} \left( \frac{Q^R_j - q \chi Q^R_j}{Q^R_j} - \frac{Q^R_j}{D^R_j} \right) dq \right\}
= VD^R_j - \frac{V(D^R_j)^2}{\chi \gamma Q^R_j} \left( 1 - e^{-\chi \gamma Q^R_j D^R_j} \right).
$$

(3.12)

It is intractable to find the optimal solution of the model due to the complexity of the resulting $R(Q^R_j)$. Therefore, we replace $e^{-\chi \gamma Q^R_j D^R_j}$ with its second-order Taylor-series expansion

$$
e^{-\chi \gamma Q^R_j D^R_j} \approx 1 - \frac{\chi \gamma Q^R_j}{D^R_j} + \frac{\chi^2 \gamma^2 (Q^R_j)^2}{2 (D^R_j)^2}.
$$

Then, equation (3.12) can be approximated as follows:

$$
R(Q^R_j) \approx \hat{R}(Q^R_j) = \frac{V \chi \gamma}{2} Q^R_j.
$$

(3.13)

Let us define $R_I^R(Q^R_j) = WI^R_j(Q^R_j) + W \cdot R(Q^R_j)$ which represents the sum of working inventory costs and value loss of returned products. We next present the following property regarding this expression that we continue to study experimentally in section 3.5.3.
Property 1. $RI^R(Q_j^R)$ is unimodal in $Q_j^R$.

Proof. See Appendix. From the approximation in (3.13) we have that

$$RI^R_j(Q_j^R) \approx \hat{RI}^R_j(Q_j^R) = WI^R_j(Q_j^R) + W \cdot \hat{R}(Q_j^R).$$

In order to examine the accuracy of the approximation, we define

$$ERR(Q_j^R) = RI_j(Q_j^R) - \hat{RI}^R_j(Q_j^R) = W \left( R(Q_j^R) - \hat{R}^R_j(Q_j^R) \right),$$

which measures the error between $RI(Q_j^R)$ and its approximation, $\hat{RI}^R_j$.

Property 2. $ERR(Q_j^R)$ is a concave function of $Q_j^R$.

Proof. See Appendix.

Owing to Property 2, the quantities of returned products with respect to the maximal error, denoted by $Q_{j,max}^R$, can be determined by using search algorithms such as golden section method or bisection method. Since the optimal shipment quantity of returned products with capacity constraints must be less than that without consideration of capacity constraints, we also calculate the later one, denoted by $Q_j^{R*}$. Due to Property 1, search algorithms are employed as well. Then, $Q_j^R = \min\{Q_{j,max}^R, Q_j^{R*}\}$ is used to examine the accuracy of the approximation.

We perform 1,000,000 numerical experiments with the parameters drawn uniformly from the range given in Table 3.1. The values of $Q_{j,max}^R$ and $Q_j^{R*}$ are found by bisection method. The results are summarized in Table 3.2, in which,

$$err = \frac{|ERR|}{RI_j^R}.$$

The results show that $\hat{RI}^R_j$ is a quite tight approximation of $RI_j^R$.

Equation (3.13) has similar form as equation (3.11). Therefore, we can adopt equation (3.11) to approximate the loss in value of returned products with exponential price decay function.

### 3.4 Model properties and reformulation

The problem is modeled as a nonlinear mixed-integer program, which, in general, its optimal solutions are very hard to find in a reasonable amount of time. However, we note that our model could be identified as a novel version of the family of joint location-inventory models first time introduced by Shen et al. (2003).
Table 3.1: Parameter intervals for numerical experiments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interval</th>
<th>Parameter</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F^R_j$</td>
<td>[5, 15]</td>
<td>$V$</td>
<td>[1, 10]</td>
</tr>
<tr>
<td>$g^R_j$</td>
<td>[5, 15]</td>
<td>$\chi$</td>
<td>[1, 10]</td>
</tr>
<tr>
<td>$a^R_j$</td>
<td>[1, 10]</td>
<td>$\gamma$</td>
<td>[1, 10]</td>
</tr>
<tr>
<td>$h$</td>
<td>[1, 5]</td>
<td>$\theta$</td>
<td>[0.01, 1]</td>
</tr>
<tr>
<td>$W$</td>
<td>[0.1, 100]</td>
<td>$\beta$</td>
<td>[0.001, 0.1]</td>
</tr>
<tr>
<td>$D^R_j$</td>
<td>[1, 1000000]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Statistical results for numerical experiments.

<table>
<thead>
<tr>
<th>err (%)</th>
<th>number of experiments</th>
<th>percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.01</td>
<td>291,687</td>
<td>29.2%</td>
</tr>
<tr>
<td>&lt;0.02</td>
<td>513,487</td>
<td>51.3%</td>
</tr>
<tr>
<td>&lt;0.05</td>
<td>854,207</td>
<td>85.4%</td>
</tr>
<tr>
<td>&lt;0.1</td>
<td>951,443</td>
<td>95.1%</td>
</tr>
<tr>
<td>&lt;0.2</td>
<td>1,000,000</td>
<td>100%</td>
</tr>
</tbody>
</table>

average error=0.031%, maximal error=0.126%, minimal error=0.0054%

From Chapter 2 we can define an equivalent conic quadratic mixed-integer program that will be directly solved via commercial optimization packages. Further, Chapter 2 suggests that some cuts can be beneficial valid inequalities for models of the mentioned family. In the current chapter, we show how the polymatroid cuts are beneficial for the specific model of this chapter.

The following proposition provides an equivalent CQMIP formulation of problem $(P)$.

**Proposition 4.** Problem $(P)$ is equivalent to the following (CQMIP):
\[
\begin{align*}
\min_{X,Y} Z^S &= \sum_{j \in J} \left\{ f_j^F X_j^F + \theta h z_\omega \omega_j + \sum_{i \in I} \beta(d_{ij} + a_j^F) \chi \mu_i^F Y_{ij}^F + \frac{\theta h}{2} u_j \right\} \\
&\quad + \sum_{j \in J} \left\{ f_j^R X_j^R + \sum_{i \in I} \beta(d_{ij} + a_j^R) \chi \mu_i^R Y_{ij}^R + \frac{W \gamma \chi V + \theta h}{2} v_j \right\} - \sum_{j \in J} S_j^C X_j^C,
\end{align*}
\]
\[
\text{s.t.} \quad \sum_{j \in J} Y_{ij}^F = 1, \sum_{j \in J} Y_{ij}^R = 1, \quad \forall i \in I, \quad (3.14)
\]
\[
Y_{ij}^F \leq X_j^F, Y_{ij}^R \leq X_j^R, \quad \forall i \in I, \forall j \in J, \quad (3.15)
\]
\[
X_j^C \leq X_j^F, X_j^C \leq X_j^R, \quad \forall j \in J, \quad (3.16)
\]
\[
\omega_j^2 \geq L_j \sum_{i \in I} \mu_i^F (Y_{ij}^F)^2, \quad \forall j \in J, \quad (3.17)
\]
\[
\frac{1}{2}(u_j + Q_j^F)^2 \geq H_j^F \chi \sum_{i \in I} \mu_i^F (Y_{ij}^F)^2 + \frac{3}{2}(Q_j^F)^2 + \frac{1}{2} u_j^2, \quad \forall j \in J, \quad (3.18)
\]
\[
\frac{1}{2}(v_j + Q_j^R)^2 \geq H_j^R \chi \sum_{i \in I} \mu_i^R (Y_{ij}^R)^2 + \frac{3}{2}(Q_j^R)^2 + \frac{1}{2} v_j^2, \quad \forall j \in J, \quad (3.19)
\]
\[
Q_j^F + z_\omega \omega_j + L_j \sum_{i \in I} \mu_i^F Y_{ij}^F + Q_j^R \leq C_j, \quad \forall j \in J, \quad (3.20)
\]
\[
\omega_j, u_j, v_j, Q_j^F, Q_j^R \geq 0, \quad \forall j \in J, \quad (3.21)
\]
\[
X_j^F, X_j^R, X_j^C, Y_{ij}^F, Y_{ij}^R \in \{0, 1\}, \quad \forall i \in I, \forall j \in J, \quad (3.22)
\]

where \( H_j^F = \frac{2(F_j^F + \beta g_j^F)}{\theta h} \) and \( H_j^R = \frac{2(F_j^R + \beta g_j^R)}{W \gamma \chi V + \theta h} \).

**Proof.** A conic transformation is employed to linearize the objective of problem (P) in order to convert it into a CQMIP model. First, three sets of auxiliary variables \( \omega_j, u_j \) and \( v_j \) are introduced, which satisfy the following inequalities:

\[
\omega_j \geq \sqrt{L_j \sum_{i \in I} \mu_i^F Y_{ij}^F}, \quad (3.23)
\]
\[
\frac{\theta h}{2} u_j \geq (F_j^F + \beta g_j^F) \frac{D_j^F}{Q_j^F} + \frac{\theta h}{2} Q_j^F, \quad (3.24)
\]
\[
\left( \frac{W \gamma \chi + \frac{\theta h}{2} }{2} \right) v_j \geq (F_j^R + \beta g_j^R) \frac{D_j^R}{Q_j^R} + \left( \frac{W \gamma \chi + \frac{\theta h}{2} }{2} \right) Q_j^R. \quad (3.25)
\]
Recall $Y_{ij}^2 = Y_{ij}$ if $Y_{ij}$ is a binary variable, so we transform the above inequalities as follows:

\[
\omega_j^2 \geq L_j \sum_{i \in I} \mu_i^F (Y_{ij}^F)^2,
\]

\[
\frac{1}{2} (u_j + Q_j^F)^2 \geq \frac{2(F_j^F + \beta g_j^F)}{\theta h} \chi \sum_{i \in I} \mu_i^F (Y_{ij}^F)^2 + \frac{3}{2} (Q_j^F)^2 + \frac{1}{2} u_j^2,
\]

\[
\frac{1}{2} (v_j + Q_j^R)^2 \geq \frac{2(F_j^R + \beta g_j^R)}{W \gamma \chi V + \theta h} \chi \sum_{i \in I} \mu_i^R (Y_{ij}^R)^2 + \frac{3}{2} (Q_j^R)^2 + \frac{1}{2} v_j^2.
\]

Then, the objective of problem (P) is reformulated as:

\[
\sum_{j \in J} \left\{ f_j^F X_j^F + \theta h z_\omega \omega_j + \sum_{i \in I} \beta (d_{ij} + a_j^F) \chi \mu_i^F Y_{ij}^F + \frac{\theta h}{2} u_j \right\}
\]

\[
+ \sum_{j \in J} \left\{ f_j^R X_j^R + \sum_{i \in I} \beta (d_{ij} + a_j^R) \chi \mu_i^R Y_{ij}^R + \frac{W \gamma \chi V + \theta h}{2} v_j \right\} - \sum_{j \in J} S_j^C X_j^C.
\]

The set of capacity constraints (3.5) is linearized by substituting the nonlinear term by $z_\omega \omega_j$ obtaining the set of constraints (3.20).

The rest of constraints of problem (P) remain untransformed since they are linear.

3.4.1 Extremal extended polymatroid inequalities:

Utilizing submodularity, the conic quadratic constraints (3.17) ~ (3.19) lead to the class of valid inequalities called extremal extended polymatroid inequalities that can improve the performance of the solution algorithm (we refer to Section 2.5.3 for an introduction of this type of inequalities). To simplify the notation, we drop the superscripts $F$ and $R$ in this subsection.

**Proposition 5.** Let $Q_f$ denote the lower convex envelope of the sets of solutions which satisfy constraints (3.17), i.e.

\[
Q_f = \text{conv} \left\{ (Y_j, \omega_j) \in \{0, 1\}^{|I|} \times R : \omega_j \geq f(S) = \sqrt{L_j \sum_{i \in S} \mu_i}, \forall S \subseteq I \right\}.
\]

Then, the inequality $\sum_{i \in I} \pi_i Y_{ij} \leq \omega_j$ is valid for $Q_f$. 

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where \( \pi_i = \sqrt{L_j \sum_{i \in S(i)} \mu_i} - \sqrt{L_j \sum_{i \in S(i-1)} \mu_i} \in EP_f, S = \{i \mid Y_{ij} = 1\}, \) and \( S(i) = \{(1), (2), \cdots, (i)\}, 1 \leq i \leq |I| \) for some permutation. This valid inequality is an extremal extended polymatroid inequality.

**Proof.** See Appendix. \( \square \)

A similar result can be derived for the set of constraints (3.18) and (3.19).

**Proposition 6.** Let \( Q_u \) denote the lower convex envelope of the sets of solutions which satisfy constraints (3.18) and (3.19), i.e.,

\[
Q_u = \text{conv} \left\{ (Y_j, u_j, Q_j) \in \{0, 1\}^{|I|} \times \mathbb{R} : \frac{1}{2} (u_j + Q_j)^2 \geq H_j \chi \sum_{i \in I} \mu_i (Y_{ij})^2 + \frac{3}{2} (Q_j)^2 + \frac{1}{2} u_j^2 \right\},
\]

Then, \( \sum_{i \in I} \pi_i Y_{ij} \leq u_j + Q_j \) is a valid inequality for \( Q_u \) where \( \pi_i = \sqrt{8H_j \chi \sum_{i \in S(i)} \mu_i} - \sqrt{8H_j \chi \sum_{i \in S(i-1)} \mu_i}, S = \{i \mid Y_{ij} = 1\}, \) and \( S(i) = \{(1), (2), \cdots, (i)\}, 1 \leq i \leq |I| \) for some permutation. This valid inequality is an extremal extended polymatroid inequality of \( Q_u = \text{conv} \left\{ (Y_j, \pi_j) \in \{0, 1\}^{|I|} \times \mathbb{R} : \frac{1}{2} \pi_j^2 \geq 4H_j \chi \sum_{i \in I} \mu_i (Y_{ij})^2 \right\}. \)

**Proof.** See Appendix. \( \square \)

To find these valid inequalities, we will need to solve its respective separation problem introduced in Chapter 2. This problem for the extremal extended polymatroid inequality can be computed by a greedy algorithm described in Edmonds (1971). This algorithm is described in Chapter 2. The greedy algorithm will find valid extremal extended polymatroid inequalities of the types described in Propositions 5 and 6 and we will add them to our formulation to speed up the solution process.

