Heat Pipe Performance Enhancement with Binary Mixture Fluids that Exhibit Strong Concentration Marangoni Effects

by

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A dissertation submitted in partial satisfaction of the requirements for the degree of Doctor of Philosophy in Engineering – Mechanical Engineering with the Designated Emphasis in Energy Science and Technology in the Graduate Division of the University of California, Berkeley

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Abstract

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This research investigates the impact of Marangoni phenomena, with low mixture concentrations of alcohol and water, to enhance thermal transport capability of gravity-assisted heat pipes. The use of binary mixture working fluids in gravity-assisted heat pipes are shown to improve the critical heat flux (CHF) and operating performance, more so than with pure fluids. The CHF is responsible for dryout when the pumping rate of a liquid flow structure is not sufficient to provide enough fluid to the evaporator section.

In the first study, heat pipe performance experiments were conducted for pure water and 2-propanol solutions with varying concentrations. Initial tests with pure water determined the optimal working fluid charge for the heat pipe; subsequent performance tests over a wide range of heat input levels were then conducted for each working fluid at this optimum value. The results indicated that some mixtures significantly enhance the heat transfer coefficient and heat flux capability of the heat pipe evaporator. For the best mixture tested, the maximum evaporator heat flux carried by the coolant without dryout was found to be 52% higher than the value for the same heat pipe using pure water as a coolant under comparable conditions. Peak evaporator heat flux values above 100 W/cm² were achieved with some mixtures. Evaporator and condenser heat transfer coefficient data are presented and the trends are examined in the context of the expected effect of the Marangoni mechanisms on heat transfer.

Analytical modeling effort was also conducted investigating the impact of Marangoni phenomena for low concentrations of 2-propanol/water and methanol/water mixtures. In real systems the addition of small levels of surface-active contaminants can affect the surface tension of the liquid-vapor interface and thermodynamic conditions in this region. Analysis was performed for three widely accepted binary mixture correlations to predict heat flux and superheat values for subatmospheric experimental data using bulk fluid and film thermodynamic properties. Due to the non-ideal nature of these alcohol/water mixtures, this study employs an average pseudo single-component (PSC) coefficient in place of an ideal heat transfer coefficient.
(HTC) to improve the correlation predictions. This investigation evaluates the ability for these correlations to predict strong Marangoni effects of mixtures that have large surface tension variation with concentration under subatmospheric conditions. It is not always clear that evaluation of bulk fluid properties will satisfactorily account for Marangoni effects. Analysis is also performed to assess correlation predictions for interfacial film properties rather than that of the bulk fluid. The results indicate that the use of film properties along with the PSC coefficient improves heat flux model predictions of subatmospheric experimental data by as much as 59.3% for 0.015M 2-propanol and 49.1% for 0.04M methanol/water mixtures, where strong Marangoni effects are believed to be more evident.

A second experimental study was also performed of a 37° inclined, gravity-assisted, brass heat pipe with a 0.05M 2-Propanol/water binary mixture. The device design was developed from the first study by enlarging the evaporator and condenser surface areas. Strip heaters were also employed to provide larger input heat flux levels, for enhanced heat pipe performance testing. These experiments were carried out for varying liquid charge ratios between 30% and 70%, to determine an optimal value that would enhance heat transport performance by maximizing the critical heat flux (CHF) condition, while reducing the evaporator wall superheat. A 45% fill ratio was found to have the lowest overall superheat and highest thermal conductance by as much as 7.5W/K, as well as an enhanced CHF condition of 114.8W/cm². A heat pipe analytical model, that characterizes binary mixture pool boiling is also presented, which was developed based on modeling efforts presented in studies 1 and 2. Model results with a 45% liquid charge ratio were found to provide good correspondence with the experimental data with an average rms evaporator vaporization heat flux deviation of 6.5%.

The final study of this investigation assesses the cooling of single and dual-junction solar cells with the inclined, gravity-assisted, brass heat pipe, with a 0.05M 2-propanol/water mixture. Thermal behavior of this heat pipe solar collector system was investigated theoretically and semi-empirically through experimentation of varying input heat loads from attached strip-heaters to simulate waste heat production of single-junction monocrystalline silicon (Si), and multijunction GaInP/GaAs solar cells. It was also found that the 45% liquid charge was capable of achieving the lowest superheat levels at a predicted solar concentration of 162 suns. Solar cell semiconductor theory was employed to evaluate the effects of increasing temperature and solar concentration on solar cell performance. Results showed that a combined PV/heat pipe system had a 1.7% higher electrical efficiency, at a concentration ratio 132 suns higher than a stand-alone PV system. The dual-junction system also exhibited enhanced performance at elevated system temperatures with a 2.1% greater electrical efficiency, at an operational concentration level of 560 suns higher than a stand-alone PV system. Waste heat recovery analysis of the silicon solar cell, revealed respective thermal and system efficiencies as high as 56.3% and 66.3%, as the incident solar radiation and corresponding condenser heat removal factor increased to 82 suns.
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NOMENCLATURE

\( C_p \) Specific heat \( V \) Voltage
\( k \) Thermal Conductivity \( A \) Area
\( \dot{Q}^* \) Heat Flux \( H \) Head Loss
\( C \) Concentration \( P \) Pressure
\( x \) Liquid mass fraction \( q \) Electric Charge
\( y \) Vapor mass fraction \( R \) Universal gas constant
\( \dot{x} \) Liquid mole fraction \( Re \) Reynolds number
\( \dot{y} \) Vapor mole fraction \( Pr \) Prandtl number
\( h \) Enthalpy \( Nu \) Nusselt number
\( D \) Diffusion Coefficient \( N \) Number
\( d \) Bubble departure diameter \( u \) Velocity
\( k_B \) Boltzmann constant \( T \) Temperature
\( \dot{Q} \) Heat load \( Ja \) Jakob number
\( L \) Characteristic length \( V \) Volume
\( CHF \) Critical heat flux \( G \) Conductance
\( Z \) Liquid height \( FF \) Fill Factor
\( S \) Shape factor \( I \) Current
\( M \) Mass \( E \) Energy
\( h \) Heat transfer coefficient
\( g \) Gravitational constant
\( m \) Mass flow rate

x
### SUBSCRIPTS

- **c**: Critical property  
- **fins**: Condenser external fins  
- **j**: mixture component  
- **w**: Wall  
- **r**: Reduced property  
- **i**: Interface  
- **th**: Thermal  
- **tot**: Total  
- **elec**: Electrical  
- **∞**: Ambient  
- **NB**: Nucleate boiling  
- **avg**: Average  
- **sat**: Saturation condition  
- **m**: Mixture  
- **id**: Ideal  
- **oc**: Open circuit  
- **b**: Bubble  
- **sc**: Short circuit  
- **s**: Interface  
- **sys**: System  
- **v**: Vapor  
- **cell**: Solar cell  
- **l**: liquid  
- **ref**: Reference  
- **bp**: Bubble point  
- **g**: Band Gap  
- **dp**: Dew point  
- **inc**: Incident  
- **e**: Evaporator  
- **c**: Condenser  
- **Br**: Brass  
- **adia**: Adiabatic  
- **in**: input  
- **cold**: Condenser external surface  
- **out**: removal  
- **Al**: Aluminum  
- **cond**: Condenser  
- **sys**: System
### GREEK SYMBOLS

<table>
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<tr>
<td>$\eta$</td>
<td>Efficiency</td>
<td>$\rho$</td>
<td>Density</td>
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<tr>
<td>$\sigma$</td>
<td>Surface Tension</td>
<td>$\hat{\rho}$</td>
<td>Molar density</td>
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<tr>
<td>$\mu$</td>
<td>Kinematic viscosity</td>
<td>$\gamma$</td>
<td>Activity coefficient</td>
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<tr>
<td>$\alpha$</td>
<td>Thermal diffusivity</td>
<td>$\theta$</td>
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CHAPTER 1 - INTRODUCTION

1.1 Nucleate Pool Boiling Enhancement

Nucleate pool boiling heat transfer has great capability for providing thermal transport and waste heat recovery in high-technology areas such as electronics and energy generation industries. Nucleate boiling heat exchange devices benefit from elevated heat transfer coefficients, attained from transport of latent heat. This allows a device reduction in size, weight and volume to improve thermal performance of thermo-semiconducting systems, such as microprocessors and photovoltaics (PV’s). Nucleate pool boiling of a liquid medium begins when the superheated state is reached, whereby a nucleated bubble will grow rapidly from an active surface cavity to a diameter large enough to release and transport thermal energy away from the heated surface. Although nucleate pool boiling of pure single-component fluids are able to attain high vaporization heat flux levels [1], many binary mixtures have demonstrated the attainment of higher critical heat flux (CHF) conditions with reduced superheat levels [4]. The characterization and number of nucleate pool boiling models has steadily increased over the last decade, with single and binary mixture correlations that have provided more accurate predictions of boiling experiments [2]. A chronological summary for nucleate pool boiling correlations for binary mixtures can be seen in Table (A.1) of the Appendix, with the most recent contribution by Táboas et. al. [3]. These improved predictions have promoted the design and proliferation of high performance heat transfer equipment, such as heat pipes, which are capable of exploiting large heat transfer coefficients resulting from evaporation and condensation processes.

1.2 Surface Conditioning

Nucleate pool boiling performance metrics, typically assessed in literature [5], are the superheat required for bubble incipience, the maximum CHF condition and the fully developed nucleate boiling heat transfer coefficient. To optimize these parameters, prior studies have examined the reduction of saturation pressure and surface modification of boiling surfaces, among others [6]. Collier and Thome [7] note that methods for improving the performance of nucleate boiling depend on achieving sustained nucleation at lower superheat levels, which can be facilitated through the provision of more nucleation sites which are stable for prolonged periods. Jakob and Fritz [8] as well as Corty and Foust [9] showed that surface roughening of horizontal surfaces can provide greater numbers of nucleation sites. The authors noted that nucleation occurs at gas or vapor-filled cavities in the heated surface, and that the superheat at which nucleation occurs depends critically on the size and distribution of these cavities, as well as the temperature gradient away from the heated surface. However, their study showed that after prolonged boiling much of the heat transfer improvement due to surface roughness alteration disappeared due to instability in newer larger cavities that were formed. Another study by McGillis and Carey [6] investigated boiling enhancement techniques for water at low pressures over a copper finned apparatus. The authors found that although roughened surfaces significantly reduced wall superheat and provided larger cavities required for low pressure boiling, gas embryos that exist
in large cavities were easily washed away, leading to a long waiting time before boiling would reoccur. Additionally, they also found that because low pressure boiling relies on a small number of large cavities, the boiling heat transfer enhancement with rough surfaces may not be as great as the enhancement at lower pressures where there is a significant increase in the number of active sites. Therefore boiling performance-enhancing limitations exist with surface modifications in low pressure systems, as increased surface roughness does not necessarily create larger active nucleation cavities.

1.3 Low Pressure Boiling and the Marangoni Effect

Bubble nucleation also depends on fluid thermodynamic properties of nucleate embryo formation and propagation. Lower superheat values are desired for high-performance heat transfer equipment, which enable the removal of thermal energy at lower wall surface temperatures. The time necessary to facilitate nucleate pool boiling is dependent on fluid properties, the thermal boundary layer, and subcooling of the bulk liquid. Studies by Raben et. al. [9], Van Stralen et. al. [10] as well as McGillis [11] have shown that bulk fluid pressure reduction can improve boiling performance, through an increase in bubble departure frequency. This frequency improvement has been shown to occur at low pressures, where resultant low vapor densities facilitate relatively large bubble departure diameters [5]. These departing bubbles create a large wake which induces mixing and a removal of superheated liquid from the surface. As an example, Fig. (1) presents nucleate pool boiling data, by McGillis and Carey [5], for pure water on a copper surface where enhanced transport heat flux values were found at lower bulk fluid pressures due to resultant lower wall temperatures.

![Figure 1.1. Nucleate pool boiling data by McGillis and Carey [5] for pure water over a copper surface at varying pressures between 4kPa – 101kPa.](image)
Many nucleate boiling investigators have also discovered that for the same input heat loads, the CHF condition for binary mixture fluids can be significantly larger than that of their respective pure components. This is especially true for low concentration levels of alcohol/water mixtures. A study by McGillis and Carey [12] investigated the boiling performance of binary mixtures of water mixed with varying concentrations of methanol, 2-Propanol and ethanol. Their study found that small additions of a volatile alcohol component were able to increase the CHF condition above that of pure water, while higher concentrations decreased the CHF to that of the respective pure alcohol. Specifically, the results showed that small additions of 2-propanol reduced fluctuations in the evaporator wall temperature from that of pure water, possibly due to a reduction in the bubble departure diameter. It was found that although the average superheat did not vary significantly with concentration, the CHF did, reaching a maximum at approximately 0.05 M 2-propanol, with a respective value of approximately 200W/cm². They concluded that correlations based on effective subcooling [13] were not able to accurately predict nucleate boiling and the CHF condition for all alcohol/water mixtures, and the influence of the Marangoni effect was the principle cause for the CHF variation of boiling liquid mixtures. Their study of Marangoni phenomena and the use of a surface tension gradient model was found to provide more consistent predictions of the CHF condition for all studied alcohol/water binary mixtures, better than those of a traditionally-used subcooling-based model.

Research of interfacial phenomena between liquid and vapor species has shown that thermodynamic surface tension properties of alcohol/water mixtures can vary dramatically with temperature and species concentrations in the liquid phase [14]. Justification can be attributed to temperature and species composition variations, which can create non-uniform variations of interfacial tension that result with liquid being pulled from regions of low surface tension toward regions with higher values. As non-uniformities continue to exist, steady surface tension-gradient liquid flow motion becomes established, also known as the Marangoni effect. This effect creates circulation cells, as shown in Fig. (2) [15], where steady flow of warm liquid is driven upwards by buoyancy toward the liquid-vapor interface. This warmer liquid is then convectively cooled by the surrounding vapor, which create both a temperature and resultant surface tension gradient toward cooler interfacial regions. These gradients then drive liquid flow across the interface and down toward the heated surface, where the process is repeated.

![Figure 1.2. Graphical representation of Marangoni circulation flow over a heated surface.](image)

According to Carey [15], cellular flow due to large surface-tension gradients, can cause subsequent wave deflections, where lower fluid becomes progressively heated as it passes toward the heated surface, while the vertex becomes cooler as it moves away. These deflections
can further increase the surface-tension gradients thus increasing the liquid flow to maintain a deflected configuration. This effect can be further intensified with multi-component mixtures where preferential evaporation of the more volatile component, occurs at the liquid-vapor interface [15]. The magnitude of surface interfacial surface tension is strongly dependent on a solution’s concentration. For multicomponent liquid mixtures, variation of surface tension with concentration usually dominates over temperature variation [20], which becomes a dominant factor in interfacial turbulence and consequential heat and mass transfer across the liquid vapor interface. Temperature and surface tension gradients can also cause vapor bubbles within a liquid solution to move toward a higher temperature region, since thermodynamically favorable motion in this direction would reduce the interfacial free energy of the bubble [15]. Marangoni phenomena can also affect the spreading of an evaporating binary mixture liquid droplet over a heated surface, where the thinner advancing layer of the liquid forms at the perimeter of the droplet. Since the perimeter of the droplet is thinner, preferential evaporation of the more volatile component takes place here causing motion of the droplet towards the thinner, higher surface-tension perimeter, reducing the superheat in this area.

1.4 Heat Pipes for Waste Heat Recovery

Many electrical power-producing processes produce large quantities of thermal energy, much of which is eventually exhausted to the environment. Recovering this waste heat represents one of the largest opportunities for reducing industrial energy consumption in the U.S. Since the majority of industrial waste heat sources have temperatures less than 1000ºF [22], it is particularly important, from an energy efficiency standpoint, that waste-heat recovery systems be developed within this operational temperature limit. Previously, industrial heat recovery systems were generally thought to be cost effective for temperatures of at least 1000ºF [22]. Today, increasing energy prices, greater industrial production and a more sensitive environment have necessitated waste heat recovery systems to improve heat transfer equipment within this temperature limit. According to Roos [22] heat recovery options can be broadly classified into three categories:

1. Recycling energy back into the process
2. Recovering energy for other secondary usage
3. Using it to generate electricity in combined heat and power systems

Waste heat recovery technologies can be classified as either passive or active. Passive heat recovery makes use of heat transfer devices, such as heat pipes, to transfer thermal energy from a higher temperature source to a lower temperature stream. Passive heat recovery technologies do not require significant mechanical or electrical input for their operation. Alternatively, active heat recovery technologies require energy input to facilitate waste heat transport.

Due to their high heat transport capability, with no additional input power requirements, heat pipes are used in passive heat exchanger applications. Major advantages of heat pipe devices,
compared to conventional heat exchangers, include nearly isothermal operation and can be assembled from conventional materials depending on their respective compatible working fluid. Heat pipe heat exchangers [10] can serve as compact waste heat recovery systems which require no power, low pressure drop and are simple to install [16]. In particular, gravity-assisted heat pipes have been proven to facilitate high heat transport capability through successive evaporation and condensation processes of an internal working fluid in a closed-loop system [21], as shown in Fig.(3). During operation, heat is transferred from the evaporator heated surface to an internal working fluid, typically under low pressure conditions. This liquid absorbs heat and undergoes phase change into a vapor that rises with buoyancy into a condenser. Here the vapor releases its latent heat and returns to a liquid state, passing back to the heated section by gravity.

![Figure 1.3. Gravity-assisted heat pipe with liquid flow toward heated evaporator section and vapor flow toward cooler condenser section.](image)

One application for waste recovery with heat pipe devices is with concentrating photovoltaic (PV) systems. PV semiconductor materials traditionally are expensive, leading to intrinsically high costs, which motivate the development of concentrating systems. Concentration of sunlight onto photovoltaic cells, with replacement of photovoltaic materials with less expensive concentrating mirrors or lenses, is seen as one method of lowering the cost of solar electricity [17]. However, only a fraction of the incoming sunlight striking the cell is converted into electrical energy. The remainder of the absorbed energy will be converted into thermal energy and may cause an internal p-n junction temperature to rise unless the heat is efficiently dissipated to the environment [110]. Therefore, heat pipe solar cell cooling can be an attractive technology for improving power production, while allowing for secondary energy use.

This study investigates the performance of a gravitationally-driven heat pipe with working fluids of water and low concentration 2-Propanol/water mixtures that exhibit strong Marangoni effects. Liquid charge optimization of the internal working fluid is conducted to determine an operational point capable of mitigating the dryout condition, while ensuring adequate condensing surface area, necessary for maximizing waste heat transport. An experimental analysis was performed for an optimal molar concentration ratio of 2-Propanol and pure water that maximizes the CHF condition, while minimizing the evaporator wall superheat. Broader analytical modeling of binary mixture pool boiling of low concentration alcohol-water mixtures was also conducted. It was found that methanol water mixtures were correlated well with nucleate boiling models by Fugita et. al. [18], while 2-Propanol/water mixtures found good correspondence with a modification of Kandlikar’s model [19]. This model included thermodynamic properties of a
nucleated bubble’s interface rather than bulk fluid properties, and a surface tension property correlation by McGillis [5]. This pool boiling model was next applied to a more comprehensive heat pipe model to characterize thermal transport within the evaporator section. The heat pipe model found good correspondence with experimental data for heat pipes in horizontal and 37° inclined orientations, with a 0.05M 2-Propanol/water working fluid. The attractive CHF condition of 114.8W/cm² found with the inclined heat pipe, with a 45% liquid charge, was applied to evaluate performance enhancement of single-junction silicon, and dual-junction GaInP/GaAs solar cells. A predictive solar cell model of a specified solar concentration ratio and cell temperature was developed based on input heat loads and measured temperatures of strip-heaters. This model used experimental data to simulate cell waste heat generation, which would be extracted by the heat pipe evaporator section. Results predicted significant electrical and thermal efficiency improvements of a combined PV/heat pipe configuration over a stand-alone PV system.
2.1 Introduction

Estimation of thermo-fluid properties at a specified temperature, pressure and composition is essential to perform accurate engineering calculations. From a practical standpoint, design of boiling heat transfer equipment depends on accurate property model predictions of published correlations, to avoid degradation and failure at elevated temperatures. Additionally, multicomponent evaporation, condensation and solidification phase-change processes require that physical properties depend on mixture composition. Current methods for predicting most binary mixture boiling phenomena require interpretations of single-component properties and respective mixture equilibrium thermodynamics. Thermophysical properties of binary mixtures are often nonlinear with composition, and can have significant impacts on heat and mass transport.

This chapter is intended to provide accurate means for predicting properties through a review of respective formulations for respective liquid and vapor species, of both single and binary-component fluids. Thermophysical properties were evaluated for single and binary-component mixtures. Here, modeling is performed for thermo-fluid properties based on physical theory and empirical correlations, where correlation accuracy, operational range and methodology were accounted for. The fluids examined were pure distilled water, isopropanol and methanol, which are common working fluids in a variety of heat transfer equipment [16]. Computer subroutines were built to accurately predict properties at any temperature, pressure and mixture composition.