### 3.5 Computational experiments

In this section, we perform computational experiments to test the model and check how the addition of valid inequalities can speed up the computation. We start by varying the inventory and transportation weights (Table 3.3) and varying the loss in value of returned weights (3.4) but without adding valid inequalities. The subsequent analysis studies the effect of the valid inequalities over a range of
different DC capacity values (Tables 3.5, 3.6, and 3.7). The second subsection is devoted exclusively to evaluating the impact of the DC capacities (Table 3.8). We continue by studying the impact of marginal value of time of returned products (Table 3.9 and 3.10), and the trade-off between inventory and value loss of returned products (Figures 3.4 and 3.5).

All experiments have a maximum limit CPU time opt 3,600 seconds and are based on two of the data sets presented in Chapter 2 (from Daskin (1995)): the 49-city data set and the 88-city data set. For the first data set, the mean demand of new products is obtained by dividing the first group of demand data by 100 and the fixed forward facility location costs are obtained by dividing the facility location costs by 100. For the second data set, the mean demand is obtained by dividing the first group of demand data by 1000 and the fixed forward facility location costs are obtained as in the first data set. The mean quantity of returned products is calculated by multiplying the return rate with the second group demand data from each data set. The return rate is identical among all retailers. The fixed reverse facility location costs are equal to the fixed forward facility location costs. Each retailer location is also a candidate DC location. The cost savings of joint DCs are set to 0.2 min(f^F_j, f^R_j) in all experiments. The capacities of all DCs are equal to each other for the same experiment. The parameter values and descriptions of this model are listed in Tables B.1 and B.2.

We directly employ these data in all the experiments except for Tables 3.5, 3.6, 3.7, and 3.4 that are build to show computational performance. In these tables we report the average of ten random instances per row. In turn, each instance is generated by adding noise to some of the main parameters defined above. In particular, we multiply the values of the mean demand, standard deviation, and fixed costs by (1+ \epsilon) where \epsilon is drawn from a uniform [-0.1, 0.1].

The computational experiments are conducted on a SUN Ultra40 workstation running the CentOS5.4 operating system. We used the MIQCP solver of CPLEX 12.1, which solves CQMIP relaxations at the nodes of the branch-and-bound tree, with CPLEX heuristics turned off.

3.5.1 Computational performance of the algorithm

In this section, we confirm the validity of the model and the efficiency of the algorithm. In Table 3.3, we report the results of numerical experiments carried out over different values of \theta and \beta (the second and third columns, respectively). Note that the quantity of returned products is less than the demand of new products as defined in both data sets. The capacities of the DCs are set to 31000 and 7700 for the 49-city and 88-city data sets, respectively. They are 1.05 times the maximum
daily demand of new products. Total costs are listed in the fourth column. DC usage is shown in the next three columns, by displaying the number of forward DCs, reverse DCs and joint DCs, respectively (labeled DCs$_F$, DCs$_R$ and DCs$_C$). The algorithm’s performance is measured in terms of CPU time, the gap between the upper and lower bounds, and the number of nodes searched.

Observation 5. From Table 3.3, we observe that:

- **Total costs increase when weight factors ($\theta$ or $\beta$) increase.**
- **More forward/reverse DCs are opened if unit transportation cost is expensive (larger $\beta$). In contrast, some forward/reverse DCs are closed due to the fact that holding cost becomes expensive (larger $\theta$).**
- **In most cases, joint DCs are preferred due to cost savings but we can find some cases in which the numbers of reverse DCs and joint DCs are different due to capacity restrictions.**
- **Computational times have an increasing trend when we increase the value of $\theta$ while they decrease when $\beta$ increases.**

Furthermore, in Table 3.4 we test performance by varying the weight factor associated to the loss in value of returned products ($W$). CPU times, % GAP, and number of nodes tend to increase when we increase the value of $W$ because this parameter is factorizing a set of decision variables that is related to a set of conic constraints.

Finally, we present Tables 3.5, 3.6, and 3.7 to confirm the computational benefits of the extremal extended polymatroid inequalities. In these tables, “C” reports the capacities of the DCs, “CPU time” reports the average running time, and “Nodes” the average number of nodes in branch and bound tree. The last three columns report the number of valid inequalities added to the root node of the corresponding branch and bound tree. “W”, “U” and “V” are the average number of extremal extended polymatroid inequalities generated based on constraints (3.18), (3.19), and (3.17), respectively. Note that the valid inequalities contribute to finding the optimal solution in less time. Even in instances where the optimal solutions cannot be found in the time limit specified, adding the valid inequalities improves the quality of the solutions by providing a solution closer to the optimal (smaller % GAP).

### 3.5.2 The impact of DC capacity

In this section, we study the effect of DC capacity on both the number of open DCs and the operations at the DCs, as illustrated in Table 3.8. From this table,
### Table 3.3: Performance of the model without adding valid inequalities.

<table>
<thead>
<tr>
<th>Num. cities</th>
<th>$\theta$</th>
<th>$\beta$</th>
<th>Total cost</th>
<th>DCs$^c$</th>
<th>DCs$^R$</th>
<th>DCs$^C$</th>
<th>CPU time (s)</th>
<th>GAP (%)</th>
<th>Nodes</th>
</tr>
</thead>
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<td>0.001</td>
<td>116554</td>
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<td>1</td>
<td>606.70</td>
<td>0.0098</td>
<td>32609</td>
</tr>
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<td>0.005</td>
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<td>0.0070</td>
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</tr>
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<td>20997</td>
<td>10</td>
<td>5</td>
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<td>57.10</td>
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<td>50279</td>
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<td>11</td>
<td>10</td>
<td>9.28</td>
<td>0.0070</td>
<td>28</td>
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<td>0.005</td>
<td>53196</td>
<td>22</td>
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<td>10</td>
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</tbody>
</table>

*: For related computational experiments, see Table 3.5 and 3.6.
Table 3.4: Performance of the model with varying $W$.

<table>
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<th>Num.</th>
<th>$W$</th>
<th>CPU time (s)</th>
<th>GAP(%)</th>
<th>Nodes</th>
</tr>
</thead>
<tbody>
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<td>39.08</td>
<td>0.0099</td>
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</tr>
<tr>
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<td>837</td>
</tr>
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<td>0.0099</td>
<td>3057</td>
</tr>
<tr>
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<td>50</td>
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<td>8</td>
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<td>110.02</td>
<td>0.0096</td>
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</tr>
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<td>17</td>
<td>150</td>
<td>625.12</td>
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</table>

we see that if we tighten the capacities the number of forward DCs increases and more DCs are utilized at capacity.

The impact of DC capacity on joint DCs is summarized in the following property:

**Property 3.** Given a joint DC $j$,

(a) If capacity at DC $j$ is binding, the optimal order quantities of new products ($Q_j^{F*}$) and returned products ($Q_j^{R*}$) are strictly less than the corresponding EOQ quantities.

(b) If capacity at DC $j$ is not binding, the optimal order quantities are the corresponding EOQ quantities:

\[
Q_j^{F*} = Q_{j,EOQ}^{F} = \sqrt{\frac{2(F_j^F + \beta g_j^F)D_j^F}{\theta h}},
\]

\[
Q_j^{R*} = Q_{j,EOQ}^{R} = \sqrt{\frac{2(F_j^R + \beta g_j^R)D_j^R}{\theta h + W\gamma}},
\]

*Proof.* See Appendix. \(\square\)
Table 3.5: Instances with and without valid equalities I.

<table>
<thead>
<tr>
<th>C</th>
<th>CPU time (s)</th>
<th>Nodes</th>
<th>GAP (%)</th>
<th>CPLEX</th>
<th>CPU time (s)</th>
<th>Nodes</th>
<th>GAP (%)</th>
<th>W</th>
<th>U</th>
<th>V</th>
</tr>
</thead>
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Table 3.6: Instances with and without valid equalities II.

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<th>Nodes</th>
<th>GAP (%)</th>
<th>CPLEX</th>
<th>CPU time (s)</th>
<th>Nodes</th>
<th>GAP (%)</th>
<th>W</th>
<th>U</th>
<th>V</th>
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<td>0.0608</td>
<td>235</td>
<td>111</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>7600</td>
<td>3602.13</td>
<td>31178</td>
<td>0.3668</td>
<td>3596.21</td>
<td>32726</td>
<td>0.0506</td>
<td>241</td>
<td>115</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>7500</td>
<td>3602.37</td>
<td>36580</td>
<td>0.4605</td>
<td>3428.99</td>
<td>36004</td>
<td>0.0371</td>
<td>250</td>
<td>95</td>
<td>35</td>
<td></td>
</tr>
</tbody>
</table>

3.5.3 The impact of returned products’ marginal value of time

Returned products’ marginal value of time is associated with the degree of time sensitivity of the product’s price. If we consider it along with DC capacity it leads to distinct characterizations of DC location decisions. Tables 3.9 and 3.10 report some results with different return rates (60\% and 100\%, respectively). The marginal value of time, \( \gamma \), in the experiments is set to 1\%, 10\%, 30\%, 50\%, 70\%, and 90\%. The columns labeled “Cost\(^F\)” and “Cost\(^R\)” list the costs associated with the forward and reverse flows, respectively. We find that:
Table 3.7: Instances with and without valid equalities III.

<table>
<thead>
<tr>
<th>CPLEX</th>
<th>CPLEX+CUTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>CPU time (s)</td>
<td>Nodes GAP (%)</td>
</tr>
<tr>
<td>10000</td>
<td>3605.21 9592  1.5118</td>
</tr>
<tr>
<td>9000</td>
<td>3603.44 12387 1.2710</td>
</tr>
<tr>
<td>8000</td>
<td>3602.97 13860 1.1848</td>
</tr>
<tr>
<td>7900</td>
<td>3602.28 15100 1.0743</td>
</tr>
<tr>
<td>7800</td>
<td>3601.32 18979 1.0752</td>
</tr>
<tr>
<td>7700</td>
<td>3602.28 14336 1.2851</td>
</tr>
<tr>
<td>7600</td>
<td>3601.96 16096 1.2715</td>
</tr>
<tr>
<td>7500</td>
<td>3603.78 16658 1.0263</td>
</tr>
</tbody>
</table>

Observation 6. 1. Fewer reverse DCs are needed for highly time-sensitive returned products (higher $\gamma$). Intuitively, storage time of time-sensitive returned products has been reduced in order to retrieve more salvage value of them. This implies more shipments with smaller quantity so the storage space needed decreases.

2. Returned products impact the forward DCs in number and location. Fewer forward DCs may be constructed in the case of highly time-sensitive returned products. Even if the number of forward DCs is identical, the locations of some forward DCs are different. For instance, a DC is opened in Atlanta as a forward DC in experiment 2 while Charlotte is constructed in experiment 3 and Atlanta is closed. Similar results are also observed in reverse DCs. However, the number of the stand-alone forward DCs increases for highly time-sensitive returned products (the difference between the sixth column and the eighth column in Table 3.9 and 3.10).

3. Reverse flows impact forward flows’ decisions not only on facility location but also on inventory management. In Table 3.10, it is interesting to note that the forward flow costs slightly diminish for highly time-sensitive returned products. This shows an opposite trend with the total costs and reverse flow costs.

Figure 3.3 aims to describe the trade-off between working inventory costs and value loss of returned. As shown, the working inventory cost decreases while the loss in value increases in the range of $[0, Q^{R}_{j}]$. Note that $Q^{R}_{j}$ denotes the optimal shipment quantity of returned products without the consideration of the loss in
Chapter 3. Applied area I: Reverse Flows

Table 3.8: Impact of DC capacity.

<table>
<thead>
<tr>
<th>Num.</th>
<th>Capacity</th>
<th>Total cost</th>
<th>DCs(F)</th>
<th>DCs(R)</th>
<th>DCs(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50000</td>
<td>20043</td>
<td>9 (0)</td>
<td>5 (0)</td>
<td>5 (0)</td>
</tr>
<tr>
<td>2</td>
<td>20000</td>
<td>20043</td>
<td>9 (0)</td>
<td>5 (0)</td>
<td>5 (0)</td>
</tr>
<tr>
<td>3</td>
<td>15000</td>
<td>20043</td>
<td>9 (0)</td>
<td>5 (0)</td>
<td>5 (0)</td>
</tr>
<tr>
<td>4</td>
<td>14000</td>
<td>20069</td>
<td>9 (1)</td>
<td>5 (1)</td>
<td>5 (1)</td>
</tr>
<tr>
<td>5</td>
<td>13000</td>
<td>20249</td>
<td>9 (1)</td>
<td>5 (1)</td>
<td>5 (1)</td>
</tr>
<tr>
<td>6</td>
<td>12000</td>
<td>20430</td>
<td>10 (1)</td>
<td>5 (1)</td>
<td>5 (1)</td>
</tr>
<tr>
<td>7</td>
<td>11000</td>
<td>20489</td>
<td>10 (1)</td>
<td>5 (1)</td>
<td>5 (1)</td>
</tr>
<tr>
<td>8</td>
<td>10000</td>
<td>20598</td>
<td>10 (1)</td>
<td>5 (1)</td>
<td>5 (1)</td>
</tr>
<tr>
<td>9</td>
<td>9000</td>
<td>20677</td>
<td>10 (2)</td>
<td>5 (1)</td>
<td>5 (1)</td>
</tr>
<tr>
<td>10</td>
<td>8000</td>
<td>20778</td>
<td>10 (2)</td>
<td>5 (1)</td>
<td>5 (1)</td>
</tr>
<tr>
<td>11</td>
<td>7900</td>
<td>20815</td>
<td>10 (2)</td>
<td>5 (1)</td>
<td>5 (1)</td>
</tr>
<tr>
<td>12</td>
<td>7800</td>
<td>20877</td>
<td>10 (2)</td>
<td>5 (1)</td>
<td>5 (1)</td>
</tr>
<tr>
<td>13</td>
<td>7700</td>
<td>20997</td>
<td>10 (2)</td>
<td>5 (1)</td>
<td>5 (1)</td>
</tr>
</tbody>
</table>

The numbers in brackets in the last three columns indicate how many of the DCs opened have binding capacities.