2.2 Pure Component Properties

Pure component properties underlie much of the observed behavior of mixtures [14]. This section evaluates pure fluid property correlations developed by Polling et al. [14], as well as other investigators [3, 14, 25, 31, 44], who used experimental measurements to validate predictive models.

2.2.1 Pressure-Temperature

Correlations by Wagner [42, 43] have been found to correlate single-component temperature-pressure behavior well for liquids and vapor species of water and various alcohols. Wagner developed Eq. (2.1) using an elaborate statistical method, based on experimentally determined data. This pressure-temperature virial relationship was used in this analysis as the basis for determining other thermodynamic properties for both pure and mixture solutions.

\[
P_l = P_{c,l}exp \left[ \frac{1}{1-z} (a_l z + b_l z^{1.5} + c_l z^3 + d_l z^6) \right] = 1 - \frac{T}{T_c}, \quad z = 1 - \frac{T}{T_{c,l}} \quad (2.1)
\]
The indice \( i \) pertains to the single component species and \( T/T_{c,i} \) is the reduced temperature of the fluid. The virial constants in Eq. (2.1) for the three fluids analyzed in this investigation can be found in Table (2.1).

**Table 2.1.** Pressure/temperature pure fluid virial constants for water, 2-Propanol and methanol

<table>
<thead>
<tr>
<th>Fluid</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>( T_c ) [K]</th>
<th>( P_c ) [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>-7.764</td>
<td>1.45838</td>
<td>-2.77580</td>
<td>-1.14303</td>
<td>647.3</td>
<td>22.12x10^6</td>
</tr>
<tr>
<td>2-Propanol</td>
<td>-8.737</td>
<td>2.16240</td>
<td>-8.70778</td>
<td>4.77927</td>
<td>508.3</td>
<td>4.76x10^6</td>
</tr>
<tr>
<td>Methanol</td>
<td>-8.548</td>
<td>0.76982</td>
<td>-3.10850</td>
<td>1.54481</td>
<td>512.0</td>
<td>8.09x10^6</td>
</tr>
</tbody>
</table>

**2.2.2 Latent Heat**

When the vapor and liquid phases of a pure liquid are in equilibrium with each other, the relation between enthalpy, temperature and pressure can be characterized by the Clausius-Clapeyron equation:

\[
\frac{dT}{dp} = \frac{h_{lv}}{(RT^2/Z_{lv})} = \frac{h_{lv}}{T\Delta V_v}
\]  

(2.2)

where \( h_{lv} \) and \( Z_{lv} \) are the respective differences of enthalpy and compressibility factors of saturated vapor and liquid. According to McGillis [3] for fluids at subatmospheric pressures, \( \Delta V_v \) is accurately approximated by \( 1/\rho_v \). Through rearrangement, the latent heat can be calculated as:

\[
h_{lv} = T\left(\frac{\partial p}{\partial T}\right)_{\rho_v}
\]

(2.3)

where \( \partial P/\partial T \) is calculated by numerically differentiating Eq. (2.1).

**2.2.3 Heat Capacity**

The pure liquid predictions evaluated in this investigation were determined from definition of heat capacity in terms of the Prandtl number, thermal conductivity and the dynamic viscosity.

\[
C_{P,l} = \frac{Pr_lk_l}{\mu_l}
\]

(2.4)

Air was evaluated as an ideal gas, defined by the equation of state:

\[
C_p + C_v = R
\]

(2.5)
where values for the heat capacity at constant pressure \( (C_P) \) and the heat capacity at constant volume \( (C_v) \) were determined by measured values by Green and Perry [45].

### 2.2.4 Liquid Thermal Conductivity

Temperature dependent liquid thermal conductivities can be accurately determined using an expression developed by Baroncini et al. [3]. The following expression applies to liquid thermal conductivities for prescribed experimentally determined constants [14] as shown in Table (2.2).

\[
k_l = A + T(B + CT)
\]  

(2.6)

where temperature \( T \) is in [K] and thermal conductivity, \( k \) is in the units of [W/m-K].

**Table 2.2.** Thermal conductivity pure fluid virial constants for water, 2-Propanol and methanol

<table>
<thead>
<tr>
<th>Fluid</th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>( Z_{RA} )</th>
<th>( M ) [g/mol]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>-3.838x10^{-1}</td>
<td>5.254x10^{-3}</td>
<td>-6.369x10^{-5}</td>
<td>0.1438</td>
<td>18.02</td>
</tr>
<tr>
<td>2-Propanol</td>
<td>3.225x10^{-1}</td>
<td>-3.847x10^{-4}</td>
<td>2.211x10^{-7}</td>
<td>0.2493</td>
<td>60.01</td>
</tr>
<tr>
<td>Methanol</td>
<td>1.854x10^{-1}</td>
<td>-3.366x10^{-5}</td>
<td>-2.215x10^{-7}</td>
<td>0.1434</td>
<td>32.04</td>
</tr>
</tbody>
</table>

The thermal conductivity of air was determined using an experimentally determined correlation by Kannuik and Carman [40], which found good accuracy between -183 ºC and 218ºC.

\[
k_{v, \text{air}} = 5.75 \times 10^{-5}(1 + 0.00317T - 0.0000021T^2)
\]  

(2.7)

where temperature \( T \) is in [°C] and thermal conductivity \( k \) is in [cal/cm-s-°C].

### 2.2.5 Viscosity

Viscosity is a measure of the internal fluid friction, which opposes any dynamic change in fluid motion. The viscosities of liquids decrease with increasing temperature under isobaric or saturated liquid conditions [14]. Andrade [23] proposed a correlation for liquid viscosity, where \( ln(\mu_l) \) was linearly proportional to the reciprocal of the absolute temperature. Further experimentation by Alder [44] later provided a more accurate prediction with an extended form of this correlation:

\[
ln(\mu_l) = A + \frac{B}{T} + CT + DT^2
\]  

(2.8)

where the experimentally determined constants can be found for the studied fluids in Table (2.3).
### Table 2.3. Viscosity pure fluid virial constants for water, 2-Propanol and methanol

<table>
<thead>
<tr>
<th>Fluid</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>ω</th>
<th>κ</th>
<th>$V_c$ [cm$^3$/mol]</th>
<th>ζ [debyes]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>-24.71</td>
<td>4209</td>
<td>4.52x10$^{-2}$</td>
<td>-3.38x10$^{-5}$</td>
<td>0.344</td>
<td>0.076</td>
<td>57.1</td>
<td>1.8</td>
</tr>
<tr>
<td>2-Propanol</td>
<td>-8.114</td>
<td>2624</td>
<td></td>
<td></td>
<td>0.665</td>
<td>0.143</td>
<td>220</td>
<td>1.7</td>
</tr>
<tr>
<td>Methanol</td>
<td>-39.35</td>
<td>4826</td>
<td>1.09x10$^{-1}$</td>
<td>-1.13x10$^{-4}$</td>
<td>0.565</td>
<td>0.215</td>
<td>118</td>
<td>1.7</td>
</tr>
</tbody>
</table>

The vapor viscosities were determined using a theoretically accurate model by Chung et al. [35] where the authors developed their correlation using a shape parameter $F_c$ to account for molecular shapes and polarities of dilute gases. Their model is expressed as:

$$\mu_v = 40.785 \frac{F_c(MT)^{0.5}}{V_c^{2/3} \omega}$$

where $V_c$ is the critical volume of the fluid in [cm$^3$/mol], $T$ is in [K] and $F_c$ is determined from the expression:

$$F_c = 1 - 0.2756 \omega + 0.059035 \zeta_r^4 + \kappa$$

In Eqn. (2.11), $\omega$ is the acentric factor and $\kappa$ is a special correction parameter for highly polar substances such as alcohols and acids. The term $\zeta_r$ is a dimensionless dipole moment with the unit of [debyes].

$$\zeta_r = 131.3 \frac{\zeta}{(V_cF_c)^{1/2}}$$

To solve for the Lennard-Jones molecular collision parameter $\Omega_v$, Neufeld et al. [24] proposed empirically determined Eq. (2.12),

$$\Omega_v = [1.16(T^*)^{-0.15}] + 0.52[\exp(-0.77T^*)] + 2.16[\exp(-2.44T^*)], 0.3 \leq T^* \leq 100$$

where:

$$T^* = \frac{kT}{\epsilon}$$

#### 2.2.6 Density

In their boiling experiments of various pure alcohols and water, McGillis and Carey [3] found accurate vapor density correlation of their experimental data with that of the ideal gas law, Eq. (2.14)

$$\rho_v = \frac{p}{RT} M$$

The liquid density was determined using a modified form of the Racket equation [26], which provides the expression:

$$\rho_l = \frac{M}{V_b}$$
where \( V_b \) is the liquid molal volume expressed as:

\[
V_b = \frac{RT_c}{P_c} \left( Z^{1-\frac{(1-T_c)^2}{7}} \right)
\]  

(2.16)

where \( Z_{RA} \) is a constant determined by Rackett [26], provided in Table (2.2).

### 2.2.7 Surface Tension

Surface tension at the interface of a liquid and vapor decreases as the temperature is increased, and approaches zero at the critical point [27]. Most methods of predicting surface tension of alcohols and water are based on their critical temperature and their values at Standard Temperature and Pressure (STP) conditions [3], where \( T_0 \) is 298.15K and \( \sigma_0 \) values can be found in Table (2.4).

\[
\sigma = \sigma_0 \left[ \frac{T_c - T}{T_c - T_0} \right]
\]  

(2.17)

**Table 2.4. STP Surface Tension Constants**

<table>
<thead>
<tr>
<th>Fluid</th>
<th>( \sigma_0 ) [dynes/cm] at ( T=25^\circ C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>68</td>
</tr>
<tr>
<td>2-Propanol</td>
<td>24</td>
</tr>
<tr>
<td>Methanol</td>
<td>22</td>
</tr>
</tbody>
</table>

### 2.2.8 Prandtl Number

The Prandtl number is a dimensionless number that approximates the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity through the expression:

\[
Pr_l = \frac{\nu}{\alpha} = \frac{\mu c_{p,l}}{k_l}, \quad \alpha_l = \frac{k_l}{\rho_l c_{p,l}}
\]  

(2.18)

For low pressure boiling conditions with water, McGillis and Carey [5] experimentally showed that reduced property correlations generally provide more simplified expressions that do not require extensive property information and are less susceptible to calculation error. For water, this investigation used the Prandtl correlation by McGillis for reduced pressures:

\[
Pr_l = P_r^{-0.49} T_r^{1.51} (1 - T_r)^{-0.64} 10^{-0.771}
\]  

(2.19)
2.3 Mixture Properties

Unlike pure fluids, most mixture properties are not accurately determined by linear estimation methods of pure components [28]. To predict their intrinsic nature, phase equilibrium diagrams are used to determine conditions at which thermodynamically distinct phases can occur. Binary mixture equilibrium phase diagrams are computed based on species composition with liquid and vapor phases represented with respective bubble point and dew point curves. For this investigation, Fig. (2.1) and Fig. (2.2) present respective liquid-vapor phase equilibrium diagrams for 2-Propanol/water and Methanol/water mixtures:

**Fig. 2.1.** Liquid-vapor equilibrium phase diagram with mole-fraction-dependent surface tension distribution for 2-propanol/water at 101kPa.

**Fig. 2.2.** Liquid-vapor equilibrium phase diagram with mole-fraction-dependent surface tension distribution for methanol/water at 101kPa.
When equilibrium conditions exist for a mixture, respective values of temperature, pressure and chemical potential have to be equal between each of its respective components [28]. Mixture thermodynamic properties are dependent on activity coefficients of its respective components. According to McGillis [3], for isothermal low pressure conditions, the Gibbs-Duhem equation can be expressed in terms of activity coefficients:

$$\sum_i x_i d(ln\gamma_i) = 0$$  \hspace{1cm} (2.20)

In addition, for binary mixtures Prausnitz et al. [28] related activity coefficients to molar excess gibbs free energy using the Margules equation:

$$\bar{g}^E_i = [A_1 + 2(A_2 - A_1)x_i]x_j^2$$  \hspace{1cm} (2.21)

where the Gibbs energy can be defined as:

$$\bar{g}^E_i = RTln\gamma_i$$  \hspace{1cm} (2.22)

Combining Eqns. (2.20) through (2.22) the binary mixture activity coefficients can be determined as:

$$ln\gamma_1 = [A_{12} + 2(A_{21} - A_{12})x_1]x_2^2$$  \hspace{1cm} (2.14)

$$ln\gamma_2 = [A_{21} + 2(A_{12} - A_{21})x_2]x_1^2$$  \hspace{1cm} (2.24)

where species activity coefficient constants $A_{12}$ and $A_{21}$ are experimentally determined values [3], which are presented in Table (2.5) for the respective mixtures considered here.

<table>
<thead>
<tr>
<th>Solution Mixture</th>
<th>$A_{12}$</th>
<th>$A_{21}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-Propanol/Water</td>
<td>0.9768</td>
<td>2.4153</td>
</tr>
<tr>
<td>Methanol/Water</td>
<td>0.6552</td>
<td>0.8106</td>
</tr>
</tbody>
</table>

Table 2.5. Mixture Activity Coefficient Constants

Thermodynamic properties of binary mixtures are dependent on respective species fractions and concentrations. The liquid mass fraction can be characterized by Eq. (2.26), while the liquid mole fraction can be described by Eq. (2.26).

$$c_i = \frac{m_i}{m_i + m_j}$$  \hspace{1cm} (2.25)

$$x_i = \frac{-c_i M_j}{c_i(M_i - M_j) - M_i}$$  \hspace{1cm} (2.26)
Additionally, the liquid and vapor molar volume fractions can be described by Eq. (2.27) and Eq. (2.28) respectively.

\[ \nu_{l,i} = x_i \frac{v_{b,l}}{v_{b,m}} \]  \hspace{1cm} (2.27)

\[ \nu_{v,i} = y_i \frac{M_m \rho_{v,i}}{M_i \rho_{v,m}} \]  \hspace{1cm} (2.28)

Additionally, properties are also calculated using molar-weighted averages for both liquid and vapor mixtures. The molecular weight of a mixture can be predicted using Eq. (2.29) for vapor and Eq. (2.30) for a liquid.

\[ MW_{v,m} = y_1 MW_1 + (1 - y_1) MW_2 \]  \hspace{1cm} (2.29)

\[ MW_{l,m} = x_1 MW_1 + (1 - x_1) MW_2 \]  \hspace{1cm} (2.30)

### 2.3.1 Density

Similar to pure liquids, the ideal gas equation of state is often accurate for many binary mixtures [14]. The mixture vapor density is therefore described by the expression:

\[ \rho_{v,m} = \frac{p}{RT} M_{v,m} \]  \hspace{1cm} (2.31)

Virial equations have been shown to accurately predict the liquid density of polar liquids mixtures considered in this investigation [14]. Spencer et. al. [29] recommended the virial expression:

\[ \rho_{l,m} = \frac{MW_{l,m}}{V_{l,m}} \]  \hspace{1cm} (2.32)

where:

\[ V_{l,m} = R \left( \sum_i x_i \frac{T_{ci}}{p_{ci}} \right) Z_{RA,m}^{1+(1-T_r)^{0.2857}} \]  \hspace{1cm} (2.33)

In Eq. (2.34), the Rackett constant, \( Z_{RA,i} \) for mixtures is defined by Eq. (2.34). A listing of Rackett constants for various pure fluids can be found in literature by Poling [14]:

\[ Z_{RA,m} = \sum_i x_i Z_{RA,i} \]  \hspace{1cm} (2.34)

The reduced temperature parameter, \( T_r \) in Eq. (2.33) is defined by Spencer et. al. [29] as:

\[ T_r = \frac{T}{T_{cm}} \]  \hspace{1cm} (2.35)

where,

\[ T_{c,m} = \sum_i \sum_j \phi_i \phi_j T_{c,ij} \]  \hspace{1cm} (2.36)
\[
T_{c,ij} = (1 - k_{ij}) (T_{c,i} T_{c,j})^{1/2} 
\]

\[
1 - k_{ij} = \frac{\theta (V_{c,i} V_{c,j})^{1/2}}{(V_{c,i}^{1/3} + V_{c,j}^{1/3})^3} 
\]

\[
\phi_i = \frac{x_i V_{c,i}}{\Sigma_j x_j V_{c,j}} 
\]

2.3.2 Latent Heat

During thermal equilibrium conditions of a binary mixture, the latent heat is defined as the difference in enthalpy values of liquid and vapor [14], at the same concentration. Here, the latent heat equation defined earlier for pure liquids is extended for mixtures by the following relation:

\[
h_{lv,m} = \frac{T (\frac{\partial P}{\partial T})_m}{\rho_{lv,m}} 
\]

where \((\partial P/\partial T)_m\) is defined by differentiation of Eq. (1.1), for a specified mixture temperature.

2.3.3 Liquid Thermal Conductivity

According to Poling, traditional empirical thermal conductivity models of most organic liquid mixtures are typically less accurate than mole or weight fraction average models, however correlated deviations are often found to be small [14]. The correlation by Filippov and Novoselova [30] has been shown to have good accuracy with empirical data for many binary fluid mixtures:

\[
k_{l,m} = w_1 k_1 + w_2 k_2 - 0.72 w_1 w_2 (k_2 - k_1) 
\]

where, \(w_1\) and \(w_2\) are the weight fractions and \(k_1\) and \(k_2\) are the respective thermal conductivities of components 1 and 2. This equation has the restriction of \(k_2 \geq k_1\).

2.3.4 Mass Diffusivity

Mass diffusion occurs by the spread of particles through random motion from regions of higher concentration to that of lower concentration. Liquid diffusion occurs slower than in a gas phase, as molecules generally move slower resulting in lower diffusion coefficients. A highly regarded correlation for liquid diffusion is provided by Vignes [31] who was able to predict the diffusion of many alcohol liquid mixtures to good accuracy [14]:
\[ D_{12} = [(D_{12}^o)^{x_2}(D_{21}^o)^{x_1}] \alpha_1 \] (2.42)

where \( D_{12} \) and \( D_{21} \) are the mutual diffusion coefficients. These coefficients can be determined using a correlation developed by Tyn and Calus [32], where the binary mixture diffusion coefficients can be determined in terms of the component surface tension and mixture temperature.

\[ D_{12}^o = 8.93 \times 10^{-8} \frac{V_2^{0.267}}{V_1^{0.433}} \frac{T}{\mu_{l,2}} \left( \frac{\sigma_2}{\sigma_1} \right)^{0.15} \] (2.43)

\[ D_{21}^o = 8.93 \times 10^{-8} \frac{V_1^{0.267}}{V_2^{0.433}} \frac{T}{\mu_{l,1}} \left( \frac{\sigma_1}{\sigma_2} \right)^{0.15} \] (2.44)

where \( V_1 \) and \( V_2 \) are the respective component molar volumes. The activity coefficient \( \alpha \) in Eq. (2.42) can was determined using mixture empirical data for water-alcohol solutions [33].

### 2.3.5 Viscosity

Currently, most liquid-mixture viscosity correlations are based on general mixing rules relating pure component viscosities to their component concentrations [3]. Herráez and Belda [34] showed empirically that liquid-mixture viscosities can be computed to good accuracy for alcohol liquid mixtures. For this investigation, the liquid-mixture dynamic viscosities are approximated as the weight-fraction averages of the respective viscosities for the pure components.

\[ \mu_{l,m} = \bar{v}_{l,1} \mu_{l,1} + (1 - x_1) \bar{v}_{l,2} \] (2.45)

For vapor mixtures containing an alcohol, as molecular interactions increase, the separation between molecules decrease, enhancing their mutual force of attraction [40]. These molecular collisions also cause an interchange of momentum throughout the fluid that distributes the bulk motion velocity throughout the fluid. For a given fluid system, the bulk fluid velocity vector is high near the source of applied stress. Although, as molecules move away from the source, their respective velocities decrease due to random molecular collisions, facilitating a momentum interchange that causes the viscosity of a gas to increase [14].

Two prominent theoretical approaches are used to predict gas transport viscosity: the law of corresponding states described by Chung et. al. [35], or by an elementary kinetic theory of gases approach, described by Chapman et. al. [36]. To predict the gas-phase binary mixture viscosity, this investigation used a modified form of the kinetic theory approach by Wilke [37], which further simplified the prediction by Chapman and Enskog [36] by neglecting second-order effects. Eq. (2.46) by Wilke has been extensively tested [14], where comparisons of his model with various binary systems have provided good correspondence within 1% deviation.

\[ \mu_{v,m} = \sum_{i=1}^{n} \sum_{j=1}^{n} y_i y_j \phi_{ij} \] (2.46)
where:

\[
\phi_{12} = \frac{1 + (\mu_1/\mu_2)^{1/2}(M_2/M_1)^{1/4})^2}{[8(1 + M_1/M_2)]^{1/2}} \tag{2.47}
\]

\[
\phi_{21} = \frac{\mu_2 M_1}{\mu_1 M_2} \phi_{12} \tag{2.48}
\]

### 2.3.6 Surface Tension

Generally for aqueous organic mixtures at a given temperature, the surface tension in liquid mixtures is typically less than a mole-weighted average of pure component surface tension values, since the concentration at the liquid-vapor interface has been shown to differ from that of the bulk fluid [3]. For a variety of organic-water mixtures, the correlation by Tamura et. al. [38] has been found to match well with experimental observations [44]:

\[
\sigma_w^{1/4} = \psi_w \sigma_w^{1/4} + \psi_O \sigma_O^{1/4} \tag{2.49}
\]

where the \( \sigma_w \) and \( \sigma_O \) are the respective component surface tension values for pure water and the organic fluid. The superficial volume fraction \( \psi \) can be determined for pure water and the organic alcohol component using Eqs. (2.50) and (2.51) respectively:

\[
\psi_w = \frac{x_w v_w}{x_w v_w + x_O v_O} \tag{2.50}
\]

\[
\psi_O = \frac{x_O v_O}{x_w v_w + x_O v_O} \tag{2.51}
\]

where \( v_w \) and \( v_O \) are the molal volumes of pure water and the organic component respectively. However, improved accuracy with 2-Propanol/water mixtures was found experimentally [3] using a predictive model by Meissner and Michaels [39] for small liquid mole fractions of 2-propanol (x<0.01).

\[
\sigma_m = \sigma_w \left[ 1 - 0.411 \log \left( 1 + \frac{x_O}{0.0026} \right) \right] \tag{2.52}
\]

To provide a more comprehensive surface tension correlation, applicable to all concentrations of isopropanol-water binary mixtures, McGillis [3] prescribed a correlation, Eq. (2.53):

\[
\sigma_m = (\sigma_w - \sigma_O) \exp(-30\sqrt{x_O}) + \sigma_O \tag{2.53}
\]

which was used extensively throughout this investigation for 2-Propanol/water mixtures.
CHAPTER 3 – AN EXPERIMENTAL STUDY OF HEAT PIPE PERFORMANCE USING BINARY MIXTURE FLUIDS THAT EXHIBIT STRONG CONCENTRATION MARANGONI EFFECTS

3.1 Introduction

Heat pipe thermal devices are able to transport large quantities of thermal energy efficiently through phase change processes of an internal working fluid. Gravity-assisted heat pipes are designed with a condenser section that is elevated above an evaporator section to utilize a hydrostatic pressure gradient rather than a capillary pressure difference for liquid transport. These devices have been used in many industrial processes, as they are advantageous with their low cost and ease of manufacture. The operating performance of a gravity-assisted heat pipe may be enhanced using working fluids that are able to increase the CHF condition. The CHF is responsible for dryout when the pumping rate of a liquid flow structure is not sufficient to provide enough fluid to the evaporator section. Investigations [49, 51] have shown that optimizing the charge volume within a heat pipe is important for optimal heat pipe performance. Duncan et al. [51] showed experimentally that increasing the liquid charge from 10% to 50% by volume, increased the measured heat transport by approximately 81%. Strel’tsov [49] also showed that increasing liquid charge enhanced heat transport, but only until the condensate did not create an optimal uniform film. [52] showed that overfilling a heat pipe would eventually reduce the condensation rate as the condenser area decreased beyond that needed for an optimal liquid charge. However, condenser performance is also limited to the amount of exposed condenser area not submerged by water. Heat pipe liquid charging studies typically only consider heat pipes that utilize capillary pressure for liquid transport and not gravity-assisted heat pipes that require hydrostatic pressure differences to facilitate liquid flow. This investigation evaluates the performance of a 17.2cm x 6.9cm x 1cm gravity-assisted heat pipe with pure deionized water used as the working fluid, for three different liquid charge values: 35%, 45% and 70.

Boiling studies [12] have also shown that working fluid mixtures of water with a volatile fluid can have marked improvements on increasing the CHF. Further experimental research of heat transfer equipment has shown [50] the use of binary mixtures to improve performance more than that of a single working fluid. Kiatsiriroat et al. [50] investigated water mixtures containing ethanol and triethylene glycol (TEG) for thermosyphon applications. Their study found that mixtures with small amounts of TEG, with its high boiling point (278°C, at 1atm), was able to increase the CHF and improve thermosyphon performance. A study by McGillis and Carey [12] investigated the boiling performance of several alcohol/water binary mixtures and compared them to their respective pure components. Their study found that small additions of a volatile alcohol were able to increase the CHF condition above that for pure water, while higher concentrations decreased the CHF to that of pure alcohol. Results showed that small additions of 2-propanol reduced fluctuations in the evaporator wall temperature from that of pure water, possibly due to a reduction in the bubble departure size. It was found that although the average superheat did not vary significantly with concentration, the CHF did, reaching a maximum at approximately 0.05M 2-Propanol. They concluded that correlations based on effective subcooling [61] were not able to accurately predict the CHF condition for all alcohol/water
mixtures and the influence of the Marangoni effect was the principle cause for the CHF variation of boiling liquid mixtures. Their use of a surface tension gradient model was found to provide more consistent predictions of the CHF condition for all studied alcohol/water binary mixtures, better than those of a subcooling-based model.

In addition to liquid water, the work performed here investigates two low concentrations of 2-propanol in deionized water as working fluids. 0.05M and 0.2M concentrations will be investigated since low values of 2-propanol in water were found [3] to have high CHF values. This study however, only focuses on the thermal-fluid modeling of pure water. Based on the material properties and heat pipe geometry of this investigation, Stephan-Abdelsalam and Nusselt analysis correlations for nucleate pool boiling and film condensation will be considered in this study. In addition to nucleate pool boiling, the total evaporator heat transfer coefficient was determined by superposition, which also included thermal heat conduction through liquid water. An idealization of a flat meniscus and specific liquid water geometry within the evaporator channel was made based on experimental observations and contact angle analysis. An idealization of a flat meniscus and specific liquid water geometry within the evaporator channel was made based on experimental observations and contact angle analysis. Since a vacuum was used to reduce the operating pressure, this model development also approximates negligible air resistance. From experimentation, steady state temperatures within the evaporator and condenser sections were found to occur after approximately 5 minutes. Therefore modeling considered steady state conditions and adiabatic boundary conditions for all external surfaces, with the exceptions of the heat input and extraction areas. This investigation first performs an analysis to determine an optimal liquid charge, used to evaluate the CHF and performance of the gravity-assisted heat pipe with pure water and two binary mixtures as the working fluids.

3.2 Experimental Setup and Procedures

The experimental apparatus and corresponding schematic diagram for this investigation are shown in Fig. (3.1), where the orientation of the heat pipe remained horizontal throughout this study. The heat pipe was machined from of an aluminum alloy and had a rubber o-ring to seal the top plate to the bottom section. The top plate consisted of a machined evaporator heating element, which fit within the evaporator-portion of the bottom section.
As shown in section A-A of Fig. (3.2), the evaporator heating element had four 4mm horizontal cylindrical channels to facilitate evaporation. The heat pipe consisted of an axially-grooved structure with 8 channels that facilitated flow necessary to replenish liquid on the evaporator heated surface. The narrow 2mm liquid channels facilitated a hydrostatic pressure difference of 5mm between the condenser and adiabatic sections that enabled liquid flow.
Several k-type thermocouples, with a bead diameter of approximately 0.06cm were used for temperature measurements. Two were connected to the bottom surface of the condenser-section external fins, as well as two locations within the copper heating element, located centrally atop the evaporator section. Six thermocouples were also used to measure heat losses across the insulation that covered the external surfaces of all three sections. The thermocouples were connected in series to a control switch, AEM 4-channel/k-type thermocouple amplifier and a National Instruments data acquisition (DAQ) board. The DAQ would scan and record thermocouple measurements after the system had reached a thermal steady state level for each consecutive power input. The heating element was made of oxygen-free, high purity copper, which ensured the thermophysical properties of the element were defined to high accuracy. Two 1cm holes were machined into the top portion to accommodate two electrical heating cartridges, each 2.5cm long. The bottom portion of the heating element had a 3cm long finger machined with a cross-sectional area of 1.44cm². In this section, two holes were drilled to the center to hold thermocouple wires. The depth and diameter of the holes were 0.6cm and 0.08cm, respectively. The heating cartridges were connected to a 120V autotransformer that was used to control the heat input during the experiment. This device was connected in series to an AC power source.

To reduce contact resistance between the heating element and the heat pipe surface, a spring loaded system was constructed with a downward force of 7.4N. This was the force necessary to sustain a constant effective thermal conductivity across the heat pipe, void of a working fluid, at low power input levels (<10W). Additionally, an Omega high thermal conductivity paste was used to reduce contact resistance. Prior to filling the heat pipe, each respective working fluid was boiled to remove dissolved gases within the solution. After this process the fluid was sealed and allowed to cool to room temperature. The heat pipe was then filled to a predetermined liquid charge and sealed with an o-ring gasket and several bolts. To ensure surface conditions remained constant for each subsequent test, the heat pipe was thoroughly inspected for oxidation and cleaned appropriately prior to filling. To reduce air thermal resistance within the heat pipe, an aspirator vacuum system was used to lower the heat pipe evaporator operating pressure to approximately 10kPa, monitored with an Ashcroft vacuum gauge. Once this low pressure was reached, the vacuum was engaged for approximately 1.5 minutes more to remove residual air in the system, whereby two valves connected between the aspirator and pressure gauge were then closed. Since mixture vapor pressures, according to the equilibrium phase diagram [3] for
0.05M, 0.2M 2-propanol/water mixtures and pure water, are respectively 92kPa, 88kPa and 101kPa, these vapor pressures would exceed the system pressure displacing residual air out through the vacuum port during this time. Lab grade de-ionized water was used throughout the experiment for pure water and binary mixture test cases.

During each consecutive power input trial the auto transformer was tuned to a specified level and a multimeter was used to measure the corresponding electrical resistance within the cartridge heaters. Using Ohm’s law the power input level was set. To invoke a constant low condenser temperature, an ice bath was connected to a pumping system which was mounted just beneath the external cooling fins of the heat pipe condenser section. As power input was increased during the experiment, the system pressure with corresponding saturation temperature \( T_{sat,e} \), increased due to amplified evaporation. The evaporator heat transfer coefficient was determined from physical measurements of \( T_{in}, T_{sat,e} \) and \( \dot{Q}_{in} \) using Eq. (3.1).

\[
h_{w,e} = \frac{\dot{Q}_{in}}{T_{w,e} - T_{sat,e}} \tag{3.1}
\]

where

\[
T_{w,e} = T_{in} - \frac{\dot{Q}_{in}}{S_{evap}k_{Al}} \tag{3.2}
\]

In Eq. (3.2) \( S_{evap} \) was determined from a shape factor provided by Rohsenow [54], and \( k_{Al} \) was determined from Fourier conduction analysis with measurements of \( T_{in}, T_{cold} \) and \( \dot{Q}_{in} \) of a heat pipe void of a working fluid. Experimental uncertainties in the pressure and temperature measurements were ±0.4kPa (0.5%) and ±1.1°C (0.4%) respectively.

### 3.3 Liquid Charge Analysis

Prior work has shown that a major transport constraint is the dryout limitation, associated with the CHF, where the liquid pressure gradient across the heat pipe decreases below that of the vapor pressure drop [16, 47, 55], reducing liquid coverage along the heated section of the evaporator. The liquid pressure gradient at high input heat flux levels decreases because the volume of liquid water within the heat pipe reduces with increased evaporation. This will lessen the hydrostatic pressure difference between the condenser and evaporator sections, decreasing liquid transport. Experimental observation of a sudden increase in evaporator temperature and decrease in the overall effective thermal conductivity indicates that the CHF has been attained. This analysis investigates the CHF for three liquid charge values (defined as the volume of liquid per volume of internal chamber void space) to determine an optimal charge for experimentation with pure water and binary mixtures. The heat pipe orientation was held in a horizontal orientation during the duration of these liquid fill tests.

To mitigate the onset of dryout within heat pipes for high heat input loads, an increased liquid volume of a working fluid can improve the CHF condition. Andros and Florschuetz [47], Cohen and Bayley [48] as well as Strel’tsov [49] presented experimental data that showed that
increasing the fill ratio increased the critical heat flux for a given operational temperature. Heat pipe evaporator input transport heat flux was evaluated for heat input levels between 1 to 140W, with respective liquid charge levels of 35%, 45% and 70%. The results in Fig. (3.3), for pure water as the working fluid, with added trend lines, show that increasing the liquid charge level facilitates the CHF condition (as indicated by the sudden increase in superheat values) to occur at higher evaporator input heat flux levels.

![Figure 3.3](image)

**Figure 3.3.** Evaporator surface superheat vs. evaporator input heat flux for pure water with fill ratios of 35%, 45% and 70%. The input transport heat flux is based on the heater surface area.

A liquid charge increase to 70% from 35% and 40%, increased the CHF condition by 75% and 17% respectively. Similar results were found in experimental work by Strel’tsov [49], Duncan [51], and Shiraishi et al. [61], where the critical heat flux also rose by increasing the liquid charge. Higher CHF values can be attributed to hydrostatic pressure increases with greater charge levels, which would likely improve liquid flow to the evaporator section and reduce the superheat and the onset of dryout. Subsequent experimental work and analytical model development considered the liquid charge of 70%.

### 3.4 Analytical Model

The gravity-assisted heat pipe evaluated in this experiment transfers thermal energy from a heating element positioned atop the evaporator section to its condenser section via an internal cavity. This heat transfer process for the heat pipe is shown in Fig. (3.2), where the bottom portion of the condenser section was cooled via an ice water bath, held at an observed constant temperature of 5°C. The condenser surface temperature $T_{\text{cold}}$ was assumed constant at 5°C and all other external heat pipe surfaces, with the exceptions of those exposed to direct heat input and output, were considered adiabatic. Heat loss to the environment was mitigated by using a 3cm thick Unifrax thermal blanket which covered the external surface of the heat pipe and copper
heating element. Heat loss was determined experimentally from thermal heat conduction analysis and was found to be less than 4% of the total heat input, validating the adiabatic assumption.

Heat transfer from the evaporator to the condenser section was facilitated by liquid and vapor transport within the inner chamber and by heat conduction through the aluminum casing,

\[ \dot{Q}_{\text{tot}} = \dot{Q}_{l/v} + \dot{Q}_{\text{al}} \]  

(3.3)

Consistent with our experimental setup \( \dot{Q}_{\text{tot}} \) and \( T_{\text{cold}} \) are specified controlled parameters. The following model is then used to derive \( T_{w,c} \) and evaluate the performance of this heat pipe design. Since \( T_{w,e} \) depends on both \( \dot{Q}_{l/v} \) and \( \dot{Q}_{\text{al}} \) in the heat transfer process and the relation between \( \dot{Q}_{l/v} \) and \( \dot{Q}_{\text{al}} \) is unknown, an iterative scheme is required to solve for \( T_{w,e} \). This procedure is described below, where parameters are iterated for each \( i \) superscript value:

1: \( \dot{Q}_{l/v}^0 \) was initially set as \( \dot{Q}_{\text{tot}} \).
2: Heat transfer analysis was performed for liquid/vapor transport within the inner chamber, and \( T_{w,e}^0 \) was determined.
3: With calculated values for \( T_{w,e}^0 \) and \( T_{w,e} \), heat conduction analysis was performed to obtain \( \dot{Q}_{\text{al}}^0 \).
4: \( \dot{Q}_{l/v}^{i+1} \) was determined based on the difference between \( \dot{Q}_{\text{tot}} - \dot{Q}_{\text{al}}^i \).
5: If the difference between \( \dot{Q}_{l/v}^{i+1} \) with \( \dot{Q}_{l/v}^i \) was less than a specified tolerance of 0.1W, the corresponding iterative value of \( T_{w,e} \) was output. Otherwise steps 3-5 were repeated until \( \dot{Q}_{l/v} \) converged.

The heat transfer analysis for liquid/vapor transport within the inner chamber in step 2 was carried out for the 3 sections: the condenser section, the evaporator section, and the flow channels within the adiabatic section. For the condenser section, using Fourier conduction analysis, the lower internal wall condenser temperature \( T_{w,c} \), which is assumed to be approximately constant through the very small and narrow internal condenser fins, can be determined from the external condenser temperature \( T_{\text{cold}} \) and material thermal resistance. Experimental Sessile drop contact angle analysis, of liquid water droplets on the surface of the evaporator heated section and condenser section, revealed a value of approximately 81°±3°, which is slightly wetting thus permitting the use of the Nusselt analysis [15] heat transfer correlation for film condensation,

\[ F_{\text{cond}} = 0.943 \left( \frac{k_l}{l_h} \right) \left[ \rho_l (\rho_l - \rho_v) g h_v^* l_h^3 \right]^{1/4} \]

(3.4)

\[ h_v^* = h_v \left( 1 + 0.68 \left[ \frac{c_p \left( T_{\text{sat,c}} - T_{w,c} \right)}{h_v} \right] \right) \]

(3.5)

where \( l_h \) is the vertical length along the condenser surface, exposed to condensing vapor. Using a more detailed analysis by Rosenhow [64] that accounts for the effects of subcooling and energy convection along the vertical condenser surface, Eq. (3.5) was used with Eq. (3.4). Using the
heat transfer correlation in Eq. (3.6) the condenser saturation temperature $T_{sat,c}$ and $h_{cond}$ can be determined by iteration.

$$\dot{Q}_{out} = h_{cond} A_{cond} (T_{sat,cond} - T_{w,cond})$$  \hspace{1cm} (3.6)

In Eq. (3.6) $h_{cond}$ and $A_{cond}$ correspond to the unsubmerged portions of the internal condenser channels. The mass flow rate between the evaporator and condenser section was calculated as the input power divided by the latent heat, where the idealization $Q_{tot} = \dot{Q}_{out}$ was made. Further idealizations were made such that liquid and vapor mass flow rates, as well as heat input and output values were approximately equal.

$$\dot{m} = \frac{\dot{Q}_{out}}{h_{lv}}$$  \hspace{1cm} (3.7)

Liquid flow is modeled from the condenser section to the interface between the adiabatic and evaporator sections. To determine the evaporator saturation pressure, Bernoulli’s equation was used to characterize liquid flow driven by a hydrostatic pressure difference between liquid height levels in the condenser section $Z_{cond}$ and adiabatic section $Z_{adia}$. For this analytical model, these parameter values were determined from initial system measurements.

$$\frac{P_{sat,e}}{\rho g} + z_{adia} = \frac{P_{sat,e}}{\rho g} + z_{cond} + H_i$$  \hspace{1cm} (3.8)

where

$$H_i = f \frac{L}{D_h} \frac{V^2}{2g}, \quad f = \frac{64}{Re}, \quad Re = \frac{\rho V D_h}{\mu_v}, \quad V = \frac{\dot{m}}{\rho A}, D_h = \frac{4A}{P}$$

$P_{sat,e}$ and $P_{sat,c}$ are saturation pressure values corresponding to the saturation temperatures at their respective locations. Within the evaporator section, heat transfer from the heated surface of the cylindrical vapor channels is due to bubble nucleation and conduction through the fluid to the liquid/vapor interface. The liquid-vapor interface within the cylindrical evaporator channels was idealized to be flat such that the liquid pressure is equal to the saturated vapor. This assumption was verified through liquid height measurements of pure water within the evaporator channels for the 70% liquid charge case. Observations revealed a flat meniscus for water that occupied approximately half the channel.

Heat conduction through the liquid water within the evaporator channels was performed using the shape factor $S_{evap}$, found by numerically solving the Laplace equation with two isothermal boundary conditions. In Fig. (3.4), the temperature at the bottom face of the evaporator channels is $T_{w,e}$ and the temperature of the liquid-vapor interface is $T_{sat,e}$.
Figure 3.4. Finite difference analysis diagram for a 3-dimensional half cylindrical channel shape factor.