Table 3.9: Impact of returned products’ marginal time value I.

<table>
<thead>
<tr>
<th>Num.</th>
<th>(\gamma) (%)</th>
<th>Total cost</th>
<th>Cost(F)</th>
<th>Cost(R)</th>
<th>DCs(F)</th>
<th>DCs(R)</th>
<th>DCs(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>54755</td>
<td>32033</td>
<td>25124</td>
<td>23</td>
<td>20</td>
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<td>2</td>
<td>10</td>
<td>56489</td>
<td>32033</td>
<td>25521</td>
<td>23</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>58439</td>
<td>32033</td>
<td>26360</td>
<td>23</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>59751</td>
<td>32033</td>
<td>26940</td>
<td>23</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>70</td>
<td>60745</td>
<td>32033</td>
<td>27438</td>
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<td>12</td>
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<tr>
<td>6</td>
<td>90</td>
<td>61605</td>
<td>32033</td>
<td>27863</td>
<td>23</td>
<td>13</td>
<td>12</td>
</tr>
</tbody>
</table>

value. From the proof of property 1, \(Q_{j}^{R*}\) refers to the optimal shipment quantity of returned products when considering the loss in value. Once taking the loss in value of returned produces into account, \(Q_{j}^{R*}\) must be less than \(Q_{j}^{R}\). This leads to different decisions due to different priorities/preferences of the decision-makers. It is therefore interesting to find the corresponding non-inferior solutions.

One commonly used procedure for accomplishing this is the weighting method (Cohon (2004)), which creates a single objective by weighting and summing the multiple objectives. The resulting single objective problem is then solved, yielding one point on the trade-off curve. It is generally used to approximate the non-inferior set rather than to find an exact representation. Many different weights are used until an adequate representation of the non-inferior set is obtained.
Table 3.10: Impact of returned products’ marginal time value II.

<table>
<thead>
<tr>
<th>Num.</th>
<th>γ (%)</th>
<th>Total cost</th>
<th>Cost&lt;sup&gt;F&lt;/sup&gt;</th>
<th>Cost&lt;sup&gt;R&lt;/sup&gt;</th>
<th>DCs&lt;sup&gt;F&lt;/sup&gt;</th>
<th>DCs&lt;sup&gt;R&lt;/sup&gt;</th>
<th>DCs&lt;sup&gt;C&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>60957</td>
<td>32412</td>
<td>31708</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>63351</td>
<td>32387</td>
<td>32639</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>66230</td>
<td>32376</td>
<td>33983</td>
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<td>24</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>68176</td>
<td>32370</td>
<td>34876</td>
<td>23</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>5</td>
<td>70</td>
<td>69722</td>
<td>32349</td>
<td>35616</td>
<td>23</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>90</td>
<td>71054</td>
<td>32330</td>
<td>36320</td>
<td>23</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

Figure 3.3: The loss in value and the working inventory cost of returned products.

We employ the weighting method, varying the weight factor \( W \), and plot the trade-off curves corresponding to working inventory cost and value loss of returned products (Figure 3.4). As shown in Figure 3.4, we find that fewer DCs are constructed if we try to reduce the loss in value of returned products (or, in other words, we try to retrieve more salvage value from returned products).

Figure 3.5 reports the trade-off curves between working inventory cost and loss in value of returned products with different \( γ \) (i.e. returned products with different marginal values of time) and DC capacity=7700. According to the figure, given a fixed change of loss in value, the change of working inventory cost of time-sensitive returned products (higher \( γ \)) is smaller than that of time-insensitive ones (lower \( γ \)). Therefore, and confirming our intuition, it makes more sense to salvage the value of time-sensitive returned products. Note that for small \( γ \) the corresponding
Chapter 3. Applied area I: Reverse Flows

Figure 3.4: Trade-off curve between working inventory costs and value loss of returned products.

The curve’s domain is smaller than that of larger values of $\gamma$ since the loss in value is defined as $\frac{1}{2}Q_j^R$ and $Q_j^R$ is in the range $(0, EOQ\ value]$.

Figure 3.5: Trade-off curves for returned products with different $\gamma$.

3.6 Conclusions and further research

This chapter studies the capacitated facility location problem with bidirectional flows, which is starting to receive much attention in the literature. This model minimizes the fixed location costs, the working inventory, and the transportation costs. Moreover, we consider the loss in value of returned products.
when making location decisions. We transform the model into a conic quadratic mixed integer program. The model can be solved efficiently in most cases by using CPLEX. Some valid inequalities are added to improve the efficiency of the branch and cut algorithm and the quality of the solutions.

We perform an extensive computational study and observe the following interesting results:

1. DC capacity has impact on facility location decisions and inventory operations. The optimal order quantities of new/returned products at a joint DC are the EOQ quantities, if DC capacity is non-binding, and below the EOQ level if capacity is binding.

2. Marginal value of time of returned products impacts the location and inventory decisions not only of reverse facilities but also of forward facilities. Fewer joint DCs and more stand-alone forward DCs are constructed for highly time-sensitive returned products.

3. In order to retrieve more salvage value from returned products, it is necessary to tolerate higher working inventory costs of returned products. Thus, it is helpful to consider the salvage value of time-sensitive returned products when making location decisions. This consideration results in smaller number of opened DCs.

We suggest some avenues of further research. First, this model can naturally be extended to incorporate multiple products (Shen (2005)). Second, it is interesting to explore the impact of pricing on the design of the integrated network with returned products (Shen (2006)). Finally, other decisions, such as the vehicle routing decisions for collecting returned products (e.g., Berger et al. (2007) and Shen and Qi (2007)), can be integrated into the capacitated facility location problem.
Chapter 4

Applied area II: the Nonprofit Setting

4.1 Introduction

This chapter’s main focus is the study of nonprofit supply chain design problems. A nonprofit supply chain consists of all parties involved in fulfilling a beneficiary need with the objective of achieving an overall nonprofit goal. Thus, the supply chain does not necessarily aim to maximize the difference between its revenue (which is generated from donors and beneficiaries) and its costs. This differs from commercial supply chains, which define their success based on supply chain profitability. The success of a nonprofit supply chain is measured in terms of the achievement of its specific nonprofit goal.

Next we present two initiatives that are pertinent examples of this type of supply chains. The first example is the supply chain that embraces the Weatherization Assistance Program (WAP). The program’s main goal is to provide energy efficient services free of charge to low-income families in the United States. The program has served more than 6.2 million families since its inception in 1976 and receives funds from the Departments of Energy and Health and Human Services. These funds are proportionally assigned to each U.S. state; the states, in turn, distribute the money to subgrantees. The subgrantees are community-based organizations (CBOs) or public service providers at the county level, which might also receive funds from private donors. Finally, each subgrantee administers the services to the households that have applied and qualify for the program.

The second example of a nonprofit supply chain is the set of relief efforts made by Wal-Mart during the aftermath of Hurricane Katrina in 2005. Wal-Mart donated about $20 million in cash donations, plus 1,500 truckloads of free
merchandise, food for 100,000 meals, and the promise of a job for every one of its displaced workers. This initiative’s fast response has been recognized by scholars as more efficient, at least in the short run, than that of the Federal Emergency Management Agency (FEMA) (Hayes (2005) and Horwitz (2008)).

Despite being quite different, both initiatives share a common characteristic, which is a focus on the provision of goods or services to beneficiaries in a nonprofit way. This objective can be implemented by a nonprofit organization (NPO), as it is its natural goal, or by the government or a for-profit organization, as illustrated by our second example. Furthermore, other commonalities can be encountered, especially when studying their main challenges.

Multiple NPOs and for-profit and public entities can be part of the nonprofit supply chain. Donors can be present at any point of the chain and the downstream party is the end beneficiary (see Figure 4.1).

Figure 4.1: The nonprofit supply chain.

Some supply chains in the nonprofit context have been already studied in the literature, and they can be considered as specific examples of nonprofit supply chains. The “aid chain” is employed in the third sector literature and it is defined as “the series of links through which aid (funds) flows on its way from donors to recipients” (Bornstein (2003)). The analysis of aid chains focuses on the relationship between different stakeholders that act as donors and/or recipients but less attention is payed to the flow of information, materials, or services. Another example is the notion of the “humanitarian relief chain” that appears in the OR/MS literature. Beamon and Balcik (2008) state that the goal of a humanitarian relief chain is to minimize suffering and death by delivering emergency supplies to those affected by disasters. We note that this is a particular nonprofit goal and, hence, we can classify this type of supply chains as nonprofit supply chains. Oloruntoba and Gray (2006) describe a common humanitarian supply chain, where govern-
ment and international agencies fund an international NPO that coordinates with local NPOs and CBOs.

The main contributions of this chapter are the study of the nonprofit supply chain and its challenges addressed from an OR/MS perspective. Furthermore, from a motivational example of a particular nonprofit supply chain, we model a supply chain design problem employing the approach showed in Chapter 2. This example describes how the World Food Program (WFP) distributes food in an unsecured area of Somali in Ethiopia.

The remainder of the chapter is organized as follows. In Section 4.2, we present the main challenges in the nonprofit supply chain. In Section 4.3 a motivational example about a humanitarian organization that distributes food aid in a less secure area is presented. From this example and its major challenges in Section 4.4 we model its corresponding integrated supply chain design problem. The model is solved via conic programming. A numerical analysis is produced in Section 4.5 to illustrate some parameter effects on the design of the optimal supply chain. In Section 4.6 we conclude with a brief recap.

4.2 The main challenges in the nonprofit supply chain

The challenges of some specific nonprofit supply chains have already been studied in the literature. For example, Oloruntoba and Gray (2006), Beamon (2004), Van Wassenhove (2005), Tomasini and Van Wassenhove (2010), and Ergun et al. (2010) list challenges by comparing humanitarian supply chains with commercial ones. Similarly, Yadav et al. (2010) characterize the drug distribution supply chain by comparing medicine and consumer product supply chains in the developing world.

From these supply chain comparisons and the compilation of OR/MS work related to different types of nonprofit supply chains, we have been able to derive the most important nonprofit supply chain challenges in the following list.

- Lack of single performance measure
- Limited/uncertain funds, supply, and resources
- Allocation when demand exceeds supply
- Weak demand forecasts
- High value of loss and stock-out costs
Chapter 4. Applied area II: the Nonprofit Setting

- Lack of intra- and inter-collaboration

This list provides the framework for the next paragraphs, in which we briefly describe some OR/MS tools that are used to analyze each type of challenge in the nonprofit setting through a comprehensive (non-exhaustive) review of literature.

4.2.1 Lack of single performance measure

While profit is a good overall measure for any for-profit organization, there is no single measure that is suitable for all nonprofit supply chains. Anthony and Young (1988) claim that the lack of a single nonprofit performance measure makes it impossible to decentralize and delegate decisions, fairly compare different entities, and even conduct a simple broad quantitative analysis.

The lack of a single performance measure implies a tendency to employ multi-objective models. In turn, there are different multi-objective optimization techniques to be used, such as goal programming (GP), the weighted sum method, or the $\epsilon$-constrained method. To illustrate, we have selected two papers that employ the GP resolution methodology to solve multiple and, at times, conflicting objectives for a nonprofit supply chain. One of the first papers using the GP technique was related to resource allocation problems in higher education, Lee and Clayton (1972). The objective function measures deviations from certain prioritized goals, such as accreditation, salary increase, faculty/student ratio, faculty/staff ratio, faculty/distribution ratio, faculty/graduate assistant ratio, and cost minimization. Ramudhin et al. (2008) present a green supply chain network design problem with the conflicting objectives of minimizing total logistics costs and minimizing carbon emissions.

To emphasize this important challenge, we present in Appendix C.1 a list of nonprofit performance measures that we can find in the OR/MS and economics literature.

4.2.2 Limited/uncertain funds, supply, and resources

Funds, supply, and resources are the parts of the supply chain affected by uncertainty and scarcity. We will examine each of these three challenges separately.

- Limited or uncertain funds

Individuals and private firms make donations for a variety of reasons in which there may not be a formal commitment or contract involved. In addition to commitment unreliability, there are uncertainties in the timing of grant reimbursement that also affect grant availability. All this translates into fund unreliability for those nonprofit supply chains that depend on donations and grants.
Funding providers can take steps to ensure NPO efficiency that will in turn improve trustworthiness and funders' commitment. Privett and Erhun (2011) state that current funding methods (e.g., report-based funding) do not allocate funds efficiently. They develop a principal-agent framework to analyze the use of audit contracts so fund providers can audit the NPO after grants have been awarded. They further argue that such contracts benefit the nonprofit sector by encouraging efficient allocation.

Even if funds are secured, the amounts guaranteed can be very limited and a budget constraint might need to be added. For example, in the context of humanitarian relief, Salmerón and Apte (2010) employ a stochastic optimization budget constraint in a two-stage model that predicts the strategic allocation of resources for humanitarian responses to future disasters. McCoy and Brandeau (2011) analyze, via dynamic programming, how to partition a total budget that is employed for maintaining a stockpile and for shipping items from that stockpile to the relief location. Duran et al. (2011) study how to best allocate existing inventory given an operating budget. This budget constraint appears in two different forms: as the maximum number of warehouses to open and as the maximum inventory amount to keep throughout the network.

Given that funds are highly unreliable and limited, one can think of two actions to raise and preserve funds: revenue generation and cost reduction, respectively.

**Revenue generation**

While revenue generation for NPOs has been extensively studied by economists (e.g., Froelich (1999)), this is a novel topic in OR/MS.

NPOs can take steps to increase funding. McCardle et al. (2009) show that a tiered structure allows an NPO to receive larger donations because of different reputations associated with different tiers. This is a decision analysis paper that develops a utility function to model how donors choose tiers.

Nonprofit supply chains can benefit from funds in the form of subsidies. This occurs, for example, in the areas of environmental protection and drug distribution in developing countries. In the case of environmental protection, Lobel and Perakis (2011) study the optimal subsidy value that the government can provide to achieve a desired adoption target of solar panels by customers by using a policy design model followed with an empirical study. On a general note, managers should evaluate reuse, remanufacturing, or recycling to compensate for resource scarcity. See Mayorga and Subramanian (2010) for a review on how to recast supply chain models to take these aspects into account. Taylor and Yadav (2011) study subsidies in the private-sector distribution of essential products (e.g., medicines in the developing world). They employ a supply chain contract framework to model the
problem and study the effectiveness of a purchase subsidy versus a sales subsidy from the perspective of the donor and society.

Some NPOs engage in for-profit activities to generate revenues and subsidize their nonprofit activities. Despite being a common strategy in practice, this situation has not been sufficiently studied in the OR/MS literature. To our knowledge, the only pertinent article is de Véricourt and Lobo (2009). This paper is inspired by the work done in Aravind Eye Hospitals in India, where profitable fee-charging hospitals are used to subsidize care at free hospitals. This work dynamically allocates hospital assets between for-profit and nonprofit activities to maximize social benefit.

Cost reduction

Cost is a nonprofit performance metric classified under efficiency measures in Appendix C.1.1. Besides considering the regular costs in a supply chain, nonprofit managers should study the potential reduction of other costs that are more particular to some nonprofit supply chains such as fundraising, service, and environmental costs.

In the context of environmentally responsible inventory models, Bonney and Jaber (2011) present the concept of environmental cost. This is composed of the traditional costs (order, unit purchase, and holding costs) and costs concerning the delivery and collection of returned items, emissions costs from transportation, and waste costs produced by the inventory. They suggest that traditional inventory models should be reworked to include these new costs (an example with the EOQ model is shown). MRP and WIP levels are other aspects to be calibrated.

Pedraza-Martinez and Van Wassenhove (2011) initiated the study of humanitarian fleet management in developing countries. These scholars claim that transportation is one of the most important cost factors in humanitarian operations. The study finds the optimal replacement policy via a dynamic programming model and compares it with the currently used standard commercial vehicle replacement policy. An empirical analysis shows the potential savings of a policy readjustment. The authors recommend further study of humanitarian fleet management, arguing that it is the key to cost savings.

- Limited or uncertain supply

Facing limited or unreliable supply can be a direct consequence of experiencing limited or unreliable funds. Further, some nonprofit supply chains operate in areas that are prone to man-made or natural instability, and this can cause supply unreliability. This has been thoroughly studied in the humanitarian disaster context (e.g., Van Wassenhove (2005)).
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Related to the latter case, most of the current OR/MS papers in relief routing consider supply uncertainty. This can be caused by several factors, including physical damages to the supply by the disaster itself, inaccessible infrastructures, an undercapacitated fleet, miscoordination between different agents, delays in customs, lack of safety that can lead to robberies or path variations, etc. For a review of work in relief routing and supply uncertainty, we refer the readers to de la Torre et al. (2011).

Recently, we have seen research on reliable supply chain design that deals with supply chain disruptions. Typically, supply disruptions are modeled via scenario analysis or assigning failure probabilities to the nodes of the network. In scenario modeling, each scenario describes a different disruption situation. This methodology has been applied to the location-inventory model with risk pooling by Snyder et al. (2007), which presents a stochastic version. The $\alpha$-reliable mean-excess model for strategic facility location planning presented by Chen et al. (2006) is another example of scenario modeling that minimizes the expected regret with respect to a set of worst-case scenarios.

We can also analyze supply disruption by the second approach, which uses disruption probabilities at the nodes (i.e., facilities). Snyder and Daskin (2005) is the first paper to handle facility disruptions in the uncapacitated fixed charge location problem (UFL). Later on, Berman et al. (2007), Shen et al. (2011), and Cui et al. (2010) provide more generalized models and efficient algorithms for this type of location problems. For recent reviews, we refer the reader to Snyder et al. (2006) and Snyder et al. (2010).

There is an OR/MS stream of literature that designs contracts to share supply risks. This can include minimum purchase commitments, quantity flexibility contracts, buyback contracts, revenue sharing, and real options contracts (e.g., Tsay (1999)). Chick et al. (2008) is one of the few OR/MS publications that considers supply contracts in nonprofit settings. In particular, this work studies the cost-sharing contract applied to the influenza vaccine supply chain.

- Limited or unreliable resources

The activities of the nonprofit supply chain are often restricted by financial limitations and earmarking. Hence, the chain can become resource-constrained in aspects such as storage capacity, beds (in the hospital setting), or staff. The OR/MS practice most recurrently used to describe this situation is the addition of resource constraints to the model. For example, Balcik and Beamon (2008) present a facility location and inventory decision model for a humanitarian relief chain with budget and storage capacity restrictions. Griffin et al. (2008) maximize the weighted demand coverage of the population in need of health care subject to budget and capacity constraints.
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Staff and beds are commonly cited as particularly scarce resources in the hospital setting. Cochran and Roche (2009) use Split Patient Flow queuing theory to manage emergency department staff and beds. Facility storage space also tends to be a limited resource in the hospital setting. Lapierre and Ruiz (2007) present two scheduling models that coordinate the procurement and distribution operations through a supply chain approach with storage capacity constraints. The first scheduling model minimizes inventory costs under a human resource constraint, and the second one has a two-criteria objective composed by a quadratic metric of workload equilibrium and the same inventory objective of the first model.

An example of unreliable resources is the unstable state of the roads studied in the humanitarian relief setting by Salmerón and Apte (2010). These authors model different transportation times depending on different road state scenarios.

### 4.2.3 Allocation when demand exceeds supply

If one could satisfy all demand with on-hand supply, there would be no allocation issues because all beneficiaries would receive the service they need when they need it. Unfortunately, this is not the case in most of the nonprofit supply chains, and some priority policy should be established, based on fairness or other nonprofit measures.

When demand is larger than supply, rationing is regularly employed. It has been studied for customers of different priority classes that arrive in sequence (e.g., Topkis (1968)) and when there is asymmetric information with no classes (e.g., Cachon and Lariviere (1999)). Usually, this literature has been devoted to the supplier-customer relationship, where profits or costs have been the main objectives. As an example of a nonprofit supply chain application, we refer to Deshpande et al. (2003), which studies the systems employed to manage consumable service parts for the U.S. military. This paper considers a static threshold-based stock-rationing policy for low-priority and high-priority demands, which is numerically proven to perform well in military environments.

As noted by de la Torre et al. (2011), there is no publication in humanitarian relief that provides standard priority guidelines when demand exceeds supply. However, a common practice is to give preference to the most vulnerable, especially in the case of malnourished children. Some humanitarian organizations follow the Sphere Handbook to satisfy minimum needs for a selected subset or the overall population (Project (2011)).

The organ allocation problem is a pertinent resource allocation problem since there is scarcity in the supply. This type of problem has been extensively studied in OR/MS. As an example, Zenios et al. (2000) study a dynamic resource allocation...
problem that assigns kidneys to wait-listed patients. A three-criteria objective is developed with one efficiency and two inequity measures. Another example is Alagöz et al. (2004), which employ a Markov decision process (MDP) to study the optimal timing for a living-donor liver transplant. This work maximizes the patient’s health measured as quality-adjusted life expectancy.

- **Increasing accessibility**

  In many situations, given the allocation problem, assignments will be dictated by the maximization of demand satisfaction. For example, OR/MS has extensive work on location and routing analysis with the main purpose of increasing accessibility. If we restrict it to nonprofit supply chains, the literature is still abundant. For example, in the field of humanitarian logistics, Balcı and Beamon (2008) present a variant of the maximal covering location model to improve response time and the proportion of demand satisfied. In the context of community services, Francis et al. (2006) study the routing efficiency in interlibrary loan delivery with a model that describes a period vehicle routing problem (PVRP) and incorporates different levels of service (represented by levels of delivering frequencies). In the context of health care in the developing world, mobile facilities can provide care to rural areas. Doerner et al. (2007) present a multi-objective combinatorial optimization formulation for a location-routing problem related to mobile facilities in which one of the objectives is to minimize the percentage of the population unable to reach a tour stop within a predefined maximum distance.

### 4.2.4 Weak demand forecasts

Demand uncertainty is a challenge for any supply chain. In the nonprofit supply chain, this lack of information is particularly common due to abrupt changes in demand, lack of information sharing between multiple stakeholders, and a shortage of funding available to collect data or to monitor or forecast demand. Most NPOs do not appreciate the long-term payoff of gathering and analyzing data for forecasting, and this contributes to a shortage of funding for this purpose (Bradley et al. (2003)).

OR/MS models employ random variables to represent stochastic demand. Stochastic dynamic programming is a recurrent mathematical method in which we can learn each period’s demand level and use this information for our next event sequence. To illustrate, scholars use dynamic programming for nonprofit supply chains in areas such as humanitarian relief (e.g., McCoy and Brandeau (2011)) and community-based action (e.g., Lien et al. (2009)).

Demands for products such as the influenza vaccine are naturally hard to predict. As a consequence, simulation is frequently used to describe this type of
demand (e.g., Das et al. (2008)). Similarly, Fleßa (2003a) simulates the spread of HIV and AIDS in eastern Africa via a systems dynamic model. This work helps to forecast the future demand of AIDS control programs and, hence, the planning for scarce resources related to these programs.

Pre-positioning and stockpiling are studied in OR/MS as location and inventory management strategies, respectively, to mitigate stock-outs when demand is unknown. Regarding location decisions, Akkihal (2006) identifies optimal locations for warehouses by minimizing the average distance between the warehouses and the people whom forecasts suggest could become homeless because of hazards. Duran et al. (2011) collaborate with CARE International to optimize a pre-positioning network by considering: upfront investment, operating costs, and average response time. This paper includes estimates of potential demand based on historical data from the International Disaster Database (EM-DAT 2007). Apte (2009) states that for pre-positioning, two optimization models are usually used: the set covering problem (SCP) and the facility location problem (FLP).

High demand variability at one place can be offset by low demand variability at another place, so aggregating demand from both places in one location reduces the overall demand variability. This is the principle behind the risk pooling strategy. To illustrate, Jack and Powers (2004) describe risk pooling as an effective tool to increase delivery of patient services under demand uncertainty. Besides location pooling, risk pooling can also be implemented for high and low demand variability across different projects. For example, in the humanitarian context, relief projects have highly stochastic and short duration demands while development projects’ demands are less uncertain and long term. Pooling funds and/or supply for both types of projects can potentially mitigate unserved demand.

### 4.2.5 High value of loss and stock-out costs

If a stock-out occurred in some of the nonprofit supply chains introduced so far, the loss and stock-out costs would be dramatically high. Some of the goods and services provided in these settings can be classified as meeting critical needs (i.e. as being essential to the survival of the population, Nagurney et al. (2009)). If we wonder how this situation affects the modeling of the supply chain, OR/MS scholars come up with different answers:

- Assigning a very high penalty value to shortage situations (i.e. implying that the model highly prioritize the avoidance of an under-stocking situation)
- Contemplating alternative solutions if the stock-out is about to happen (e.g., emergency shipping, outsourcing, etc.)
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Pierskalla (2005) studies the supply chain management of blood banks and defines the shortage costs at the CBCs based on the cost of processing, handling, and transporting blood in an emergency. The shortage costs at the HBBs are based on the cost of keeping a buffer stock at the site or at other sites.

Nagurney et al. (2009) design supply chain networks in the case of critical needs. In this paper, in addition to incurring associated penalties if demand is not met, the organization has the option of outsourcing the “production/storage/delivery of the critical product” to other companies.

In practice, managers implement preventive actions to reduce the chances to face loss and stock-out situations. Pre-positioning, stockpiling, and risk pooling (as previously described to overcome weak demand) are also valid for this purpose.

4.2.6 Lack of intra- and inter-collaboration

As mentioned, the nonprofit supply chain can be composed of multiple stakeholders: NPOs, public entities, and private companies. This situation naturally leads to decentralized control, in which much of the supply often comes from the private sector, while demand comes from the public or nonprofit sector. Indeed, the decentralized structure of the chain has been shown to make NPOs more responsive to local needs, but at the same time they could remain disconnected from the rest of the chain, triggering inefficiency. Moreover, a lack of intra-entity information visibility is common at the NPO level due to low investment in information technologies.