Using evaporator geometrical parameters and material properties, the value of $S_{\text{evap}}$ was found to be 0.31. The heat transferred through the water by conduction in the evaporator was then calculated using Eq. (3.9).

$$\dot{Q}_t = k_i S_{\text{evap}} (T_{w,e} - T_{\text{sat},e}) = h_i A_{\text{evap}} (T_{w,e} - T_{\text{sat},e})$$  \hspace{1cm} (3.9)

The interfacial temperature between the copper heating element and the external evaporator surface was determined from thermal heat conduction analysis using a shape factor provided by Rohsenow [54],

$$S_e = \frac{2\pi l}{\ln \left( \frac{2l}{\pi D} \sinh \left( \frac{\pi \delta}{l} \right) \right)}$$  \hspace{1cm} (3.10)

where $l$ is half the length of the evaporator channels, $D$ is the channel diameter, $L$ is the channel length and $\delta$ is the distance between the channel and the external isothermal surface, determined by measurement. Using the Stephan and Abdelsalam [20] mathematical heat transfer correlation for nucleate pool boiling we can determine the evaporator heat flux for water as the working fluid.

$$\frac{\dot{Q}_{NB}^* D_b}{k_i (T_w - T_{\text{sat}})} = 2.46 \times 10^4 \left( \frac{\dot{Q}_{NB}^* D_b}{k_i T_{\text{sat}}} \right)^{0.673} \left( \frac{h_b D_b^2}{\alpha_i^2} \right)^{-1.58} \left( \frac{c_p T_{\text{sat}} D_b^2}{\alpha_i^2} \right)^{1.26} \left( \frac{\rho_l - \rho_v}{\rho_l} \right)^{5.22}$$  \hspace{1cm} (3.11)

To determine the evaporator wall temperature a second iteration scheme was performed by combining Eqns. (3.9) through (3.11). To perform heat conduction analysis in step 3 of the iteration procedure, the thermal resistance of the aluminum casing was determined using a three-dimensional finite difference analysis with adiabatic boundary conditions set for all external heat pipe surfaces, with the exceptions of the heat input and removal areas. The heat input area was defined as the cross-sectional area of the heating element and the extraction area was set as the area of the internal condenser section. The numerically calculated thermal resistance of the casing was 1.95K/W.
3.5 Pure Water Analysis

Experimental heat pipe performance results with water as the working fluid revealed that a superheat increase of 10°C would provide an average increase of 90% in transport heat flux across the device. Fig. (3.5) presents experimental data based on heat transfer contributions of internal liquid/vapor phase change and thermal conduction due to the aluminum casing.

![Graph of Pure Water experimental data and comprehensive analytical model comparison](image)

**Figure 3.5.** Experimental data and comprehensive analytical model comparison of evaporator vaporization transport heat flux for pure water with contribution due to the aluminum casing.

A comprehensive theoretical model is also included that accounts for these two heat transfer processes. The contributions due to conduction through the aluminum casing were found to be significant given the proportionally thick walls that surrounded the inner chamber. As the superheat increased, this contribution decreased up to 50% with increased evaporation. These results suggest that tandem utility of liquid/vapor transport with heat conduction can be used to improve the performance over a wide range of input heat flux levels.

Experiments were performed with the heat pipe void of liquid water to determine the thermal resistance of its casing material. Under the same experimental conditions as the studies with working fluids, an average experimental thermal resistance of 1.67K/W was found with an average standard deviation of 9.5x10^{-2} between trials. Using thermal heat conduction analysis, this value was used to determine the heat flux due to liquid/vapor transport only. Fig. (3.6) presents boiling performance experimental data of liquid/vapor transport, with the model prediction up to the CHF of 53.5W/cm². The model, which only predicts nucleate boiling up to the CHF condition found good correspondence with the experimental data, with an rms deviation...
of 6%. Experimental results show an average increase of 43% from a superheat of 26.2°C to 48.5°C.

![Figure 3.6. Evaporator vaporization transport heat flux experimental data and theoretical model for pure water. The evaporator vaporization heat flux is based on the evaporator passages wetted surface area.](image)

The heat transfer coefficient for the evaporator section was determined as the superposition of nucleate pool boiling and heat conduction through liquid water within the evaporator channels, modeled as half-cylinders. Fig. (3.7) results reveal that heat transfer contributions due to conduction were relatively constant and quite low compared to nucleate pool boiling heat transfer. These low values however are based on the idealization of a flat meniscus and fluid height equal to half the evaporator channel, based on actual observations performed prior to experimentation. Since the vacuum system was only connected to the evaporator section of the heat pipe, condenser operating pressures and temperatures were not measured during experimentation. Analytical modeling however predicted an average saturation temperature of 4.0°C between the condenser and evaporator.
3.6 Binary Mixture Experimental Analysis

Studies by Reddy et al. [13] have shown that binary mixtures are able to enhance the CHF condition, especially for a 2-propanol/water mixture in low concentration [50]. Our study investigates the performance of a gravity-assisted heat pipe by measuring the CHF value for two different concentration mixtures of 2-propanol and pure water. Fig. (3.8) presents device performance results for low concentration levels of 2-propanol and pure water. This data comparison reveals that low concentration levels of 2-propanol reduced the superheat, from that of pure water by as much as 55% and 50% for 0.05M and 0.2M respectively. This difference was found to grow significantly for low heat flux values. Similar results were also found in a previous study with 2-propanol/water mixtures [58], where the authors showed a decrease in superheat with decreasing alcohol concentration. Justification for this superheat reduction can be attributed to an increase in the critical radius of curvature of nucleated bubbles as the 2-propanol/water mixture surface tension increases with decreasing concentration [3].
Figure 3.8. Performance experimental results for a liquid charge of 70% and for 2-Propanol/water binary mixtures of 0.2M, 0.05M as and pure water.

As prescribed by Eq. (3.12), an increase in the critical radius of curvature $r^*$ would reduce the number of active nucleation sites and the wall superheat, which is further explained by Carey [3].

\begin{equation}
    r^* = \frac{2\sigma T_{sat,e} v_{lv}}{h_{lv} [T_{w,e} - T_{sat,e}]} \tag{3.12}
\end{equation}

These results also show a significant increase in the evaporator outer surface superheat at approximately 80°C for 0.2M 2-propanol and pure water. This increase occurs as the CHF condition is reached within the evaporator section.

Experimental evaporator vaporization heat flux results are shown in Fig. (3.9). In comparison with pure water, the CHF for 0.05M and 0.2M concentrations increased by as much as 52% and 21% respectively.
Figure 3.9. Experimental binary mixture transport heat flux, for a liquid charge of 70% and for 2-Propanol/water binary mixtures of 0.2M, 0.05M and for pure water. Critical heat flux reached in each case.

Similar trends were found by McGillis and Carey [3], who demonstrated through alcohol-water mixture experiments that the CHF can be greatly enhanced by the Marangoni effect. In the boiling of binary mixtures, preferential evaporation of the more volatile 2-propanol component intensifies near the heated surface. This facilitates a surface tension gradient with respect to its concentration distribution that increases towards the liquid-vapor interface, inducing Marangoni convection [58]. Hovestreudt [59] postulated that for positive mixtures, where the volatile component has a lower surface tension, the CHF increases as Marangoni convection inhibits coalescence around incipient bubbles. Furthermore, Tsukinari et. al. [60] showed that reduced concentrations of alcohol in water can significantly enhance the surface tension of the mixture and the Marangoni number up to approximately 0.05M. This confirms the larger CHF obtained with 0.05M from that of pure water and larger 0.2M concentration where Marangoni convection heat transfer would be less.

Model development is currently underway to characterize these Marangoni effects and their impact on the CHF for the two concentrations of 2-propanol/water mixtures. Currently, nearly flat meniscus observations have been made for the two low concentrations. However, sessile drop experimentation is being conducted to determine the contact angles of the two concentrations to determine their wettability and influence on boiling heat transfer within the evaporator section.
3.7 Summary

An analytical model was developed for heat transport through a gravity-assisted heat pipe with water as the working fluid. Good correspondence was found between the liquid/vapor transport heat flux experimental data and the analytical prediction. Experimental data was found to agree with parametric trends within an rms deviation of 6%. The analytical model included the superposition contributions of nucleate pool boiling and thermal heat conduction through the liquid water within the evaporator channels to determine the evaporator heat transfer coefficient. The heat conduction contribution was based on the numerical calculation of a shape factor for a half cylinder. The results of the liquid charge study revealed that increased liquid charge levels improved the CHF by as much as 69% from a charge of 35% to 70%. Heat conduction through the aluminum casing was found to be a significant component of the total heat transfer across the heat pipe due to thick walls that surrounded the inner chamber. However, its impact was shown to decrease with increasing superheat as the evaporation rate increased. Pure water experiments revealed a CHF of 53.6W/cm² for a superheat of 48.5°C. Experimental analysis was also conducted for water mixtures with 2-propanol as an additive. Investigation of 0.05M and 0.2M 2-Propanol molar fractions found a CHF increase of 52% and 45% from the case of pure water respectively. Decreasing the concentration of 2-propanol from 0.2M to 0.05M also improved the evaporator heat transfer coefficient by 11% at the CHF condition. It is proposed that Marangoni effects become stronger with decreasing 2-propanol concentrations that could allow interfacial surface tension gradients to enhance flow and improve the performance of the gravity-assisted heat pipe.
CHAPTER 4 – PREDICTION OF BINARY MIXTURE BOILING HEAT TRANSFER IN SYSTEMS WITH STRONG MARANGONI EFFECTS

4.1 Introduction

Binary mixtures of alcohol and water play an important role in the performance of heat transfer equipment. A heat pipe study by Armijo and Carey [62] demonstrated the use of low concentrations of 2-propanol/water mixtures, when compared to pure water, can increase the critical heat flux (CHF) by as much as 52%, while lowering the evaporator wall superheat. The Marangoni effect was further explored by McGillis and Carey [3] who investigated the boiling performance of several alcohol/water binary mixtures and compared them to their respective pure components. Their study found that small additions of a volatile alcohol were able to increase the CHF condition above that for pure water, while higher concentrations decreased the CHF to that of the pure alcohol. Results showed that small additions of 2-propanol reduced fluctuations in the evaporator wall temperature from that of pure water, possibly due to a reduction in the bubble departure size. It was found that although the average superheat did not vary significantly with concentration, the CHF did, reaching a maximum at approximately 0.05M 2-Propanol. They concluded that correlations based on effective subcooling [3] were not able to accurately predict the CHF condition for all alcohol/water mixtures and the influence of the Marangoni effect was the principle cause for the CHF variation of boiling liquid mixtures.

Carey [15] explained that the stability mechanism and cellular flow in real systems resulting from the Marangoni effect can be significantly altered by the presence of small amounts of surface-active contaminants that tend to flow from regions of high to low temperature. Concurrently, the Marangoni effect also causes vapor bubbles in a liquid medium with an imposed temperature gradient to move toward the high temperature region as the liquid flow that causes this motion is driven by interfacial surface tension gradients.

A number of investigators (e.g., Stephan and Körner [63], Fujita et al. [65], Kandlikar, [19], Jungnickel et. al. [67], Calus and Leonidopoulus [56]) have developed correlations for binary mixtures containing a solute and a volatile solvent. Table (4.1) presents three correlations selected for comparison study with subatmospheric experimental data by McGillis [11] for binary mixtures of 2-propanol/water and methanol/water.

The pioneering work of Stephan and Körner [63] developed a heat transfer coefficient correlation based on the absolute value of the difference between a mixture’s more volatile component vapor and liquid molar concentrations. These authors considered the relationship between the temperature difference driving force required to obtain the same heat flux with a binary mixture, and with each of its two pure components. They provided experimental evidence that the wall superheat for a mixture varies linearly with the concentration if the liquid-vapor equilibrium relationship approaches ideality when the mass transfer driving force is very small. $A_0$ is a binary constant determined empirically at atmospheric pressure. The authors stated that $A_0$ can be regarded as a constant for the range of mixture concentrations exhibiting ideal liquid-
vapor equilibrium behavior. This correlation considered bulk thermodynamic properties and assumed complete miscibility between pure liquids.

Fujita et al. [65] developed an empirical correlation based on a previous model by Fujita and Tsutsui [18] by replacing the heat flux term with one that included an ideal wall superheat, $\Delta T_{id}$. They compared their correlation with literature data sets and their own experimental mixture results of several alcohol/water mixtures at atmospheric pressure. Although their correlation predicted alcohol/water mixture data to within less than 20%, larger deviations were observed with refrigerant mixture data from Jungnickel et al. [67] for R-22/R-12 and R23/R-12 systems. Component 1 within the forcing term in Eq. (4.2) corresponds to the more volatile component with a lower boiling point than the solute component 2.

Rather than using an ideal weighted-average coefficient, a predictive model was developed by Kandlikar [19], which is based on an average pseudo single-component (PSC) coefficient which accounts for property effects in binary mixtures.

$$h_{psc} = h_{avg} \left( \frac{T_{sat,m}}{T_{sat,avg}} \right)^{-0.674} \left( \frac{\Delta h_{lv,m}}{\Delta h_{lv,avg}} \right)^{0.371} \left( \frac{\rho_{v,m}}{\rho_{v,avg}} \right)^{0.371} \left( \frac{\sigma_{m}}{\sigma_{avg}} \right)^{-0.317} \left( \frac{k_{lm}}{k_{Lavg}} \right)^{0.284} \quad (4.1)$$

To account for real mixture properties, the average heat transfer coefficient is corrected with the form,

$$h_{avg} = 0.5 \left[ x_1 h_{l,1} + (1-x_1) h_{l,2} + \left( \frac{x_1}{h_{l,1}} + \frac{(1-x_1)}{h_{l,2}} \right)^{-1} \right] \quad (4.2)$$

Kandlikar’s theoretical model, as outlined by Carey [15], was developed to calculate equilibrium concentrations at the interface of the bubble as it approaches the asymptotic growth condition. In his work, Kandlikar presented comparisons of his correlation with those of Fujita et. al. [65] and Calus and Leonidopolous [68] for experimental data of various binary mixtures. His binary mixture heat transfer coefficient correlation was derived solely on a theoretical basis by applying a one-dimensional model for heat and mass transfer at the bubble liquid-vapor interface under diffusion-controlled growth conditions. Kandlikar also suggested that the correction factor for mass diffusion effects, $F_D$, varies for different volatility parameter ranges.

The three correlations used here were originally investigated for pool boiling of binary mixtures under atmospheric conditions with thermodynamic properties evaluated based on that of the bulk fluid. However during nucleate pool boiling, as the system pressure or temperature are reduced below atmospheric conditions, boiling and dew point temperatures decrease, affecting other system thermodynamic properties. Additionally, most nucleate pool boiling binary mixture correlations have traditionally evaluated thermodynamic properties based on bulk fluid conditions. However, as shown by Kandlikar [19] the volatile component mole fraction can vary at the liquid-vapor interface of a bubble from that of the bulk fluid.

In most heat pipes, boiling of the working fluid occurs under low pressure conditions. This investigation first analyzes model predictions of all three correlations with bulk fluid properties for subatmospheric pressure data, for both weak (0.1M) and strong (<0.1M) Marangoni effect.
concentrations where large surface tension variation are evident. It is not always clear that evaluation of the bulk mixture properties will allow the correlations to effectively predict mixture fluids where strong Marangoni effects are present. Therefore for these same concentrations, this study then investigates correlation predictions using boundary layer film properties to improve accuracy between the models and experimental data. These are determined by an average of the properties at the liquid-vapor interface and those of the bulk fluid. An idealization is made that the temperature at the liquid-vapor interface is approximately equal to the vapor temperature inside the bubble.

Traditionally, an ideal HTC, which often is based on an averaging technique of mole or mass fractions of pure component fluids, is used to represent the heat transfer coefficient in the absence of mass diffusion effects. However, since pool boiling heat transfer can be highly non-ideal, this study also employs Kandlikar’s PCS coefficient in place of the mole-weighted ideal HTC for both Stephan Kömer and Fugita et al. models, using mixture surface tension correlations Eq. (4.14) for 2-propanol/water and Eq. (4.15) for methanol/water mixtures in place of $\sigma_m$.

4.2 Mixture Theory and Discussion

For correlations (4.3) and (4.4) listed in Table (4.1), the ideal superheat $\Delta T_{id}$, and ideal HTC $h_{l,id}$ were calculated based on activity coefficients since the mixture fluids in this study are non-ideal. As prescribed in chapter 2, the activity coefficients $\gamma_1$ and $\gamma_2$ were determined by equating the molar excess Gibbs energy with the two-parameter Margules equation [3].

**Table 4.1** Nucleate pool boiling heat transfer coefficient correlations for binary mixtures.

<table>
<thead>
<tr>
<th>Author(s) and Year</th>
<th>Correlation Scheme</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stephan and Kömer [63]</td>
<td>$\frac{h}{h_{l,id}} = \frac{1}{1 + \gamma_1 - \gamma_2 (A_2 (0.88 + 0.12))}$</td>
<td>(4.3) $A_2$ is specific to particular mixtures in the range (0.43-0.56), $P$ is in bar $h_{l,id} =$ Reciprocal mole-fraction average</td>
</tr>
<tr>
<td>Y. Fujita et. AL [65]</td>
<td>$\frac{h}{h_{l,id}} = \left[1 + \left(1 - \exp \left(\frac{-2.8}{\Delta T_{id}} \left(\frac{T_{id}}{T_{id,1}} - \frac{T_{id}}{T_{id,2}}\right)\right)\right)^{-1}\right.$</td>
<td>(4.4) $\Delta T_{id} =$ Reciprocal mole-fraction $h_{l,id} =$ Reciprocal mole-fraction average</td>
</tr>
<tr>
<td>S.G. Kandlikar [19]</td>
<td>$\frac{h}{h_{\text{PC5}}} = \begin{cases} \frac{1}{1 - 6 \Psi_1} &amp; \text{for } \Psi_1 \leq 0.03 \ \frac{0.67}{1 + \left(\frac{\alpha_2}{\alpha_{1,2}}\right)^2 \left(\frac{T_{id}}{T_{id,1}} \frac{T_{id,2}}{T_{id}}\right)} &amp; \text{for } 0.03 &lt; \Psi_1 \leq 0.2 \end{cases}$</td>
<td>(4.5) $h_{\text{PC5}}$ = Psuedo single phase coefficient $\Psi_1 =$ Volatility parameter $V_1 =$ for $\Psi_1 &gt; 0.2$</td>
</tr>
</tbody>
</table>
For non-ideal mixtures, the vapor pressure is related to the vapor mole fraction by:

\[ P = \frac{x_i \gamma_i P_{\text{sat},i}}{y_i} \]  

(4.7)

Using the fact that \( y_1 + y_2 = 1 \) and \( x_1 + x_2 = 1 \), the saturation pressure of the mixture can be expressed as:

\[ P = x_2 \gamma_2 P_{\text{sat},2} + x_1 \gamma_1 P_{\text{sat},1} \]  

(4.8)

Equating together equations (4.7) and (4.8) provides equilibrium concentrations in the liquid and vapor for any system pressure and temperature. Using these calculated liquid and vapor mole fractions, equilibrium phase diagrams are presented by Figs. (2.1) and (2.2) for 2-propanol/water and methanol/water mixtures respectively.

To determine the system HTC, an ideal temperature difference was defined by a mole-fraction-weighted average of the superheats for pure component fluids at the same system temperature and pressure, and for a specified heat flux,

\[ \Delta T_{id} = \Delta T_1 x_1 + \Delta T_2 (1 - x_1) \]  

(4.9)

where \( \Delta T_1 \) and \( \Delta T_2 \) are determined from the Stephan-Abdelsalam [20] mathematical correlation for pure fluids.

\[ \frac{\dot{Q}_{\text{NB}}^* d_b}{k_i (T_u - T_{\text{sat}})} = 0.23 \left( \frac{\dot{Q}_{\text{NB}}^* d_b}{k_i T_{\text{sat}}} \right)^{0.643} \left( \frac{h_i d_b^2}{\alpha_i^2} \right)^{0.371} \left( \frac{\rho_i - \rho_v}{\rho_i} \right)^{-1.73} \left( \frac{\alpha_i^2 \rho_i}{\sigma_j d_b} \right)^{0.35} \]  

(4.10)

The bubble departure diameter from Eq. (4.11) is defined by the Borishansky et al. [70] correlation:

\[ d_b \left( \frac{\dot{P}_e M}{k_B T_c} \right)^{1/3} = 5.0 \times 10^5 \left( \frac{P}{P_c} \right)^{-0.46} \]  

(4.11)

The ideal HTC is determined by the reciprocal mole fraction-weighted-average of the pure components, where \( x_j \) is the mass fraction of the volatile component.

\[ h_{i,id} = \left( \frac{x_i}{h_i} + \frac{(1 - x_i)}{h_2} \right)^{-1} \]  

(4.12)

The mixture HTC is then determined based on the product of the ideal HTC, \( h_{i,id} \) and a correction factor, \( F_D \) which accounts for mass diffusion effects.

\[ h_i = h_{i,id} F_D \]  

(4.13)
For low concentration conditions of the mixtures considered here, properties such as surface tension, vapor density and saturation temperature may exhibit non-ideal behavior. Through experimentation of various water mixtures containing methanol as well as 1 and 2-propanol, Vázquez et al. [69] presented empirical evidence indicating the non-ideal behavior of surface tension properties for low concentration systems. They further showed that deviation from ideality increases with the length of an organic molecule and decreases with rising system temperature. The non-ideal nature of mixture surface tension properties can be further reflected in the surface tension curves in Figs. (2.1) and (2.2). In this investigation the mixture surface tension for the 2-propanol/water mixture was evaluated based on Eq. (4.14) while the methanol/water mixture was evaluated based on Eq. (4.15), as outlined by McGillis [3].

\[
\sigma_m = (\sigma_2 - \sigma_1) \exp(-30\sqrt{x_1}) + \sigma_1
\]  

\[
\sigma_m = \psi_2 \sigma_2^{1/4} + \psi_1 \sigma_1^{1/4}
\]  

where \(\psi\) is the surface layer superficial volume fraction as outlined by Tamura et al. [75]. Kandlikar [19] developed an expression that determines the interfacial concentration at the surface of a bubble growing on a heated wall within a binary mixture system:

\[
x_{1,s} = x_1 \frac{2.13}{\pi} J_{a_0} \left( \frac{\alpha_1}{D_{12}} \right)^{1/2} \frac{\rho_v}{\rho_l} \left( y_{1,s} - x_1 \right)
\]

where \(J_{a_0}\) is the modified Jakob number that accounts for mass diffusion effects and the rise of the interfacial temperature,

\[
J_{a_0} = \frac{(T_w - T_{sats})}{\left( \frac{\rho_v}{\rho_l} \right) \left( \frac{h_v}{C_{p,l}} \right) \left( \frac{\alpha_1}{D_{12}} \right)^{1/2} \left( \frac{T_{sats} - T_{sat}}{g_k} \right)}
\]

\[
g_k = \frac{x_1 - x_{1,s}}{y_{1,s} - x_{1,s}}
\]

The temperature at the liquid-vapor interface of the bubble, \(T_{sats}\), was approximated to be equal to the vapor temperature inside it. To determine the interface concentration, an iterative numerical scheme was developed that first solved for the wall superheat for each respective binary mixture correlation, and then determined \(x_{1,s}\) by solving simultaneously Eqs. (2.24) and (4.7) with Eqs. (4.16)-(4.18). The calculated interfacial concentrations were then used to find respective thermodynamic properties at the liquid-vapor interface.

The experimental pool boiling data [3] used in this investigation includes 0.1M concentrations of 2-propanol and methanol water mixtures in addition to two lower concentrations of 0.015M 2-propanol and 0.04M methanol water mixtures. The mixture data was obtained from pool boiling experiments on a copper heated surface under a subatmospheric pressure of 7kPa for the 0.1M 2-propanol case and a system temperature of 39.8°C for all other cases.
4.3 Analytical Results

The results indicate that for subatmospheric system conditions with bulk fluid properties, the Kandlikar model correlates adequately for 2-propanol/water mixtures, especially for conditions where weak Marangoni effects are present.

![Graph showing comparison of experimental data with correlations for 2-propanol/water mixtures.]

**Figure 4.1** Comparison between McGillis [12] experimental data and mixture pool boiling correlations for 0.1M 2-propanol/water at 7kPa with bulk fluid properties.

In Fig. (4.1) the Fugita model shows more significant deviations with the larger 0.1M data, though as the concentration was reduced to 0.015M the rms error for superheat and heat flux was reduced to 2.7°C and 4.9W/cm² respectively. The Stephan Körner model was found to have large deviation for both 2-propanol concentrations. Both models by Fugita and Stephan Körner were originally developed for atmospheric conditions or higher, which can explain the large errors found for both alcohol/water mixtures for the concentrations considered here.
Figure 4.2 Comparison between McGillis [12] experimental data and mixture pool boiling correlations for 0.015M 2-propanol/water at 38.9°C with bulk fluid properties.

From Fig. (4.2) large deviation was found by all three models for the 0.1M methanol/water mixture, evaluated with bulk fluid properties at subatmospheric conditions. Kandlikar’s model had an 8.6°C rms error while the Fugita model error decreased with larger superheat values. This deviation further increased as the concentration reduced to 0.04 M, with superheat rms values of 12.9°C and 5.2°C for the models of Kandlikar and Fugita respectively. Similarly the Stephan Körner model also had large deviation with the experimental data for both strong and weak Marangoni concentrations of methanol/water.

Figure 4.3 Comparison between McGillis [12] experimental data and mixture pool boiling correlations for 0.1M methanol/water at 38.9°C with bulk fluid properties.
This study also explored the modeling impact of using film thermodynamic properties rather than those of the bulk fluid in addition to the use of the PSC coefficient in place of the traditional mole-weighted ideal HTC for the Fugita and Stephan Körner models. This particular analysis found a reduction in superheat rms error for the Kandlikar model of 27.0% and 4.2% for respective strong and weak concentrations of 2-propanol/water mixtures. Similarly, the Fugita model found an improvement in respective heat flux rms errors of 17.6% and 3.67% for these respective mixtures. In addition, the strong 2-propanol/water Marangoni concentration accuracy was found to improve for the Stephan Körner model to a superheat rms error of 4.3°C. These results indicate the use of average properties, using the PSC in place of the ideal HTC, can improve 2-propanol/water mixture predictions for both strong and weak Marangoni concentrations.

Figure 4.4 Comparison between McGillis [12] experimental data and mixture pool boiling correlations for 0.04M methanol/water at 38.9°C with bulk fluid properties.
This same analysis found a greater impact for the methanol/water mixtures. For the 0.1M weaker Marangoni concentration, the superheat rms error for the Kandlikar and Fujita models was reduced to 4.9°C and 2.4°C, corresponding to a respective 28.2% and 54.8% reduction from the results of the bulk properties using the ideal HTC. For the 0.04M case, the rms values for Kandlikar, Fugita and Stephan Körner models were reduced to 2.8°C, 3.1°C and 2.4°C, indicating the use of average properties in place of bulk properties as well as the use of the PSC in place of the ideal HTC has a major impact on both strong and weak Marangoni concentrations of methanol/water mixtures. The results of this study indicates that although Kandlikar’s model was able to more accurately predict both strong and weak Marangoni concentrations, Fugita’s model was overall able to predict those for methanol/water best.
Figure 4.6  Comparison between McGillis [12] experimental data and mixture pool boiling correlations for 0.1M methanol/water at 38.9°C with film fluid properties and PSC coefficient.

Figure 4.7  Comparison between McGillis [12] experimental data and mixture pool boiling correlations for 0.04M methanol/water at 38.9°C with film fluid properties and PSC coefficient.
4.4 Summary

Binary mixture model comparisons were performed with subatmospheric pressure heat flux and superheat data, for concentrations of 2-propanol/water and methanol/water mixtures that exhibit strong and weak Marangoni phenomena. The results indicate that for bulk fluid properties, the Kandlikar correlation was able to predict well the low pressure 2-propanol/water mixture data for varying concentrations between 0.1M and 0.015M, better than the models of Fugita and Stephan Körner, with rms superheat deviations of 1.8°C and 5.8°C respectively. However under the same pressure and input heat flux conditions, all three models did not correlate well with the 0.1M and 0.04M mixtures of methanol/water, where respective superheat deviations of 5.7°C and 6.1°C were found for the more predictive Fugita model.

As the concentration of the more volatile fluid decreases, the variation of surface tension at the liquid-vapor interface can be quite large affecting the saturation conditions and heat transfer in the film region. The use of film properties and the PSC coefficient for all models improved the heat flux and superheat predictions for strong Marangoni concentrations. The results suggest a heat flux rms error reduction of up to 59.3% for 0.015M 2-propanol and 49.1% for 0.04M methanol, from their respective bulk fluid properties. The film thermodynamic properties were based on an average approximation of bulk fluid properties and those calculated at the liquid-vapor interface. An idealization was made that the temperature at the liquid-vapor interface was approximately equal to the vapor temperature, for a respective mixture concentration. This method also found greater accuracy predicting superheat values for both strong and weak Marangoni concentrations for 2-propanol/water using Kandlikar’s model, with superheat deviations up to 1.8°C. Fugita’s model however was found to better predict both strong and weak concentrations of methanol/water mixtures with superheat deviations up to 1.6°C.
CHAPTER 5 – AN ANALYTICAL AND EXPERIMENTAL STUDY OF AN INCLINED GRAVITY-DRIVEN HEAT PIPE WITH STRONG-MARANGONI FLUIDS FOR ENHANCED THERMAL TRANSPORT

5.1. Introduction

Phase change heat transfer through heat pipe devices is an attractive solution for heat dissipation from concentrating solar photovoltaic [53] and microelectronic [76, 77] devices, due to the very high heat transfer coefficients that can be achieved. Wickless, gravity-assisted heat pipes have been proven to facilitate high heat transport capability through successive evaporation and condensation processes, in a closed-loop system [78]. During operation, heat is transferred from the evaporator heated surface to an internal working fluid, typically under low pressure conditions. This liquid absorbs heat and undergoes phase change into a vapor that rises with buoyancy into a condenser. Here the vapor releases its latent heat and returns to a liquid state, passing back to the heated section by gravity. Operationally however, evaporation and condensation limitations have been shown to impact performance based on liquid charge and selection of a working fluid [16].

There has been much work to enhance the heat transport of gravity-driven heat pipes through investigation of performance-enhancing intermediate working fluids [3, 13, 50]. However, many of these studies consider only pure fluids [78, 80-82], while few have examined alcohol/water binary mixtures that exhibit strong Marangoni phenomena. This phenomena has been shown [12] to induce convection due to preferential evaporation of the more volatile component, which facilitates a surface tension gradient with respect to its concentration distribution that increases toward the liquid–vapor interface [62]. This convection has been shown to suppress the CHF condition [3] thereby improving the heat transport capacity of the device [62]. Armijo and Carey [62] demonstrated that an optimal liquid charge of a 0.05M 2-propanol/water binary mixture, that exhibits strong concentration Marangoni effects, can improve the critical heat flux (CHF) condition and heat transport capacity by as much as 52% more than a heat pipe using pure water as a working fluid, under comparable conditions. Despite this enhanced performance, the authors maintained a horizontal orientation throughout their experiments and didn’t take full advantage of the buoyancy-driven flow capability of their gravity-assisted device. Han et al. [83] analyzed thermosyphon heat transport with water, methanol and ethanol as working fluids, for various volumetric liquid charge ratios and inclination angles between 0 and 90°. Their study found that the inclination angle of a thermosyphon has a notable influence on the condensation heat transfer coefficient, and improves the overall thermosyphon heat transport capacity, with an optimal angle lying between 25°-30°.

The liquid charge, defined as the ratio of working fluid volume to total heat pipe void space, is an important parameter that impacts evaporation and condensation processes in closed-loop heat pipe devices [85]. Liquid charge optimization has been shown to impact thermal transport capacity [3, 16, 61, 78] where under-filling can reduce the CHF condition, and overfilling can cause excess fluid to accumulate in the condenser section, increasing thermal resistance and limiting heat removal. Duncan et al. [51] showed experimentally that increasing the liquid charge from 10% to 50% by volume increased the measured heat transport by approximately 81%.
Strel’tsov [49] also showed that increasing liquid charge enhanced heat transport, but only until the condensate did not create an optimal uniform film. Y. Cao et al. [52] showed that overfilling a heat pipe would eventually reduce the condensation rate as the condenser area decreased beyond that needed for an optimal charge.

This study investigates the heat transport performance of a gravity-driven heat pipe in an inclined orientation of 37° for several liquid charge ratios between 30%-70%. The working fluid used was a low concentration mixture of 0.05M 2-propanol and de-ionized water. This mixture was used since low 0.05M values of 2-propanol in water were found [3, 62] to have high CHF conditions and low superheat values. This investigation also assesses thermal-fluid transport modeling for this heat pipe design. To date, few heat pipe models with binary mixture working fluids exist that characterize heat transfer performance with regard to Marangoni effects. Armijo and Carey [62] developed an accurate binary mixture model, adapted from a correlation by Kandlikar [19], to account for Marangoni effects in binary mixture pool boiling. Kandlikar noted that it is not always clear that evaluation of bulk binary mixture properties using traditional mole-weighted ideal heat transfer coefficient (HTC) correlations, effectively predict mixture fluids where strong Marangoni effects are present. Therefore this study incorporates work by Armijo and Carey [62] by employing Kandlikar’s PCS coefficient [19] correlation, while also using boundary layer film properties instead of bulk fluid properties to improve accuracy between the model and experimental data. Since a vacuum was used to reduce the operating pressure, this model approximates negligible air resistance. From experimentation, steady state temperatures within the evaporator and condenser sections were found to occur after approximately 5 min. Therefore, model development considered steady state conditions and adiabatic boundary conditions for all external surfaces, with the exceptions of the heat input and extraction areas.

5.2. Experimental Setup and Procedures

The experimental apparatus and corresponding schematic diagram for this study are shown in Fig. (5.1). The heat pipe was machined from a naval brass 464 alloy and had a rubber o-ring to seal the top plate to the bottom section. The top plate consisted of a machined evaporator heating element, which fit within the evaporator-portion of the bottom section.
As shown in section A-A of Fig. (5.2), the evaporator heating element had nine 4.76mm horizontal cylindrical channels to facilitate evaporation. The inclined heat pipe consisted of an axially-grooved structure, with 14 2.5mm channels, that facilitated condensation and resultant flow necessary to replenish liquid on the evaporator heated surface. The inclination of the heat pipe was set to 37°, relevant for application analysis involving waste heat recovery of solar energy systems in Berkeley, CA [89].

Several k-type thermocouples were used for temperature measurements. Two were connected to the bottom surface of the condenser-section external fins, as well as two locations within the aluminum interface block, located atop the evaporator section. The depth and diameter of the holes were 43.3cm and 1.09mm respectively. Three thermocouples were used to measure heat losses across the insulation that covered the external surfaces of all three sections. The thermocouples were connected in series to a control switch, and a National Instruments data acquisition (DAQ) board. The DAQ would scan and record thermocouple measurements after the system had reached a thermal steady state level for each consecutive power input test. The
four heating elements that were attached atop the aluminum interfacial block were constructed of a chrome-steel alloy that spanned a total cross-sectional area of 243.4 cm². The four heating elements were connected to a 120V autotransformer that was used to control the heat input during the experiment. These devices were connected in series to an AC power source.

To reduce contact resistance between the heating element and the heat pipe surface, a spring loaded system was constructed with a downward force of 7.4N. This was the force necessary to sustain a constant effective thermal conductivity across the heat pipe void of a working fluid, at low power input levels (<10W). Additionally, an Omega Engineering® high thermal conductivity paste was used to reduce contact resistance. Prior to filling the heat pipe, each respective working fluid was boiled to remove dissolved gases within the solution. After this process the fluid was sealed and allowed to cool to room temperature. To ensure that surface conditions remained constant for each subsequent test, the heat pipe was thoroughly inspected for oxidation and cleaned appropriately prior to filling. The heat pipe was then sealed with an o-ring gasket and several bolts. To reduce air thermal resistance within the heat pipe, an aspirator vacuum system was used to lower the heat pipe operating pressure to approximately 10kPa, monitored with an Ashcroft vacuum gauge. Once this low pressure was reached, the vacuum was engaged for approximately 1.5 minutes more to remove residual air from the system, whereby two valves connected between the aspirator and pressure gauge were then closed. Finally, the heat pipe was filled to a predetermined liquid charge through a second external valve connected to the condenser section. Since the mixture vapor pressures, according to the equilibrium phase diagram Fig. (2.1), for 0.05M 2-propanol and pure water are respectively 92kPa and 101kPa, these vapor pressures would exceed the system pressure displacing residual air out through the vacuum port during this time. Lab grade de-ionized water was used throughout the experiment for pure water and binary mixture test cases.

During each consecutive power input trial the auto transformer was tuned to a specified level and a multimeter was used to measure the corresponding electrical resistance within the cartridge heaters. Using Ohm’s law the power input level was set. To cool the external condenser surface, a compact air compressor system was connected to the external cooling fins portion of the heat pipe condenser section. As power input was increased during the experiment, the system pressure with corresponding saturation temperature $T_{sat,e}$ increased due to amplified evaporation. The evaporator heat transfer coefficient was determined from physical measurements of $T_{in}$, $T_{sat,e}$ and $\frac{\dot{Q}_{Liq/Vap}}{\dot{m}}$ using Eq. (5.1).

$$h_{w,e} = \frac{\dot{Q}_{l/v}}{T_{w,e} - T_{sat,e}}$$  \hspace{1cm} (5.1)

where

$$T_{w,e} = T_{in} - \frac{\dot{Q}_{l/v}}{S_{e} k_{Br}}$$  \hspace{1cm} (5.2)

In Eq. (5.2) $S_{evap}$ was determined from a shape factor provided by Rohsenow [54], and $k_{Al}$ was determined from Fourier conduction of the heat pipe void of a working fluid. Experimental uncertainties in the pressure and temperature measurements were ±0.5kPa (0.5%) and ±2.2°C.
(0.8%) respectively. \( \dot{Q}_{\text{Liq/Vap}} \) was determined from Eq. (5.3) and thermal resistance analysis of heat transfer by phase change and conduction across the heat pipe wall, with system approximations of steady state and negligible heat generation.

\[
\dot{Q}_{\text{tot}} = \dot{Q}_{l/v} + \dot{Q}_{\text{Cond}} \tag{5.3}
\]

5.3. Analytical Model

The gravity-assisted heat pipe evaluated in this investigation transfers thermal energy from a heating element positioned atop an aluminum interface block and the evaporator section to its condenser section via an internal cavity. This heat transfer process for the heat pipe is shown in Fig. (5.2), where the bottom portion of the condenser section was cooled via an air duct system, held at an observed average temperature of 17 °C. All external heat pipe surfaces, with the exceptions of those exposed to direct heat input and output, were considered adiabatic. Heat loss to the environment was mitigated by using a 2.5mm thick Unifrax thermal blanket which covered the external surface of the heat pipe and heating elements. Heat loss was determined experimentally from thermal heat conduction analysis and was found to be less than 5% of the total heat input, validating the adiabatic assumption.

Heat transfer from the evaporator to the condenser section was facilitated by liquid and vapor transport within the inner chamber and by heat conduction through the brass casing as prescribed by Eq. (5.3). Consistent with our experimental setup, \( \dot{Q}_{\text{tot}} \) and \( T_{\text{cold}} \) are specified controlled parameters. The following heat pipe transport model is then used to derive \( T_{w,e} \) and evaluate the performance of this heat pipe design. Since \( T_{w,e} \) depends on both \( \dot{Q}_{l/v} \) and \( \dot{Q}_{Br} \) in the heat transfer process, and the relation between \( \dot{Q}_{l/v} \) and \( \dot{Q}_{Br} \) is unknown, and an iterative scheme is required to solve for \( T_{w,e} \):

1: \( \dot{Q}^{0}_{l/v} \) was initially set as \( \dot{Q}_{\text{tot}} \).
2: Heat transfer analysis was performed for liquid/vapor transport within the inner chamber, and \( T_{w,e}^{0} \) was determined.
3: With calculated values for \( T_{w,e}^{0} \) and \( T_{w,e} \), heat conduction analysis was performed to obtain \( \dot{Q}_{Br}^{0} \).
4: \( \dot{Q}_{l/v}^{i+1} \) was determined based on the difference between \( \dot{Q}_{\text{tot}}^{i} - \dot{Q}_{Br}^{i} \).
5: If the difference between \( \dot{Q}_{l/v}^{i+1} \) with \( \dot{Q}_{l/v}^{i} \) was less than a specified tolerance of 0.1W, the corresponding iterative value of \( T_{w,e} \) was output. Otherwise steps 3-5 were repeated until \( \dot{Q}_{l/v} \) converged.