Regarding intra-entity collaboration, Bradley et al. (2003) state that acquiring sophisticated information systems boosts communication between different divisions and trims administrative costs. Tomasini and Van Wassenhove (2010) add that it brings more visibility, transparency, and accountability. Deshpande et al. (2006) prove that information collaboration across different divisions of the same organization (in this case USCG) is valuable. In particular, they employ part-age dependent supply replenishment policies to empirically show that sharing information between the maintenance and inventory systems can generate significant benefits. Pedraza-Martinez et al. (2011) study incentive alignment and asymmetric information related to fleet management between the headquarters and the program managers of an NPO that is in charge of development activities.

There is also lack of inter-collaboration in the nonprofit supply chain, which is explained by multiple factors. In the humanitarian relief context, it has been claimed to be due to differing objectives, diversity of management structures, missing or wrong information related to other institutions, differing information technologies, limited funding and earmarking that limits flexibility, costs related
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to collaboration (e.g. travel costs, salaries, etc.), and competition (Schulz (2009), Tomasini and Van Wassenhove (2010), Balçik et al. (2010), and Blecken (2010)). On top of all these challenges, the urgency in the response needed in the humanitarian relief settings is an obstacle (Dolinskaya et al. (2011)).

The most common benefits of inter-collaboration are the possibilities of group purchasing and capacity and information sharing. These benefits can be reached, if the collaboration is stable, by establishing a contractual framework or membership mechanisms.

4.3 Motivational example: food delivery by the WFP in a less secure area

The World Food Program (WFP) distributes food in the Somali region of Ethiopia. This area is located adjacent to Somalia and due to its proximity it is highly influenced by this neighboring country. Among other influences, political instability, rebel activity, ethnic tensions, and poor infrastructures directly affect the flow of food through the WFP supply chain (Goentzel et al. (2009)). From the work by Goentzel et al. we have learned more details about this distribution of food aid. In 2008 the WFP delivered, via the port of Djibouti through a small set of Extended Delivery Points (EDPs), an approximate amount of 80 metric tones of food per month at each one of the 212 Final Delivery Points (FDPs) around this region. If the food was distributed just once a month more official storage areas would need to be conditioned in the FDPs by the WFP. In lieu, the WFP handed responsibility for the FDP’s inventory to the local government. As a result of this shift of control, proper storage was not ensured and the inventory was kept by unofficial distributors who took for themselves part of the food (around a 16 % of total monthly food distributed, Goentzel et al. (2009)). This resulted in loss deliveries for the rest of beneficiaries. From this situation, we can observe that an increment in the frequency of deliveries would incur benefits for the end beneficiaries since less aid would be stored at the FDPs and, hence, less food would be diverted. This would ensure a more fair distribution of the food. Shipments from the EDPs to each FDP were fully responsibility of the carriers that charged depending on travel times among other aspects. Hence, a higher shipping frequency of two or four deliveries per month as opposed to one per month would increase transportation costs proportionally. Observe that the insecurity of the region makes the problem particularly interesting since it impacts in two opposite ways (1) it affects food security at the FDPs if less frequent deliveries and (2) it affects transportation costs if more frequent deliveries. This chapter addresses
these tradeoffs to come up with the optimal design of the supply chain within the context of humanitarian aid distribution.

4.3.1 The supply chain structure

A multi-tiered structure is appropriate for the design of the humanitarian supply chain since it allows organizations to be more respondent to the end beneficiaries needs by settling closer to them. In particular, we study a three-tiered supply chain (Figure 4.2) with one central warehouse, several distribution centers (DCs, a.k.a. EDPs), and points-of-demand (PODs, a.k.a. FDPs). DCs are located in some pre-established candidate sites and supply goods to different PODs in its surrounding areas. We determine the optimal supply chain design by deciding DC location, the assignments between DCs and PODs, and the selection of delivery frequency between each DC and POD.

![Three-tiered supply chain network](image)

**Figure 4.2:** The three-tiered supply chain network III.

4.3.2 Challenges of this nonprofit supply chain

In the following lines, we review how some of the challenges described above are relevant in the specific context of the WFP food supply chain distribution in the Somali area. First of all, it is unclear what WFP managers consider as best performance metrics for the problem described. However, from Goentzel et al. (2009), we can determine that some of the most important concerns are (a) costs and (b) ensuring service delivery of aid. While it is a fact that the WFP has limited capital resources, the second metric/goal is also really important in the
context of the example explained because food is a critical need for beneficiaries suffering from “hunger periods\(^1\)”. Besides limited and uncertain funds, there are other restrictions in supply and resources such as warehousing space limitations. Food insecurity is part of the global endemic insecurity problem in the area. Nonetheless, a better coordination between the WFP and the local government, that is in charge of controlling the food at the last distribution point, could lessen the problem. Finally, demand is hard to forecast because the Somali area is a pastoral region with high population mobility.

In the following section we take into account the mentioned challenges to define the supply chain design model that corresponds to this particular example.

### 4.4 The model

Demand occurs at the POD level and is assumed to be Gaussian. In particular, we define \(\mu_i\) as the mean of daily demand at POD \(i\), \(\sigma_i\) as the standard deviation of daily demand at POD \(i\), and \(\rho_{ik}\) as the correlation factor between demands in POD \(i\) and POD \(k\). A schedule \(s\) refers to the set of days per month in which a specific delivery is provided from DC to POD. Hence, besides determining location and assignment decisions, our model decides which service frequency to assign to the delivery between DC and POD. The more often the supply is delivered the larger are its transportation costs and larger the delivery benefit since this reduces the risk of supply diversion.

Below, we describe the two sets of decision variables employed in this model. The set of subindices \(j \in J\) refers to the set of candidate DC locations, the set of subindices \(i \in I\) refers to the set of fixed POD locations, and \(s \in S\) refers to the set of different (monthly) schedules.

\[
x_j = \begin{cases} 
1, & \text{if a distribution center is located at site } j, \\
0, & \text{otherwise}; 
\end{cases}
\]

\[
y_{ij}^s = \begin{cases} 
1, & \text{if POD } i \text{ is assigned to DC located at site } j \text{ on schedule } s, \\
0, & \text{otherwise}. 
\end{cases}
\]

Now, we describe the different components of the objective function of the model.

\(^1\)this periodically occurs in the Somali area.
4.4.1 Costs at the DC level

The set of cost measures at the DC level can be described as a composition of fixed costs of locating DCs, shipment costs, fixed costs of placing and shipping orders, and costs of holding average inventory and safety stocks.

DCs carry inventory and follow a periodic review policy characterized by a single parameter, the base-stock level (Simchi-Levi et al. (2003)). We believe that the periodic review policy is the most realistic approach for humanitarian supply chains because, under this context, there is scarce investment in information systems technologies. In particular, we assume that DCs order up to level every month (i.e., every four weeks), whereas PODs order up to level with the same frequency or more often (e.g., every other week or every week). From these conditions, we have the following result related to the average inventory level at each DC:

**Proposition 7.** The average inventory at DC $j$ is

$$\sum_{i \in I} \sum_{s \in S} (r_j - r^*) \mu_{ij} y_{is}^2 ,$$

where $r_j$ is the length of the review period at DC $j$ and $r^*$ is the length of the review period on schedule $s$.

**Proof.** See Appendix.

Orders at the DC level might arrive before or after the expected receiving time and for this reason we assume stochastic lead times between the central warehouse and each DC $j$. This uncertainty can be due to a large number of reasons such as availability from suppliers, customs delays, transportation incidences (e.g., escort requirements and road conditions), and problems with handling operations. To model this variability we assume that the lead time is a normal distribution with mean $L_j$ and standard deviation $\sigma_{L_j}$. Furthermore, we assume that successive lead times are independent and orders do not cross (Nahmias (1993)).

**Proposition 8.** (Nahmias (1993)) Given possible stochastic lead times and correlated demands between PODs, the safety stock at any DC $j$ is

$$z_\alpha \sqrt{(r_j + L_j) \sigma^2_{D_j} + \sigma^2_{L_j} \mu^2_{D_j}},$$

where $\sigma^2_{D_j} = \sum_{i \in I} \sum_{s \in S} \sigma^2_i y_{ij}^s + 2 \sum_{i=1}^{I-1} \sum_{k=i+1}^I \sum_{s=1}^{S} \sum_{s'=-1}^{S} \sigma_i \sigma_k \rho_{ik} y_{ij}^s y_{kj}^{s'}$, and $\mu_{D_j} = \sum_{i \in I} \sum_{s \in S} \mu_i y_{ij}^s$.

Note that lead time variability affects our safety stock at the DC level in which the larger the variability the larger the safety stock required for the same service level. In a similar manner, when demand at the POD level is positively correlated safety stock increases.

Next, we list all the costs terms at the DC level:

- the yearly fixed costs of locating DC $j$, $f_jx_j$. 

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- the annual expected shipment costs from the central warehouse to each DC $j$, $\sum_{i \in I} \sum_{s \in S} c_{aij}^s \mu_i y_{ij}^s$.

- the expected fixed cost of placing an order at DC $j$ and the expected fixed cost per shipment from the central warehouse to DC $j$, $(F_j + g_j)\eta_j x_j$.

- the cost of holding average inventory at DC $j$, $h_i \sum_{s \in S} (r_j - r_s) \mu_i y_{ij}^s$.

- the safety stock cost, $z_\alpha h \sqrt{(r_j + L_j)\sigma_{D_j}^2 + \sigma_{\mu_j}^2 \mu_{D_j}^2}$. This term is based on the fact that the lead times are variable and demands can be correlated among PODs.

4.4.2 Costs and lost service at the POD level

The present model is also composed by costs and a “lost supply” term at the POD level. To model the latter term we define the coefficient of reliability at each POD $i$, $\theta_i^s$, which is the parameter that represents the percentage amount of supply that would be diverted from POD $i$ if schedule $s$ is assigned. In practice, the diversion rate per each schedule at each POD can be derived from historical data by calculating the average proportion between the amount of supply distributed to the end beneficiaries at the POD and the initial amount of supply delivered to that POD under that schedule. Here is a summary of all these terms:

- the annual expected shipment costs from DC $j$ to POD $i$ on schedule $s$, $d_{ij} \gamma^s r^s \mu_i y_{ij}^s$.

- the expected fixed cost of shipping to POD $i$ on schedule $s$ from DC $j$, $g_{2i} \gamma^s y_{ij}^s$.

- the value related to the loss supply at POD $i$ on schedule $s$, $\theta_i^s \gamma^s r^s \mu_i y_{ij}^s$.

The delivery of food aid at the POD level is not monitored by the WFP and the only information received from the third party responsible is the amount of food delivered to the population. Thus, we do not explicitly account for average inventory and safety stock levels at the POD.

4.4.3 The model

The model belongs to the family of integrated supply chain design problems:
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\[
\begin{align*}
\min & \sum_{j \in J} \left( f_j x_j + \sum_{i \in I} \sum_{s \in S} \chi x_j + (F_j + g_j) \eta_j x_j + h \sum_{i \in I} \sum_{s \in S} (r_j - r_j) \mu_i y_{ij} + \chi \sum_{i \in I} \sum_{s \in S} \sum_{j \in J} d_{ij} \gamma_{rs} \mu_i y_{ij} + g_{2i} \gamma_{rs} y_{ij} + \theta_{ij} \gamma_{rs} \mu_i y_{ij} \right) \\
\text{s.t.} & \sum_{s \in S} \sum_{j \in J} y_{ij} = 1, \quad i \in I, \quad (4.2) \\
& y_{ij} \leq x_j, \quad i \in I, j \in J, s \in S, \quad (4.3) \\
& z_\alpha \sqrt{(r_j + L_j) \sigma^2_{D_j} + \sigma^2_{L_j} \mu^2_{D_j}} + (r_j + L_j) \sum_{i \in I} \sum_{s \in S} \mu_i y_{ij} \leq C_j, \quad j \in J, \quad (4.4) \\
& \sum_{j \in J} \sum_{s \in S} r^s \mu_i y_{ij} \leq cap_i, \quad i \in I, \quad (4.5) \\
& x_j, y_{ij} \in \{0, 1\}, \quad i \in I, j \in J, s \in S. \quad (4.6)
\end{align*}
\]

The first set of constraints (4.2) ensures that each POD is assigned to exactly one DC. Constraints (4.3) guarantee that PODs are only assigned to open DCs. Constraints (4.4) define the capacity of each DC to be the sum of the order quantity and the reorder point that is defined as the sum of the safety stock and the expected demand during the lead time. Constraints (4.5) define the maximum capacity of each POD. Finally, the set of constraints (4.6) defines the domain of the decision variables.

We transform the model into an equivalent conic quadratic mixed-integer problem.
Proposition 9. Problem (P1) is equivalent to the following (CQMIP11):

\[
\min \sum_{j \in J} \left\{ f_j x_j + \sum_{i \in I} \sum_{s \in S} \chi a_j \mu_i y_{ij}^s + (F_j + g_j) \eta_j x_j \right. \\
+ \frac{h}{2} \sum_{i \in I} \sum_{s \in S} (r_j - r^s) \mu_i y_{ij}^s \left. \right\} + \alpha h s_j \tag{4.7}
\]

\[+ \chi_2 \sum_{i \in I} \sum_{s \in S} \sum_{j \in J} \left( d_{ij} \gamma^s r^s \mu_i y_{ij}^s + g_{2i} \gamma^s y_{ij}^s + \theta_{i}^s \gamma^s r^s \mu_i y_{ij}^s \right) \]

s.t. \[z_\alpha s_j + (r_j + L_j) \sum_{i \in I} \sum_{s \in S} \mu_i y_{ij}^s \leq C_j x_j \quad \forall j \in J, \tag{4.8}\]

\[\sum_{i=1}^I \sum_{k=1}^I \sum_{s \neq s'} V_{ikj} (w_{ikj}^{s, s'})^2 \leq s_j^2 \quad \forall j \in J, \tag{4.9}\]

\[w_{ikj}^{s, s'} \leq y_{ij}, \quad w_{ikj}^{s, s'} \leq y_{kj}, \quad y_{ij} + y_{kj} \leq 1 + w_{ikj}^{s, s'} \quad \forall i, k \in I, s, s' \in S, j \in J, \tag{4.10}\]

\[w_{ikj}^{s, s'} \in \{0, 1\}, \quad s_j \geq 0 \quad \forall i, k \in I, s, s' \in S, j \in J, \tag{4.11}\]

Where \(V_{ikj} = (r_j + L_j) \sigma_i \sigma_k \rho_{ik} + \sigma_{ik}^2 \mu_i \mu_k\).