The heat transfer analysis for liquid-vapor transport within the inner chamber in step 2 was carried out for the three respective heat pipe sections. For the condenser section, the lower internal wall condenser temperature \( T_{w,c} \), was assumed to be approximately constant through the narrow internal condenser fins. Its value was determined using Fourier conduction analysis, which considered material thermal resistance and the external condenser temperature \( T_{\text{fins}} \). This temperature was determined from measurement and analysis of air flow through a bank of fins, connected externally to the condenser section. The forced convection heat transfer coefficient, Eq. (5.4), for crossflow between the rectangular fins was determined based on an approximation
of laminar flow through a tube bank with an aligned configuration, as prescribed by Zhukauskas [86].

$$\overline{h}_{Ext} = \frac{\overline{Nu} \cdot k}{L} \quad (5.4)$$

$$\overline{Nu} = C \text{Re}_{D,max}^m \text{Pr}^{0.36} \left( \frac{\text{Pr}}{\text{Pr}_L} \right)^{\frac{1}{4}} \quad (5.5)$$

with the prescribed limitations:

$$\left\{ \begin{array}{l}
N_t \geq 20 \\
0.7 < \text{Pr} < 500 \\
1000 < \text{Re}_D < 2 \times 10^6
\end{array} \right.$$ 

Next, $T_{w,\text{cond}}$ was solved from heat conduction analysis for a rectangular fin, as prescribed by Mills [90]:

$$T_{w,\text{cond}} = (T_L - T_\infty) \cos(\beta L) + T_\infty \quad (5.6)$$

where $P$ and $A$ are the respective fin perimeter and area, and $\beta$ is defined as:

$$\beta = \sqrt{\frac{hP}{kA}} \quad (5.7)$$

For this analysis, air was used as the external cooling fluid with properties based on the boundary layer film temperature, which was determined as the average of the ambient temperature $T_\infty$ and external surface temperature $T_b$. Internally, experimental Sessile drop contact angle analysis of droplets on the surface of the evaporator heated section and condenser section, revealed a value of approximately 44°±3° for water and 26°±2° for isopropanol, which indicates a wetting surface and the permitted use of Nusselt analysis [3] heat transfer correlation for film condensation,

$$\overline{h}_{\text{cond}} = 0.943 \left( \frac{k_f}{l_v} \right) \left\{ \frac{\rho_f (\rho_f - \rho_v) g h_v^* l_v^3}{k_f \mu_f (T_{\text{sat,c}} - T_{w,\text{cond}})} \right\}^{1/4} \quad (5.8)$$

$$h_v^* = h_v \left\{ 1 + 0.68 \left( \frac{c_{\text{m}} (T_{\text{sat,c}} - T_{w,c})}{h_v} \right) \right\} \quad (5.9)$$

where $l_v$ is the vertical length along the condenser surface, exposed to condensing vapor. Using a more detailed analysis by Rosenhow [54] that accounts for the effects of subcooling and energy convection along the vertical condenser surface, Eq. (5.9) was used in Eq. (5.8). Using the heat transfer correlation Eq. (5.10), $T_{\text{sat,c}}$ and $\overline{h}_{\text{cond}}$ can be determined by iteration.

$$\dot{Q}_{\text{out}} = \overline{h}_{\text{cond}} A_{\text{cond}} \left( T_{\text{sat,cond}} - T_{w,\text{cond}} \right) \quad (5.10)$$
where $\bar{h}_{\text{cond}}$ and $A_{\text{cond}}$ correspond to the unsubmerged portions of the internal condenser channels. Idealizations between the condenser and evaporator sections were made such that $\dot{Q}_{\text{sat}} = \dot{Q}_{\text{out}}$, $T_{\text{sat},e} = T_{\text{sat},c}$ and liquid and vapor mass flow rates were approximately equal. The interfacial temperature between the aluminum heating element and the external evaporator surface was determined from Fourier heat conduction analysis using a shape factor provided by Rohsenow [54],

$$S_{\text{evap}} = \frac{2\pi \bar{g} L}{\ln \left( \frac{2l}{\pi D} \right)}$$

(5.11)

where $l$ is half the length of the evaporator channels, $D$ is the channel diameter, $L$ is the channel length and $\delta$ is the distance between the channel and the external isothermal surface, determined by measurement. To determine the evaporator heat transfer coefficient for binary mixtures, a number of investigators [19, 56, 63, 65 and 67] have developed correlations containing a solute and a volatile solvent. Rather than using an ideal weighted-average coefficient, a predictive model was developed by Kandlikar [19], which is based on an average PSC coefficient which accounts for property effects in binary mixtures.

$$h_{\text{psc}} = h_{\text{avg}} \left( \frac{T_{\text{sat},m}}{T_{\text{sat},avg}} \right)^{-0.674} \left( \frac{\Delta h_{v,m}}{\Delta h_{v,avg}} \right)^{0.371} \left( \frac{\rho_{v,m}}{\rho_{v,avg}} \right)^{0.297}$$

(5.12)

$$\times \left( \frac{\sigma_m}{\sigma_{avg}} \right)^{-0.317} \left( \frac{k_{l,m}}{k_{l,avg}} \right)^{0.284}$$

For real mixture properties, the average heat transfer coefficient is corrected with the form,

$$h_{\text{avg}} = 0.5 \left[ x_i h_{i,1} + (1-x_i) h_{i,2} + \frac{x_i}{h_{i,1}} + \frac{(1-x_i)}{h_{i,2}} \right]^{-1}$$

(5.13)

where $h_{1,1}$ and $h_{1,2}$ are respective component heat transfer coefficients, obtained from superheat temperatures $\Delta T_1$ and $\Delta T_2$ using the Stephan and Abdelsalam [20] mathematical heat transfer correlation for nucleate pool boiling of pure fluids:

$$\frac{\dot{Q}_{\text{stab}}^* d_b}{k_i(T_w - T_{\text{sat}})} = 0.23 \left( \frac{\dot{Q}_{\text{stab}}^* d_b}{k_i T_{\text{sat}}} \right)^{0.641} \left( \frac{h_{w}}{\alpha_i^2} \right)^{0.371} \left( \frac{\rho_i - \rho_x}{\rho_i} \right)^{1.73} \left( \frac{\sigma^2 \rho_i}{\sigma_d d_b} \right)^{0.35}$$

(5.14)

The bubble departure diameter from Eq. (5.14) is defined by the Fritz [87] correlation:

$$d_b = \sqrt{\frac{0.0208 \sigma}{g \cos(\alpha)(\rho_i - \rho_x)}}$$

(5.15)
Kandlikar’s theoretical model, Eqn. (5.16) as outlined by Carey [15], was derived solely on a theoretical basis by applying a one-dimensional model for heat and mass transfer at the bubble liquid-vapor interface under diffusion-controlled growth conditions, where the correction factor for mass diffusion effects, \( F_D \) varies for different volatility parameter ranges. Kandlikar’s correlation was developed for pool boiling of binary mixtures under atmospheric conditions with thermodynamic properties evaluated based on that of the bulk fluid. However, Armijo and Carey [21] argued that during nucleate pool boiling, as system pressure and temperature are reduced below atmospheric conditions, boiling and dew point temperatures decrease, affecting other system thermodynamic properties.

Since the volatile component mole fraction has been shown to vary at a bubble’s liquid-vapor interface from that of the bulk fluid [22], the properties in this study are taken at the boundary film layer. These are determined by an average of the properties at the liquid-vapor interface and those of the bulk fluid. Due to the non-ideal nature of pool boiling of 2-propanol/water mixtures [15], this study employs Kandlikar’s PCS coefficient.

\[
h = \begin{cases} 
1 & \text{for } V_1 \leq 0.03 \\
1 - 64V_1 & \text{for } 0.03 < V_1 \leq 0.2 \\
0.678 \left[ 1 + \left( \frac{\alpha_l}{D_{12}} \right)^{1/2} \left( \frac{T_{lp}(x_{1,s}) - T_{lp}(x_1)}{g_k} \right) \right]^{-1} & \text{for } V_1 > 0.2
\end{cases}
\]

Kandlikar [19] developed an expression that determines the interfacial concentration at the surface of a bubble growing on a heated wall within a binary mixture system:

\[
x_{1,s} = x_1 - \frac{2.13}{\pi} J_{a_0} \left( \frac{\alpha_l}{D_{12}} \right)^{1/2} \frac{\rho_v}{\rho_l} \left( y_{1,s} - x_1 \right)
\]

where \( J_{a_0} \) is the modified Jakob number that accounts for mass diffusion effects and the rise of the interfacial temperature,

\[
J_{a_0} = \left( \frac{\rho_v}{\rho_l} \right) \left( \frac{h_{pv}}{C_{p,l}} \left( \frac{\alpha_l}{D_{12}} \right)^{1/2} \frac{T_{sat,s} - T_{sat}}{g_k} \right)
\]

where

\[
g_k = \frac{x_1 - x_{1,s}}{y_{1,s} - x_{1,s}}
\]

Through experimentation of low concentration mixtures, Vazquez et al. [67] demonstrated non-ideal behavior of surface tension properties for alcohol/water mixtures. The authors also presented evidence indicating that deviation from ideality increases with the length of an organic molecule, while decreasing with rising system temperature. For mixtures of 2-Propanol and water, the non-ideal nature of mixture surface tension properties has been shown to become more
significant as the more volatile 2-Propanol concentration is reduced [15]. For this investigation the 2-Propanol/water mixture surface tension was evaluated based on Eq. (5.20) as outlined by McGillis [3]:

$$\sigma_m = (\sigma_2 - \sigma_1) \exp(-30\sqrt{x_1}) + \sigma_1$$  \hspace{1cm} (5.20)

who explained that for non-ideal alcohol/water binary mixtures, the vapor pressure is related to the vapor mole fraction by

$$P = \frac{x_j \gamma_j P_{satj}}{\gamma_j}$$  \hspace{1cm} (5.21)

The activity coefficient $\gamma_j$ for each respective $j$ component, is determined by equating the molar excess Gibbs energy with the two-parameter Margules Eq. (2.23)-(2.24), where $A_{12}$ and $A_{21}$ are experimentally determined constants [3]. An idealization is made here that the liquid-vapor interface temperature, $T_{sat,s}$, is approximately equal to the vapor temperature inside the bubble. The concentration at this location was determined using an iterative numerical scheme that first solved for the wall superheat and then determined $x_{1,s}$ by solving simultaneously Eqs. (5.17)-(5.19) with Eqs. (5.21)-(5.23). The calculated interfacial concentrations were then used to find respective thermodynamic properties at the liquid-vapor interface.

To determine the evaporator wall temperature a second iteration scheme was performed by combining Eqns. (5.13) through (5.23). To perform heat conduction analysis in step 3 of the heat pipe transport iteration procedure, the thermal resistance of the brass casing was determined using a three-dimensional finite difference analysis with adiabatic boundary conditions set for all external heat pipe surfaces, with the exceptions of the heat input and removal areas. The heat input area was defined as the cross-sectional area of the heating element, and the extraction area was set as the area of the internal condenser section. The numerically calculated thermal resistance of the casing was 1.15 K/W.

5.4. Liquid Charge Analysis

Maximum heat pipe transport capability was evaluated for heat input levels between 40W-1000W, with respective liquid charge levels between 30% and 70%, for a 0.05M 2-Propanol/water binary mixture. The results in Fig. (5.3), with added trend lines, show that increasing the liquid charge level facilitates the CHF condition (as indicated by the sudden increase in superheat values) to occur at higher evaporator input heat flux levels.
Prior work has shown that a major thermal transport constraint is the dryout limitation, associated with the CHF condition, where the liquid pressure gradient across the heat pipe decreases below that of the vapor pressure drop [53, 76-78], reducing liquid coverage along the heated section of the evaporator. The liquid pressure gradient at high input heat flux levels decreases because the volume of liquid water within the heat pipe reduces with increased evaporation. This will lessen the hydrostatic pressure difference between the condenser and evaporator sections, decreasing liquid transport, leading to a sudden increase in wall superheat indicating the CHF condition has been reached. Conversely, elevated superheat temperatures with higher charge ratios beyond 45% can also be attributed to condenser limitations. It has been shown [80] that overfilling a heat pipe can mitigate performance as internal condensing surface area is reduced by liquid coverage of a working fluid, which consequently increases the condenser temperature, decreasing heat removal.

The results of each respective liquid charge test for the 0.05M 2-propanol/water mixture showed a steady increase in input heat flux with growing wall superheat, with a tapering for all five cases at approximately 100 W/cm². The results indicate respective CHF conditions of 96.5 W/cm², 112.9 W/cm² and 114.8 W/cm² for liquid charge ratios 30%, 40% and 45%. However, although higher respective heat flux values of 117.3W/cm² and 118.2W/cm² were attained for 50% and 70% fill ratios, the CHF condition was not reached for the input heat loads considered in this investigation due to high experimental temperature limitations. Additionally, the fill ratio of 45% had the lowest overall superheat values and highest CHF condition than any other liquid charge ratio tested. Similar results were found in experimental work by Strel’tsov [49], Duncan [51], and Shiraishi et al. [61], where the critical heat flux also rose by increasing the liquid charge.

**Figure 5.3.** Evaporator surface superheat vs. evaporator vaporization heat flux for pure water with fill ratios of 30%, 40%, 45%, 50% and 70%, for a 37° heat pipe orientation and with CHF conditions reached only for fill ratios of 45% and lower.
Higher CHF values can be attributed to hydrostatic pressure increases with greater charge levels, which would likely improve liquid flow to the evaporator section and reduce the superheat and the onset of dryout. Similar CHF and superheat improvements were also found by Pal and Joshi [78], who found a maximum CHF condition with a liquid charge of 40%.

Comprehensive heat transport performance, from the external evaporator heated surface to the external condensing fins, was evaluated in terms of a thermal conductance parameter, Eq. (5.22).

\[ G_{HP} = \frac{q_{in}}{T_{input}-T_{fins}} \]  

(5.22)

The heat transfer rates at different working liquid charge ratios are shown in Fig. (5.4). Heat transfer was found to increase with liquid charge and reached a maximum thermal conductance at a value of 45%, for the range of input heat loads evaluated. Above 45%, the heat transfer performance decreased as the liquid charge was increased. For the 45% liquid charge, the maximum conductance was found to occur for an input heat load of 600W, which was 42.7% and 5.6% greater than respective heat loads of 200W and 800W. Thermal conductance values at other liquid charge ratios were also found to be greater overall for the 600W input heat load, when compared to low 200W and 400W heat loads. However, only average marginal differences of 1.0% and 2.7% were found between input load comparisons of 600W with 650W and 800W respectively. Similar results were found in the experimental work of Jia and Li [88] as well as Kannan and Natarajan [80], who found maximum heat transfer rates for liquid charge ratios of approximately 50%. The authors found that liquid charge ratios beyond the maximum increases the overall thermal resistance, suppressing thermal cycling performance by only facilitating heat transfer by conduction and not phase change.

![Figure 5.4. Heat pipe thermal conductance vs. fill ratio for heat input loads of 200W-800W, with 0.05M 2-Propanol/Water binary mixture, for a 37° heat pipe orientation and fill ratios between 30%-70%.](image)
5.5. Empirical Model Validation

Experimental heat pipe evaluation was conducted with a 0.05M 2-propanol/water binary mixture and performed with a liquid charge of 45%. To determine the transport contributions due only to phase change, experiments were performed with the heat pipe void of liquid water to determine the thermal resistance of the casing material. Under the same experimental conditions, an average experimental thermal resistance of 0.96K/W was found with an average standard deviation of 0.14 between trials. Using thermal heat conduction analysis and Eqn. (5.3), this value was used to determine the heat flux due to liquid-vapor transport only.

Experimental heat pipe performance results with water as the working fluid revealed that a superheat increase of 12.8°C would provide an average increase of 111.5W/cm² in transport heat flux across the device. Using the aforementioned model development discussed earlier, Fig. (5.5) presents good model correspondence to experimental data with an average superheat and heat flux deviation of 1.3°C and 4.3W/cm² respectively. Armijo and Carey [62] showed similar model correspondence when examining pool boiling mixtures containing low concentrations of alcohol and water. Correspondingly, the use of film properties and the PSC coefficient, in place of the traditional mole-weighted ideal heat transfer coefficient, improved the heat flux and superheat predictions for the low concentration Marangoni working fluid used in the gravitational heat pipe. Due to the non-ideal nature of surface tension for low 2-propanol/water concentrations, additional correspondence can also be attributed to the use of the non-ideal surface tension property correlation developed by McGillis [3], in place of traditional mole-weighted averages of both mixture components [65]. It is proposed that the non-ideal nature of the mixture surface tension gives rise to the elevated CHF condition through Marangoni convection, which is facilitated by a surface tension gradient [3] with respect to the concentration distribution that increases toward the liquid–vapor interface.

Figure 5.5. Theoretical solar cell electrical output based experimental waste heat evaluation for a heat pipe with a 37° orientation, with a liquid charge level of 45% and 0.05M 2-Propanol/Water binary mixture.
5.6 Summary

Experimental analysis was conducted for an inclined 37°, gravity-assisted heat pipe with a binary mixture concentration of 0.05M 2-Propanol/water as the working fluid. For a range of input heat load values, the results of the liquid charge study revealed an optimum value of 45%, which had lower overall superheat levels and increased the CHF condition as high as 114.8W/cm². For respective 50% and 70% fill ratios, although the CHF was not reached, higher heat flux values of 117.3W/cm² and 118.2W/cm² were attained, while the superheat was found to be respectively higher by 4.6% and 15.5%. Further analysis also showed that higher liquid charge ratios of 50% and 70% decreased the thermal conductance by 21.7% and 34.0% respectively from a 45% liquid charge. Justification of decreased performance with higher charge levels can be attributed to suppression of thermal cycling, where only heat transfer is facilitated by conduction and not by phase change, which would likely promote higher transfer capability [34]. Thermal conductance analysis across the heat pipe also showed an optimal heat load of 600W, which was 42.7% and 5.6% greater than respective heat loads of 200W and 800W.

The analytical model developed in this investigation found good correspondence between the liquid-vapor transport heat flux experimental data and the analytical prediction. Experimental data were found to agree with parametric trends with an rms deviation of 6.5%. Deviations between the model predictions and experimental data also found an average superheat and heat flux deviation of 1.3°C and 4.3W/cm² respectively. Justification for the correspondence can be attributed to the use of film properties, Kandlikar’s PSC coefficient and McGillis’ surface tension correlation for non-ideal binary mixtures. It is proposed that the non-ideal nature of the mixture surface tension, which is sensitive to temperature variation, facilitates Marangoni flow that improves gravity heat pipe performance.
CHAPTER 6 – GRAVITY-ASSISTED HEAT PIPE WITH STRONG-MARANGONI FLUIDS FOR WASTE HEAT RECOVERY OF SINGLE AND DUAL-JUNCTION PHOTOVOLTAIC CELLS

6.1 – Introduction

Efficient means of heat rejection in single and multijunction solar photovoltaic (PV) power generation systems is critical for achieving optimal performance for medium (10-100 suns) and high (>100 suns) solar concentration levels [66]. Solar cells, like other semiconductor devices, are thermal-sensitive whereby temperature elevations can decrease the semiconductor band gap and open circuit voltage, which reduces photoelectric conversion and solar cell efficiency [68, 91]. Wang et al. [92] demonstrated that decreasing the temperature of an average solar cell by 1°C can increase its output power by 0.2%-0.5%. Operationally, low cell temperatures must be maintained to avoid efficiency degradation. This is especially true for concentrator photovoltaic (CPV) systems where temperature variations can be quite substantial [101]. Therefore, as solar concentration increases, CPV systems require prodigious means of heat rejection.

Various thermal photovoltaic (PV/T) system designs have been developed to cool solar cells [93] through waste thermal energy transport via a fluid medium. An attractive passive cooling method for moving large quantities of heat during high concentration operation is through heat pipes. Heat pipe solar collectors have several advantages over traditional flat-plate collectors that allow it to have higher thermal and combined system efficiencies [95]. Unlike traditional solar flat-plate collectors, the position of the condenser within a heat pipe may not be restricted to any specific orientation. Additionally, heat pipes are unidirectional conductors that act as a thermal diode, allowing heat to be transferred only from the evaporator to the condenser, but never in the reverse direction. This is advantageous in reducing heat losses when the cell temperature is lower than that of the liquid in the heat exchanger [53]. Since heat pipes are sealed, compatible material and working fluids can significantly minimize corrosion and freezing issues that occur with traditional flat-plate collectors [53, 97].

There has been much work to enhance the heat transport capacity of gravity-driven heat pipes through investigation of performance-enhancing intermediate working fluids [49, 51, 62]. Recent investigations [51, 14] have shown that optimizing the charge volume within a heat pipe is important for optimal performance. Duncan et al. [51] showed experimentally that increasing the liquid charge from 10% to 50% by volume increased the measured heat transport by approximately 81%. Strel’tsov [49] also showed that increasing liquid charge enhanced heat transport, but only until the condensate did not create an optimal uniform film. Y. Cao et al. [131] showed that overfilling a heat pipe would eventually reduce the condensation rate as the condenser area decreased beyond that needed for an optimal charge. For binary mixtures, Armijo and Carey [62] demonstrated that an optimal liquid charge of a 0.05M 2-propanol/water mixture that exhibits strong concentration Marangoni effects, can improve the critical heat flux (CHF) condition and heat transport capacity by as much as 52% more than a heat pipe using pure water as a working fluid under comparable conditions.
This study evaluates the performance of a gravity-assisted heat pipe for improving electrical efficiencies of a single and dual-junction solar cell. To achieve high PV waste heat recovery, the heat pipe contained a 0.05M 2-Propanol/water working fluid, which is known to have strong Marangoni thermo-fluid transport characteristics [62]. A complete description of the setup and procedures for this experiment is outlined in section 5.1. An analysis was first conducted to determine an optimal liquid charge for the internal heat pipe working fluid, used to evaluate the lowest evaporator superheat for varying incident solar concentration levels. Experimentally, strip-heaters were attached atop an aluminum interface block that conducted heat away from the heat pipe evaporator-section. Measured heater temperatures and input heat loads were used to simulate solar cell temperatures and incident solar radiation respectively. These metrics were used to assess the effects of cell temperatures and concentration levels on semiconductor electrical conversion properties and performance. Thermal equilibrium approximations were made within the analytical model at each respective solar concentration level, with all waste heat transported solely through the heat pipe.