Proof. See Appendix.

4.5 Numerical experiments

The goal of this section is to show a preliminary study of the impact of some parameter values on the design of the optimal supply chain. In particular, we study the impact of the diversion rates, lead time stochasticity, positive demand correlation, and shipping costs on schedule assignments. The numerical experiments of this section employ the same data set used for the other chapters of this dissertation. In particular, experiments are done with the 25-node data set and the main parameter values are described in C.1. We assume two different schedules, a weekly distribution to POD from DC (schedule 1) and a three times a week distribution (schedule 2).

We start with presenting the base case instance in which if the delivery of aid from a DC to any POD is as frequent as in schedule 1 a 14% of supply will be diverted (i.e., \(\theta_1^i = 0.14 \forall i\)). On the other hand, if the delivery frequency is that of schedule 2 there would not be any diversion (\(\theta_2^i = 0 \forall i\)). For the purpose of simplicity, we assume that there is no diversion if following schedule 2 (this might
be unrealistic) and that there is the same diversion rate at each POD (in reality the diversion rate can change depending on location).

The optimal supply chain design for the base case scenario is described in Figure 4.3, in which there is a total of 8 DCs opened. The circles and links colored in green describe the DC-POD assignments that follow schedule 1 whereas the ones colored in garnet color follow schedule 2. To describe some trade-offs of the model we should observe that not all DCs that are located at a POD site deliver as frequently as they can. The San Francisco, Phoenix, and Nashville PODs receive supply with lower frequency than the rest of PODs with DCs located at the same site. Naturally, one might think that since there are no shipping costs between DC-POD these three PODs should receive supply with a schedule 2 frequency that also assures no loss supply. However, there are other drivers in the model that play a stronger role for these three DCs. These drivers are the fixed shipping costs, the capacity restrictions at the DC level, and the inventory holding costs at DC level. The latter is due to the fact that when following schedule 2 part of the stock needs to be kept at the DC for later deliveries.

**Figure 4.3:** Optimal supply chain design of the base case scenario.

The solution in Figure 4.3 is in the “threshold zone”. This zone represents the set of optimal solutions that follow schedule 1 for a subset of DCs and schedule 2 for the remaining set of DCs. The threshold zone is depicted in Figure 4.4 as the fine line between both colored areas. The X-axis in this figure represents $\theta^1$. 
and the Y-axis is $\theta^2$. The area colored in green describes all optimal designs from different pairs of $(\theta_1, \theta_2)$ that follow schedule 1 and the area colored in garnet follows schedule 2. Note that the optimal supply chain design for the base case instance belongs to the “threshold zone” and it represents the extreme point $(0.14, 0)$ placed on the X-axis. Note that when $\theta_1 \leq \theta_2$ the optimal solution follows schedule 1 for all DCs. Hence, in general:

**Observation 7.** When $\min_{s \in S} \theta^s$ is the schedule with the lowest frequency the optimal solution always follows this schedule.

Clearly, there is no incentive to follow other schedules since the rest of them will have more supply diverted and are more costly in terms of shipping and inventory holding costs. Moreover, note that schedule 1 is the optimal one for a portion of the zone in which $\theta_1 > \theta_2$.

Table 4.1 confirms Observation 7 since it describes the proportion of diverted supply of the optimal solution given different combinations of diversion rate in schedules 1 and 2, $(\theta_1, \theta_2)$. Cells colored in gray indicate that the schedule followed by all PODs of the optimal solution is 2. The two cells colored in blue represent solutions in the threshold zone.

Now, we run an experiment with stochastic lead time and demand correlation between PODs. In particular, we assume that the lead time standard deviation is of 1 day and that there is 80% demand correlation between beneficiaries among different PODs. By comparing this experiment with the results for the base

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*Figure 4.4: Diversion rates and optimal schedule.*
Table 4.1: Supply diverted in percentage per optimal design with \((\theta^1, \theta^2)\).

| \(\theta^1\) | 0.0  | 0.05 | 0.1  | 0.15 | 0.2  | 0.25 | 0.3  | 0.35 | 0.4  | 0.45 | 0.5  | 0.55 | 0.6  |
| | 0    | 5     | 5    | 5    | 5    | 5    | 5    | 5    | 5    | 5    | 5    | 5    | 5    |
| 0.05 | 10   | 10   | 10   | 10   | 10   | 10   | 10   | 10   | 10   | 10   | 10   | 10   | 10   |
| 0.1  | 0    | 15   | 15   | 15   | 15   | 15   | 15   | 15   | 15   | 15   | 15   | 15   | 15   |
| 0.15 | 0    | 5    | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   |
| 0.2  | 0    | 5    | 10   | 25   | 25   | 25   | 25   | 25   | 25   | 25   | 25   | 25   | 25   |
| 0.25 | 0    | 10   | 15   | 30   | 30   | 30   | 30   | 30   | 30   | 30   | 30   | 30   | 30   |
| 0.3  | 0    | 5    | 10   | 15   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   |
| 0.35 | 0    | 10   | 15   | 20   | 25   | 25   | 25   | 25   | 25   | 25   | 25   | 25   | 25   |
| 0.4  | 0    | 5    | 10   | 15   | 20   | 25   | 25   | 25   | 25   | 25   | 25   | 25   | 25   |
| 0.45 | 0    | 10   | 15   | 20   | 25   | 25   | 25   | 25   | 25   | 25   | 25   | 25   | 25   |
| 0.5  | 0    | 5    | 10   | 15   | 20   | 25   | 25   | 25   | 25   | 25   | 25   | 25   | 25   |
| 0.55 | 0    | 5    | 10   | 15   | 20   | 25   | 25   | 25   | 25   | 25   | 25   | 25   | 25   |
| 0.6  | 0    | 5    | 10   | 15   | 20   | 25   | 25   | 25   | 25   | 25   | 25   | 25   | 25   |

Case scenario the major impacts observed are some changes of schedule type for some DC-POD assignments and a reduction of diverted supply from 4.6% to 3.3%. There is a 0.05% overall objective value raise, which is consistent with Observation 4.

Finally, Figure 4.5 shows the impact of a raise in transportation costs. The subfigure on the left represents the base case experiment. The subfigure on the right assumes an increment of shipping costs of ten times more than the one defined for the base case. The effects of this raise show an increment of the number of opened DCs (and this is consistent with Observation 2) and a preference of schedule 1 versus schedule 2 for those PODs that are supplied by a DC from another site (since schedule 1 requires less transportation costs).

4.6 Conclusion and future research

Some nonprofit supply chain challenges identified in this chapter need a more thorough OR/MS perspective. For example, scholars could study revenue generation, where specific pricing and funding strategies are topics to be further analyzed. A second challenge to be further studied by OR/MS scholars is the reduction of unserved demand via the employment of risk pooling practices. This is specially pertinent in the hospital (location pooling) and humanitarian relief (fund and supply pooling) settings. A third challenge to be studied in depth
Chapter 4. Applied area II: the Nonprofit Setting

(a) Base case

(b) Modified case

Figure 4.5: Impact of transportation costs.

is collaboration between different parties within the nonprofit supply chain. In particular, the contractual mechanisms between NPOs and funders, governments, beneficiaries, and other NPOs.

Related to the particular model presented in this chapter, we believe that different versions of this model are pertinent to different situations, not only to the humanitarian delivery of food aid. For example, delivery of health care aid with the adjunct provision of services is a problem that could also be implemented with a similar model. Under this situation, besides modeling the loss of supply depending on delivery frequency, we could also be concerned about the increment of waiting times when the health care service is provided less frequently.

This chapter’s next steps are to compare the model presented with other models that explicitly track inventory at the POD level. In that case, supply security would be warranted but an investment related to monitoring is required. A more complete numerical analysis is planned in which parameters would ideally be estimated from WFP’s data in the Somali area.

In general, we believe that the development of OR/MS models appropriate for the nonprofit setting has immense potential. From the classic location and routing optimization problems to the supply contract theory, OR/MS scholars can adapt these problems to the nonprofit supply chain. Francis et al. (2006) and Chick et al. (2008) are good inspirational examples of these OR/MS problems, respectively.

Recent special issues in the OR/MS literature motivate work on nonprofit supply chains (e.g., “Special Issue on Operations Research for the Public Interest” in Operations Research and “Community-based Operations Research” in International Series in Operations Research & Management Science). According to
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this trend, the study of the nonprofit supply chain is an emerging and important research area that is still underserved by OR/MS. This chapter aims to stimulate future OR/MS research by providing some basic highlights of this type of supply chain and its major challenges.
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BIBLIOGRAPHY


BIBLIOGRAPHY


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Appendix A

Chapter 2

A.1 Parameter values

Table A.1: Parameters values in all experiments of Chapter 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{ij}$</td>
<td>great circle distance</td>
</tr>
<tr>
<td>$F_j, g_j$</td>
<td>10</td>
</tr>
<tr>
<td>$a_j$</td>
<td>5</td>
</tr>
<tr>
<td>$h, \chi, L_j$</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha, z_\alpha$</td>
<td>0.975,1.96</td>
</tr>
</tbody>
</table>
### Table A.2: Parameters values in experiments of Chapter 2.

<table>
<thead>
<tr>
<th>Table/Figure</th>
<th>Parameter/Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 2.1 (88, 150 nodes)</td>
<td>$f_j$ in Daskin (1995) divided by 100, if 88 nodes</td>
<td>100, if 150 nodes</td>
</tr>
<tr>
<td></td>
<td>$\mu_i, \sigma_i^2$ demand 1 in Daskin (1995) divided by 1000, $\mu_i$</td>
<td>100000, if 150 nodes</td>
</tr>
<tr>
<td>Table 2.3 (15, 88, 150 nodes)</td>
<td>$f_j$ in Daskin (1995) divided by 100, if 15 or 88 nodes</td>
<td>10000, if 150 nodes</td>
</tr>
<tr>
<td></td>
<td>$\mu_i, \sigma_i^2$ description in Özsen et al. (2008), $\mu_i$</td>
<td>(0.00001, 0.001)</td>
</tr>
<tr>
<td>Table 2.4 (88 nodes)</td>
<td>$f_j$ uniform [40,000, 50,000]</td>
<td>10,000</td>
</tr>
<tr>
<td></td>
<td>$\mu_i, \sigma_i^2$ in Özsen et al. (2008) $(1 + \epsilon_i), \mu_j(1 + \epsilon_i)$</td>
<td>(0.0004, 10)</td>
</tr>
<tr>
<td>Figure 2.1 (25 nodes)</td>
<td>$f_j$ demand 1, demand 2 (Daskin (1995))</td>
<td>10,000</td>
</tr>
<tr>
<td>Figures 2.2, 2.3, 2.4 (25 nodes)</td>
<td>$f_j$ demand 1, demand 2 (Daskin (1995))</td>
<td>10,000</td>
</tr>
<tr>
<td></td>
<td>$\mu_i, \sigma_i$ $(\beta, \theta), C_j$</td>
<td>(0.00001, 0.001), 170000000</td>
</tr>
<tr>
<td>Table 2.5 (25 nodes)</td>
<td>$f_j$ uniform [40,000, 50,000]</td>
<td>1,000</td>
</tr>
<tr>
<td></td>
<td>$\mu_i, \sigma_i$ demand 1$(1 + \epsilon_i), \mu_j(1 + \epsilon_i)$ Daskin (1995)</td>
<td>(0.00001, 0.001), 200000000</td>
</tr>
<tr>
<td>Table 2.6 (25 nodes)</td>
<td>$f_j$ demand 1, demand 2 (Daskin (1995))</td>
<td>10,000</td>
</tr>
<tr>
<td></td>
<td>$\mu_i, \sigma_i$ $(\beta, \theta), C_j$</td>
<td>(0.00001, 0.001), 170000000</td>
</tr>
<tr>
<td>Table 2.7 (25 nodes)</td>
<td>$f_j$ uniform [40,000, 50,000]</td>
<td>6,000</td>
</tr>
<tr>
<td></td>
<td>$\mu_{i1}, \sigma_{i1}^2$ demand 1 from Daskin (1995) divided by 100, $\mu_{i1}$</td>
<td>(0.01, 0.1), 200000000</td>
</tr>
<tr>
<td>Figure 2.5 (25 nodes)</td>
<td>$f_j$ demand 2 from Daskin (1995) divided by 100, $\mu_{i2}$</td>
<td>(0.01, 0.1), 200000000</td>
</tr>
</tbody>
</table>
Appendix B

Chapter 3

B.1 Proofs of Chapter 3

Proof of PROPERTY 1

Proof. $RI_j^R$ is the sum of a concave function ($R(Q_j^R)$) and a convex function ($WI_j^R(D_j^R, Q_j^R)$). The second-order derivative of $RI_j^R$ with respect to $Q_j^R$ is:

$$
\frac{\partial^2 RI_j^R}{\partial (Q_j^R)^2} = \frac{1}{\chi \gamma (Q_j^R)^3} \left\{ -2D_j^R [D_j^R VW - (F_j^R + \beta g_j^R)\chi \gamma] \right. \\
+ \chi \gamma D_j^R Q_j^R \\
\left. + e^{-\chi \gamma} VM \left[ 2(D_j^R)^2 + 2D_j^R Q_j^R \chi \gamma + (Q_j^R)^2 \chi^2 \gamma^2 \right] \right\}. \tag{B.1}
$$

We cannot determine whether it is positive or not. As such, $RI_j^R$ is neither convex nor concave.