6.2 – Thermal performance enhancement of single-junction solar cells

Heat pipe thermal absorbers in solar PV systems have several distinct advantages over other passive cooling methods. They have excellent heat transport capability during high heat flux conditions, with notable thermal transport capacities as high as 90W/cm² [62], using minimal amount of fabrication material, which provides quick responses to changes in incident radiation intensity. Additionally, their inherent thermal diode design also provides the benefit of minimizing heat loss when the collector temperature is above ambient [97]. These two attributes have demonstrated conversion efficiency improvements and the ability to provide a secondary source of thermal energy for heating applications or thermal electric power generation [99]. In this investigation, a gravity-assisted heat pipe was connected to four strip heaters to simulate PV thermal power, \( \dot{Q}_{th} \). For each successive heat load, the interface temperature between the aluminum block and strip heaters was taken as the cell temperature. Operationally, the heat pipe working fluid absorbs heat from the internal evaporator heated surface and passes this thermal energy to the condenser section where the heat is dissipated to an external cooling fluid via rectangular fins. Air was used as the external cooling fluid, and was measured to be approximately 20°C, with a constant 3.3m/s velocity. The theoretical solar concentration factor, \( C \) was computed from an energy balance of total incident solar energy \( \dot{Q}_{inc} \),

\[
\dot{Q}_{inc} = \dot{Q}_{th} + \dot{Q}_{elec} \quad (6.1)
\]

\[
C \cdot \dot{Q}_{inc,ref} = \dot{Q}_{th} + \eta_{elec,ref}(C \cdot \dot{Q}_{inc,ref}) \quad (6.2)
\]

through further algebra:

\[
C = \frac{\dot{Q}_{th}}{\dot{Q}_{inc,ref}(1-\eta_{ref,elec})} \quad (6.3)
\]
where $\eta_{\text{ref,elec}}$ is the reference solar cell efficiency and $\dot{Q}_{\text{inc,ref}}$ is the reference incident solar cell radiation, taken to be 1000 W/m$^2$ based on standard AM-1.5 test conditions [102], with an ambient temperature of 25°C.

Several investigators have shown that heat pipe charge optimization has a direct impact on electrical conversion efficiency improvement of solar energy equipment [92, 100]. Wang et. al. [92] assessed heat pipe thermal performance and PV electrical conversion efficiency under varying concentrated solar illumination. They found the evaporator temperature rises with increasing illumination to an optimal concentration factor of 160 suns, which provided the highest electrical conversion efficiency with a 30% liquid charge ratio. The authors found that low liquid charge levels resulted in early dryout conditions, while excessive charging delayed nucleate boiling and increased the evaporator wall superheat.

In this work, the heat pipe evaporator superheat was evaluated based on varying liquid charge levels between 30%-70%, of a 0.05M 2-propanol/water mixture. Using Eqns. (6.1)-(6.3) the solar concentration factor was calculated for each successive heat input level as presented in Fig. (6.1). The reference cell efficiency, $\eta_{\text{cell}}$ was taken to be 14% based on a mono-Si solar cell used by Meneses-Rodriguez et al [101] that had an area of 72 cm$^2$.

**Figure 6.1.** Heat pipe evaporator wall superheat vs. incident solar concentration for a 37° system orientation, 0.05M 2-propanol/water mixture, 30%, 40%, 45%, 50% and 70% liquid charge ratios and with a $\eta_{\text{cell}}$=14% mono-Si solar cell.

Fig. (6.1) results show that lower 30% and 40% liquid charge ratios had higher overall superheat levels and reached respective critical heat flux (CHF) conditions of 96 W/cm$^2$ and 112 W/cm$^2$, for
incident solar concentration levels of 122 suns and 145 suns respectively. Higher 50% and 70% liquid charge ratios were also found to have higher overall superheat values due to reduced surface area within the condenser to facilitate condensation. However, the optimum liquid charge level of 45% was found to have the lowest overall superheat values for varying solar concentration, with a CHF condition of 114.8 W/cm², reached at 162 suns.

Within a solar cell, incident radiation generates electron-hole pairs on both sides of an internal semiconductor junction that comprises an n-type emitter and p-type base. The generated minority carriers, which include electrons from the base and holes from the emitter, diffuse to the junction and are swept away by an electric field, producing an electric current across the device [102]. The p-n junction therefore separates the carriers with opposite charge, and transforms the generation current, \( I_\text{g} \), between bands into electric current, \( I \) across the p-n junction.

\[
I = I_\text{g} - I_\text{o} \exp \left( \frac{nqV}{k_BT} \right) - 1
\]  

(6.4)

In an open circuit configuration when \( I=0 \) and \( I=I_\text{sc} \), all the generated current passes through the diode. Conversely, under a short circuit (\( V=0 \)) configuration all the current passes through the external load.

\[
V_\text{oc} = \frac{k_BT}{nq} \ln \left( \frac{I_\text{sc}}{I_\text{o}} + 1 \right)
\]  

(6.5)

where \( n \) (typically between 1<\( n < 2 \)) is the ideality factor [103], \( I_\text{o} \) is the reverse-saturation current, \( q \) is the charge of an electron and \( k \) is the Boltzmann constant. To compute \( I_\text{o} \), the equation for reverse saturation current is given by [104]:

\[
I_\text{o} = \delta A_\text{cell} T^3 \exp \left( \frac{-E_g}{k_BT} \right)
\]  

(6.6)

where \( A_\text{cell} \) is the active photosensitive area and \( \delta \) is the pre-exponential factor dependent on the band gap energy \( E_g \). The temperature dependence of the bang gap can be expressed as:

\[
E_g(T) = E_g(0) - \frac{\alpha T^2}{T+\beta}
\]  

(6.7)

where \( E_g(0) \) is the linearly extrapolated bandgap at zero kelvin [105] and \( \alpha \) and \( \beta \) are empirically determined parameters. Table 1 lists the values of \( E_g(0) \), \( \alpha \) and \( \beta \) for Si [106], InGaP [107] and GaAs [107].

<table>
<thead>
<tr>
<th>Energy band gap parameters for Si, InGaP and GaAs</th>
</tr>
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<tbody>
<tr>
<td>( E_g(0) ) (eV)</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Si</td>
</tr>
<tr>
<td>InGaP</td>
</tr>
<tr>
<td>GaAs</td>
</tr>
</tbody>
</table>
For silicon solar cells, $\delta=1.5 \times 10^{-12}$ is a widely adopted experimentally determined value [91, 108, 109]. The maximum theoretical power produced is obtained from Eq. (6.8), where $I_m$ and $V_m$ are the current and voltage at the maximum power point of the solar cells I-V power curve [110].

$$P_{\text{max}} = I_m V_m = FF \frac{kT_{\text{cell}}}{q} V_{oc}$$  \hspace{1cm} (6.8) \\

$$I_m \approx I_{sc} - I_o \exp \left( \frac{qV_{oc}}{kT_{\text{cell}}} - \ln \left( \frac{qV_{oc}}{kT_{\text{cell}}} + 1 \right) \right)$$  \hspace{1cm} (6.9) \\

$$V_m = V_{oc} - \frac{kT_{\text{cell}}}{q} \ln \left( \frac{qV_{oc}}{kT_{\text{cell}}} + 1 \right)$$  \hspace{1cm} (6.10)

According to Lee [110], the fill factor $FF$ is an ideality measure of the solar cell I-V curve that accounts for series and shunt resistances, which tend to degrade this parameter and resulting output power with increasing cell temperatures. Through mathematical rearrangement of Eqns. (8)-(10), Lee [110] provided an explicit form of $FF$ as a function of temperature:

$$FF = \frac{qV_{oc} - \ln \left( \frac{qV_{oc}}{kT_{\text{cell}}} + 1 \right)}{\frac{kT_{\text{cell}}}{q} + 1}$$  \hspace{1cm} (6.11)

For silicon solar cells, typically measured fill factors fall in the range of 0.75-0.85 [111]. It has been shown that increasing solar cell temperatures decrease electrical conversion efficiency [102, 112, 137]. The main reason for this decrease is the drop in open circuit voltage and fill factor with increasing temperature. Coventry [113] found a measured reduction in solar cell efficiency to be approximately 0.35%/°C [113] due to an open-circuit voltage drop of 1.9mV/°C. Additionally, Meneses-Rodriguez et al. [101] studied the effect of concentration on cell parameters at a constant temperature of 300K, and concluded that the increase in short circuit current with concentration is more sensitive than that of open-circuit voltage.

A widely used figure of merit for solar cells is its power conversion efficiency [115]. The variation of solar cell efficiency with temperature, under standard AM 1.5 test conditions is typically prescribed by the first-order relationship [112]:

$$\eta_{\text{elec}} = \eta_{\text{ref}} \left[ 1 - \beta (T_{\text{cell}} - T_{\text{ref}}) \right]$$  \hspace{1cm} (6.12) \\

where $\eta_{\text{ref}}$ and $T_{\text{ref}}$ are the respective reference efficiency and temperature, while $\beta$ is a cell reference parameter varying between 0.004 and 0.006°C$^{-1}$ for silicon cells [115]:

$$\beta = \frac{1}{T_{\eta=0} - T_{\text{ref}}}$$  \hspace{1cm} (6.13)

In Eq. (6.13), $T_{\eta=0}$ is the temperature at which the PV module’s electrical efficiency drops to zero [115]; 270°C for typical silicon solar cells [111]. However, for solar cells under concentrated illumination, Fan [106] provided a relationship for efficiency based on short-circuit current and open-circuit voltage as a function of both temperature and concentration:
where Xiao et al. [103], Luque [132] and Nelson [133] equate the short-circuit current and open-circuit voltage proportional to the solar concentration factor:

\[ I_{sc}(C) = CI_{sc}(1) \]  \hspace{1cm} (6.15)

\[ V_{oc}(C, T) = \frac{n k_B T}{q} \ln \left( \frac{I_{sc}(C)}{I_0(T)} \right) \]  \hspace{1cm} (6.16)

For this analysis, a mono-crystalline silicon solar cell with an aperture area of 85x85 mm\(^2\) and conversion efficiency of 14\%, at room temperature [101], was considered. To comprehensively evaluate the range of solar cell operating conditions for single-junction silicon solar cells, a parametric analysis, as shown in Fig. (6.2), was performed to evaluate electrical conversion efficiency with respect to a range of cell temperatures and solar concentration levels.

**Figure 6.2.** Mono-Si single-junction solar cell electrical efficiency as a function of temperature and solar concentration, for a gravity-assisted heat pipe with a 37\(^\circ\) orientation, 0.05M 2-propanol/water mixture and 45\% liquid charge ratio.

The figure indicates an efficiency improvement with solar concentration by an average 0.02\%/suns, up to a maximum operational potential limit of 15.4\% at 160 suns and 20\(^\circ\)C. The results also show an average electrical efficiency reduction with cell temperature by -0.06\%/\(^\circ\)C. In addition, a computed cell efficiency prediction is included based on waste heat extraction with an inclined gravity-assisted heat pipe. This prediction, with respective error bars, is based on measured experimental interfacial temperatures of the aluminum-block and strip heaters.

Semiconductor parameters pertaining to the empirical prediction curve in Fig. (6.2) are presented in Fig. (6.3). Here, the electrical efficiency decreases with diminishing open-circuit voltage and
solar concentration by an average -24.8%/V. This voltage drop with temperature in silicon solar cells has been shown to occur with increasing temperature, regardless of cell quality [136]. Studies by Conventry and Franklin [116] showed that cells under open-circuit conditions that are non-uniformly illuminated have significant thermal-induced internal current flows, which result in voltage losses overall. In a later study, Coventry [113] further demonstrated that voltage sensitivity to cell temperature is due more specifically to the reverse saturation current increasing rapidly with temperature, as prescribed by Eq. (6.6).

Figure 6.3. Projected single p-n junction monocrystalline silicon solar cell electrical properties vs. solar concentration.

The results of the single-junction silicon solar cell, revealed an average $V_{oc}$ decrease of 6mV/suns and an $I_{sc}$ increase of 0.02A/suns from 6 suns to 154 suns. Skoplaki [115] noted that that as cell temperature increases, open circuit voltage and fill factor decreased as thermally excited electrons dominated electrical properties of the semi-conductor. It has been shown that short circuit current increases with temperature [101, 113, 117, 131], due to an increased diffusion length, allowing a higher probability of carriers reaching the p-n junction [113].

To further evaluate cell performance enhancement with a gravity heat pipe compared to that of a stand-alone cell during typical operating conditions, an empirical model by King [58] was employed, where cell temperature is computed based on incident radiation and wind velocity:

$$T_{cell} = T_{oo} + \frac{\dot{Q}_{inc}}{\dot{Q}_{inc,ref}} (0.071u_{wind}^2 - 2.41u_{wind} + 32.96) \quad (6.17)$$

where $u_{wind}$ is the wind velocity and $\dot{Q}_{inc,ref}$ is the reference incident solar flux. King developed this empirical model based on silicon photovoltaic cell measurements of $I_{sc}$ and $V_{oc}$, referenced to
a known temperature during thermal equilibrium conditions. King further included wind velocity parameters in his correlation to account for environmental affects, to provide more accurate temperature predictions for outdoor PV modules, for wind speeds less than 18 m/s. The model did not account for cloud cover and wind direction, and is only applicable to open rack or roof-integrated mounting configurations. However, the model’s uncertainty with experimental conditions was found to be within 3%. For this analysis, the empirical prediction model of a stand-alone silicon solar cell was computed based on the cell temperature using Eq. (6.17) and semiconductor formulations of Eqns. (6.14)-(6.16). For the empirical prediction with a heat pipe attached to the back-side surface of a cell, input heat load and interfacial Al-block/heater measurements were used to predict operational cell temperatures and waste heat loads. Anemometer flow measurements of air passing through the external condenser section were used for computation of Eq. (6.17).

As shown in Fig. (6.4), the inclusion of a gravity-assisted heat pipe containing a strong Marangoni fluid, was found to have significant performance improvement over a stand-alone PV system. For the same 30 suns solar concentration, the heat pipe PV system was found to have a 1.3% efficiency improvement with a 27.2°C temperature reduction. This deviation between the two systems was found to increase with temperature up to 177°C, where an efficiency improvement of 1.7% was found at a concentration ratio of 132 suns higher than the stand-alone PV system.

![Figure 6.4](image-url)

**Figure 6.4.** Empirical electrical efficiency projections, for a single p-n junction mono-Si solar cell, with and without a heat pipe containing a 0.05M 2-Propanol/water working fluid with a 45% liquid charge.
6.3 – PV waste heat recovery performance analysis

Combination energy conversion systems have much promise for improving the electrical conversion efficiency of photovoltaics through waste heat removal that could be used to provide secondary energy conversion processes in low temperature applications. Such applications could be used in building efficiency improvements through space heating, refrigeration, industrial process heat [68], thermoelectric generation [99, 117], among others [93]. Thermal efficiency of thermal collectors has previously [89, 120, 121] been defined as the ratio of useful work to incident solar energy [93]:

\[
\eta_{th} = \frac{\dot{m}C_p(T_{\text{cond.out}}-T_{\text{cond.in}})}{\dot{Q}_{\text{inc}}} \tag{6.18}
\]

where \( \dot{m} \) is the mass flow rate per square meter, \( C_p \) is the heat capacity of air, \( T_{\text{cond.in}} \) and \( T_{\text{cond.out}} \) are the entrance and exit temperatures of the external condenser fin bank, and \( \dot{Q}_{\text{inc}} \) is the solar irradiation flux [W/m²]. Several researchers [122-125, 138] have used total system efficiency to evaluate PV/T performance:

\[
\eta_{sys} = \eta_{elec} + \eta_{th} \tag{6.19}
\]

To assess the heat removal performance of the external heat transfer fluid, performance efficiencies are conventionally assessed against a heat removal factor [95]:

\[
F_r = \frac{T_{\text{cond.in}}-T_{\text{amb}}}{\dot{Q}_{\text{inc}}} \tag{6.20}
\]

where \( T_{\text{amb}} \) is the ambient temperature, which experimentally was fixed at approximately 20°C. Thermal and system efficiency results for a single-junction monocrystalline silicon solar cell with a 14% reference efficiency is shown in Fig. (6.5).
Figure 6.5. Condenser thermal and total system efficiency vs. heat removal factor, with a passive cooling heat pipe device in a 37° orientation, containing a 0.05M 2-propanol/water working fluid and 45% liquid charge ratio, for a $\eta_{\text{cell}}=14\%$ mono-Si solar cell.

It is observed that both thermal and total system efficiency decrease with $F_r$ and incident radiation. The combined system efficiency was shown to reach a maximum of 66.3% for a heat removal factor of $9.82\times10^{-5}$ K-m$^2$/W, which corresponds with a solar concentration ratio of 82 suns. At this operating point, system efficiency was found to be 75.5%, which is a 49% enhancement over the $1.8\times10^{-4}$ K-m$^2$/W total range of heat removal factors assessed. Similar trends were also found by Huang et al. [126] who found large thermal efficiencies up to 45% for an integrated PV/T collector, with a combined system efficiency of up to 54%, which provided improved performance over a stand-alone solar absorber system. However, the promising thermal efficiency found here is limited by a number of thermal-loss factors explained by da Silva et al. [121], who found that loss factors responsible for hybrid PV/T collectors having a lower thermal efficiency were attributed to optical losses from the PV glass cover and high thermal resistances associated with the PV encapsulation and adhesive layers.

Justification for this thermal efficiency improvement, with decreasing heat removal factor, can be attributed to Marangoni effects of the internal working fluid, which has been found to enhance evaporator vaporization heat flux while reducing evaporator wall superheat [3]. Armijo and Carey [62] demonstrated enhanced heat pipe thermal transport due to elevated CHF conditions involving heat pipe experiments with 2-Propanol/water mixtures, which had high surface tension properties at low concentrations. Their work demonstrated that boiling enhancement resulted from induced Marangoni convection that occurs when preferential evaporation of the more volatile 2-propanol component intensifies near the heated surface. This then facilitates a surface tension gradient with respect to its concentration distribution that increases toward the liquid–
vapor interface, inhibiting coalescence around incipient bubbles [59], improving the CHF condition and thermal transfer across the heat pipe.

6.4 – Thermal performance enhancement of dual-junction solar cells

Multijunction solar cells in concentrator systems are widely attractive for improving PV performance due to their high conversion efficiencies and comparatively low cost [107]. Despite greater materials costs for multijunction cells over monocrystalline silicon, the necessary cell size decreases with increasing concentration ratio, thus reducing the total cost of the module [127]. Previous studies however have shown concentrating PV performance limitations due to the thermal management issues [129] and shunt current [130]. It has been shown [101] that the temperature of the solar cell rises considerably under elevated concentration conditions. Nishioka et. al. [127] showed that insulated solar cells under concentrations of up to 500 suns could raise the cell temperature of upwards of 1400°C. This provides great incentive for use of high heat flux transport devices, such as heat pipes, for high solar concentration applications of multijunction cells.

Tandem cells consisting of InGaP and GaAs have been shown previously to have higher conversion efficiencies than traditional single-junction cells [106, 131]. Friedman analytically assessed the temperature dependence on semiconductor operating parameters, for a GaInP/GaAs series-connected tandem cell for both terrestrial and space applications. His model considered standard AM-0 and AM-1.5 direct spectrum test conditions, with temperature variation between 300K and 390K. The author’s results showed that the GaAs bottom-cell had a much smaller temperature variation than an individual unfiltered GaAs cell, as well as an efficiency drop of approximately 5% over the 90K temperature increase. A second experimental study by Lei et. al. [107] assessed the performance of the same tandem cell, fabricated using a metal-organic chemical vapor deposition technique, and tested over a measured temperature range of 25-160°C. Their results indicated that the quantum efficiencies of the cells increased slightly with increasing temperature to provide an overall system conversion efficiency of 25.2%. They also found that dual-junction short-circuit current was limited by the GaInP top cell over the entire range of temperatures explored.