Let $Q_j^{R*}$ denotes $Q_j^R$ such that

$$
\frac{\partial RI_j^R}{\partial Q_j^R} = \frac{1}{2\chi \gamma (Q_j^R)^2} \left\{ -2e^{-\chi \gamma} D_j^R V W (D_j^R + Q_j^R \chi \gamma) \\
+ [2(D_j^R)^2 V W - 2D_j^R (F_j^R + \beta g_j^R)\chi \gamma + h(Q_j^R)^2 \chi \gamma \theta] \right\} = 0.
$$
Substituting $Q_j^{R*}$ into equation (B.1), we find

$$\frac{\partial^2 R_j^R}{\partial (Q_j^R)^2} |_{Q_j^R = Q_j^{R*}} = \frac{1}{\chi \gamma (Q_j^{R*})^3} \left\{ h(Q_j^{R*})^2 \chi \gamma \theta - 2e \frac{\gamma \chi}{D_j^R} Q_j^{R*} D_j^R V W(D_j^R + Q_j^{R*} \chi \gamma) \right. \\
+ e \frac{\chi \gamma}{D_j^R} Q_j^{R*} V M \left[ 2(D_j^R)^2 + 2D_j^R Q_j^{R*} \chi \gamma + (Q_j^{R*})^2 \chi^2 \gamma^2 \right] \right\} \\
= \frac{1}{\chi \gamma (Q_j^{R*})^3} \left\{ h(Q_j^{R*})^2 \chi \gamma \theta + e \frac{\chi \gamma}{D_j^R} Q_j^{R*} V W(Q_j^{R*})^2 \chi^2 \gamma^2 \right\} \geq 0.$$

It shows that $R_j^R(Q_j^R)$ is unimodal in $Q_j^R$ and $Q_j^{R*}$ is global minimum. \hfill \Box

**Proof of PROPERTY 2**

*Proof.* Taking the first- and second-order derivative of $R(Q_j^R)$ with respect to $Q_j^R$, we can show that $R(Q_j^R)$ is an increasing and concave function of $Q_j^R$.

$$\frac{dR(Q_j^R)}{dQ_j^R} = V(D_j^R)^2 e - \frac{Q_j^R \chi \gamma}{(Q_j^R)^3 \chi} \frac{Q_j^R \chi}{D_j^R} \left( \frac{Q_j^R \chi}{D_j^R} - 1 - \frac{Q_j^R \chi \gamma}{D_j^R} \right) > 0.$$

$$\frac{d^2 R(Q_j^R)}{d(Q_j^R)^2} = V e - \frac{\gamma \chi Q_j^R}{(Q_j^R)^3 \gamma \chi} \left[ -2(D_j^R)^2 \left( 1 + e \frac{\gamma \chi Q_j^R}{D_j^R} \right) + 2D_j^R Q_j^R \gamma \chi + (Q_j^R)^2 \chi^2 \gamma^2 \right] \\
= \frac{2(D_j^R)^2 V e - \gamma \chi Q_j^R}{(Q_j^R)^3 \gamma \chi} \left( -e \frac{\gamma \chi Q_j^R}{D_j^R} + 1 + \frac{Q_j^R \gamma \chi}{D_j^R} + \frac{(Q_j^R)^2 \gamma^2 \chi^2}{2(D_j^R)^2} \right) < 0.$$

$\hat{R}_j^R(Q_j^R)$ is linear. Therefore, $ERR(Q_j^R)$ is a concave function of $Q_j^R$. \hfill \Box

**Proof of PROPERTY 3**
Proof. The shipment quantities of new and returned products can be obtained exogenously by solving the following program:

\[
W_j(D_j^F, D_j^R) = \begin{cases} 
\text{Min} & F_j^F \frac{D_j^F}{Q_j^F} + \beta (g_j^F + a_j^F Q_j^F) \frac{D_j^F}{Q_j^F} + \theta \frac{hQ_j^F}{2} + \theta h z_{\alpha} \sqrt{L_j \frac{D_j^F}{\chi}}, \\
+ F_j^R \frac{D_j^R}{Q_j^R} + \beta (g_j^R + a_j^R Q_j^R) \frac{D_j^R}{Q_j^R} + \theta h + W_{\gamma} Q_j^R, \\
\text{s.t.} & Q_j^F + z_{\alpha} \sqrt{L_j \frac{D_j^F}{\chi}} + L_j \frac{D_j^F}{\chi} + Q_j^R \leq C_j, \\
& Q_j^F, Q_j^R \geq 0.
\end{cases}
\]

Because \(W_j(D_j^F, D_j^R)\) is convex, we apply KKT conditions and obtain the equations:

\[
\begin{cases} 
- \frac{(F_j^F + \beta g_j^F) D_j^F}{(Q_j^F)^2} + \frac{h \theta}{2} + \lambda_j = 0, \\
- \frac{(F_j^R + \beta g_j^R) D_j^R}{(Q_j^R)^2} + \frac{h \theta + W_{\gamma}}{2} + \lambda_j = 0, \\
\lambda \left( Q_j^F + L_j \frac{D_j^F}{\chi} + z_{\alpha} \sqrt{L_j \frac{D_j^F}{\chi}} + Q_j^R - C_j \right) = 0, \\
Q_j^F, Q_j^R, \lambda \geq 0,
\end{cases}
\]

where \(\lambda_j\) is a nonnegative Lagrangian multiplier. If the capacity constraint is strictly satisfied, then \(\lambda_j=0\) and the shipment quantities of new and returned products can be determine by the economic order quantities of them, i.e.,

\[
Q_j^F = \sqrt{\frac{2(F_j^F + \beta g_j^F) D_j^F}{h \theta}}, \\
Q_j^R = \sqrt{\frac{2(F_j^R + \beta g_j^R) D_j^R}{h \theta + W_{\gamma}}},
\]

If the capacity constraint is binding, then \(\lambda_j > 0\) and the shipment quantities of new and returned products are:

\[
Q_j^F = \sqrt{\frac{2(F_j^F + \beta g_j^F) D_j^F}{h \theta + 2 \lambda_j}}, \\
Q_j^R = \sqrt{\frac{2(F_j^R + \beta g_j^R) D_j^R}{h \theta + W_{\gamma} + 2 \lambda_j}}.
\]
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Note that both of them are less than the respective economic order quantities.

Proof of Proposition 5

Proof. \( f(S) = \sqrt{L_j \mu(S)} \), where \( \mu(S) = \sum_{i \in S} \mu_i \), is a submodular function due to its concavity based on Proposition 1. Hence, \( \pi_i = \sqrt{L_j \mu(S(i))} - \sqrt{L_j \mu(S(i-1))} \) is an extreme point of the extended polymatroid \( EP_f \) based on Edmonds (1971). That is, \( \pi_i \in EP_f \). Therefore, \( \pi(S) \leq f(S) \leq \omega_j \), which completes the proof.

Proof of Proposition 6

Proof. Let \( \pi_j = u_j + Q_j \), from constraints (3.18) and (3.24) we obtain the following relaxed form of constraints (3.18)

\[
\bar{u}_j^2 \geq \frac{4(F_j + \beta g_j)}{\theta h} D_j + 3(Q_j)^2 + u_j^2 \\
\geq \frac{4(F_j + \beta g_j)}{\theta h} D_j + 3(Q_j)^2 + \left( \frac{2(F_j + \beta g_j)}{\theta h} \frac{D_j}{Q_j} + Q_j \right)^2. \tag{B.2}
\]

Taking the derivative of the right-hand side of the above inequality with respect to \( Q_j \), we obtain

\[
6Q_j + 2 \left( 1 - \frac{2D_j(F_j + \beta g_j)}{h \theta (Q_j)^2} \right) \left( Q_j + \frac{2D_j(F_j + \beta g_j)}{h \theta Q_j} \right) = 0.
\]

Solving for \( Q_j \), we obtain \( Q_j = \sqrt{\frac{(F_j + \beta g_j)D_j}{h \theta}} \). Substituting this into the inequality (B.2) we obtain the following relaxed constraint

\[
\bar{u}_j^2 \geq \frac{4(F_j + \beta g_j)}{\theta h} D_j + 3(Q_j)^2 + u_j^2 \\
\geq \frac{4(F_j + \beta g_j)}{\theta h} D_j + 3(Q_j)^2 + \left( \frac{2(F_j + \beta g_j)}{\theta h} \frac{D_j}{Q_j} + Q_j \right)^2 \\
\geq \frac{4(F_j + \beta g_j)}{\theta h} D_j + \frac{12(F_j + \beta g_j)}{\theta h} D_j \\
= \frac{16(F_j + \beta g_j)}{\theta h} D_j.
\]

According to Proposition 5, we can get a valid inequality (that is also an extremal extended polymatroid inequality), \( \sum_{i \in I} \pi_i Y_{ij} \leq u_j + Q_j \), for the lower convex envelope of the relaxed constraint, that is
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\[ Q_\pi = \text{conv} \left\{ (Y_j, u_j) \in \{0, 1\}^{|I|} \times R : \frac{1}{2}u_j^2 \geq 4H_j\chi \sum_{i \in I} \mu_i (Y_{ij})^2 \right\} \]

where \( \pi_i = \sqrt{8H_j\chi \sum_{i \in S(i)} \mu_i} - \sqrt{8H_j\chi \sum_{i \in S(i-1)} \mu_i} \).

Note that the suggested inequality is also valid for \( Q_u \) of constraints (3.18), where

\[ Q_u = \text{conv}\{(Y_j, u_j, Q_j) \in \{0, 1\}^{|I|} \times R^2 : \frac{1}{2}(u_j + Q_j)^2 \geq H_j\chi \sum_{i \in I} \mu_i (Y_{ij})^2 + \frac{3}{2}(Q_j)^2 + \frac{1}{2}u_j^2 \}. \]

The same proof can be derived for constraints (3.19) and (3.25).

\[ \square \]

B.2 Parameter values

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td>Inventory holding cost per unit per year for each DC</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>97.5% Service level</td>
</tr>
<tr>
<td>( z_\alpha )</td>
<td>1.96 Standard normal deviate such that ( P(z \leq z_\alpha) = \alpha )</td>
</tr>
<tr>
<td>( F_{j,F,R} )</td>
<td>10 Fixed order costs</td>
</tr>
<tr>
<td>( g_{j,F,R} )</td>
<td>10 Fixed transportation costs between the DCs and supplier</td>
</tr>
<tr>
<td>( a_{j,F,R} )</td>
<td>5 Per unit shipment costs between the DCs and supplier</td>
</tr>
<tr>
<td>( L_j )</td>
<td>1 Lead time in days</td>
</tr>
<tr>
<td>( \chi )</td>
<td>1 Number of days worked in a year</td>
</tr>
<tr>
<td>( W )</td>
<td>1 Weight associated with loss in value of returned products</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>10% Marginal value of time of returned products</td>
</tr>
</tbody>
</table>

Return rate 40%
Table B.2: Parameter values in experiments of Chapter 3.

<table>
<thead>
<tr>
<th>Table or Figure</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 3.4</td>
<td>88-city, $\theta = 0.1, \beta = 0.005$</td>
</tr>
<tr>
<td>Table 3.5</td>
<td>88-city, $W=1, \theta = 2, \beta = 0.005$</td>
</tr>
<tr>
<td>Table 3.6</td>
<td>88-city, $W=1, \theta = 5, \beta = 0.005$</td>
</tr>
<tr>
<td>Table 3.7</td>
<td>88-city, $W=1, \theta = 10, \beta = 0.005$</td>
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<tr>
<td>Table 3.8</td>
<td>$\theta = 0.1, \beta = 0.001$</td>
</tr>
<tr>
<td>Tables 3.9 and 3.10</td>
<td>88-city, $W=10, \theta = 0.1, \beta = 0.005$, Capacity=7700</td>
</tr>
<tr>
<td>Figures 3.4 and 3.5</td>
<td>$\gamma=50$, return rate=80%, $\theta=0.1, \beta=0.005$, Capacity=7700</td>
</tr>
</tbody>
</table>
Appendix C

Chapter 4

C.1 Nonprofit performance metrics

C.1.1 Efficiency

Efficiency is defined as the ratio of service output to total input. Total input is typically measured in dollars or manpower while the definition of service output varies and depends on the application.

In OM, productivity is a common measure of efficiency. Data envelopment analysis (DEA) and stochastic frontier analysis (SFA) are heavily used to empirically evaluate productivity. DEA was introduced by Charnes et al. (1978) and is a nonparametric method for estimating production frontiers. SFA is defined by Aigner et al. (1977) and is a parametric method estimating the production frontier via regression techniques. In a nonprofit context, these techniques are mainly employed to measure hospital efficiency (e.g., Kuntz and Scholtes (2000), Jacobs (2001)) and education efficiency (e.g., Mancebon and Molinero (2000)).

Yadav (2010) studies health care in developing countries. This work aims to highlight many general issues in this topic from an operations research perspective. Some key public health metrics are suggested as output measures related to efficiency, such as mortality, morbidity, life expectancy, quality-adjusted life year (QALY), and disability-adjusted life year (DALY).

Cost is a predominant resource metric to evaluate service output. For example, in humanitarian relief, Beamon and Balcik (2008) state that the three main costs are the costs of supplies, distribution, and inventory holding. The number of relief workers employed per aid recipient or the total dollars spent per aid recipient are also efficiency measures.
C.1.2 Effectiveness

For Savas (1978), effectiveness is defined as a measure of the level of satisfaction of a need and the alleviation of its adverse impacts. Measures of effectiveness within the public service field include expected aggregate utility, level of citizen satisfaction, mortality rates, and environmental factors. Hence this measure can be found in multiple forms depending on the context.