This investigation considers the performance characteristics of a GaInP/GaAs tandem cell system, as prescribed by Ohmori [96], with a reference system photo-electric conversion efficiency of 32.3% and an aperture area of 4 cm². Using experimentally determined heater temperatures to simulate an operational cell system temperature, this analysis considers a dual-junction GaInP/GaAs tandem cell with system specifications outlined by Lei et al [107]. The voltage for the tandem device is the sum of the individual voltages from both cells:

\[ V_{\text{tot}} = V_{\text{InGaP}} + V_{\text{GaAs}} \]  \hspace{1cm} (6.21)

Since the top and bottom cells are connected in series, the system current is limited by the smaller of the two electrical currents produced by the respective cells. According to Friedman [131], the system fill factor is sensitive to how close the top and bottom subcells are to current
matching, and is equal to the \( FF \) of the current-limiting subcell. For multijunction cells, Wanlass et al. [104] developed a semi-empirical model to calculate the reverse-saturation current that spans a wide-range of materials and their respective band-gaps within the range of \((0.75-1.93\text{eV})\). Their model computed \( I_0 \) using Eq. (6.6), with prescribed bang gap parameters outlined in Table (6.1), and Eq. (6.21) for the pre-exponential factor \( \delta \).

\[
\delta = 3.165 \exp(2.912E_g)
\]  

(6.22)

The tandem multi-junction InGaP/GaAs system output parameters are shown in Fig. (6.6). Similar to the single-junction study, tandem solar cells, under increasing solar concentration and resultant elevated temperatures, showed an average \( V_{oc} \) decrease of \(-0.7\text{mV/suns}\) and a marked system \( I_{sc} \) increase to a value of 35.4A at 686suns. Results by Friedman [131] also found a decrease in \( V_{oc} \) of approximately 3.2V, with a 5% short-circuit current increase over the 90°C temperature range studied. Additionally, the results showed the electrical conversion efficiency to decrease over the solar concentration range to a value of 22.4% at 686 suns.

![Figure 6.6. Projected Multijunction InGaP/GaAs junction solar cell electrical properties vs. solar concentration with a gravity-assisted heat pipe in a 37° orientation and a 0.05M 2-Propanol/water working fluid mixture at 45% liquid charge.](image)

Performance comparisons are again made of a solar cell heat pipe system and a stand-alone multijunction solar cell system. Although, few studies have developed temperature correlations for the tandem solar cell system, Antón et. al. [134] introduced an expression that computes operational cell temperature as a function of concentration, based on an approximation of linear \( V_{oc} \) dependence on temperature:
\[ T_{\text{cell}} = T_{\text{ref}} + \frac{V_{\text{oc}}(T_0, C) - V_{\text{oc}}(T_{\text{ref}}, C)}{\beta(C)} \]  

(6.23)

where \( V_{\text{oc}}(T_0, C) \) is computed using Eqn. (6.16) and the temperature coefficient \( \beta(C) \), computed using a formulation by Antón et al. [57]:

\[ \beta(C) = \frac{dV_{\text{oc}}}{dT} = \beta(C_{\text{ref}}) + \frac{mk}{q} \ln \left( \frac{C}{C_{\text{ref}}} \right) \]  

(6.24)

Fig. (6.7) results show that the inclusion of a heat pipe improves electrical efficiency performance for all tandem cell system temperatures assessed. For a 30 suns concentration ratio, the PV/heat pipe system was found to have a 1.2% efficiency improvement over a stand-alone system, with a 25°C temperature reduction. For an operational temperature of 189°C, the efficiency difference between the two systems was found to be 2.1%, with the PV/heat pipe system operating 560 suns higher than a stand-alone tandem PV system.

![Image](image-url)

**Figure 6.7.** Empirical electrical efficiency projections, for a tandem multijunction GaAs/GaInP solar cell, with and without a heat pipe containing a 0.05M 2-Propanol/water working fluid with a 45% liquid charge.

### 6.5 – Summary

The inclusion of a gravity-assisted heat pipe, with a strong Marangoni working fluid, was investigated to improve the electrical conversion efficiencies of a single-junction monocrystalline silicon solar cell, as well as a dual-junction GaInP/GaAs solar cell. A naval brass heat pipe was constructed and evaluated under varying input heat loads of up to 1000W,
from connected heaters that were connected atop an aluminum interface block, attached to the evaporator-section of the heat pipe. Evaporator superheat analysis revealed an optimum liquid charge level of 45%, which was found to have the lowest overall superheat values for varying solar concentration, with a 114.8 W/cm² CHF maximum performance condition reached at 162 suns.

Performance analysis of a single-junction monocrystalline silicon solar cell with an attached gravity assisted heat pipe, showed a marked improvement over a stand-alone PV system. At an operational temperature of 177°C, efficiency improvement of 1.7% was found with concentration ratios 132 suns higher than the stand-alone single-junction system. Further solar concentration electrical efficiency analysis revealed an average $V_{oc}$ decrease of 6mV/suns and an $I_{sc}$ increase of 0.02A/suns over the concentration ratio range from 6 suns to 154 suns. PV waste heat recovery analysis predicted that thermal and total system efficiencies both increased by an average of 10% with decreasing heat removal factor, $F_r$. The combined system efficiency was shown to reach a maximum of 66.3% for a removal factor of $9.82\times10^{-5}$ K-m²/W, which corresponds with a solar concentration ratio of 82 suns. These elevated thermal and system efficiencies show great promise for developing combined PV/heat pipe energy generation devices.

Thermo-electric evaluation of a multijunction GaInP/GaAs solar cell system also revealed improved performance through a PV/heat pipe system configuration. At a low solar concentration ratio of 30 suns, the PV/heat pipe system was found to have a 1.2% efficiency improvement over a stand-alone system, with a 25°C temperature reduction. At an operational temperature of 189°C, the electrical efficiency difference between the two systems was found to be 2.1% with the PV/heat pipe system operating 560 suns higher than the stand-alone PV system. This marked improvement for both single and multijunction systems demonstrate that combined PV/heat pipe systems have tremendous potential for enhancing photoelectric power generation through reduced cell temperatures, while allowing each respective system to operate at significantly higher solar concentration levels.
CHAPTER 7 – CONCLUSIONS

This investigation studied the performance enhancement of gravity-assisted heat pipes containing working fluids that exhibited strong Marnagoni phenomena. Aluminum and brass heat pipes were developed and tested for respective horizontal and inclined orientations, with experimental working fluids that consisted of water, methanol and 2-propanol.

Experimental results from the 1st study of an aluminum heat pipe, with water as the working fluid, found agreement with parametric trends within an rms deviation of 6%. The analytical model included the superposition contributions of nucleate pool boiling and thermal heat conduction through the liquid water within the evaporator channels to determine the evaporator heat transfer coefficient. The heat conduction contribution was based on the numerical calculation of a shape factor for a half cylinder. The results of the liquid charge study revealed that increased liquid charge levels improved the CHF by as much as 69% from a charge of 35% to 70%. Heat conduction through the aluminum casing was found to be a significant component of the total heat transfer across the heat pipe due to thick walls that surrounded the inner chamber. However, its impact was shown to decrease with increasing superheat as the evaporation rate increased. Pure water experiments revealed a CHF of 53.6W/cm² for a superheat of 48.5°C. Experimental analysis was also conducted for pure water mixtures with 2-propanol as an additive. Investigation of 0.05M and 0.2M 2-Propanol molar fractions found a CHF increase of 52% and 45% from the case of pure water respectively. Decreasing the concentration of 2-propanol from 0.2M to 0.05M also improved the evaporator heat transfer coefficient by 11% at the CHF condition. It was proposed that Marangoni effects become stronger with decreasing 2-propanol concentrations that could allow interfacial surface tension gradients to enhance flow and improve the performance of the gravity-assisted heat pipe.

Binary mixture model comparisons were performed with subatmospheric pressure heat flux and superheat data, for concentrations of 2-propanol/water and methanol/water mixtures. The results indicated that for bulk fluid properties, a nucleate pool boiling correlation by Kandlikar [1] was able to predict well the low pressure 2-propanol/water mixture data for varying concentrations between 0.1M and 0.015M, better than the models of Fugita [2] and Stephan Körner [3], with rms superheat deviations of 1.8°C and 5.8°C respectively. However under the same pressure and input heat flux conditions, all three models did not correlate well with the 0.1M and 0.04M mixtures of methanol/water, where respective superheat deviations of 5.7°C and 6.1°C were found for the more predictive Fugita model.

As the concentration of the more volatile fluid decreases, the variation of surface tension at the liquid-vapor interface can be quite large affecting the saturation conditions and heat transfer in the film region. The use of film properties and the PSC coefficient for all models improved the heat flux and superheat predictions for strong Marangoni concentrations. The results suggest a heat flux rms error reduction of up to 59.3% for 0.015M 2-propanol and 49.1% for 0.04M methanol, from their respective superheat deviations of 5.7°C and 6.1°C were found for the more predictive Fugita model.

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An idealization was made that the temperature at the liquid-vapor interface was
approximately equal to the vapor temperature, for a respective mixture concentration. This method also found greater accuracy predicting superheat values for both strong and weak Marangoni concentrations for 2-propanol/water using Kandlikar’s model, with superheat deviations up to 1.8°C. Fugita’s model however was found to better predict both strong and weak concentrations of methanol/water mixtures with superheat deviations up to 1.6°C.

A second performance study was conducted for an inclined, gravity-assisted, brass heat pipe with a 0.05M 2-Propanol/water working fluid as the working fluid. For a range of input heat load values, the results of the liquid charge study revealed an optimal liquid charge of 45%, which had lower overall superheat levels, and a CHF condition as high as 114.8W/cm². For higher 50% and 70% liquid charge ratios, although the CHF was not reached, higher respective heat flux values of 117.3W/cm² and 118.2W/cm², as well as superheat values of 4.6% and 15.5% were attained. Further analysis showed that higher charge ratios of 50% and 70%, decreased thermal conductance by 21.7% and 34.0% respectively from that of a 45% liquid charge. Justification of decreased performance with charge levels greater than 45% can be attributed to suppression of thermal cycling, where heat transfer is only facilitated by conduction and not phase change, which would likely promote higher heat transport [5]. Thermal conductance analysis across this heat pipe also showed an optimal heat load of 600W, which was 42.7% and 5.6% greater than respective heat loads of 200W and 800W.

The analytical model developed in this investigation found good correspondence between the liquid-vapor transport heat flux experimental data and the analytical model prediction. Experimental data were found to agree with parametric trends within an rms deviation of 6.5%. Deviations between the model predictions and experimental data also found an average superheat and heat flux deviation of 1.3°C and 4.3W/cm² respectively. Justification for the correspondence can be attributed to the use of film properties, Kandlikar’s PSC coefficient [1], and McGillis’ surface tension correlation [4] for non-ideal binary mixtures. It is proposed that the non-ideal nature of the 2-propanol/water mixture surface tension, which is sensitive to temperature variation, facilitates Marangoni flow that improves the CHF condition.

In the final study, heat transport performance of the second heat pipe was assessed for improving electrical conversion efficiencies of a single-junction monocrystalline silicon, and dual-junction GaInP/GaAs solar cell. With this, varying input heat loads of up to 1000W, from connected heaters simulated waste heat production of the respective PV solar cells. Performance analysis of the single-junction monocrystalline silicon solar cell with attached heat pipe, showed a marked improvement over a stand-alone PV system. At an operational temperature of 177°C, efficiency improvement of 1.7% was found with a concentration of 132 suns higher than the stand-alone PV system. Further solar concentration electrical efficiency analysis revealed an average $V_{oc}$ decrease of 6mV/suns and an $I_{sc}$ increase of 0.02A/suns over the assessed concentration ratio range, from 6 suns to 154 suns. PV waste heat recovery analysis predicted that thermal and total system efficiencies both increased by an average of 10% with decreasing heat removal factor $F_r$. The combined system efficiency was shown to reach a maximum of 66.3% for a removal factor of 9.82x10⁻² K-m²/W, which corresponds with a solar concentration ratio of 82 suns. These elevated thermal and system efficiencies show great promise for developing combined PV/heat pipe energy generation devices.
Thermo-electric evaluation of a multijunction GaInP/GaAs solar cell system also revealed improved performance through a PV/heat pipe system configuration. At a low solar concentration ratio of 30 suns, the PV/heat pipe system was found to have a 1.2% efficiency improvement over a stand-alone system, with a 25°C temperature reduction. At an operational temperature of 189°C, the electrical efficiency difference between the two systems was found to be 2.1% with the PV/heat pipe system operating 560 suns higher than the stand-alone PV system. This marked improvement for both single and multijunction systems demonstrate that combined PV/heat pipe systems have tremendous potential for enhancing photoelectric power generation through reduced cell temperatures, while allowing each respective system to operate at significantly higher solar concentration levels.
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# APPENDIX

## Table A.1. Binary Mixture Nucleate Boiling Heat Transfer Correlations Chronology

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Correlation</th>
<th>Year Published</th>
</tr>
</thead>
</table>
| Van Sralen | \[
\frac{h}{h_{id}} = \frac{1}{1-x_1 \left( y_1, \frac{\rho_{id}}{\rho}, \frac{\mu_{id}}{\mu}, \frac{\lambda_{id}}{\lambda}, \frac{\gamma_{id}}{\gamma} \right)}
\] | 1966 |
| Stephan and Komer | \[
\frac{h}{h_{id}} = 1 + A \left( 0.14 + 0.12 \left( \frac{\Delta T}{\Delta T_{id}} \right)^2 \right)
\] | 1969 |
| W.F. Caluc and P. Rice | \[
\frac{h}{h_{id}} = 1 + \left( \frac{y_1}{y_1 - x_1} \right) \sqrt{\frac{4}{D_{id}}} \left( \frac{\rho_{id}}{\rho}, \frac{\mu_{id}}{\mu}, \frac{\lambda_{id}}{\lambda}, \frac{\gamma_{id}}{\gamma} \right)
\] | 1972 |
| W.F. Caluc and D.J. Leonidopolous | \[
\frac{h}{h_{id}} = \frac{1}{1 - \left( \frac{y_1}{y_1 - x_1} \right) \sqrt{\frac{4}{D_{id}}} \left( \frac{\rho_{id}}{\rho}, \frac{\mu_{id}}{\mu}, \frac{\lambda_{id}}{\lambda}, \frac{\gamma_{id}}{\gamma} \right)}
\] | 1974 |
| O. Happel and K. Stephan | \[
\frac{h}{h_{id}} = \frac{1}{1 + \left( \frac{y_1}{y_1 - x_1} \right) \left( \frac{\rho_{id}}{\rho}, \frac{\mu_{id}}{\mu}, \frac{\lambda_{id}}{\lambda}, \frac{\gamma_{id}}{\gamma} \right)}
\] | 1974 |
| K. Stephan and Preussner | \[
\frac{h}{h_{id}} = \frac{1}{1 + \left( \frac{y_1}{y_1 - x_1} \right) \left( \frac{\rho_{id}}{\rho}, \frac{\mu_{id}}{\mu}, \frac{\lambda_{id}}{\lambda}, \frac{\gamma_{id}}{\gamma} \right)}
\] | 1979 |
| H. Jungrickel, P. Wassilew and W.E. Kraus | \[
\frac{h}{h_{id}} = \frac{1}{1 + \left( \frac{y_1}{y_1 - x_1} \right) \left( \frac{\rho_{id}}{\rho}, \frac{\mu_{id}}{\mu}, \frac{\lambda_{id}}{\lambda}, \frac{\gamma_{id}}{\gamma} \right)}
\] | 1980 |
| J.R. Thome | \[
\frac{h}{h_{id}} = \frac{1}{1 + \left( \frac{y_1}{y_1 - x_1} \right) \left( \frac{\rho_{id}}{\rho}, \frac{\mu_{id}}{\mu}, \frac{\lambda_{id}}{\lambda}, \frac{\gamma_{id}}{\gamma} \right)}
\] | 1983 |
| E.U. Schröder | \[
\frac{h}{h_{id}} = \left( \frac{y_1}{y_1 - x_1} \right) \left( \frac{\rho_{id}}{\rho}, \frac{\mu_{id}}{\mu}, \frac{\lambda_{id}}{\lambda}, \frac{\gamma_{id}}{\gamma} \right)
\] | 1983 |
| K. Bier, J. Schmad and D. Gorenflo | \[
\frac{h}{h_{id}} = \frac{1}{1 + \left( y_1 - y_2 \right) \left( \frac{\rho_{id}}{\rho}, \frac{\mu_{id}}{\mu}, \frac{\lambda_{id}}{\lambda}, \frac{\gamma_{id}}{\gamma} \right)}
\] | 1984 |
| H.C. Uattle | \[
\frac{h}{h_{id}} = \left( \frac{y_1}{y_1 - x_1} \right) \left( \frac{\rho_{id}}{\rho}, \frac{\mu_{id}}{\mu}, \frac{\lambda_{id}}{\lambda}, \frac{\gamma_{id}}{\gamma} \right)
\] | 1986 |
| J.R. Thome and S. Shakir | \[
\frac{h}{h_{id}} = \frac{1}{1 + \left( \frac{y_1}{y_1 - x_1} \right) \left( \frac{\rho_{id}}{\rho}, \frac{\mu_{id}}{\mu}, \frac{\lambda_{id}}{\lambda}, \frac{\gamma_{id}}{\gamma} \right)}
\] | 1987 |
| Y. Fujita and M. Tsutsui | \[
\frac{h}{h_{id}} = \frac{1}{1 + \left( \frac{y_1}{y_1 - x_1} \right) \left( \frac{\rho_{id}}{\rho}, \frac{\mu_{id}}{\mu}, \frac{\lambda_{id}}{\lambda}, \frac{\gamma_{id}}{\gamma} \right)}
\] | 1994 |
| Y. Fujita and M. Tsutsui | \[
\frac{h}{h_{id}} = \frac{1}{1 + \left( \frac{y_1}{y_1 - x_1} \right) \left( \frac{\rho_{id}}{\rho}, \frac{\mu_{id}}{\mu}, \frac{\lambda_{id}}{\lambda}, \frac{\gamma_{id}}{\gamma} \right)}
\] | 1997 |
<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Equation</th>
<th>Year</th>
</tr>
</thead>
</table>
| Bajorek and Lloyd | \( \frac{h}{h_{ld}} = 1 + \frac{1}{1 + \left( \frac{2D}{D_{ld}} \right)^2} \) \times \left[ 1 - \left( \phi_1 \exp^{-\phi_1 \left( \frac{L}{L_{ld}} \right)} + \phi_2 \exp^{-\phi_2 \left( \frac{L}{L_{ld}} \right)} \right) \right]^{-1} \)  
where, \( \phi_1 = D_{ld} - D_{ld} \left( \frac{L_{ld}}{L} \right) - \zeta \)  
\( \phi_2 = D_{ld} - D_{ld} \left( \frac{L_{ld}}{L} \right) - \zeta \)  
\( \zeta = \frac{k_h}{\lambda} \frac{\rho \alpha \rho c_p}{\lambda^2} \)  
\( \psi_1 = \frac{k_h}{\lambda} \frac{\rho \alpha \rho c_p}{\lambda^2} \)  
\( \psi_2 = \frac{k_h}{\lambda} \frac{\rho \alpha \rho c_p}{\lambda^2} \)   | 1997 |
| T. Inoue, N. Kawae and M. Monde | \( h = \frac{1}{1 + \frac{1}{h_{ld}}} \) \( \alpha = 1 - 0.75 \exp \left(-0.75 \times 10^{-4} q \left( W/m^2 \right) \right) \)   | 1998 |
| S.G. Kandlikar    | \( h = h_{m} F_D \)  
\( F_D = \begin{cases} 1 - 64V_e & \text{for } V_e < 0.03 \\ 0.678 \left[ 1 + \left( \frac{V_e}{V_e} \right)^{0.5} \left( \frac{L}{L_{ld}} \right) \right]^{-1} & \text{for } 0.03 < V_e < 0.2 \\ 1 & \text{for } V_e > 0.2 \end{cases} \)  
\( V_e = \frac{0.4 \left( \frac{L}{L_{ld}} \right)^{0.5} \left( \frac{L}{L_{ld}} \right) \left( \frac{L}{L_{ld}} \right)^{-1} \right) \)  
\( \frac{\left( \frac{L}{L_{ld}} \right)^{-1} \left( \frac{L}{L_{ld}} \right)^{-1} \right) \)  
\( \frac{\left( \frac{L}{L_{ld}} \right)^{-1} \left( \frac{L}{L_{ld}} \right)^{-1} \right) \)  
\( g = g_{m} \left( \frac{L}{L_{ld}} \right)^{1.5} \)  
\( r_{ll} = \frac{2.83}{\pi} \left( \frac{L}{L_{ld}} \right)^{1.5} \)  
\( \lambda_{ld} = \frac{T_{ld}}{T_{ld}} \left( \frac{T_{ld}}{T_{ld}} \right)^{2} \)   | 1998 |
| R.J. Benjamin and A.R. Balakrishnan | \( \frac{h}{h_{ld}} = 1 - \left[ 1 - \frac{L}{L_{ld}} \right] \)   | 1999 |
| F. Tallois, N. Vallée, M. Bourguis, A. Coronas | \( h = \frac{1}{1 + K} \)  
\( K = A \left( \frac{h_{ld}}{q} \right) \left( T_{ld} - T_{in} \right) \left( \frac{L}{L_{ld}} \right) \left( 1 - \exp \left( \frac{B_{ld}}{A_{ld} \lambda_{ld}} \right) \right) \)  
\( + A \left( 1 - \frac{h_{ld}}{q} \right) \left[ 1 - \exp \left( \frac{B_{ld}}{A_{ld} \lambda_{ld}} \right) \right] \)   | 2007 |