In health care, Griffin et al. (2008) maximize the weighted demand coverage of the population in need. Fle‰a (2003b) advises some German and international agencies, the local government, and Médecins Sans Frontières (MSF) by studying the optimal allocation of health care resources in Tanzania. The author suggests measuring effectiveness by analyzing five metrics: number of deaths, years of life lost, incidence of disease, prevalence of disease, and loss of quality of life.

In education, Jauch and Glueck (1975) provide an empirical evaluation of the research performance of university professors. The metric employed is the number of publications in respectable journals.

In relief chains, Beamon and Balcik (2008) claim that effectiveness is correlated with output metrics such as response time, number of items supplied, and supply availability.

C.1.3 Equity

According to equity theory, human beings believe that rewards and punishments should be distributed according to recipients’ inputs or contributions (see Leventhal (1976)). Rawls (1999) suggests, as a measure of fairness, the maximum of the minimum individual utility function \( \max \min \{u_1, \ldots, u_n\} \), where \( u_i \) is the utility function of individual \( i \). In other words, this model maximizes the poorest individual’s utility. However, this is just one measure of equity, out of a large set of proposed measures in the economics literature.

In the public service literature, Savas (1978) emphasizes equity over the other two performance measures. This author states that despite citywide services being efficient and effective, they could be inequitable if all beneficiaries are not treated similarly. Leclere et al. (2011) review literature that models equity for allocating resources in public service systems with a special focus on the allocation of Emergency Medical Services (EMS).

Related to theoretical OM models, Marsh and Schilling (1994) provide a rich theoretical review of equity principles along with twenty different equity measures for facility location problems. Balcik et al. (2009) claim that the issue of equity is relevant for vehicle routing problems and review this topic in the context of the nonprofit and public sectors. Bertsimas et al. (2011) define the “price of fairness”
in resource allocation problems as the relative loss between a fair allocation and the allocation that maximizes the sum of player utilities. They study two fairness criteria, max-min and proportional fairness, and they provide bounds on the “price of fairness” related to each of these measures. Chan et al. (2011) suggest a measure of fairness to serve queues when the system is temporarily overloaded. They focus on MaxWeight scheduling policies, which define the proportion of aggregate backlog each queue contributes.

In humanitarian relief, Ogryczak (2000) studies the distribution of travel distances among service recipients (beneficiaries). This paper develops a bi-criteria optimization problem for mean distance and absolute inequality measures. Some bounded trade-offs are defined to assure that solutions are equitable. Campbell et al. (2008) study vehicle routing in disaster response problems. The model minimizes the maximum arrival time and the average arrival time. These two equity measures are equivalent to minimizing the makespan and the sum of completion times in scheduling systems.

In the community action field, Mandell (1991) designs a multiobjective program to allocate new books among the branches of a public library system. This program maximizes output level and minimizes inequity. Inequity is represented by the Gini coefficient, which in this case is defined as the average “perceived net envy level” associated with the distribution of public services. Lien et al. (2009) study food distribution by designing inventory and routing policies for a U.S. food bank. They maximize the minimum expected fill rate (the ratio of amount allocated to demand). The resulting model is a sequential resource allocation (SRA) problem, which is solved using dynamic programming. Near-optimal heuristics are suggested for large problem instances.

In health care, Kaplan and Merson (2002) find a middle-ground solution that addresses both equity rules and cost-efficiency in the study of HIV-prevention strategies. Equity is ensured by earmarking a fraction of the total budget for equity alone. The remaining funds are allocated according to cost-effectiveness. Su and Zenios (2006) analyze the social benefits of organ transplants. They study the trade-off between two measures: the expected aggregate utility (an efficiency measure) and the minimum utility across all candidates (an equity measure). Zenios et al. (2000) suggest two measures of inequity regarding the allocation of kidneys to patients: the likelihood of transplant for various types of patients and the differences in mean waiting times across patient types. Yadav (2010) suggests the concentration curve and the Gini coefficient as good measures of equity for health care in developing countries.

To conclude this section, we note that most of the studies that employ equity as an objective are accompanied by other measures of efficiency or effectiveness.
C.1.4 Other suggested measures

- **Net social benefit**

  The idea of maximizing net social benefit originated in welfare economics. Baumol (1965) studies the resource allocation problem in terms of the entire community’s welfare and two approaches are used: the maximization of consumers’ and producers’ surpluses and the Pareto optimality approach. The surplus is understood as the sum of net gains (the area between the demand curve and the horizontal line indicating the price paid for the commodity). The Pareto optimality model maximizes an arbitrary person’s utility while ensuring that no one else’s utility diminishes.

  We refer to Feldstein (1972) for an example of customer surplus and social utility formulations employed in economics. This paper solves a public pricing problem that incorporates customer distributional aspects. In particular, the welfare objective is the weighted sum of the household consumer surpluses.

  The OM literature also employs net social benefit as an objective measure. To illustrate, Erlenkotter (1977) solves the uncapacitated facility location problem when prices and location are decided simultaneously (i.e. price-sensitive demands). Three solution techniques are offered: the private sector solution, which maximizes profits, the public sector solution, which maximizes net social benefits, and a quasi-public approach, which constrains revenues to the level required to cover costs.

- **Sustainability**

  Sustainability is relevant for long-term projects and can be defined either as covering all costs or covering all costs except capital. Leonard et al. (2007) study sustainability for Opportunity International, a microfinance NPO, and they define it in two domains: operational and financial sustainability. Operational sustainability is the ability of the NPO to cover its lending expenses with income earned by its lending operations. So, when operational sustainability is 100%, the NPO earns what it spends, without considering inflation or the cost of borrowing at market interest rates. Financial sustainability is the capability of an NPO to cover lending expenses and its cost of capital. This definition assumes that the organization takes grants and subsidies as if they were obtained from commercial sources. If financial sustainability is more than 100%, the NPO earns a surplus and can expand.

  Zhen and Routray (2003) study NPOs in developing countries. This work reviews relevant literature on operational indicators for agricultural sustainability. When spatial and temporal characteristics are determined, a selection of indicators is prioritized based on three subgroups: ecological, economic, and social indicators.
Appendix C. Chapter 4

Sustainability in supply chains is reviewed by Clift (2003) and Seuring et al. (2005).

• Poverty reduction

Let \( y = (y_1, y_2, ..., y_n) \) be a vector of household incomes in increasing order. Two simple poverty reduction measures have been proposed in the literature, the headcount ratio \( H = \frac{q}{n} \) and the income-gap ratio \( I = \sum_{i \in S(z)} \frac{g_i}{q} \), where \( q \) is the number of people with income \( y_i \leq z \), \( n \) is the total population size, \( z > 0 \) is the predetermined poverty line, \( S(z) \) is the set of people with income no higher than \( z \), and \( g_i = z - y_i \) is the income shortfall of the ith household.

Sen (1976) analyzes the shortcomings of these and similar measures and introduces an axiomatic approach to poverty measures. Foster et al. (1984) study a popular class of poverty measures called decomposable poverty measures. These measures accomplish some of Sen’s axioms. One example is \( P_\alpha(y; z) = \frac{1}{n} \sum_{i=1}^{q} \left( \frac{g_i}{z} \right)^\alpha \), a normalized weighted sum of the income shortfalls of the poor where \( \alpha \geq 0 \) is a measure of poverty aversion. Clark et al. (1981), Atkinson (1987), and Kanbur (1987) study convenient poverty measures.

There is a body of macroeconomic literature on aid allocation and poverty reduction strategies for governments and international agencies. Collier and Dollar (2002) develop a poverty-efficient aid allocation scheme and compare it with real data. To measure poverty, these authors use three poverty measures: headcount, poverty gap, and squared poverty gap. The corresponding optimization problem maximizes poverty reduction, \( \sum_i G_i \alpha^i h_i N_i \), subject to total available aid, where \( G \) is growth, \( \alpha \) is the elasticity of poverty reduction with respect to income, \( h \) is one of the three aforementioned measures of poverty, \( N \) is population. Mosley et al. (2004) present a similar optimization problem where poverty elasticities vary depending on corruption and inequality.

• Vulnerability

There is a new stream of literature on vulnerability-based poverty measures. Ligon and Schechter (2003) define vulnerability as the level of a household’s well-being, which depends on income or consumption, as well as the risks it faces. They take a utilitarian approach and construct the mathematical definition per household \( i \), \( V_i(c) = U_i(z) - EU_i(c^i) \), where \( c^i \) is the consumption of household \( i \), \( U_i \) is a strictly increasing, weakly concave function mapping consumption expenditures into the real line, and \( z \) is the poverty line. This measure decomposes across poverty, aggregate risk, and idiosyncratic risk measures and it is equivalent to maximizing the utilitarian social welfare function \( (\sum_{i=1}^{n} EU_i(c^i)) \) subject to an aggregate resource constraint.
Another technique to measure vulnerability adapts the standard measures of poverty ($P_{\alpha}$) to a non-deterministic setting through an expected poverty measure (Foster et al. (1984)). In particular, Kamanou and Morduch (2004) present the vulnerability measure $EP_{\alpha,t+1} - P_{\alpha,t}$. Calvo and Dercon (2005) review both kinds of approaches and suggest a different measure of individual vulnerability. This paper constructs a list of desirable welfare-economic axioms at the individual level.

C.2 Proofs of Chapter 4

Proof of PROPOSITION 7

Proof. Note that $r_s = \frac{r_j}{n_s}$, where $n_s$ is an integer that describes the frequency of schedule $s$. If $n_s = 1$, there is no inventory to be stored at DC $j$ for POD $i$ since all inventory for this POD will be immediately shipped. If $n_s = 2$, just half of the inventory will be stocked at DC $j$ for half of the review period, $r_j$. So the average inventory during period $r_j$ at the DC is $\frac{1}{2}r_j \mu_i$. If we generalize by any $n_s$, the average inventory at DC $j$ for POD $i$ during period $r_j$ is $\frac{n_s}{2}r_j \mu_i = (n_s - 1)\frac{r_j}{2} \mu_i$. Note that by employing the formula introduced in the proposition for a specific $i$ and $s$ we obtain the same expression $(r_j - r_s) \mu_i = (1 - \frac{1}{n_s})r_j \mu_i = (n_s - 1) \frac{r_j}{n_s} \mu_i$. \hfill $\square$

Proof of PROPOSITION 9

Proof. From Chapter 2 we note that the square root portion of the safety stock at the DC level can be written in a much simple way by using auxiliary variables ($w_{ikj}$). Also, we further employ another set of auxiliary variables, $s_j$, to substitute this nonlinear square root portions in the objective. As a consequence, we need to add the set of equations (2.10).

$$\sqrt{r_j + L_j} \left( \sum_{i \in I} \sum_{k=1}^{I} \sum_{s=1}^{S} \sum_{s'=1}^{S} \sigma_i \sigma_{ik} \rho_{ik} y_{ij}^s y_{kj}^s \right) + \sigma_i^2 \left( \sum_{i \in I} \sum_{s \in S} \mu_i y_{ij}^s \right)^2 \leq s_j$$

$$\equiv$$

$$\sqrt{\sum_{i=1}^{I} \sum_{k=1}^{I} \sum_{s=1}^{S} \sum_{s'=1}^{S} \left( (r_j + L_j) \sigma_i \sigma_{ik} \rho_{ik} + \sigma_i^2 \mu_i \mu_k \right) y_{ij}^s y_{kj}^s } \leq s_j$$

$$\equiv$$

$$\sum_{i=1}^{I} \sum_{k=1}^{I} \sum_{s=1}^{S} \sum_{s'=1}^{S} V_{ikj} (w_{ikj}^s s_j)^2 \leq s_j^2$$
where $V_{ikj} = (r_j + L_j)\sigma_i\sigma_k\rho_{ik} + \sigma_{Lj}^2\mu_i\mu_k$. The auxiliary variables $w^{ss'}_{ikj}$ are defined as $w^{ss'}_{ikj} = y^s_{ij}y^{s'}_{kj}$ and we use that $(w^{ss'}_{ikj})^2 = w^{ss'}_{ikj}$ and $(y^s_{ij})^2 = y^s_{ij}$ since they are binary variables.

Note that with this substitution the objective of (CQMIP1) is linear and the constraints are either conic quadratic or linear and we have a conic quadratic MIP.

\[\square\]

## C.3 Parameter values

**Table C.1:** Parameter values in experiments of Chapter 4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{ij}$</td>
<td>great circle distance</td>
</tr>
<tr>
<td>$F_j, g_j$</td>
<td>1000</td>
</tr>
<tr>
<td>$a_j$</td>
<td>5</td>
</tr>
<tr>
<td>$h, L_j$</td>
<td>20</td>
</tr>
<tr>
<td>$\alpha, z_{\alpha}$</td>
<td>0.975, 1.96</td>
</tr>
<tr>
<td>$r_j, \chi_2, \chi$</td>
<td>7, 52, 364</td>
</tr>
<tr>
<td>$f_j$</td>
<td>$5 \cdot 10^9$</td>
</tr>
<tr>
<td>$\mu_i, \sigma_i^2$</td>
<td>demand from Daskin (1995) divided by 300, $\mu_{i2}$</td>
</tr>
<tr>
<td>$(\beta, \theta)$</td>
<td>(1, 1)</td>
</tr>
<tr>
<td>$C_j, cap_i$</td>
<td>$3 \cdot 10^{12}, 3 \cdot 10^{12}$</td>
</tr>
</tbody>
</table>